GRAVITY INDUCED BY QUANTUM SPACETIME

Shahn Majid, Queen Mary Univ of London

(new part is joint with E.J. Beggs, arXiv:1305.2403)

2nd International Conference on New Frontiers in Physics, Crete 9/2013

- Review of flat space quantum spacetime, VSL etc
- Why this does not work ...

  ➡ Why there is gravity!

  ➡ Illustrates rigidity of even a small amount of `quantum' noncommutavity; can explain things otherwise unconnected in physics

- nice example of quantum Riemannian geometry

Review of quantum spacetime

- next-to-classical emergent geometry should appear as O(λ) `quantum' corrections to spacetime, λ=Planck scale

Classical geometry ⟵ Quantum diffl geometry ⟵ Quantum Gravity???

- avoids the `continuum problem' (fuzzy below the planck scale)

- most-studied 1994 `Majid-Ruegg' bicrossproduct model

$$[x_i, t] = \imath\lambda x_i, \quad [x_i, x_j] = 0$$

space, time not simultaneously measurable

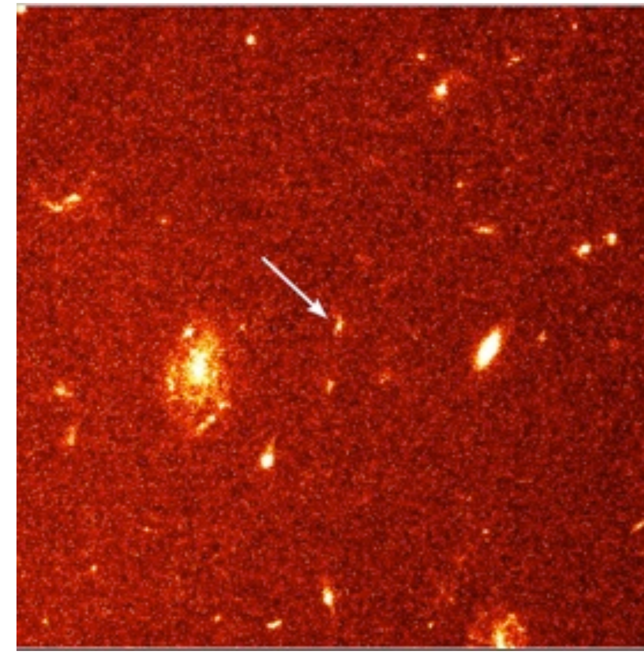
⟹ quantum Poincare symmetry $H = U(so(1,3))\triangleright\!\!\blacktriangleleft\mathbb{C}[\mathbb{R}\triangleright\!\!<\mathbb{R}^3]$

$$[p^i, N_j] = -\frac{\imath}{2}\delta^i_j\left(\frac{1-e^{-2\lambda p^0}}{\lambda}+\lambda\vec{p}^2\right)+\imath\lambda p^i p_j,$$

cf SM 1988
cf. Lukierski et al '91

wave operator on plane waves $e^{i\vec{x}\cdot\vec{p}}e^{itp_0}$

$$||p||^2_\lambda = \vec{p}^2 e^{-\lambda p^0} - \frac{2}{\lambda^2}(\cosh(\lambda p^0) - 1)$$



variable Speed Light

$$|\frac{\partial p^0}{\partial p^i}| = e^{-\lambda p^0}$$

$$\Delta_T \sim \lambda \Delta_{p^0} \frac{L}{c} \sim 10^{-44}\,\mathrm{s} \times 100\,\mathrm{MeV} \times 10^{10}\,\mathrm{y} \sim 1\,\mathrm{ms},$$

- differential arrival time of gamma-ray bursts is experimentally testable by Fermi-Glast satellite (SM+GAC'2000)
- momentum space (given by product of plane waves) is a nonabelian group or `curved'

	Position	Momentum
Gravity	Curved	Noncommutative
Cogravity	Noncommutative	Curved
Quantum Gravity	Both	Both

Quantum gravity restores `quantum Born reciprocity' (SM '88)

- space of 1-forms, i.e. `differentials dx'

Ω^1 a((db)c)=(a(db))c `bimodule'

$\mathrm{d}: A \to \Omega^1$ d(ab)=(da)b+a(db) `Leibniz rule'

$\{a\mathrm{d}b\} = \Omega^1$ $\ker \mathrm{d} = \mathbb{C}.1$ connectedness(optional)

$$\Omega = \langle A, \Omega^1 \rangle / \ldots = \oplus_n \Omega^n$$

- For bicrossproduct model A is $[x_i, t] = \imath\lambda x_i$ algebra and

$$\Omega^1 = A.\{\mathrm{d}x_i, \mathrm{d}t\}$$

$$[\mathrm{d}x_i, x_j] = 0 = [\mathrm{d}x_i, t], \quad [(\ ), \mathrm{d}t] = \imath\lambda \mathrm{d}(\ )$$

$$\mathrm{d}f(x_i, t) = \frac{\partial f}{\partial x_i}\mathrm{d}x_i + (\partial_0 f)\mathrm{d}t, \quad \partial_0 f = \frac{f(x_i, t) - f(x_i, t - \imath\lambda)}{\imath\lambda}$$

$\partial_0^2 - \sum_i (\frac{\partial}{\partial x_i})^2$ ⟹ nc plane waves ⟹ VSL as before

SM+GAC '00

Quantum Riemannian Geometry

bimodule connection $\nabla : \Omega^1 \to \Omega^1 \underset{A}{\otimes} \Omega^1$ $\sigma : \Omega^1 \underset{A}{\otimes} \Omega^1 \to \Omega^1 \underset{A}{\otimes} \Omega^1$

(Michor, Dubois-Violette)

$$\nabla(f\omega) = \mathrm{d}f \otimes \omega + f\nabla\omega \qquad \nabla(\omega f) = \sigma(\omega \otimes \mathrm{d}f) + (\nabla\omega)f$$

action on 2-tensor

$$\omega \otimes \eta \in \Omega^1 \underset{A}{\otimes} \Omega^1 \qquad \nabla(\omega \otimes \eta) = \nabla\omega \otimes \eta + (\sigma \otimes \mathrm{id})(\omega \otimes \nabla\eta)$$

metric $g \in \Omega^1 \underset{A}{\otimes} \Omega^1$ $\wedge(g) = 0$ $\nabla g = 0$

$$(\ ,\ ) : \Omega^1 \underset{A}{\otimes} \Omega^1 \to A \qquad ((\ ,\ ) \otimes \mathrm{id})g = (\mathrm{id} \otimes (\ ,\ ))g = \mathrm{id} : \Omega^1 \to \Omega^1$$

curvature $R_\nabla : \Omega^1 \to \Omega^2 \underset{A}{\otimes} \Omega^1$

$$R_\nabla = (\mathrm{d} \underset{A}{\otimes} \mathrm{id} - (\wedge \underset{A}{\otimes} \mathrm{id})(\mathrm{id} \underset{A}{\otimes} \nabla))\nabla$$

torsion $T_\nabla : \Omega^1 \to \Omega^2$ $T_\nabla = \wedge\nabla - \mathrm{d}$

to be able to contract/ `raise/lower' via metric we need it to be central i.e. (,) a bimodule map to have well defined contraction

1+1 dimensional model

Look at 1+1 bicrossproduct model

$$[r,t]=\lambda r \qquad [r,\mathrm{d}t]=\lambda\mathrm{d}r, \quad [t,\mathrm{d}t]=\lambda\mathrm{d}t$$

Theorem: this has up to normalisation a 1-parameter moduli of quantum metrics, $b\in\mathbb{R}$, $b\neq 0$

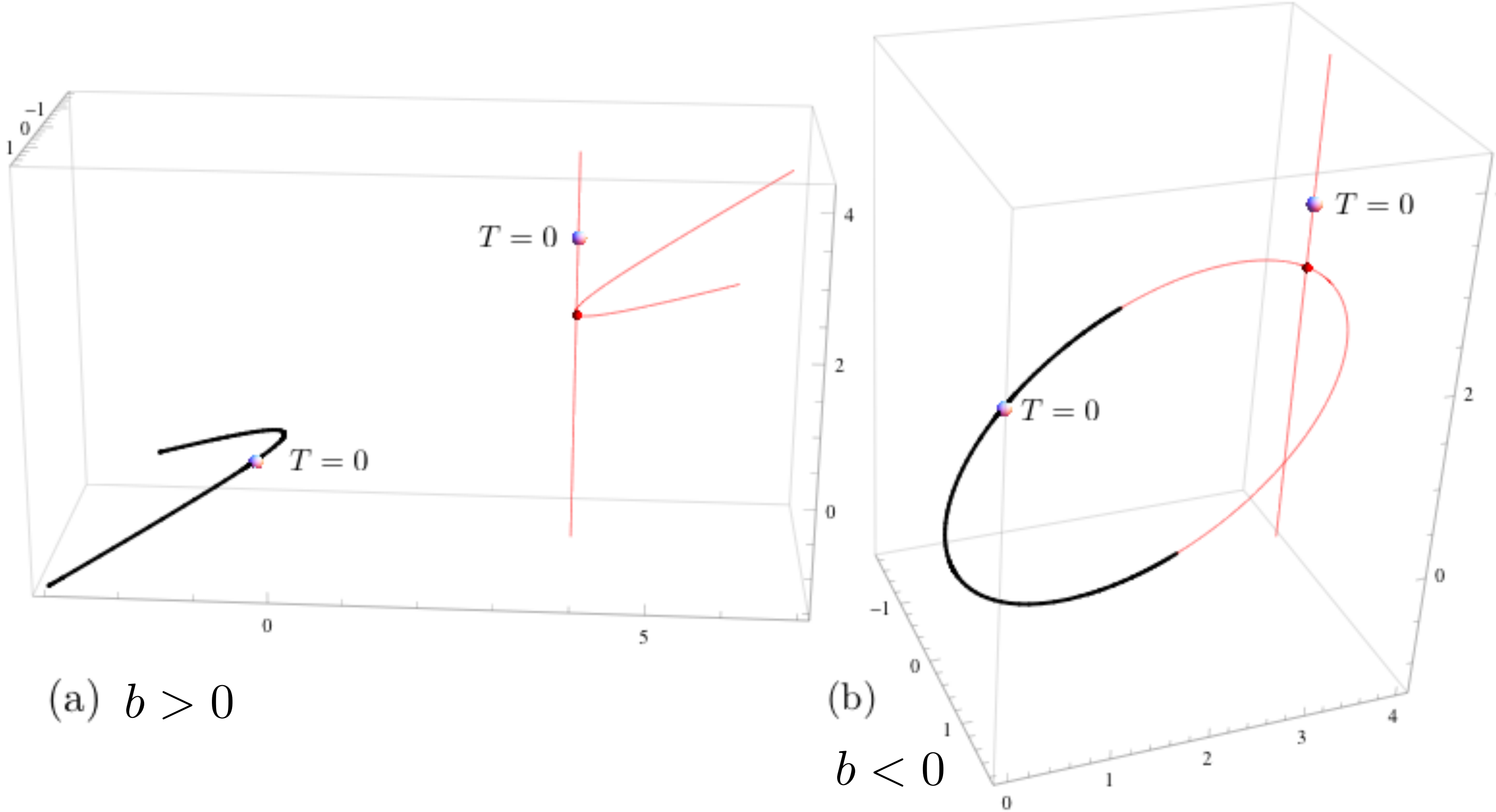
$$g=\mathrm{d}r\otimes\mathrm{d}r+b(v^*\otimes v+\lambda(\mathrm{d}r\otimes v-v^*\otimes\mathrm{d}r))$$

$v=r\mathrm{d}t-t\mathrm{d}r$, $v^*=(\mathrm{d}t)r-t\mathrm{d}r$

(mainly get this from $[g,r]=0,\ [g,t]=0$ and some implicit reality)

- for fixed b lets solve for the quantum Levi-Civita connection
- and compute its Ricci curvature

- moduli of real metric-compatible ∇ form a line + conic



- black parts have classical limit as $\lambda \to 0$
- red parts blow up as $\lambda \to 0$ so not visible classically
- in each case a unique `Levi-Civita point' where torsion T=0

- formulae for unique Levi-Civita soln with classical limit

$$\nabla \mathrm{d}r = \frac{1}{r}\left(v - \frac{\lambda \mathrm{d}r}{2}\right) \otimes \left((\frac{8b}{4+7b\lambda^2})v - (\frac{12b\lambda}{4+7b\lambda^2})\mathrm{d}r \right)$$

$$\mathrm{Ricci} = \left(\frac{4+7b\lambda^2}{4-9b\lambda^2}\right)\frac{g}{r^2}$$

$$\mathrm{Ricci} - \frac{1}{(\ ,\ )(g)} S = 0, \quad S = (\ ,\ )\mathrm{Ricci} \qquad (\ ,\ )(g) = \frac{2+b\lambda^2}{1+b\lambda^2}$$

quantum dimension

⇒ `quantum Einstein'=0

⇒ `usual Einstein' $\mathrm{Ricci} - \frac{1}{2}S = b\lambda^2 \frac{g}{2r^2} + O(\lambda^3)$

nonconstant vacuum energy??

<u>Aside:</u> $\mathrm{Ricci} = ((\ ,\ ) \otimes \mathrm{id})(\mathrm{id} \otimes i \otimes \mathrm{id})((\mathrm{id} \otimes R)(g)$

where $i : \Omega^2 \to \Omega^1 \otimes_A \Omega^1$ is uniquely determined by $\wedge i = \mathrm{id}$ and requirement that Ricci has same symmetry and reality as the metric

- formulae for unique Levi-Civita soln without classical limit

$$\nabla \mathrm{d}r = \frac{bv}{r} \otimes \left((\frac{1}{1+b\lambda^2})v - (\frac{2}{\lambda})\mathrm{d}r \right) + (\frac{2+b\lambda^2}{r(1+b\lambda^2)})\mathrm{d}r \otimes \left(-(\frac{1}{\lambda})v + (\frac{3}{2})\mathrm{d}r \right)$$

blows up as $\lambda \to 0$ and its entangled

$$\mathrm{Ricci} = -3\frac{(4+3b\lambda^2)(-2+b\lambda^2)}{20r^2\lambda^2\left(1+b\lambda^2\right)}(v^* \otimes v + \lambda(\mathrm{d}r \otimes v - v^* \otimes \mathrm{d}r) + \frac{\left(1+b\lambda^2\right)\left(14+3b\lambda^2\right)}{b\left(-2+b\lambda^2\right)}\mathrm{d}r \otimes \mathrm{d}r)$$

nothing like the metric

It all works in 1+1as toy model of `quantum' Riemannian geometry, but the metric can't be flat and there is a second non-perturbative solution for the Levi-Civita connection

3+1 dimensional model $[x_i, t] = \imath\lambda x_i, \quad [x_i, x_j] = 0$

Theorem: no central metrics exist at all on this differential algebra

but can solve restricted to functions in $r = |\vec{x}|, t$
=> 2-parameter family $a, b \in \mathbb{R}$

$$g = r^2 \mathrm{d}\Omega + a\mathrm{d}r \otimes \mathrm{d}r + b\left(v^* \otimes v + \lambda(\mathrm{d}r \otimes v - v^* \otimes \mathrm{d}r\right))$$

=> Forced classically to

$$G_{ij} = R_{ij} - \tfrac{1}{2} S\, g_{ij} = \begin{pmatrix} \left(\frac{1}{a} - 1\right) b & \frac{(a-1)bt}{ar} & 0 & 0 \\ \frac{(a-1)bt}{ar} & \frac{-a^2 - bt^2 a + 5a + bt^2}{ar^2} & 0 & 0 \\ 0 & 0 & \frac{4}{a} & 0 \\ 0 & 0 & 0 & \frac{4\sin^2(p)}{a} \end{pmatrix}$$

$$\left.\begin{aligned} G^{ij} &= \frac{8\pi G}{c^4} T^{ij} \\ T^{ij} &= p\, g^{ij} + (p+\rho)\, u^i u^j \end{aligned}\right\} \Rightarrow$$

a=1: $p = 1/(2\pi G r^2), \quad \rho = 0$

a=-3: $p = -1/(6\pi G r^2), \quad \rho = 1/(3\pi G r^2)$

(cf quintessence at $\frac{p}{\rho} = -\frac{1}{2}$)

=> null geodesics spiral out of r=0 to r=0 / r=infinity respectively.
=> metric conformal scaling of flat $\mathbb{R}^{1,3}$ metric in new coordinates.