Detecting critical fluctuations of the proton density in ion collisions at the NA49 experiment through improved intermittency analysis

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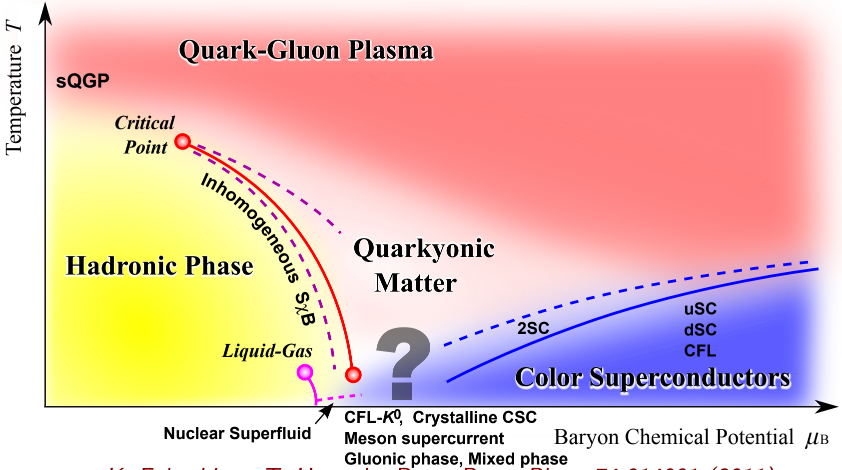
Phase diagram of QCD

- Objective: Detection / existence of the QCD Critical Point (CP)



K. Fukushima, T. Hatsuda, Rept. Prog. Phys. 74:014001 (2011)

Observables for critical fluctuations

- Detection of "chiral" critical point (CP) $\Rightarrow$ critical fluctuations of the order parameter
- Order parameter = "chiral" condensate

$$\sigma(x) = \langle \bar{q}(x) q(x) \rangle$$

($q(x)$ = quark field, sigma-field $\sigma(x)$=quantum state (wave function) describing the "chiral" condensate)

- In medium (finite baryon density) sigma-field mixes with net baryon density
- (Critical) fluctuations of the sigma field transferred to the net baryon density
- Look for observables tailored for CP search in ion collisions. Scan the phase diagram for the existence and location of the CP by varying the energy and size of the collision system.

Self-similar density fluctuations

- In our analysis, we use local observables $\Rightarrow$ not sensitive to experimental acceptance, contrary to global observables.
- Local observable $\Rightarrow$ self-similar density fluctuations of the order parameter in transverse configuration space (random fractal) $\Rightarrow$

Power-law dependence of the density-density correlation functions in transverse momentum space	$\Leftrightarrow$	Intermittency analysis (critical opalescence, correlation length vs. size)

[F.K. D., N.G. Antoniou and G. Mavromanolakis, PoS(CPOD2006)010, Florence]

Critical exponents

- Power-law exponents are determined by universality class (critical exponents). For 3-D Ising:

  $$\langle n_\sigma(k) n_\sigma(k') \rangle \sim |k - k'|^{-4/3} \quad ; \quad \langle n_B(k) n_B(k') \rangle \sim |k - k'|^{-5/3}$$

  $n_\sigma(k) = \sigma^2(k)$,
  n_B = net baryon density at midrapidity,
  k, k' are transverse momenta.

- The coupling of the (isospin zero) σ-field with protons transfers critical fluctuations to the net proton density

  [Y. Hatta and M. A. Stephanov, PRL **91**, 102003 (2003).]

Observing power-law fluctuations

Experimental observation of local, power-law distributed fluctuations

$\Downarrow$

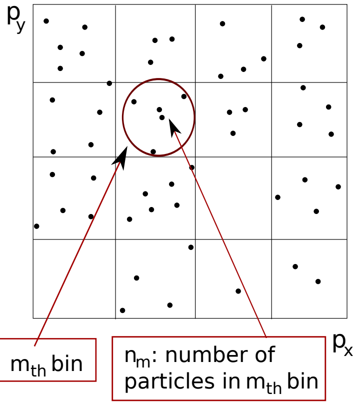
Intermittency in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions)

- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size $\Leftrightarrow$ number of cells M:

$$F_2(M) \equiv \frac{\sum_m \langle n_m(n_m-1)\rangle}{\sum_m \langle n_m\rangle^2},$$

where $\langle \ldots \rangle$ denotes averaging over events.



Scaling of factorial moments – Subtracting mixed events

Factorial moments of mixed events must be subtracted from raw events in order to remove background correlations:

$$\Delta F_2(M) = F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

For a critical system, ΔF_2 scales with cell size (number of cells, M) as:

$$\Delta F_2(M) \sim \left(M^2\right)^{\varphi_2}$$

where φ_2 is the intermittency index.

Theoretical predictions for φ_2

universality class, effective actions

$$\varphi_{2,cr}^{(\sigma)} = \frac{2}{3}\ (0.66\ldots)$$

sigmas (neutral isoscalar dipions)

[N. G. Antoniou **et al**, Nucl. Phys. A **693**, 799 (2001)]

$$\varphi_{2,cr}^{(p)} = \frac{5}{6}\ (0.833\ldots)$$

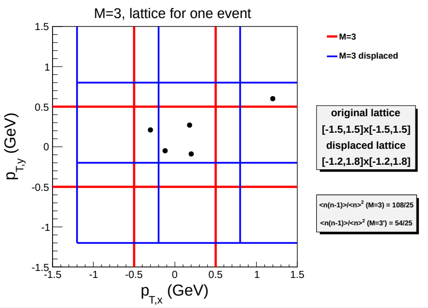
net baryons (protons)

[N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis, K. S. Kousouris, Phys. Rev. Lett. **97**, 032002 (2006)]

Improving calculation of $F_2(M)$ via lattice averaging

- Problem: With low statistics/multiplicity, lattice boundaries may split pairs of neighboring points, affecting $F_2(M)$ values (see example below).
- Solution: Calculate moments several times on different, slightly displaced lattices (see example)
- Average corresponding $F_2(M)$ over all lattices. Errors can be estimated by variance over lattice positions.
- Lattice displacement is larger than experimental resolution, yet maximum displacement must be of the order of the finer binnings, so as to stay in the correct p_T range.

Displaced lattice — a simple example



Improved confidence intervals for ϕ_2 via resampling

- In order to estimate the statistical errors of $\Delta F_2(M)$, we need to produce variations of the original event sample. This, we can achieve by using the statistical method of resampling (bootstrapping) $\Rightarrow$
  - Sample original events with replacement, producing new sets of the same statistics (# of events)
  - Calculate $\Delta F_2(M)$ for each bootstrap sample in the same manner as for the original.
  - The variance of sample values provides the statistical error of $\Delta F_2(M)$.

[W.J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).]

- Furthermore, we can obtain a distribution $P(\varphi_2)$ of φ_2 values. Each bootstrap sample of $\Delta F_2(M)$ is fit with a power-law:

$$\Delta F_2(M;\lambda,\varphi_2) = e^{\lambda}\cdot(M^2)^{\varphi_2}$$

and we can extract a confidence interval for φ_2 from the distribution of values. [B. Efron, *The Annals of Statistics* **7**,1 (1979)]

Analysed data sets & cuts

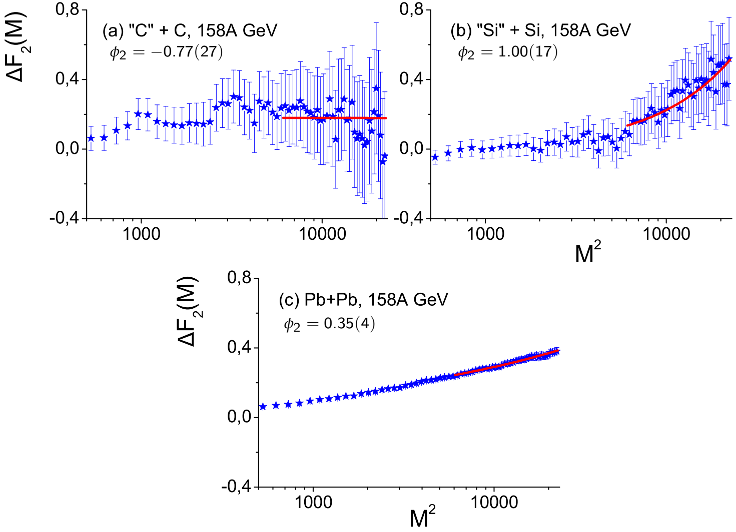
A	"C"+C*	"Si"+Si*	Pb+Pb
# Bootstrap samples	1000		200
beam energy	158 A GeV		40, 158 A GeV
centrality range	$0 \to 11.5\%$		$0 \to 12.5\%$
rapidity range	$-0.75 \leq y_{CM} \leq 0.75$		
Lattice range (GeV)	$[-1.529, 1.471] \to [-1.471, 1.529]$		
# of events	201 189	175 943	1 480 587
$\langle p_{data} \rangle$ (after cuts)	1.6 ± 0.9	3.1 ± 1.7	10.9 ± 3.8

** Beam Components: "C" = C,N, "Si" = Si,Al,P*

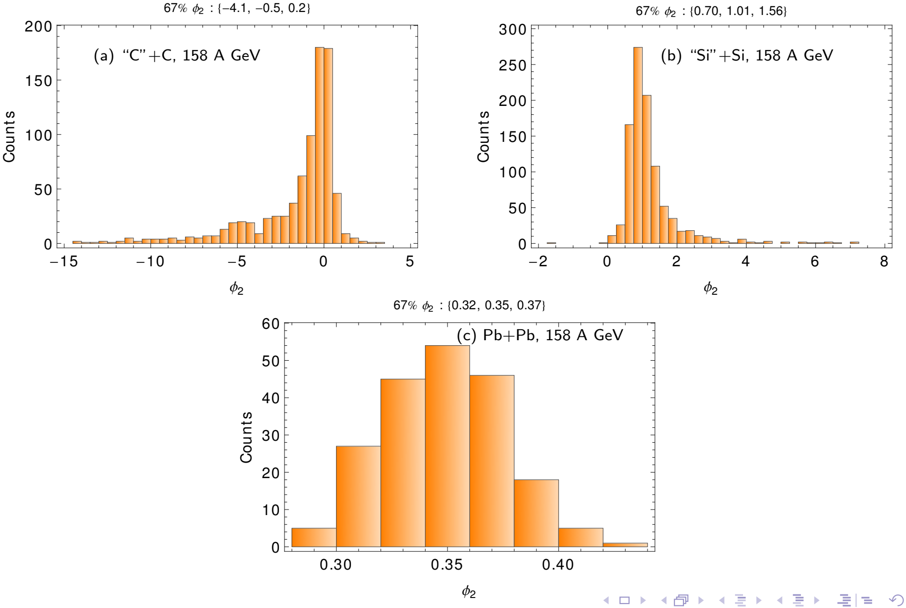
- Standard event and track selection cuts of NA49 experiment, [T. Anticic *et al*, PR**C 81**, 149 (2010)].
- Particle (protons, pions) identification with purity $> 80\%$.
- Neglect antiprotons (much fewer than protons).
- Mid-rapidity selected because of approximately constant proton density in rapidity in this region.

[N.G. Antoniou, F.K. Diakonos, A.S. Kapoyannis and K.S. Kousouris, PRL.**97**, 032002 (2006)]

Analysis results - $\Delta F_2(M)$ for protons



Analysis results - φ_2 bootstrap distribution



Split tracks & the q_{inv} cut

- Events may contain split tracks: sections of the same track erroneously identified as a pair of tracks that are close in momentum space.
- Intermittency analysis is based on pairs distribution $\Rightarrow$ split tracks can create a false positive, and so must be reduced or removed.
- Standard cuts remove part of split tracks. In order to estimate the residual contamination, we check the q_{inv} distribution of track pairs:

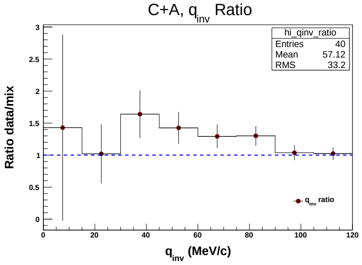
$$q_{inv}(p_i, p_j) \equiv \sqrt{-(p_i - p_j)^2},$$

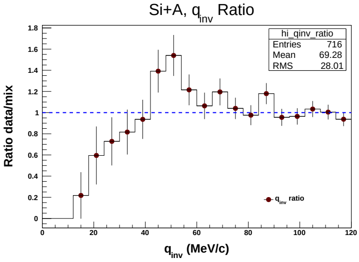
p_i : 4-momentum of i^{th} track.

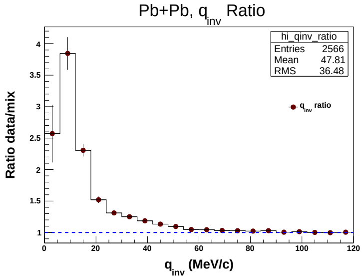
- We calculate the ratio of $q_{inv}^{data} / q_{inv}^{mixed}$. A peak at low q_{inv} (below 20 MeV / c) indicates a possible split track contamination that must be removed.

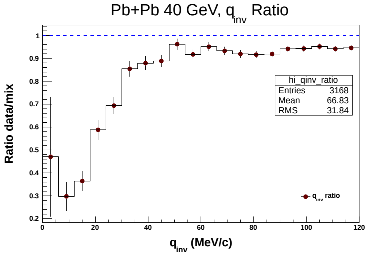
q_{inv} test – Analysed datasets

- Only Pb+Pb, 158A GeV exhibits a possible contamination.

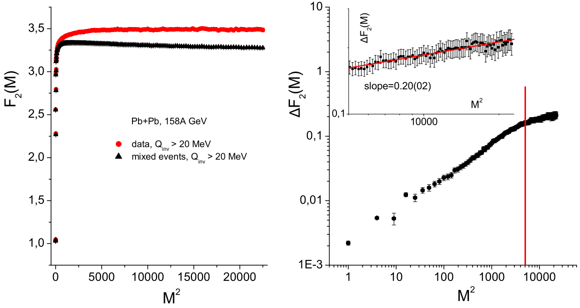


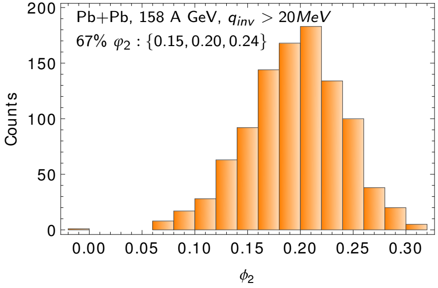






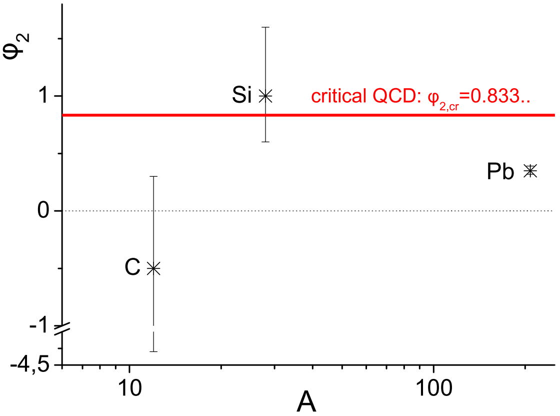
q_{inv} cut – Pb+Pb 158A GeV





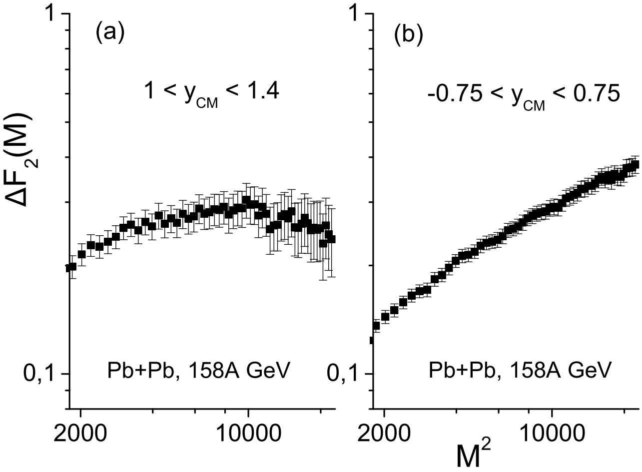
- $\Delta F_2(M)$ shows intermittent behaviour for $M^2 > 5000$ with $\varphi_2 = 0.20(02)$
- Bootstrap φ_2 distribution is almost symmetric and centered around 0.20
- q_{inv} cut, being aggressive, distorts the signal, however the intermittency effect remains, with a reduced φ_2 value.

Analysis results – Size dependence of ϕ_2



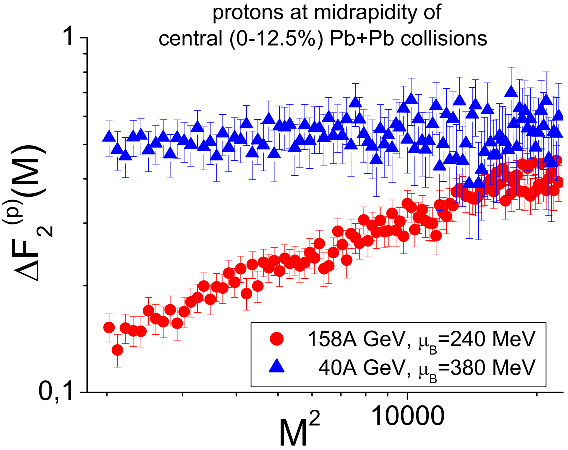
Significant power-law fluctuations for Si and Pb systems at 158A GeV. The intermittency index ϕ_2 for **Si overlaps** with the critical QCD prediction.

Analysis results – Rapidity dependence of ϕ_2 for Pb+Pb



The intermittency effect is confined to protons at mid-rapidity.

Analysis results – Energy dependence of $\phi_2^{(p)}$



No trace of power-law fluctuations in Pb+Pb at 40A GeV (not enough statistics for "C"+C and "Si"+Si) ⇒ CP closer to 158A GeV freeze out conditions.

Summary and outlook

Intermittency analysis in transverse momentum space of NA49 data for central "C"+C, "Si"+Si and Pb+Pb collisions has been performed.

- For protons at midrapidity we have found significant power-law fluctuations in Si and Pb at 158A GeV.
- The **intermittency index ϕ_2** for the **Si system overlaps with the critical QCD prediction**.
- Although Pb+Pb shows good quality intermittency, due to larger statistics and event multiplicity, the q_{inv} analysis gives us a lower bound of 0.20 for φ_2, well below the theoretically predicted value for systems that freeze out exactly at the critical point.

Summary and outlook

- **First experimental evidence** for the **approach to the vicinity of the critical point**.
- No power-law behaviour is observed for protons of Pb+Pb system at 40A GeV.
- The critical baryochemical potential seems closer to 240 MeV than to 380 MeV.

Exploring peripheral Pb+Pb collision data of NA49 at 158 A GeV and performing a systematic **intermittency study in lighter systems** (Be+Be, Ar+Ca, Xe+La) as function of energy **in NA61** will hopefully lead to an **accurate determination of the critical point location**.

Thank you!

Back Up Slides

Event & track cuts for Si+A

Event cuts:

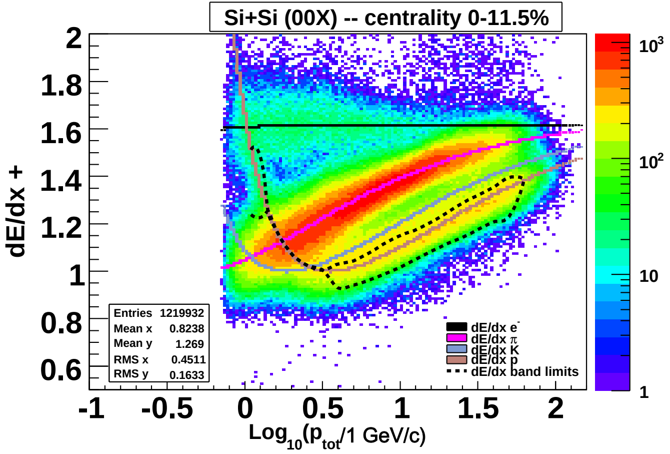
- Iflag $= 0$, $\mathrm{chi}^2 > 0$
- Beam charge cuts (Al,Si,P)
- Vertex cuts:
  - -0.4 cm $\leq V_x \leq 0.4$ cm
  - -0.5 cm $\leq V_y \leq 0.5$ cm
  - -580.3 cm $\leq V_z \leq -578.7$ cm

Track cuts:

- Iflag $= 0$
- Npoints ≥ 30
  (for the whole detector)
- Ratio $\frac{\mathrm{Npoints}}{\mathrm{NMaxPoints}} \geq 0.5$
- ZFirst ≤ 200
- Impact parameters:
  $|B_x| \leq 2$, $|B_y| \leq 1$
- dE/dx cuts for particle identification
- p_{tot} cuts (via dE/dx cut)
- rapidity cut

Analysed data sets - proton identification

- dE/dx histograms for small p_{tot} "slices" – fit with sum of 4 gaussians (e,p,K,π).



- Width & overlap of fitted peaks $\Rightarrow$ Selection bands around proton theoretical curve that maximize purity.