Yang-Mills Thermodynamics: an Effective Theory Approach

Chihiro Sasaki
Frankfurt Institute for Advanced Studies

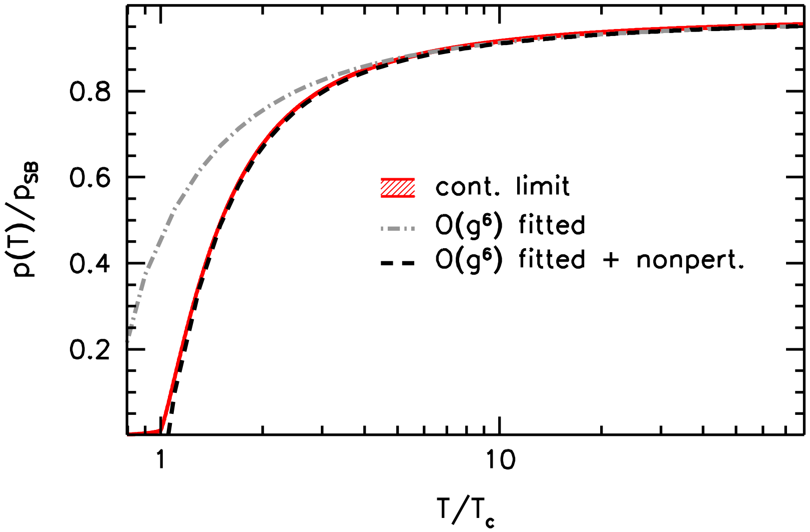
based on:

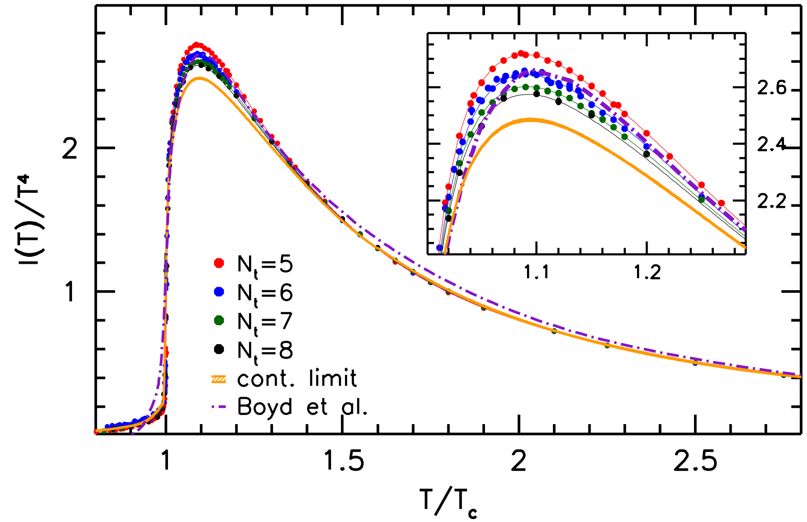
C.S. and K. Redlich, Phys. Rev. D **86**, 014007 (2012).
C.S., I. Mishustin and K. Redlich, arXiv:1308.3635 [hep-ph].

I. Modeling QCD Thermodynamics

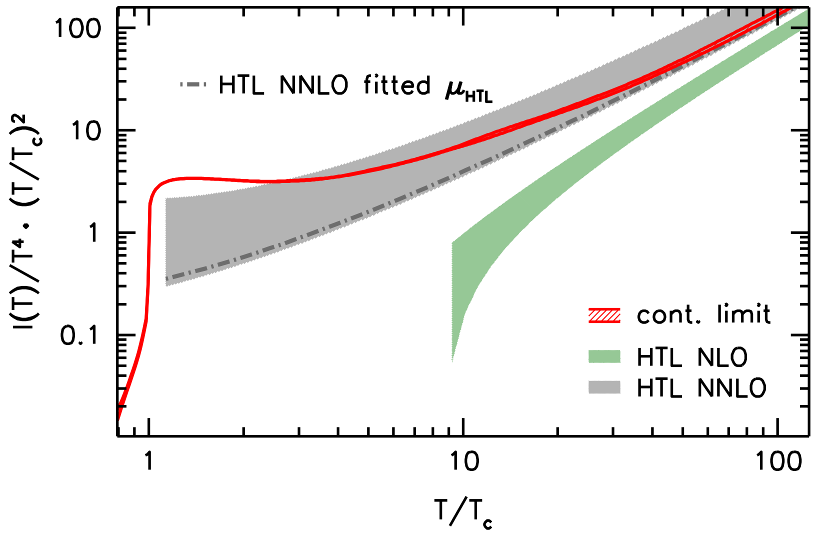
Yang-Mills thermodynamics from Lattice QCD [Borsanyi et al. (12)]

- EoS *does not reach SB limit.* $I = \mathcal{E} - 3P$ *does not vanish.*





- $I/T^2T_c^2 \sim$ *constant in intermediate temperatures.*



cf. $I^{\mathrm{pert}} \propto (gT)^4$

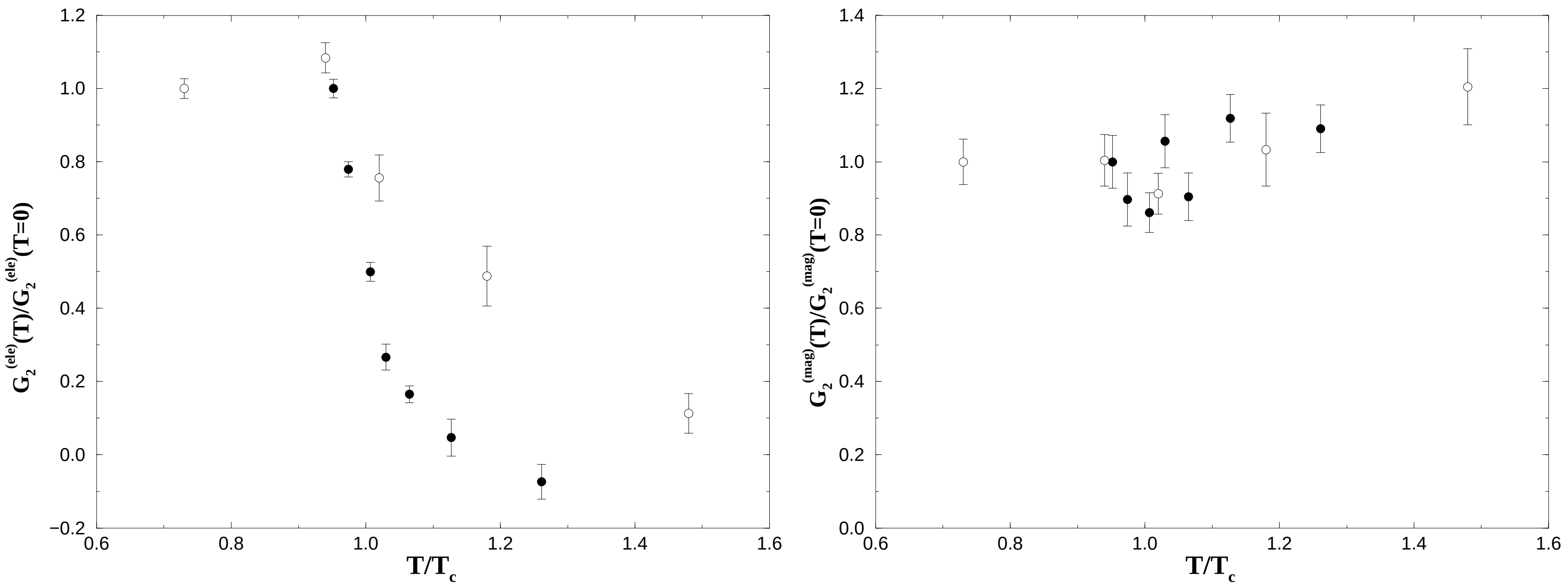
$\Rightarrow \quad I^{\mathrm{pert}}/T^2 \propto (gT)^2$

hot gluon matter in deconf. phase:

still nontrivial!

Residual interaction due to magnetic confinement

- *electric sector* $(G_{0i}^2 \sim E^2)$ changes qualitatively with T: deconfinement phase transition driven by electric gluons
- *magnetic sector* $(G_{ij}^2 \sim B^2)$ unaffected: [Borgs (85), Manousakis-Polonyi (87)] spatial string tension non-vanishing at any T $\Rightarrow$ confined



[D'Elia-Di Giacomo-Meggiolaro (02)]

- how to model these aspects? $\Rightarrow$ effective theory approach

- **Polyakov-loop model** [Pisarski (00)]
  - $Z(N_c)$ symmetries ($m_{\rm quark} \to \infty$) and Polyakov loop ($A_4 = iA_0$)

$$\hat{L}(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} d\tau A_4(\vec{x}, \tau) \right] , \quad \Phi = \mathrm{Tr}\hat{L}/N_c \,, \quad \bar{\Phi} = \mathrm{Tr}\hat{L}^\dagger/N_c$$

  * $\Phi \to e^{i\frac{2\pi j}{N_c}}\Phi$: its VEV is an order parameter of $Z(N_c)$ breaking
  * heavy-quark potential [McLerran-Svetitsky (81)]

$$\langle \Phi(\vec{x}) \rangle \sim e^{-F_q(\vec{x})/T} \begin{cases} = 0 & \text{confined phase} \\ \neq 0 & \text{deconfined phase} \end{cases}$$

  - effective potential ($N_c = 3$):

$$\mathcal{U} = a\bar{\Phi}\Phi + b(\bar{\Phi}^3 + \Phi^3) + c(\bar{\Phi}\Phi)^2 + \cdots$$

- **thermodynamics of pure gauge theory** (color-electric part) made based on $Z(3)$ symmetry:

$$\mathcal{U} = a(T)\bar{\Phi}\Phi + b(T)\left(\bar{\Phi}^3 + \Phi^3\right) + c(T)\left(\bar{\Phi}\Phi\right)^2 + \cdots$$

  - T-dep. coefficients *unconstrained* by $Z(3)$
    $\Rightarrow$ putting T-dep. by hand & fitting Lattice EoS **not unique!**
  - drawback: insufficient $\mathcal{U}$ for fluctuations [CS-Friman-Redlich (06)]
  - where T-dep. comes from? $\cdots$ thermal gluon excitations $\sim \mathcal{L}_{\rm YM}$
    $\Rightarrow$ closer contact with the underlying theory

- **issues**

  - how thermal gluon distributions appear from $\mathcal{L}_{\rm YM}$?
  - how to model the interplay between chromoelectric $(deconf.)$ and chromomagnetic $(conf.)$ aspects of YM?

II. Effective Gluon Potential

Deriving partition function from YM Lagrangian

- background field method, a constant uniform background $\bar{A}_0$

$$A_\mu = \bar{A}_\mu + g\check{A}_\mu$$
$$\bar{A}^a_\mu = \bar{A}^a_0 \delta_{\mu 0}\,, \quad \bar{A}_0 = \bar{A}^3_0 T^3 + \bar{A}^8_0 T^8$$

$$\sum_n \ln\det\left(D^{-1}\right) = \ln\det\left(1 - \hat{L}_A e^{-|\vec{p}|/T}\right)$$

[Gross-Pisarski-Yaffe (81)]

- Polyakov loop matrix in adjoint representation (8x8 matrix)

$$\hat{L}_A = \text{diag}\left(1\,,1\,,e^{i(\phi_1-\phi_2)}\,,e^{-i(\phi_1-\phi_2)}\,,e^{i(2\phi_1+\phi_2)}\,,e^{-i(2\phi_1+\phi_2)}\,,e^{i(\phi_1+2\phi_2)}\,,e^{-i(\phi_1+2\phi_2)}\right)$$

rank of SU(3) group $= 2$ $\Rightarrow$ elements expressed in 2 variables

- express in terms of

$$\Phi = \text{tr}\hat{L}_F/3\,,\ \bar{\Phi} = \text{tr}\hat{L}_F^\dagger/3$$

- full thermodynamic potential:

$$\Omega = \underbrace{\Omega_g}_{\sim a(T)\bar{\Phi}\Phi?} + \underbrace{\Omega_{\text{Haar}}}_{\text{responsible for Z(3) breaking}}$$

- full thermodynamics potential: $\Omega = \Omega_g + \Omega_{\rm Haar}$

$$\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \ln \left(1 + \sum_{n=1}^{8} C_n(\Phi, \bar{\Phi})\, e^{-n|\vec{p}|/T} \right) ,$$

$$\Omega_{\rm Haar} = -a_0 T \ln \left[1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2 \right] ,$$

$$C_1 = C_7 = 1 - 9\bar{\Phi}\Phi\,, \quad C_2 = C_6 = 1 - 27\bar{\Phi}\Phi + 27\left(\bar{\Phi}^3 + \Phi^3\right) ,$$

$$C_3 = C_5 = -2 + 27\bar{\Phi}\Phi - 81\left(\bar{\Phi}\Phi\right)^2 ,$$

$$C_4 = 2\left[-1 + 9\bar{\Phi}\Phi - 27\left(\bar{\Phi}^3 + \Phi^3\right) + 81\left(\bar{\Phi}\Phi\right)^2 \right] , \quad C_8 = 1$$

$\Rightarrow$ energy distributions solely determined by group characters of SU(3)

– no free parameter in Ω_g

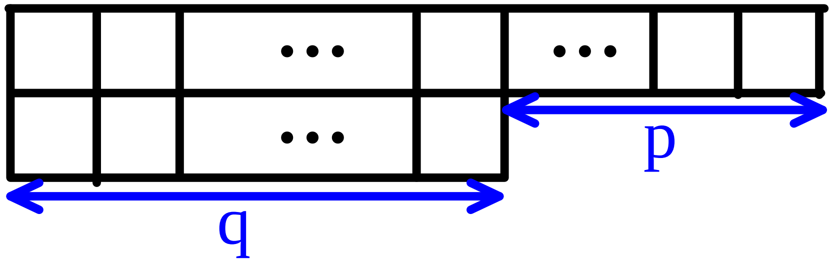
– one parameter in $\Omega_{\rm Haar}$: $a_0 \Leftrightarrow T_c^{\rm lat} = 270$ MeV

Character expansion of Ω_g

- **all the phenomenological potentials deduced from Ω_g!**
- effective action in the strong coupling exp. [Wozar-Kaestner-Wipf-Heinzl-Pozsgay (06)]

$$\mathcal{S}_{\rm eff}^{\rm (SC)} = \lambda_{10} S_{10} + \lambda_{20} S_{20} + \lambda_{11} S_{11} + \lambda_{21} S_{21}$$

S_{pq}: products of SU(3) characters $\sim$ a series of $Z(3)$-inv. operators



$$C_{1,7} = S_{10}\,, \quad C_{2,6} = S_{21}\,,$$
$$C_{3,5} = S_{11}\,, \quad C_4 = S_{20}$$

- a "minimal" model: $\mathcal{S}_{\rm eff} = \lambda S_{10} \sim \lambda \bar{\Phi}\Phi$ plus $\mathcal{S}_{\rm Haar}$
  $\Rightarrow$ 1st-order phase transition
- coefficient λ can be deduced from Ω_g! $\quad \Omega_g \simeq \mathcal{F}(T)\bar{\Phi}\Phi$
  - correct sign $\Rightarrow$ a 1st-order phase transition
  - strength of the phase transition $\Rightarrow$ $\langle \Phi \rangle_c^{\rm lattice} \simeq 0.4$
    $\langle \Phi \rangle_c = 0.39$ from Ω_g

III. Thermodynamics

Thermodynamics

- any finite temperature in confined phase: $\Phi = 0$ thus $\Omega_{\rm Haar} = 0$

$$\Omega_g(\Phi = \bar{\Phi} = 0) \sim 2T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-|\vec{p}|/T}\right)$$

wrong sign! $\Rightarrow$ unphysical EoS $s, \epsilon < 0$

Gluons are NOT correct dynamical variables below T_c!

- cf. PNJL/PQM: quarks are suppressed but exist at any T.

[Meisinger-Ogilvie (96), Fukushima (03), Ratti et al. (06)]

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ - A\!\!\!/\right) q + G\left(\bar{q}q\right)^2 - \mathcal{U}(\bar{\Phi}, \Phi)\,, \quad A_\mu = \delta_{\mu 0} A^0$$

$$\Omega_q = -d_q\, T \int \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3\Phi e^{-E_+/T} + 3\bar{\Phi} e^{-2E_+/T} + e^{-3E_+/T}\right]$$

$\langle \Phi \rangle \simeq 0$ at low T: 1- and 2-quark states *thermodynamically* irrelevant
$\Rightarrow$ *mimicking confinement*

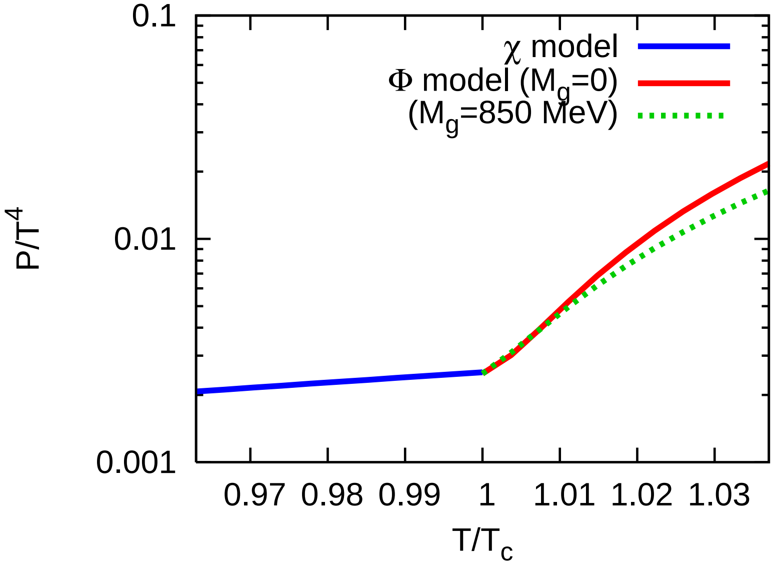
A hybrid approach for Yang-Mills thermodynamics

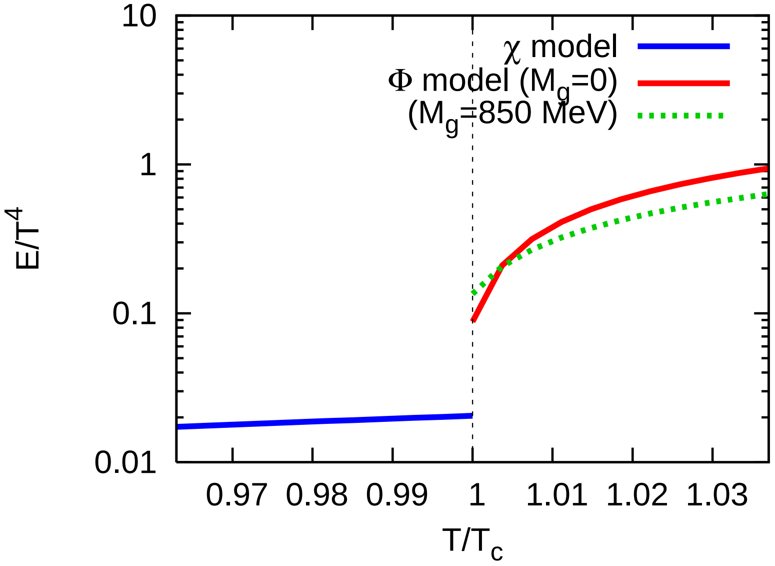
- below T_c: no gluons but **glueballs** $\Rightarrow$ introduce dilatons χ
- **QCD trace anomaly**:
  $T^\mu_\mu \sim \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle$ scale symmetry breaking due to gluon condensate
  $\Rightarrow$ this is encoded with dilaton potential such that $T^\mu_\mu \sim \chi^4$ [Schechter (80)]

$$V_\chi = \frac{B}{4} \left(\frac{\chi}{\chi_0} \right)^4 \left[\ln \left(\frac{\chi}{\chi_0} \right)^4 - 1 \right]$$

- switching dynamical variables at T_c

$$\Omega = \Theta(T_c - T)\, \Omega(\chi) + \Theta(T - T_c)\, \Omega(\Phi)$$





IV. Chromoelectric and Chromomagnetic Dynamics

Chromoelectric vs. chromomagnetic gluons

- electric sector ($G_{0i}^2 \sim E^2$) changes qualitatively with T: electric screening
  deconfinement phase transition driven by electric gluons
- magnetic sector ($G_{ij}^2 \sim B^2$) unaffected: no magnetic screening
  string tension non-vanishing at any T [Borgs (85), Manousakis-Polonyi (87)]

dilatons can survive above T_c!

$\Phi \Leftrightarrow A_0$: electric $\chi \Leftrightarrow G_{\mu\nu}G^{\mu\nu}$: electric plus magnetic

- **how to transmute $\langle E^2 \rangle$-$\langle B^2 \rangle$ interaction into Φ-χ interaction?**
  - effective theory above T_c:

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_\chi + \underbrace{\mathcal{L}_{\rm mix}}_{\sim\chi^4\left(a\bar{\Phi}\Phi + b\left(\bar{\Phi}^3 + \Phi^3\right) + \cdots\right)}$$

  - effective gluon mass? $\sim G^2\chi^2 A_\mu A^\mu$ would appear when hard modes integrated out $\cdots$ $m_g \sim \langle B^2 \rangle$ in deconfined phase

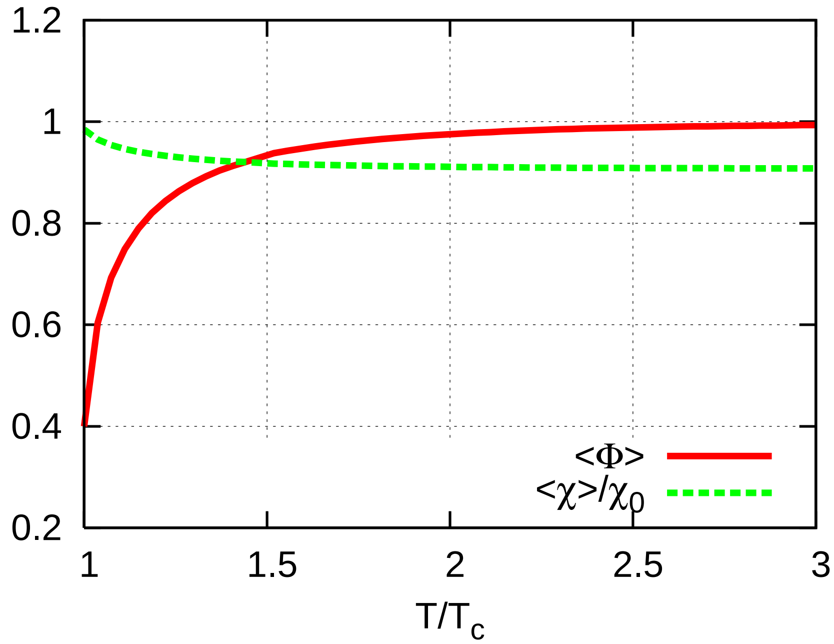
An effective theory [CS-Mishustin-Redlich (13)]

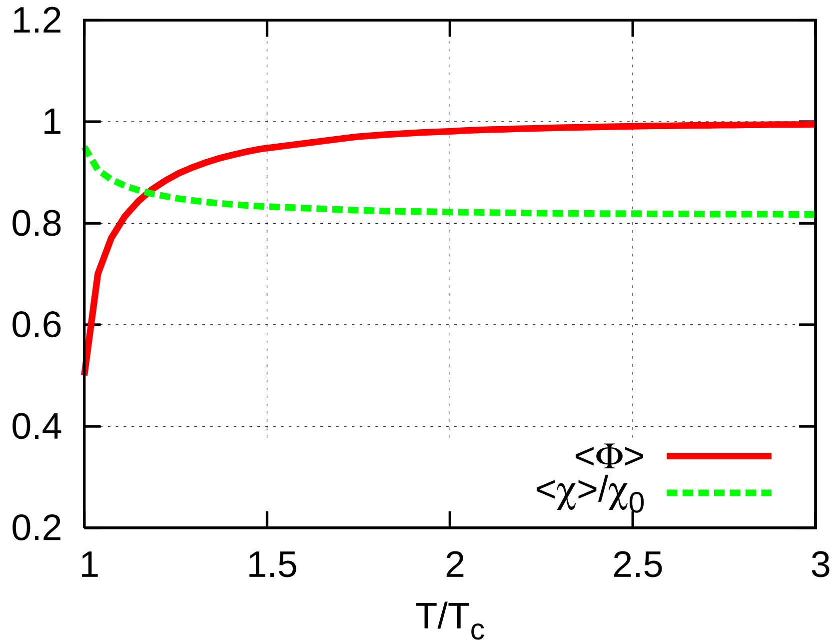
- effective potential

$$\Omega(\Phi,\chi;T) = \Omega_g(\Phi;T) + \Omega_{\mathrm{Haar}}(\Phi;T) + V_\chi(\chi) + V_{\mathrm{mix}}(\Phi,\chi)\,,$$

$$V_{\mathrm{mix}}(\Phi,\chi) = G_{\phi\chi}\left(\frac{\chi}{\chi_0}\right)^4 \bar{\Phi}\Phi$$: invariant under Z(3) and scale sym.

- two condensates: $\chi/\chi_0 = \exp\left[-G_{\phi\chi}\bar{\Phi}\Phi/B\right]$





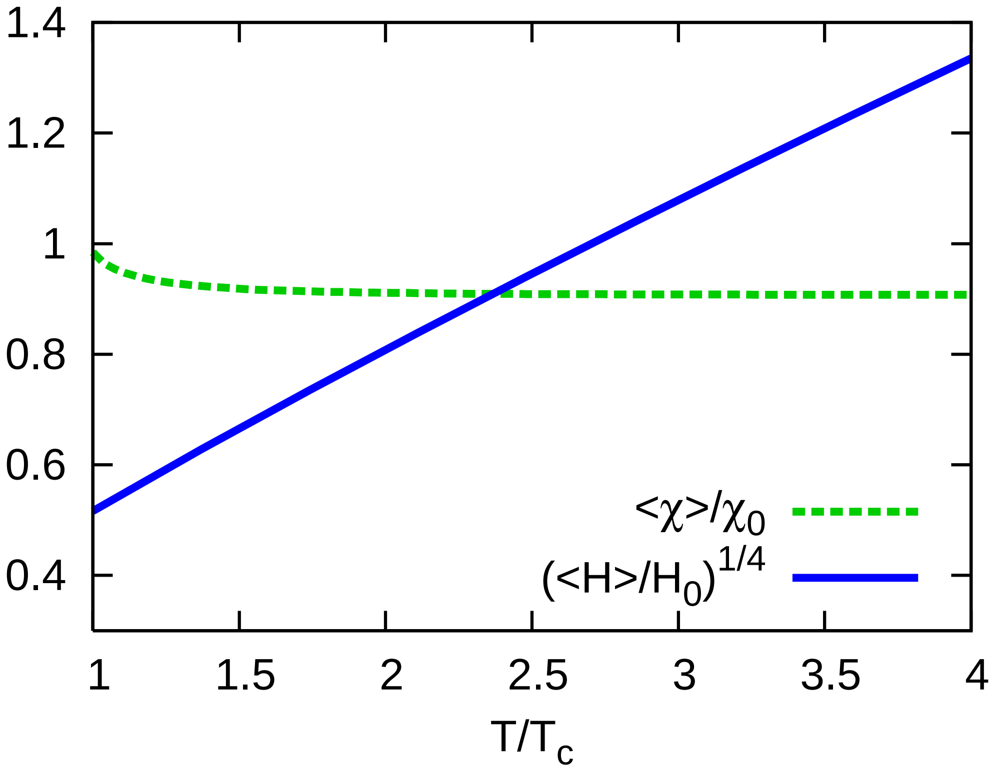
- $\langle\chi\rangle$ at higher temperature? $\cdots$ magnetic scale $g^2(T)T$!

- residual interaction at high temperature: *magnetic confinement*
  - mathcing to 3d YM theory (high T effective theory)

$$\mathcal{D} \sim \langle B^2 \rangle e^{-|\vec{x}|/\xi}, \quad \sigma_s \sim \langle B^2 \rangle \xi^2 \quad \Leftrightarrow \quad \text{3-dim YM: } \sqrt{\sigma_s} = c g^2(T) T$$

$$\langle B^2 \rangle = c_B \left(g^2(T) T \right)^4 \qquad \text{[Agasian (03)]}$$

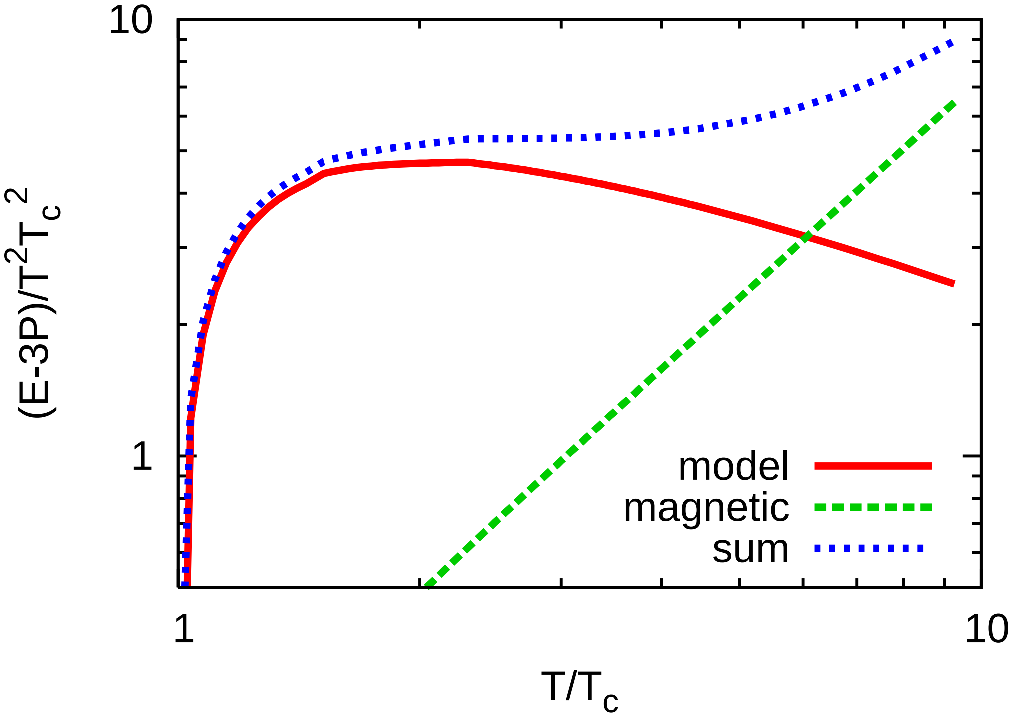
  - identify $\langle B^2 \rangle$ with $\langle \chi \rangle^4$ at $T \gg T_c$: $V(\chi) \to V(g^2(T)T)$

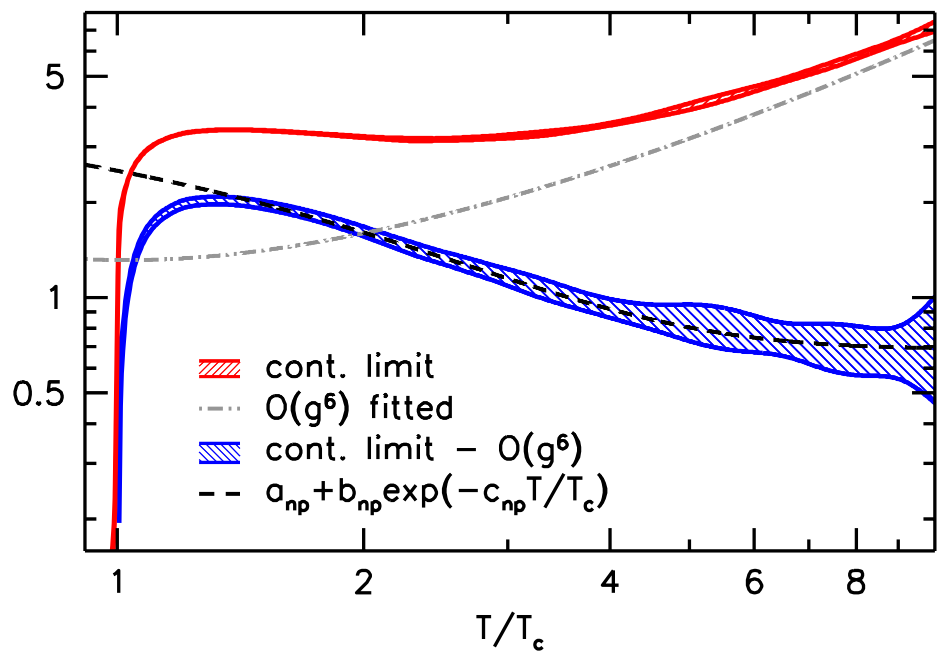


at which T does $\langle \chi \rangle$ start to go like $g^2(T)T$?
$\Rightarrow$ level crossing at $T_0 \sim 2.5\,T_c$

$H \equiv B^2$

- interaction measure: this model (L) & lattice [Borsanyi et al. (12)] (R)





$$\Omega = \underbrace{\Omega_g}_{\sim T^4} + \underbrace{\Omega_{\rm Haar}}_{\sim T} + \underbrace{V_{\chi+{\rm mix}}}_{\sim{\rm const}\,\Rightarrow\,\sim(g^2T)^4}$$

$$I_1(T<T_0) = \mathcal{E} - 3P \sim \Omega_g \sim 0\times T^4 \quad\Rightarrow\quad I_2(T_0<T)\sim V_{\chi+{\rm mix}}\sim T^4$$

− decreasing I_1/T^2 + increasing $I_2/T^2 \Rightarrow I/T^2 \sim$ constant

− tendency of $I^{\rm lat}/T^2$ is reproduced.

Summary

- **derivation of gluon partition function from YM Lagrangian**
  - higher representations of Polyakov loop: mean field treatment
  - Polyakov loops naturally appear representing group character.
  - gluons are forbidden below T_c dynamically.
  - a hybrid approach.
- **interplay between chromoelectric and chromomagnetic gluon dynamics**
  - identify $\langle B^2 \rangle$ with $\langle \chi^4 \rangle$
  - matching to 3-dim theory: magnetic scale $g^2(T)T$ comes in.
  - reproducing lattice interaction measure I/T^2