PHYSICS OF ELECTROWEAK SYMMETRY BREAKING

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based on a lecture course given at ETH

PARTICLE PHYSICISTS OF THE 20TH CENTURY

- Pieced together an almost perfect theory of particle interactions
- Guided by symmetry and mathematical consistency
- Predicted new particles to be found many years later!

SYMMETRY MAKES ALL LIGHT

- We have a theory which predicts accurately probabilities for quark, lepton, gluon, W and Z boson interactions.

- High energy symmetries.
  We can reshuffle W, Z bosons and the photon reshuffling also up and down quarks without altering hypothetical measurements at very high energies

- Symmetries render quantum field theories predictive.

PUZZLE OF MASS

- Why then the W and Z bosons have mass and the photon is massless?

- What is the origin of mass for elementary particles?

$$m_\gamma = 0$$

$$m_{gluon} = 0$$

$$m_W \approx 86 \times m_{proton}$$

$$m_Z \approx 97 \times m_{proton}$$

$$m_e \approx 0.00054 \times m_{proton}$$

$$m_\mu \approx 0.113 \times m_{proton}$$

$$m_\tau \approx 1.894 \times m_{proton}$$

$$m_{strange} \approx 0.1 \times m_{proton}$$

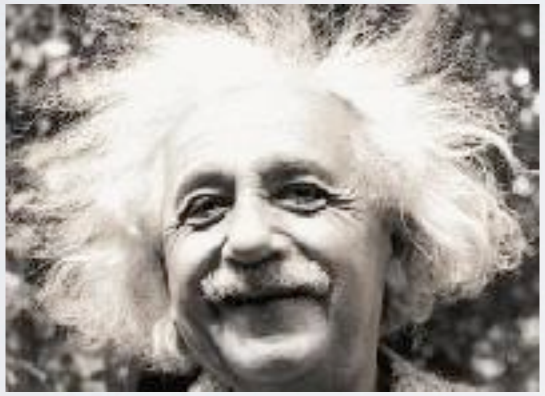
$$m_{charm} \approx 1.3 \times m_{proton}$$

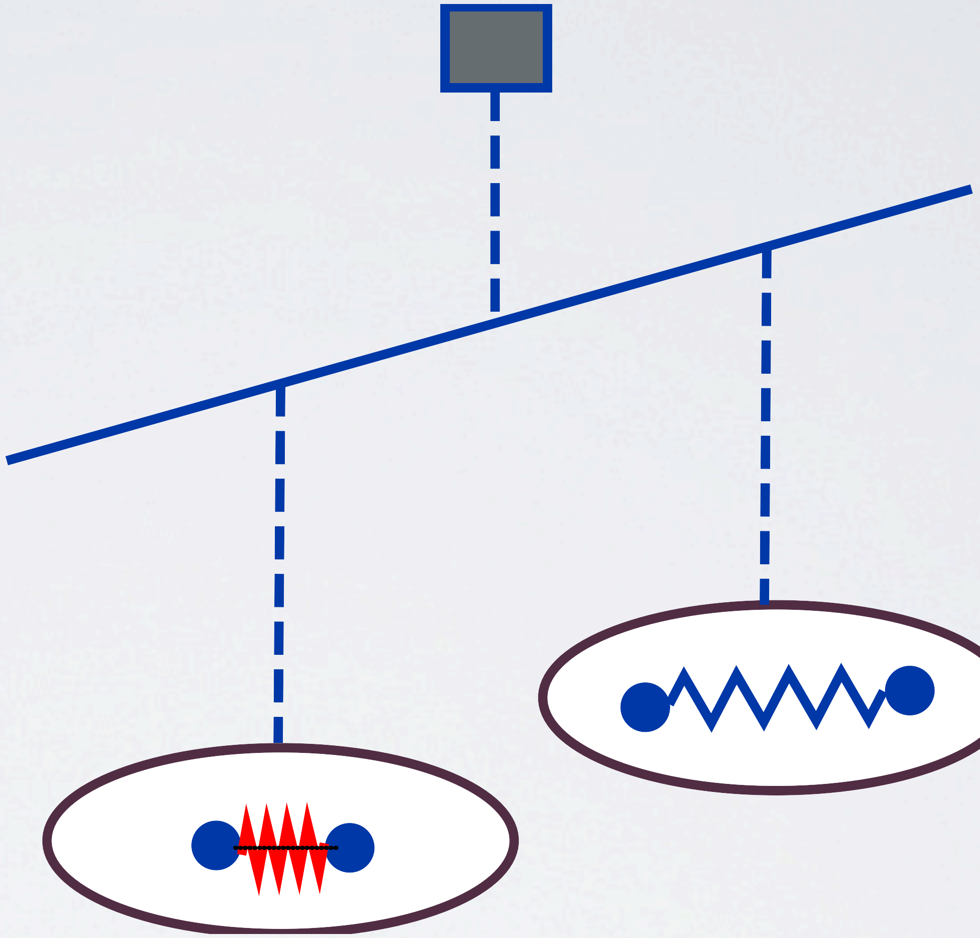
$$m_{bottom} \approx 4.5 \times m_{proton}$$

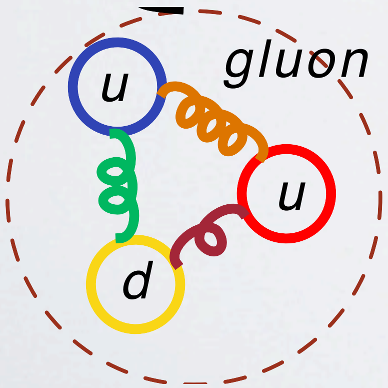
$$m_{top} \approx 184 \times m_{proton}$$

MASS IS STATIC ENERGY

$$Mass = \frac{Energy}{c^2}$$





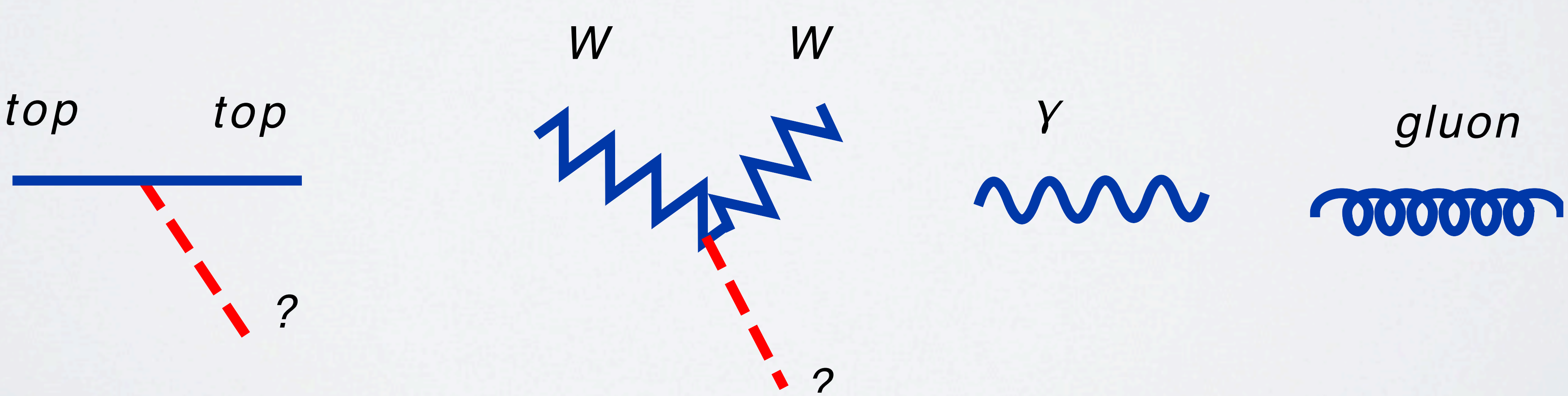


$$2m_{up} + m_{down} \approx 1\% \, m_{proton}$$

$$\text{Gluon Interactions} \approx 99\% \, m_{proton}$$

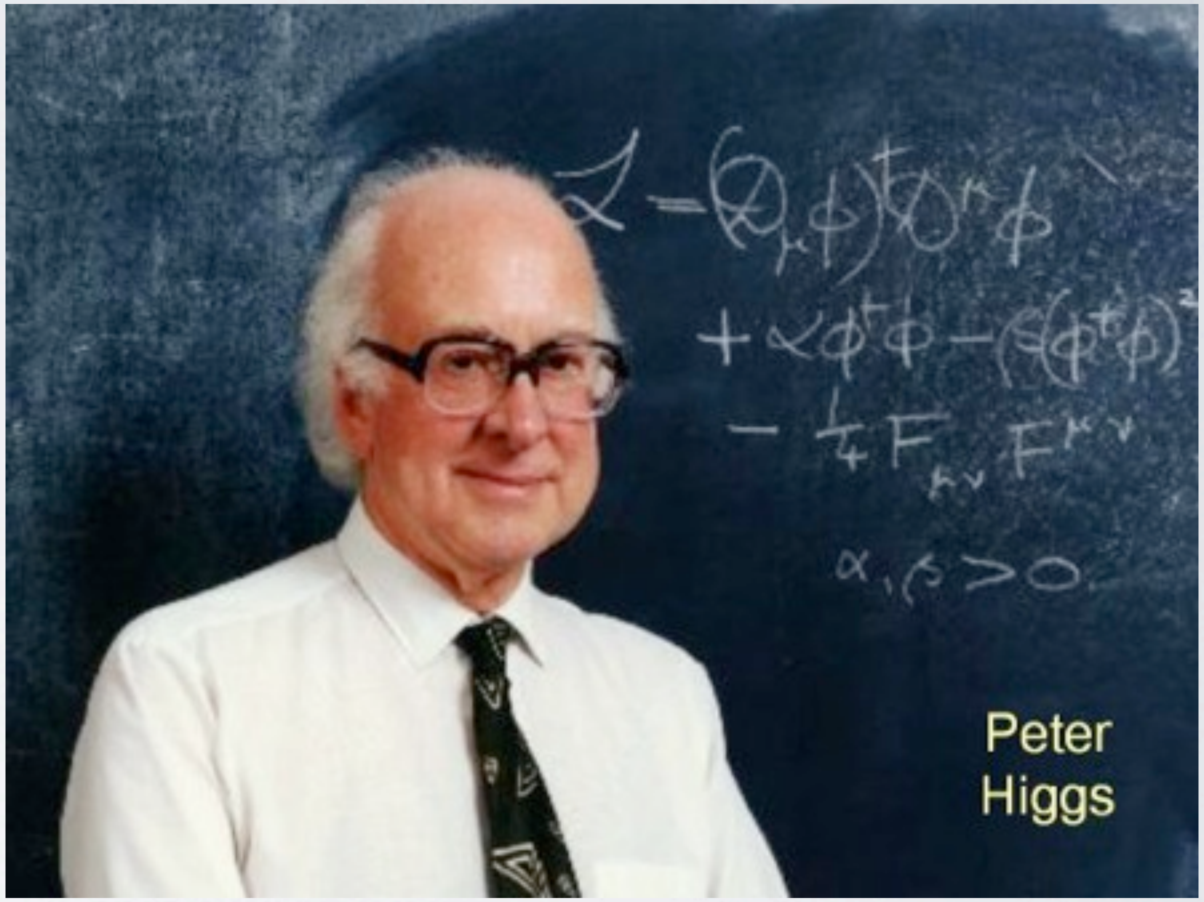
A FIFTH FORCE! WHAT IS IT?

- Known forces cannot create sufficient potential energy to account for the measured masses of elementary particles.
- What is the new force? This is the tightest tied Gordian knot of particle physics.



THE HIGGS HYPOTHESIS

- The simplest known solution. Introduced in 1964.
- The "Standard Model" of particle interactions is inconsistent without it!
- All probed Standard Model predictions have been proven correct.
- Higgs hypothesis still awaits verification!



Unknown Force = Undiscovered particle (Higgs boson)!

TOPICS IN THIS LECTURE

- classic and quantum action and their symmetries
- breaking of global symmetries and goldstone bosons
- breaking of approximate global symmetries and pseudo-goldstone bosons
- identification of Goldstone boson fields and fields orthogonal to them.
- breaking of local gauge symmetries and the mass spectrum of electroweak gauge bosons.

CLASSICAL ACTION

- Classical laws of physics are derived by a principle of least action.

$$\delta S = \delta \int d^3\vec{x}dt\,\mathcal{L} = 0$$

- For example, classical electrodynamics is described by a Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + j_\mu A^\mu$$

- The physical value of the electromagnetic field corresponds to a minimum value of the action, satisfying Maxwell equations

$$\delta S = 0 \rightsquigarrow \partial_\mu F^{\mu\nu} = j^\nu$$

QUANTUM FIELDS

- In our quantum world, we compute expectation values of physical observables through a path integral over fields

$$Z[J] = e^{iW[J]} = \int \mathcal{D}\phi\, e^{iS[\phi]+i\int d^4x J(\phi)\phi(x)}$$

$$\langle \text{vacuum}|\text{vacuum}\rangle_J = Z[J]$$

$$\left\langle \hat{\phi} \right\rangle_J \equiv \frac{\left\langle \text{vacuum}|\hat{\phi}(x)|\text{vacuum} \right\rangle_J}{\langle \text{vacuum}|\text{vacuum}\rangle_J} = \frac{\delta}{\delta J(x)} W[J]$$

- This encodes the uncertainty principle. All field values, not only the extrema of the action, contribute to probability amplitudes.

QUANTUM ACTION

- Theorem of least quantum effective action:
  "the physical expectation values of quantum fields in the ground state for a closed system (e.g. the universe) where the sources take a zero value minimize the quantum action"

$$\Gamma\left[\langle\phi\rangle_J\right] = W[J] - \int d^4x J(x)\left\langle\phi(x)\right\rangle_J \qquad \frac{\delta\Gamma\left[\langle\phi\rangle_0\right]}{\delta\left\langle\phi\right\rangle_0} = 0$$

- Every probability amplitude for any physical process can be derived from the quantum action provided that the latter can be computed.

- Quantum fields "create" and "annihilate" particle and other states from the vacuum.

- Intuitively, the expectation value of fields in the vacuum (ground state with zero number of particles) must be zero. Does it have to be always like this?

SYMMETRIES OF THE QUANTUM ACTION

- Symmetries of physical systems give rise to conservation laws.
- Suppose we have identified a number of symmetries which leave invariant the partition function Z[J].
- These symmetries constrain probability amplitudes and relative rates of particle interactions.
- The symmetries of the partition function are not guaranteed to be symmetries of the quantum action

SYMMETRIES OF THE QUANTUM ACTION

- Assume that the partition function is invariant under a field transformation
  $$\phi \to \phi + \delta\phi \quad \rightsquigarrow \quad W[J] \to W[J]$$
- The quantum action is symmetric under a transformation
  $$\langle\phi\rangle \to \langle\phi\rangle + \langle\delta\phi\rangle \quad \rightsquigarrow \quad \Gamma\left[\langle\phi\rangle\right] \to \Gamma\left[\langle\phi\rangle\right]$$
- This is not necessarily the same symmetry. It may be
  $$\langle\delta\phi\rangle \neq \delta\left\langle\phi\right\rangle$$
- But it is clear how to find the symmetries of the quantum action if we know the symmetries of the partition function.
- The latter may also be related to the symmetries of the classical action.

SPONTANEOUS SYMMETRY BREAKING OF GLOBAL SYMMETRIES

- Definition of spontaneous symmetry breaking:
  "Symmetries of the quantum action are not symmetries of the physical states and especially the vacuum state."

- SSB is associated with a degeneracy of the vacuum.

$$\Gamma\left[\langle\phi\rangle\right] = \Gamma\left[S\,\langle\phi\rangle\right] = \text{minimum} \quad \rightsquigarrow \quad \begin{array}{ll} A\rangle : & \left\langle A|\hat{\phi}|A\right\rangle = \langle\phi\rangle \\ B\rangle : & \left\langle B|\hat{\phi}|B\right\rangle = S\,\langle\phi\rangle \end{array}$$

- Multiple states with a field expectation value corresponding to a minimum of the quantum action

DEGENERATE VACUUM STATES

- The ground state of a physical system is one of the many possible vacuum states.
- Once one of all possible (orthonormal) vacuum states has been chosen, a tunneling through an infinite volume is required in order to move to another (of the same energy).
- True vacuum states are stable.

SSB OF GLOBAL SYMMETRIES AND GOLDSTONE BOSONS

- Broken global symmetries constrain propagators and other Green's functions.

$$\Gamma\left[\langle\phi_n\rangle\right] = \Gamma\left[\langle\phi_n\rangle + \delta\langle\phi_n\rangle\right]$$

$$\rightsquigarrow \qquad \int d^4x \frac{\delta^2\Gamma}{\delta\langle\phi_l\rangle\,\delta\langle\phi_n\rangle}\,\delta\langle\phi_n\rangle = 0$$

$$\int d^4z \frac{\delta^2\Gamma}{\delta\langle\phi_n(x)\rangle\,\delta\langle\phi_k(z)\rangle}\,\langle\Omega|T\phi_k(z)\phi_m(y)|\Omega\rangle = -\delta(x-y)\delta_{nm}$$

GOLDSTONE BOSONS

- The spectrum of a SSB theory has a massless state for each symmetry of the quantum action which does not leave the vacuum state invariant (for each broken symmetry generator)
- These massless states are one-particle states
- They are invariant under rotations: spin-0 states
- These states have the same quantum numbers as the corresponding conserved current of the broken symmetry generators.

EXAMPLE OF THEORY WITH SPONTANEOUSLY BROKEN GLOBAL SYMMETRIES

- A system of N real scalar fields

$$\mathcal{L} = \frac{1}{2}\sum_n (\partial_\mu \phi_n)^2 - \frac{M^2}{2}\sum_n \phi_n^2 - \frac{g}{4}\left(\sum_n \phi_n^2\right)^2$$

- The Lagrangian, action, and quantum action are all symmetric under rotations

$$\phi_n \to \phi'_n = R_{nm}\phi_m, \quad R^2 = \mathbf{1}$$

- Up to loop corrections, the quantum action is:

$$\frac{\Gamma}{\text{space-time volume}} = -V_{\text{eff}} \qquad V_{\text{eff}} = \frac{M^2}{2}\sum_n \langle\phi_n\rangle^2 + \frac{g}{4}\left(\sum_n \langle\phi_n\rangle^2\right)^2$$

EXAMPLE OF A THEORY WITH BROKEN GLOBAL SYMMETRIES

- Extrema of quantum action satisfy

$$\left[M^2 + g\left(\sum_n \langle\phi_n\rangle^2\right)\right]\langle\phi_l\rangle \approx 0$$

- A symmetric vacuum under rotations for a "choice" of Lagrangian parameters

$$M^2, g > 0 \rightsquigarrow \langle\phi_l\rangle = 0$$

- Spontaneous symmetry breaking occurs for:

$$M^2 < 0, g > 0 \rightsquigarrow \sum_n \langle\phi_n\rangle^2 \approx -\frac{M^2}{g}$$

- Degenerate vacua of which one is chosen spontaneously.

MASS SPECTRUM

- The corresponding mass matrix of the theory is

$$M^2_{nm} = \frac{\partial^2 V_{\text{eff}}}{\partial \langle \phi_m \rangle \partial \langle \phi_n \rangle} \approx 2g \langle \phi_m \rangle \langle \phi_n \rangle$$

- Diagonalizing,

$$det\left(M^2_{nm} - \mu^2 \delta_{nm}\right) = 0$$

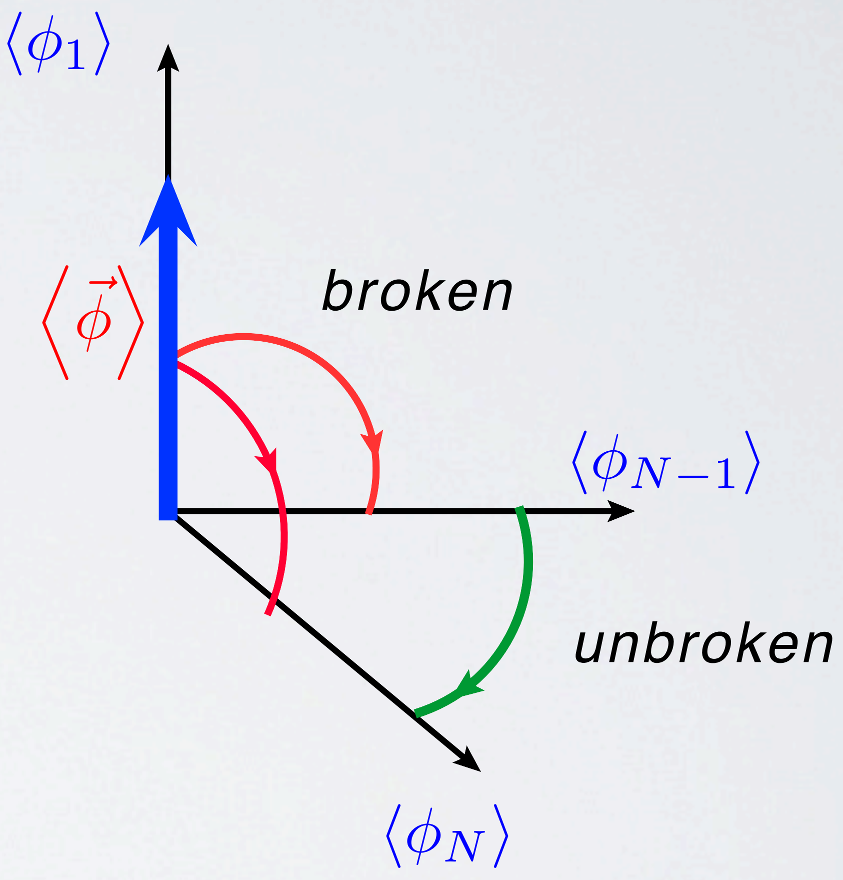
$$\rightsquigarrow \left(\mu^2\right)^{N-1}\left(\mu^2 - 2g\sum_i \langle \phi_i \rangle^2\right) = 0$$

- we find **N-1** particles with zero mass and one massive particle with mass:

$$\mu^2 \approx 2g\sum_i \langle \phi_i \rangle^2 = 2|M|^2$$

MASS SPECTRUM

- The action is symmetric under the O(N) group with N(N-1)/2 generators
- Spontaneous symmetry breaking picks up one direction in the space of $\langle \phi_i \rangle$
- The linear combinations of fields which are orthogonal to this direction can be freely rotated to each other.
- The surviving symmetry group is O(N-1) with (N-1)(N-2)/2 generators which do not transform the spontaneously chosen vacuum expectation value.
- N(N-1)/2-N-1)(N-2)/2=(N-1) generators, equal to the number of Goldstone bosons, transform the vacuum expectation value and are "broken".



APPROXIMATE GLOBAL SYMMETRIES AND SPONTANEOUS SYMMETRY BREAKING

- Consider an effective potential which is approximately symmetric:

$$V_{eff} = V_0(\phi) + V_I(\phi)$$

- Under a transformation,

$$\phi \to \phi + \epsilon\delta\phi$$

$$\rightsquigarrow$$

$\delta V_0(\phi) = 0$	$\delta V_I(\phi) \neq 0$
symmetric	***breaks symmetry explicitly***

- Does spontaneous symmetry breaking take place for approximate symmetries?

VACUUM ALIGNMENT

- Expanding around the vacuum of the symmetric action,

$$\langle \phi_n \rangle = \left\langle \phi_n^{(0)} \right\rangle + \left\langle \phi_n^{(I)} \right\rangle$$

- we find that at leading order the expectation value of the fields is "aligned" to the symmetry breaking term. For a linear symmetry transformation,

$$\delta \phi_n = \sum_m T_{nm} \phi_m$$

- the vacuum expectation values of the fields have a certain direction in the field-coordinate space

$$\sum_{nm} \frac{\partial V_I}{\partial \left\langle \phi_n^{(0)} \right\rangle} T_{nm} \left\langle \phi_m^{(0)} \right\rangle$$

VACUUM ALIGNMENT

- For a quantum action with exact-symmetry, spontaneous symmetry breaking picks up randomly one of the degenerate vacuum states.

- For an action with an approximate symmetry, the degeneracy is lifted (at least partially) and the vacuum states of the theory with exact symmetry yield different values for the effective action.

- The vacuum-alignment condition selects a state which minimizes the value of the symmetry-breaking term in the effective action.

PSEUDO-GOLDSTONE BOSONS

- The mass matrix for a theory with a spontaneously broken approximate symmetry has the form:

$$M_{ab}^2 = \sum_{cd} F_{ac}^{-1} \frac{\partial^2 V_I}{\partial\theta_c \partial\theta_d} F_b^{-1}$$

- This is positive definite since vacuum alignment requires that Vi is minimized.

- Goldstone bosons (termed pseudo-Goldstone bosons) have mass.

WHICH FIELDS ARE GOLDSTONE BOSONS AND WHICH NOT?

- Let's assume a spontaneous global symmetry breaking pattern

$$G \to H, \quad H \subset G$$

- The quantum action is invariant under the G group

$$\psi_n \to \psi'_n = \sum_m g_{nm} \psi_m \rightsquigarrow \mathcal{L}(\psi_n) = \mathcal{L}(\psi'_n), \quad \forall\, g \in G$$

- The vacuum expectation values of the fields are invariant under the H group

$$\langle \psi_n \rangle = \sum_m h_{nm} \langle \psi_m \rangle\,, \quad \forall\, h \in H$$

WHICH FIELDS ARE GOLDSTONE BOSONS AND WHICH NOT?

- The massless eigenstates of the mass-matrix

$$\Gamma\left[\langle\psi_n\rangle\right] = \Gamma\left[\langle\psi_n\rangle + \delta\langle\psi_n\rangle\right] \rightsquigarrow \int d^4x \underbrace{\frac{\delta^2\Gamma}{\delta\langle\psi_l\rangle\,\delta\langle\psi_n\rangle}}_{\sim M^2_{ln}} \delta\langle\psi_n\rangle = 0$$

- are: $\delta\langle\psi_n\rangle \sim \sum_m T_{nm}\langle\psi_m\rangle$. The non-Goldstone fields should be orthogonal:

$$\sum_n \tilde{\psi}_n \delta\psi_n = \sum_{nm} \tilde{\psi}_n T_{nm}\langle\psi_m\rangle = 0.$$

- Starting from an arbitrary labeling of fields, ψ_n, we can find the fields orthogonal to Goldstone bosons $\tilde{\psi}_n$ by applying a **local** gauge transformation.

$$\psi_n(x) = \sum_m \gamma_{nm}(x)\tilde{\psi}_m(x), \quad \gamma \in G$$

WHICH FIELDS ARE GOLDSTONE BOSONS AND WHICH NOT?

- This local transformation is constructed specially at each space-time point to maximize the quantity

$$V[g] = \sum_{nm} \psi_n(x) g_{nm}(x) \langle \psi_m \rangle \qquad V[\gamma] \geq V[g], \quad \forall\, g \in G$$

- To expose the Goldstone and non-Goldstone fields it remains to substitute in the Lagrangian

$$\psi_n(x) = \sum_m \gamma_{nm}(x) \tilde{\psi}_m(x)$$

- This transformation, had it been global, would let the Lagrangian invariant.

WHICH FIELDS ARE GOLDSTONE BOSONS AND WHICH NOT?

- However, a local transformation introduces new terms.

$$\gamma(x) = \gamma(x_0) + (x - x_0) \cdot \partial\gamma(x_0) + \ldots = \gamma(x_0) + F[\partial\gamma]$$

$$\begin{aligned} \mathcal{L}\left[\psi(x)\right] &= \mathcal{L}\left[\gamma(x)\tilde{\psi}(x)\right] \\ &= \mathcal{L}\left[\gamma(x_0)\tilde{\psi}(x)\right] + (\text{derivatives of } \gamma(x)) \\ &= \mathcal{L}\left[\tilde{\psi}(x)\right] + (\text{derivatives of } \gamma(x)) \end{aligned}$$

- The Goldstone fields correspond to the functions which parameterize the local transformation $\gamma(x)$

GOLDSTONE BOSON MASS AND INTERACTIONS

- The Lagrangian is a a functional of non-Goldstone fields and derivatives of Goldstone boson fields

$$\mathcal{L}\left[\tilde{\psi}, \partial B, \partial^2 B, \ldots\right]$$

- This forbids a mass-term for the Goldstone bosons

~~$m_B^2 B^2$~~

- Also, Feynman rules for Goldstone interactions should be proportional to their momenta. At low energies, Goldstone interactions vanish.

SPONTANEOUSLY BROKEN LOCAL SYMMETRIES

- Exact global symmetries and the inevitable massless Goldstone bosons have been feared by particle theorists. Such particles should be easy to detect experimentally.

- Goldstone's theorem can be evaded if the action is symmetric under a local symmetry.

$$\psi_n(x) \to \psi'_n(x) = g_{nm}(x)\psi_m(x) \rightsquigarrow \mathcal{L}\left[\psi_n\right] = \mathcal{L}\left[\psi'_n\right]$$

- Then, the local gauge transformation which makes explicit the fields orthogonal to the Goldstone bosons leave the Lagrangian intact. Goldstone bosons disappear!

$$\mathcal{L}\left[\psi(x)\right] = \mathcal{L}\left[\gamma(x)\tilde{\psi}(x)\right] = \mathcal{L}\left[\tilde{\psi}(x)\right]$$

GAUGE SYMMETRY

- In order to achieve a local symmetry, derivatives in the Lagrangian must appear in combinations with gauge fields, forming covariant derivatives.

$$\partial_\mu \to (D_\mu)_{nm} = \partial_\mu \delta_{nm} - igT^a_{nm}A^a_\mu$$

- "Kinetic energy" terms in the Lagrangian appear in the form,

$$\left(\partial_\mu \tilde{\psi}_n\right)^2 \to \left(D_\mu \tilde{\psi}_n\right)^2$$

MASSIVE GAUGE FIELDS

- We now decompose fields into their vacuum expectation value and an excitation

$$\tilde{\psi}_n = \left\langle \tilde{\psi}_n \right\rangle + \phi_n$$

- The covariant derivative becomes

$$(D_\mu)_{nm}\tilde{\psi}_m = (D_\mu)_{nm}\tilde{\phi}_m - igA^a_\mu T^a_{nm}\left\langle \tilde{\psi}_m \right\rangle$$

- The kinetic energy term (which is the square of the above) contains potential mass terms for gauge bosons.

$$\mu^2_{ab}\, g^{\mu\nu} A^a_\mu A^b_\nu \qquad\qquad \mu^2_{ab} = -g^2 T^a_{nm} T^b_{nl}\left\langle \tilde{\psi}_m \right\rangle \left\langle \tilde{\psi}_l \right\rangle$$

MASSES OF GAUGE BOSONS

- The mass-matrix for gauge bosons satisfies,

$$\mu^2_{ab} \geq 0$$

  (generators are imaginary for real fields)

- Masses of gauge bosons are proportional to couplings. Ratios of masses are constrained by symmetry. Can be tested experimentally!

- This is a mechanism in which gauge bosons can obtain a mass. We believe that the W and Z electroweak gauge bosons acquire their mass in this way and plenty of data verifies this.

MASSES OF GAUGE BOSONS AND SYMMETRY GENERATORS

- If there is a linear combination of symmetry generators which is unbroken,

$$\tilde{T}_{nm} = \sum_a c_a T^a_{nm} \;\; \text{with} \;\; \tilde{T}_{nm} \left\langle \tilde{\psi}_m \right\rangle = 0$$

then the mass matrix of the gauge bosons has a zero eigenvalue.

- Conversely, if there is a massless gauge boson then there is a linear combination of generators which is unbroken.

- Massive gauge bosons correspond to broken generators.

SYMMETRY OF ELECTROWEAK INTERACTIONS

- Experimental data meticulously collected in the last 50 years has verified in a spectacular fashion that the electroweak interactions possess an SU(2) x U(1) local gauge symmetry.

- This symmetry group has four generators.

- Three (W^+, W^-, Z) of the four corresponding gauge bosons are massive. There is a linear combination of generators, corresponding to the photon, which is unbroken.

.

$$SU(2) \otimes U_Y(1) \rightarrow U_{em}(1)$$

Quantum action symmetry **Vacuum symmetry**

CAUSES OF SPONTANEOUS SYMMETRY BREAKING?

- We have seen that spontaneous symmetry breaking manifests itself with a characteristic mass spectrum of Goldstone bosons (exact global symmetries), pseudo-Goldstone bosons (approximate global symmetries) or massive gauge bosons (local gauge symmetries).
- There is no mathematical or experimental doubt about the consequences of spontaneous symmetry breaking.
- But now we need to start to speculate.
- What causes electroweak symmetry breaking at the first place?

THE SIMPLEST HYPOTHESIS

- The commonly known as Standard Model of electroweak interactions is built using the simplest mechanism for the spontaneous breaking of the SU(2) x U(1) gauge group.

- It introduces a sector of scalar fields and interactions,

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i\pi_1 + \pi_2 \\ \sigma - i\pi_3 \end{pmatrix}$$

$$\mathcal{L} = (D_\mu \phi)(D^\mu \phi)^\dagger - \lambda \left(|\phi|^2 - \frac{v^2}{2} \right)^2 + \ldots$$

$$D_\mu = \partial_\mu + i\frac{g}{2} T^a A^a$$

- It is a renormalizable theory.

A NEW PARTICLE

- It gives rise to a single new particle, the infamous Higgs boson.
- The mass of the Higgs boson is the only undetermined parameter in this theory.
- Other mechanisms for electroweak symmetry breaking are also a possibility.
- These introduce a richer spectrum of new particles..
- An exciting time for LHC whose purpose is to explore the exact mechanism for EWSB.