Highly-anisotropic hydrodynamics and the early thermalization puzzle in relativistic heavy-ion collisions

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1. Motivation

1.1 Early thermalization puzzle at RHIC

- experimental data from RHIC (and the LHC) are commonly interpreted as the evidence for very fast local equilibration of matter, success of perfect fluid hydrodynamics ($\tau_{\rm th} \leq 1$ fm/c)
- fast equilibration contradicts the results of microscopic models of heavy-ion collisions, string models, color glass condensate, pQCD kinetic calculations, ...
- thermalization of transverse degrees of freedom understood as the effect of fluctuations of the string tension (A. Białas)
- strongly or weakly interacting QGP?
  sQGP (E. Shuryak,...) or wQGP (S. Mrówczyński,...)
- successful description of data with models relaxing the assumption of fast equilibration
  perfect-fluid preceded by free streaming
  W. Broniowski, WF, M. Chojnacki, A. Kisiel, Phys. Rev., C80 (2009) 034902,
  S. V. Akkelin, Y. M. Sinyukov, Phys. Rev., C81 (2010) 064901
  perfect-fluid preceded by transverse hydrodynamics
  A. Bialas, M. Chojnacki, WF, Phys. Lett. B661 (2008) 325
  R. Ryblewski, WF, Phys. Rev. C 82, 024903 (2010)

1. Motivation

1.2 Forms of the energy-momentum tensor of matter produced in HIC

- color glass condensate:

$$T^{\mu\nu}\Big|_{\tau \ll 1/Q_s} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & -\varepsilon \end{pmatrix}$$

T. Lappi, K. Fukushima

$$T^{\mu\nu}\Big|_{\tau \gg 1/Q_s \sim 0.2 fm} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon/2 & 0 & 0 \\ 0 & 0 & \varepsilon/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Y. V. Kovchegov, A. Krasnitz
Y. Nara, R. Venugopalan

- perfect-fluid hydrodynamics:

$$T^{\mu\nu}_{\rm perfect\ hydro} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

- relativistic heavy-ion collisions can be described by the energy momentum-tensor with essentially diagonal form (in the local rest frame (LRF))

1. Motivation

1.3 High initial anisotropy of pressure

- at the early times, $\tau < 1$ fm/c, when the transverse distribution of matter in nuclei is known and may be used to model the initial energy/entropy density for the hydrodynamic calculations, the system exhibits high pressure anisotropy, typically $P_\perp \gg P_\parallel$
- at the later times, $\tau > 1$ fm/c, the pressure is almost isotropic but the transverse distribution of matter is not known
- need for an effective hydrodynamic approach which describes a transition from a highly-anisotropic system to perfect-fluid regime, THIS TALK
- more realistically, a transition from a highly-anisotropic system to viscous hydrodynamics with small viscosity to entropy ratio, WORK IN PROGRESS
- one cannot apply viscous hydrodynamics too early after the impact – the corrections to the energy-momentum tensor are very large due to the initial rapid longitudinal expansion

1. Motivation

1.4 ADHYDRO: highly-Anisotropic and strongly-Dissipative HYDROdynamics

WF, R.Ryblewski, Phys. Rev. C 83, 034907 (2011), arXiv:1007.0130
R.Ryblewski, WF, J. Phys. G 38, 015104 (2011); Acta Phys. Polon. B 42, 115 (2011)

- $P_\perp \neq P_\parallel$

$$T^{\mu\nu} = (\varepsilon + P_\perp)\, U^\mu U^\nu - P_\perp\, g^{\mu\nu} - (P_\perp - P_\parallel) V^\mu V^\nu$$

- $P_\perp = P_\parallel \rightarrow$ isotropic fluid, $\quad T^{\mu\nu} \rightarrow T^{\mu\nu}_{\rm perfect\ hydro}$

$$U^\mu = \gamma(1, v_x, v_y, v_z), \quad \gamma = (1 - v^2)^{-1/2} \quad \text{hydrodynamic flow}$$

$$V^\mu = \gamma_z(v_z, 0, 0, 1), \quad \gamma_z = (1 - v_z^2)^{-1/2} \quad \text{longitudinal axis}$$

$$U^2 = 1, \quad V^2 = -1, \quad U \cdot V = 0$$

- local rest frame: $U^\mu = (1,0,0,0)$ and $V^\mu = (0,0,0,1)$

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P_\perp & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\parallel \end{pmatrix}$$

2. Dynamic equations

2.1 Energy-momentum conservation and entropy production

- in analogy to perfect-fluid hydrodynamics we assume:

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu S^\mu = \Sigma$$

$S^\mu = \sigma U^\mu$ – entropy flow
Σ – internal entropy source

- one has to specify:

generalized EOS $\quad \epsilon = \epsilon(P_\perp, P_\parallel)$
entropy production term $\quad \Sigma = \Sigma(P_\perp, P_\parallel)$

- system of 5 equations for 5 unknown functions: $\vec{v}$, $P_\perp$, $P_\parallel$
- in particular, for massless partons the condition $T^\mu{}_\mu = 0$ gives

$$\varepsilon(P_\perp, P_\parallel) = 2P_\perp + P_\parallel$$

3. Microscopic interpretation

3.1 Parton distribution function

- locally anisotropic systems of particles $\rightarrow$ two different scales $\lambda_\perp$ and $\lambda_\parallel$, may be interpreted as the transverse and longitudinal temperature

$$f_{LRF} = f\left(\frac{p_\perp}{\lambda_\perp}, \frac{|p_\parallel|}{\lambda_\parallel}\right)$$

- generalization of Boltzmann equilibrium distribution, Romatschke-Strickland (RS) form

$$f_{LRF} = g_0 \exp\left(-\sqrt{\frac{p_\perp^2}{\lambda_\perp^2} + \frac{p_\parallel^2}{\lambda_\parallel^2}}\right) = g_0 \exp\left(-\frac{1}{\lambda_\perp}\sqrt{p_\perp^2 + x\, p_\parallel^2}\right)$$

where $x = 1 + \xi = \left(\frac{\lambda_\perp}{\lambda_\parallel}\right)^2$ is the anisotropy parameter

- covariant RS form

$$f = g_0 \exp\left(-\frac{1}{\lambda_\perp}\sqrt{(p \cdot U)^2 + \xi\, (p \cdot V)^2}\right)$$

3. Microscopic interpretation

3.2 Energy-momentum tensor and entropy flux

- moments of anisotropic distributions

$$T^{\mu\nu} = \int \frac{d^3p\, p^\mu p^\nu}{(2\pi)^3 E_p} f(p\cdot U, p\cdot V) = (\varepsilon + P_\perp) U^\mu U^\nu - P_\perp g^{\mu\nu} - (P_\perp - P_\parallel) V^\mu V^\nu$$

$$S^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} f(p\cdot U, p\cdot V) \left[1 - \ln\left(\frac{f(p\cdot U, p\cdot V)}{g_0}\right)\right] = \sigma U^\mu$$

- for further analysis most convenient two independent parameters are x (anisotropy parameter) and σ (non-equilibrium entropy density)

$$(P_\perp, P_\parallel) \text{ or } (\lambda_\perp, \lambda_\parallel) \longrightarrow (\sigma, x)$$

3. Microscopic interpretation

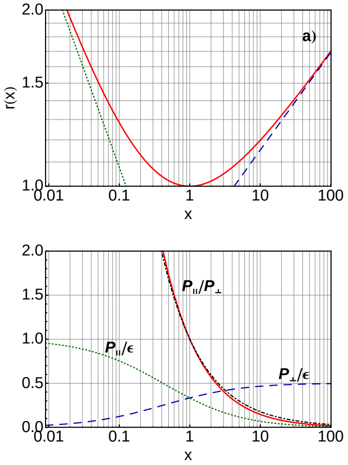
3.3 Energy, pressure, entropy for RS form

$$\begin{aligned} \varepsilon(\sigma,x) &= \varepsilon_{\rm id}(\sigma) r(x) \\ P_\perp(\sigma,x) &= P_{\rm id}(\sigma)\left[r(x)+3xr'(x)\right] \\ P_\parallel(\sigma,x) &= P_{\rm id}(\sigma)\left[r(x)-6xr'(x)\right] \end{aligned}$$

$$r(x) = \frac{x^{-\frac{1}{3}}}{2}\left[1+\frac{x\arctan\sqrt{x-1}}{\sqrt{x-1}}\right]$$

in equilibrium ($x=1$):

$$\begin{aligned} \varepsilon_{\rm id} &= 3g_0T^4/\pi^2 \\ P_{\rm id} &= g_0T^4/\pi^2 \\ \sigma_{\rm id} &= 4g_0T^3/\pi^2 \end{aligned}$$



3. Microscopic interpretation

3.4 Phenomenological ansatz for Σ

- the simplest ansatz for Σ (positive, correct dimension, symmetric with respect to the interchange of $\lambda_\perp$ and $\lambda_\parallel$, vanishes in equilibrium) has the form

$$\Sigma = \frac{(\lambda_\perp - \lambda_\parallel)^2}{\lambda_\perp \lambda_\parallel} \frac{\sigma}{\tau_{\rm eq}} = \frac{(1-\sqrt{x})^2}{\sqrt{x}} \frac{\sigma}{\tau_{\rm eq}}$$

  $\tau_{\rm eq}$ is a timescale parameter

- consistent with Israel-Stewart theory for small $|x-1|$ and for purely longitudinal boost-invariant motion

$$\Sigma \approx \frac{(x-1)^2}{4\tau_{\rm eq}} \sigma$$

- for large $|x-1|$ various forms of the Σ are conceivable, results of microscopic models may be used to introduce time dependence of x, in particular one may use the AdS/CFT correspondence correspondence

3. Microscopic interpretation
3.5 Connection to kinetic theory

M.Martinez, M.Strickland, Nucl. Phys. A 848 (2010), Nucl. Phys. A 856 (2010)
M.Martinez, R. Ryblewski, M.Strickland, PRC in press
zeroth moment of the Boltzmann equation = entropy production = gluon emission

$$p^\mu \partial_\mu f = C \approx -p \cdot U \, \Gamma \, (f - f_{\rm eq}), \quad \partial_\mu \int dP \, p^\mu f = \int dP \, C$$

Γ is the inverse relaxation time, for the covariant RS form, $\sigma = 4n$, one gets

$$\partial_\mu \left(\sigma U^\mu\right) = \frac{1}{4} \int dP \; C = \Sigma \approx \frac{\Gamma}{4} (n_{\rm eq} - n) \quad \text{(one equation)}$$

first moment of the Boltzmann equation, energy-momentum conservation

$$\partial_\mu \int dP \, p^\nu p^\mu f = \int dP \, p^\nu C = \partial_\mu T^{\nu\mu} = 0 \quad \text{(four equations)}$$

$$\int dP \, p^\nu C = - \int dP \, p \cdot U \, p^\nu \, \Gamma \, (f - f_{\rm eq}) = 0 \quad \text{(Landau matching for } T(P_\perp, P_\parallel) \text{ in } f_{\rm eq})$$

5 equations for 5 unknown functions: $\vec{v}$, $P_\perp$, $P_\parallel$ similarly as in the phenomenological approach introduced earlier

4. Purely-longitudinal boost-invariant motion

4.1 Implementation of boost-invariance

- boost-invariant ansatz for U and V

$$U^\mu = (\cosh\eta, 0, 0, \sinh\eta), \qquad V^\mu = (\sinh\eta, 0, 0, \cosh\eta)$$

$$\tau = \sqrt{t^2 - z^2}, \qquad \eta = \frac{1}{2}\ln\frac{t+z}{t-z}$$

- leads to the two equations of motion

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + P_\parallel}{\tau}, \qquad \frac{d\sigma}{\sigma d\tau} + \frac{1}{\tau} = \frac{\Sigma}{\sigma}$$

- the first equation is equivalent to:

$$r'(x)\left(\frac{dx}{d\tau} - \frac{2x}{\tau}\right) = -\frac{4}{3}r(x)\left(\frac{d\sigma}{\sigma d\tau} + \frac{1}{\tau}\right)$$

$\Sigma = 0 \longrightarrow \quad x = 1 \quad$ or $\quad dx/d\tau = 2x/\tau \quad$ (local equilibrium or free streaming)

4. Purely-longitudinal boost-invariant motion

4.2 Anisotropy evolution

- for the simple phenomenological ansatz

$$\frac{dx}{d\tau} = \frac{2x}{\tau} - \frac{4H(x)}{3\tau_{\rm eq}}$$

where

$$H(x) = \frac{r(x)}{r'(x)} \frac{(1-\sqrt{x})^2}{\sqrt{x}} \approx \frac{45}{16}(x-1) + \frac{195}{112}(x-1)^2 + \ldots$$

- in Martinez-Strickland's approach

$$\frac{dx}{d\tau} = \frac{2x}{\tau} - \frac{4\Gamma H_{MS}(x)}{3}$$

where

$$H_{MS}(x) \approx \frac{3}{8}(x-1) + \frac{17}{84}(x-1)^2 + \ldots$$

- the two approaches are equivalent close to equilibrium if

$$\Gamma = \frac{15}{2\tau_{\rm eq}}$$

4. Purely-longitudinal boost-invariant motion

4.3 Connection with the Israel-Stewart theory

- close to equilibrium, $|\xi| \ll 1$, $P_{\parallel}(x) = P_{\rm eq} - \bar{\pi}$, $P_{\perp}(x) = P_{\rm eq} - \frac{\bar{\pi}}{2}$

$$\frac{\bar{\pi}}{\varepsilon_{\rm eq}} = \frac{8}{45}(x-1) = \frac{8}{45}\xi$$

- our equations agree with the evolution equation for $\bar{\pi}$ in 0+1 I-S theory:

$$\frac{d\overline{\pi}}{d\tau} + \frac{4\overline{\pi}}{3\tau} - \frac{16}{45}\frac{\varepsilon}{\tau} = -\frac{15\overline{\pi}}{4\tau_{\rm eq}} \qquad \longrightarrow \qquad \frac{d\overline{\pi}}{d\tau} = -\frac{4\overline{\pi}}{3\tau} + \frac{4\eta_\pi}{3\tau_\pi \tau} - \frac{\overline{\pi}}{\tau_\pi}$$

for the identification

$$\frac{1}{\tau_{\rm eq}} = \frac{4}{15\tau_\pi}, \qquad \tau_\pi = \frac{5\eta_\pi}{T\sigma_{\rm eq}}$$

- similar agreement for the entropy production with I-S:

$$\partial_\mu S^\mu = \sigma_{\rm eq}\frac{\xi^2}{4\tau_{\rm eq}} \qquad \longrightarrow \qquad \partial_\mu S^\mu = \frac{3\bar{\pi}^2}{4\eta_\pi T}$$

5. Non-boost-invariant (3+1)D case

5.1 Initial conditions

- Initial evolution time $\tau_0 = 0.25$ fm, equilibration time $\tau_{\rm eq} = 0.25$ fm and 1 fm
- initial anisotropy choices: $x_0 = 100$ (transverse thermalization), $x_0 = 1$ (perfect fluid), and $x_0 = 0.032$ (longitudinal thermalization)
- initial energy density profile (tilted source by P.Bozek)

$$\varepsilon_0(\tau_0, \eta, \mathbf{x}_\perp) = \varepsilon_{\rm i}\, \tilde{\rho}(b, \eta, \mathbf{x}_\perp) \qquad \tilde{\rho}(b, \eta, \mathbf{x}_\perp) = \frac{\rho(b, \eta, \mathbf{x}_\perp)}{\rho(0,0)}$$

$$\rho(b, \eta, \mathbf{x}_\perp) = (1-\kappa)\left[\rho_W^+(b, \mathbf{x}_\perp) f^+(\eta) + \rho_W^-(b, \mathbf{x}_\perp) f^-(\eta)\right] + \kappa \rho_B(b, \mathbf{x}_\perp) f(\eta)$$

- initial longitudinal profile

$$f(\eta) = \exp\left[-\frac{(\eta - \Delta\eta)^2}{2\sigma_\eta^2}\theta(|\eta| - \Delta\eta)\right] \qquad \Delta\eta = 1, \sigma_\eta^2 = 1.3$$

- mixing factor $\kappa = 0.14$, initial energy density in the center $\varepsilon_{\rm i}$ chosen separately for each pair of x and $\tau_{\rm eq}$

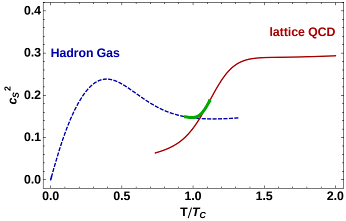
5. Non-boost-invariant (3+1)D case

5.2 Generalized EOS – inclusion of the phase transition

to connect the isotropization with the process of formation of the equilibrated quark-gluon plasma we may consider the following ansatz

$$\begin{aligned}
\varepsilon(\sigma, x) &= \varepsilon_{\rm qgp}(\sigma) r(x) \\
P_\perp(\sigma, x) &= P_{\rm qgp}(\sigma)\left[r(x) + 3x r'(x)\right] \\
P_\parallel(\sigma, x) &= P_{\rm qgp}(\sigma)\left[r(x) - 6x r'(x)\right]
\end{aligned}$$

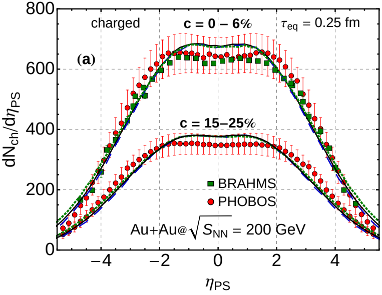
Here, the functions $\varepsilon_{\rm qgp}(\sigma)$ and $P_{\rm qgp}(\sigma)$ describe the realistic equation of state :
M. Chojnacki and WF, Acta Phys. Pol. B38 (2007) 3249.

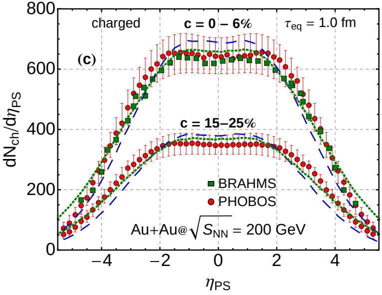


5. Non-boost-invariant 3+1D case

5.3 $dN/d\eta$ of charged particles

Initial anisotropy : $x_0 = 1$ (black), $x_0 = 100$ (blue), and $x_0 = 0.032$ (green)

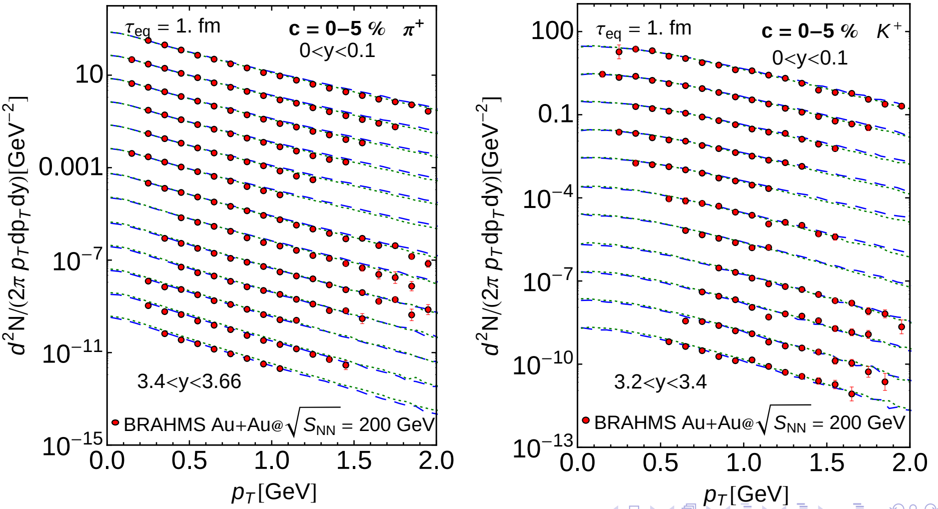




5. Non-boost-invariant (3+1)D case

5.4 $p_\perp$ spectra in different y windows

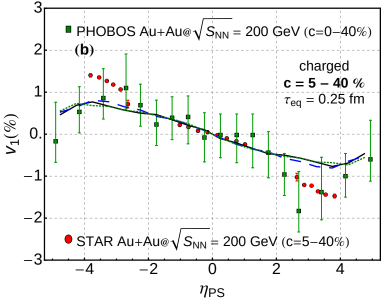
Initial anisotropy : x_0 = 1 (black), x_0 = 100 (blue), and x_0 = 0.032 (green)

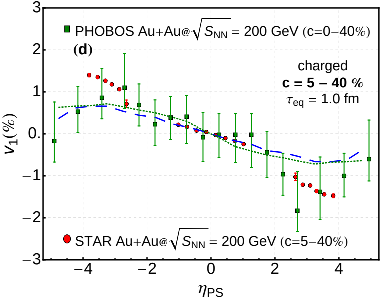


5. Non-boost-invariant (3+1)D case

5.5 pseudorapidity dependence of v_1 for charged particles

Initial anisotropy : x_0 = 1 (black), x_0 = 100 (blue), and x_0 = 0.032 (green)

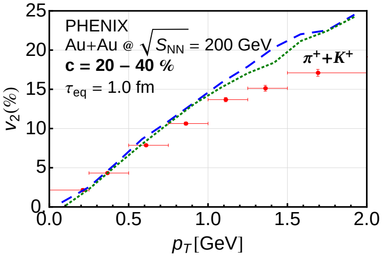




5. Non-boost-invariant (3+1)D case

5.6 $v_2(p_T)$ in midrapidity

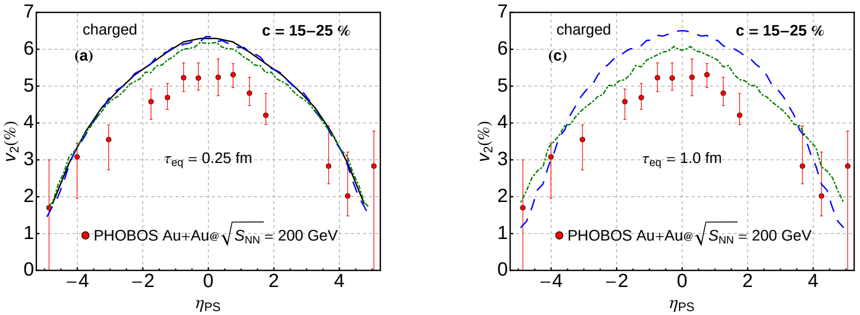
Initial anisotropy : x_0 = 1 (black), x_0 = 100 (blue), and x_0 = 0.032 (green)



5. Non-boost-invariant (3+1)D case

5.7 pseudorapidity dependence of v_2 for charged particles

Initial anisotropy : x_0 = 1 (black), x_0 = 100 (blue), and x_0 = 0.032 (green)



6. Conclusions

- A framework of highly-anisotropic hydrodynamics with strong dissipation (ADHYDRO) has been introduced. The effects of dissipation are defined by the form of the entropy source. The RHIC soft hadronic data described using 2+1D and 3+1D code.
- Initial conditions with extremely different anisotropies lead to similar results, provided the initial conditions of the evolution are properly readjusted.
- Complete thermalization of the system may be delayed to easily acceptable times of about 1-2 fm/c. The early-thermalization puzzle may be circumvented.