The Thermal Model at the LHC.

Jean Cleymans

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Kolymbari, Crete
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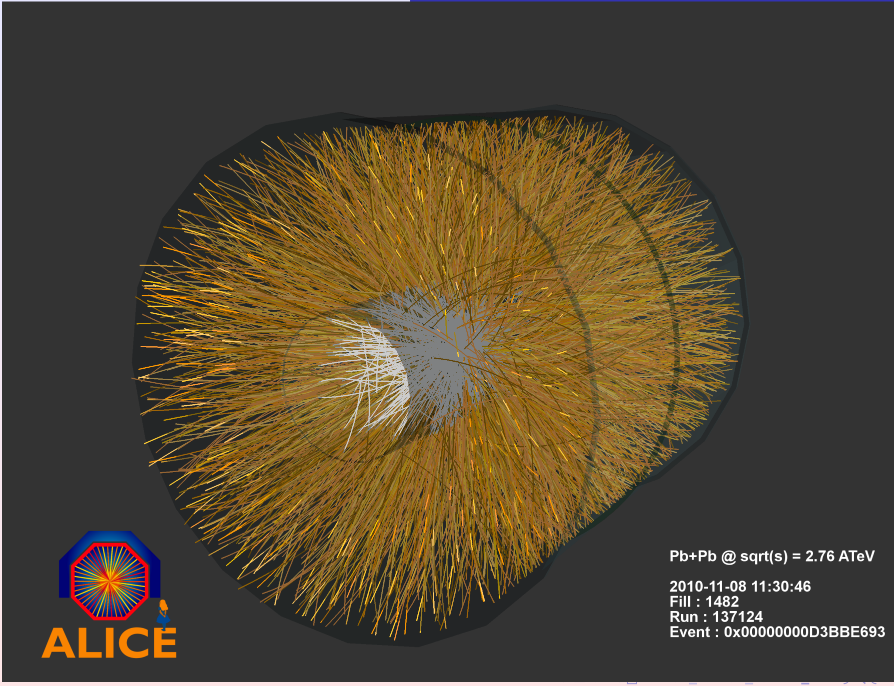

ALICE
Pb+Pb @ sqrt(s) = 2.76 ATeV
2010-11-08 11:30:46
Fill : 1482
Run : 137124
Event : 0x00000000D3BBE693

The energy is

$$\sqrt{s} = 2760.0 \ \mathrm{AGeV}$$

yet the temperature seen in the particle ratios is only

$$T \approx 0.16 \ \mathrm{GeV}$$

What is the story behind this?

Outline

1. Temperature from Number of Particles - Hagedorn Temperature
2. Temperature from Particle Yields - Chemical Equilibrium
3. Temperature from Transverse Momenta Spectra - Tsallis?

Temperature

The temperature can be obtained from:

- Mass spectrum of hadrons: simply adding up the number of hadronic resonances (Hagedorn, Ranft) (Hagedorn temperature),
- Lattice QCD at finite temperature (phase transition temperature),
- Transverse momentum spectra (kinetic or thermal freeze-out temperature).
- Hadronic ratios (chemical freeze-out temperature),

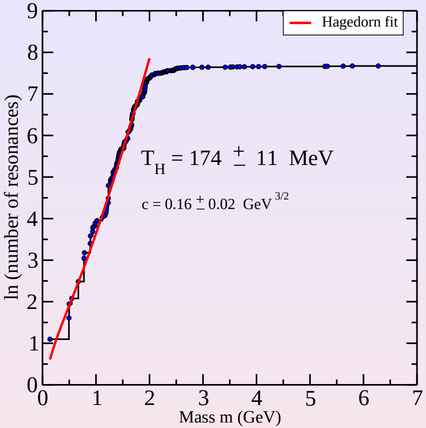
Are they all the same?

Hagedorn Temperature

Uncertainties in determining T_H:

- Analytic formula to be used?
- Possibly a different T_H for Mesons and Baryons ?

W. Broniowski, W. Florkowski and L. Y. Glozman,
Phys. Rev. D70 117503 (2004),
S. Chatterjee, S. Gupta and R. M. Godbole,
Phys. Rev. C81 044907 (2010),
J.C. and Dawit Worku, Mod. Phys. Lett. A26 1197 (2011).



Keep on adding the number of hadronic resonances.

J.C. and Dawit Worku, Mod. Phys. Lett. A26 (2011) 1197; arXiv: 1103.1463

HADRONS DO NOT EXIST ABOVE THE HAGEDORN TEMPERATURE.

	Equilibrium
π^0	$\exp\left[-\frac{E_\pi}{T}\right]$
p	$\exp\left[-\frac{E_p}{T}+\frac{\mu_B}{T}+\frac{\mu_Q}{T}\right]$
n	$\exp\left[-\frac{E_n}{T}+\frac{\mu_B}{T}\right]$
Λ	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_B}{T}-\frac{\mu_S}{T}\right]$
$\overline{\Lambda}$	$\exp\left[-\frac{E_\Lambda}{T}-\frac{\mu_B}{T}+\frac{\mu_S}{T}\right]$
K^+	$\exp\left[-\frac{E_K}{T}+\frac{\mu_S}{T}+\frac{\mu_Q}{T}\right]$
$\overline{K^0}$	$\exp\left[-\frac{E_K}{T}-\frac{\mu_S}{T}\right]$

Chemical Equilibrium

- Particle multiplicities (integrated over 4π) are Lorentz invariant, independent of flow.
- No need for “instantaneous” freeze-out. Freeze-out time can be very complicated. The only requirement is that the chemical freeze-out happens when the cell reaches the chemical freeze-out temperature. This normally happens at different times for different cells (on the surface or in the center).

SPS data.

	Measurement
Pb–Pb 158A GeV	
$(\pi^+ + \pi^-)/2.$	600±30
K^+	95 ±10
K^-	50 ±5
K^0_S	60 ±12
p	140±12
$\bar{p}$	10 ±1.7
ϕ	7.6±1.1
Ξ^-	4.42±0.31
$\overline{\Xi^-}$	0.74±0.04
$\bar{\Lambda}/\Lambda$	0.2±0.04

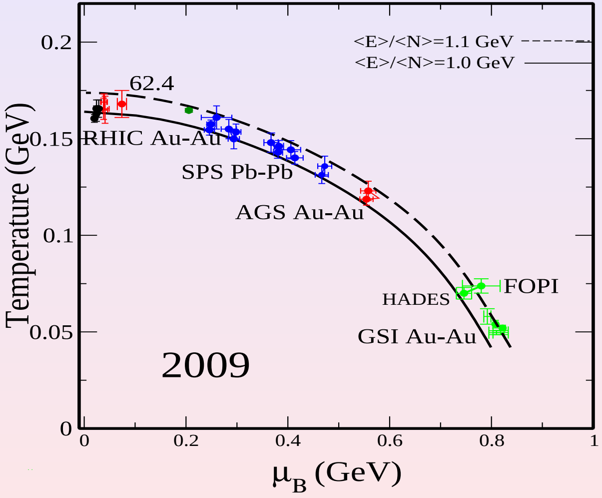
SPS data.

SPS: Freeze-Out Parameters:

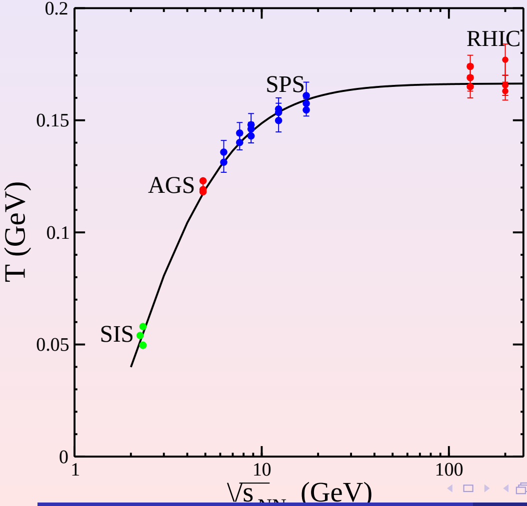
$$T = 156.0 \pm 2.4\mathrm{MeV}$$
$$\mu_B = 239 \pm 12\mathrm{MeV}$$

F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich
Physical Review C64 (2001) 024901.

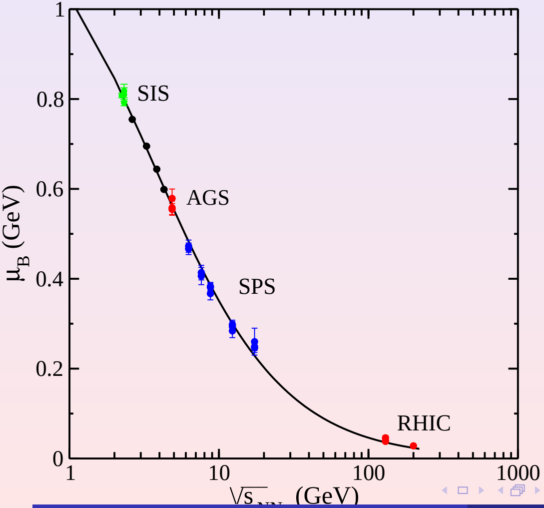
Chemical Equilibrium



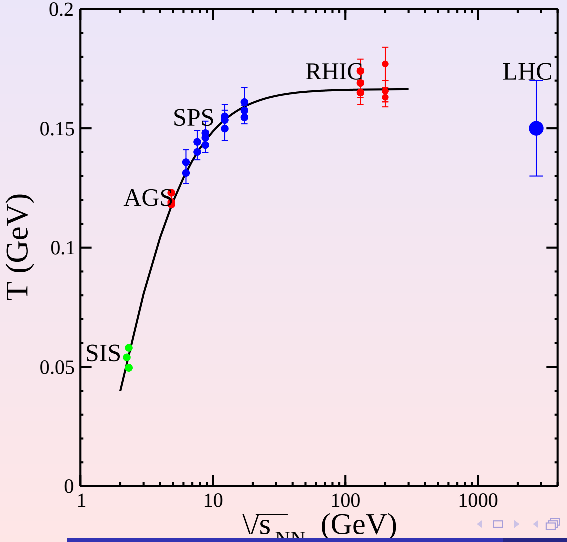
Chemical Freeze-Out Temperature

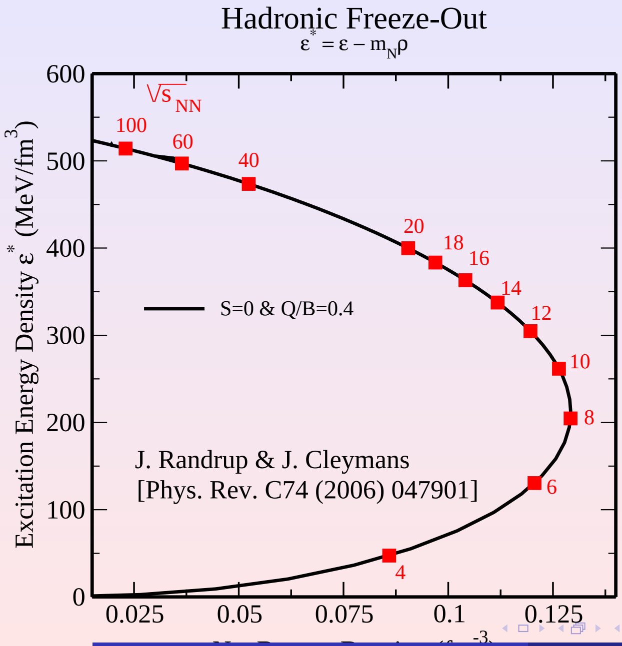


Chemical Freeze-Out μ_B

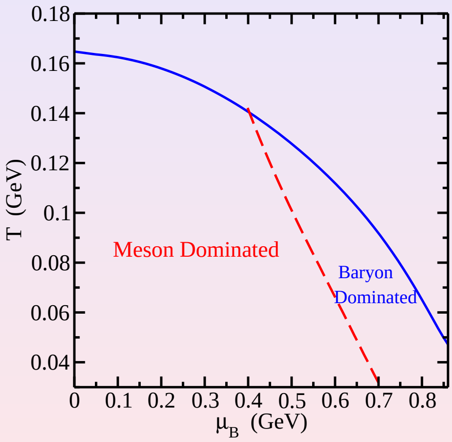


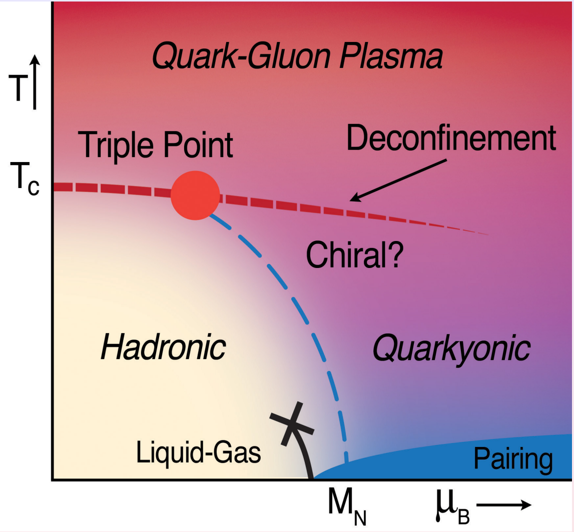
Chemical Freeze-Out Temperature




Hadronic Freeze-Out
ε* = ε − m_N ρ
√s_NN
100
60
40
20
18
16
14
12
10
8
6
4
S=0 & Q/B=0.4
J. Randrup & J. Cleymans
[Phys. Rev. C74 (2006) 047901]
Excitation Energy Density ε* (MeV/fm³)
600
500
400
300
200
100
0
0.025
0.05
0.075
0.1
0.125

Transition


0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
T (GeV)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
μB (GeV)
Meson Dominated
Baryon
Dominated



R. Pisarski and L. McLerran

Transverse Momentum Distribution

STAR collaboration, B.I. Abelev at al.
arXiv:nucl-ex/0607033; Phys. Rev. C75, 064901 (2007)
PHENIX collaboration, A. Adare et al.
Phys. Rev. **C83**, 064903 (2011)
ALICE collaboration, K. Aamodt et al.
arXiv:1101.4110 [hep-ex]
CMS collaboration, V. Khachatryan et al.
arXiv: 1102.4282 [hep-ex]
ATLAS collaboration, G. Aad et al.
New J. Phys. **13** (2011) 053033.

All use the Tsallis distribution.

Tsallis Distribution

Possible generalization of Boltzmann-Gibbs statistics

Constantino Tsallis
Rio de janeiro, CBPF
J. Stat. Phys. 52 (1988) 479-487

Citations: 1 389
However:
Citations in HEP: 403

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Notas de Física

CBPF-NF-062/87

POSSIBLE GENERALIZATION OF BOLTZMANN-GIBBS STATISTICS

by

Constantino TSALLIS

CBPF-NF-062/87

Multifractal concepts and structures are quickly acquiring importance in many active areas (e.g., non-linear dynamical systems, growth models, commensurate/incommensurate structures). This is due to their utility as well as to their elegance. Within this framework, the quantity which is normally scaled is p_i^q, where p_i is the probability associated to an event and q any real number [1]. We shall use this quantity to generalize the standard expression of the entropy S in information theory, namely $S = -k\sum_{i=1}^{W} p_i \ln p_i$, where $W \in \mathbb{N}$ is the total number of possible (microscopic) configurations and $\{p_i\}$ the associated probabilities. We *postulate* for the entropy

$$S_q \equiv k\,\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \qquad (q \in \mathbb{R}) \qquad (1)$$

where k is a conventional positive constant and $\sum_{i=1}^{W} p_i = 1$. We immediately verify that

$$S_1 \equiv \lim_{q\to 1} S_q = k \lim_{q\to 1} \frac{1-\sum_{i=1}^{W} p_i e^{(q-1)\ln p_i}}{q-1} = -k\sum_{i=1}^{W} p_i \ln p_i \qquad (1')$$

where we have used the replica-trick type of expansion. We illustrate definition (1) in Fig. 1. S_q may be rewritten as follows:

Transverse Momentum Distribution

STAR, PHENIX, ALICE, CMS, ATLAS use:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{t}\mathrm{d}y} = p_\mathrm{t} \times \frac{\mathrm{d}N}{\mathrm{d}y} \frac{(n-1)(n-2)}{nC(nC + m_0(n-2))} \left(1 + \frac{m_\mathrm{t} - m_0}{nC}\right)^{-n}$$

Direct connection with Tsallis distribution.

In the Tsallis distribution the total number of particles is given by:

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{q/(1-q)} ,$$

The corresponding momentum distribution is given by

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{q/(1-q)} ,$$

which, in terms of the rapidity and transverse mass variables, becomes (for $\mu = 0$)

$$\left.\frac{d^2N}{dp_t\, dy}\right|_{y=0} = gV\frac{p_t m_t}{(2\pi)^2} \left[1 + (q-1)\frac{m_t}{T}\right]^{q/(1-q)} ,$$

J.C. and D. Worku, arXiv:1106.3405[hep-ph]

Rewrite the Tsallis distribution using

$$[1+(q-1)x]^{1/(1-q)} = \exp\left(\frac{1}{1-q}\ln\left[1+(q-1)x\right]\right),$$

and consider the limit $q \to 1$

$$\begin{aligned}\lim_{q\to 1} \quad & [1+(q-1)x]^{1/(1-q)} \\ &= \exp\frac{1}{(1-q)}(q-1)x \\ &= \exp\left(-x\right), \end{aligned} \tag{1}$$

The Tsallis distribution reduces to the Boltzmann distribution in the limit where $q \to 1$

$$\begin{aligned}&\lim_{q\to 1}\frac{d^2N}{dp_t\, dy} = \\ &gV\frac{p_t m_t \cosh y}{(2\pi)^2}\exp\left(-\frac{m_t\cosh y-\mu}{T}\right).\end{aligned} \tag{2}$$

In all cases q is close to one, typically between 1.05 and 1.2.

Comparison of Tsallis with STAR, ALICE, CMS distributions

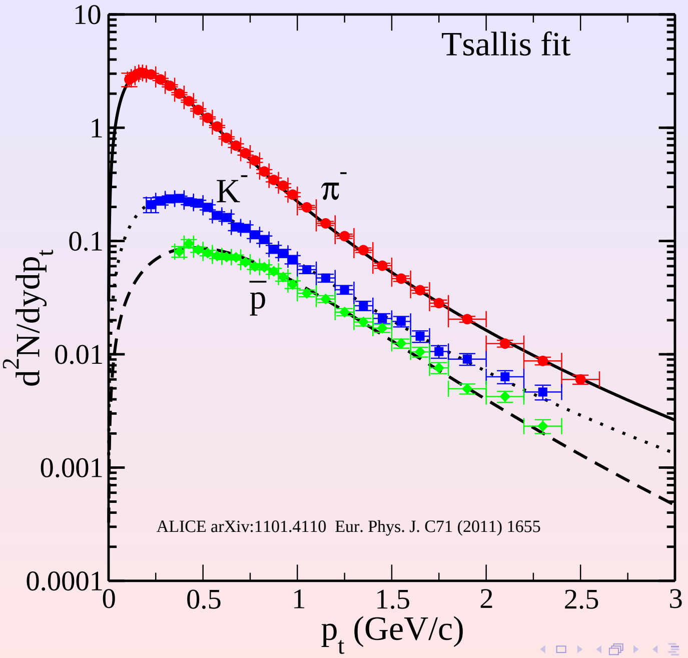
$$\frac{d^2N}{dp_t dy} = gV\frac{p_t m_t}{(2\pi)^2}\left[1+(q-1)\frac{m_t}{T}\right]^{q/(1-q)}, \tag{3}$$

$$\frac{d^2N}{dp_t dy} = p_t \times \frac{dN}{dy}\frac{(n-1)(n-2)}{nC(nC+m_0(n-2))}\left[1+\frac{m_t-m_0}{nC}\right]^{-n} \tag{4}$$

$$n \rightarrow \frac{q}{q-1}$$

$$nC \rightarrow \frac{T+m_0(q-1)}{q-1}$$

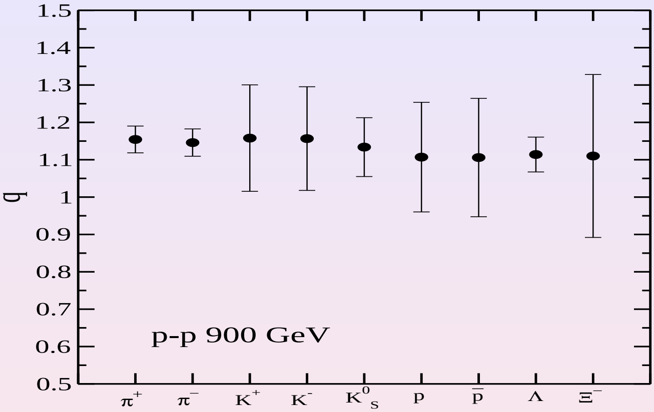
Only a factor of m_T differs!


Tsallis fit
K^-
π^-
$\bar{p}$
ALICE arXiv:1101.4110 Eur. Phys. J. C71 (2011) 1655
$d^2N/dydp_t$
p_t (GeV/c)
10
1
0.1
0.01
0.001
0.0001
0
0.5
1
1.5
2
2.5
3

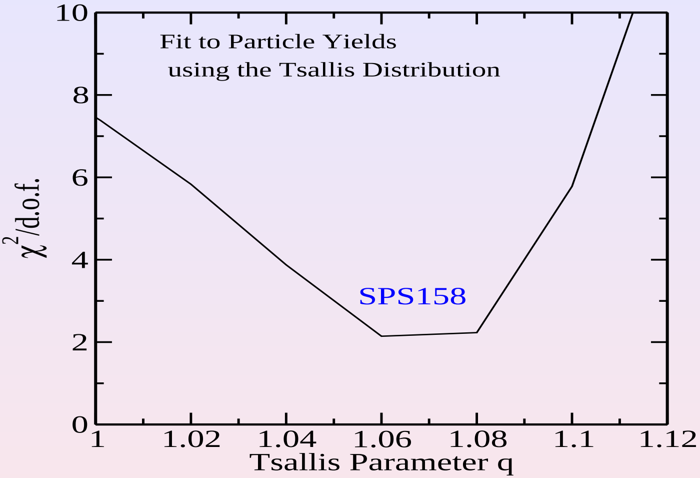
$p-p$ 900 GeV		
Particle	q	T
π^+	1.154 $\pm$0.036	0.0682 $\pm$0.0026
π^-	1.146 $\pm$0.036	0.0704 $\pm$ 0.0027
K^+	1.158 $\pm$0.142	0.0690 $\pm$0.0223
K^-	1.157 $\pm$0.139	0.0681 $\pm$ 0.0217
K^0_S	1.134 $\pm$0.079	0.0923 $\pm$0.0139
p	1.107 $\pm$0.147	0.0730 $\pm$ 0.0425
$\bar{p}$	1.106 $\pm$0.158	0.0764 $\pm$0.0464
Λ	1.114 $\pm$0.047	0.0698 $\pm$ 0.0148
Ξ^-	1.110 $\pm$0.218	0.0440 $\pm$ 0.0752

Table: Fitted values of the T and q parameters measured in $p-p$ collisions by the ALICE and CMS collaborations using the Tsallis form for the momentum distribution.

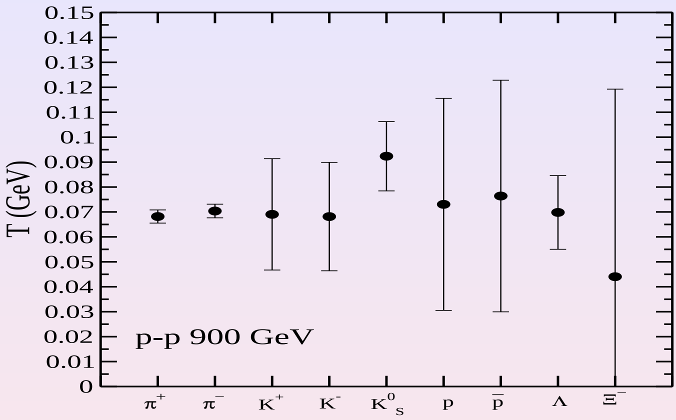
$p-p$ 900 GeV		
Particle	T Tsallis vs C ALICE (MeV)	q
π^+	70 (126)	1.147
K^+	70 (160)	1.156
p	73 (196)	1.110



Values of the Tsallis parameter *q* for different species of hadrons.



J. C., G. Hamar, P. Levai, S. Wheaton
Journal of Physics **G 36** (2009) 064018.



Values of the Tsallis temperature T for different species of hadrons.

J.C. and D. Worku e-Print: arXiv:1110.5526 [hep-ph]

In conclusion, temperatures obtained from

- Hagedorn spectrum,
- Lattice QCD,
- Particle Ratios

are compatible.
Transverse momentum distributions give a much lower temperature in $p - p$ (thermal freeze-out temperature); higher in $Pb - Pb$ (flow).