Some recent results from lattice QCD

Christian Hoelbling
Bergische Universität Wuppertal

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BERGISCHE UNIVERSITÄT WUPPERTAL

Purpose of lattice QCD

- QCD fundamental objects: quarks and gluons
- QCD observed objects: protons, neutrons (π, K, . . .)

! Huge discrepancy: not even the same particles observed as in the Lagrangean

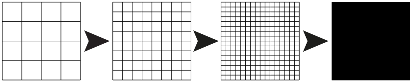
➛ Perturbation theory has no chance

- Need to solve low energy QCD to:
  - Compute hadronic and nuclear properties
    "people who love QCD"
    - Masses, decay widths, scattering lengths, thermodynamic properties, . . .
  - Compute hadronic background
    "people who hate QCD"
    - Non-leptonic weak MEs, quark masses, g-2, . . .

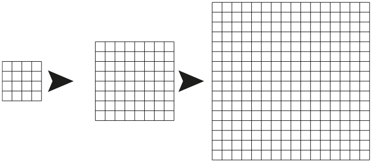
Lattice

Lattice QCD=QCD when

- Cutoff removed (continuum limit)



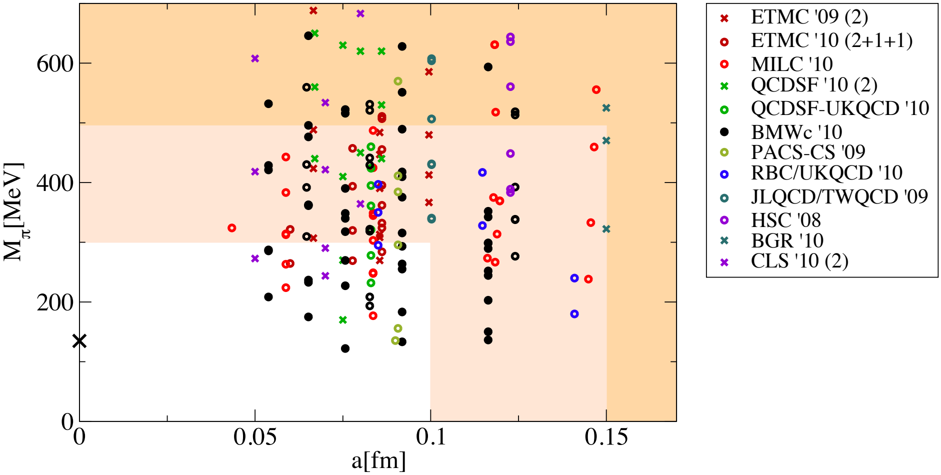
- Infinite volume limit taken



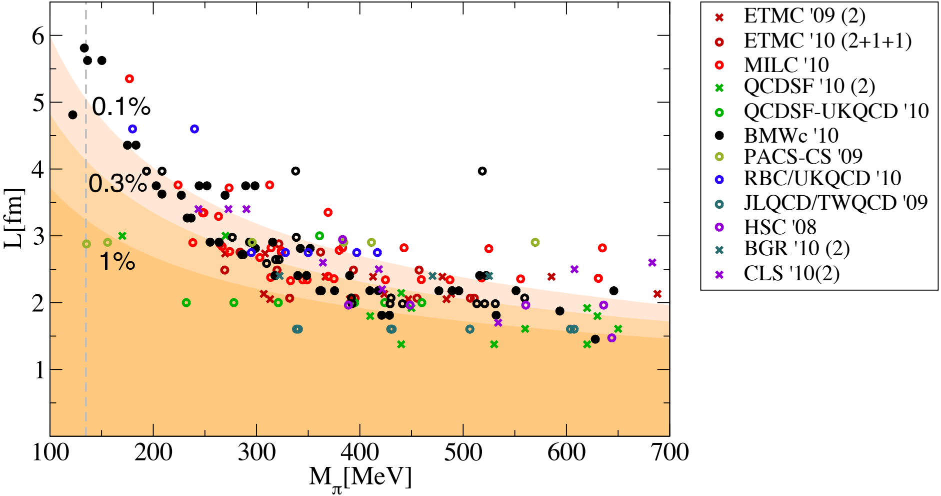
- At physical hadron masses (Especially π)
  - Numerically challenging to reach light quark masses

Statistical error from stochastic estimate of the path integral

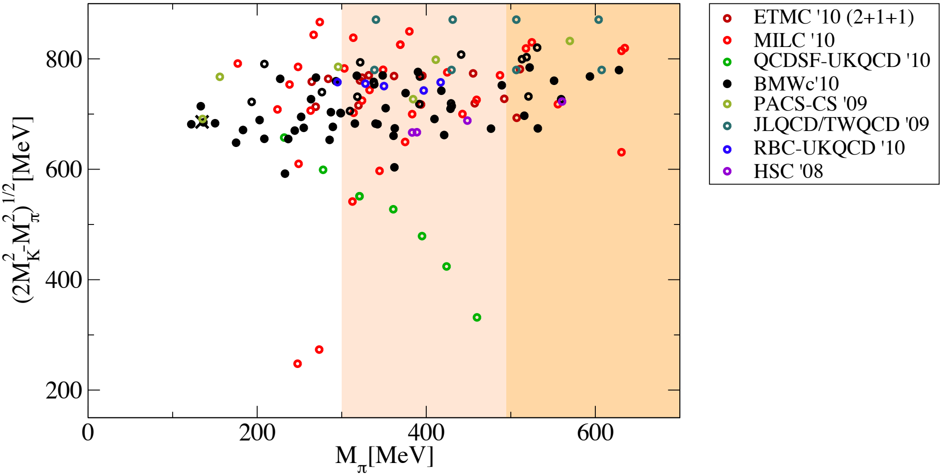
Landscape M_π vs. a



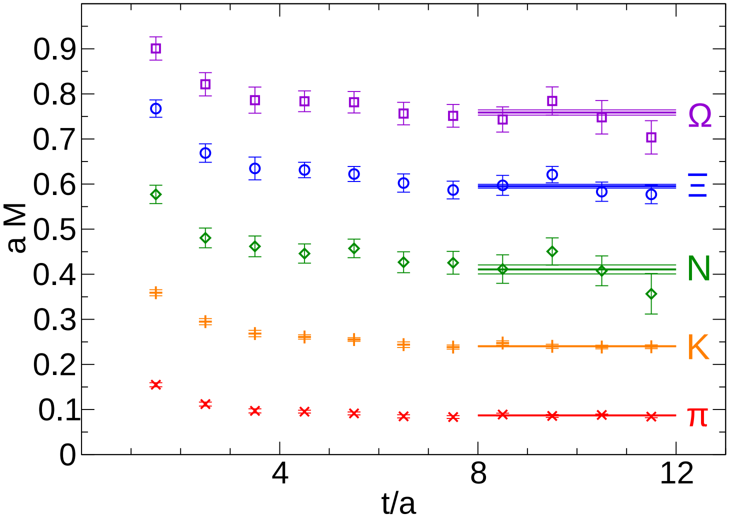
Landscape L vs. M_π



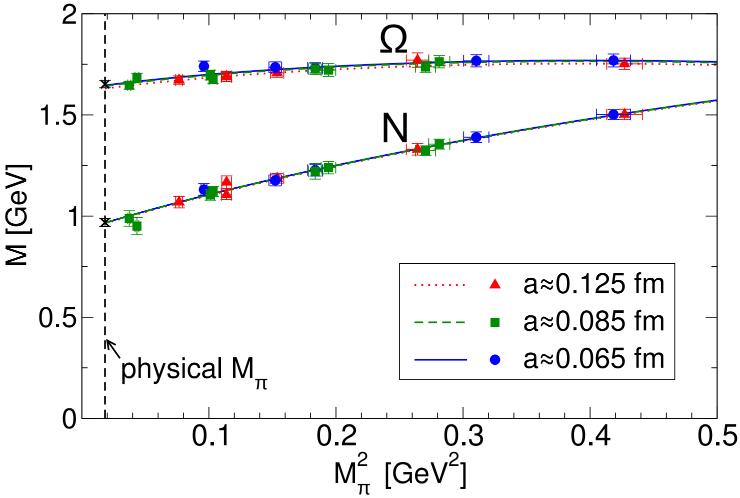
Landscape M_K vs. M_π



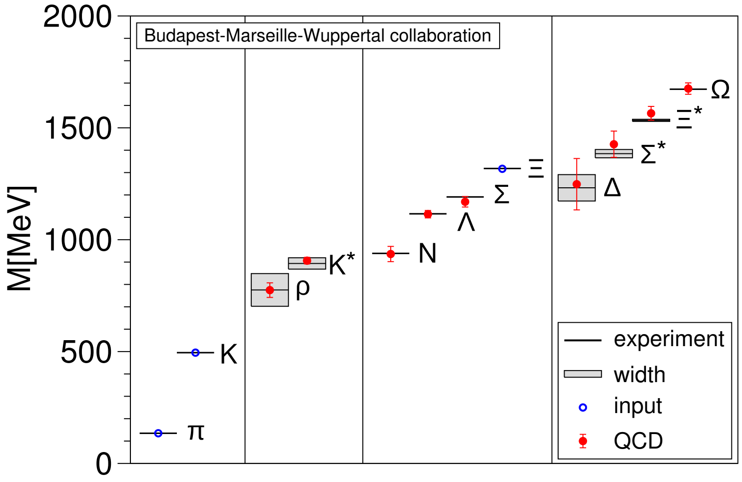
Effective masses and correlated fits



Chiral fit


2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
M [GeV]
M$_\pi^2$ [GeV^2]
Ω
N
physical M$_\pi$
a≈0.125 fm
a≈0.085 fm
a≈0.065 fm

The light hadron spectrum

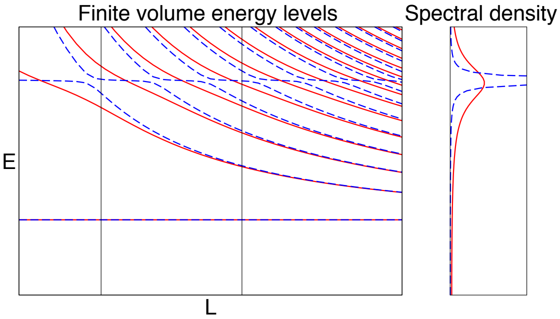


(Science 322:1224,2008)

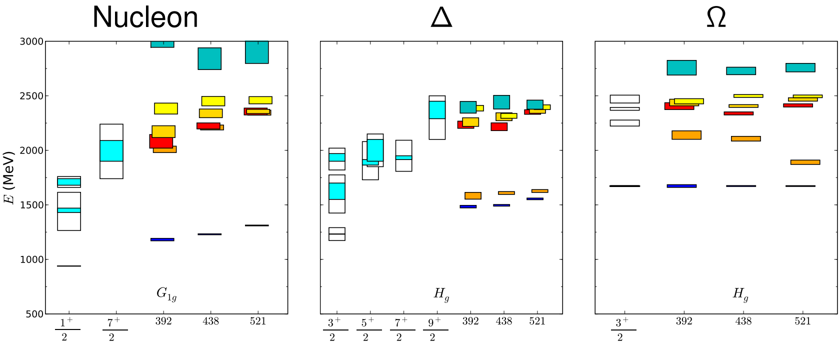
Excited states

Extracting excited states is much tougher:

- ☞ Extraction of energy levels is harder:
  Die out at large t ➠ need to use small t correlators
- ☞ Once extracted, relation to $V \to \infty$ is nontrivial:
  Disentangle resonances and scattering states at finite volume



High lying resonances

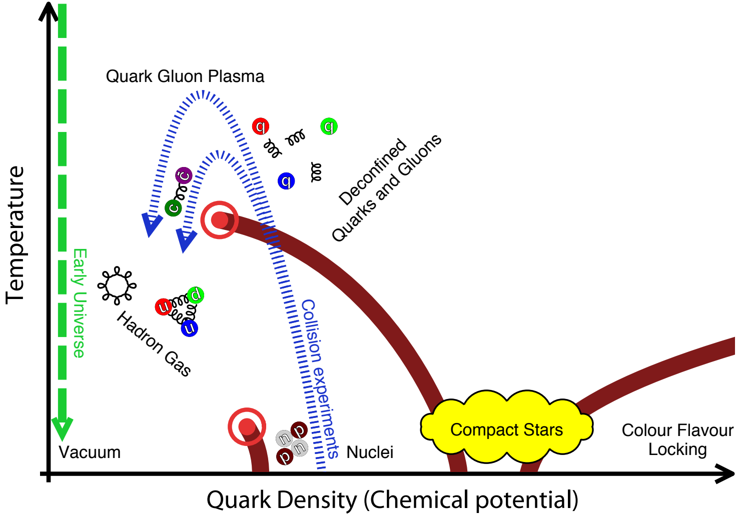


(Bulava, et al., 2010)

✓ Qualitative understanding of experimental spectrum

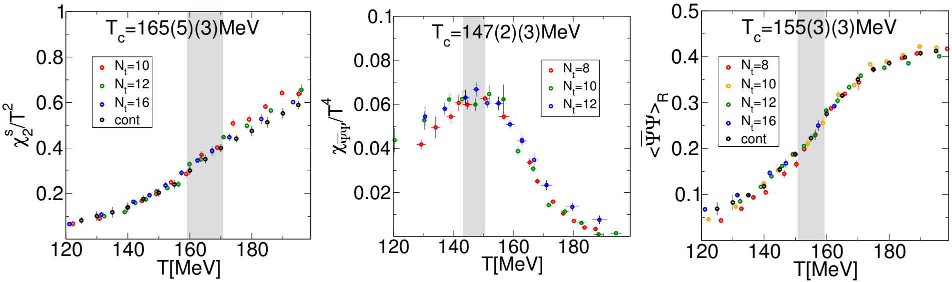
✗ No extrapolation to physical point, continuum

QCD phase diagram



QCD at $\mu = 0$

- No phase transition but crossover (Aoki et. al. '06)
  - At physical quark masses
- Spread in pseudocritical temperatures for different observables (Borsanyi et. al. '10)



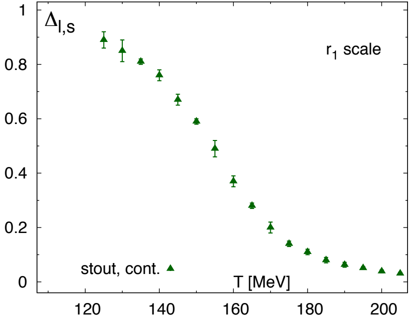
Transition temperature

Discrepancies in T_c

- Cheng et. al. '06:
  - Unique T_c
  - $T_c = 192(7)(4)$MeV
- Aoki et. al. '06:
  - Spread in T_c
  - $T_c(\chi_{\bar{\psi}\psi}) = 151(3)(3)$MeV
  - $T_c(\chi_2^s) = 175(2)(4)$MeV

Has been resolved:

- Lattice artifacts
- Unphysical quark masses



(HotQCD (Bazavov et. al.) 2012)

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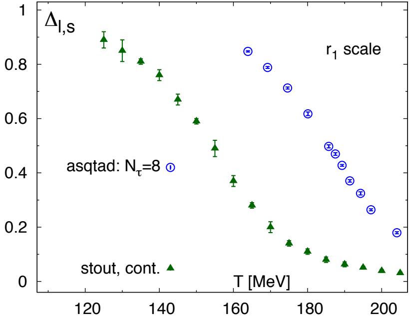

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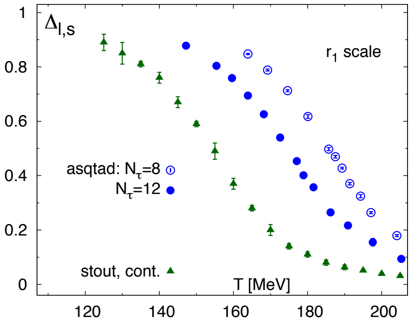

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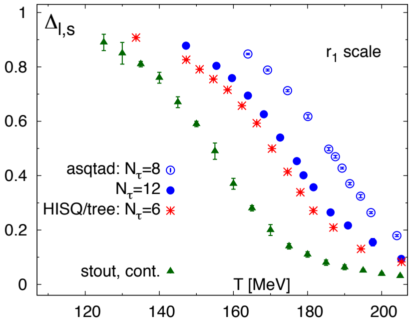

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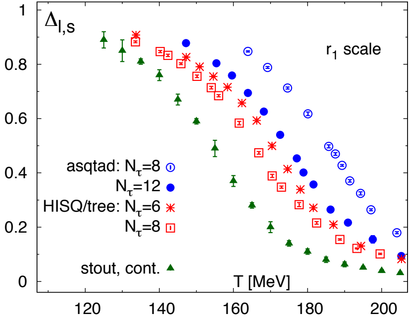

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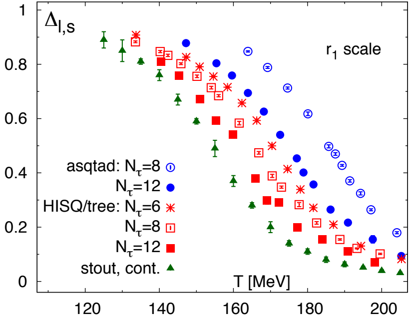

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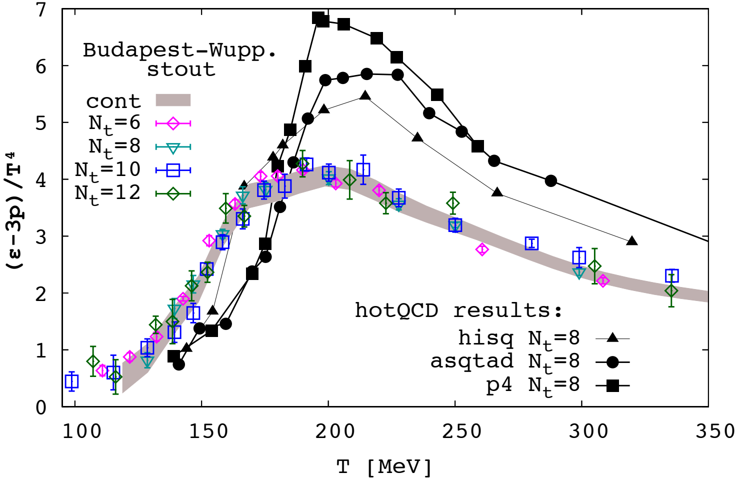
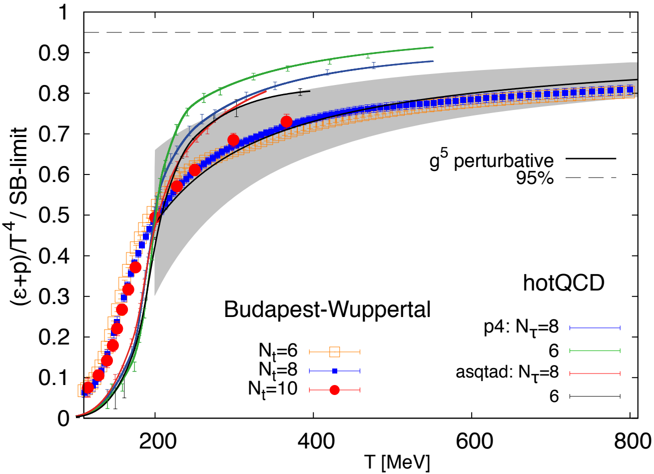
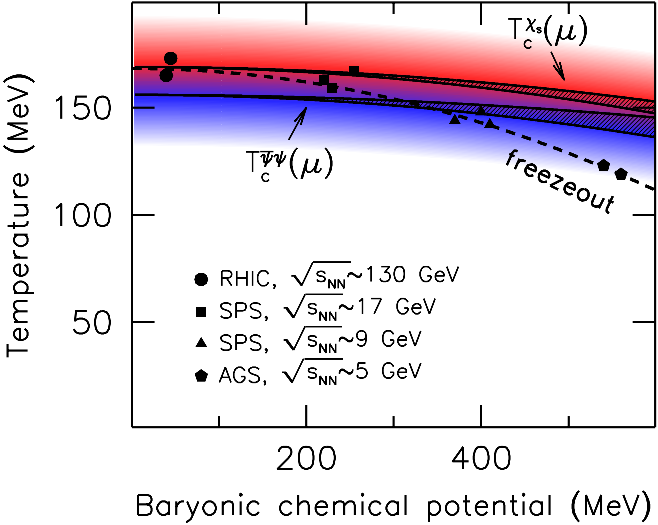
QCD equation of state at $\mu = 0$



Entropy at $\mu = 0$



Curvature of the transition line at $\mu = 0$



(Endrodi et.al., 2011)

Light quark masses

Goal:

- Compute light quark masses ab initio

Method:

- Go to the physical point
- Read off input quark masses and renormalize

Challenge:

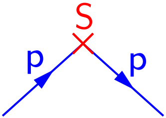
- Minimize and control all systematics
  - 2+1 dynamical fermion flavors
  - Physical quark masses
  - Continuum extrapolation
  - Infinite volume
  - Nonperturbative renormalization

Renormalization

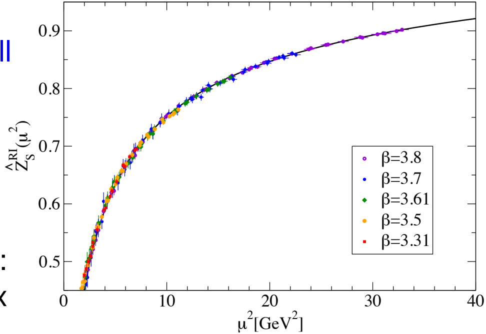
- Quark masses logarithmically divergent ($a \to 0$) ➛ renormalization
- Usually $\overline{\text{MS}}$ scheme: only perturbatively defined

☞ RI-MOM scheme

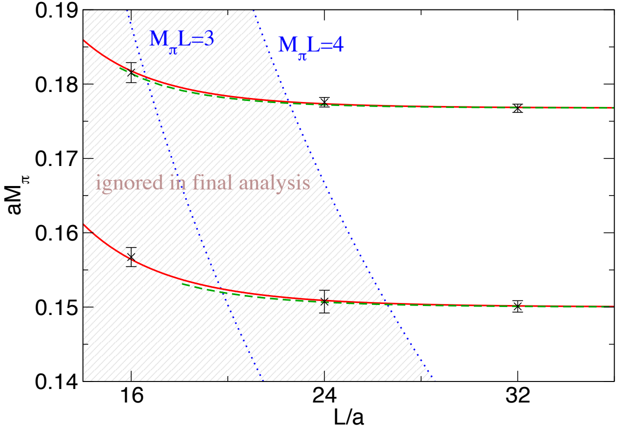
- matrix elements of off-shell quarks in fixed gauge



- Renormalization condition: at $p^2 = \mu^2$ tree level matrix element

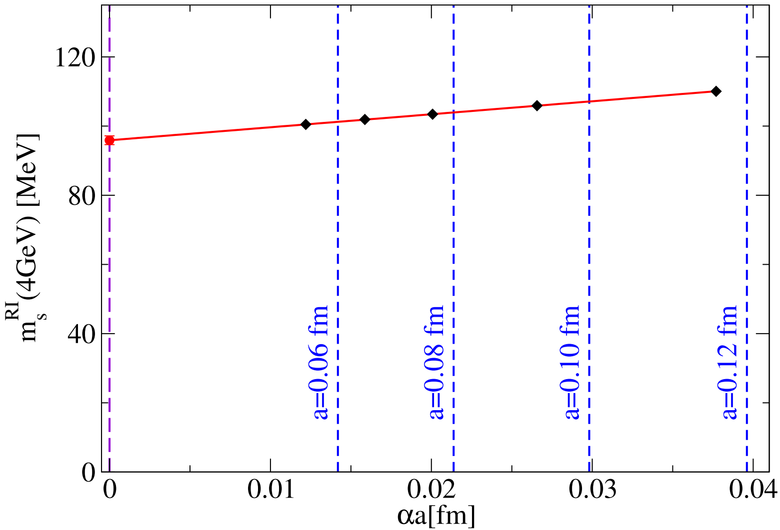


Tiny finite volume effects

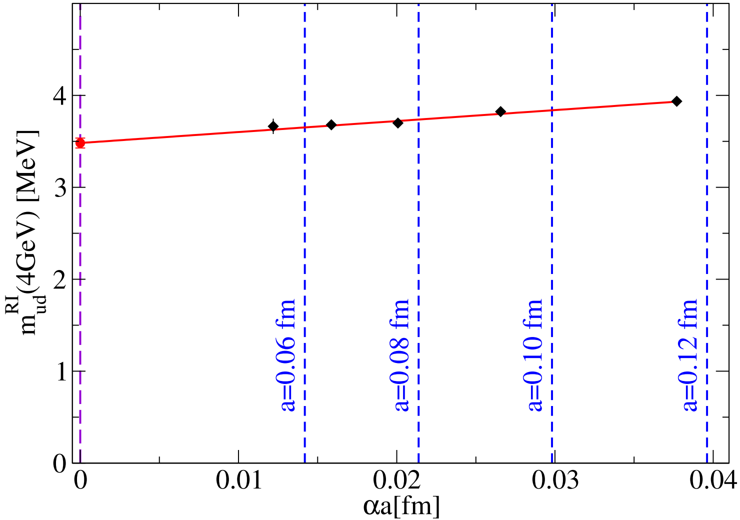


- FV effects tiny
- Dedicated FV runs
- Perfect agreement with FV χPT (Colangelo et. al. 2005)

Strange quark mass



Light quark masses



Individual m_u and m_d

- Goal:
  - Compute m_u and m_d separately
- Method:
  - Need QED and isospin breaking effects in principle
  - Alternative: use dispersive input -Q from $\eta \to \pi\pi\pi$

    $Q^2 = \frac{1}{2}\left(\frac{m_s}{m_{ud}}\right)^2 \frac{m_d - m_u}{m_{ud}}$
  - ✓ Transform precise m_s/m_{ud} into $(m_d - m_u)/m_{ud}$
  - We use the conservative $Q = 22.3(8)$ (Leutwyler, 2009)

Systematic error treatment

- Goal:
  - Reliably estimate total systematic error
- Method:
  - 288 full analyses (2000 bootstrap on each)
    - 2 plateaux regions
    - 2 continuum forms: $O(\alpha_s a)$, $O(a^2)$
    - 3 chiral forms: $2 \times SU(2)$, Taylor
    - 2 chiral ranges: $M_\pi < 340, 380$ MeV
    - 3 renormalization matching procedures
    - 2 NP continuum running forms
    - 2 scale setting procedures
  - All analyses weighted by fit quality
    - Mean gives final result
    - Stdev gives systematic error
  - Statistical error from 2000 bootstrap samples

Final result

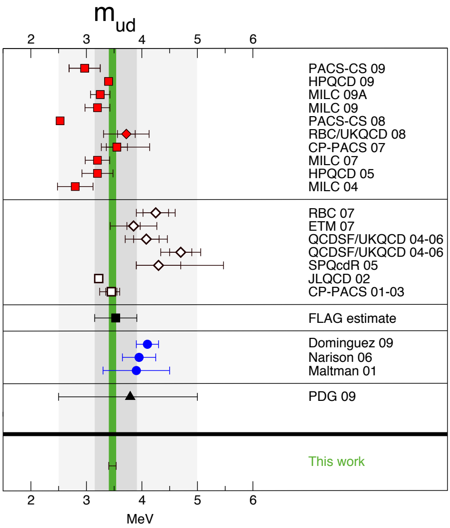
	RI @ 4 GeV	RGI	$\overline{\text{MS}}$ @ 2 GeV
m_s	96.4(1.1)(1.5)	127.3(1.5)(1.9)	95.5(1.1)(1.5)
m_{ud}	3.503(48)(49)	4.624(63)(64)	3.469(47)(48)
m_s/m_{ud}		27.53(20)(8)	
m_u	2.17(04)(10)	2.86(05)(13)	2.15(03)(10)
m_d	4.84(07)(12)	6.39(09)(15)	4.79(07)(12)

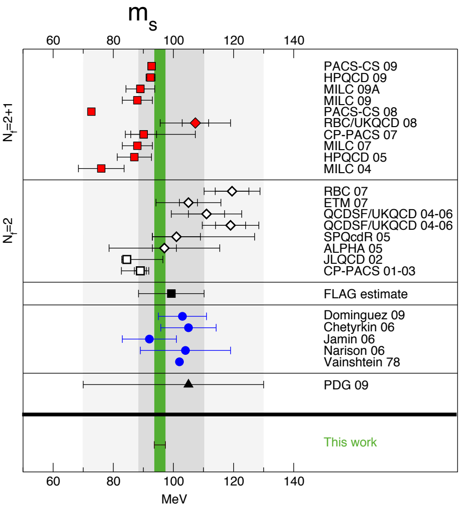
Relative error budget:

	stat.	plateau	scale set	mass	renorm.	cont.
m_s	0.702	0.148	0.004	0.064	0.061	0.691
m_{ud}	0.620	0.259	0.027	0.125	0.063	0.727
m_s/m_{ud}	0.921	0.200	0.078	0.125	—	0.301

(JHEP 1108:148,2011; PLB 701:265,2011)

Comparison


m_{ud}
2 3 4 5 6
PACS-CS 09
HPQCD 09
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS 07
MILC 07
HPQCD 05
MILC 04
RBC 07
ETM 07
QCDSF/UKQCD 04-06
QCDSF/UKQCD 04-06
SPQcdR 05
JLQCD 02
CP-PACS 01-03
FLAG estimate
Dominguez 09
Narison 06
Maltman 01
PDG 09
This work
MeV


m_s
60 80 100 120 140
N_f=2+1
N_f=2
PACS-CS 09
HPQCD 09
MILC 09A
MILC 09
PACS-CS 08
RBC/UKQCD 08
CP-PACS 07
MILC 07
HPQCD 05
MILC 04
RBC 07
ETM 07
QCDSF/UKQCD 04-06
QCDSF/UKQCD 04-06
SPQcdR 05
ALPHA 05
JLQCD 02
CP-PACS 01-03
FLAG estimate
Dominguez 09
Chetyrkin 06
Jamin 06
Narison 06
Vainshtein 78
PDG 09
This work
MeV

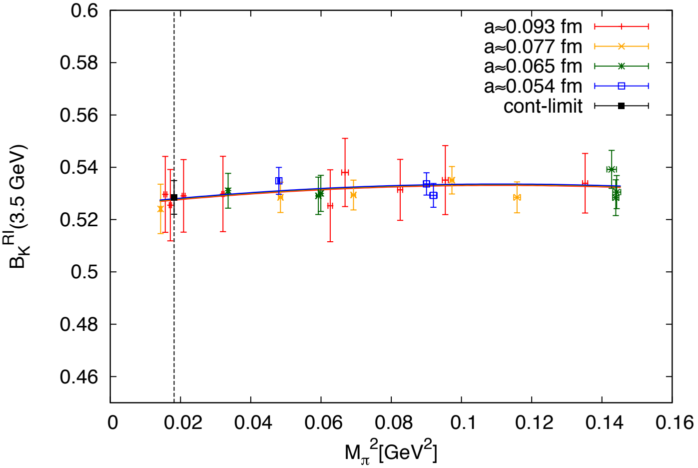
Future perspective

- Goal:
  - Decrease precision below percent level
- Method:
  - Take light masses non-degenerate
  - Include QED effects
  - Include charm quark effects
- Challenge:
  - Stability near $m_u < m_{ud}$
  - Finding the physical point
  - Non gauge invariant final states
  - Quark mass ratios renormalize

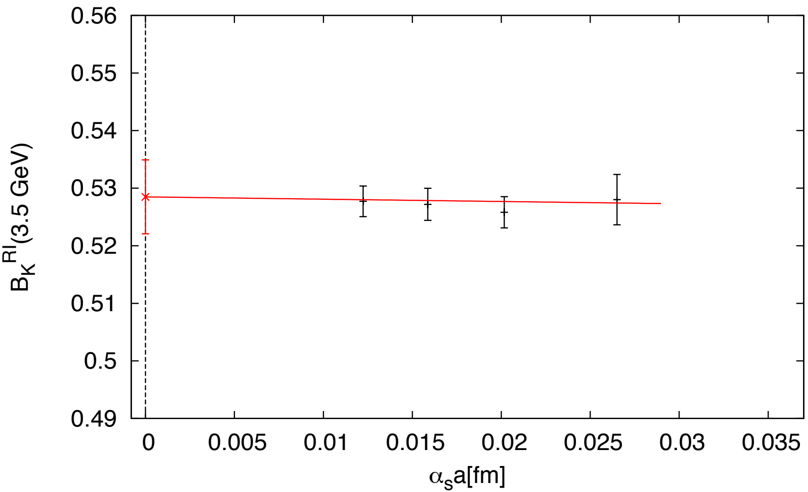
Standard model neutral K mixing

- Goal:
  - Check SM CP violation in neutral K system
- Method:
  - Compute effective weak matrix element $B_K = \frac{3\langle \bar{K}|O_{\Delta S=2}|K\rangle}{8 f_K^2 M_K^2}$
  - Relate kaon CP violation to CKM phase
    ➛from CKM unitarity: $\hat{B}_K = 0.83^{+0.21}_{-0.15}$ (CKMfitter, 2011)
- Challenge:
  - Minimize and control all systematics
    - 2+1 dynamical fermion flavors
    - Physical quark masses
    - Mixing of unphysical operators
    - Continuum
    - Infinite volume

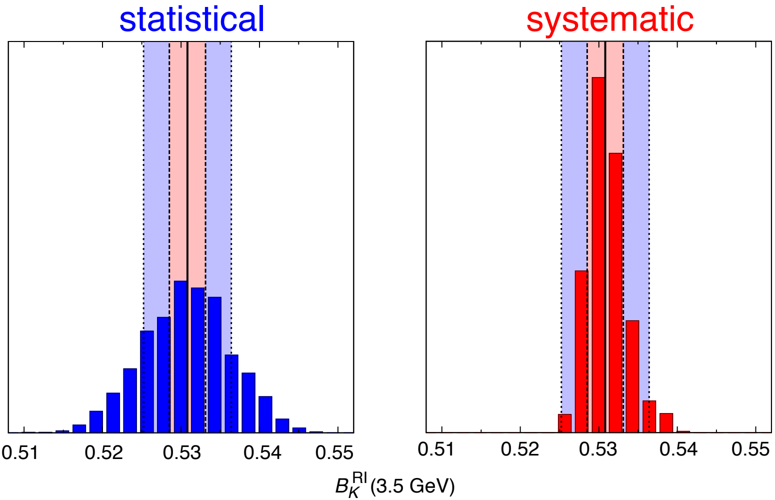
Physical point



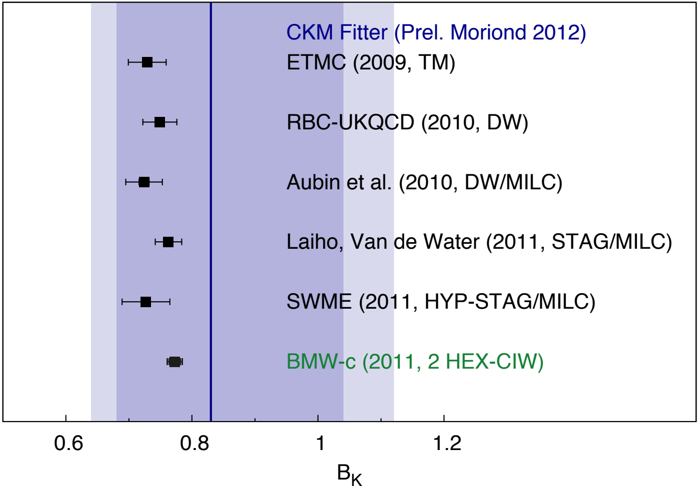
Continuum extrapolation



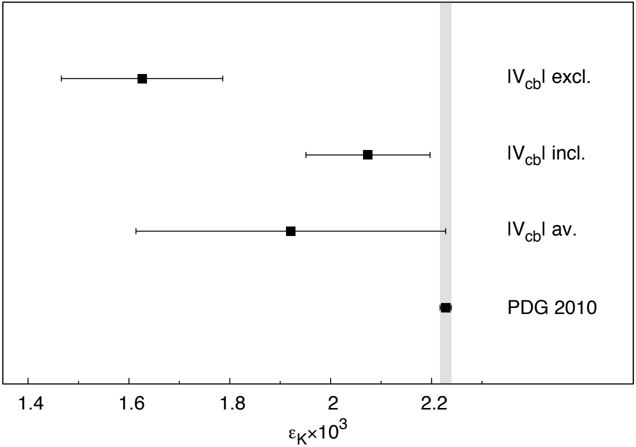
Errors



Comparison


CKM Fitter (Prel. Moriond 2012)
ETMC (2009, TM)
RBC-UKQCD (2010, DW)
Aubin et al. (2010, DW/MILC)
Laiho, Van de Water (2011, STAG/MILC)
SWME (2011, HYP-STAG/MILC)
BMW-c (2011, 2 HEX-CIW)
0.6
0.8
1
1.2
B_K

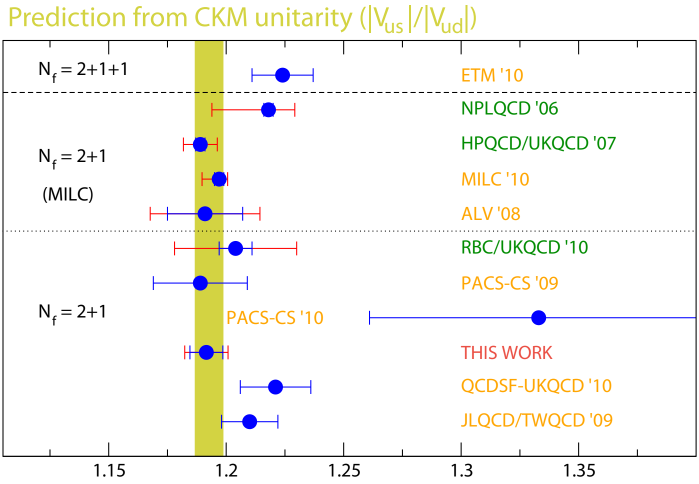
Implications

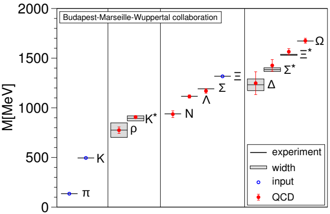


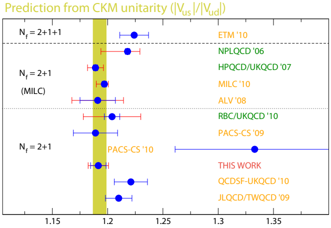
Pseudoscalar decay constant ratio

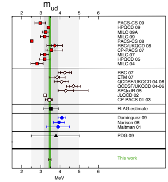
- Goal:
  - Check first row unitarity of CKM matrix
- Method:
  - Compute F_K/F_π
  - Perturbative relation to $|V_{us}|^2/|V_{ud}|^2$ with 0.4% accuracy
- Challenge:
  - Minimize and control all systematics
    - 2+1 dynamical fermion flavors
    - Physical quark masses
    - Continuum
    - Infinite volume

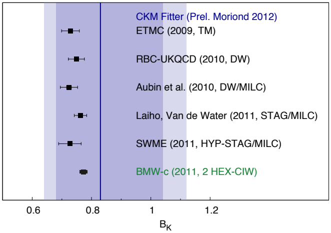
Decay constant ratio




Budapest-Marseille-Wuppertal collaboration
M[MeV]
2000
1500
1000
500
0
π
K
ρ
K*
N
Λ
Σ
Ξ
Δ
Σ*
Ξ*
Ω
experiment
width
input
QCD


Prediction from CKM unitarity (|V_us|/|V_ud|)
N_f = 2+1+1
N_f = 2+1
(MILC)
N_f = 2+1
ETM '10
NPLQCD '06
HPQCD/UKQCD '07
MILC '10
ALV '08
RBC/UKQCD '10
PACS-CS '09
PACS-CS '10
THIS WORK
QCDSF-UKQCD '10
JLQCD/TWQCD '09
1.15
1.2
1.25
1.3
1.35


m_ud
MeV
FLAG estimate
PDG 09
This work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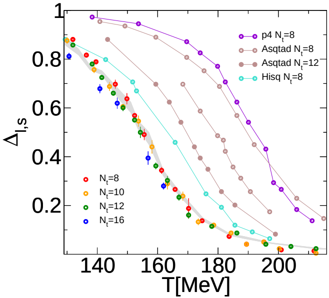
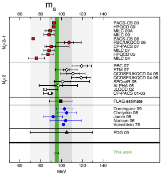
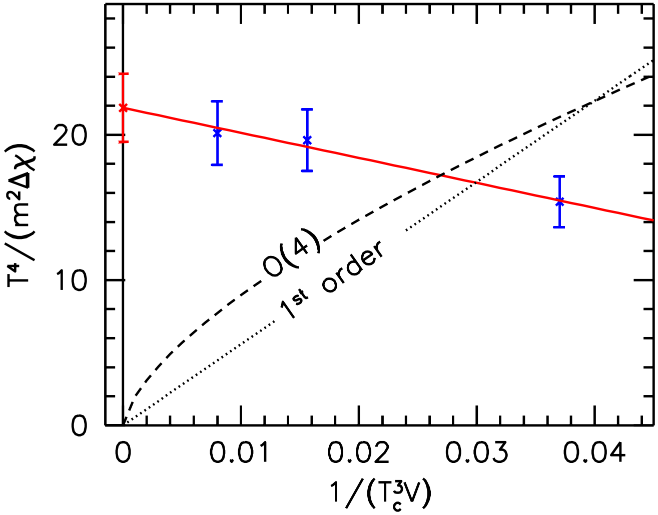

Δ_l,s
1
0.8
0.6
0.4
0.2
140
160
180
200
T[MeV]
p4 N_t=8
Asqtad N_t=8
Asqtad N_t=12
Hisq N_t=8
N_t=8
N_t=10
N_t=12
N_t=16

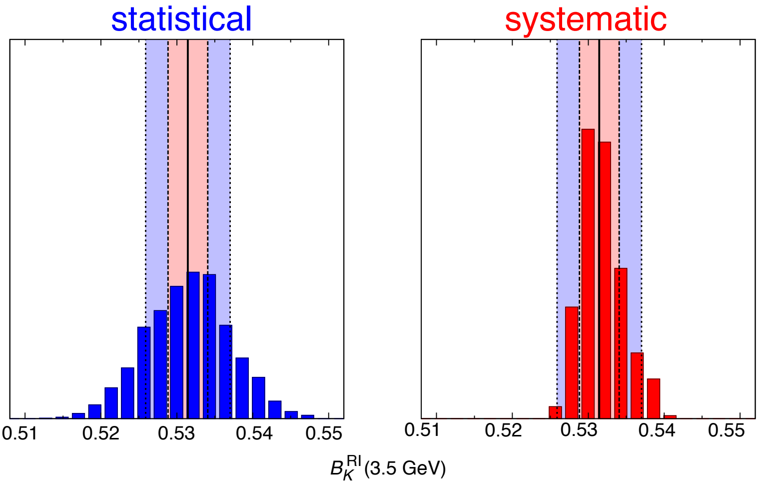

m_s
MeV
FLAG estimate
PDG 09
This work

BACKUP

Order of QCD phase transition



Errors - unit weight



Action details

Goal:

- Optimize physics results per CPU time
- Conceptually clean formulation

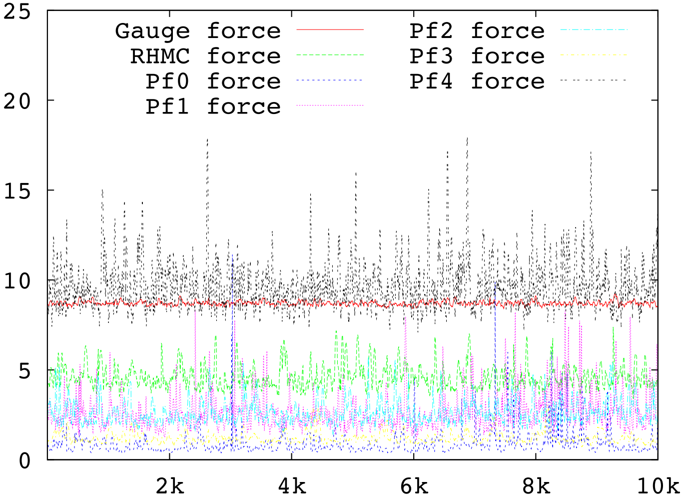
Method:

- Dynamical $2+1$ flavor, Wilson fermions at physical M_π
- 3-5 lattice spacings $0.053\text{ fm} < a < 0.125\text{ fm}$
- Tree level $O(a^2)$ improved gauge action (Lüscher, Weisz, 1985)
- Tree level $O(a)$ improved fermion action (Sheikholeslami, Wohlert, 1985)
  - Why not go beyond tree level?
    - Keeping it simple (parameter fine tuning)
    - No real improvement, UV mode suppression took care of this
  - This is a crucial advantage of our approach
- UV filtering (APE coll. 1985; Hasenfratz, Knechtli, 2001; Capitani, Durr, C.H., 2006)

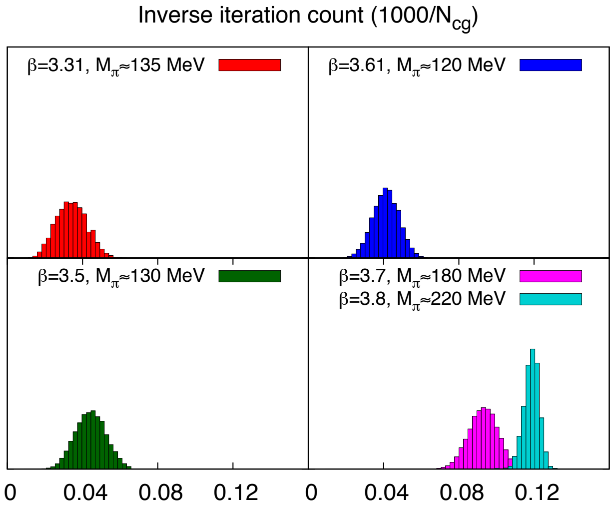
➛ Discretization effects of $O(\alpha_s a, a^2)$

✓ We include both $O(\alpha_s a)$ and $O(a^2)$ into systematic error

Algorithm stability

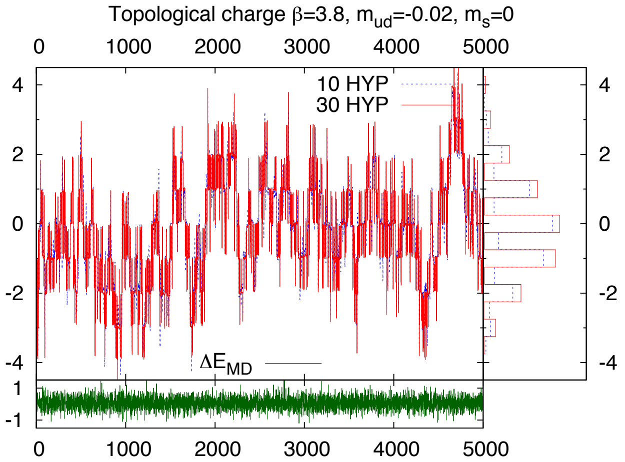


No exceptional configs



Topological sector sampling

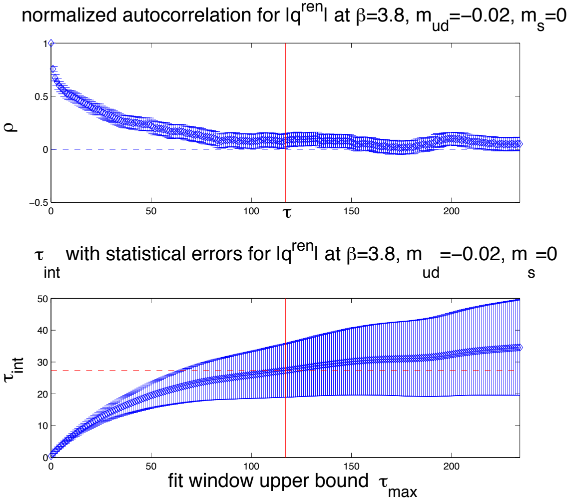
worst case



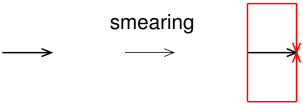
Autocorrelation time (finest lattice, small mass)

$\tau_{\text{int}} = 27.3(7.4)$

(MATLAB code from Wolff, 2004-7)



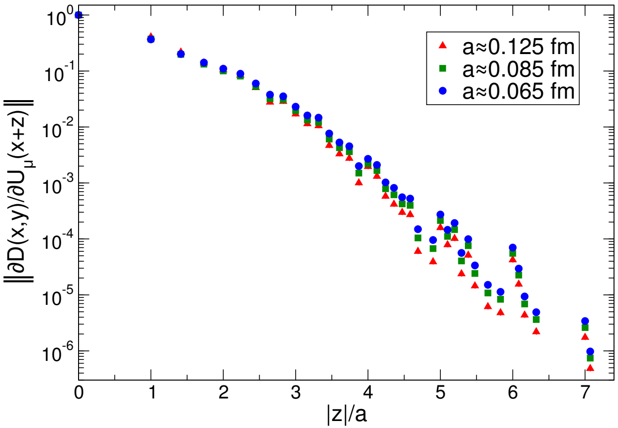
Locality properties



- locality in position space:
  $|D(x,y)| < \text{const}\, e^{-\lambda|x-y|}$ with $\lambda = O(a^{-1})$ for all couplings.
  Our case: $D(x,y) = 0$ as soon as $|x-y| > 1$
  (despite smearing)
- locality of gauge field coupling:
  $|\delta D(x,y)/\delta A(z)| < \text{const}\, e^{-\lambda|(x+y)/2-z|}$ with $\lambda = O(a^{-1})$ for all couplings.
  Our case: $\delta D(x,x)/\delta A(z) < \text{const}\, e^{-\lambda|x-z|}$ with $\lambda \simeq 2.2a^{-1}$ for $2 \leq |x-z| \leq 6$

Gauge field coupling locality

6-stout case:



Light hadron spectrum

- Goal:
  - Firmly establish (or invalidate?) QCD as the theory of strong interaction in the low energy region
- Method:
  - Post-diction of light hadron spectrum
    - Octet baryons
    - Decuplet baryons
    - Vector mesons
- Challenge:
  - Minimize and control all systematics
    - 2+1 dynamical fermion flavors
    - Physical quark masses
    - Continuum
    - Infinite volume (treatment of resonant states)

Scale setting

Goal:

- Unambiguous, precise scale setting

Method:

- We set the scale via a baryon mass
- Desirable properties:
  - experimentally well known
  - small lattice error (Octet better than Decuplet)
  - independent of light quark mass $\rightarrow$ large strange content
- Best candidates:
  - Ξ: largest strange content of the octet
  - Ω: member of the decuplet, but no light quarks

Quark mass dependence

Goal:

- Extra-/Interpolate M_X (baryon/vector meson mass) to physical point (M_π, M_K)

Method:

- Fundamental parameters: g, m_{ud}, m_s
  - Experimentally inaccessible (confinement!)
  - Must be set via 3 experimentally accessible quantities
- Use M_Ξ or M_Ω and M_π, M_K to set parameters
- Variables to parametrize M_π^2 and M_K^2 dependence of M_X:
  - Use bare masses aM_y, $y \in \{X, \pi, K\}$ and a (bootstrapped)
  - Use dimensionless ratios $r_y := \frac{M_y}{M_{\Xi/\Omega}}$ (cancellations)

We use both procedures ➛systematic error

Quark mass dependence (ctd.)

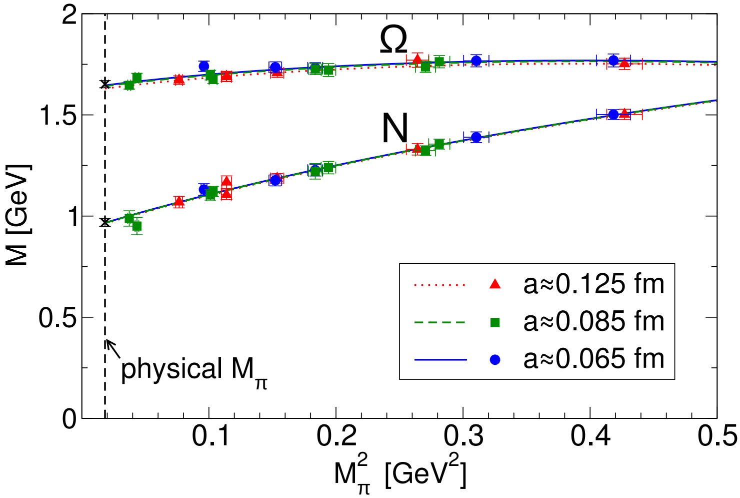
Method (ctd.):

- Parametrization: $M_X = M_X^{(0)} + \alpha M_\pi^2 + \beta M_K^2 +$ higher orders
  - Leading order sufficinet for M_K^2 dependence
  - We include higher order term in M_π^2
    - Next order χPT (around $M_\pi^2 = 0$): $\propto M_\pi^3$
    - Taylor expansion (around $M_\pi^2 \neq 0$): $\propto M_\pi^4$

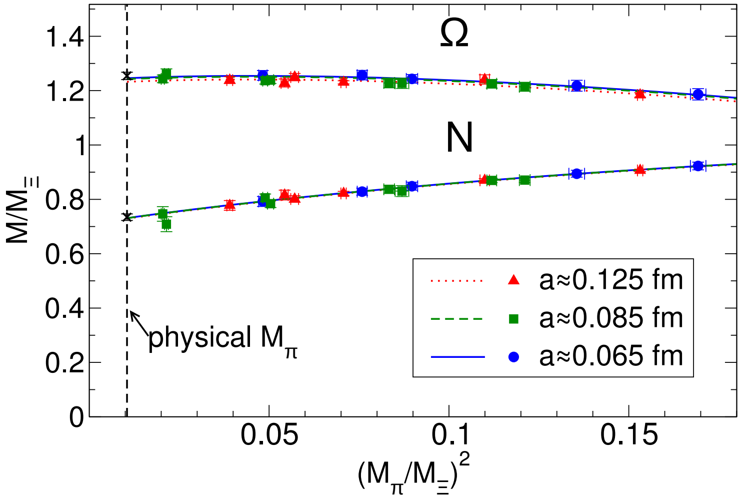
  Both procedures fine ➛systematic error
  No sensitivity to any order beyond these
- Vector mesons: higher orders not significant
- Baryons: higher orders significant
  - Restrict fit range to further estimate systematics:
    - full range, $M_\pi < 550/450$MeV

  We use all 3 ranges ➛systematic error

Chiral fit


Ω
N
M [GeV]
M_π^2 [GeV^2]
physical M_π
a≈0.125 fm
a≈0.085 fm
a≈0.065 fm
0
0.5
1
1.5
2
0.1
0.2
0.3
0.4
0.5

Chiral fit using ratios



Continuum extrapolation

Goal:

- Eliminate discretization effects

Method:

- Formally in our action: $O(\alpha_s a)$ and $O(a^2)$
- Discretization effects are tiny
  - Not possible to distinguish between $O(a)$ and $O(a^2)$

➛include both in systematic error

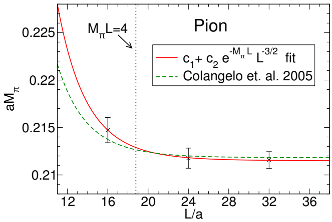
Finite volume effects from virtual pions

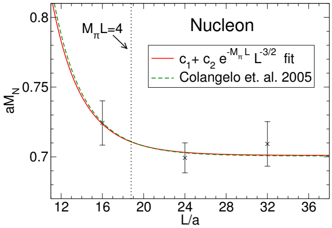
Goal:

- Eliminate virtual pion finite V effects
  - Hadrons see mirror charges
  - Exponential in lightest particle (pion) mass

Method:

- Best practice: use large V
  - Rule of thumb: $M_\pi L \gtrsim 4$
  - Leading effects $\frac{M_X(L)-M_X}{M_X} = cM_\pi^{1/2} L^{-3/2} e^{M_\pi L}$ (Colangelo et. al., 2005)





Finite volume effects in resonances

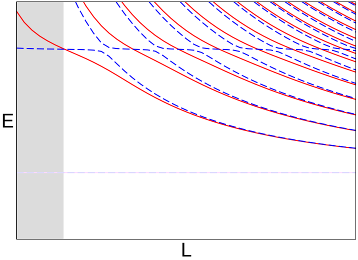
Goal:

- Eliminate spectrum distortions from resonances mixing with scattering states

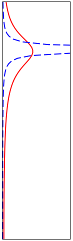
Method:

- Stay in region where resonance is ground state
    - Otherwise no sensitivity to resonance mass in ground state

Finite volume energy levels



Spectral density



- Treatment as scattering problem

  (Lüscher, 1985-1991)
    - Parameters: mass and coupling (width)
    - Alternative approaches suggested

Systematic uncertainties

Goal:

- Accurately estimate total systematic error

Method:

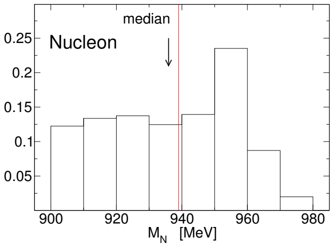
- We account for all the above mentioned effects
- When there are a number of sensible ways to proceed, we take them: Complete analysis for each of
  - 18 fit range combinations
  - ratio/nonratio fits (r_X resp. M_X)
  - $O(a)$ and $O(a^2)$ discretization terms
  - NLO χPT M_π^3 and Taylor M_π^4 chiral fit
  - 3 χ fit ranges for baryons: $M_\pi < 650/550/450$ MeV

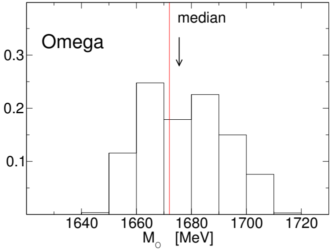
resulting in 432 (144) predictions for each baryon (vector meson) mass with each 2000 bootstrap samples for each Ξ and Ω scale setting

Systematic uncertainties II

Method (ctd.):

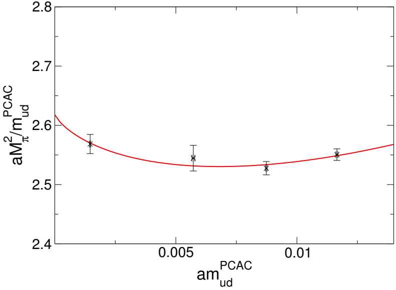
- Weigh each of the 432 (144) central values by fit quality Q
  - Median of this distribution ➛final result
  - Central 68% ➛systematic error
- Statistical error from bootstrap of the medians

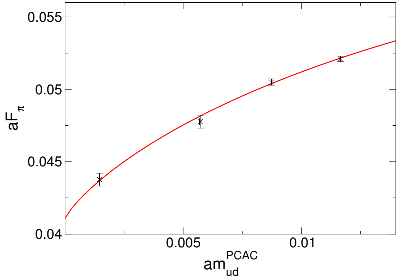




Chiral interpolation

- Simultaneous fit to NLO $SU(2)$ χPT (Gasser, Leutwyler, 1984)
- Consistent for $M_\pi \lesssim 400$ MeV



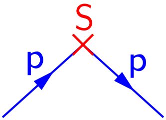


➢ We use 2 safe chiral interpolation ranges: $M_\pi < 340, 380$ MeV

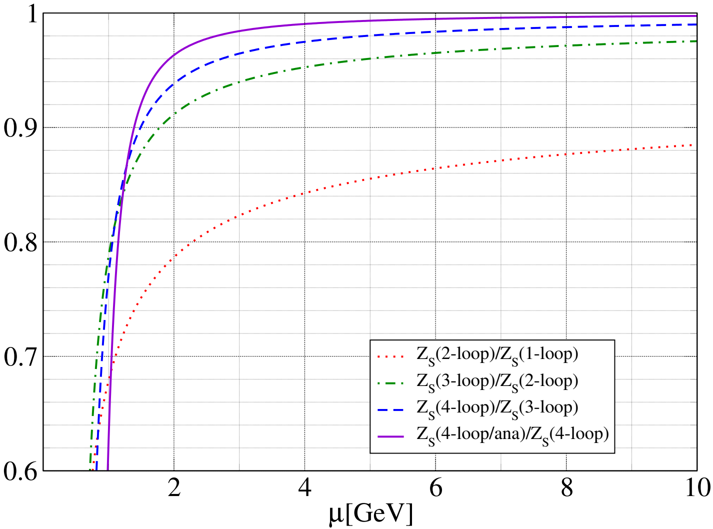
➢ We use $SU(2)$ χPT and Taylor interpolation forms

Renormalization strategy

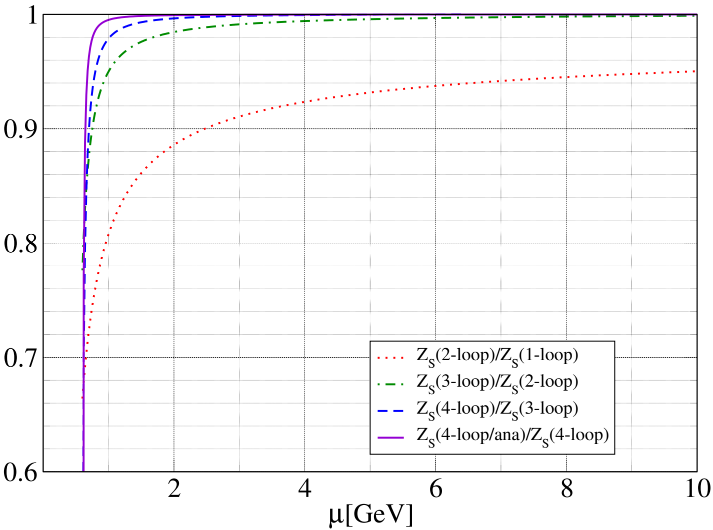
- Goal:
  - Full nonperturbative renormalization
  - Optional accurate conversion to perturbative scheme
- Method:
  - We use RI-MOM scheme (Martinelli et. al., 1993)
    - $O(a)$ correction (Maillart, Niedermayer, 2008)
  - Compute m_q at low scale $\mu \ll 2\pi/a \sim 11 - 24$ GeV
    - $\mu = 2.1 GeV$
    - $\mu = 1.3 GeV$
  - Do continuum non-perturbative running to high scale $\mu' \gg \Lambda_{QCD}$
  - Further conversion in 4-loop PT



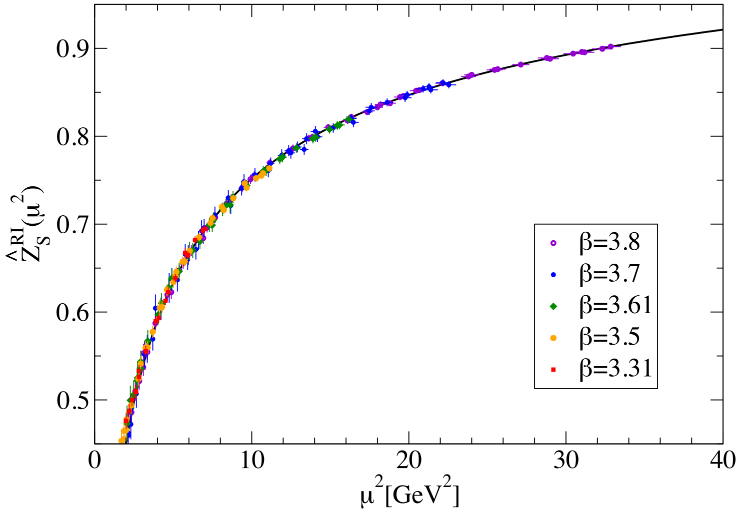
Desired scale in RI-MOM scheme



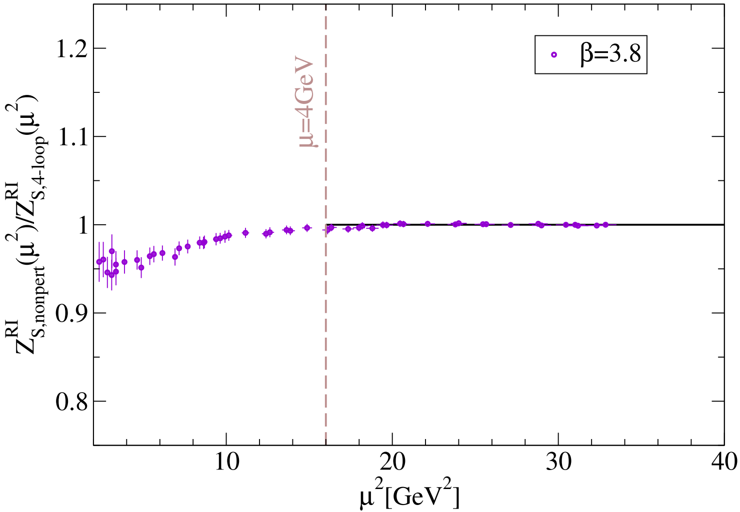
Optional conversion to $\overline{\text{MS}}$



Nonperturbative running



Reaching the perturbative regime



Quark mass definitions

- Lagrangian mass m^{bare}
- $m^{\text{ren}} = \frac{1}{Z_S}(m^{\text{bare}} - m^{\text{bare}}_{\text{crit}})$

- m^{PCAC} from $\frac{\langle \partial_0 A_0(t) P(0)\rangle}{\langle P(t) P(0)\rangle}$
- $m^{\text{ren}} = \frac{Z_A}{Z_P} m^{\text{PCAC}}$

Better use

- $d = m_s^{\text{bare}} - m_{ud}^{\text{bare}}$
- $d^{\text{ren}} = \frac{1}{Z_S} d$

- $r = m_s^{\text{PCAC}} / m_{ud}^{\text{PCAC}}$
- $r^{\text{ren}} = r$

and reconstruct

- $m_s^{\text{ren}} = \frac{1}{Z_S} \frac{rd}{r-1}$
- $m_{ud}^{\text{ren}} = \frac{1}{Z_S} \frac{d}{r-1}$

✓ No additive mass renormalization and ambiguity in m_{crit}

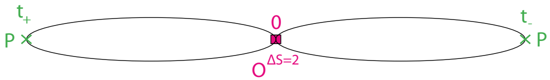
✓ Only Z_S multiplicative renormalization (no pion poles)

☞ Works with $O(a)$ improvement (we use this)

Observable

Matrix element of the effective weak operator $\langle \bar{K}|O|K\rangle$:

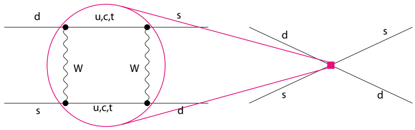
$$\langle P^\dagger(t_+)O(0)P(t_-)\rangle \stackrel{t_\pm\to\pm\infty}{\longrightarrow} \frac{|\langle K|P|0\rangle|^2}{(2M_K)^2}\langle\bar{K}|O|K\rangle e^{-M_K(t_+-t_-)}$$



where

$$P = [\bar{s}d]_P = \bar{s}\gamma_5 d$$

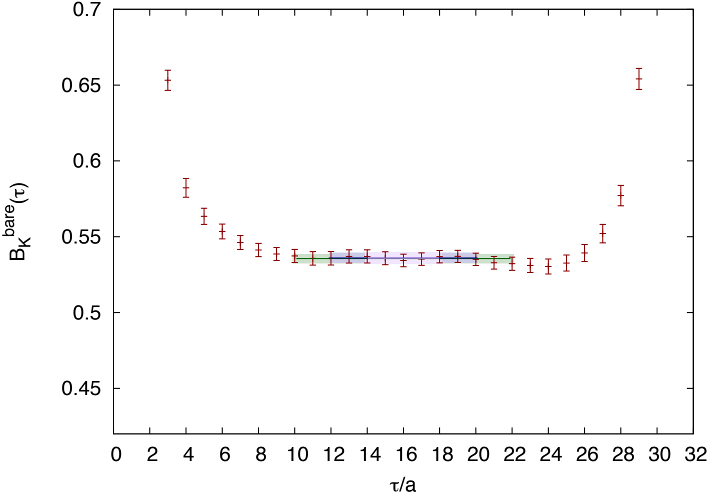
$$O_{\Delta S=2} = [\bar{s}d]_{V-A}[\bar{s}d]_{V-A}$$



Norm from:

$$\langle P^\dagger(t)P(0)\rangle \stackrel{t\to\infty}{\longrightarrow} \frac{|\langle K|P|0\rangle|^2}{2M_K}e^{-M_K t}$$

Signal


0.7
0.65
0.6
0.55
0.5
0.45
$B_K^{bare}(\tau)$
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
τ/a

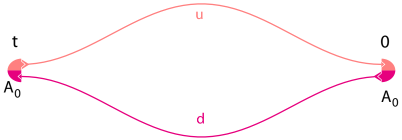
Observable

With the axial vector current

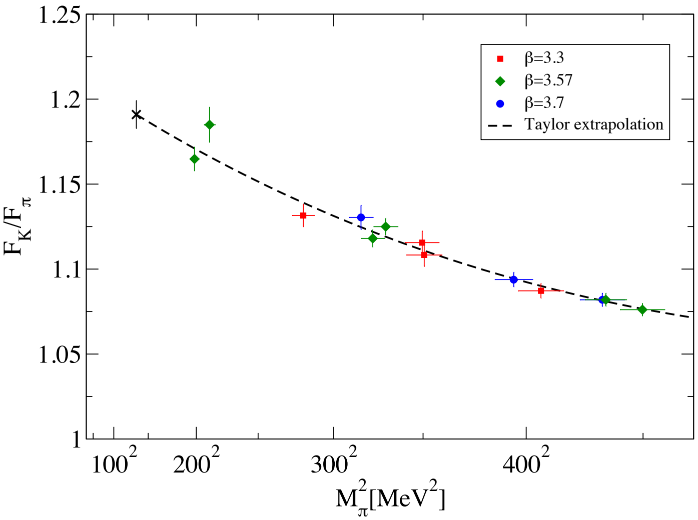
$$A_\mu(t) = \sum_{\vec{x}} \left(\bar{\Psi}^d \gamma_\mu \gamma_5 \Psi^u\right)(\vec{x}, t)$$

one obtains

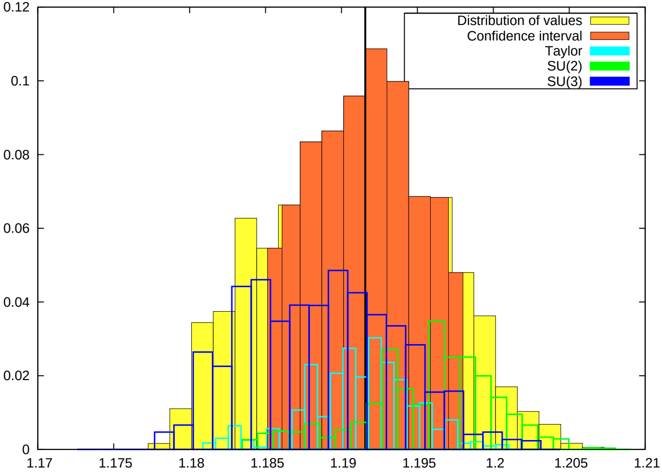
$$\langle A_0^\dagger(t) A_0(0)\rangle \xrightarrow{t\to\infty} \frac{|\langle \pi | A_0 | 0\rangle|^2}{2M_\pi} e^{-M_\pi t} = \frac{M_\pi^2 {F_\pi^{\text{bare}}}^2}{2M_\pi} e^{-M_\pi t}$$



Chiral extrapolation

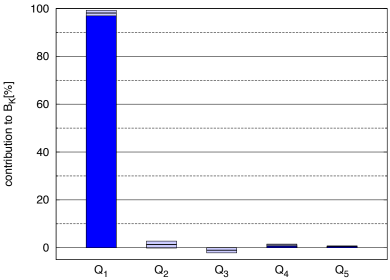


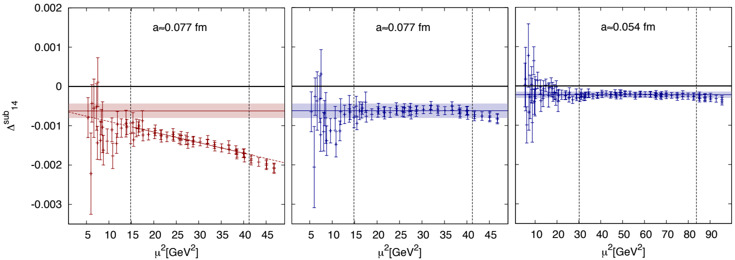
Errors


Distribution of values
Confidence interval
Taylor
SU(2)
SU(3)
0.12
0.1
0.08
0.06
0.04
0.02
0
1.17
1.175
1.18
1.185
1.19
1.195
1.2
1.205
1.21

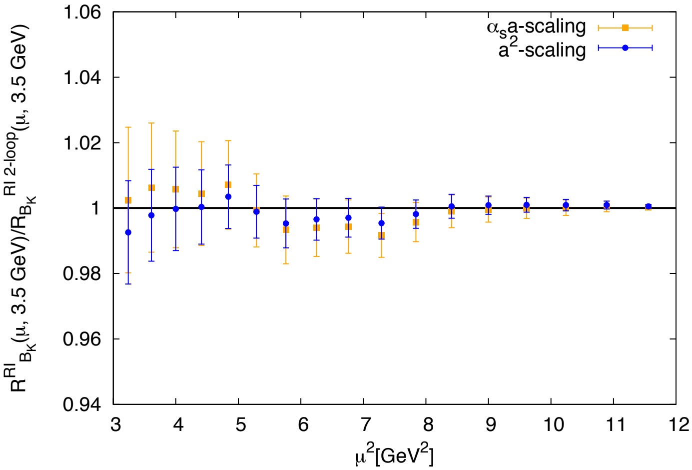
Unphysical operator mixing

- χSB induces mixing with 4 unphysical operators
- Mixing terms chirally enhanced
- ✓ Small even below physical m_π
- ✓ Good chirality of our action

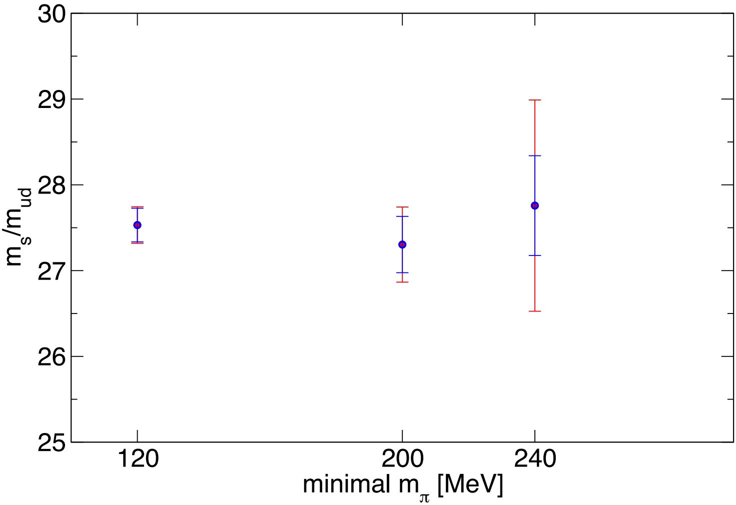




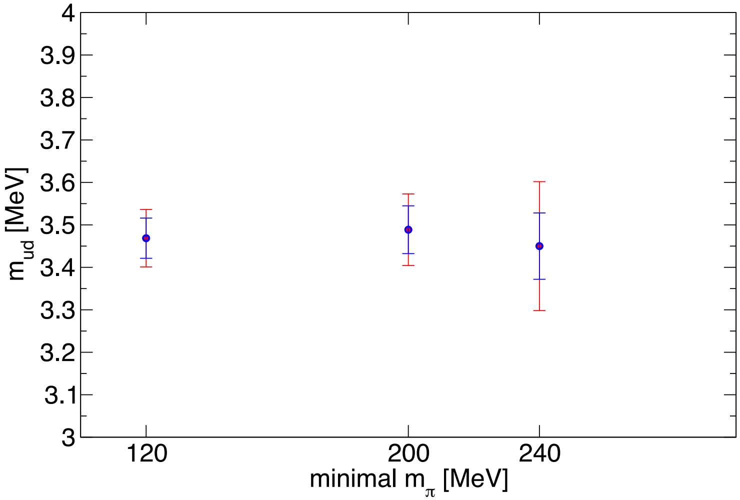
Running


1.06
1.04
1.02
1
0.98
0.96
0.94
3
4
5
6
7
8
9
10
11
12
α_sa-scaling
a^2-scaling
$R^{RI}_{B_K}(\mu, 3.5\text{ GeV})/R_{B_K}^{RI\text{ 2-loop}}(\mu, 3.5\text{ GeV})$
$\mu^2[\text{GeV}^2]$

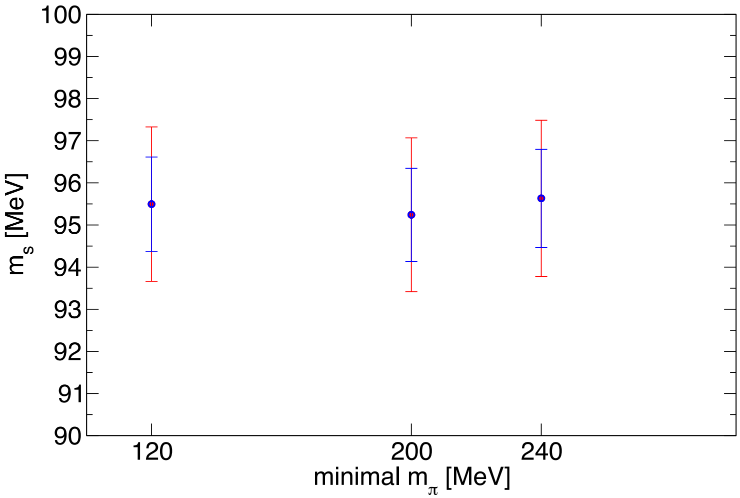
Chiral cuts



Chiral cuts



Chiral cuts


100
99
98
97
96
95
94
93
92
91
90
m_s [MeV]
120
200
240
minimal m_π [MeV]