NONEXTENSIVE STATISTICAL MECHANICS AND HIGH ENERGY PHYSICS

Constantino Tsallis

Centro Brasileiro de Pesquisas Físicas

and Instituto Nacional de Ciência e Tecnologia de Sistemas Complexos

Rio de Janeiro, Brazil

and

Santa Fe Institute, New Mexico, USA

CBPF

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investing in knowledge society
MINISTRY OF EDUCATION, LIFELONG LEARNING AND RELIGIOUS AFFAIRS
MANAGING AUTHORITY
European Union
European Social Fund
Co- financed by Greece and the European Union
NSRF 2007-2013
programme for development
EUROPEAN SOCIAL FUND

Kolimbari, September 2013

J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, (1981), page 35

In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92) [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space […]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. […]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*

Enrico FERMI *Thermodynamics* (Dover, 1936)

The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role.

ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ equiprobability	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy (*q* =*1*)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* (*q real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable

Topsoe-factorizable (unique)

Amari-Ohara-Matsuzoe conformally invariant geometry (unique)

Biro-Barnafoldi-Van thermostat universal independence (unique)

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

[C.T., J. Stat. Phys. **52**, 479 (1988)]

DEFINITIONS : *q-logarithm* : $$\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\ \ln_1 x = \ln x)$$

q-exponential : $$e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$$

Hence, the entropies can be rewritten :

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\ \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

TYPICAL SIMPLE SYSTEMS:

e.g., $W(N) \propto \mu^N \;\; (\mu > 1)$

Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), **Ergodic**, Riemannian geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear/homogeneous Fokker-Planck equations, Gausssians

→ **Boltzmann-Gibbs entropy (additive)**

→ **Exponential dependences (Boltzmann-Gibbs weight, ...)**

TYPICAL COMPLEX SYSTEMS:

e.g., $W(N) \propto N^\rho \;\; (\rho > 0)$

Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), **Nonergodic**, Multifractal geometry

Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear/inhomogeneous Fokker-Planck equations, *q*-Gaussians

→ **Entropy S*q* (nonadditive)**

→ ***q*-exponential dependences (asymptotic power-laws)**

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B)=S(A)+S(B)$$

Therefore, since

$$S_q(A+B)=S_q(A)+S_q(B)+(1-q)\,S_q(A)\,S_q(B)\ ,$$

S_{BG} and $S_q^{Renyi}\,(\forall q)$ are additive, and $S_q\ (\forall q\neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma\equiv A_1+A_2+...+A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0<\lim_{N\to\infty}\frac{S(N)}{N}<\infty\ ,\ i.e.,\ S(N)\propto N\ \ (N\to\infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

If $W(N) \sim \mu^N \quad (\mu > 1)$

$$\Rightarrow S_{BG}(N) = k_B \ln W(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim N^\rho \quad (\rho > 0)$

$$\Rightarrow S_q(N) = k_B \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow S_{q=1-1/\rho}(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim \nu^{N^\gamma} \quad (\nu > 1;\ 0 < \gamma < 1)$

$$\Rightarrow S_\delta(N) = k_B \left[\ln W(N)\right]^\delta \propto N^{\gamma\,\delta}$$

$$\Rightarrow S_{\delta=1/\gamma}(N) \propto N \qquad \text{OK!}$$

IMPORTANT: $\mu^N >> \nu^{N^\gamma} >> N^\rho \quad \text{if} \quad N >> 1$

PHYSICAL REVIEW E **78**, 021102 (2008)

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

Filippo Caruso[1] and Constantino Tsallis[2,3]
[1]*NEST CNR-INFM and Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa, Italy*
[2]*Centro Brasileiro de Pesquisas Fisicas, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil*
[3]*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*
(Received 16 March 2008; revised manuscript received 16 May 2008; published 5 August 2008)

The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d-dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1}. Here we show, for $d=1,2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q \neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q.

SPIN ½ XY FERROMAGNET WITH TRANSVERSE MAGNETIC FIELD:

$$\hat{\mathcal{H}} = -\sum_{j=1}^{N-1} \left[(1+\gamma)\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + (1-\gamma)\hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + 2\lambda \hat{\sigma}_j^z \right]$$

$|\gamma| = 1 \quad \rightarrow$ *Ising ferromagnet*

$0 < |\gamma| < 1 \quad \rightarrow$ *anisotropic XY ferromagnet*

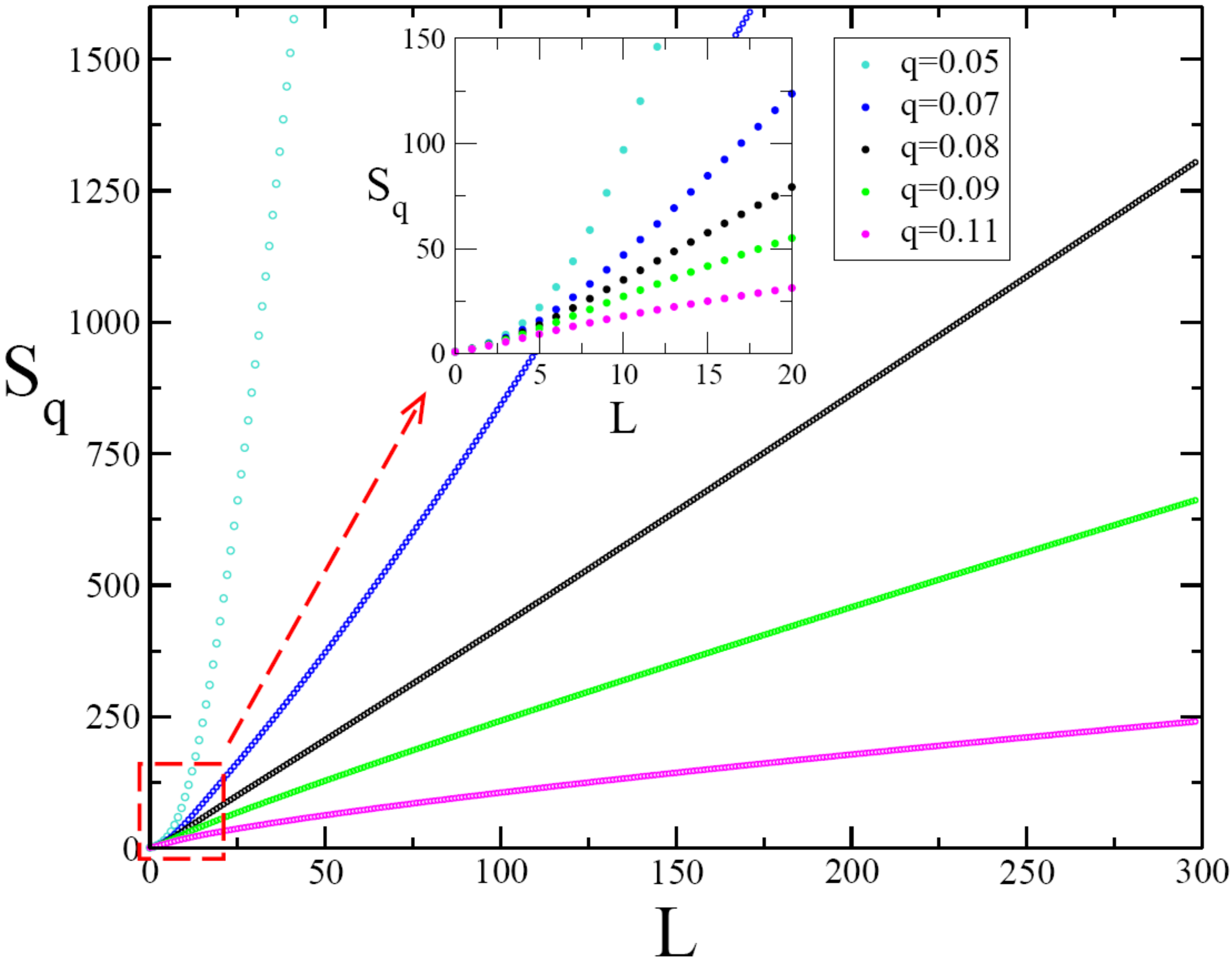
$\gamma = 0 \quad \rightarrow$ *isotropic XY ferromagnet*

$\lambda \equiv$ *transverse magnetic field*

$L \equiv$ *length of a block within a* $N \rightarrow \infty$ *chain*

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

ISING MODEL



F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

Using a Quantum Field Theory result

*in P. Calabrese and J. Cardy, JSTAT P*06002 (2004)

we obtain, at the critical transverse magnetic field,

$$q_{ent} = \frac{\sqrt{9+c^2}-3}{c}$$

with $c \equiv$ *central charge in conformal field theory*

Hence

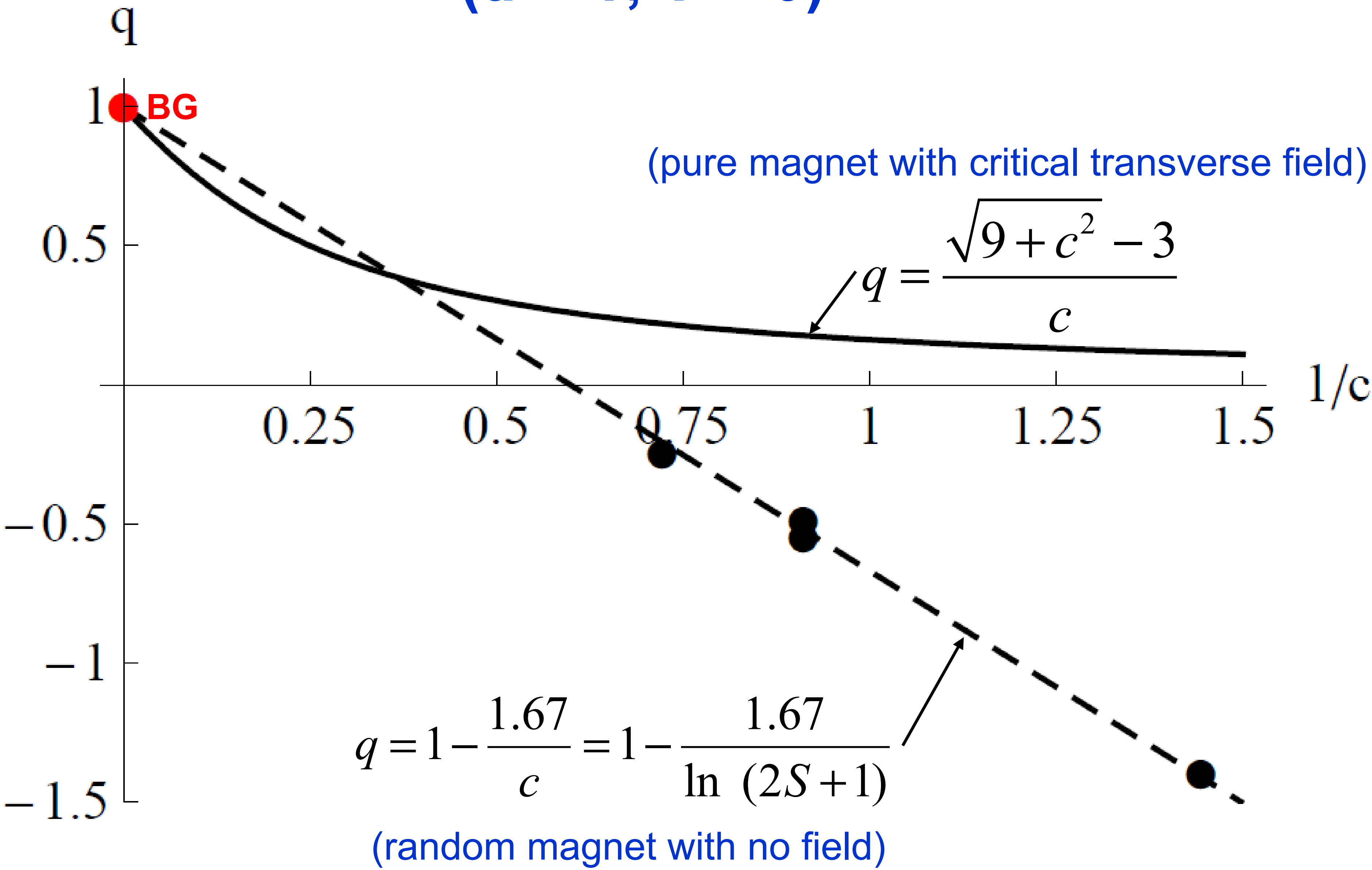
Ising and anisotropic XY ferromagnets $\Rightarrow\ c = \frac{1}{2}\ \Rightarrow\ q_{ent} = \sqrt{37}-6\ \approx 0.0828$

and

Isotropic XY ferromagnet $\Rightarrow\ c = 1\ \Rightarrow\ q_{ent} = \sqrt{10}-3\ \approx 0.1623$

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

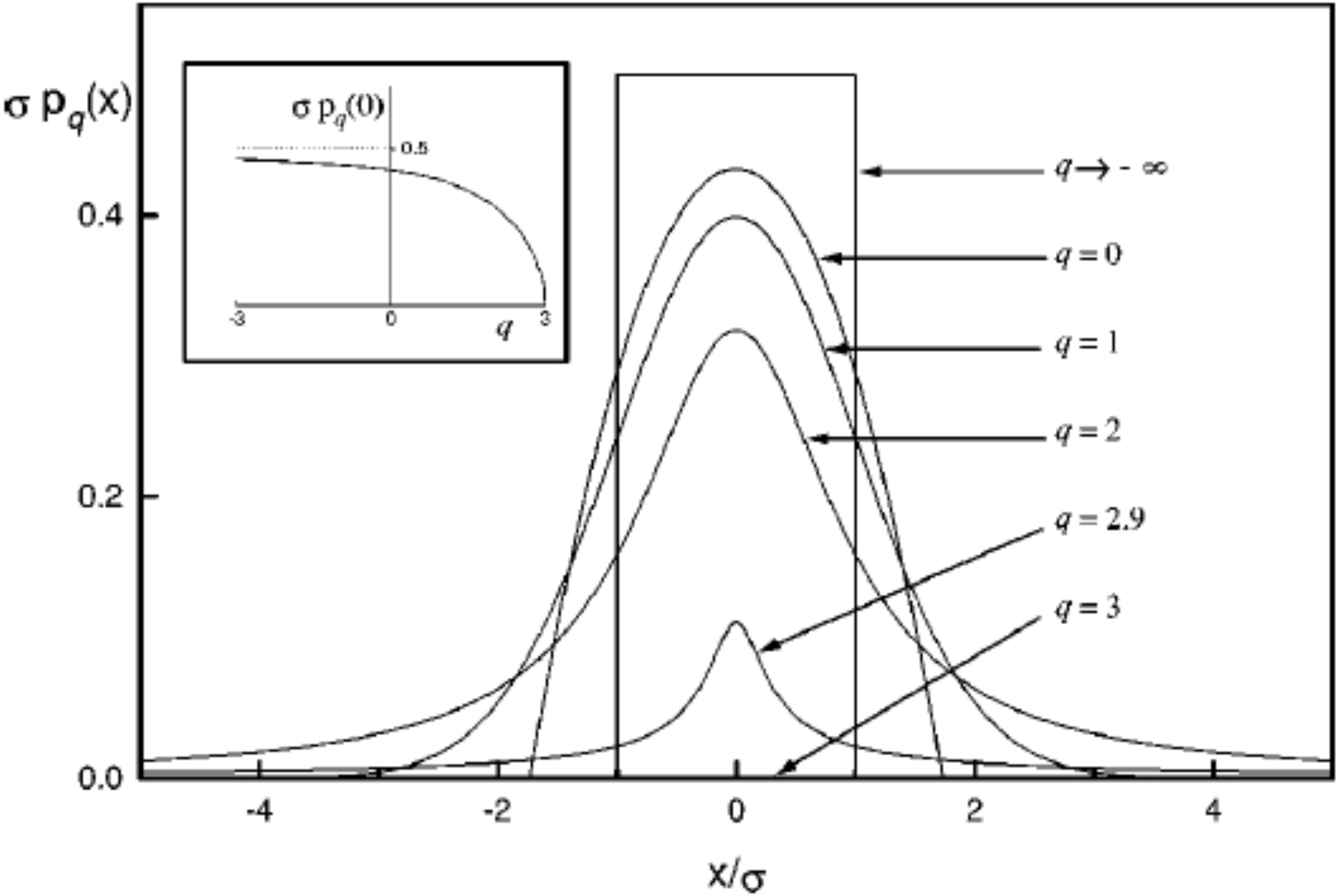
(*d* = 1; *T* = 0)



A Saguia and MS Sarandy, Phys Lett A **374**, 3384 (2010)

q-GAUSSIANS:

$$p_q(x) \propto e_q^{-(x/\sigma)^2} \equiv \frac{1}{\left[1+(q-1)\,(x/\sigma)^2\right]^{\frac{1}{q-1}}} \qquad (q<3)$$



D. Prato and C. T., Phys Rev E **60**, 2398 (1999)

Milan j. math. 76 (2008), 307–328
© 2008 Birkhäuser Verlag Basel/Switzerland
1424-9286/010307-22, *published online* 14.3.2008
DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

Sabir Umarov,[1,a)] Constantino Tsallis,[2,3,b)] Murray Gell-Mann,[3,c)] and Stanly Steinberg[4,d)]
[1]*Department of Mathematics, Tufts University, Medford, Massachusetts 02155, USA*
[2]*Centro Brasileiro de Pesquisas Fisicas and National Institute of Science and Technology for Complex Systems, Rua Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil*
[3]*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*
[4]*Department of Mathematics and Statistics, University of New Mexico, Albuquerque, New Mexico 87131, USA*

(Received 10 November 2009; accepted 4 January 2010; published online 3 March 2010)

See also:
H.J. Hilhorst, JSTAT P10023 (2010)
M. Jauregui and C. T., Phys Lett A **375**, 2085 (2011)
M. Jauregui, C. T. and E.M.F. Curado, JSTAT P10016 (2011)
A. Plastino and M.C. Rocca, Physica A and Milan J Math (2012)
A. Plastino and M.C. Rocca (2013)

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ *-scaled attractor* $\mathrm{F}(x)$ *when summing* $N \to \infty$ q*-independent identical random variables with symmetric distribution* $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\; q_1 = \frac{1+q}{3-q} \right)$

	$q=1$ *[independent]*	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) *[globally correlated]*
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathrm{F}(x) =$ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathrm{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x \rvert << x_c(q,2) \\ f(x) \sim C_q / \lvert x \rvert^{2/(q-1)} & \text{if } \lvert x \rvert >> x_c(q,2) \end{cases}$ *with* $\lim_{q\to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$\mathrm{F}(x) =$ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x \rvert \to \infty$ *behavior* $\mathrm{L}_\alpha(x) \sim \begin{cases} G(x) \\ \quad \text{if } \lvert x \rvert << x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x \rvert^{1+\alpha} \\ \quad \text{if } \lvert x \rvert >> x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$\mathrm{F}(x) = L_{q,\alpha}$, *with same* $\lvert x \rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\,\alpha}(x) \sim C^*_{q,\alpha} / \lvert x \rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \quad (\textit{intermediate regime}) \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\,2}(x) \sim C^L_{q,\alpha} / \lvert x \rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \quad (\textit{distant regime}) \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

PHYSICAL REVIEW E **84**, 021121 (2011)

Group entropies, correlation laws, and zeta functions

Piergiulio Tempesta*
Departamento de Física Teórica II, Facultad de Físicas, Ciudad Universitaria, Universidad Complutense, E-28040 Madrid, Spain
(Received 15 February 2011; revised manuscript received 3 May 2011; published 10 August 2011)

The notion of group entropy is proposed. It enables the unification and generaliztion of many different definitions of entropy known in the literature, such as those of Boltzmann-Gibbs, Tsallis, Abe, and Kaniadakis. Other entropic functionals are introduced, related to nontrivial correlation laws characterizing universality classes of systems out of equilibrium when the dynamics is weakly chaotic. The associated thermostatistics are discussed. The mathematical structure underlying our construction is that of formal group theory, which provides the general structure of the correlations among particles and dictates the associated entropic functionals. As an example of application, the role of group entropies in information theory is illustrated and generalizations of the Kullback-Leibler divergence are proposed. A new connection between statistical mechanics and zeta functions is established. In particular, Tsallis entropy is related to the classical Riemann zeta function.

$$S_q \leftrightarrow \frac{1}{(1-q)^{s-1}}\varsigma(s) \quad (q<1)$$

$$\text{with } \varsigma(s) \equiv \sum_{n=1}^{\infty}\frac{1}{n^s} = \prod_{p\text{ prime}}\frac{1}{1-p^{-s}}$$

$$= \frac{1}{1-2^{-s}}\frac{1}{1-3^{-s}}\frac{1}{1-5^{-s}}\frac{1}{1-7^{-s}}\frac{1}{1-11^{-s}}\cdots$$

COLD ATOMS IN DISSIPATIVE OPTICAL LATTICES:

RAPID COMM

PHYSICAL REVIEW A **67**, 051402(R) (2003)

Anomalous diffusion and Tsallis statistics in an optical lattice

Eric Lutz
Sloane Physics Laboratory, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120
(Received 26 February 2003; published 27 May 2003)

We point out a connection between anomalous transport in an optical lattice and Tsallis' generalized statistics. Specifically, we show that the momentum equation for the semiclassical Wigner function which describes atomic motion in the optical potential, belongs to a class of transport equations recently studied by Borland [Phys. Lett. A **245**, 67 (1998)]. The important property of these ordinary linear Fokker-Planck equations is that their stationary solutions are exactly given by Tsallis distributions. An analytical expression of the Tsallis index q in terms of the microscopic parameters of the quantum-optical problem is given and the spatial coherence of the atomic wave packets is discussed.

(i) The distribution of atomic velocities is a q-Gaussian;

(ii) $q = 1 + \dfrac{44 E_R}{U_0}$ where $E_R \equiv$ recoil energy

$U_0 \equiv$ potential depth

Tunable Tsallis Distributions in Dissipative Optical Lattices

P. Douglas, S. Bergamini, and F. Renzoni

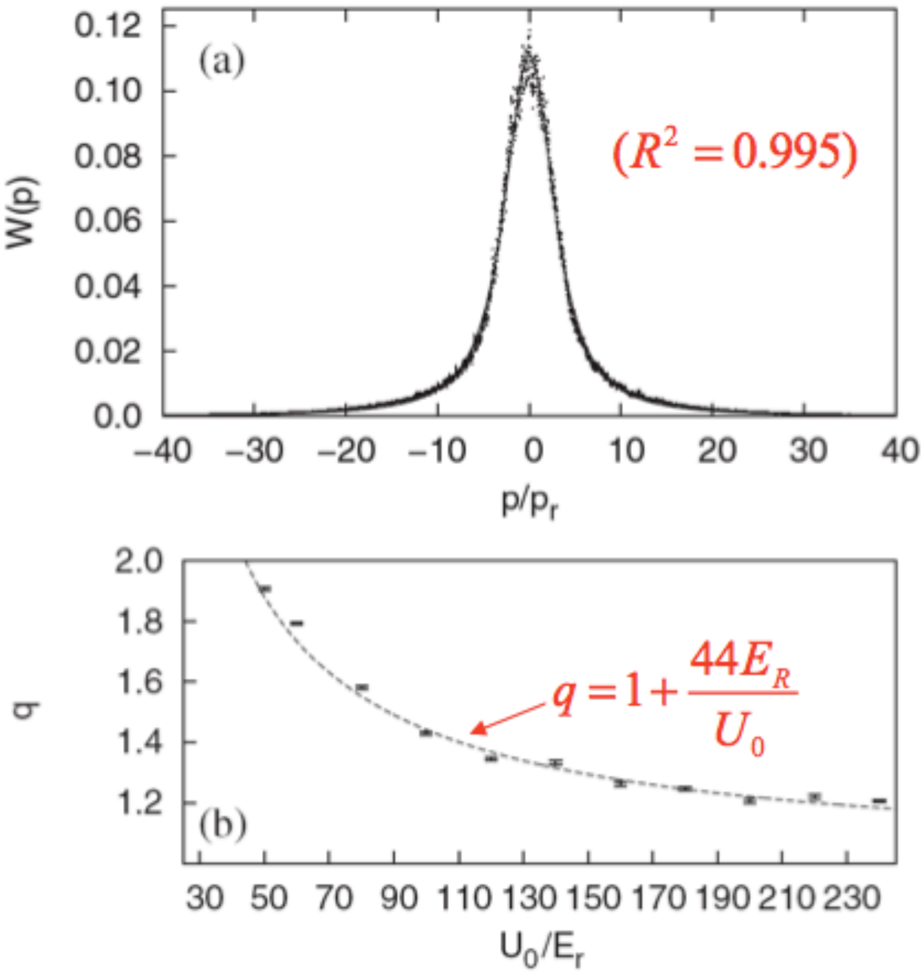
Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom

(Received 10 January 2006; published 24 March 2006)

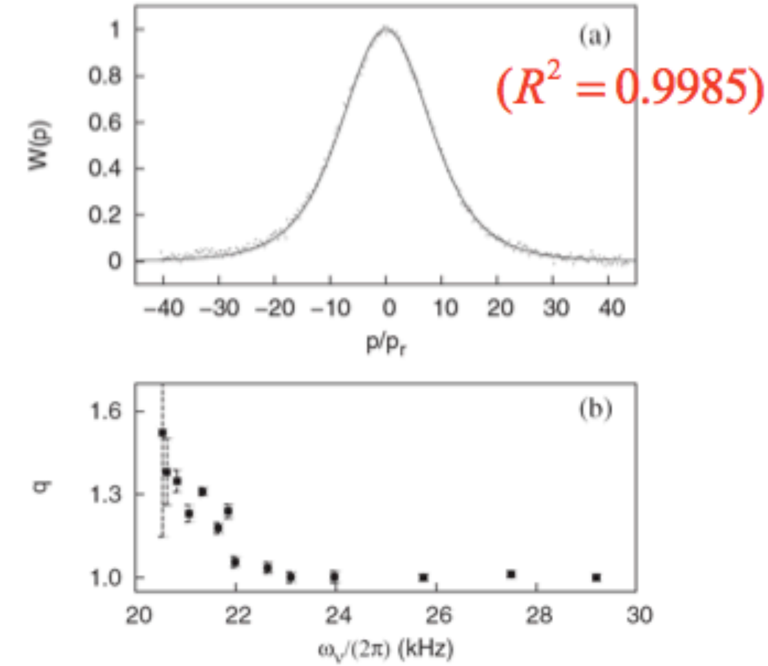
We demonstrated experimentally that the momentum distribution of cold atoms in dissipative optical lattices is a Tsallis distribution. The parameters of the distribution can be continuously varied by changing the parameters of the optical potential. In particular, by changing the depth of the optical lattice, it is possible to change the momentum distribution from Gaussian, at deep potentials, to a power-law tail distribution at shallow optical potentials.

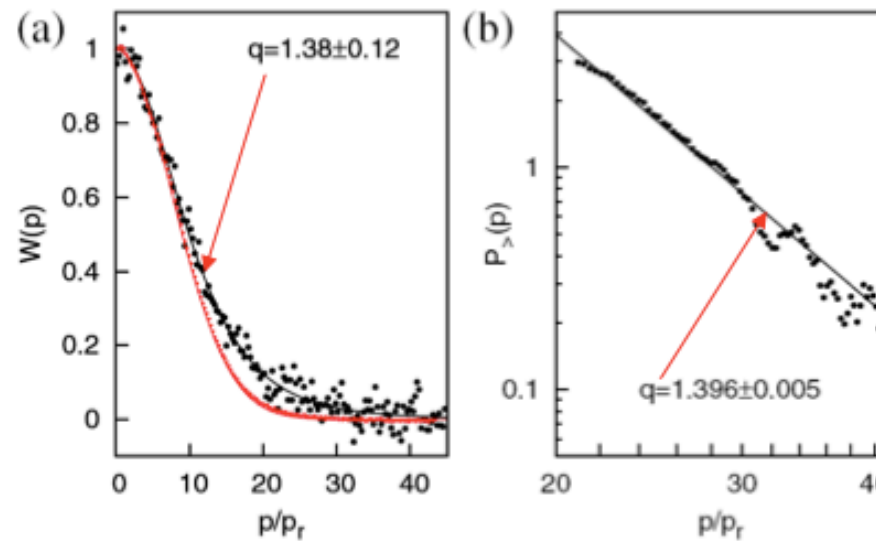
Experimental and computational verifications

by P. Douglas, S. Bergamini and F. Renzoni, Phys Rev Lett **96**, 110601 (2006)



(Computational verification: quantum Monte Carlo simulations)





(Experimental verification: Cs atoms)

International Journal of Bifurcation and Chaos, Vol. 22, No. 9 (2012) 1250208 (18 pages)

DOI: 10.1142/S0218127412502082

TIME-EVOLVING STATISTICS OF CHAOTIC ORBITS OF CONSERVATIVE MAPS IN THE CONTEXT OF THE CENTRAL LIMIT THEOREM

G. RUIZ
Departamento de Matemática Aplicada y Estadística,
Universidad Politécnica de Madrid,
Plaza Cardenal Cisneros s/n, 28040 Madrid, Spain
guiomar.ruiz@upm.es

T. BOUNTIS
Department of Mathematics and Center for Research
and Applications of Nonlinear Systems (CRANS),
University of Patras, GR26500, Rion, Patras, Greece
bountis@math.upatras.gr

C. TSALLIS
Centro Brasileiro de Pesquisas Fisicas and National Institute
of Science and Technology for Complex Systems,
Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil

Santa Fe Institute, 1399 Hyde Park Road, Santa Fe,
New Mexico 87501, USA
tsallis@cbpf.br

CONSERVATIVE MC MILLAN MAP:

$$x_{n+1} = y_n$$

$$y_{n+1} = -x_n + 2\mu \frac{y_n}{1+y_n^2} + \varepsilon y_n$$

$\mu \neq 0 \Leftrightarrow$ nonlinear dynamics

G. Ruiz, T. Bountis and C. T., Int J Bifurcat Chaos **22**, 1250208 (2012)

$(\mu,\varepsilon)=(1.6,1.2)$ $(\lambda_{max} \approx 0.05)$

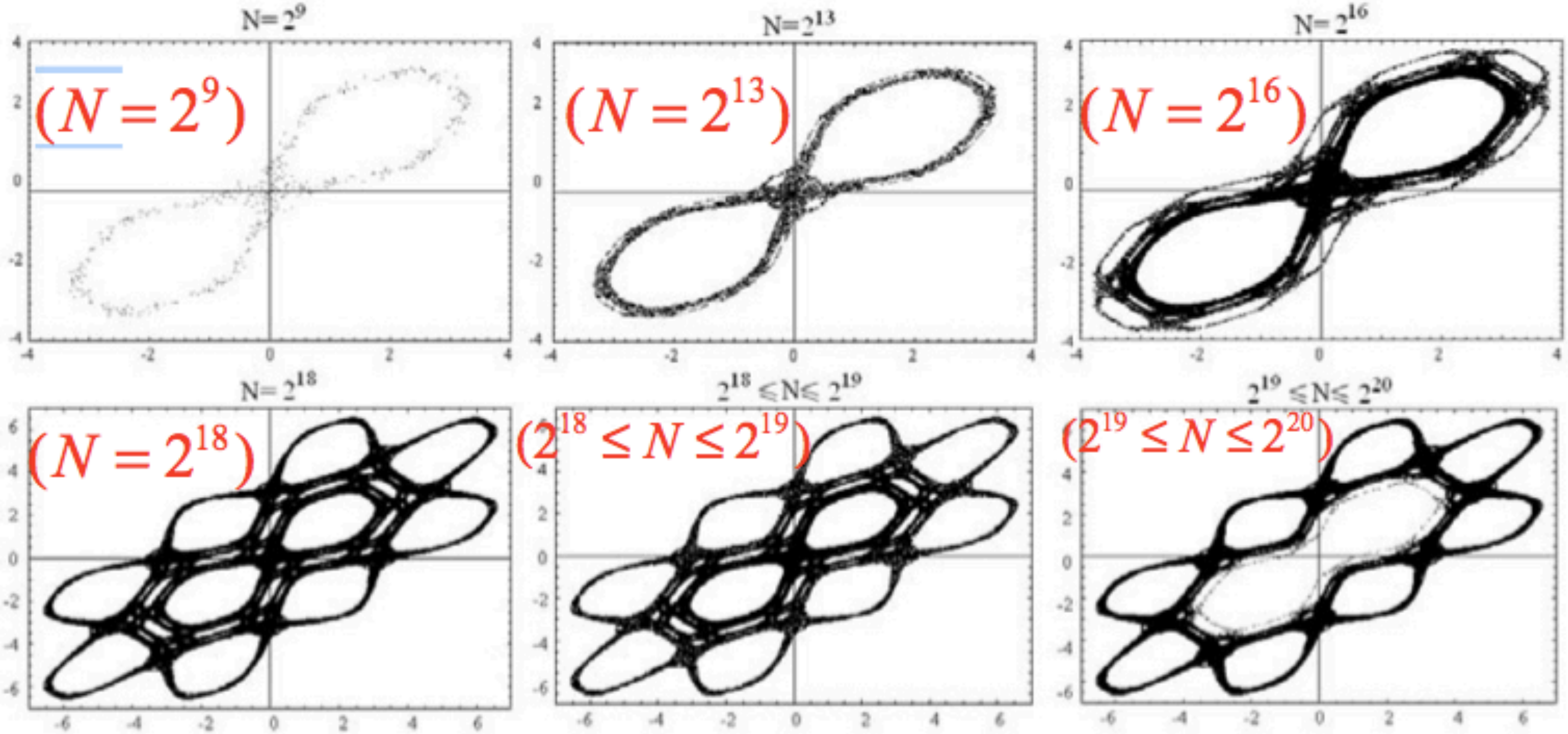
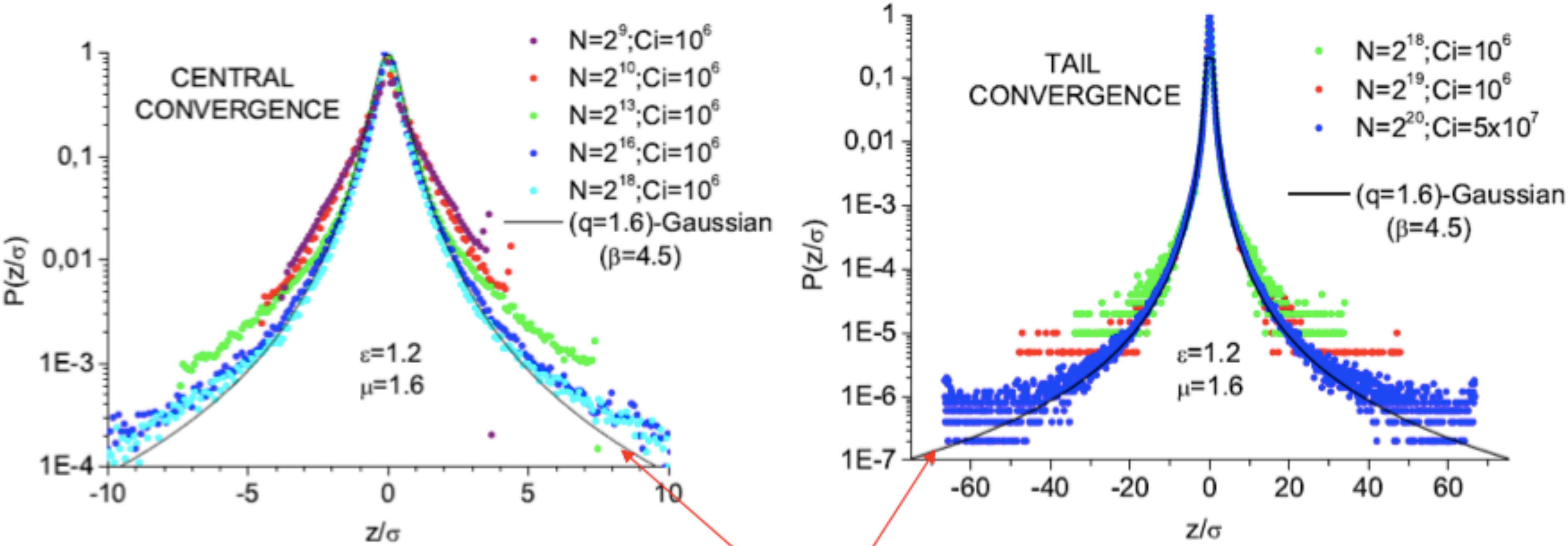


FIG. 10. Structure of phase space plot of Mc. Millan perturbed map for parameter values $\mu = 1.6$ and $\epsilon = 1.2$, starting form a randomly chosen initial condition in a square $(0, 10^{-6}) \times (0, 10^{-6})$, and for $i = 1 \ldots N$ $(N = 2^{10}, 2^{13}, N^{16}, N^{18})$ iterates.

G. Ruiz, T. Bountis and C. T., Int J Bifurcat Chaos **22**, 1250208 (2012)



$$p \propto e_q^{-\beta (z/\sigma)^2}$$

with $(q,\beta) = (1.6, 4.5)$

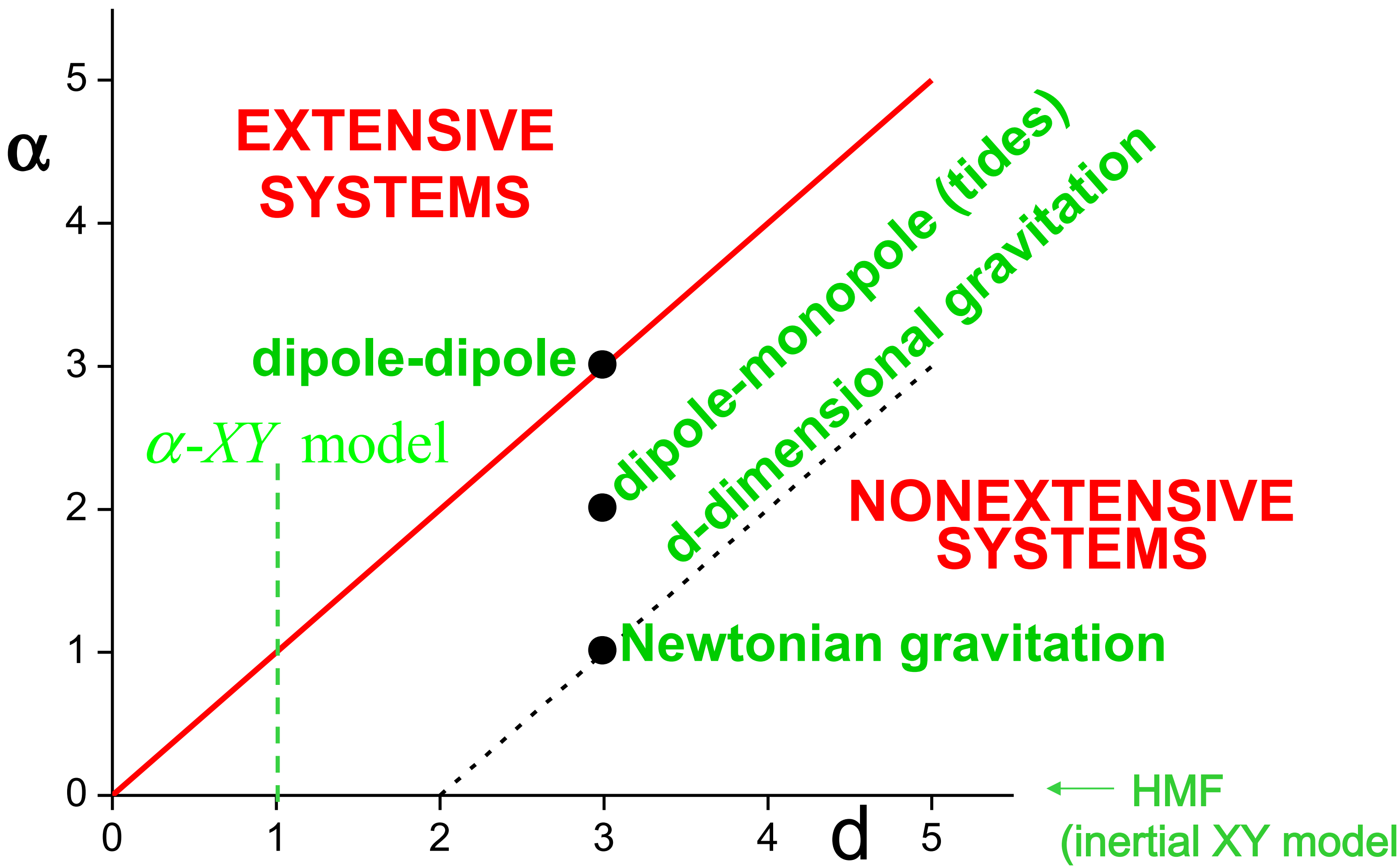
G. Ruiz, T. Bountis and C. T., Int J Bifurcat Chaos **22**, 1250208 (2012)

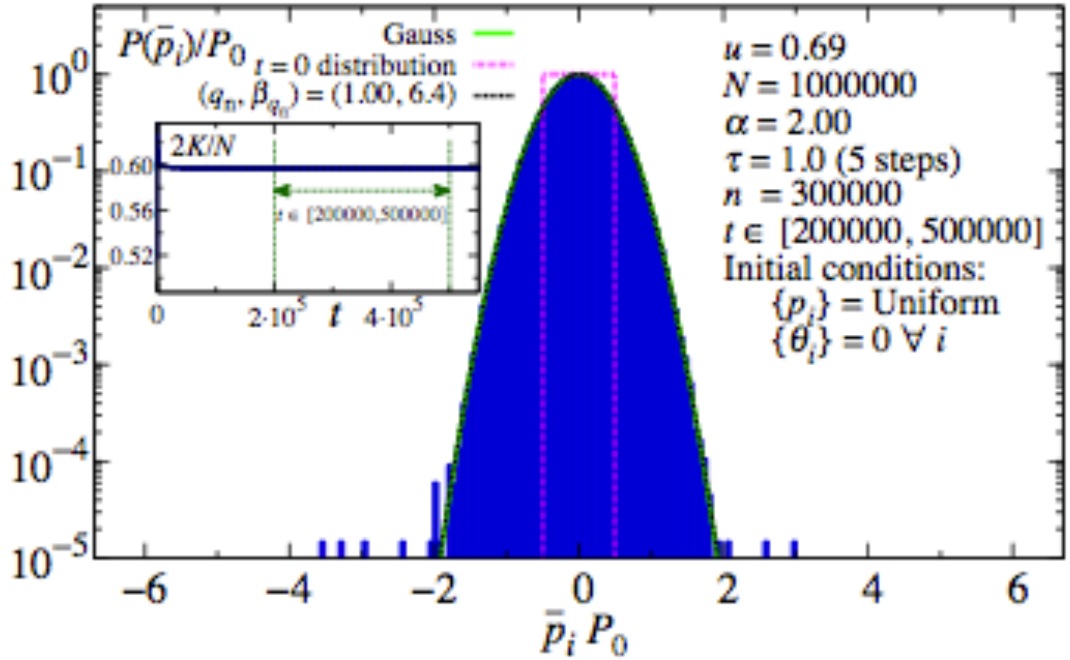
CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(r) \sim -\frac{A}{r^\alpha} \quad (r \to \infty) \qquad (A > 0, \;\; \alpha \geq 0)$$

$$\textit{integrable if} \quad \alpha / d > 1 \quad (\textit{short-ranged})$$

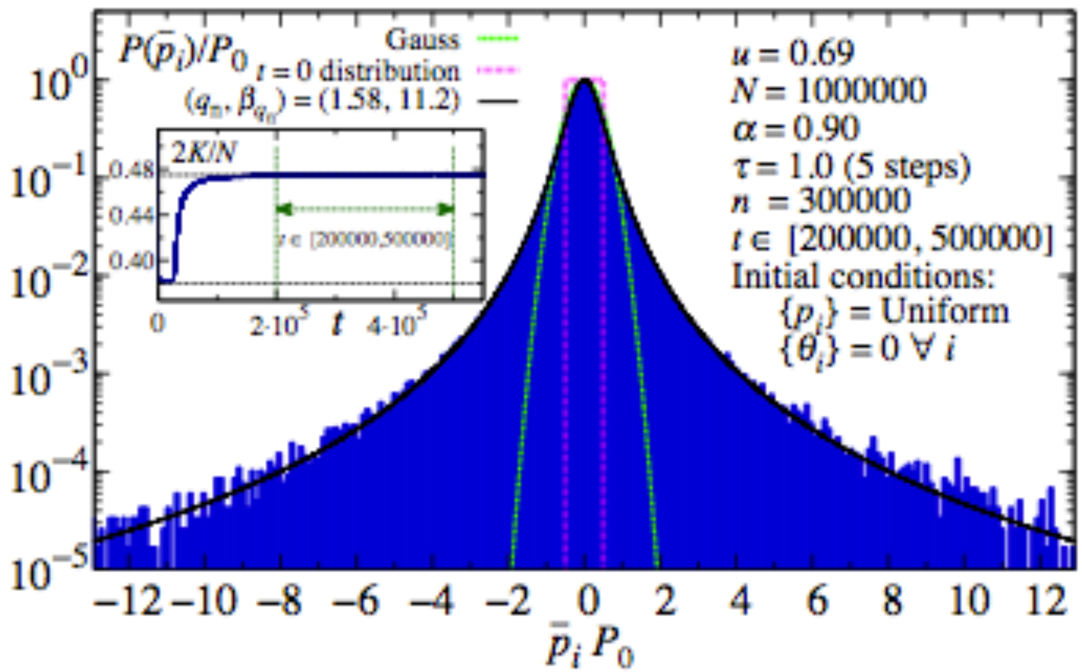
$$\textit{non-integrable if} \quad 0 \leq \alpha / d \leq 1 \quad (\textit{long-ranged})$$





$\alpha = 2$

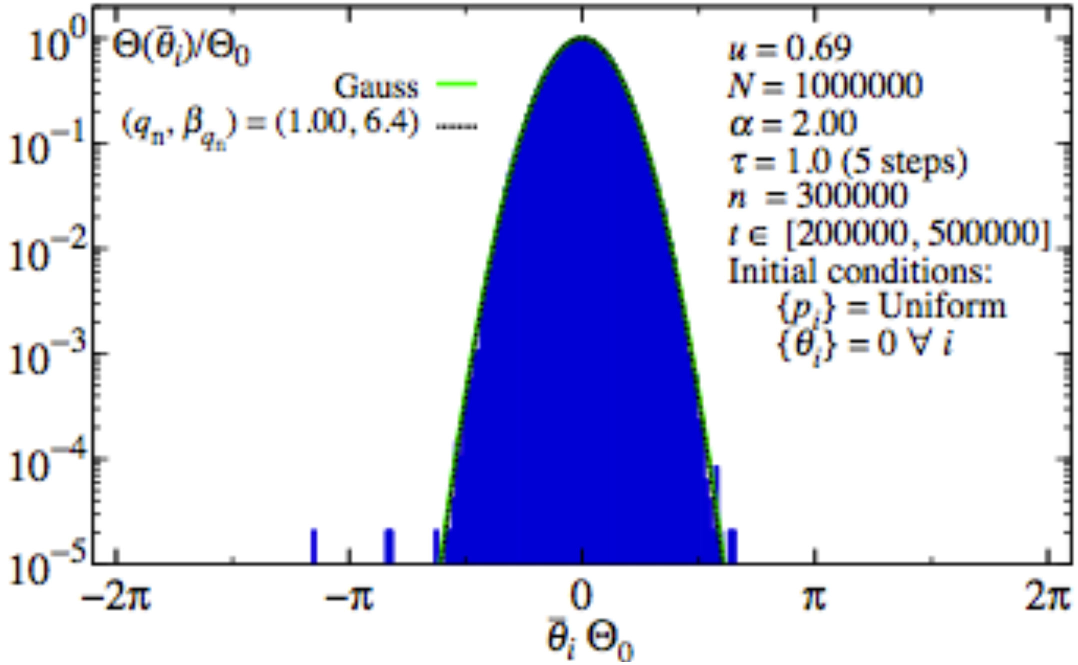
$q = 1$



$\alpha = 0.9$

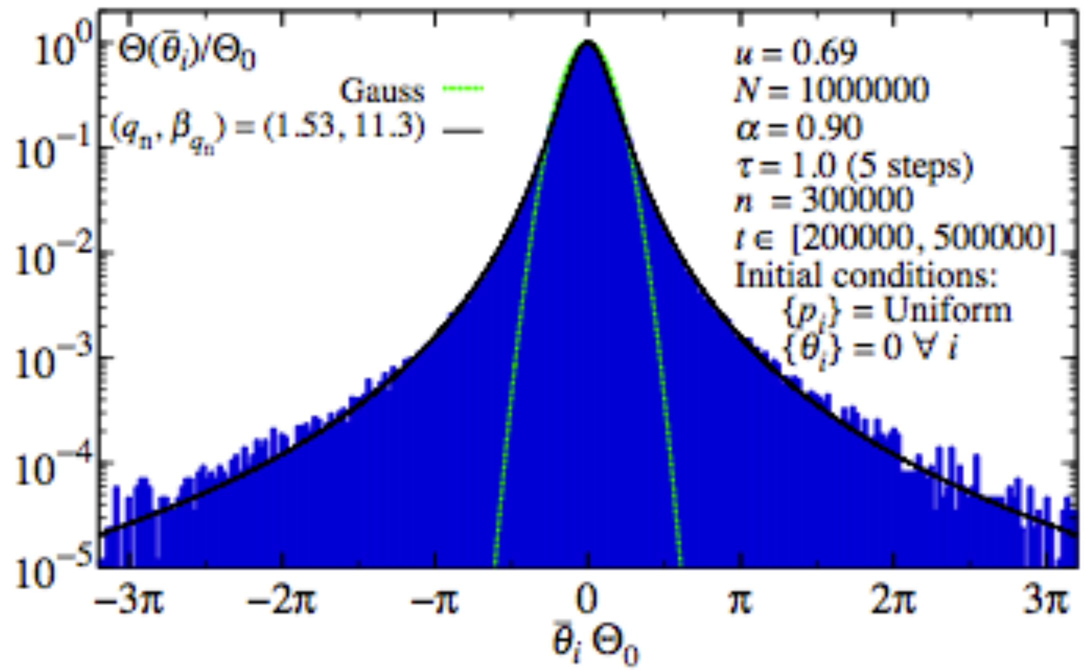
$q = 1.58$

L.J.L. Cirto, V.R.V. Assis and C. T. (2012),1206.6133 [cond-mat.stat-mech]



$\alpha = 2$

$q = 1$



$\alpha = 0.9$

$q = 1.53$

L.J.L. Cirto, V.R.V. Assis and C. T. (2012),1206.6133 [cond-mat.stat-mech]

SHEAR IN GRANULAR MATTER:

AIP Conf Proc **1542**, 453 (2013)

Experimental evidence of "Granulence"

Gaël Combe*, Vincent Richefeu*, Gioacchino Viggiani*, Stephen A. Hall†, Alessandro Tengattini* and Allbens P.F. Atman**

*UJF-Grenoble 1, Grenoble-INP, CNRS UMR 5521, 3SR Lab., B.P. 53, 38041 Grenoble Cedex 09, France.
†Lund University, Division of Solid Mechanics, P.O. Box 118, SE-221 00 Lund, Sweden
**Departamento de Física e Matemática and National Institute of Science and Technology for Complex Systems, Centro Federal de Educação Tecnológica de Minas Gerais, CEFET–MG, Av. Amazonas 7675, 30510-000, Belo Horizonte-MG, Brazil

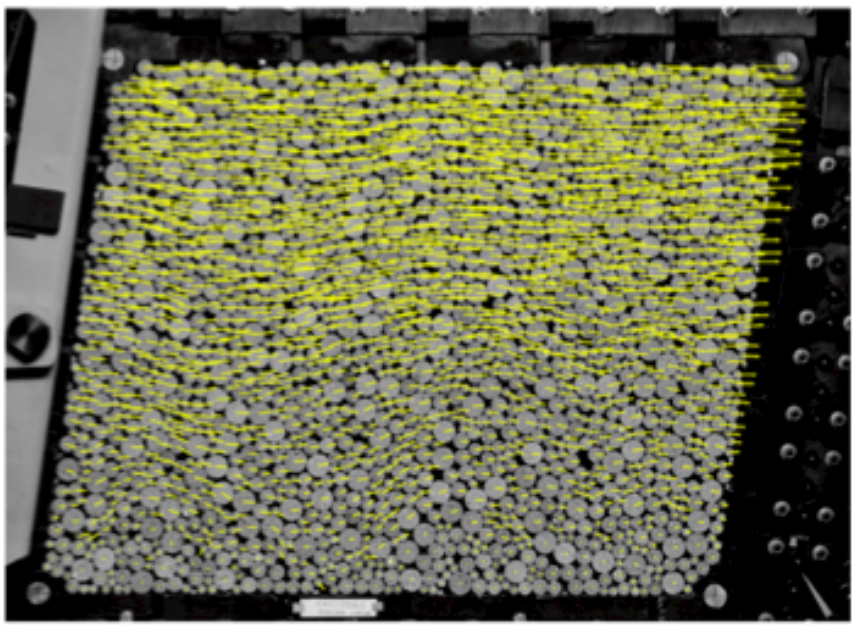


FIGURE 2. Particles displacement (maximum displacement = 46mm) in a sheared 2D granular assembly made of 2000 wooden rods. The granular packing is enclosed by a rigid frame initially rectangular ($0.56m \times 0.47m$). A speckle of black and white points is painted on each cylinder to allow the measurement of particle kinematics by means of the PIT technique.

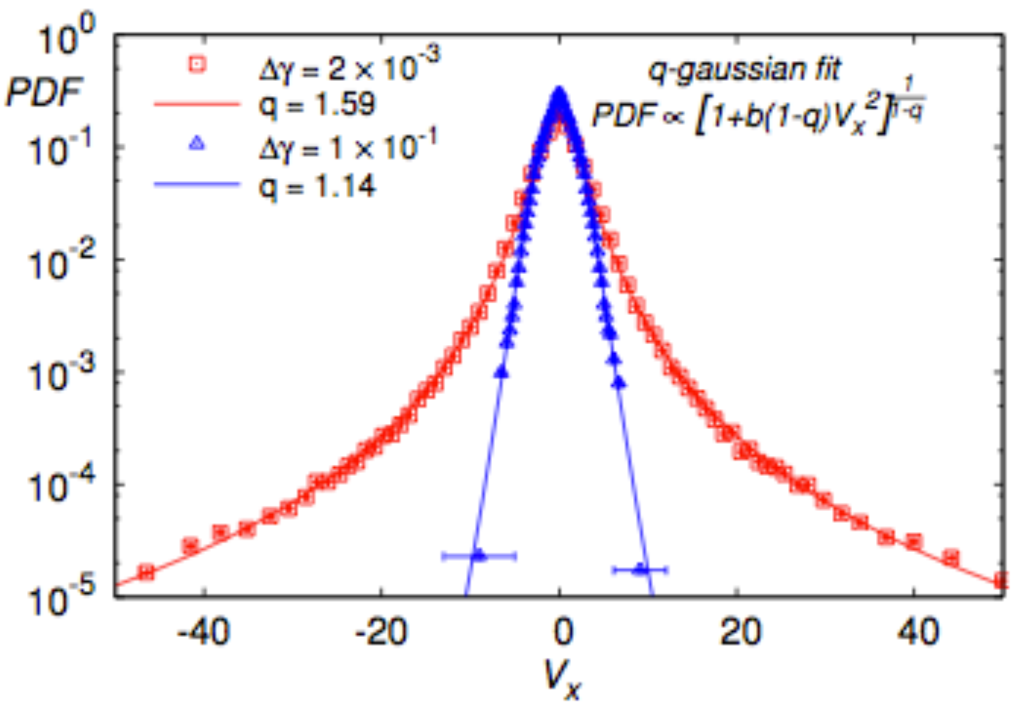


FIGURE 4. Probability density function (pdf) of normalized fluctuations V_x projected on the horizontal direction for two different strain windows, $\Delta\gamma = 2 \times 10^{-3}$ and $\Delta\gamma = 10^{-1}$.

PRL 105, 260601 (2010) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2010

Thermostatistics of Overdamped Motion of Interacting Particles

J. S. Andrade, Jr.,[1,3] G. F. T. da Silva,[1] A. A. Moreira,[1] F. D. Nobre,[2,3] and E. M. F. Curado[2,3]

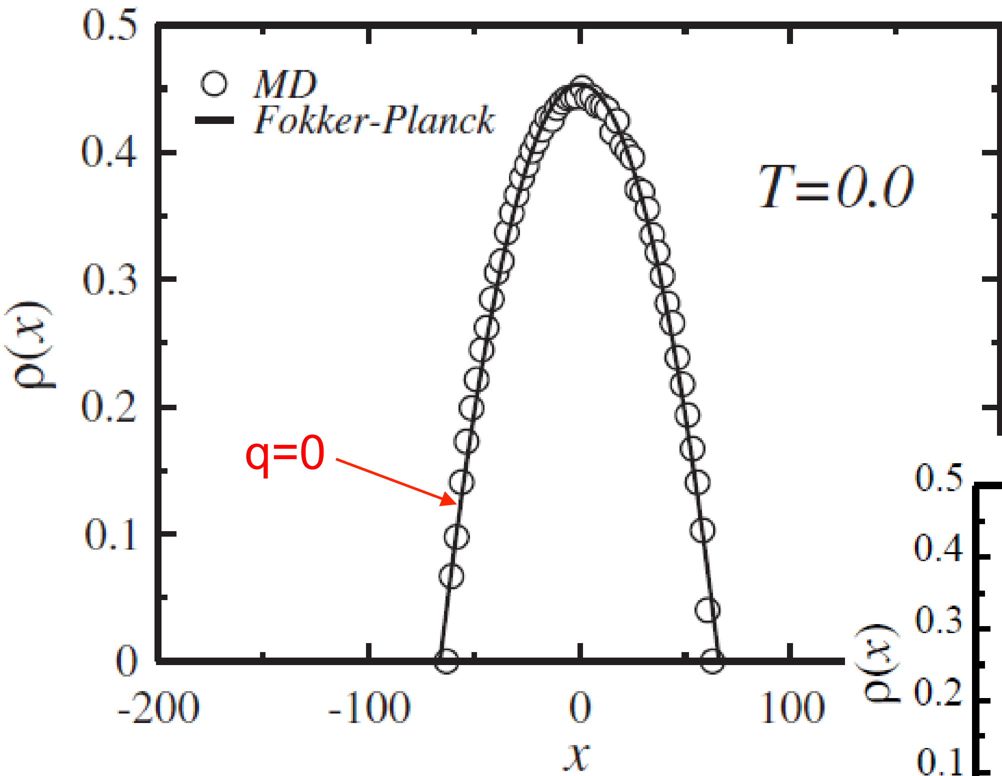
[1]*Departamento de Física, Universidade Federal do Ceará, 60451-970 Fortaleza, Ceará, Brazil*

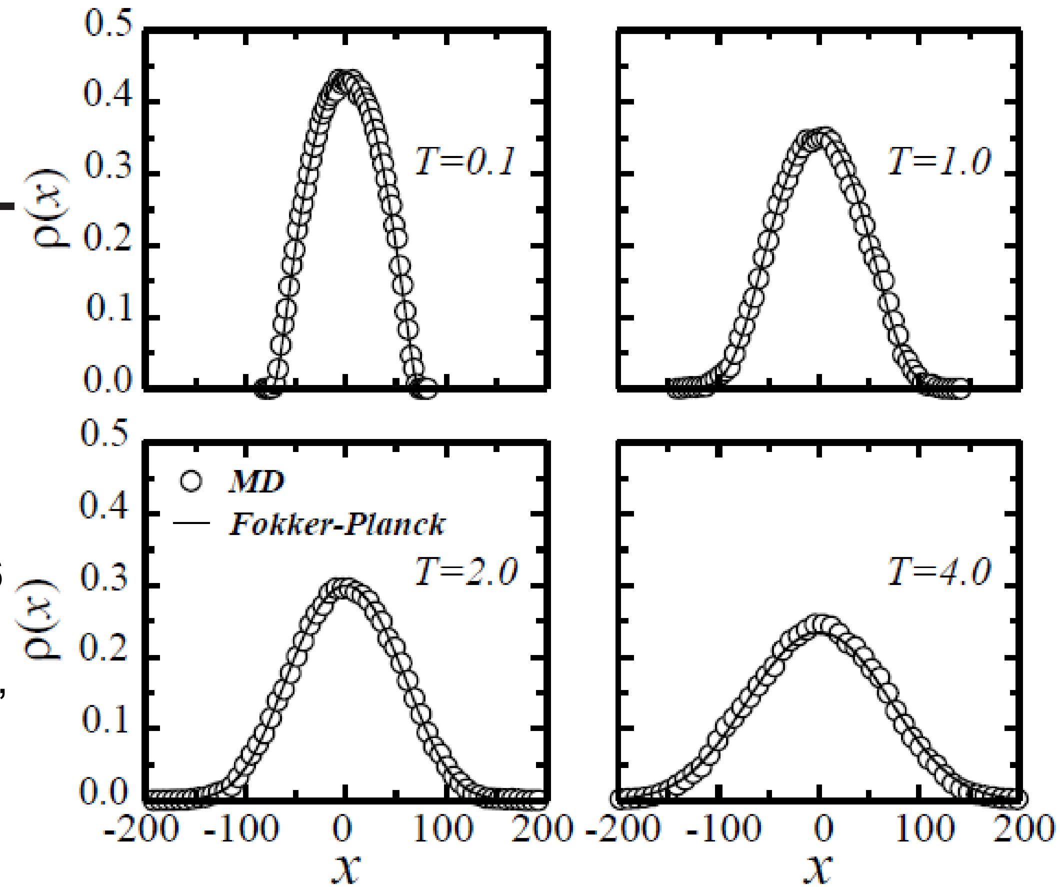
[2]*Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180, Rio de Janeiro-RJ, Brazil*

[3]*National Institute of Science and Technology for Complex Systems, Rua Xavier Sigaud 150, 22290-180, Rio de Janeiro-RJ, Brazil*

(Received 8 August 2010; published 22 December 2010)

We show through a nonlinear Fokker-Planck formalism, and confirm by molecular dynamics simulations, that the overdamped motion of interacting particles at $T = 0$, where T is the temperature of a thermal bath connected to the system, can be directly associated with Tsallis thermostatistics. For sufficiently high values of T, the distribution of particles becomes Gaussian, so that the classical Boltzmann-Gibbs behavior is recovered. For intermediate temperatures of the thermal bath, the system displays a mixed behavior that follows a novel type of thermostatistics, where the entropy is given by a linear combination of Tsallis and Boltzmann-Gibbs entropies.





See also:

Levin and Pakter, PRL **107**, 088901 (2011)

Andrade, Silva, Moreira, Nobre and Curado, PRL **107**, 088902 (2011)

Ribeiro, Nobre and Curado, PRE **85**, 121046 (2012)

Ribeiro, Nobre and Curado, Eur Phys J B **85**, 399 (2012)

Nobre, Souza and Curado, Phys Rev E **86**, 061113 (2012)

Andrade, Silva, Moreira, Nobre and Curado, Phys Rev Lett **105**, 260601 (2010)

PRL 105, 022002 (2010) PHYSICAL REVIEW LETTERS week ending 9 JULY 2010

Transverse-Momentum and Pseudorapidity Distributions of Charged Hadrons in pp Collisions at $\sqrt{s} = 7$ TeV

V. Khachatryan *et al.**
(CMS Collaboration)

(Received 18 May 2010; published 6 July 2010)