Quantumness of Gaussian discord

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QOI Quantum Optics & Quantum Information

SUPA

Disclaimer:

I'll speak about quantum correlations

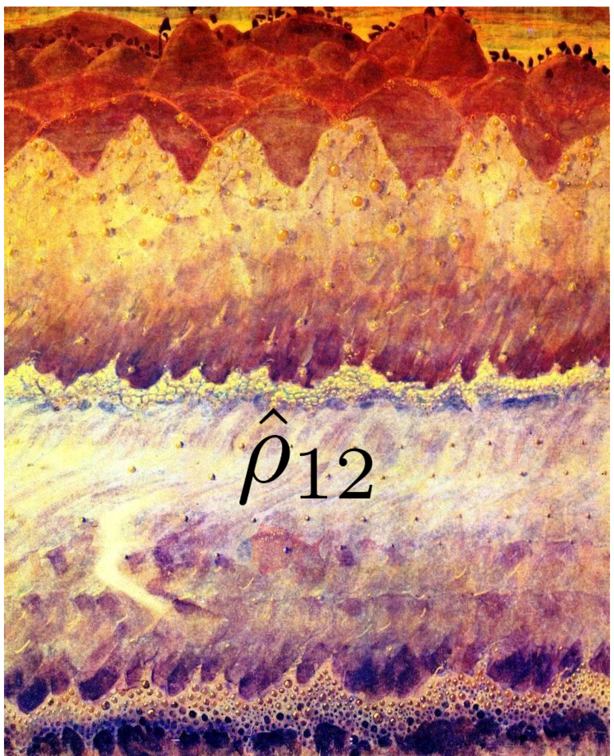
in and beyond entanglement.

They can be quantified e.g. by quantum discord.

In what follows I loosely use "discord" for
"quantum correlations in and beyond entanglement"

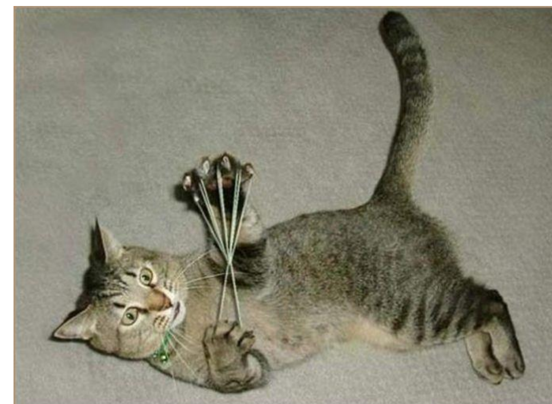
Mixed states:

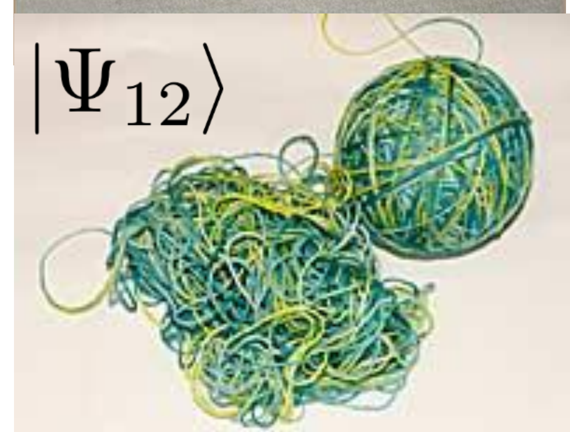
- entangled (and discordian)
- separable and have non-zero discord
- separable, no discord



Pure states:

- entangled
- separable





Entanglement – nonlocal bi-partite superposition state

$$\hat{\rho} = \sum_i p_i \hat{\rho}_{1i} \otimes \hat{\rho}_{2i}$$

- separable

$$\hat{\rho} \neq \sum_i p_i \hat{\rho}_{1i} \otimes \hat{\rho}_{2i}$$

- entangled

Bell basis

$$|\Psi^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}} \left(|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B \right)$$

$$|\Phi^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}} \left(|0\rangle_A |0\rangle_B \pm |1\rangle_A 1\rangle_B \right)$$

Entanglement $\longleftrightarrow$ **Superposition** $|0\rangle_A|1\rangle_B \pm |1\rangle_A|0\rangle_B$

Quantumness $\longleftrightarrow$ **Noncommutativity of observables**

measurements !

Entanglement/separability

Classical corrlations/quantum correlations *including* entanglement

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001);
L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

S. Luo, Phys. Rev. A 77, 022301 (2008)

Protocols without entanglement outperforming classical ones???

For example: NMR computing;
DQC1 - deterministic quantum computation with one quantum bit,
Knill and Laflamme, Phys. Rev. Lett. **81**, 5672 (1998).

NB: For pure-state:
the unbounded growth of entanglement with the system size required for exponential speed up over classical computation (Jozsa, Linden 2003)

Quantum correlations:
statistical correlations in the quantum scenario
more general than entanglement;
origin – quantum uncertainty; noncommutativity of observables

$$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{I}_q(\hat{\rho}_{AB}) - \mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB}) + \inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)$$

Quantum discord: (quantum mutual information) - (one way classical correlation)

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001); L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

Classically - equivalent definitions of mutual information:

$$\begin{aligned} I(A:B) &= H(A) + H(B) - H(A,B) = \\ J(A:B) &= H(A) - H(A|B) = \\ J(B:A) &= H(B) - H(B|A) \end{aligned}$$

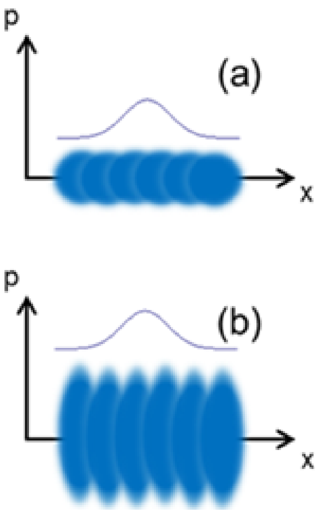
Shannon entropy: $H(A)$

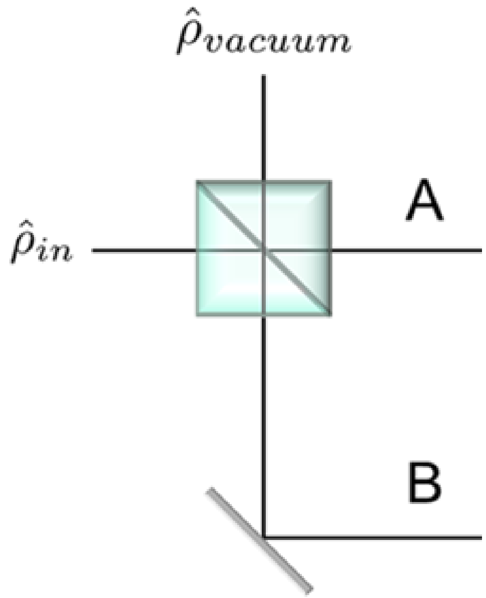
Conditional: $H(A|B)$

Quantum – they are not equivalent; mutual information:

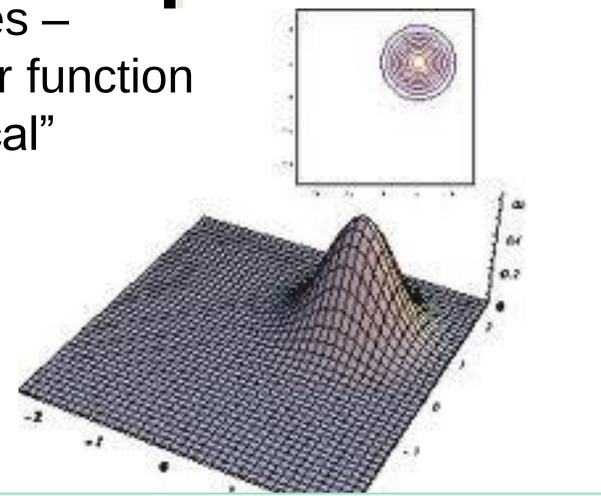
$$\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB}) \quad \longleftrightarrow \quad I(A:B)$$

von Neumann entropy: $S(\hat{\rho}_{AB})$





Gaussian states – positive Wigner function – “most classical”



$$\hat{\rho} = \int\int d^2\alpha d^2\beta P(\alpha,\beta)|\alpha\rangle\langle\alpha| \otimes |\beta\rangle\langle\beta|$$

A Gaussian mixture of coherent states = completely classical resource;

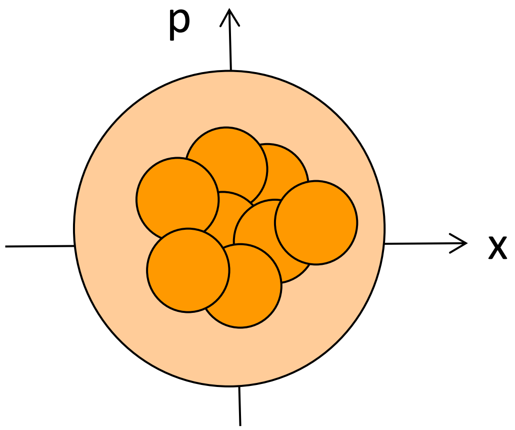
Expect no quantum features for AB. But: **non-zero Gaussian discord - ??**

All non-product (but separable) bi-partite Gaussian states have a non-zero discord

P. Giorda and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010);
G. Adesso and A. Datta, Phys. Rev. Lett. 105, 030501 (2010) ;

Quantum ???

started with a Gaussian mixture of coherent states



most classical example of "classicality"

$$\hat{\rho} = \int d^2\alpha P(\alpha) |\alpha\rangle\langle\alpha|$$

Where does quantumness come from?

Is there any quantumness?

Does Gaussian discord give me a correct picture?

Quantum features in "classical" states?

Problems with quantum discord of Gaussian states?

Definition OK! Gaussian Discord – Gaussian measurements optimal

$$\mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \underbrace{\mathcal{S}(\hat{\rho}_A)}_{\text{Total info about A}} - \underbrace{\inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)}_{\text{Quantum correlation: Info about A inferred via quantum measurement on B}}$$

- one way classical correlation

Total info about A

Quantum correlation:
Info about A inferred via quantum measurement on B

$$\mathcal{H}_{\{\hat{\Pi}_i\}}(A|B) \equiv \sum_i p_i \mathcal{S}(\hat{\rho}^i_{A|B})$$

Quantum conditional entropy related to $\hat{\rho}^i_{A|B}$ upon POVM $\{\hat{\Pi}_i\}$ on B.

$$\hat{\rho}^i_{A|B} = \mathrm{Tr}_B[\hat{\Pi}_i \hat{\rho}_{AB}]/p_i$$

Infimum: optimization to single out the least disturbing measurement on B

Indirect confirmation that Gaussiam meas. optimal:
Nonclassical correlations in continuous-variable non-Gaussian Werner states,
Tatham, Mišta, Adesso, Korolkova, Phys. Rev. A 85, 022326

$$\hat{\rho} = \sum_{ks} p_{ks} |\theta_k\rangle\langle\theta_k| \otimes |\eta_s\rangle\langle\eta_s|$$

$$\hat{\rho} = \sum_k p_k \hat{\rho}_{Ak} \otimes \hat{\rho}_{Bk}$$

CC

$$\hat{\rho} = \sum_k p_k |\theta_k\rangle\langle\theta_k| \otimes \hat{\rho}_{Bk}$$

QC

QQ

Information-theoretical approach: can I prepare state by LOCC?

All these states are separable – but:

QQ: non-zero discord, not all the information about them can be locally retrieved; cannot prepare by LOCC;

QC: zero A-discord, cannot be cloned locally (locally broadcasted)

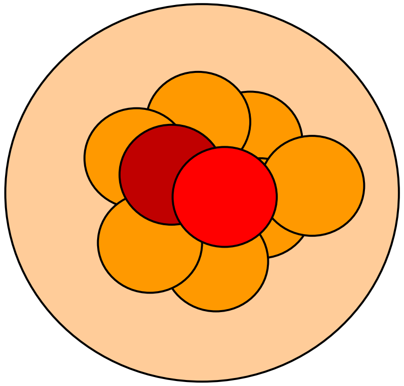
A. Ferraro, M. G. Paris, Phys. Rev. Lett. 108, 260403 (2012)

Underlying physics here:

Nonorthogonal separable states cannot be discriminated exactly

$$\hat{\rho} = \sum_k p_k |\theta_k\rangle\langle\theta_k| \otimes \hat{\rho}_{Bk}$$

a set of generic non-orthogonal states



F. Grosshans, G. van Assche, J. Wenger, R. Brouri, N. J. Cerf, P. Grangier, Nature 421, 238 (2003) – QKD using Gaussian modulated coherent states

$$\hat{\rho} = \int\int d^2\alpha d^2\beta P(\alpha, \beta)|\alpha\rangle\langle\alpha| \otimes |\beta\rangle\langle\beta|$$

$$D_{AB} \neq 0$$

“classical” definition of non-classicality in bi-partite system

Gaussian states with non-zero quantum discord are often classical according to this definition

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$$\hat{\rho} = \sum_{ks} p_{ks} |\theta_k\rangle\langle\theta_k| \otimes |\eta_s\rangle\langle\eta_s|$$

$$\hat{\rho} = \sum_{k} c_{ks} |n_k\rangle|n_s\rangle$$

Information-theoretical approach: can I prepare state by LOCC?

Gaussian states with non-zero quantum discord are non-classical according to this definition

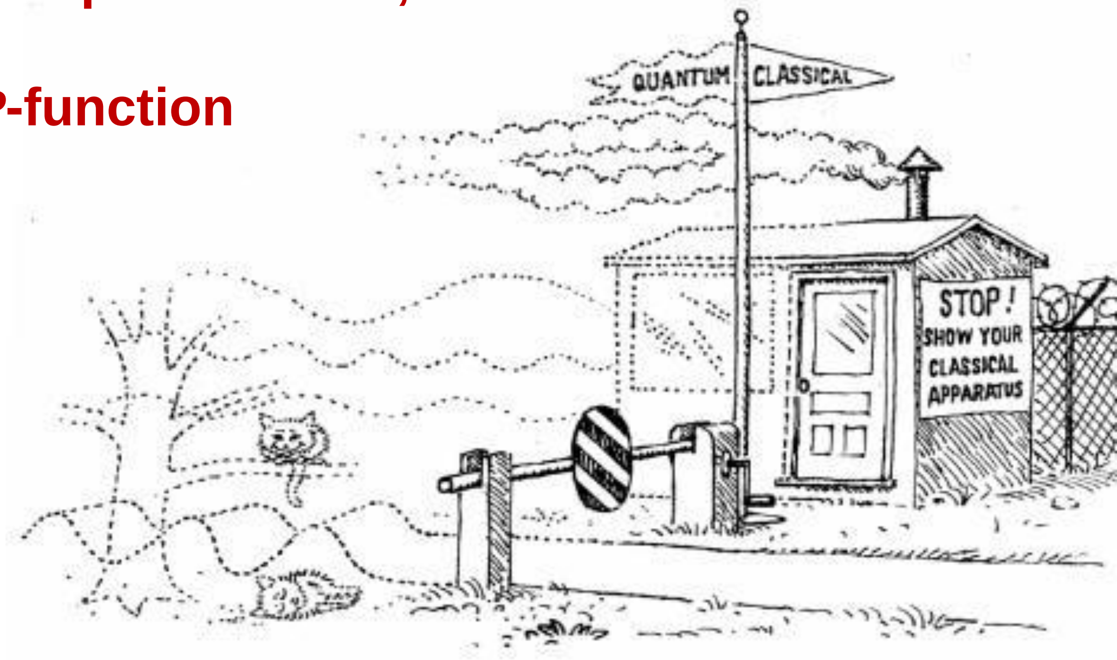
A. Ferraro, M. G. Paris, Phys. Rev. Lett. 108, 260403 (2012)

We are still working on an adequate map for the quantum – classical boarder…

And it is “custom-made”…

States with zero discord can exhibit quantum correlations according to phase-space criteria;

Classical states according to *P*-function can have positive discord and cannot be prepared by LOCC



Delineating the border between the quantum realm ruled by the Schrödinger equation and the classical realm ruled by Newton's laws is one of the unresolved problems of physics. **Figure 1**

Zurek, Physics Today (1991)

Quantum features in “classical” states

Quantum features I:

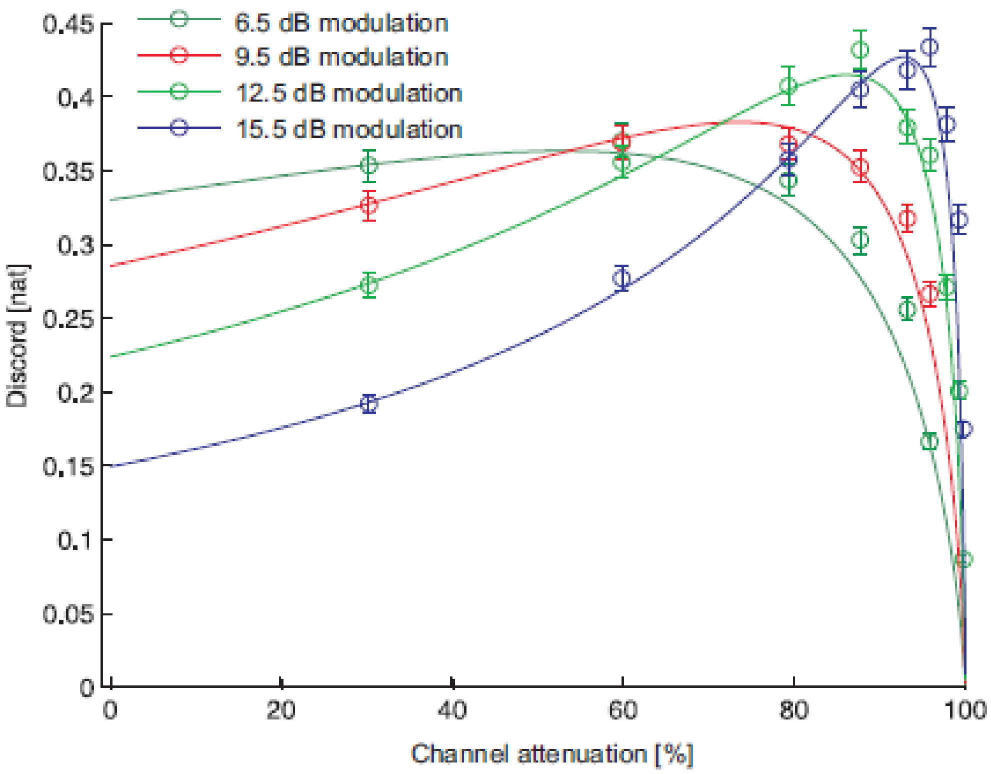
Discord increase under the action of local noise

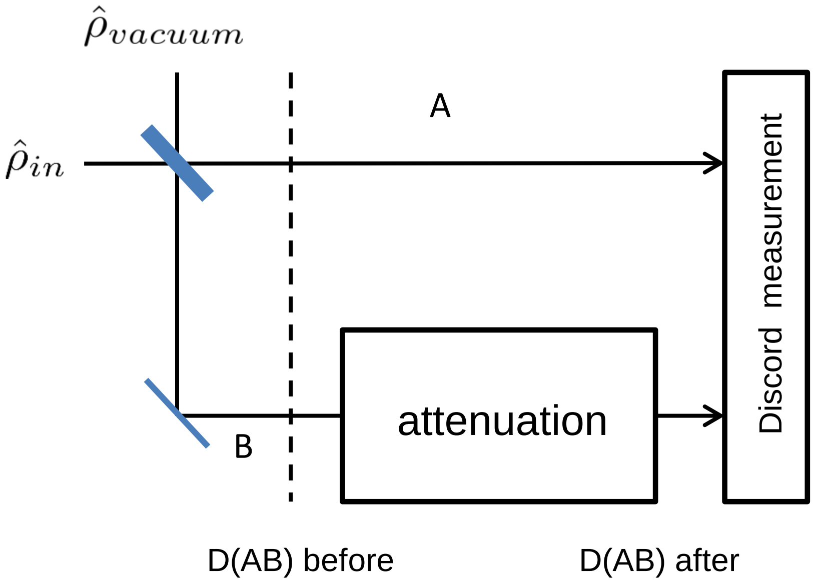
Local noisy channels, nonunital (dissipation) - **quantum correlations can increase !!!**

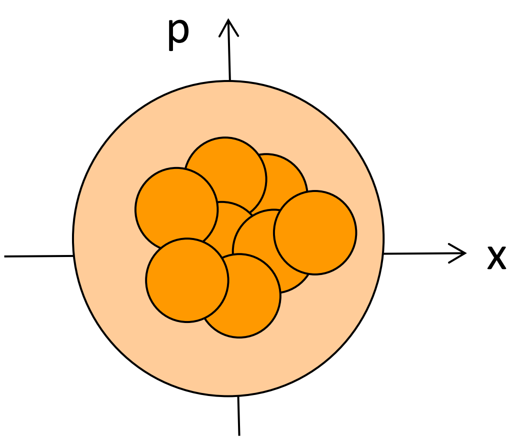
Theory: *DV: Streltsov, Kampermann, Bruss, PRL 107, 170502 (2011);*
CV: Ciccarello, Giovannetti, PRA 85, 010102 (2012)

CV version, first experimental results:

Theory:
Ciccarello, Giovannetti, PRA 85, 010102 (2012)

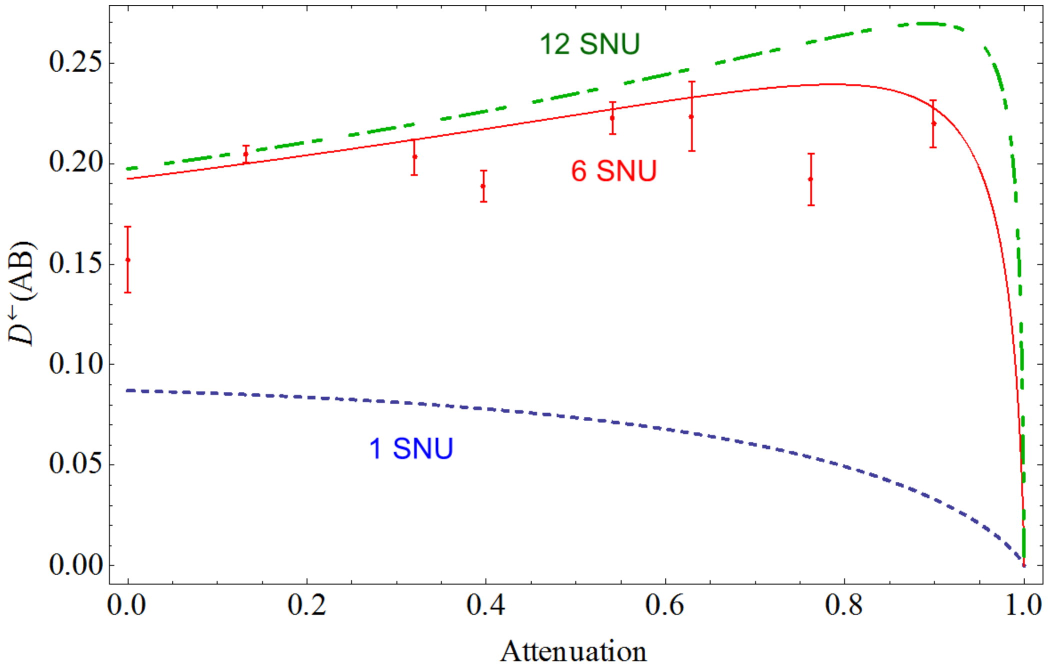


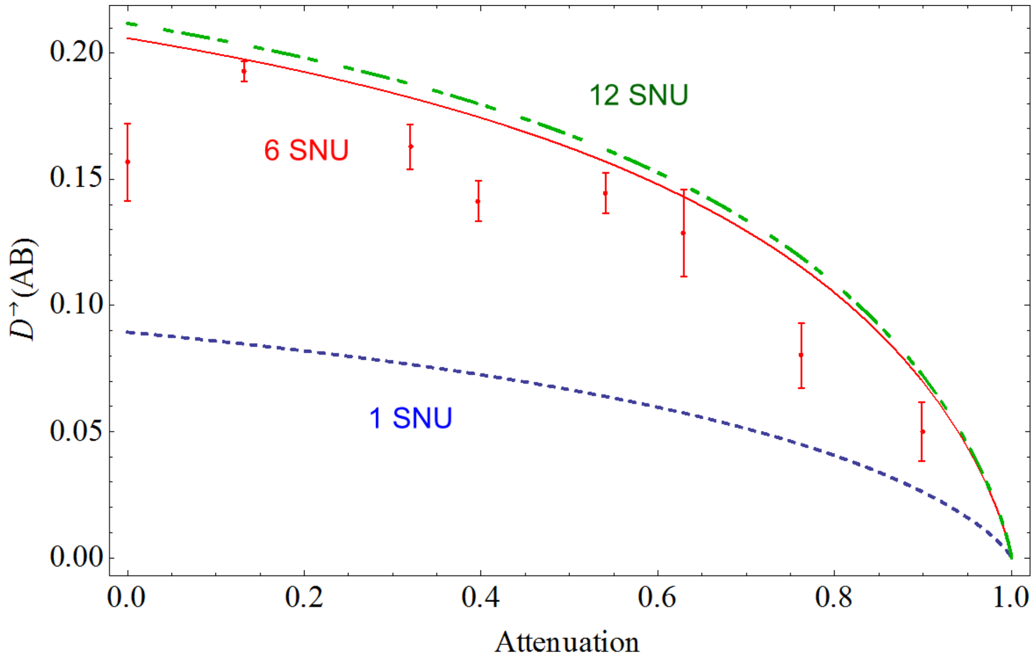




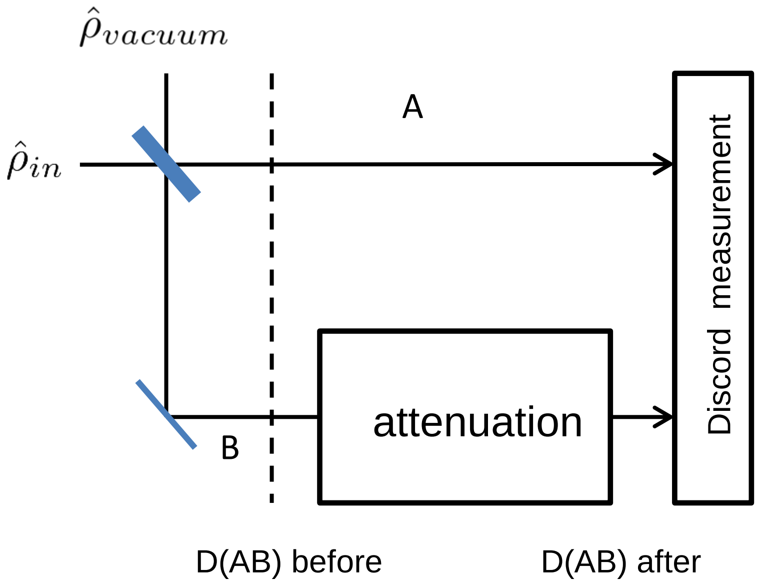
for modulated coherent state

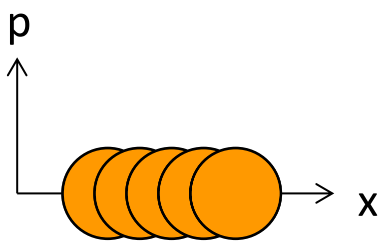
Exp: Madsen, Berni, Lassen, Andersen, Phys. Rev. Lett. 109, 030402 (2012)





Increase of quantum discord under dissipation





Experiment: V. Chille, Ch. Peuntinger, L. Mista, N. Korolkova, Ch. Marquardt, G. Leuchs, to be submitted, 2013

Koashi-Winter inequality:

$$\mathcal{S}\left(\hat{\rho}_A\right) \geq \mathcal{E}_F\left(\rho_{AB}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right)$$

Why discord Increases??

Koashi, Winter, PRA 69, 022309 (2004)

Monogamy of entanglement: a quantum system being entangled with another one limits its possible entanglement with a third system

Interplay between entanglement and classical correlation.

Perfect entanglement and perfect classical correlation are mutually exclusive

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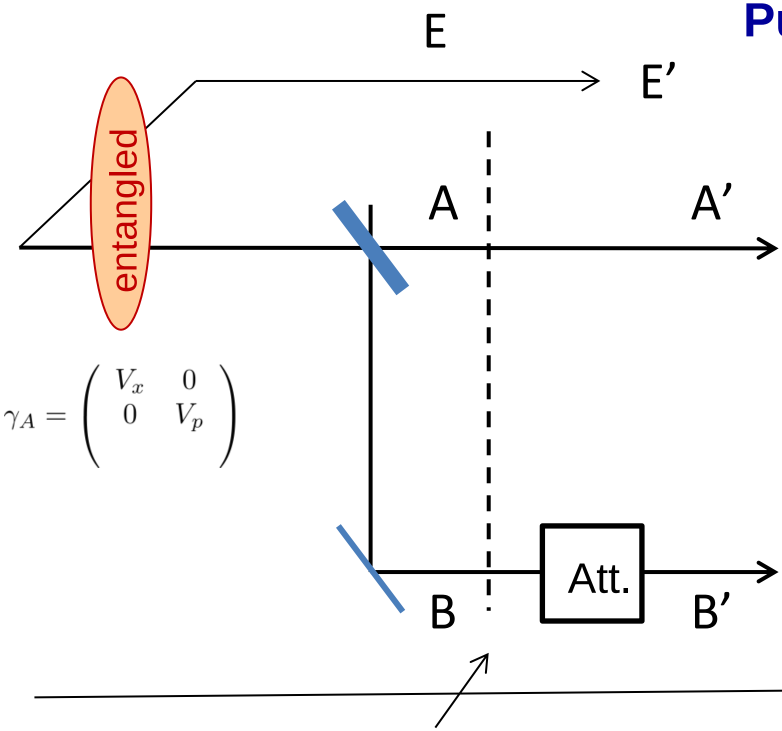
Purification:

mixed quantum state + "environmental" mode = pure quantum state

Purification: add E, replace A with TMSV for AE:



$$\gamma_A = \begin{pmatrix} V_x & 0 \\ 0 & V_p \end{pmatrix}$$

$$\gamma_{AE} = \begin{pmatrix} a\mathbf{1} & c\sigma_z \\ c\sigma_z & a\mathbf{1} \end{pmatrix}$$

$$a = \cosh(2r) = \nu = \sqrt{V_x V_p}$$

$$c = \sinh(2r) = \sqrt{V_x V_p - 1}$$

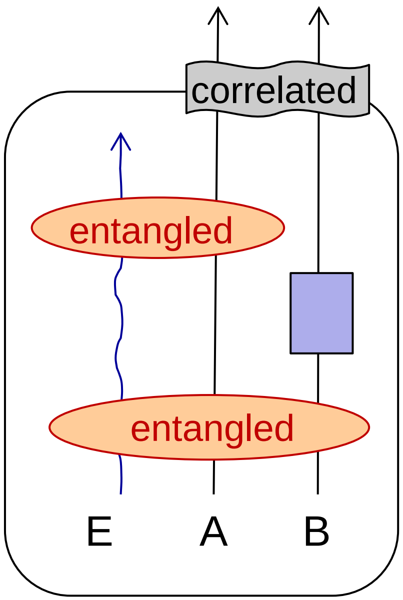
$$\gamma_{AB} = \begin{pmatrix} T^2\gamma_A + R^2\mathbf{1} & TR(\gamma_A - \mathbf{1}) \\ TR(\gamma_A - \mathbf{1}) & R^2\gamma_A + T^2\mathbf{1} \end{pmatrix} \qquad C = \sqrt{\nu^2 - 1}\begin{pmatrix} TS_A^{-1}\sigma_z \\ RS_A^{-1}\sigma_z \end{pmatrix}$$

$$\gamma_\pi = \begin{pmatrix} \gamma_{AC} & C \\ C^T & \nu\mathbf{1} \end{pmatrix}; \qquad S_A = \begin{pmatrix} \sqrt[4]{\frac{V_p}{V_x}} & 0 \\ 0 & \sqrt[4]{\frac{V_x}{V_p}} \end{pmatrix};$$

After mode B undergoes variable attenuation:

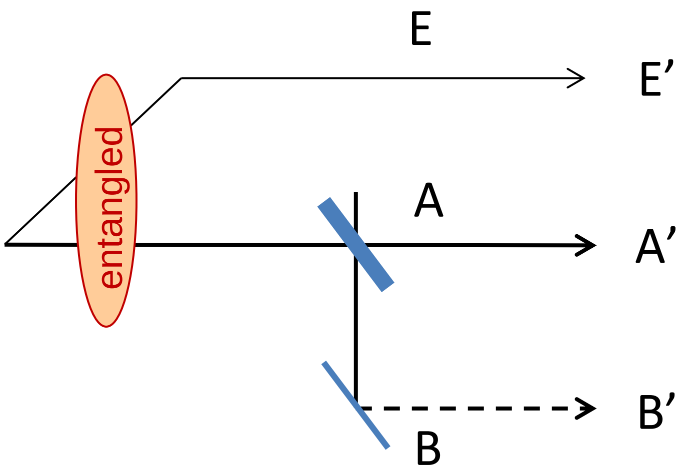
$$\mathcal{S}\left(\hat{\rho}_{A'}\right) = \mathcal{E}_F\left(\hat{\rho}_{A'E'}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{A'B'}\right) = \mathcal{S}\left(\hat{\rho}_{A}\right)$$

classical correlation $\mathcal{J}^{\leftarrow}\left(\hat{\rho}_{A'B'}\right)$
decrease



capacity for Alice's correlations
must be filled up $\mathcal{E}_F\left(\hat{\rho}_{A'E'}\right)$

entanglement between Alice's
mode and environment increase



The discord between A and B arises as
a side effect of this entanglement formation
between the subsystem unaffected
by the measurement and environment

Koashi-Winter inequality:

$$\boxed{\mathcal{S}\left(\hat{\rho}_A\right) = \mathcal{E}_F\left(\rho_{AE}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AB}\right)}$$

- for pure tripartite system

$$\mathcal{E}_F\left(\rho_{AE}\right) = \lim_{\{p_i,|\psi_i\rangle\}} \sum_i p_i S(\mathrm{Tr}_E[|\psi_i\rangle\langle\psi_i|])$$

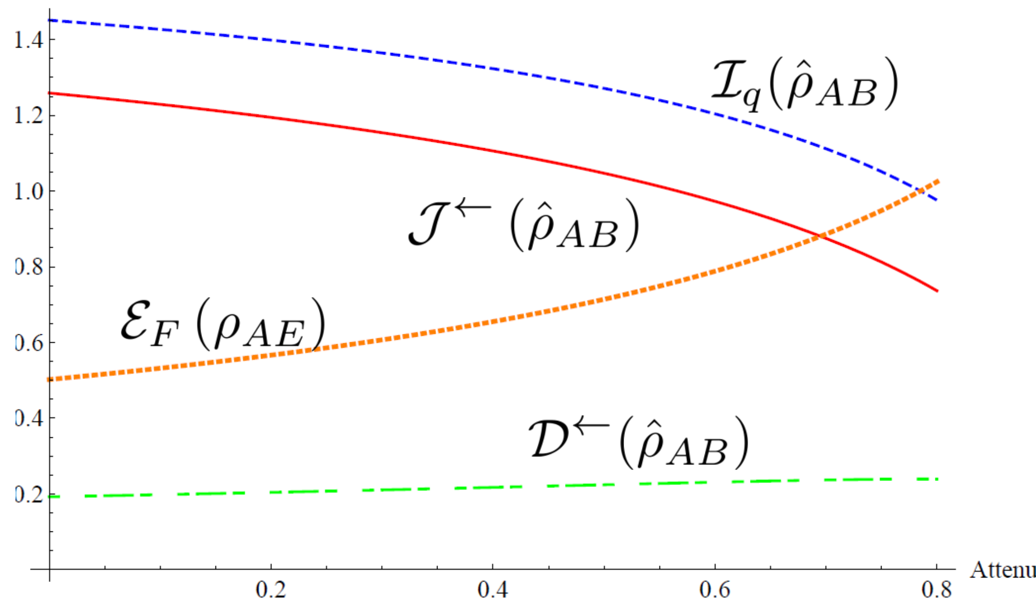
$$\{p_i, |\psi_i\rangle\} : \quad \sum_i p_i |\psi_i\rangle\langle\psi_i|] = \rho_{AE}$$

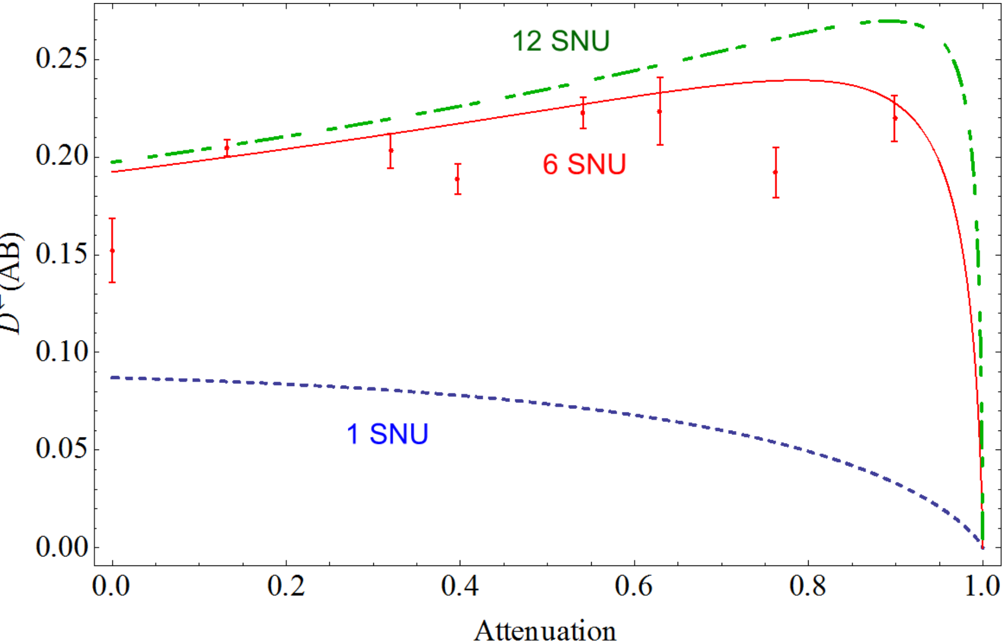
- min taken over all ensembles satisfying this

Quantum Discord:

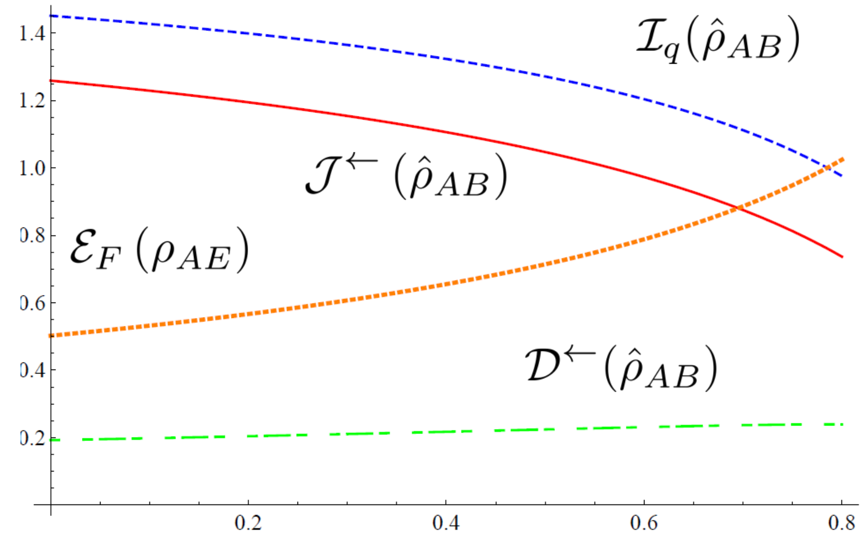
$$\boxed{\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{I}_q(\hat{\rho}_{AB}) - \mathcal{J}^{\leftarrow}(\hat{\rho}_{AB})}$$

(measurement on B)

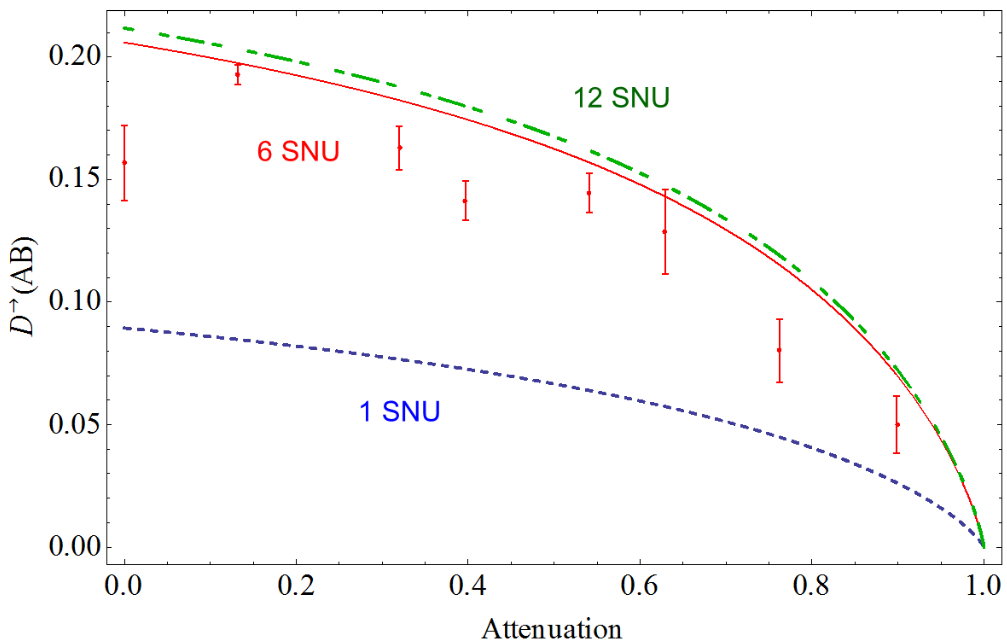




12 SNU
6 SNU
1 SNU
$D^{\leftarrow}$(AB)
Attenuation
0.00
0.05
0.10
0.15
0.20
0.25
0.0
0.2
0.4
0.6
0.8
1.0



$\mathcal{I}_q(\hat{\rho}_{AB})$
$\mathcal{J}^{\leftarrow}(\hat{\rho}_{AB})$
$\mathcal{E}_F(\rho_{AE})$
$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB})$
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.2
0.4
0.6
0.8



12 SNU
6 SNU
1 SNU
$D^{\rightarrow}$(AB)
Attenuation
0.00
0.05
0.10
0.15
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0.2
0.4
0.6
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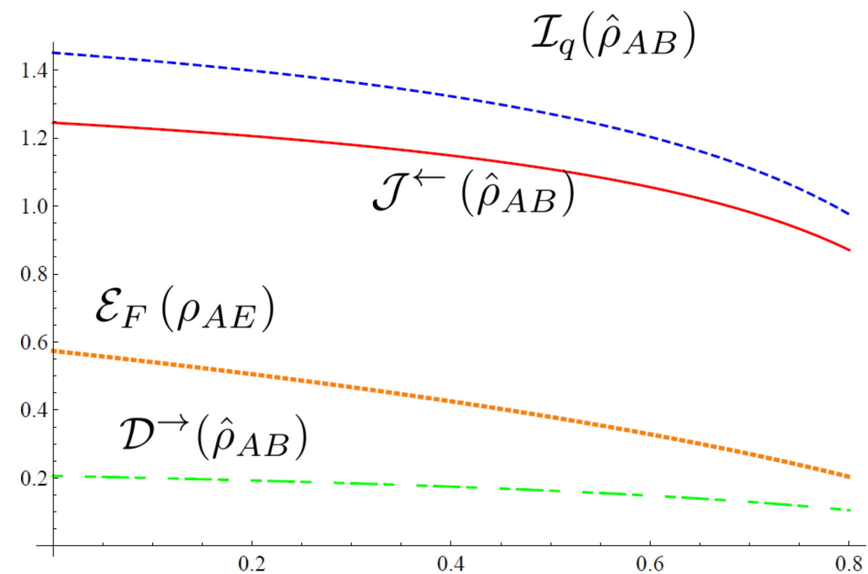
1.0

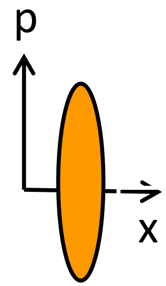


$\mathcal{I}_q(\hat{\rho}_{AB})$
$\mathcal{J}^{\leftarrow}(\hat{\rho}_{AB})$
$\mathcal{E}_F(\rho_{AE})$
$\mathcal{D}^{\rightarrow}(\hat{\rho}_{AB})$
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.2
0.4
0.6
0.8

Quantum features in “classical” states

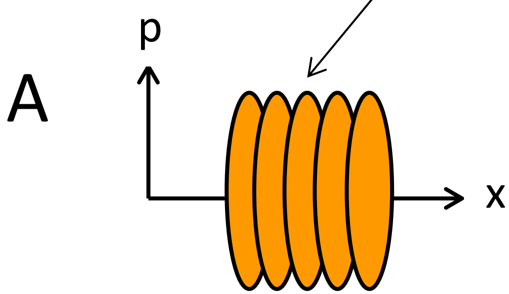
Quantum features II:

Entanglement activation from a separable discordian state



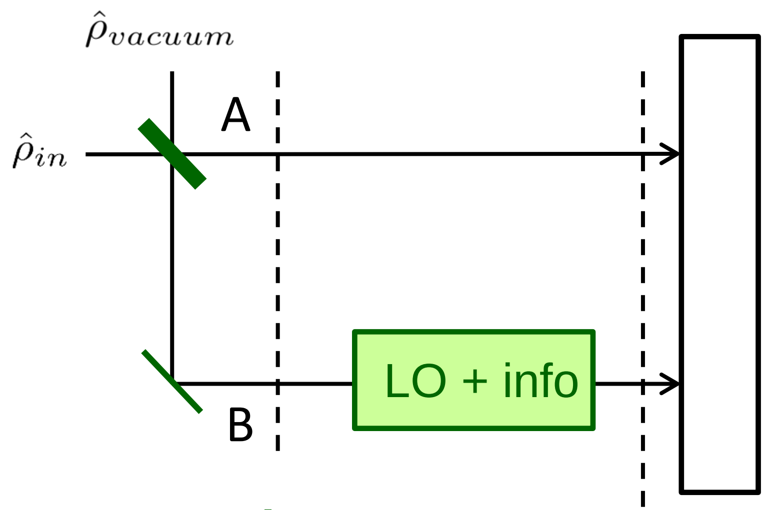
$$\gamma_A^{\text{in}} = \begin{pmatrix} V_{\text{in}}^x & 0 \\ 0 & V_{\text{in}}^p \end{pmatrix}$$

$$V_{\text{in}}^x = e^{-2r}$$



$$x_A^{\text{in}} \rightarrow x_A = x_A^{\text{in}} + \bar{x}$$

$$\gamma_A = \begin{pmatrix} V_{\text{in}}^x + V & 0 \\ 0 & V_{\text{in}}^p \end{pmatrix}$$



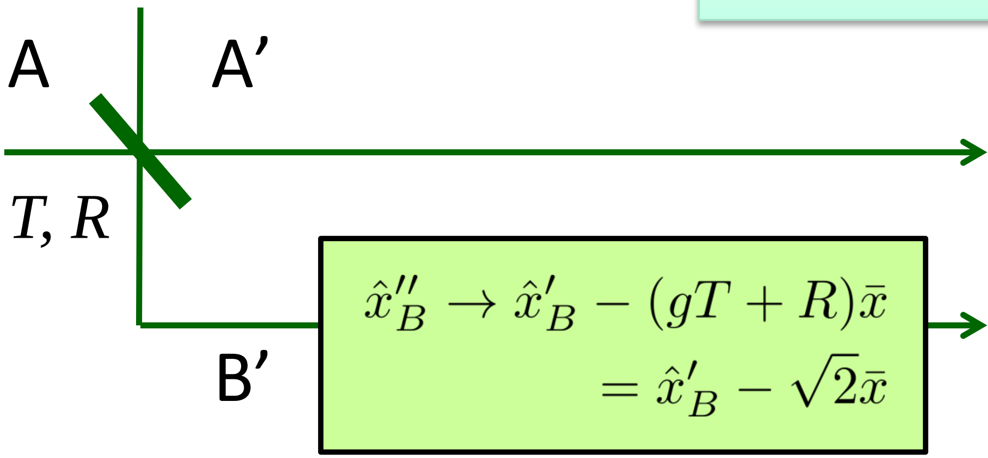
no entanglement;
non-zero discord

entanglement

correlated noise; classical info – know the structure of displacements

demodulation calculated using Duan's criterion in product form:

$$\frac{\langle (g\hat{x}'_A + \hat{x}'_B)^2 \rangle \langle (g\hat{p}'_A - \hat{p}'_B)^2 \rangle}{\frac{1}{4}(g^2+1)^2} < 1$$



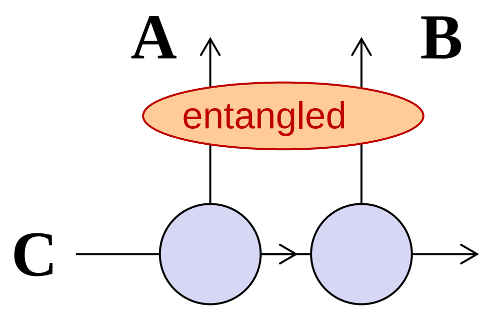
measured entanglement:

$$0.8479 \pm 0.0025 < 1$$

$$\frac{[e^{2r}(gT-R)^2 + (gR+T)^2][e^{-2r}(gT+R)^2 + (gR-T)^2]}{(g^2+1)^2} < 1$$

- criterion in terms of experimental parameters *T, R, g, r; g=0.9*

V. Chille, Ch. Peuntinger, L. Mista, N. Korolkova, Ch. Marquardt, G. Leuchs, in prep. (2013)

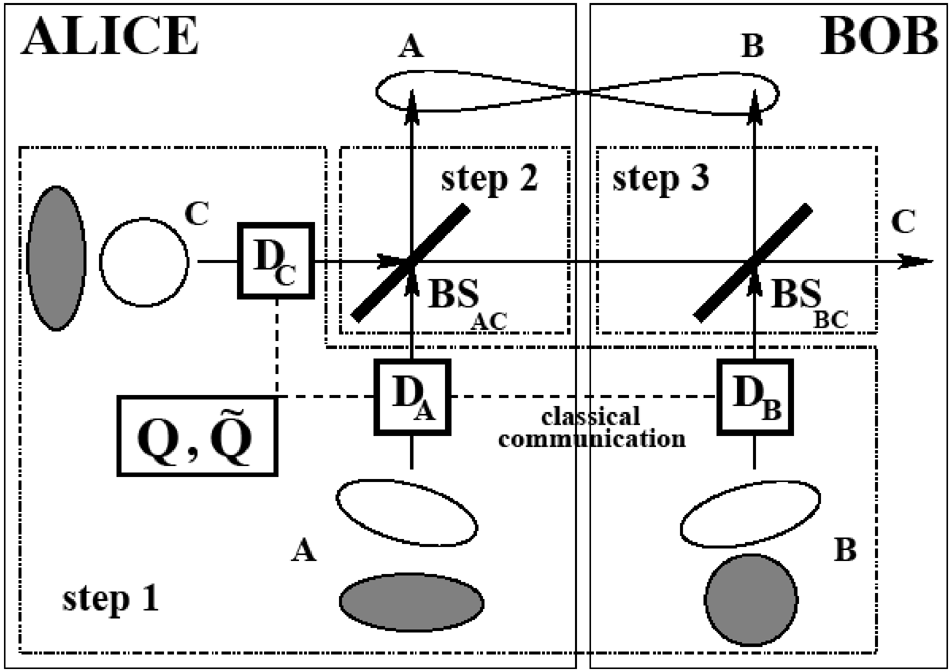


Entanglement distribution by a separable ancilla

L. Mista and N. Korolkova Phys. Rev. A 77, 050302(R) (2008), ibid 80, 032310 (2009).

Experiments:
V. Chille, Ch. Peuntinger, L. Mista, N. Korolkova, M. Förtsch, J. Korger, Ch. Marquardt, G. Leuchs, arxive 2013;

C. E. Vollmer, D. Schulze, T. Eberle, V. Händchen, J. Fiurasek, R. Schnabel, arxive 2013 (not an LOCC scheme);

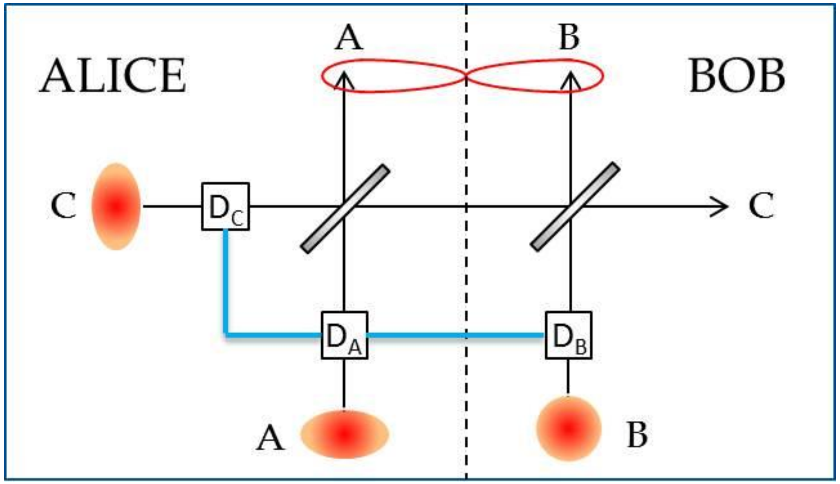


$$\gamma_A \oplus \gamma_B \oplus \gamma_C \Rightarrow$$
$$\gamma_1(x) = \gamma_{AB} \oplus \mathrm{I}_C$$

$$+x(q_1 q_1^T + q_2 q_2^T)$$
$$q_{1,2} = q_{1,2}(squeezing)$$

This term correspond to CM of specially designed (classically correlated) noise

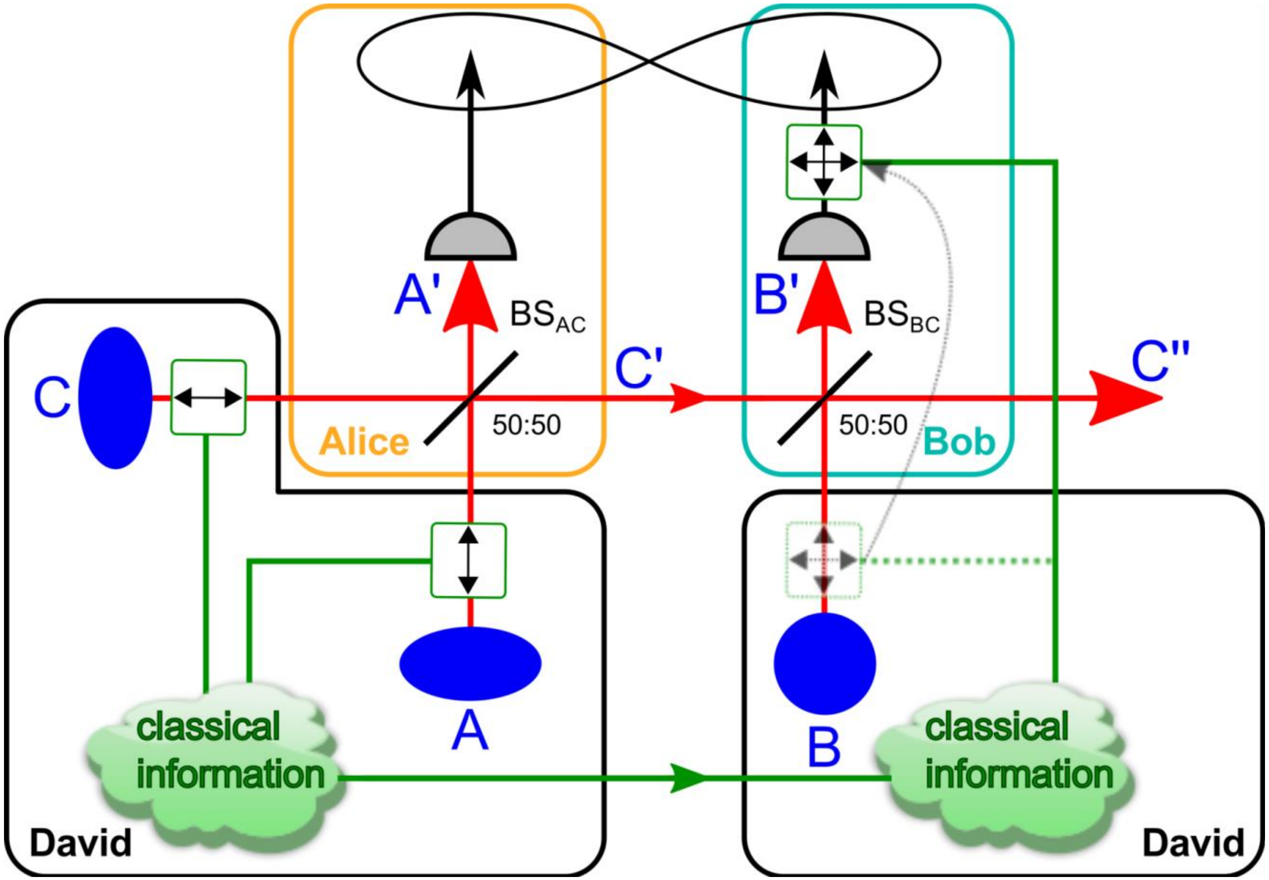
Correlated noise creates coherence



Property of David's state ABC:

- by a local operation on subsystem (AC) can be transformed to $A'BC'$ separable with respect to C' (and from B')

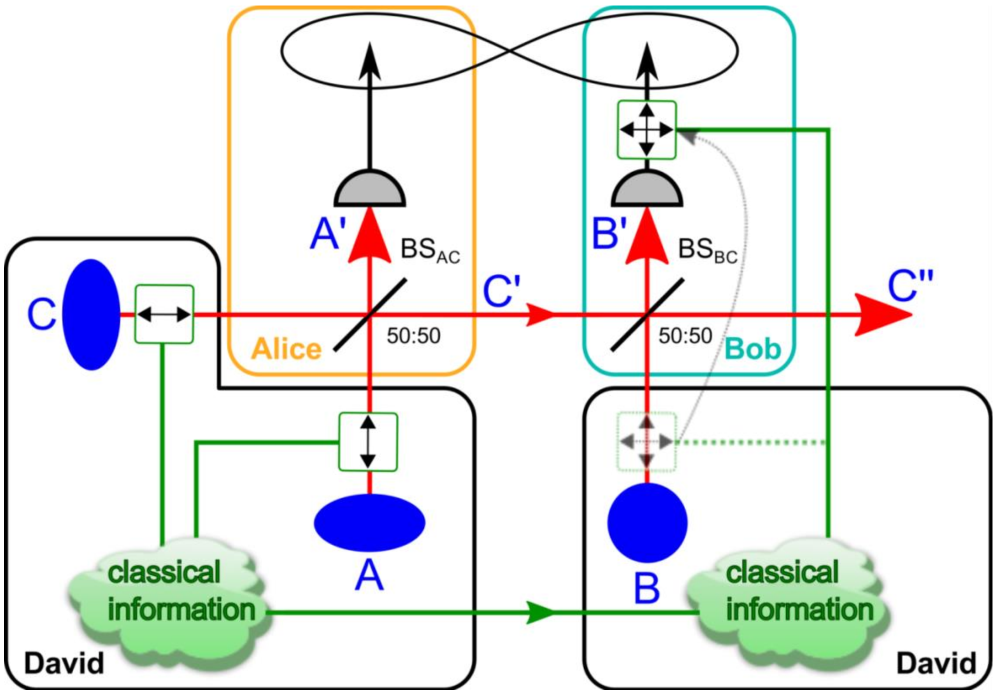
- mode A is entangled with the composite subsystem (BC')

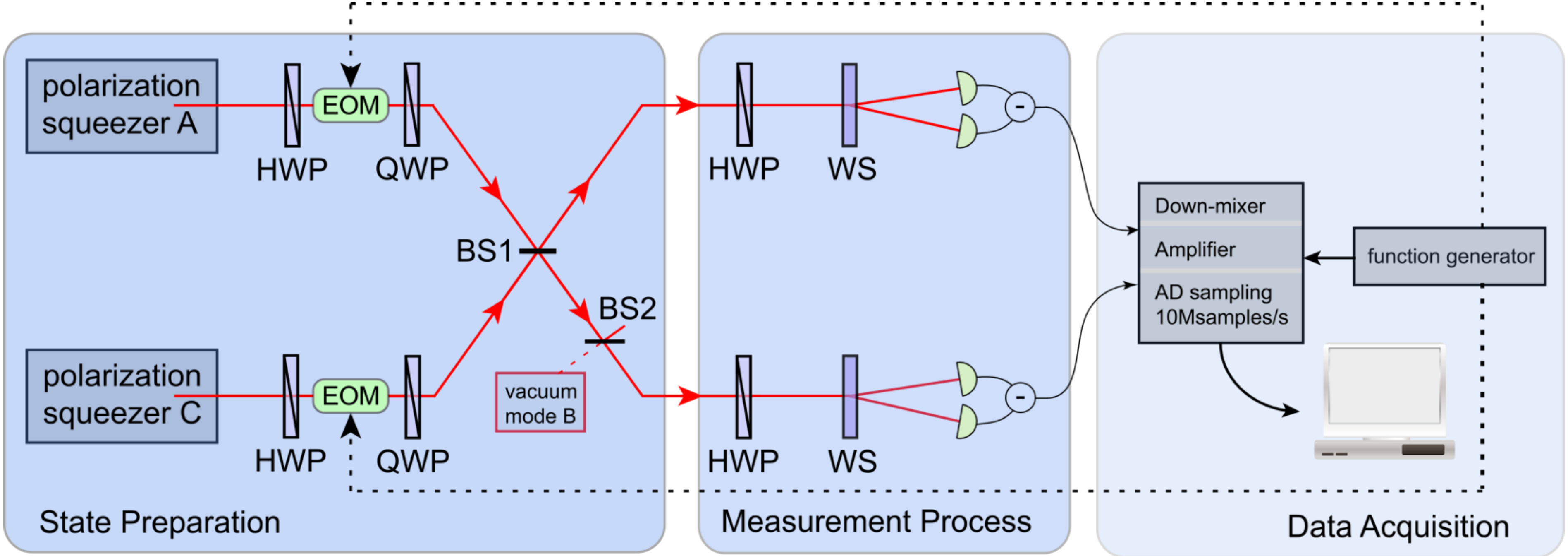


It is impossible to entangle Alice with Bob by LOCC, but we can create entanglement of mode A' with a ***delocalized*** two-mode system (BC') (C' belongs to Alice and B to Bob) by a local operation on Alice's part of the initial fully separable state.

Entanglement of mode A' with two-mode system (BC') should be localizable between modes A' and B' by a local operation on Bob's side.

Entanglement carried by the global state is bound and the localization renders its activation which crowns entanglement distribution.





State preparation by LOCC

Measurement of the Stokes observables: determination of the covariance matrix

Down-mixing process

Data Acquisition

Post Processing

V. Chille, Ch. Peuntinger, L. Mista, N. Korolkova, M. Förtsch, J. Korger, Ch. Marquardt, G. Leuchs, submitted, (2013);

covariance matrix after BS_{AC}:

$$\gamma_{A'C'} = \begin{pmatrix} 20.90 & 1.102 & -7.796 & -1.679 \\ 1.102 & 25.30 & 1.000 & 14.63 \\ -7.796 & 1.000 & 20.68 & 0.8010 \\ -1.679 & 14.63 & 0.8010 & 24.65 \end{pmatrix}$$

eigenvalues of covariance matrix → **39.84, 28.47, 13.85, 9.371**

→ **PPT criterion proves separability**

after BS_{BC}:

$$\gamma_{A'B'} = \begin{pmatrix} 19.95 & 1.025 & -4.758 & -1.063 \\ 1.025 & 22.92 & 0.9699 & 9.153 \\ -4.758 & 0.9699 & 9.925 & 0.2881 \\ -1.063 & 9.153 & 0.2881 & 11.65 \end{pmatrix}$$

→ **28.24, 21.79, 8.646, 5.756** → **PPT criterion proves separability**

C. Peuntinger et al., arXiv:1304.0504 (2013)

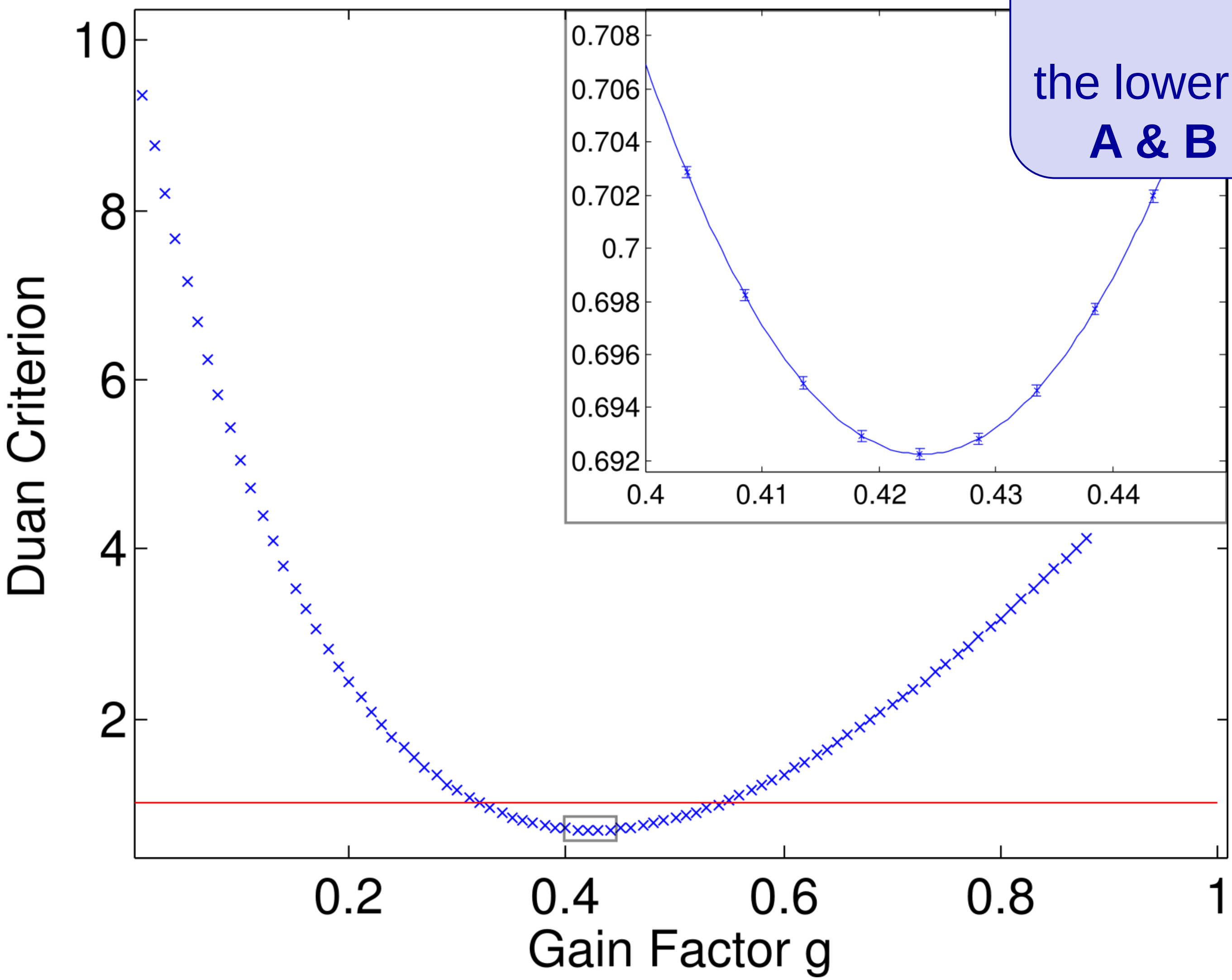
Duan's entanglement criterion

$$\Delta^2_{\mathrm{norm}}\left(\hat{S}_{X,0^\circ}\cdot g+\hat{S}_{Y,0^\circ}\right)\cdot\Delta^2_{\mathrm{norm}}\left(\hat{S}_{X,90^\circ}\cdot g-\hat{S}_{Y,90^\circ}\right)<1$$

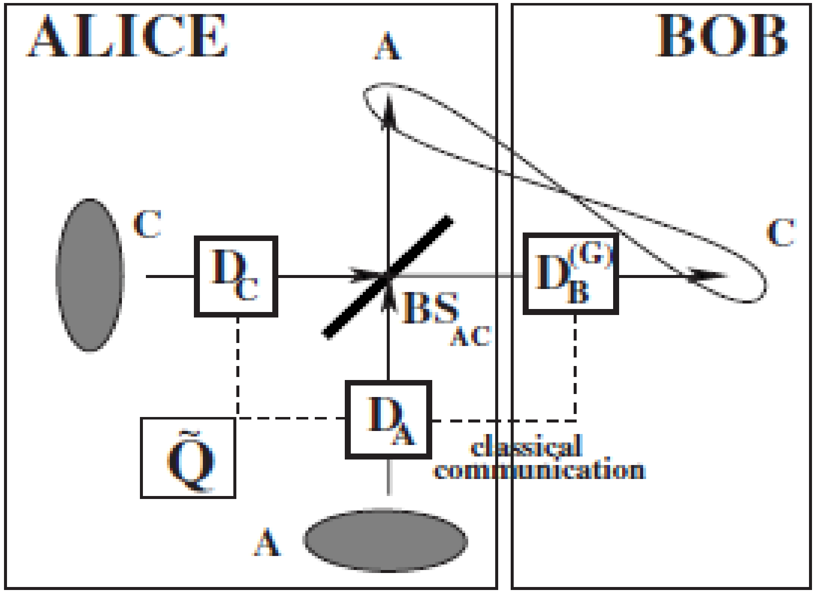
$$0.6922 \pm 0.0002 < 1$$

the lower symplectic eigenvalue

A & B are entangled



C. Peuntinger et al., arXiv:1304.0504 (2013)

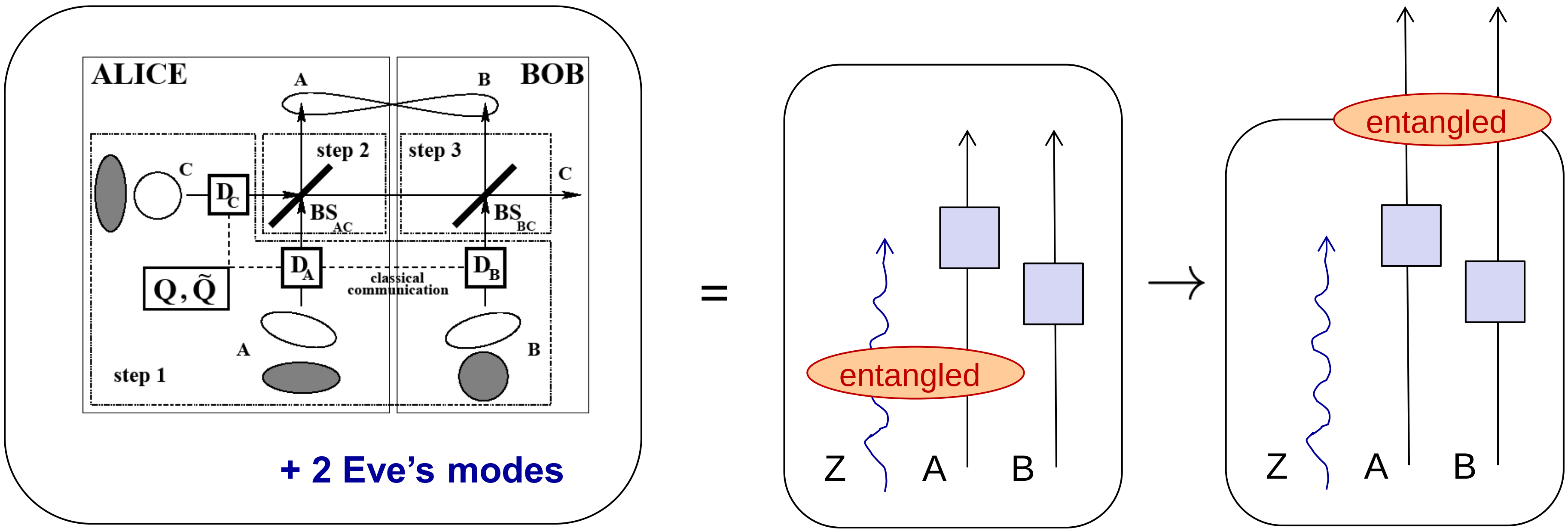


“Normal” explanation:

role of classical information

Classical information lies in our knowledge about all the correlated displacement involved.

Bob (or David for him) can recover through clever noise addition quantum resources initially present in the input quantum squeezed states.



entanglement distribution by separable ancilla

↕

entanglement swapping between the "environment" and the modes

entanglement *(ZA)* goes to entanglement *(AB)*

Quinn, Mista, Korolkova, in preparation

Role of classical communication:

we use our knowledge about initial pure product state to design correlated noise

Role of dissipation:

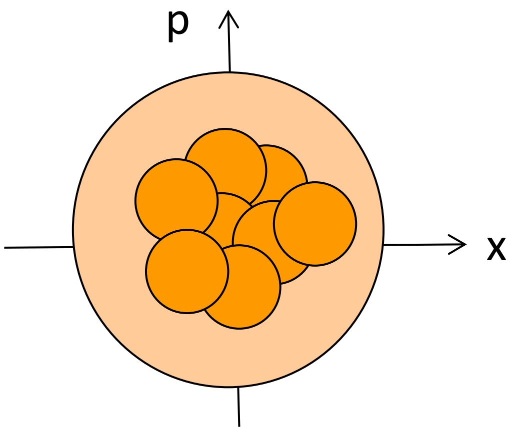
dissipation to a common reservoir, not a product state any more
(mode C viewed as “environmental mode”)

Role of discord:

need non-zero discord in order to obtain entanglement at final stage

T.K. Chuan, J. Maillard, K. Modi, T. Paterek, M. Paternostro, M. Piani, Role of quantumness of correlations in entanglement distribution, (2012);
A. Streltsov, H. Kampermann, D. Bruss, Quantum cost for sending entanglement, (2012) ;
A. Datta, Studies on the Role of Entanglement in Mixed-state Quantum Computation, PhD thesis, arxiv 2008.

Summary



- is not really classical ….

Quantum correlations in separable states can be a useful resource

Correlated dissipation can create coherence

Thank you for your attention!

Theoretical Quantum Information group:

N Korolkova, S Ivanov, (D Milne), (R Tatham), N Quinn, C Croal

L. Mista - *regular visitor from Olomouc*

www.st-andrews.ac.uk/~qoi

EPSRC Engineering and Physical Sciences Research Council

iQIT

SEVENTH FRAMEWORK PROGRAMME

SUPA

$$\mathcal{D}^{\leftarrow} = f\left[\sqrt{\beta}\right] - f[V_+] - f[V_-] + f\left[\sqrt{\det \epsilon_{\text{inf}}^{\leftarrow}}\right]$$

$$\mathcal{D}^{\rightarrow} = f\left[\sqrt{\alpha}\right] - f[V_-] - f[V_+] + f\left[\sqrt{\det \epsilon_{\text{inf}}^{\rightarrow}}\right]$$

$$f(x) = \left(\frac{x+1}{2}\right)\ln\left(\frac{x+1}{2}\right) - \left(\frac{x-1}{2}\right)\ln\left(\frac{x-1}{2}\right)$$

$$\det \epsilon_{\text{inf}}^{\leftarrow} = \begin{cases} \frac{2\gamma^2+(\beta-1)(\delta-\alpha)+2|\gamma|\sqrt{\gamma^2+(\beta-1)(\delta-\alpha)}}{(\beta-1)^2} \\ \text{if} \quad (\delta-\alpha\beta)^2 \leq (1+\beta)\,\gamma^2\,(\alpha+\delta)\,, \\ \frac{\alpha\beta-\gamma^2+\delta-\sqrt{\gamma^4+(\delta-\alpha\beta)^2-2(\gamma^2)(\alpha\beta+\delta)}}{2\beta} \\ \text{Otherwise.} \end{cases}$$

$$\det \epsilon_{\text{inf}}^{\rightarrow} = \begin{cases} \frac{2\gamma^2+(\alpha-1)(\delta-\beta)+2|\gamma|\sqrt{\gamma^2+(\alpha-1)(\delta-\beta)}}{(\alpha-1)^2} \\ \text{if} \quad (\delta-\beta\alpha)^2 \leq (1+\alpha)\,\gamma^2\,(\beta+\delta)\,, \\ \frac{\beta\alpha-\gamma^2+\delta-\sqrt{\gamma^4+(\delta-\beta\alpha)^2-2\gamma^2(\beta\alpha+\delta)}}{2\alpha} \\ \text{Otherwise.} \end{cases}$$

Zoology of the non-classicality measures

Mutual information = total correlations btw A and B

Classically - equivalent definitions of mutual information:

$$\boxed{\begin{aligned} I(A:B) &= H(A)+H(B)-H(A,B) = \\ J(A:B) &= H(A)-H(A|B) = \\ J(B:A) &= H(B)-H(B|A) \end{aligned}}$$

Shannon entropy: $H(A)$

Conditional: $H(A|B)$

Quantum – they are not equivalent; mutual information:

$$\boxed{\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})}$$

von Neumann entropy: $S(\hat{\rho}_{AB})$

$\longleftrightarrow$ $I(A:B)$

$$J(A:B), J(B:A)?$$

measurements!

$$
\begin{aligned}
I(A:B) &= H(A) + H(B) - H(A,B) = \\
J(A:B) &= H(A) - H(A|B) =
\end{aligned}
$$

$$
\mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \underbrace{\mathcal{S}(\hat{\rho}_A)}_{\substack{\text{Total info} \\ \text{about A}}} - \underbrace{\inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)}_{\substack{\text{Quantum correlation:} \\ \text{Info about A inferred via quantum measurement on B}}}
$$

- one way classical correlation

$$
\mathcal{H}_{\{\hat{\Pi}_i\}}(A|B) \equiv \sum_i p_i \mathcal{S}(\hat{\rho}^i_{A|B})
$$

Quantum conditional entropy related to $\hat{\rho}^i_{A|B}$ upon POVM $\{\hat{\Pi}_i\}$ on B.

$$
\hat{\rho}^i_{A|B} = \mathrm{Tr}_B[\hat{\Pi}_i \hat{\rho}_{AB}]/p_i
$$

Infimum: optimization to single out the least disturbing measurement on B

Quantum discord:

$$\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})$$

—

$$\mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) - \inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)$$

quantum mutual information

one way classical correlation

$$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{I}_q(\hat{\rho}_{AB}) - \mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB}) + \inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)$$

$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) = 0$ - classical

$0 < \mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) < 1$ - quantum, separable

$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) > 1$ - entangled

$$\boxed{\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})} - \boxed{\mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) - \inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)}$$

quantum mutual information **one way classical correlation**

QD = quantum mutual info - the classical mutual info of outcomes;

not unique

QD = total correlations - classical correlations;

\+ operational; conceptually easy; optimized over POVMs

\- hard to compute; asymmetric

DV: H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001);
L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

CV: G. Adesso and A. Datta, Phys. Rev. Lett. 105, 030501 (2010)
P. Giorda and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010)
L. Mista, R. Tatham, D. Girolami, N. Korolkova, G. Adesso, Phys. Rev. A. (2011)

$$\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})$$ **—** $$\mathcal{I}(A:B)$$

quantum mutual information **non-Gaussian classical correlation**

MID = quantum mutual info - the classical mutual info of outcomes of local Fock-state detectors;

+ symmetric
- no optimizations over local measurements

$\Longrightarrow$ often overestimates quantum correlations

Measurement-induced disturbance (MID)

S. Luo, Phys. Rev. A 77, 022301 (2008)

$$\boxed{\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})} - \boxed{\mathcal{I}_c(\hat{\rho}_{AB}) = \sup_{\hat{\Pi}_A \otimes \hat{\Pi}_B} \mathcal{I}(A:B)}$$

quantum mutual information

maximal classical correlation
extractable by local (Gaussian) processing

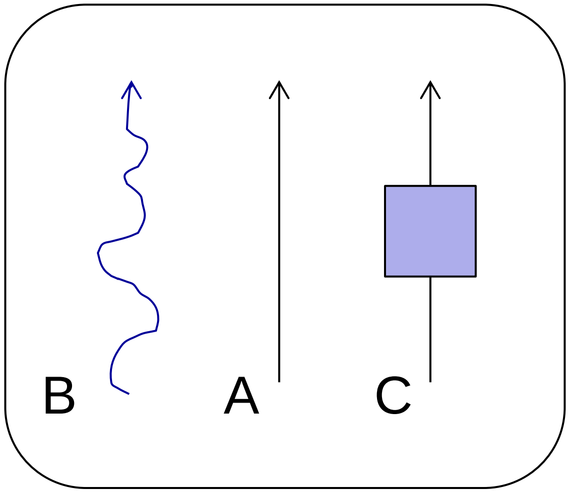
AMID (Gaussian) = quantum mutual info - the *maximal* classical mutual info obtainable by (Gaussian) local measurements

AMID – optimized as discord and symmetric as MID

L. Mista, R. Tatham, D. Girolami, N. Korolkova, G. Adesso, Phys. Rev. A (2011)

Local non-unitary measurement

Discrete example



$$\hat{\rho}_{AC} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

after measurement on C:

$$\hat{\rho}_{A'C'} = \frac{1}{2}\left(|00\rangle\langle 00| + |1+\rangle\langle +1|\right)$$

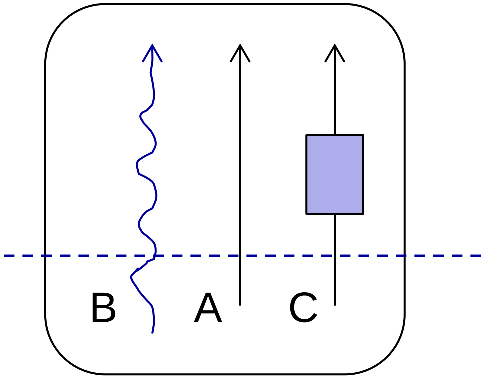
with $|+\rangle_C = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Purification: $|\psi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ - GHZ – state, max entangled

Streltsov, Kampermann, Bruss, PRL 2011 : "Are there any noisy channels that might even *increase* the amount of quantum correlations? How does dissipation influence quantum correlations, and how are they affected by decoherence?"

Initially: entanglement across any bipartition (GHZ)

$$\mathcal{E}_F\left(\hat{\rho}_{AB,C}\right) = \mathcal{E}_F\left(\hat{\rho}_{A,BC}\right) = \mathcal{E}_F\left(\hat{\rho}_{B,AC}\right) = 1$$



Now: any subsystem traced out –

no entanglement btw two remaining ones:

$$\mathcal{E}_F\left(\hat{\rho}_{AB}\right) = \mathcal{E}_F\left(\hat{\rho}_{AC}\right) = \mathcal{E}_F\left(\hat{\rho}_{BC}\right) = 0$$

classical correlations btw those are ***maximal:***

$$\mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{AC}\right) = 1$$

$$\mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AB}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{AB}\right) = 1$$

$$\mathcal{J}^{\leftarrow}\left(\hat{\rho}_{BC}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{BC}\right) = 1$$

$$\mathcal{S}\left(\hat{\rho}_{A}\right) = \mathcal{S}\left(\hat{\rho}_{B}\right) = \mathcal{S}\left(\hat{\rho}_{C}\right) = 1$$

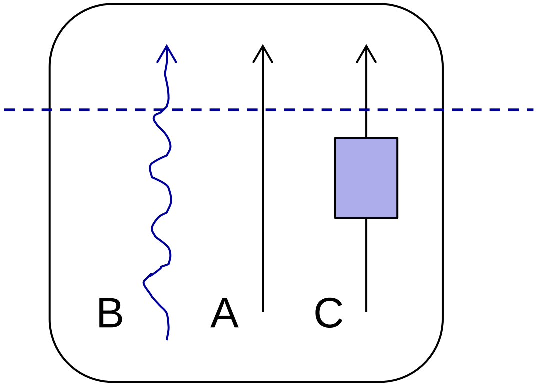
Koashi-Winter:

$$\boxed{\mathcal{S}\left(\hat{\rho}_{A}\right) = \mathcal{E}_F\left(\rho_{AB}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right)}$$

The entropy of marginal $\hat{\rho}_A$ quantifies capacity of Alice's state to form correlations

$$\boxed{\mathcal{S}\left(\hat{\rho}_A\right) = \mathcal{E}_F\left(\rho_{AB}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right)}$$

Local non-unitary measurement on C



$$\mathcal{S}\left(\hat{\rho}_{A'}\right) = \mathcal{S}\left(\hat{\rho}_A\right) = 1$$

$$\mathcal{S}\left(\hat{\rho}_{B'}\right) = \mathcal{S}\left(\hat{\rho}_B\right) = 1$$

Alice's and Bob's capacities to create correlations remain unchanged

Charlie's capacity to create correlations decrease upon the measurement on C:

$$\mathcal{S}\left(\hat{\rho}_{C'}\right) = \frac{\ln\left[8\right] - \sqrt{2}\coth^{-1}\left[\sqrt{2}\right]}{\ln\left[4\right]} = S_0^C$$

$$\boxed{\mathcal{S}\left(\hat{\rho}_{C'}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{B'C'}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{A'C'}\right) = S_0^C < \mathcal{S}\left(\hat{\rho}_C\right)}$$

After non-unitary measurement on *C:*

$$\mathcal{S}\left(\hat{\rho}_{A'}\right) = \mathcal{E}_F\left(\hat{\rho}_{A'B'}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{A'C'}\right) = 1$$

classical correlations decrease

$$\mathcal{J}^{\leftarrow}\left(\hat{\rho}_{A'C'}\right) = 1 - S_0^C$$

capacity for Alice's correlations must be filled up $\mathcal{E}_F\left(\hat{\rho}_{A'B'}\right) = S_0^C$

Alice and Bob become entangled

The discord between A and C arises as a side effect of this entanglement formation between the subsystem unaffected by the measurement and environment

Tatham, Quinn, Korolkova, in preparation

Koashi-Winter inequality:

$$\boxed{\mathcal{S}\left(\hat{\rho}_A\right) = \mathcal{E}_F\left(\rho_{AB}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right)}$$

- for pure tripartite system

$$\mathcal{E}_F\left(\rho_{AB}\right) = \lim_{\{p_i,|\psi_i\rangle\}} \sum_i p_i S(\mathrm{Tr}_B[|\psi_i\rangle\langle\psi_i|])$$

$$\{p_i, |\psi_i\rangle\} : \quad \sum_i p_i |\psi_i\rangle\langle\psi_i|] = \rho_{AB}$$

- min taken over all ensembles satisfying this

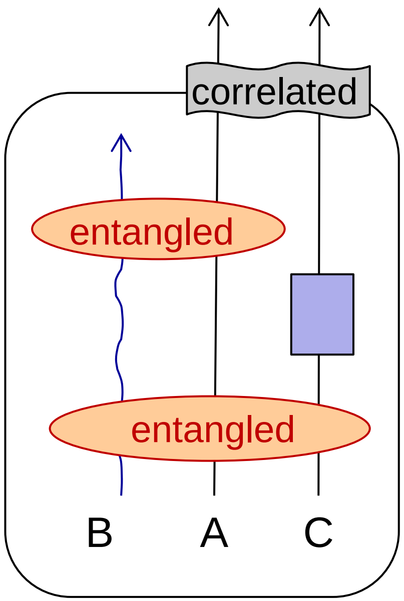
Quantum Discord:

$$\boxed{\mathcal{D}^{\leftarrow}(\hat{\rho}_{AC}) = \mathcal{I}_q(\hat{\rho}_{AC}) - \mathcal{J}^{\leftarrow}(\hat{\rho}_{AC})}$$

$$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB}) + \inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)$$

The discord between A and C arises as a side effect of the entanglement formation between the subsystem unaffected by the measurement and environment

$$\begin{aligned} \mathcal{D}^{\leftarrow}\left(\hat{\rho}_{A'C'}\right) &= \mathcal{I}\left(A':C'\right) - \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{A'C'}\right) = 2S_0^C - 1 \\ \mathcal{D}^{\rightarrow}\left(\hat{\rho}_{A'C'}\right) &= \mathcal{I}\left(A':C'\right) - \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{A'C'}\right) = 0 \end{aligned}$$



Quantum discord is not really a fundamental phenomenon but a side effect of all the changes in local entropy in a quantum system coupled to the environment

Information-theoretical concept of quantum/classical

$$\rho_{12} = \sum_i p_i \rho_1^i \otimes \rho_2^i$$

entanglement/separability according to Werner 1989

can be physically indistinguishable

not all the information about them can be locally retrieved;
Cannot prepare by LOCC.

This phenomenon has no classical counterpart,
quantumness of the correlations in separable state with positive discord