THREE P's IN COSMOLOGY: PROGRESS, PROBLEMS, AND PERSPECTIVES

(half a century in 25 minutes)

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Cosmology in the 60s:

L.D. Landau: ”Always in error but never in doubt.”

Still great works (only some of them):

Kobzarev, Okun, Pomeranchuk, Yad. Fiz. 3 (1966) 1154, mirror matter.

Zeldovich, Adv. Astron. Ap. 3 (1965) 42; Zeldovich, Okun, Pikelner, Usp. Fiz. Nauk 87, 113 (1965), freezing of species [Lee-Weinberg equation, 1977].

Gerstein, Zeldovich, Pis'ma ZhETF, 4 (1966) 174, cosmological bound: $m_\nu < 30$ eV. [Cowsick-McLelland bound, 1972, with important mistakes].

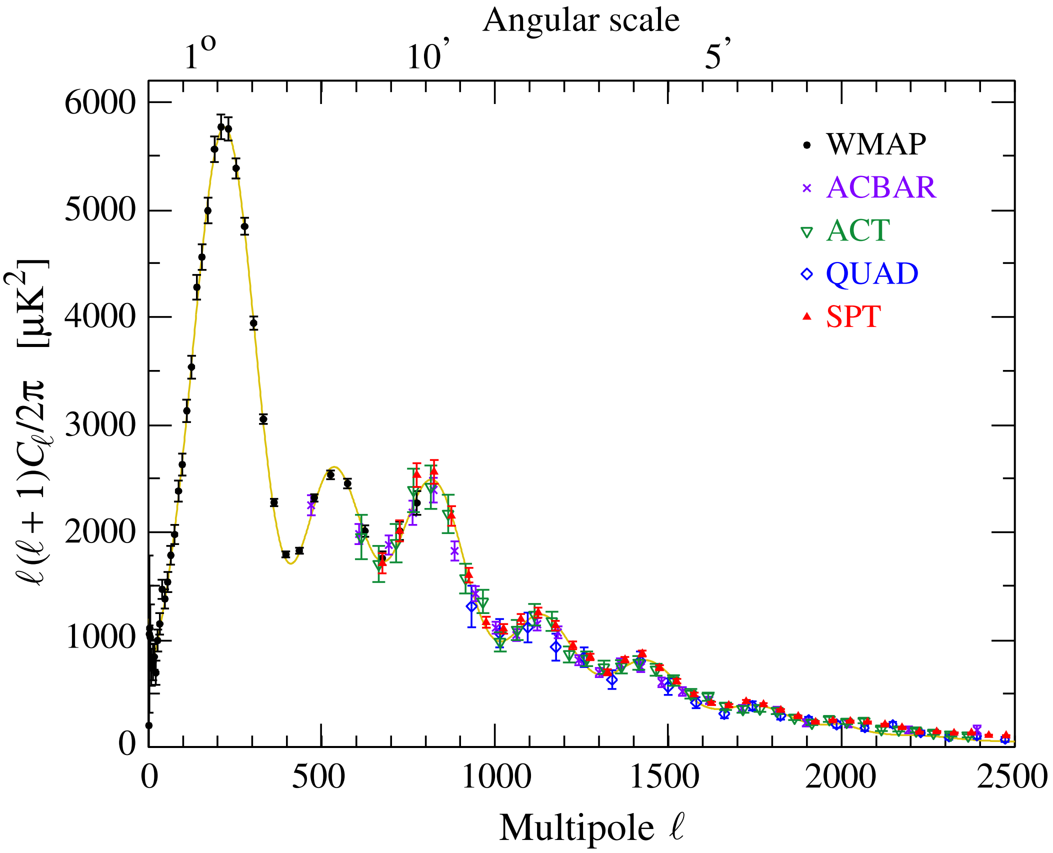
A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5, (1967) 32, baryogenesis, 3 principles: C and CP violation, breaking of thermal equilibrium, hypothesis of B-nonconservation. Nobody believed but now it is an observational fact, though indirect.

Many believed to cold universe model (e.g. Zeldovich, till CMB observation in 1965). A cornerstone of big bang: Cosmic Microwave background (CMB). Discovery: A.A. Penzias, R. Wilson, Ap. J. 142 (1965) 419.
Also: T.A Ter Shmaonov, Pribory i Tekhnica Eksperimenta, 1 (1957) 83.
Predicted by G. Gamow, Phys. Rev. 70 (1946) 572; earlier Friedman.
Detailed calculations of the spectrum: A.G. Doroshkevich, I.D. Novikov, DAN USSR, 154 (1964) 809.

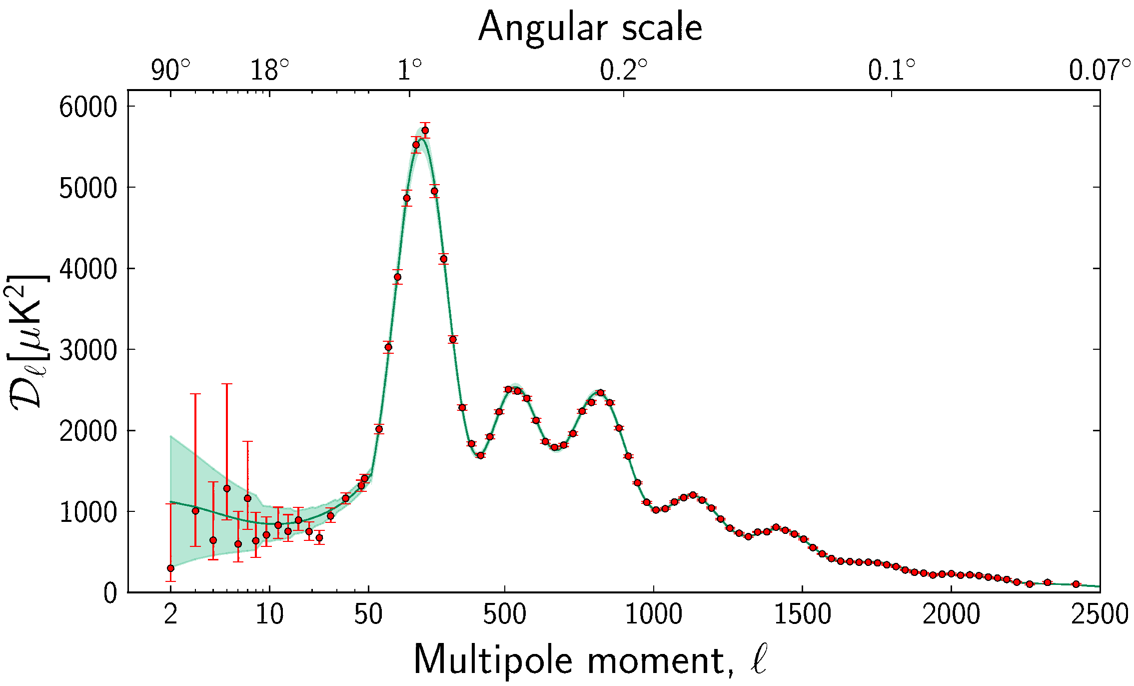
First measurements of angular fluctuations of CMB temperature: COBE: G.F. Smoot et al, Astrophys. J. 396 (1992) L1. Earlier Russian ”Relict” only quadrupole and too big.

Prior to measurements fluctuations was believed to be much larger than detected (without DM).

Later, several balloons and satellites (WMAP), very recently Planck with fantastic precision. Now it is one of the most sensitive methods to measure cosmological parameters.



Combined data of temperature fluctuations before Planck



Spectrum of angular fluctuations of CMB by Planck. Looking at these figures one can determine all cosmological parameters but it helps to use other data to resolve degeneracy.

Observational accuracy in the 60s:

Hubble parameter:

$$H = 50 - 100 \; km/sec/Mpc.$$

Baryon-to-photon ratio:

$$N_B/N_\gamma = 10^{-9\pm 1}.$$

Dark matter and dark energy unknown.

95% of matter escaped observation.

Present day precision is a few per cent for chemical content and for the values of the cosmological parameters.

First indications to DM in the 70s:
J. Einasto, A. Kaasik, E. Saar, Nature 250 (1974) 309;
J. P. Ostriker, P. J. E. Peebles, A. Yahil, Astrophys. J. 193 (1974) L1.
Earlier: J. H. Oort, Bull. Astron. Inst. Netherlands 6 (1932) 249;
F. Zwicky, Helv. Phys. Acta 6 (1933) 110, were not taken seriously.

Breakthrough in theory: inflation, early exponential expansion.

Pioneering papers:

D. Kazanas, Ap. J. 241 (1980) L59;

A.Guth, Phys. Rev. D 23 (1981) 347,

beautifully and simply solves all fundamental problems of the Friedmann cosmology: horizon, flatness, homogeneity and primordial density perturbations, origin of expansion.

Two types of most popular models: A.D. Linde's inflaton: scalar field with almost flat potential, (new inflation and chaotic inflation).

Starobinsky: R^2-inflation, (addition of R^2-term by radiative corrections).

Recent Planck measurements of $\delta T/T$ strongly restrict possible types of mechanisms of inflation, spectrum of perturbations supports R^2 inflation.

Observational predictions of inflation:
Spectrum of density perturbations, **Mukhanov, Chibisov Pisma ZhETF, 33 (1981) 549,** **power law spectrum with $n = 0.96$, very well agrees with observations.**
Gravitational waves from inflation, **Starobinsky, Pisma ZhETF, 30 (1979) 719.** **They could be the final proof of inflation,** **but intensity of GWs is model dependent and may be low.**

Inflation seems to be an experimental fact. Life is impossible without cosmological inflation (beware of danger of no-go theorems in physics). Sufficiently long inflation is impossible, if baryonic number is conserved. Thus our existence proves that protons or nuclei are unstable. Still needed: search for proton decay or neutron-antineutron oscillations.

50 years ago: ”Our existence proves that baryonic number is conserved.”
Now: we exist, so B is not conserved.
The same experimental fact but opposite conclusions.

Standard Cosmological Model (SCM) is established. It very well describes the universe but "dark clouds" still exist (maybe not as serious as two Lord Kelvin ones: UV catastrophe and Michelson-Morley problem, but maybe much more serious). Anyhow, gross features of the universe history from inflationary creation to the present day is understood and confirmed by the data and there is no doubts about big-bang cosmology AND NO DOUBTS ABOUT NEW PHYSICS BEYOND MSM.

Cornerstones of the SCM:

1. General Relativity (GR); though constantly questioned, $F(R)$-theories.
2. Initial inflationary stage. Practically experimental fact.
3. Universe heating by inflaton decay. BIG BANG.
4. Baryogenesis. Inflation proves B-nonconservation.
5. Big Bang Nucleosynthesis (BBN).
6. Cosmic Microwave Background radiation (CMB), deepest telescope.
7. LSS formation, theory and data.

Beyond SCM.

Multi-dimensional cosmologies, small extra dimensions or large extra dimensions. Do we live in domain wall? (Rubakov, Shaposhnikov, Phys. Lett. B125 (1983) 136.

Prior inflation (pre-big-bang?) Quantum gravity? Quantum space-time. Terra incognita.

Safe grounds, universe today.

Hubble parameter $H = 67.3 \pm 1.2\ \frac{km}{s\,Mpc.}$

Critical energy density:

$$\rho_c = \frac{3H^2 m_{Pl}^2}{8\pi} = 0.85 \cdot 10^{-29}\,\mathrm{g/cm^3}.$$

If $\rho > \rho_c$, the universe is closed;

$\rho < \rho_c$, the universe is open;

$\rho = \rho_c$, the universe is flat, with high school geometry. But expansion can last forever even for closed universe, due to dark energy.

Fractional energy density of different forms of matter: $\Omega_j = \rho_j / \rho_c$.
Observationally proved that:
$\Omega_{tot} = 1 \pm 0.02$,
i.e. the average geometry is Euclidean.

Usual baryonic matter: $\Omega_b \approx 0.05$.
Dark matter: $\Omega_{DM} \approx 0.27$ unknown.
Dark energy: $\Omega_{DE} \approx 0.68$, unknown and very puzzling.
Small neutrino fraction: $\Omega_\nu < 5{\cdot}10^{-3}$.

How one can see invisible: by gravity.
These results are obtained from the combined analysis of large scale structure of the universe and from CMB (especially from the recent Planck data).

Before Planck and after Planck:

$H = 71 \pm 2.5, \quad H = 67.9 \pm 1.5;$

$\Omega_B = 4.5\%, \quad \Omega_B = 4.9\%;$

$\Omega_{DM} = 22.7\%, \quad \Omega_{DM} = 26.8\%;$

$\Omega_{DE} = 72.8\%, \quad \Omega_{DE} = 68.3\%.$

Possible anomalies: low amplitudes at large scales, large scale asymmetry w.r.t. ecliptics.

Most impressive are new bounds on CνB. With new Planck data:

$$\sum m_\nu < 0.23 \text{ eV},$$

The best device for weighing neutrinos is telescope. KATRIN?

And $N_\nu^{(eff)} = 3.30 \pm 0.27$, while the standard theory says $N_\nu^{(eff)} = 3.046$, first calculated by AD and M. Fukugita. It there sterile neutrino or some other form of dark radiation?

From BBN (Izotov et al, 1308.2100, on primordial ^4He):

$$\Omega_b h^2 = 0.0234 \pm 0.0019 \;\; (68\% \; CL)$$

and effective number of neutrino species:

$$N_{eff} = 3.51 \pm 0.35 \;\; (68\% \; CL).$$

Planck:

$$\Omega_b h^2 = 0.02205 \pm 0.00028 \,.$$

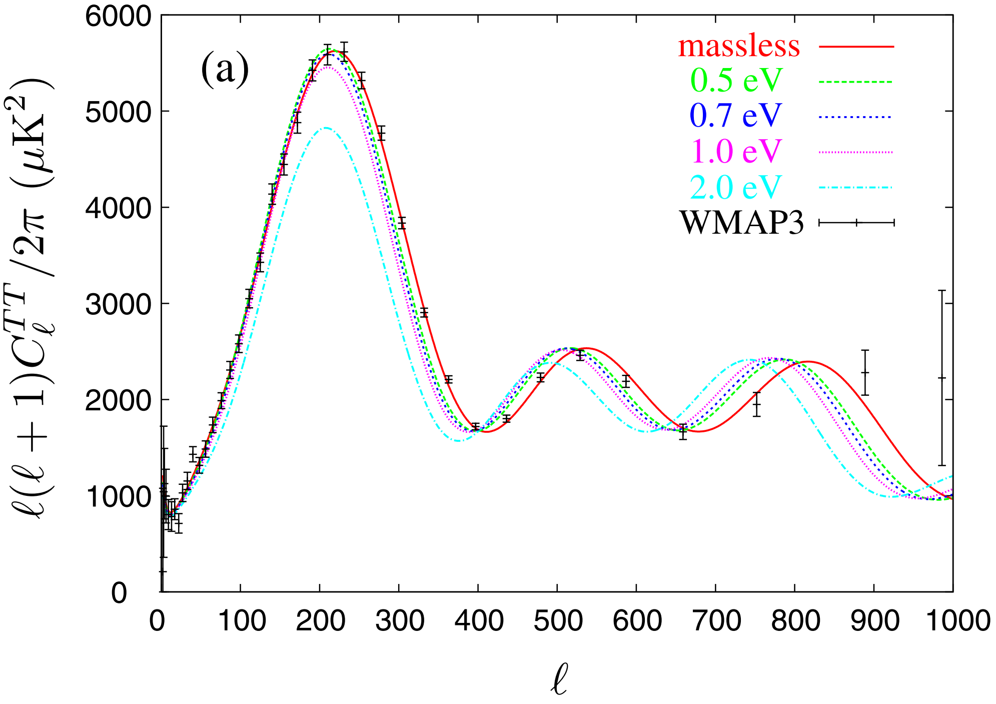
Cooke et al, 1308.3240:
$(D/H)_p = (2.53 \pm 0.04) \cdot 10^{-5}$,
corresponding to
$\Omega_b h^2 = 0.02202 \pm 0.00045$
and the effective number of neutrinos:
$N_{eff} = 3.28 \pm 0.28$.
Is there sterile neutrino or some other form of Dark Radiation?

Impact of neutrinos on angular fluctuations of CMBR

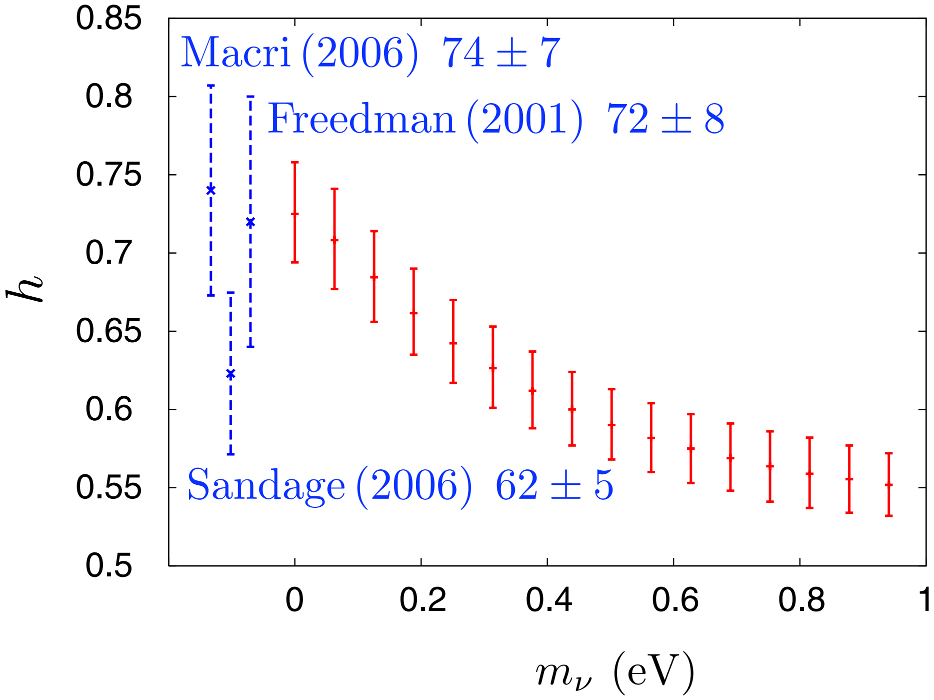
1. Shift of peaks to the left with rising mass. The larger m_ν, the earlier is the non-relativistic stage. Thus the distance to the last scatterring surface is shorter and peaks moves to smaller l

However, this shift can be compensated by a shift in H.

2. Decrease of the (1st) peak hight. **Neutrinos with $m_\nu > 0.6$ eV become nonrelativistic before recombination,** **matter radiation equality takes place earlier and the enhancement of the first peak by ISW effect is weaker.**



Effects of massive neutrinos on the CMB TT power spectrum (Ichikawa, Kawasaki, Fukugita).



The constraints on h for several fixed values of neutrino mass (red solid bars). The constraints from distance ladder measurements (blue dashed bars).

Observation of DM.

1. Flat rotational curves.

2. Gravitational lensing.

3. Equilibrium of hot gas in rich galactic clusters.

4. Cluster evolution.

5. Combined analysis of LSS (in particular, BAO) and CMB.

All agree, giving $\Omega_{DM} \approx 0.3$.

Without DM life would not exist now. **Because of low $\delta T/T$ of CMB, density fluctuations at hydrogen recombination at $T \approx 3000$ K, i.e. $z \approx 10^3$, are also small, $\delta\rho/\rho \sim 10^{-4}$. Without DM $\delta\rho/\rho$ could start rising only after recombination and rise at most as the cosmological scale factor, so today $\delta\rho/\rho < 0.1$. Stars and planets would not be created to the present time.**

Three types of DM.

1. If $M_{FS} > M_{gal} \sim 10^{12} M_{\odot}$ - hot dark matter (HDM).
Example: neutrino, $m \lesssim$ eV.
2. If $m \sim$ keV, then $M_{FS} \sim M_{gal}$ - warm dark matter (WDM).
Example: sterile neutrino, if exists, or pseudogoldstone boson.
3. Cold dark matter (CDM):
$M_{FS} < M_{gal}$.

Forms of CDM:

1. **LSP, m $\sim 100 - 1000$ GeV.**
2. **Heavy leptons, m ~ 2 GeV. Why long-lived?**
3. **Ultraheavy, quasi-stable particles, m $\sim 10^{13}$ GeV.**
4. **Axion, m $\sim 10^{-5}$ eV. Why CDM?**
5. **PBH, M $\geq 10^{16}$ g.**

6. Mirror matter, ”normal masses”, strongly interacting and dissipating.
7. Non-topological solitons, Q-balls.
8. QCD nuggets.
9. None of the above.

SCM: ΛCDM, i.e. DE+CDM.

Problems with CDM.

1. Missing satellites: CDM predicts an order of magnitude more galactic satellites than observed.

2. Destruction of galactic disk: Even if the number of the satellites is reduced by star formation winds, many smaller tightly bound DM systems would survive and destroy galactic disk by gravitational heating.

3. Central cusps: expected singularity in galactic centers, $\rho_{DM} \sim r^{-\kappa}$, $\kappa = 1 - 2$, while flat profiles are observed.

4. Excessive angular momentum: CDM predicts much smaller galactic angular momentum than observed.

Possible solutions:

1. Insufficient accuracy of numerical simulation or neglected physical effects, e.g. role of baryons
2. Dissipative and self-interacting DM (e.g. mirror). Possibly does not help.
3. WDM, or better, a mixture of WDM and CDM.

DARK ENERGY, antigravitating substance with the equation of state: $P = w\rho$ with $w = -1.13^{+0.13}_{-0.10}$. Friedman equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3P).$$

NB: pressure gravitates! Life is possible only because of that. Negative pressure is the source of the cosmological expansion, initial push, but it also (unnecessarily?) exist now. Why?

Data in favor of DE:

a) **Universe age crisis, last century.** With $H \geq 70$ km/sec/Mpc the universe would be too young, $t_U < 10$ Gyr, while stellar evolution and nuclear chronology demand $t_U \geq 13$ Gyr.

b) $\Omega_{\rm m} = 0.3$, measured by several independent ways: mass-to-light ratio, gravitational lensing, galactic clusters evolution (number of clusters for different red-shifts z).

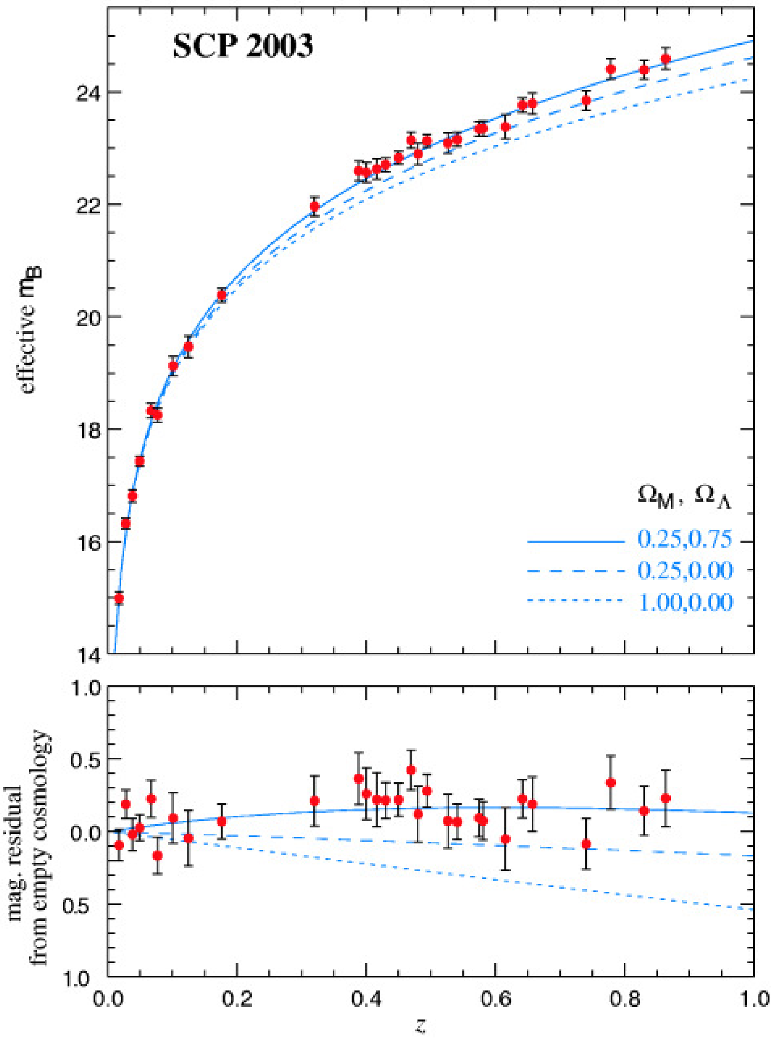
On the other hand:

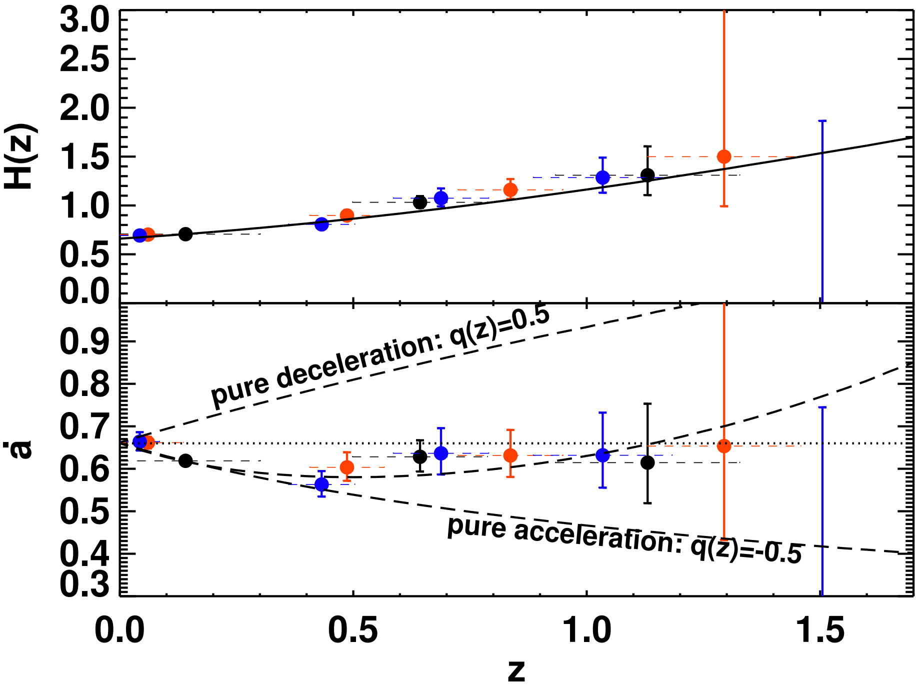
inflation predicts $\Omega_{\rm tot} = 1$ and it is indeed observed: spectrum of angular fluctuations of CMBR (position of the first peak) "measures" $\Omega_{\rm tot} = 1 \pm 0.03$.

c) Dimming of high z supernovae. Cannot be explained by dust absorption because it was found that the effect is non-monotonic in z. At larger z dimming decreases. Indeed, $\rho_m \sim 1/a^3$, while $\rho_{vac} = const.$ Equilibration at z ≈ 0.7.

d) LSS and CMBR well fit theory if $\Omega_{\rm v} \approx 0.7$. Suppression of fluctuations at large scales due to DE.

Direct measurement of acceleration. Dimming of high redshift supernova, if they are standard candles means that they are at a larger distance. i.e. the universe expands faster than expected. Nonmonotonic dependence on z excludes light absorption on the way. Nobel Prize of 2011: S. Perlmutter, B.P. Schmidt, and A.G. Riess ”for the discovery of the accelerating expansion of the Universe through observations of distant supernovae”.


SCP 2003
effective m_B
14
16
18
20
22
24
Ω_M, Ω_Λ
0.25,0.75
0.25,0.00
1.00,0.00
mag. residual
from empty cosmology
1.0
0.5
0.0
0.5
1.0
0.0
0.2
0.4
0.6
0.8
1.0
z


H(z)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\dot{a}$
0.9
0.8
0.7
0.6
0.5
0.4
0.3
pure deceleration: q(z)=0.5
pure acceleration: q(z)=-0.5
0.0
0.5
1.0
1.5
z

Equation of state $w = -1$, vacuum energy: $T_{\mu\nu} = \rho_{vac}\, g_{\mu\nu}$, and $P = -\rho$.

$$\dot{\rho} = -3H(\rho + P) = const,$$

remains constant despite cosmological expansion.
If $w < -1$, energy density rises with time and cosmological singularity with infinite ρ is reached in finite time. Recall that the central value of w is smaller than (-1). Phantom cosmology?
All will be turn apart.

Problem of vacuum energy. Maybe the observed DE is simply vacuum energy, however the estimated value of ρ_{vac} is by 50-100 orders of magnitude larger than the observed one.

Troubling mystery:
proton is a bound state of three quarks with masses about 5 MeV each. So its mass should be 15 MeV minus binding energy, instead of 938 MeV.

QCD is well established and experimentally verified science leads to conclusion that **vacuum is not empty** but filled with quark (Gell-Mann, Oakes, Renner) and gluon (SVZ) condensates:

$$\langle \bar{\mathrm{q}}\mathrm{q} \rangle \neq 0, \quad \langle G_{\mu\nu} G^{\mu\nu} \rangle \neq 0\,,$$

both having NEGATIVE vacuum energy

$$\rho_{vac}^{QCD} \approx -10^{45} \rho_c.$$

Vacuum condensate is destroyed by quarks and the proton mass is:

$$m_p = 2m_u + m_d - \rho_{vac} l_p^3$$

$m_u \sim m_d \sim 5$ **eV.**

Who adds the necessary "donation" to make the OBSERVED $\rho_{vac} > 0$ and what kind of matter is it?

Something must "live" in vacuum who donates positive contribution to ρ_{vac} compensating ρ_{QCD} with fantastic precision, 10^{-45}.

Resolution is unknown. It could be dynamical adjustment, modification of gravity in such a way that ρ_{vac} does not gravitate, infrared screening, anthropic principle with almost infinite set of subtraction constants, ... ???

Solution of the DE problem is most probably impossible without understanding of the mechanism of compensation of vacuum energy.

However, phenomenological description of DE may be instructive. The problem is aggravated by the cosmic coincidence: $\Omega_{DE} \sim \Omega_{DM} \sim 1$, while $\rho_{DM} \sim 1/a^3$ and $\rho_{DE} \sim const.$

Two main possibilities, but none solves the vacuum energy problem:

1. Slowly varying massless or very light scalar field.

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \left[(\partial \phi)^2 - U(\phi) \right].$$

2. Gravity modification at large scales:

$$S = \frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} [R + F(R)] + S_m .$$

Nonlinear function of R leads to higher order equations of motion. Care should be taken of instability, gravitational singularities, ghosts(?), tachyons(?).

There are several similar types of $F(R)$):

$$F(R) = \lambda R_0 \left[\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right] - \frac{R^2}{6m^2}.$$

The last term is added to prevent from past singularity in cosmology and future singularity in systems with rising energy/mass density.

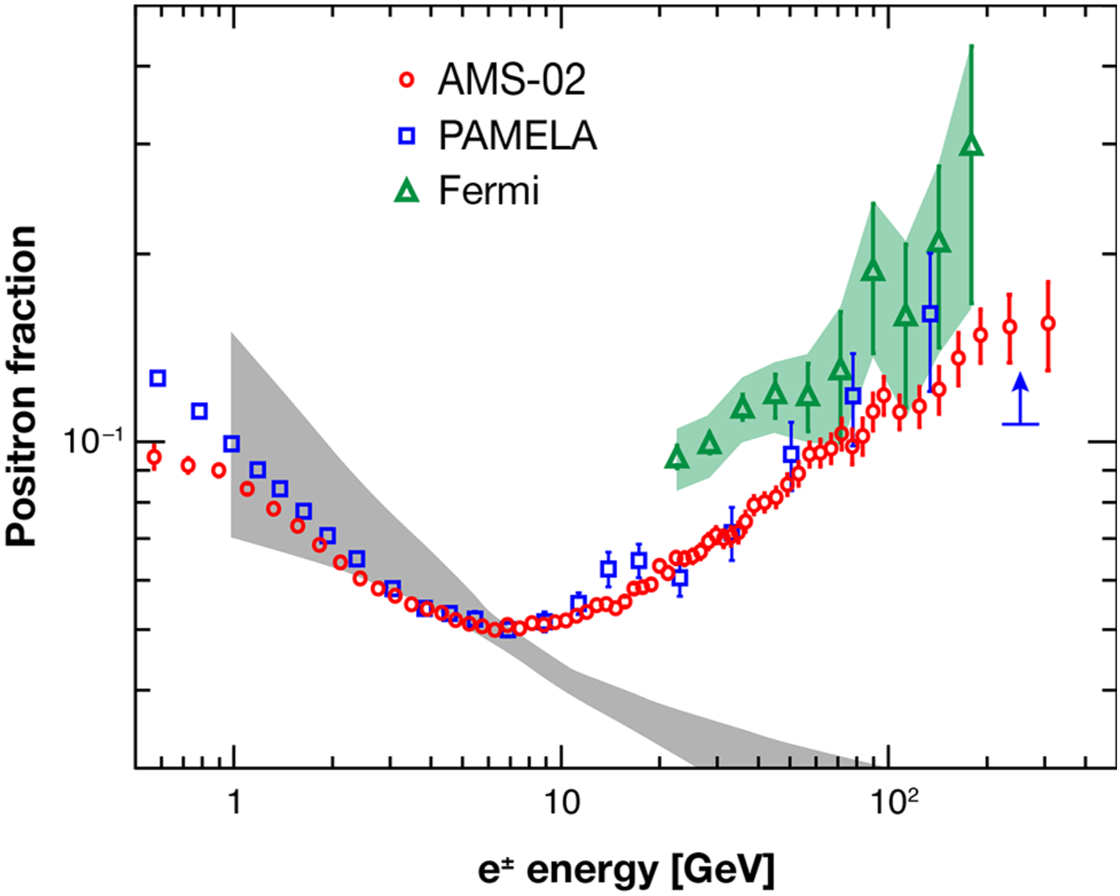
High frequency oscillations and particle production. Contribution to UHECR

Antigravity inside matter clouds ???

Is that pro or contra?

Cosmic antimatter: may there be abundant primordial antimatter, antigalaxies, antistars, antiplanets, antipeople? All observed antiprotons and positrons are compatible with secondary production. However powerful positronium annihilation line from the galactic center and excessive high energy positrons in cosmic rays are not yet explained.

Release of new AMS data.
Still no anti-helium. Confirmation of high energy positron excess discovered by PAMELA.



Plot: M. Aguilar *et al*. [1]; Grey region, see Ref. [4]

Figure 1: The positron fraction in high-energy cosmic rays. The new measurement from the AMS extends over a wider energy range and has much lower uncertainty than the earlier measurements from the PAMELA and Fermi-LAT satellites (or older balloon experiments). The AMS measurement confirms an excess in the high-energy positron fraction, above what is expected from positrons produced in cosmic-ray interactions. (The grey band indicates the expected range in the positron fraction, which is based on calculations in Ref. [4].)

Problems and questions.

1. Who are DM objects?

2. What is the mechanism of cosmological acceleration: DE or modified gravity, or what else?

3. Cosmic conspiracy. **Seemingly unrelated densities of different forms of matter have comparable magnitudes:**

$$\Omega_b \sim \Omega_{DM} \sim \Omega_{DE} \,.$$

4. The mechanism of baryogenesis?
5. Is there abundant cosmic antimatter of primordial origin? Search for antinuclei: PAMELA, BESS, AMS.
6. 0.511 - line from Galactic center?
7. Excess of high energy positrons. PAMELA, AMS.
8. Origin of UHECR.

9. Mechanism of creation of superheavy BH. Early quasar creation with eveloved chemistry?

10. Gamma-bursters, seen at high z.

11. Origin of large scale galactic and intergalactic magnetic fields.

12. Low multipole anomalies in CMB.

13. BBN and ^7Li.

14. Dark radiation; sterile neutrinos.

HUGE PROBLEM: who killed vacuum energy almost to nothing?

PERSPECTIVES:

solution of all or some of the above mentioned problems

and creation of new problems by new observational data.

THE END

Contributions to vacuum energy.

1. **Bosonic vacuum fluctuations:**

$$\langle \mathcal{H}_b \rangle_{vac} = \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2} \langle a_k^\dagger a_k + b_k b_k^\dagger \rangle_{vac}$$

$$= \int \frac{d^3k}{(2\pi)^3} \omega_k = \infty^4 .$$

2. **Fermionic vacuum fluctuations:**

$$\langle \mathcal{H}_f \rangle_{vac} = \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2} \langle a_k^\dagger a_k - b_k b_k^\dagger \rangle_{vac}$$

$$= \int \frac{d^3k}{(2\pi)^3} \omega_k = -\infty^4 .$$

Bosonic/fermionic cancellation - Zeldovich prior to SUSY.

Supersymmetry:

$N_b = N_f$ and $m_b = m_f$, then

$$\rho_{vac} = 0\,.$$

if the symmetry is **UNBROKEN**.

Soft SUSY breaking necessarily leads to

$$\rho_{vac} \sim 10^8 \text{ GeV}^4 \neq 0\,.$$

Broken SUGRA allows for $\rho_{vac} = 0$ but the natural value is

$$\rho_{vac} \sim m_{Pl}^4 \sim 10^{76}\ \mathrm{GeV}^4.$$

Plenty of phase transitions in the course of cosmological cooling

$$\delta\rho_{vac} \gg 10^{-47}\ \mathrm{GeV}^4.$$

Erasure of fluctuations of non-interacting DM particles.

Free streaming length.

Particles relativistic at decoupling, e.g. neutrinos.

$$l_{FS} = a(t)\int^{t}\frac{dt'}{a(t')} + (\text{nonrel.stage}) \approx 2t$$

if relativistic stage, $\mathrm{a}(\mathrm{t}) \sim \sqrt{\mathrm{t}}$.

Free-streaming mass:

$$M_{FS} = \frac{4\pi\,(2t_s)^3}{3}\,\frac{3m_{Pl}^2}{32\pi t^2} = m_{Pl}^2 t_s,$$

At $\mathrm{t = t_s}$ the particles became non-relativistic, i.e. at $\mathrm{T \approx m/3}$.

Since $\mathrm{t\,T^2 \approx 0.1 m_{Pl}}$:

$$\mathrm{M_{FS} = 10^{18} m_\odot \left(\frac{eV}{m}\right)^2}$$

CONTENT.

1. Cosmological acceleration, picture
2. Basic equations, expansion regimes.
3. Universe today, cosm. parameters.
4. How acceleration is measured.
5. Problem of vacuum energy.
6. Dark energy or modifed gravity.

$$S = \frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\,[R + F(R)] + S_m\,.$$

Cosmic acceleration, what does it mean: to illustrate it let us make the following gedanken (thought) experiment. Throw a stone up from the Earth surface. It would slow down and, depending upon the initial velocity, V, either returns back, $V < V_0$,

or disappears in cosmic space $V > V_0$; intermediate case, the stone goes to infinity with V → 0.

The same was believed to be true in cosmology. The universe expansion (it is similar to inertial motion as that of the stone) should slow down,

$$\ddot{a} < 0$$

and the universe would collapse back, closed universe,
or expand forever (open universe),
or, intermediate case, eternal expansion with flat Euclidean geometry.

In all the cases the speed of cosmological expansion slows down.

In this picture geometry and the ultimate destiny of the universe are rigidly connected.

A large set of independent, different types astronomical data show that this is NOT TRUE, the universe today expands with acceleration.

It is as if a stone thrown up from the Earth surface first moved with decreasing velocity but after a while it started to accelerate and never came back. With cosmological inflation, at the very beginning, the picture would be a little different: first acceleration (initial push), then normal decceleration, and lastly (today) surprising acceleration again. Antigravity at the beginning was a source of expansion - inflation. But it is unnecessary now.

BASIC EQUATIONS.

The distribution of matter in the universe is assumed to be homogeneous and isotropic, at least in the early stage, as indicated by isotropy of CMB, and even now at very large scales.

Correspondingly the metric is homogeneous and isotropic (FRW metric):

$$ds^2 = dt^2 - a^2(t)\left[f(r)dr^2 + r^2 d\Omega\right],$$

where $f(r)$ describes 3D space of constant curvature, $f(r) = 1/(1-kr^2)$.

The evolution of the scale factor $a(t)$, i.e. the expansion law, is determined by the Friedmann equations, which follow from the general GR ones for the FRW anzats:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi}{m_{Pl}^2} T_{\mu\nu} \, .$$

The derivation of Friedmann equations from the above ones is straightforward but quite tedious.

COSMOLOGICAL EQUATIONS.

Simplified derivation of the Friedmann equations.

Test body on the surface of homogeneous sphere with energy density ρ.
Energy conservation condition:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\,\rho\,G_N}{3} - \frac{k}{a^2} \tag{1}$$

Here $v^2/2 = G_N\,M/a + const$, and $M = 4\pi a^3 \rho/3$.

Energy balance of medium: $dE = -P\,dV$

where

$E = \rho V$ and $dE = V\,d\rho + 3(da/a)V\rho$

Hence:

$$\dot{\rho} + 3H(\rho + P) = 0. \tag{2}$$

Problem: derive from (1) and (2) the law for acceleration of test body:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3P)$$

NB: pressure gravitates! Life is possible only because of that. Negative pressure is the source of the cosmological expansion, cosmic antigravity.

Exercise: **prove that for $\rho > 0$ any finite objects gravitates. Antigravity is possible only for infinitely large objects (whole space, domain walls, cosmic strings).**

Hints:

1. According to the Schwazzschildt solution the gravitational field of a massive body is determined by its mass for any equation of state:

$$M = \int dr r^2 T_0^0 \, .$$

2. Integrate by parts:

$$0 = \int d^3x \partial_j (x^l T_k^j) = \int d^3x T_k^l \, .$$

SUMMARY:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\,\rho\, G_N}{3} - \frac{k}{a^2},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi\, G_N}{3}(\rho + 3P),$$

$$\dot{\rho} + 3H(\rho + P) = 0.$$

1. No stationary solutions, Einstein's disappointment.

2. If $k > 0$ and $\rho \sim 1/a^n$, n=3,4, expansion will change to contraction.

3. If $\rho > k/a^2$ expansion is forever for any k.

COMMENTS:

1. Only 2 out of 3 equations are independent.

2. Source of gravitational force $\rho + 3P$, not only ρ.

3. Third equation is just $D_\mu T^\mu_\nu = 0$.

Problem: why P gravitates, though we have used Newtonian theory?

COSMOLOGICAL REDSHIFT.

$$z = a(t_U)/a(t) - 1$$

Equation of motion of free particle: two points at distance dx with relative velocity $U = Hdx$. Doppler shift of momentum:

$$dp = dvE = -UE = -HEdx$$

Thus

$$\dot{p} = -HE\dot{x} = -Hp,$$

geodesic equation in flat FRW metric. Solution $p \sim 1/a \sim 1/(z+1)$.

More accurately:

$$\frac{dV^\alpha}{ds} = -\Gamma^\alpha_{\mu\nu} V^\mu V^\nu + \text{curvature term},$$

where $V^\alpha = dx^\alpha/ds$.

For FRW metric this equation coincides with the Doppler one presented above for 3D flat geometry.

System of units: $c = h/2\pi = k = 1$:

$G_N = 1/M_{Pl}^2$;

$M_{Pl} = 1.221 \cdot 10^{19}\,\mathrm{GeV} = 2.176 \cdot 10^{-5}\,\mathrm{g}$;

$m_p = 938\,\mathrm{MeV} = 1.67 \cdot 10^{-24}\,\mathrm{g}$;

$\mathrm{GeV}^{-1} = 1.97{\cdot}10^{-14}\,\mathrm{cm} = 0.66{\cdot}10^{-24}\mathrm{s}$;

$\mathrm{eV} = 1.16 \cdot 10^4 \mathrm{K}^o$.

Cosmological parameters:

$\mathbf{H = \dot{a}/a}$,

$\rho_c = 3H^2\, M_{Pl}^2/8\pi$,

$\Omega = \rho/\rho_c$,

$\Omega = 1$ if $\mathbf{k = 0}$.

Two equations for 3 unknowns: a, ρ, P, Equation of state $P = P(\rho)$ is necessary; sometimes does not exist but the necessary relation (not e.o.s.) can be derived from equations of motion. E.g. for a scalar field:

$$D^2\phi + U'(\phi) = 0$$

one can calculate $T_{\mu\nu}$ and find ρ and P but $P \neq P(\rho)$.

Exercise: calculate $T_{\mu\nu}(\phi)$, ρ, and P.

Simple examples, all for k=0.

1. Nonrelativistic matter: P = 0.

$$\dot{\rho} = -3H\rho$$

and $\rho \sim 1/a^3$ - dilution of massive particle at rest.

$$\dot{a}/a \sim \sqrt{\rho}$$

and thus in NR regime $a \sim t^{2/3}$.

2. Relativistic matter: $P = \rho/3$.

$$\dot{\rho} = -4H\rho$$

and $\rho \sim 1/a^4$ - dilution of particle number and redshift.

$$\dot{a}/a \sim \sqrt{\rho}$$

and thus in NR regime $a \sim t^{1/2}$.

3. Vacuum(-like) regime:

$$T_{\mu\nu} = \rho_{vac}\, g_{\mu\nu}$$
$$P_{vac} = -\rho_{vac}$$

Thus $\dot{\rho} = -3H(\rho + P) = 0$,

$$\rho_{vac} = const$$

and $a \sim \exp{(Ht)}$.
All our visible universe originated from microscopically small volume with negligible amount of matter by exponential expansion with constant ρ.

General linear equation of state:

$$P = w\rho.$$

$w = const$ is a new cosmological parameter.

Exercise:

1. Find $\rho(a)$ and $a(t)$ for any w.
2. Study the case of $w \leq -1$.

UNIVERSE TODAY.

Expansion law: $U = Hd$ with

$$H = 100\,h\,\mathrm{km/sec/Mps},$$

$\mathrm{h} = 0.73 \pm 0.05$,
$H^{-1} = 9.8\,\mathrm{Gyr}/h \approx 13.4\,\mathrm{Gyr}$.

Universe age:

$$t_U = 12 - 15\,\mathrm{Gyr}$$

from the ages of old stellar clusters and nuclear chronology.

UNIVERSE AGE:
Integrating the equation

$$\dot{a} = \left[8\pi\,\rho\,G_N\,a^2/3 - k\right]^{1/2}$$

one finds in terms of the present day values of H and Ω_j:

$$t_U = \frac{1}{H}\int_0^1 \frac{dx}{\sqrt{1 - \Omega_t + \frac{\Omega_m}{x} + \frac{\Omega_r}{x^2} + x^2\Omega_v}}$$

Exercise: 1) derive eq. for t_U;
2) find $\mathrm{t_U}$ for $\Omega_\mathrm{t} = \Omega_\mathrm{m} = 0,\ 0.3,\ 1$;
3) find $\mathrm{t_U}$ for $\Omega_\mathrm{t} = 1$, $\Omega_\mathrm{m} = 0.3$, $\Omega_\mathrm{v} = 0.7$.

Critical energy density:

$$\rho_c = 1.88 \cdot 10^{-29} h^2 \frac{\mathrm{g}}{\mathrm{cm}^3} =$$

$$10.5\, h^2 \frac{\mathrm{keV}}{\mathrm{cm}^3} \approx 10^{-47} h^2\, \mathrm{GeV}^4$$

It corresponds to 10 protons per m^3, but the dominant matter is not the baryonic one.

MATTER INVENTORY

Total energy density:
$\Omega_{tot} = 1 \pm 0.02$
from the position of the first peak of CMBR and LSS.

Usual baryonic matter:
$\Omega_B = 0.044 \pm 0.004$
from heights of CMBR peaks, BBN, and onset of structure formation with small $\delta T/T$.

Total dark matter:
$\Omega_{DM} \approx 0.22 \pm 0.04$
from galactic rotation curves, gravitational lensing, equilibrium of hot gas in rich galactic clusters, cluster evolution, LSS.

The rest:
$\Omega_{DE} \approx 0.76$, $w \approx -1$
- induces accelerated expansion;
measured from dimming of high-z supernovae, LSS, universe age.

Relativistic matter, CMB and CνB.

Photons of CMBR: perfect equilibrium Planck spectrum ($\delta < 10^{-4}$):

$$\frac{dn}{d\omega} = \frac{\omega^2}{\exp(\omega/T) - 1},$$

with $T = 2.725 \pm 0.001\,\mathrm{K}$, $\mu = 0$;
$n_\gamma = 410.4 \pm 0.5\ \mathrm{cm}^{-3}$;
$\rho_\gamma = 0.23\,\mathrm{eV}/\mathrm{cm}^3$;
$\Omega_\gamma = (4.9 \pm 0.5) \times 10^{-5}$.

Radiation is almost isotropic with precision about 10^{-4}. Fortunately the isotropy is not exact, to allow for life and for measurements of parameters.

Other ways in addition to CMB: LSS, gravitational lensing, cluster evolution, BAU, BBN, universe age are independent. It minimizes possibility for an interpretation error.

Data in favor of DM and DE or how one can see invisible.

DM might be observed directly in low background labs, but not DE.

DM: simple qualitative argument In purely baryonic universe structure formation could start only after recombination, when z $> 10^3$; matter became neutral. Due to large Thomson cross-section electrons are frozen into almost homogeneous CMB. Since $\Delta \equiv \delta\rho/\rho \sim z$, the density perturbations might rise only by 10^3. On the other hand, initial

$$\delta \sim \delta T/T \leq 10^{-5}$$

does not have time to rise to unity.

LECTURE II

In purely baryonic universe LSS formation could start at $z \leq 10^3$.

Too short time to create $\delta\rho/\rho \sim 1$.

There must be something else, DM (or invisible matter).

Amount of baryons is about $\Omega_B \sim 0.05$, out of which 90% are invisible.

Where are they?

How we see (invisible) baryons? – BBN and CMBR.

BIG BANG NUCLEOSYNTHESIS

(formation of light elements ${}^{2}H$, ${}^{3}He$, ${}^{4}He$, ${}^{7}Li$, in the early universe).

One of the pillars of the standard cosmological model.

Two steps:

1. Neutron-proton freezing,
$T \approx 1$ MeV, $t \approx 1$ s.

2. Formation of light elements,
$T \approx 65$ keV, $t \approx 200$ s.

Neutron-proton transformations:

$$n + e^{+} \leftrightarrow p + \bar{\nu}_e$$
$$n + \nu_e \leftrightarrow p + e^{-}$$

frozen at $T \approx 0.7$ MeV (see below), and decay:

$$n \leftrightarrow p + e^{-} + \bar{\nu}_e$$

determine starting value of n/p-ratio.

Neutron life-time, is it important?

When T drops down to
$T_{BBN} = 60 - 70$ keV
all neutrons quickly form ^{4}He (about 25% by mass) and ^{2}H (3×10^{-5} by number), ^{3}He (similar to ^{2}H) and ^{7}Li ($10^{-9} - 10^{-10}$) - span by 9 orders of magnitude, well agree with the data.

Problem: why T_{BBN} is much smaller than nuclear binding energy, $E_b \sim$ MeV?
Recall hydrogen recombination,
$T \sim E_b/\ln(N_\gamma/N_B)$.

Neutron-to-proton ratio as a function of time/temperature.

Onset of BBN.

based on known nuclear physics.

Chain of reactions: $\boldsymbol{p\,(n,\gamma)\,d}$, $\boldsymbol{d\,(p\gamma)\,{}^3He}$, $\boldsymbol{d\,(d,n)\,{}^3He}$, $\mathrm{d\,(d,p)\,t}$, $\boldsymbol{t\,(d,n)\,{}^4He}$, **etc.**

1. All goes through deuterium, because of low baryon density two body processes dominate.
2. No stable nuclei with A = 5, suppression of heavier nuclei production.

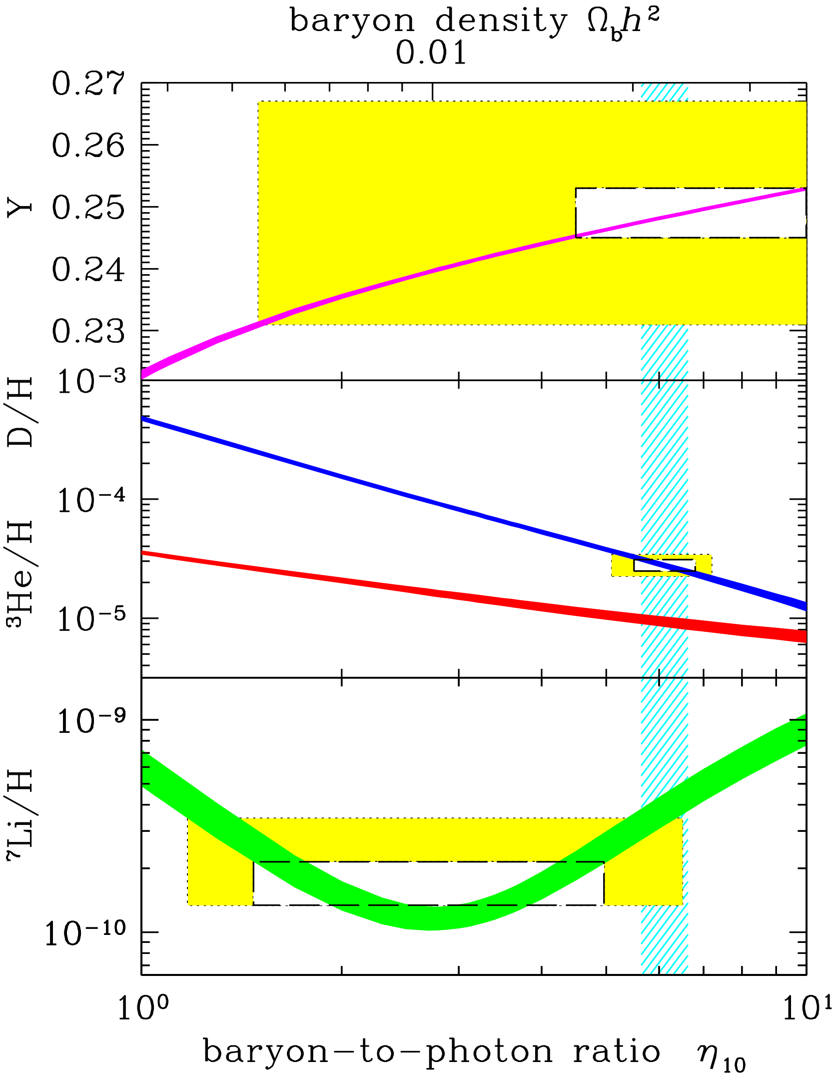


Figure 1: **He^4, D, He^3 and Li^7 predicted by the standard BBN. Boxes indicate the observed light element abundances (smaller boxes: 2σ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematic errors). The vertical band is the CMB measure of the cosmic baryon density.**

Helium-4 slowly rises with η, because the moment of BBN becomes earlier and less neutrons decayed.

Deuterium quickly drops with larger η because it is easier to proceed to heavier nuclei.

Deuterium = baryometer.

Lithium-7 is formed in two competing processes with different dependence on η.

BBN: primordial abundances of D, He^4 as functions of ρ_B demand $\Omega_B \sim 0.05$.

Total amount of gravitating matter:
Flat rotational curves.
Gravitational lensing - amount of matter along line of sight.
Equilibrium of hot gas in rich galactic clusters: $\rho_B/\rho_{DM} \sim 0.2$.
Together with BBN all that demands $\Omega_{DM} \sim 0.25$.

CMBR and Cosmological Parameters.

After hydrogen recombination at

$$z \approx 10^3$$

the CMB photons free-streamed and present to us a snapshot of the universe at that moment.

Small angular fluctuations of temperature, allow to measure cosmological parameters.

Temperature fluctuations on 2D celestial sphere can be expanded as

$$\Delta T(\theta,\phi) = T_0 \sum_{l,m} \Theta_{l,m} Y_{l,m}(\theta,\phi).$$

Multipole moments:

$$\Theta_{lm} = \int d(cos\theta) d\phi Y^*_{lm}(\theta,\phi)\,\Theta(\theta,\phi),$$

where $\Theta(\theta,\phi) \equiv \Delta T/T_0$.
Power spectrum:

$$\langle \Theta^*_{lm} \Theta_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l$$

COMMENTS:

1. The multipole number is related to angular separation as

$$\theta \sim \pi/l.$$

2. We have only one realization of stochastic quantity, $T(\theta,\phi)$, but different m are statistically independent. Thus statistical fluctuations (cosmic variance) are smaller for large l:

$$\Delta C_L/C_l = \sqrt{2/(2l+1)}$$

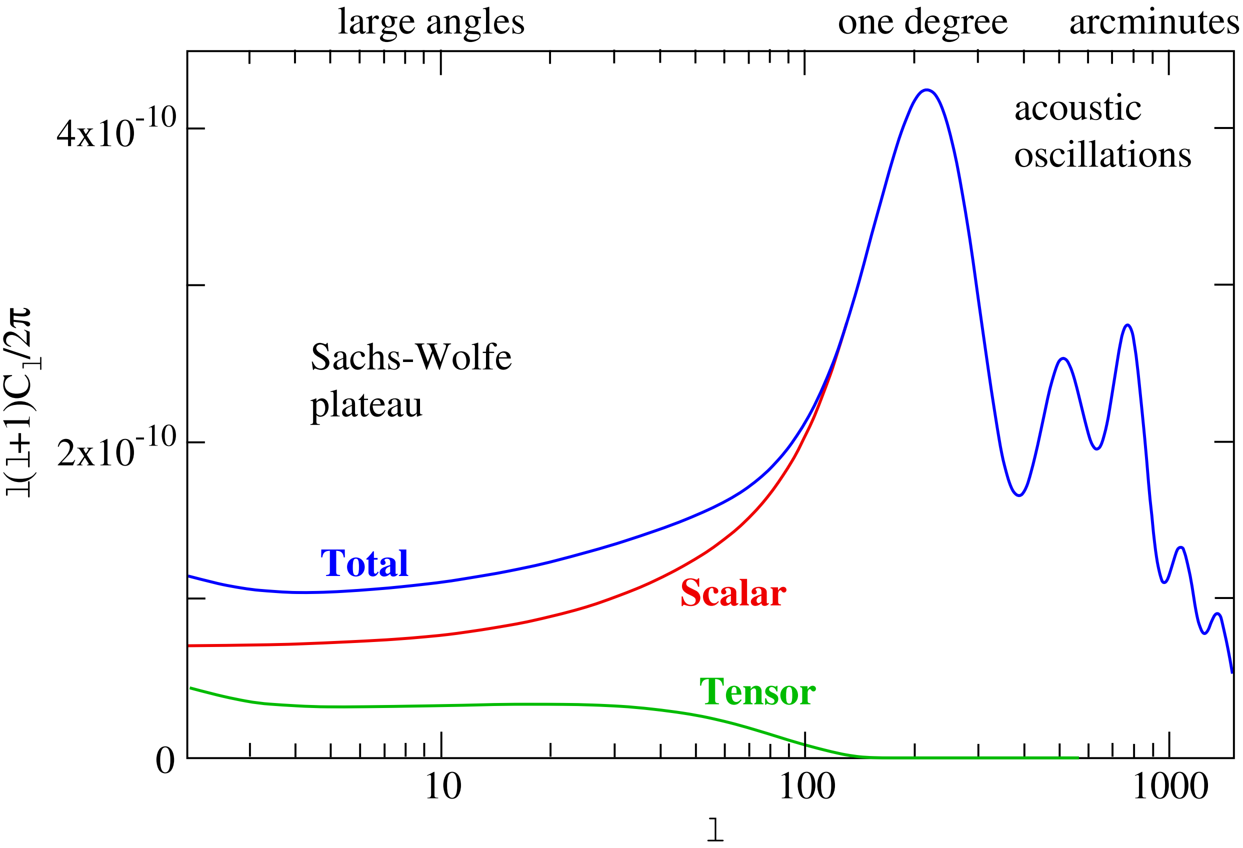


Figure 2: **Angular power spectrum for adiabatic initial perturbations and typical cosmological parameters. The scalar and tensor contributions to the anisotropies are also shown.**

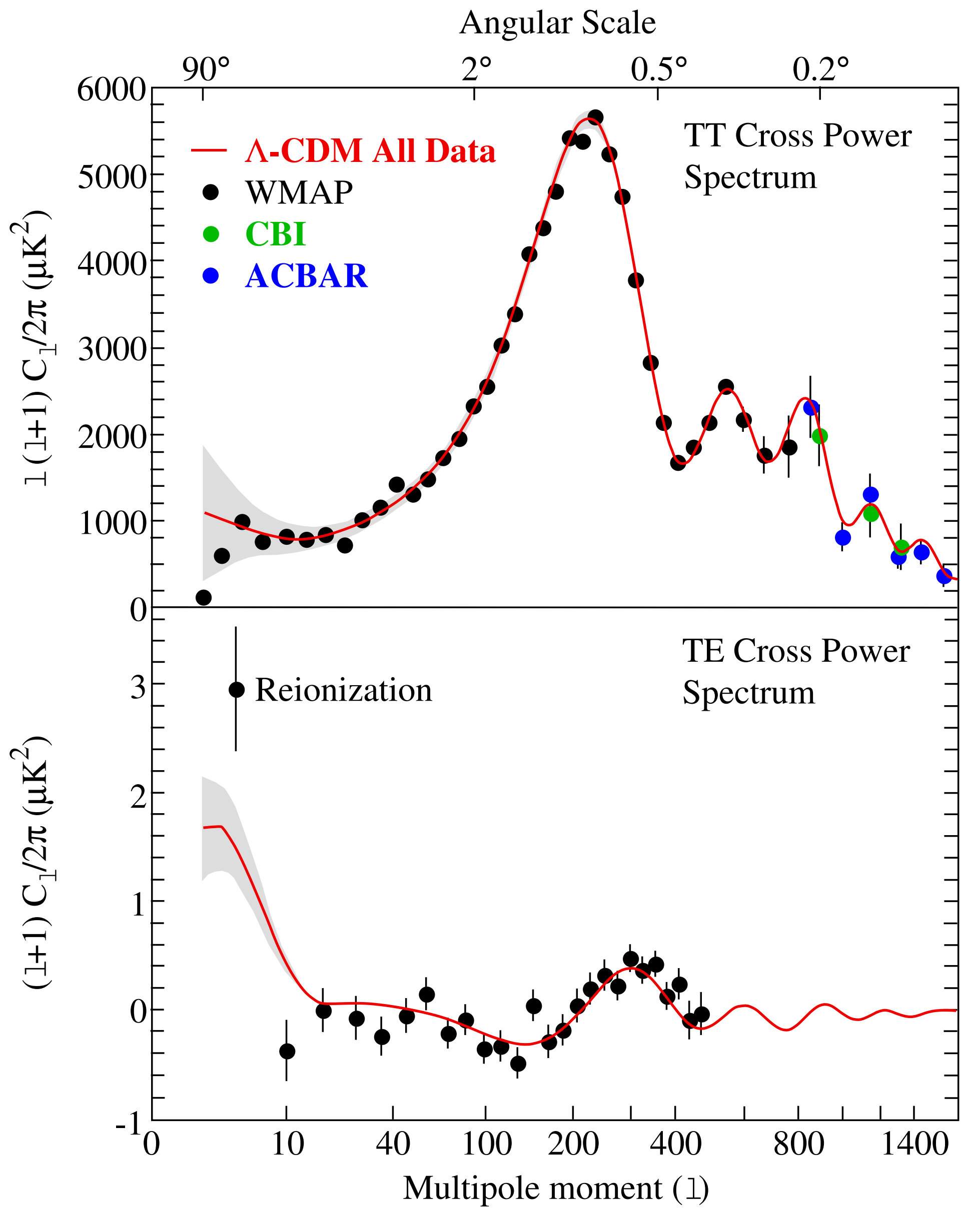


Figure 3: **Temperature-temperature (TT) and temperature-polarization TE power spectra. The best fit ΛCDM model is also shown. Alignment and small amplitudes of low multipoles. "Evil axis"? Cosmic variance.**

Features of the angular spectrum.

1. Large separation ($l < 100$), outside horizon at recombination. Flat because of flat spectrum of primordial perturbations. Slight decrease (or slow rise) is induced by the impact of "anti-gravity", leading to suppression of structure formation – a possible way to estimate amount of DE.

2. $l \geq 100$: entered horizon at recombination or earlier. Acoustic oscillations, since $M < M_{Jeans}$.

Relative amplitudes of the peaks reflects the ratio of mass to the "spring" tension, i.e. n_B/n_γ.

NB: at that stage $\rho_b \sim \rho_\gamma$, that's why peaks are so sensitive to Ω_B.

3. The position of the first peak corresponds to horizon crossing at recombination. The wave length is known. The observed angular scale depends upon 3D geometry, measurement of Ω_{tot} independently of the equation of state.

Analysis of LSS evolution allows to measure Ω_m and Ω_{DE}.
Jeans theory with Lifshitz ”relativization”. Structure formation depends upon the expansion regime.
Without expansion perturbations grow exponentially, while with expension as a power of time. Fast expansion inhibits perturbation rise. DM and DE act in the opposite way. The effect depends upon the perturbation wave length.

Perturbations in flat space-time.

Basic equations:

$$\dot{\rho} + \nabla \cdot (\rho \mathrm{v}) = 0\,,$$
$$\dot{\mathrm{v}} + (\mathrm{v} \cdot \nabla)\mathrm{v} + \frac{1}{\rho}\nabla p + \nabla \varphi = 0\,,$$
$$\nabla^2 \varphi = 4\pi G \rho\,,$$
$$\dot{\mathrm{S}} + (\mathrm{v} \cdot \nabla)\mathrm{S} = 0\,,$$
$$\mathrm{p} = \mathrm{p}(\rho, \mathrm{S})\,.$$

The first equation is the continuity equation, the second is the Euler (force) equation, the third is the Poisson equation of Newtonian gravity, the fourth expresses entropy conservation, and the last is the equation of state.

Perturbations over a constant background: ρ_o, p_0, $\mathrm{v}_0 = 0$, gravitational potential, φ_0, and entropy density S_0:

$$
\begin{aligned}
\rho &= \rho_0 + \delta\rho\,, \\
v &= \delta v\,, \\
\mathrm{p} &= \mathrm{p}_0 + \delta\mathrm{p}\,, \\
\varphi &= \varphi_0 + \delta\varphi\,, \\
\mathrm{S} &= \mathrm{S}_0 + \delta\mathrm{S}\,.
\end{aligned}
$$

$$\ddot{\delta\rho} - c_s^2\nabla^2\delta\rho - 4\pi G\rho_0\delta\rho = \sigma\nabla^2\delta S\,,$$

$$\dot{\delta S} = 0\,.$$

where equation of state reads:

$$\delta p = c_s^2\delta\rho + \sigma\delta S$$

with

$$c_s^2 = \left(\frac{\delta p}{\delta\rho}\right)_{|S}$$

Entropy fluctuations do not grow.

Adiabatic fluctuations, $\delta S = 0$, grow for wave length larger than Jeans one:

$$\mathbf{k} < \mathbf{k_J} = \left(\frac{4\pi G \rho_0}{c_s^2} \right)^{1/2}$$

Exponential instability:

$$\delta\rho_{\mathbf{k}} \sim \exp\left[(\mathbf{k_j} - \mathbf{k}) \mathbf{t} \right]$$

If $\mathbf{k} < \mathbf{k_J}$, solutions are oscillating - sound waves.

Perturbations in expanding universe.

$$\rho(t,x) = \rho_0(t)\,[1 + \Delta(t,x)]$$
$$v(t,x) = v_0(t,x) + \delta v(t,x)$$
$$p(t,x) = p_0(t) + \delta p(t,x)\,,$$

After long algebra:

$$\ddot{\Delta} + (2 + 3c_s^2 - 6w)H\dot{\Delta} + \left[c_s^2\frac{k^2}{a^2} - 4\pi G\rho(1 - 6c_s^2 + 8w - 3w^2) + 12(w - c_s^2)\frac{K}{a^2} + (3c_s^2 - 5w)\Lambda\right]\Delta = 0.$$

where $p = w\rho$, K is space curvature.

Nonrelativistic matter,
$p = 0, \; w = 0, \; \Lambda = 0$:

$$\ddot{\Delta}_m + 2H\dot{\Delta}_m = 4\pi G\rho\Delta_m - c_s^2\left(\frac{k}{a}\right)^2\Delta_m$$

If $k/a(t) < k_J = \left(4\pi G\rho_0/c_s^2\right)^{1/2}$ and ρ_0 is dominated by non-relativisitc matter i.e. $a \sim t^{2/3}$, $H = 2/3t$, $\rho_0 = 3H^2/8\pi G$, the solution is

$$\Delta_m = C_1 t^{2/3} + C_2 t^{-1} \rightarrow C_1 a(t)$$

Density contrast rises as z.

Direct measurment of acceleration. Dimming of high redshift supernova, if they are standard candles means that they are at a larger distance. i.e. the universe expands faster than expected. Nonmonotonic dependence on z excludes light absorption on the way. Nobel Prize of 2011: S. Perlmutter, B.P. Schmidt, and A.G. Riess ”for the discovery of the accelerating expansion of the Universe through observations of distant supernovae”.

Summary figure. 68.3%, 95.4%, and 99.7% confidence regions with the constraints from BAO and CMB both without (left panel) and with (right panel) systematic errors. Cosmological constant dark energy ($w = -1$) has been assumed.

68.3%, 95.4%, and 99.7% confidence regions of the (Ω, w) plane from supernovae combined with the constraints from BAO and CMB both without (left panel) and with (right panel) systematic errors. Zero curvature and constant w have been assumed.

SUMMARY

1. Baryons make 5% of the total universe mass. Found by two independent kind of data: BBN and CMB. 90% of baryons are not yet directly observed.

2. Total energy density is close to the critical one. 3D geometry is flat. Best determination from the position of the first CMB peak. Agrees with other data (LSS and the shape of CMB angular spectrum) and theory (inflation).

3. DM makes 25% as is seen from flat rotational curves, hot rich galaxy clusters, cluster evolution, LSS, BAU, gravitational lensing, CMB. 4. Remaining 70% is antigravitating DE with $P \approx -\rho$ from the iniverse age, LSS, CMB, and high redshift SNae.

LECTURE 3

PROBLEM OF VACUUM ENERGY

probably the most striking problem of contemporary physics.

It started from introduction of Λ-term:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda\, g_{\mu\nu} = 8\pi G_N T_{\mu\nu}\,.$$

Λ is equivalent to vacuum energy:

$$T_{\mu\nu}^{(vac)} = \rho_{vac} g_{\mu\nu},$$

$$\Lambda = 8\pi \rho_{vac}/m_{Pl}^2.$$

L.h.s. or r.h.s.?

Theoretically: $\Lambda \approx \infty$.
Mismatch between theory and data:
50-100 ORDERS OF MAGNITUDE.
Majority point of view during long time and maybe even now:

$$\infty = 0$$

“Corrections are infinite but small” (R. Feynman).

Biographical notes

Name(s):
Cosmological constant, Λ-term, vacuum energy, maybe, dark energy.

Date of birth: 1918

Father A. Einstein: "The biggest blunder of my life" (after Hubble's discovery of cosmological expansion).

Many times assumed dead, probably erroneously. Well alive today.

Still not safe - many want to kill it.

SOME MORE QUOTATIONS:

Lemaitre: “greatest discovery, worth to make Einstein’s name famous”.

Gamow: “λ raises its nasty head again” (after indications that quasars are accumulated near $z = 2$ in the 60s)

Due to general covariance, necessary for massless graviton, the cosmological constant must be CONSTANT:

$$\Lambda = const.$$

Models with $\Lambda = \Lambda(t)$ are not innocent, new fields to respect energy conservation condition are necessary.

or serious modifications of the theory, e.g. non-metric theories.

However, an approximate relation $P \approx -\rho$ can be achieved with light scalar field leading to $\rho = \rho(t)$.

First attempts to make time-dependent Lambda, 1935 by Bronstein (Leningrad); strongly criticized by Landau.

RISE AND FALL
OF LAMBDA-TERM

1. Universe birth: $\Omega_v \approx 1$.

2. Hubble discovery of expansion, earlier Friedman solution: $\Omega_v \equiv 0$. But Friedman studied non-zero Lambda.

3. Lemaitre, De Sitter, later Eddington: Λ is one of the most important discoveries in GR.

4. Still Lambda was not accepted by majority.

5. QSO accumulation near $z = 2$ explained by $\Omega_v \sim 1$. Later rejected.
6. From 60s to the end of the Millennium Lambda was identically zero.

Only a few treated it seriously, starting from Zeldovich.

7. End of 90s many indications to accelerated expansion:

a) Universe age crisis.

b) $\Omega_m = 0.3$, while CMB measures $\Omega = 1$ and inflation predicts $\Omega_{\rm tot} = 1$.

c) LSS + CMB.

d) Direct proof of acceleration: high red-shift SN.

Evolution of vacuum(-like) energy during cosmic history.

1. At inflation $\rho_{vac} \sim 10^{100} \rho_v^{now}$ and was dominant. It was not real vacuum energy but vacuum-like energy of almost constant scalar field inflaton.

2. At GUT p.t. (if such era existed)

$$\delta\rho_{vac} \approx 10^{60}\,\mathrm{GeV}^4\,.$$

3. At electro-weak p.t.

$$\delta\rho_{vac} \approx 10^{8}\,\mathrm{GeV}^4\,.$$

4. At QCD p.t.

$$\delta\rho_{vac} \approx 10^{-2}\,\mathrm{GeV}^4\,.$$

The magnitude of vacuum energies of gluon and chiral condensates are known from experiment!

QCD vacuum is not empty but filled with condensates of quarks and gluons with negative energy.

After inflation till almost the present epoch ρ_{vac} was always sub-dominant

ρ_{vac} started to dominate energy density only recently at $z \approx 0.3$.
Change from deceleration to acceleration: $z \approx 0.7$.

Situation became very grave after it was found that $\rho_{vac} \neq 0$ and today:

$$[\rho_{vac} = const] \sim [\rho_c \sim 1/t^2].$$

CONTRIBUTIONS TO VACUUM ENERGY

1. **Bosonic vacuum fluctuations:**

$$\langle \mathcal{H}_b \rangle_{vac} = \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2} \langle a_k^\dagger a_k + b_k b_k^\dagger \rangle_{vac}$$

$$= \int \frac{d^3k}{(2\pi)^3} \omega_k = \infty^4 .$$

2. **Fermionic vacuum fluctuations:**

$$\langle \mathcal{H}_f \rangle_{vac} = \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2} \langle a_k^\dagger a_k - b_k b_k^\dagger \rangle_{vac}$$

$$= \int \frac{d^3k}{(2\pi)^3} \omega_k = -\infty^4 .$$

Bosonic/fermionic cancellation - Zeldovich prior to SUSY.

Supersymmetry:

$N_b = N_f$ and $m_b = m_f$, then

$$\rho_{vac} = 0\,.$$

if the symmetry is UNBROKEN.

Soft SUSY breaking necessarily leads to

$$\rho_{vac} \sim 10^8 \text{ GeV}^4 \neq 0\,.$$

Broken SUGRA allows for $\rho_{vac} = 0$ but the natural value is

$$\rho_{vac} \sim m_{Pl}^4 \sim 10^{76}\ \mathrm{GeV}^4.$$

Phase transitions in the course of cosmological cooling

$$\delta\rho_{vac} \gg 10^{-47}\ \mathrm{GeV}^4.$$

Troubling mystery:

proton is a bound state of three quarks with masses about 5 MeV each. So its mass should be 15 MeV minus binding energy, instead of 938 MeV.

QCD is well established and experimentally verified science leads to conclusion that vacuum is not empty but filled with quark and gluon condensates:

$$\langle \bar{q}q \rangle \neq 0\,,$$
$$\langle G_{\mu\nu} G^{\mu\nu} \rangle \neq 0\,,$$

both having NEGATIVE vacuum energy

$$\rho_{vac}^{QCD} \approx -10^{45} \rho_c.$$

Vacuum condensate is destroyed by quarks and the proton mass is:

$$m_p = 2m_u + m_d - \rho_{vac} l_p^3$$

$m_u \sim m_d \sim 5$ **eV.**

Who adds the necessary "donation" to make the OBSERVED $\rho_{vac} > 0$ and what kind of matter is it?

Something must "live" in vacuum who donates positive contribution to ρ_{vac} compensating ρ_{QCD} with fantastic precision, 10^{-45}.

Resolution is unknown. It could be dynamical adjustment, modification of gravity in such a way that ρ_{vac} does not gravitate, infrared screening, anthropic principle with almost infinite set of subtraction constants, ... ???

INTERMEDIATE SUMMARY

1. Known and huge contributions to ρ_{vac} but unknown mechanism of their compensation down to (almost) zero.

2. Observed today $\rho_{vac} \sim \rho_c$. Why?

3. What is the nature of antigravitating matter? Consistent with $w = -1$, vacuum?

Mostly only problems 2 and 3 are addressed phenomenologically by:
a) (infrared) modification of gravity;
b) new field (quintessence) leading to accelerated expansion.

However all three problems are strongly coupled and can be solved only after adjustment of ρ_{vac} down to ρ_c is understood.

SOME POSSIBLE SOLUTIONS

1. Subtraction constant, if $w = -1$.

2. Anthropic principle, brane landscape with 10^{1000} possible vacuum states. Compare to Friedman cosmology before inflation.

3. Infrared instability of massless fields (gravitons) in DS space-time.

4. Dynamical adjustment.

5. Drastic modification of existing theory - breaking of general covariance, Lorentz invariance, rejection of the Lagrange/Hamiltonian principle, ... ???

Remember: **we need to explain only one number or a function if $w \neq -1$.**

Dynamical adjustment, as axionic solution of strong CP problem.

New field Φ (scalar of higher spin) coupled to gravity is necessary.

1) Vacuum energy → condensate of Φ
2) $\rho(\Phi)$ compensates original ρ_{vac}.
3) Negative energy density of Φ.

Generic predictions:

1. Change exponential expansion to power law one.

2. Compensation of vacuum energy is not complete but only down to terms of the order of $\rho_c(t)$.

3. Non-compensated energy may have an unusual equation of state.

Unfortunately, no realistic model found starting from 1982 till recent time, but existence of dark energy was predicted in 1982 based on this idea (AD).

EXAMPLES OF ADJUSTMENT

1. Non-minimally coupled scalar field (AD, 1982):

$$\ddot{\phi} + 3H\dot{\phi} + U'(\phi, R) = 0$$

with e.g. $U = \xi R \phi^2/2$.

Solutions are unstable if $\xi R < 0$.

Asymptotically:

$$\phi \sim t$$

and DS turns into Friedman, but

$$T_{\mu\nu}(\phi) \neq F g_{\mu\nu}$$

and the change of the regime is achieved due to weakening of gravitational coupling:

$$\mathrm{G_N} \sim 1/\mathrm{t}^2.$$

Such a rise of $\mathrm{M_{Pl}}$ was suggested as a mechanism to explain hierarchy between EW and Planck.

No-go theorem by S. Weinberg can be circumvented by more exotic coupling to curvature or higher spin fields (Rev. Mod. Phys. 1990?, translated in Uspekhi. Fiz. nauk).

2. Vector field V_μ (AD, 1985):

$$L_1 = \eta \left[F^{\mu\nu} F_{\mu\nu}/4 + (V^{\mu}_{;\mu})^2 \right] + \xi R m^2 ln \left(1 + \frac{V^2}{m^2} \right)$$

Unstable solution:

$$V_t \sim t + c/t$$

and

$$T_{\mu\nu}(V_t) \sim g_{\mu\nu} + \text{vanishing terms}$$

Logarithmic variation of gravitational coupling with time.

3. Second rank tensor field $S_{\mu\nu}$ (AD, 1994):

$$\mathcal{L}_2 = \eta_1 \mathbf{S}_{\alpha\beta;\gamma} \mathbf{S}^{\alpha\gamma;\beta} + \eta_2 \mathbf{S}^{\alpha}_{\beta;\alpha} \mathbf{S}^{\gamma\beta}_{;\gamma} + \eta_3 \mathbf{S}^{\alpha}_{\alpha;\beta} \mathbf{S}^{\gamma;\beta}_{\gamma}$$

Components $\mathbf{S_{tt}}$ and isotropic part of $\mathbf{S_{ij}} \sim \delta_{\mathbf{ij}}$ are unstable:

$$(\partial_{\mathbf{t}}^{\mathbf{2}} + \mathbf{3H}\partial_{\mathbf{t}} - \mathbf{6H^2})\mathbf{S_{tt}} - \mathbf{2H^2 s_{jj}} = \mathbf{0}$$

$$(\partial_{\mathbf{t}}^{\mathbf{2}} + \mathbf{3H}\partial_{\mathbf{t}} - \mathbf{6H^2})\mathbf{s_{tj}} = \mathbf{0}$$

$$(\partial_{\mathbf{t}}^{\mathbf{2}} + \mathbf{3H}\partial_{\mathbf{t}} - \mathbf{2H^2})\mathbf{s_{ij}} - \mathbf{2H^2}\delta_{\mathbf{ij}}\mathbf{S_{tt}} = \mathbf{0}$$

where $\mathbf{s_{tj} = S_{tj}/a(t)}$ and $\mathbf{s_{ij} = S_{ij}/a^2(t)}$.

Ill-defined theory with “non-physical” components, T_{tt} and/or T_{ii} becoming physical?

Ogievetsky and Polubarinov:
“Photon and Notoph” - gauge theory of scalar field described by t-component of vector V_μ.

In all higher spin cases after some period DS turned into Friedman
and the dominant term in $T_{\mu\nu} \sim g_{\mu\nu}$ but $\mathrm{G_N}$ is time-dependent.

More important: in all the models above expansion rate is not related to the usual matter.
Weinberg no-go theorem by scalar field adjustment is overruled by higher spins.
Recently Emelyanov and Klinkhammer, TWO vector field model.

4. Scalar with “crazy” coupling to gravity (Mukohayama, Randall, 2003; AD, Kawasaki, 2003:)

$$A = \int d^4x \sqrt{g} \left[-\frac{1}{2}(R + 2\Lambda) + F_1(R) + \frac{D_\mu \phi D^\mu \phi}{2\, R^2} - U(\phi, R) \right]$$

Solution tends to

$$R \sim \rho_{vac} + U(\phi) = 0$$

It has some nice features (“almost realistic”), $H = 1/2t$, etc
but unstable with respect to small fluctuations.

Equation of motion for Φ:

$$D_\mu \left[D^\mu \phi \left(\frac{1}{R} \right)^2 \right] + U'(\phi) = 0.$$

GR equations for the trace,
with $F_1 = C_1 R^2$:

$$-R + 3 \left(\frac{1}{R} \right)^2 (D_\alpha \phi)^2 - 4 \left[U(\phi) + \rho_{vac} \right]$$

$$-6 D^2 \left[2 C_1 R - \left(\frac{1}{R} \right)^2 \frac{(D_\alpha \phi)^2}{R} \right] = T^\mu_\mu.$$

A desperate attempt to improve the model:

$$\frac{(D\phi)^2}{R^2} \to -\frac{(D\phi)^2}{R\,|R|}.$$

More general action with scalar field (AD, Kawasaki, 2003) not yet explored:

$$\begin{aligned}A = \int d^4x\sqrt{-g}[-m_{Pl}^2(R+2\Lambda)/16\pi \\ +F_1(R) + F_2(\phi,R)D_\mu\phi D^\mu\phi \\ +F_3(\phi,R)D_\mu\phi D^\mu R - U(\phi,R)]\end{aligned}$$

Moreover $R_{\mu\nu}$ and $R_{\mu\nu\alpha\beta}$ can be also included.

LAMBDA-SUMMARY

1. Some compensating agent must exist! QCD demands that.

2. Quite natural to expect that ρ_{vac} is not completely compensated and

$$\Delta\rho \sim \rho_c\,.$$

3. Realistic model is needed, it can indicate what is w: is it (-1) or not.

4. A new form of energy lives in "empty" universe.

Still, we do not have any theory explaining small value of Lambda. Poor man substitution is phenomenology.

Two ways to phenomenologically describe accelerated expansion:

1. Dark energy.

2. Gravity modification at large distances. I will confine myself to $F(R)$ theories only:

$$S = \frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}[R + F(R)] + S_m\,.$$

Pioneering paper:
Quintessence without scalar fields,
S. Capozziello, S. Carloni, A. Troisi,
Recent Res. Develop. Astron. Astrophys. 1 (2003) 625; astro-ph/0303041.

Later:
Is Cosmic Speed-Up Due to New Gravitational Physics? S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner, Phys. Rev. D70 (2004) 043528, astro-ph/0306438.

Singular action: $F(R) = -\mu^4/R$, where $\mu^2 \sim R_c \sim 1/t_u^2$. The corresponding equation of motion reads:

$$\left(1+\frac{\mu^4}{R^2}\right) R_{\alpha\beta} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right) R g_{\alpha\beta} - \mu^4 \nabla_{(\alpha}\nabla_{\beta)}\left(\frac{1}{R^2}\right) + \mu^4 g_{\alpha\beta}\nabla_\nu\nabla^\nu\left(\frac{1}{R^2}\right) = 8\pi\, T^M_{\alpha\beta}/m_{Pl}^2\,.$$

NB: small coefficient, μ^4, in front of the highest derivative. Strong instability in matter (AD, M.Kawasaki).

Trace equation:

$$D^2 R - 3\,\frac{(D_\alpha R)\,(D^\alpha R)}{R} = \frac{R^2}{2} - \frac{R^4}{6\mu^4} - \frac{T\,R^3}{6\mu^4},$$

where $T = 8\pi T^\nu_\nu / m^2_{Pl} > 0$.

Apply this equation for perturbative calculations of the gravitational field of a celestial body $R_0 = -T + R_1$, flat metric being assumed.

Vacuum solution outside matter source: R exponentially tends to zero, as is expected in GR, if $\mu^4 > 0$. So these modified gravity theories agree with the Newtonian limit of the standard gravity for sufficiently small μ.

Let us consider the internal solution with time dependent matter density. The first order correction to the GR curvature, R_1 satisfies the equation:

$$
\begin{aligned}
&\ddot{R}_1 - \Delta R_1 - \frac{6\dot{T}}{T}\,\dot{R}_1 + \frac{6\partial_j T}{T}\,\partial_j R_1 \\
&\quad + R_1\left[T + 3\,\frac{\dot{T}^2 - (\partial_j T)^2}{T^2} - \frac{T^3}{6\mu^4}\right] \\
&\qquad = D^2 T + \frac{T^2}{2} - \frac{3 D_\alpha T\, D^\alpha T}{T}.
\end{aligned}
$$

The blue term leads to exponential instability of small fluctuations or of regular time evolution.

The characteristic time of instability:

$$\tau = \frac{\sqrt{6}\mu^2}{T^{3/2}} \sim 10^{-26}\mathrm{sec}\left(\frac{\rho_m}{\mathrm{g/cm^3}}\right)^{-3/2},$$

where ρ_m is the mass density of the body and $\mu^{-1} \sim t_u \approx 3 \cdot 10^{17}\, sec.$ This is the dominant term in the equation, since e.g.

$$T \sim (10^3\mathrm{sec})^{-2}\left(\frac{\rho_m}{\mathrm{g\ cm^{-3}}}\right).$$

Effects of inhomogeneities:

perturbation with the wave length larger than one tenth of the proton Compton wave length would be unstable.

Modified modified gravity (**Hu, Sawicki; Appleby, Battye; Starobinsky):**

$$F_{\mathrm{HS}}(R) = -\frac{R_{\mathrm{vac}}}{2}\frac{c\left(\frac{R}{R_{\mathrm{vac}}}\right)^{2n}}{1+c\left(\frac{R}{R_{\mathrm{vac}}}\right)^{2n}},$$

$$F_{\mathrm{AB}}(R) = \frac{\epsilon}{2}\log\left[\frac{\cosh\left(\frac{R}{\epsilon}-b\right)}{\cosh b}\right] - \frac{R}{2},$$

$$F(R)_S = \lambda R_0\left[\left(1+\frac{R^2}{R_0^2}\right)^{-n} - 1\right].$$

The field equations have the form:

$$f' R_\mu^\nu - \frac{f}{2}\delta_\mu^\nu + (\delta_\mu^\nu \Box - D_\mu D^\nu) f' = \frac{T_\mu^\nu}{M_{Pl}^2},$$

and their trace reads

$$3\Box f'(R) + R f'(R) - 2f(R) = M_{\rm Pl}^{-2} T_\mu^\mu ,$$

where $f = R + F(R)$. Condition of accelerated expansion in absence of matter is that the eq.

$$R f'(R) - 2f(R) = 0$$

has solution $R_1 > 0$.

Necessary conditions to avoid pathologies are to be satisfied:

1. Future stability of cosm. solutions:

$$F'(R_1)/F''(R_1) > R_1\,.$$

2. Classical and quantum stability (grav. attraction and absence of ghosts):

$$F'(R) > 0,\;\; F''(R) > 0\,.$$

3. Absence of matter instability:

$$F'(R) > 0,\;\; F''(R) > 0\,.$$

Past singularity: in cosmological background with decreasing energy density the system must evolve from a singular state with an infinite R. In other words, if we travel backward in time from a normal cosmological state, we come to singularity.

Future singularity (EA, AD). The system with rising energy density will evolve to singularity, $R \to \infty$, in finite (short) time. Consider HSS version in the limit $R \gg R_0$:

$$F(R) \approx -\lambda R_0 \left[1 - \left(\frac{R_0}{R} \right)^{2n} \right].$$

We analyze the evolution of R in massive object with time varying density, $\rho \ll \rho_{cosm}$.

Gravitational field of such objects is supposed to be weak, so the background metric is approximately flat and covariant derivatives can be replaced by the flat ones and hence:

$$(\partial_t^2 - \Delta)R - (2n+2)\frac{\dot{R}^2 - (\nabla R)^2}{R} + \frac{R^2}{3n(2n+1)}\left[\frac{R^{2n}}{R_0^{2n}} - (n+1)\right] - \frac{R^{2n+2}}{6n(2n+1)\lambda R_0^{2n+1}}(R+T) = 0\,.$$

The equation is very much simplified for $w \equiv F' = -2n\lambda\,(R_0/R)^{2n+1}$:

$$(\partial_t^2 - \Delta)w + U'(w) = 0\,.$$

Potential $U(w)$ is equal to:

$$U(w) = \frac{1}{3}\,(T - 2\lambda R_0)\,w +$$

$$\frac{R_0}{3}\left[\frac{q^\nu}{2n\nu}w^{2n\nu} + \left(q^\nu + \frac{2\lambda}{q^{2n\nu}}\right)\frac{w^{1+2n\nu}}{1+2n\nu}\right],$$

where $\nu = 1/(2n+1)$, $q = 2n\lambda$.

NB: Infinite R corresponds to $w = 0$.

If only the dominant terms are retained and if the space derivatives are neglected, the equation simplifies to:

$$\ddot{w} + T/3 - \frac{q^{\nu}(-R_0)}{3w^{\nu}} = 0\,.$$

Potential U would depend upon time, if the mass density of the object changes with time. We parametrize it as

$$T = T(t) = T_0(1 + \kappa\tau)\,,$$

where τ is dimensionless time introduced below.

With dimensionless quantities $t = \gamma\tau$ and $w = \beta z$, where

$$\gamma^2 = \frac{3q}{(-R_0)}\left(-\frac{R_0}{T_0}\right)^{2(n+1)},$$

$$\beta = \gamma^2 T_0/3 = q\left(-\frac{R_0}{T_0}\right)^{2n+1}$$

the equation further simplifies:

$$z'' - z^{-\nu} + (1 + \kappa\tau) = 0\,.$$

Quintessence, massless or very light scalar field with

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left[(\partial\phi)^2 - U(\phi)\right]$$

has $p \approx -\rho$, mimick vacuum energy, may have tracking solution, being subdominant at RD stage and becoming dominant at late MD stage, i.e. today, but does not address the problem of vacuum energy compensation.

Many models and no understanding!
Hopefully it is not the end of the story
but only the end of the lecture..