

Πάντα ρεῖ ?

The fluid dynamic paradigm of
relativistic heavy ion collisions

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From elementary interactions to collective phenomena

For any QFT of elementary particles and their interactions, we can ask:

How do macroscopic properties of matter emerge from fundamental interactions?

- QCD much richer than QED:
 - non-abelian theory
 - degrees of freedom change with Q^2
- Equilibrium QCD at high temperature $T \gg T_{crit}$ particularly interesting
 - in experimental reach in heavy ion collisions $\varepsilon_{crit} \approx (3 - 5)\varepsilon_{nuclear\ matter}$
 - many properties of hot QCD calculable from L_{QCD}
(transition to QGP, chiral symmetry restoration, viscosities, conductivities, relaxation times, susceptibilities, QED emission rates...)
- But: matter in heavy ion collisions expands quickly $\tau_{lifetime} \approx O(10\ fm)$
 - global equilibrium state not realized
 - local equilibration may be expected $\tau_{lifetime} \gg \tau_{QCD\ rates}$
- Establishing fluid dynamic behavior is prerequisite for testing equilibrium QCD in heavy ion collisions.

Ideal and viscous fluid dynamics

$$N_i^\mu = n_i u^\mu + \vec{n}_i \quad (4n \text{ comp.})$$

Ideal fluid:

(n comp.)

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p \Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu} \quad (10 \text{ comp.})$$

(5 comp.)

- Dynamics based only on E-p conservation:
(for ideal fluid)

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{and} \quad \varepsilon = \varepsilon(p, n)$$

- supplemented by 2nd law of thermodynamics:
(for viscous fluid)

$$\partial_\mu S^\mu(x) \geq 0$$

- Fluid dynamics depends only on equilibrium properties of matter that are

calculable from first principles in quantum field theory

EOS: $\varepsilon = \varepsilon(p, n)$ and **sound velocity** $c_s = \partial p / \partial \varepsilon$

transport coefficients: shear η , bulk ξ viscosity, conductivities ...

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt dx e^{i\omega t} \left\langle \left[T^{xy}(x, t), T^{xy}(0, 0) \right] \right\rangle_{eq}$$

relaxation times: $\tau_\pi, \tau_\Pi, \dots$

Lattice QCD =>

Finite Temp pQCD =>

AdS/CFT

=>

1.

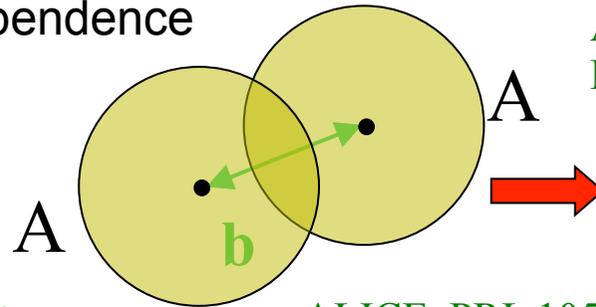
Experimental signatures of
fluid dynamics behavior
in heavy ion collisions

To separate microscopic dynamics from macroscopic properties, system size matters

- In pp, dispersion of multiplicity distribution at fixed impact parameter not factorizable from impact parameter dependence

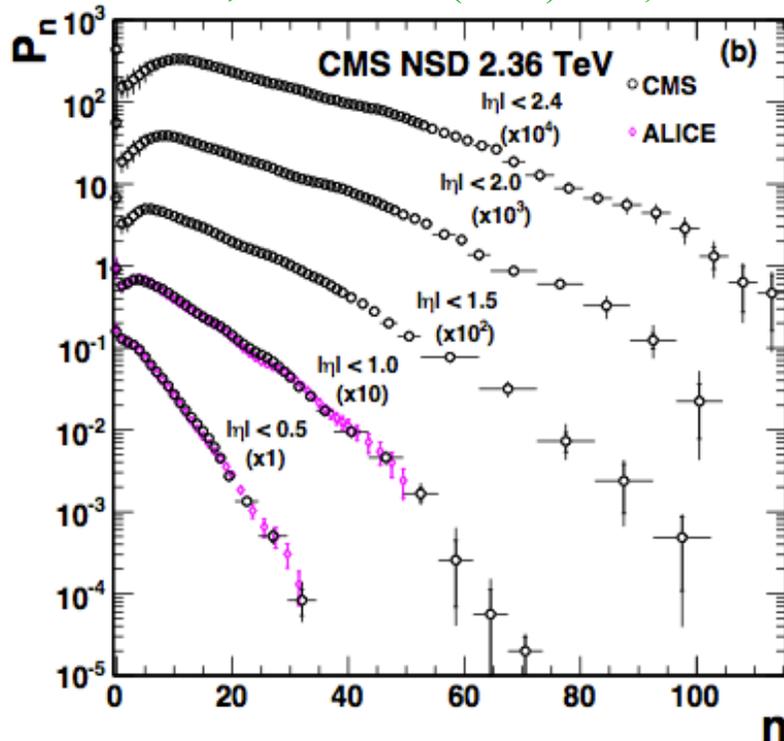
- In AA, multiplicity distribution dominated by geometry

A. Bialas and W. Czyz, Nucl. Phys. B111 (1976) 461

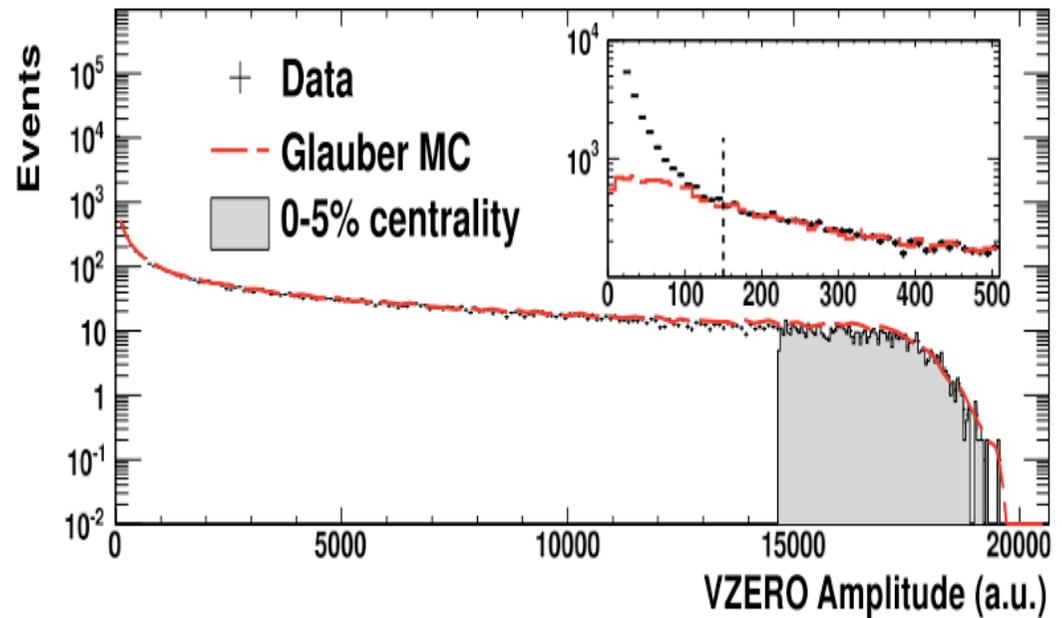


Geometry experimentally accessible

CMS, JHEP 1101 (2011) 079)



ALICE, PRL 105 (2010) 252301, arXiv:1011.3916

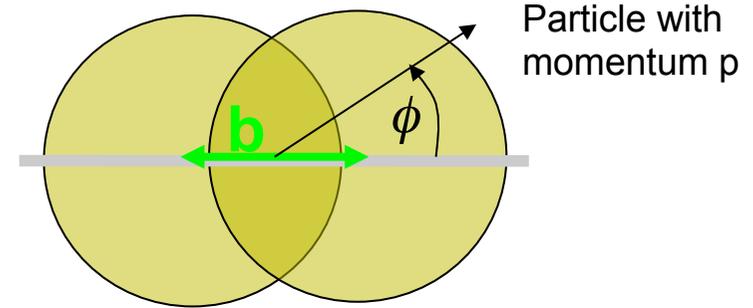


$n_{\max} \sim 20000$

Particle production w.r.t. reaction plane

- Consider harmonic analysis of inclusive spectra $f(\vec{p}) \equiv dN/E d\vec{p}$ via n-th order flow

$$v_n \equiv \langle\langle e^{in\phi} \rangle\rangle = \left\langle \frac{\int d\vec{p} e^{in\phi} f(\vec{p})}{\int d\vec{p} f(\vec{p})} \right\rangle_{\text{event average}}$$



but reaction plane is unknown ...

- Have to measure particle correlations instead:

$$\langle\langle e^{in(\phi_1 - \phi_2)} \rangle\rangle = v_n(\{2\}) v_n(\{2\}) + \underbrace{\langle\langle e^{in(\phi_1 - \phi_2)} \rangle\rangle^{\text{corr}}}_{O(1/N)}$$

Flow via 2nd order cumulants
“Non-flow effects”

But this requires signals $v_n > 1/\sqrt{N}$, $N \sim$ multiplicity

- Improve measurement with higher cumulants: Borghini, Dinh, Ollitrault, PRC (2001)

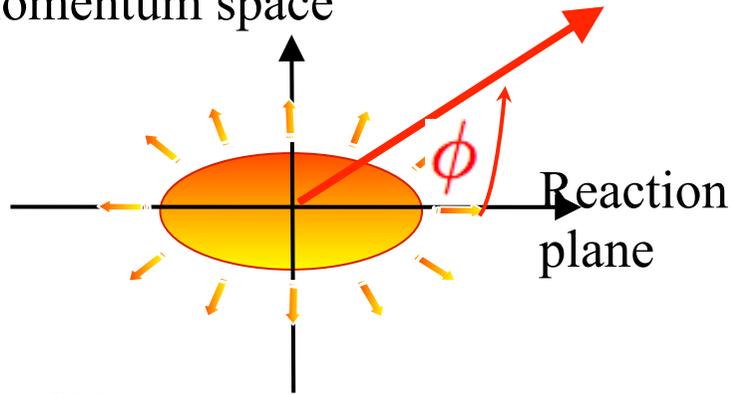
$$\langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle - \langle e^{in(\phi_1 - \phi_3)} \rangle \langle e^{in(\phi_2 - \phi_4)} \rangle - \langle e^{in(\phi_1 - \phi_4)} \rangle \langle e^{in(\phi_2 - \phi_3)} \rangle = -v_n^4 + O(1/N^3)$$

This requires signals $v_n > 1/N^{3/4}$

System size matters for flow determination

v_2 @ RHIC and LHC

- Momentum space



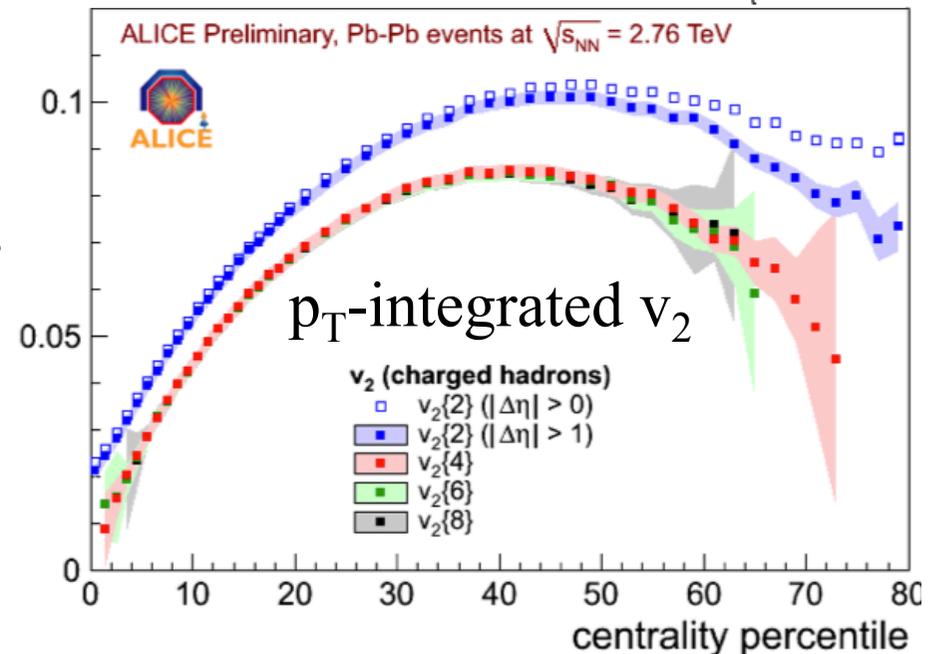
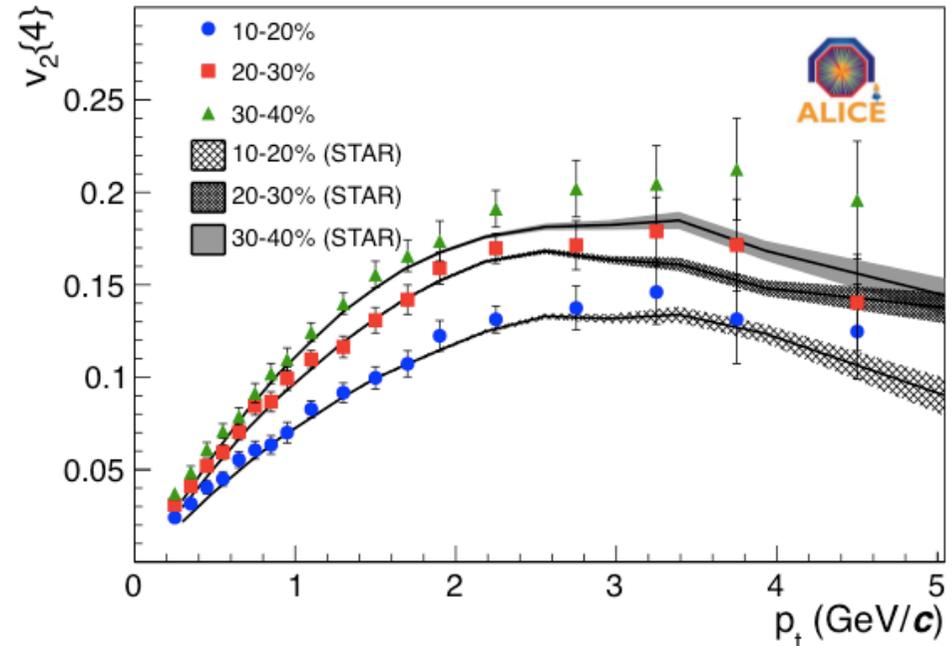
$$\frac{dN}{d\phi p_T dp_T} \propto [1 + 2v_2(p_T) \cos(2\phi)]$$

- Signal $v_2 \approx 0.2$ implies 2-1 asymmetry of particles production w.r.t. reaction plane.
- 'Non-flow' effect for 2nd order cumulants
 $N \sim 100 - 1000 \Rightarrow 1/\sqrt{N} \sim 0.1 \sim O(v_2) ??$

2nd order cumulants do not characterize solely collectivity.

$$1/N^{3/4} \sim \leq 0.03 \ll v_2$$

→ Strong Collectivity !



Spatial eccentricities drive flow

- Initial transverse energy (or entropy) densities expected to vary event-by-event

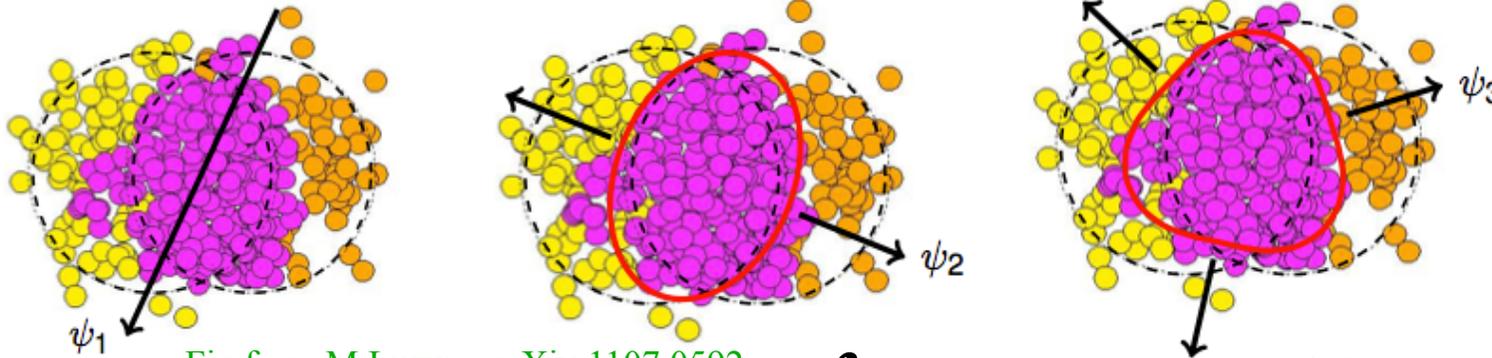


Fig from M.Luzum, arXiv:1107.0592

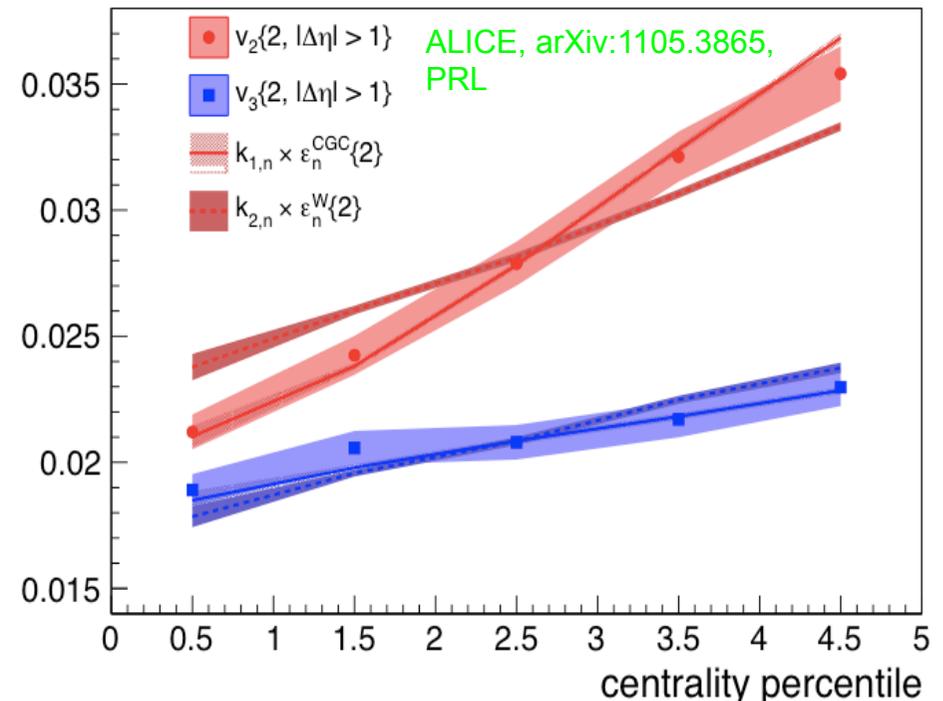
- Characterize their shape via **spatial eccentricities**

$$\varepsilon_{m,n} e^{in\phi_{m,n}} \equiv \frac{\int r dr d\phi \rho(r,\phi) r^m e^{in\phi}}{\int r dr d\phi \rho(r,\phi) r^m}, \quad \varepsilon_n \equiv \varepsilon_{n,n}$$

- Experiments establish approximately linear response between **momentum flow** and **spatial eccentricities**

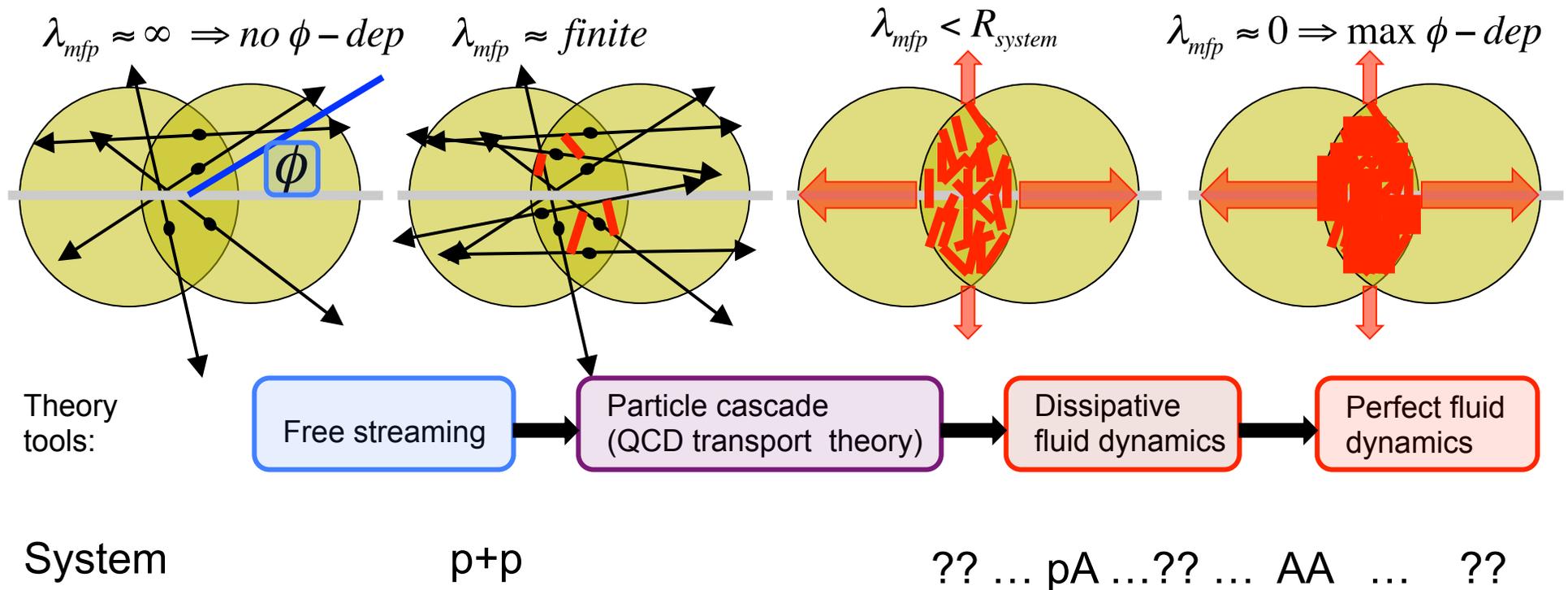
$$v_n \exp[in\psi_n] \approx k \varepsilon_n \exp[in\phi_n]$$

For tests, see e.g.
F. Gardim et al, arXiv:1111.6538



2.

Dynamical understanding of v_n



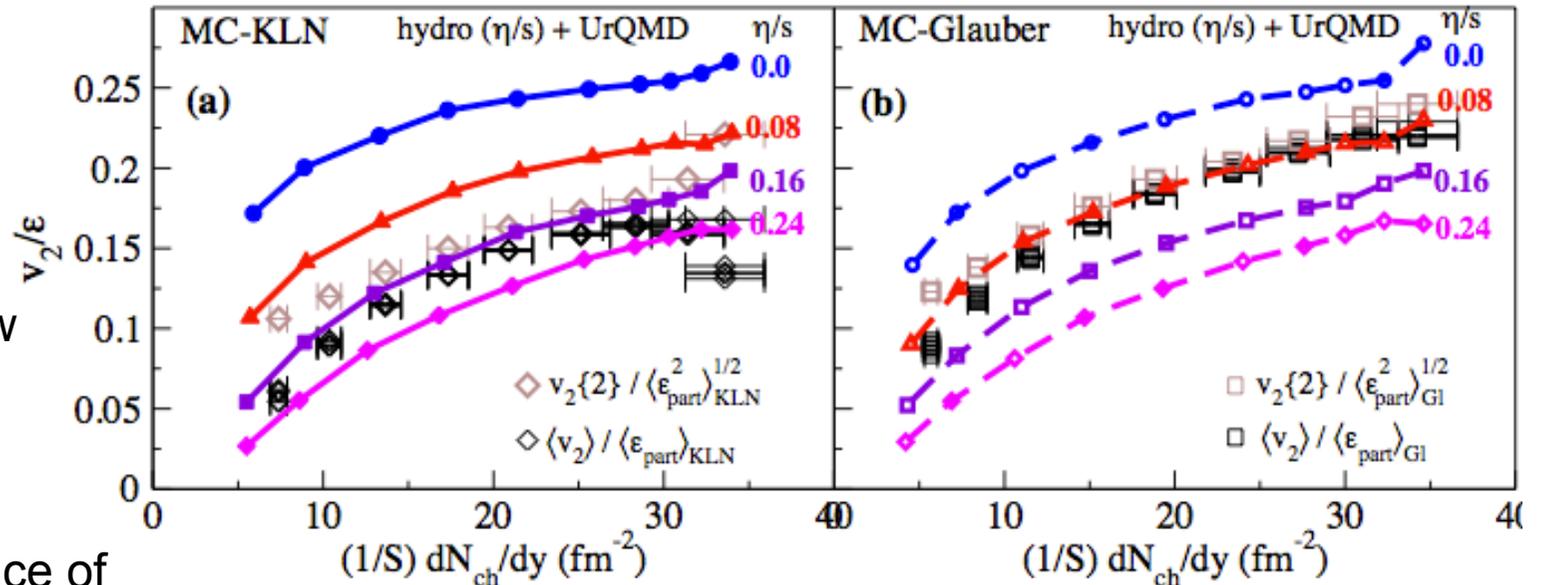
Fluid dynamics prior to LHC - results

Fluid dynamics accounts for:

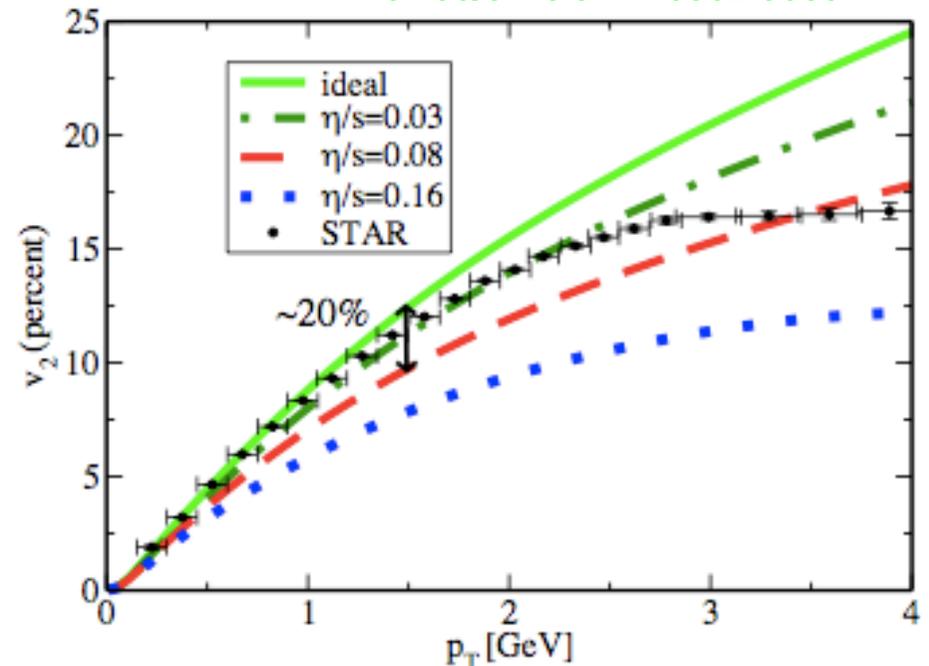
- Centrality dependence of elliptic flow
- pt-dependence of elliptic flow
- **Mass dependence** of elliptic flow (all particle species emerge from common flow field)
- Single inclusive transverse momentum spectra at pt (< 3 GeV)

In terms of fluid with QCD equation of state and minimal shear viscosity

$$\frac{\eta}{s} \ll 1$$



P. Romatschke arXiv.0902.3663



Implications of minimal viscosity

Back of envelope:

For 1-dim expanding fluid (Bjorken boost-invariant), entropy density increases like

$$\frac{d(\tau s)}{d\tau} = \frac{4}{3} \frac{\eta}{\tau T}$$

Isentropic “perfect liquid” applies if

$$\frac{\eta}{\tau T s} \ll 1$$

Put in numbers

$$\tau \sim 1 \text{ fm}/c, \quad T \sim 200 \text{ MeV} \quad \longrightarrow \quad \frac{\eta}{s} \ll 1$$

Theory

Minimal viscosity implies strongly coupled plasma.

⇒ Importance of strong coupling techniques

Arnold, Moore, Yaffe, JHEP 11 (2000) 001

Lattice QCD

H. Meyer, arXiv:0704.1801

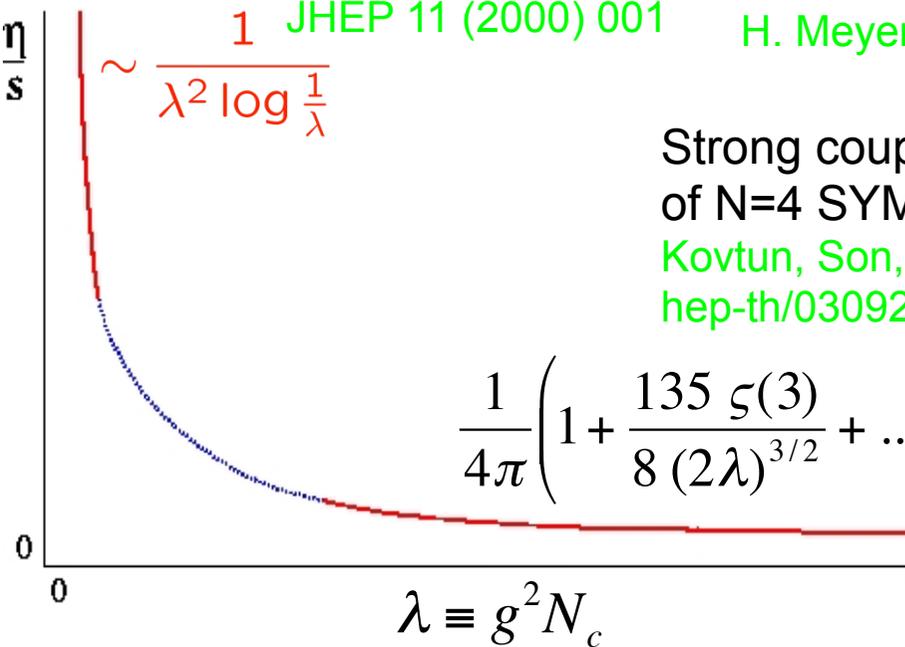
$$\frac{\eta}{s} \sim \frac{1}{\lambda^2 \log \frac{1}{\lambda}}$$

Strong coupling limit of N=4 SYM

Kovtun, Son, Starinets, hep-th/0309213

$$\frac{1}{4\pi} \left(1 + \frac{135 \zeta(3)}{8 (2\lambda)^{3/2}} + \dots \right)$$

$$\frac{\eta}{s} > \frac{1}{4\pi}$$



Phenomenological implication

Minimal dissipation \Leftrightarrow Maximal Transparency to Fluctuations

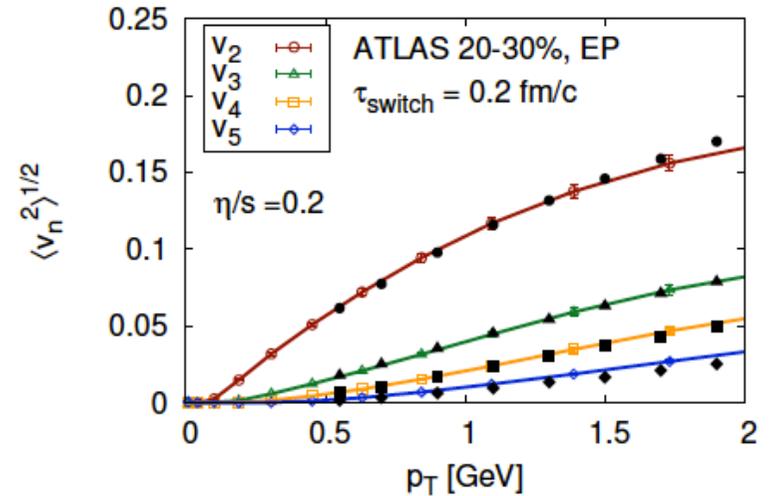
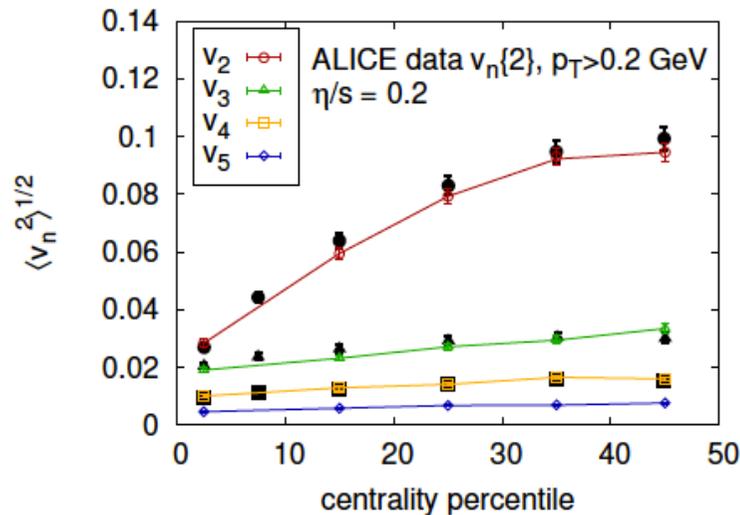
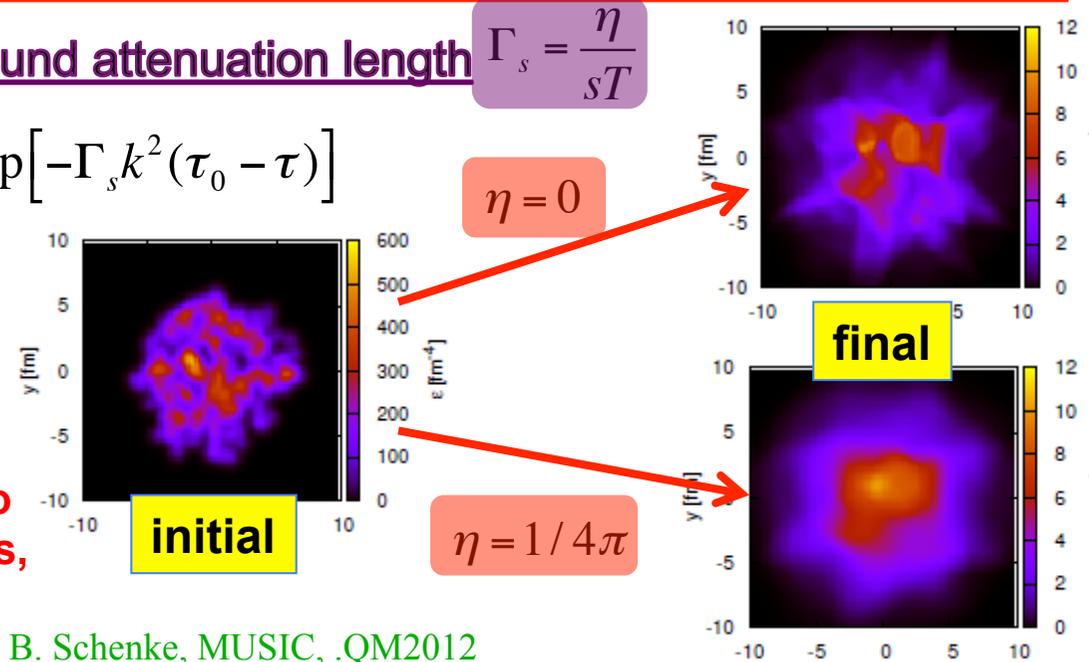
- Linear fluctuations governed by sound attenuation length $\Gamma_s = \frac{\eta}{sT}$

$$\delta v(\tau, k) = \delta v(\tau_0, k) \left(\frac{\tau_0}{\tau} \right)^* \exp[-\Gamma_s k^2 (\tau_0 - \tau)]$$

Fluctuations decay on time scale,

$$\tau_{1/e}(k) = \frac{1}{\Gamma_s k^2}$$

- Viscous fluid dynamics with close to minimal shear viscosity, realistic eos, and suitable initial conditions accounts for v_n in p_T and centrality.** B. Schenke, MUSIC, .QM2012



Fluid dynamical modeling of heavy ion collisions

Washington (2012)

Compilation by J.-Y. Ollitrault: Plenary

Many independent model studies, one general conclusion:

Viscous fluid dynamics with

- **Minimal η/s**
- **realistic eos**
- **suitable initial conditions**

accounts for v_n in p_T , and centrality.

Author/Presenter	QM2012	arXiv	initial fluctuations	3+1d	viscous	afterburner
Huichao Song	ID	1207.2396			✓	✓
Teaney/Yan	IA	1206.1905			✓	
Chun Shen	IA	1202.6620			✓	
Sangyong Jeon	2A		✓	✓	✓	✓
Matt Luzum	2A				✓	
Piotr Bozek	2C	1204.3580	✓	✓	✓	
Björn Schenke	3A	1109.6289	✓	✓	✓	
Dusling/Schaefer	3A	1109.5181			✓	
Chiho Nonaka	3A	1204.4795	✓	✓	✓	
Ryblewski/Florkowski	3D	1204.2624		✓		
Longgang Pang	4D	1205.5019	✓	✓		
Hannah Petersen	VA	1201.1881	✓	✓		✓
Fernando Gardim	6D	1111.6538	✓	✓		
Zhi Qiu	29	1208.1200	✓		✓	
Gardim/Grassi	52	1203.2882	✓	✓		
Katya Retinskaya	57	1203.0931			✓	
Hirano/Murase	255	1204.5814	✓	✓		✓
Holopainen/Huovinen	284	1207.7331	✓			
Asis Chaudhuri		1112.1166	✓		✓	
Iurii Karpenko		1204.5351		✓		✓
Yu-Liang Yan		1110.6704		✓		✓
Josh Vredevoogd		1202.1509		✓	✓	
Ron Soltz		1208.0897			✓	✓
Rafael Derradi de Souza		1110.5698	✓	✓		

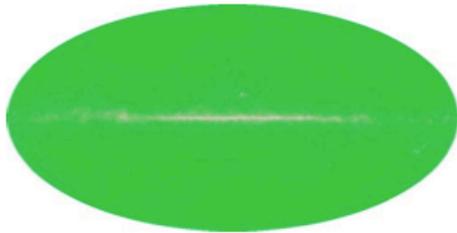
3.

‘Flow’

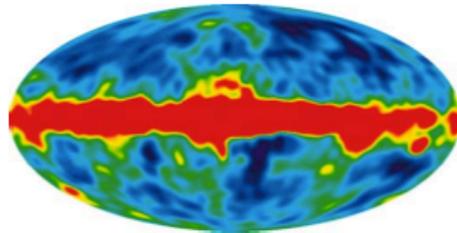
New Frontiers/new challenges

3.1. A (valid) analogy – how far can it be pushed?

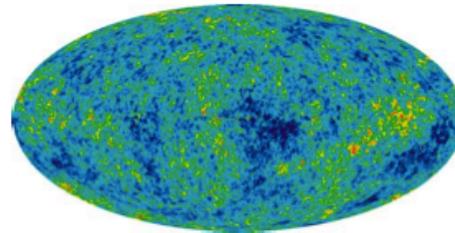
Penzias/Wilson
1965



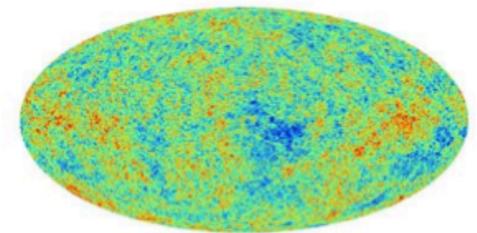
COBE
2003



WMAP
2007

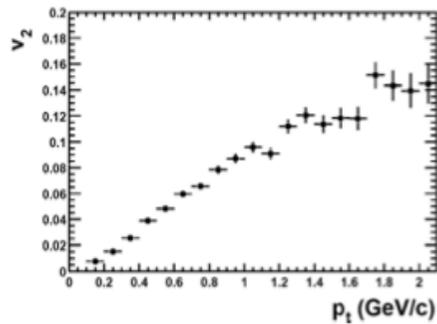


Planck
2012

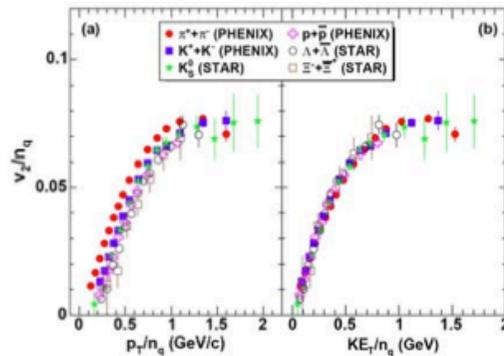


From a signal ... via fluctuations

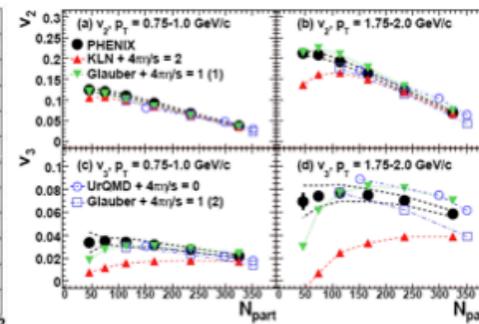
.... to properties of matter



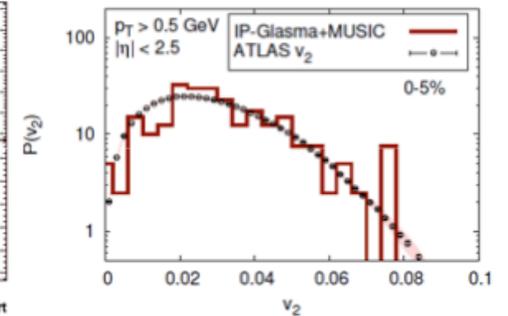
2001



2004



2008



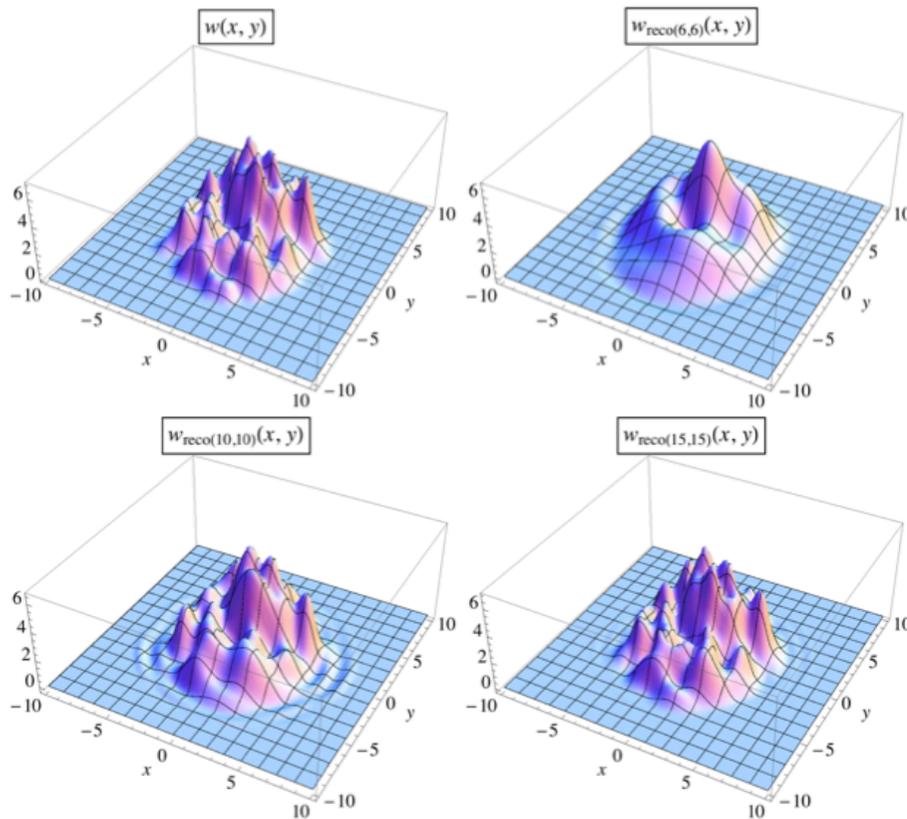
2012

Slide adapted from W. Zajc

More differential studies of the role of fluctuations

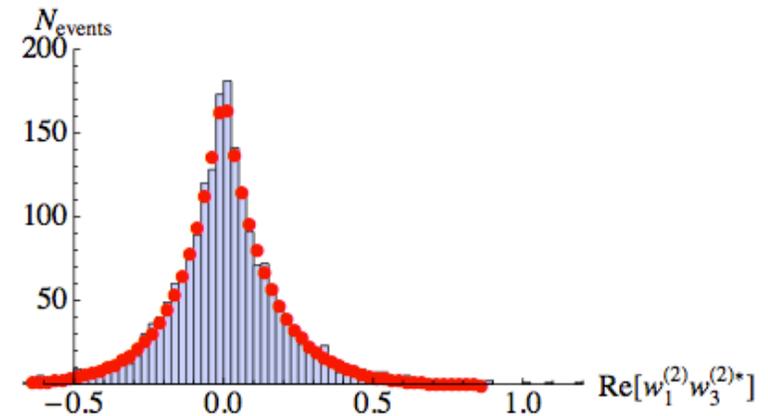
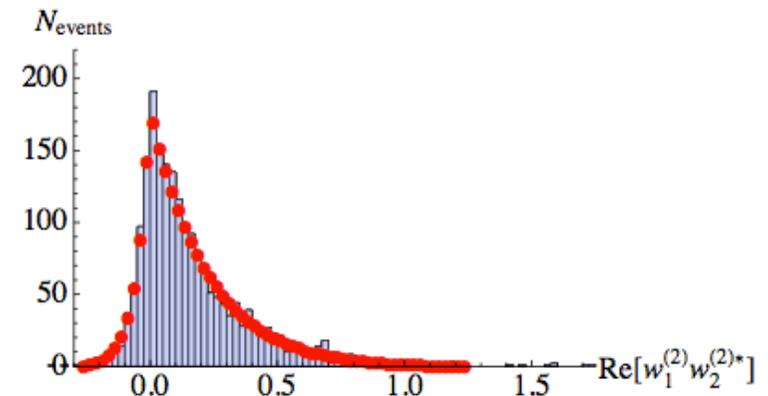
- Characterizing initial conditions in terms of orthonormal basis of fluctuating modes with radial (l) and azimuthal (m) wave number
- Characterizing probability distributions

$$w^{(m)}(r) \approx \sum_{l=1}^{N_l} w_l^{(m)} J_m(k_l^{(m)} r)$$



$$P[\{w_l^{(m)}\}]$$

of event classes



=> S. Flörchinger, parallel Friday

Mode-by-mode fluid dynamics

Decomposing
initial conditions
in modes

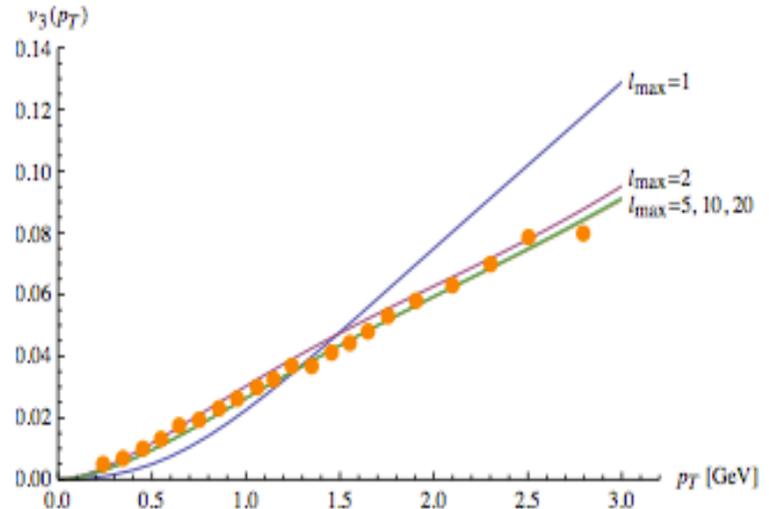
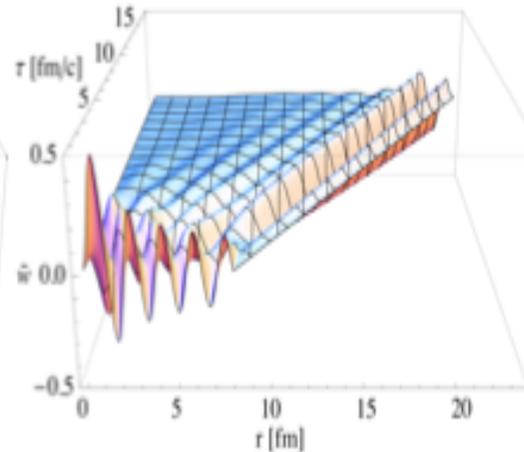
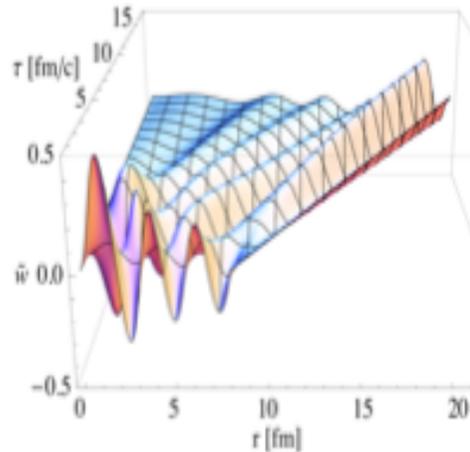
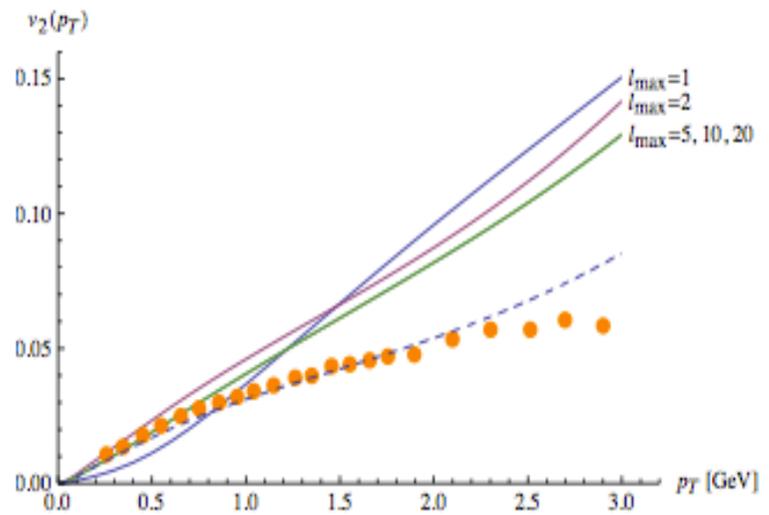
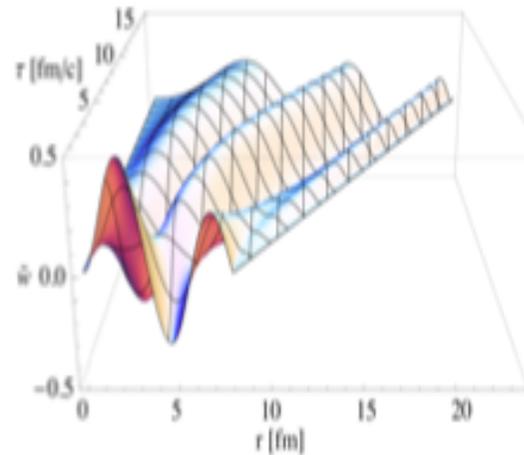
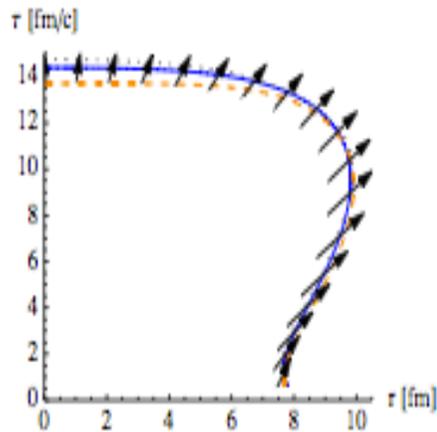


Propagating
each mode
individually



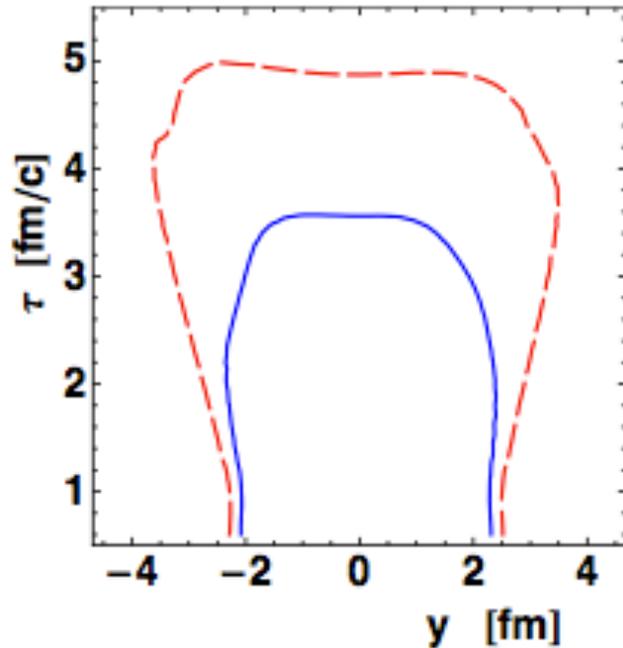
Understanding the
signal composition
mode-by-mode

S. Flörchinger, UAW, 1307.3453; 1307.7611



3.2. Do smaller systems show flow: pPb?

A fluid dynamical simulation of pPb@LHC yields [P. Bozek, 1112.0915](#)

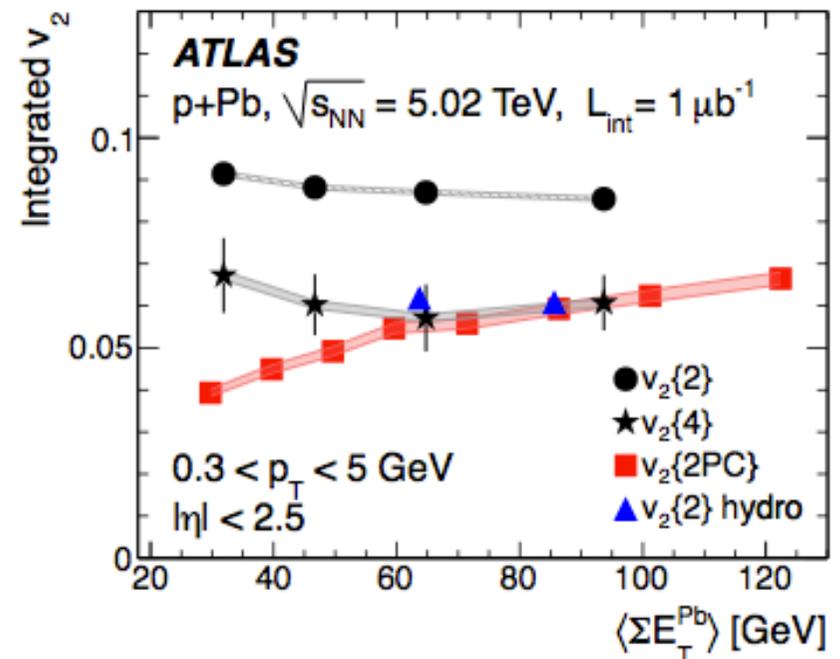
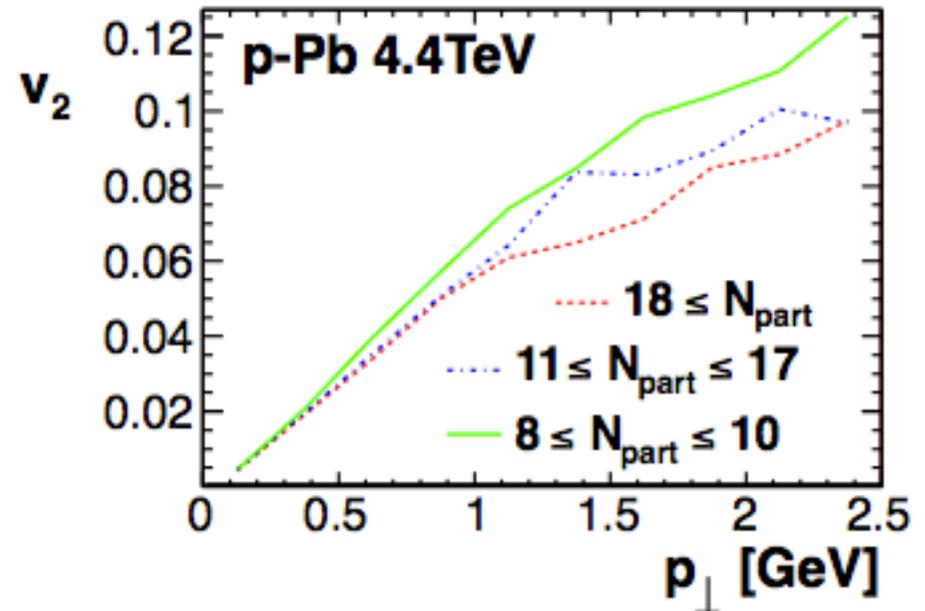


Fluid dynamics compares surprisingly well with

$$v_2 \{2\}, v_2 \{4\}, v_3 \{2\}$$

in pPb@LHC.

[ATLAS, 1303.2084](#)
[CMS, 1305.0609](#)

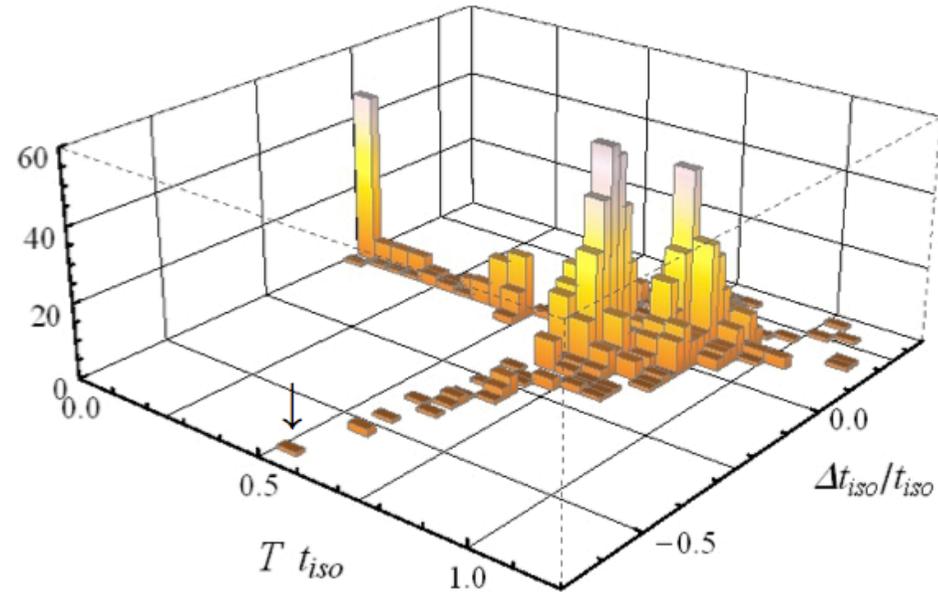


3.3 How to understand hydrodynamization in QFT?

- Model-dependent in QCD but a rigorously calculable problem of numerical gravity in AdS/CFT
- **Very fast non-perturbative isotropization**

$$\tau_{iso} < \frac{1}{T}$$

M. Heller, R. Janik et al, PRL, 1202.0981



- The first rigorous field theoretic set-up in which fluid dynamics applies at very short time scales
- These non-abelian plasma are unique in that they do not carry quasi-particle excitations:

$$\alpha_s \gg 1 \Rightarrow 0.65 \leq \tau_0 T_0$$

Chesler, Yaffe, PRL 102 (2009) 211601

perturbatively require $\tau_{quasi} \sim \frac{1}{\alpha_s^2 T} \gg \frac{1}{T}$

but $\tau_{quasi} \approx \frac{const \eta}{T s}$

3.4 Anomalous hydrodynamics

- In QFTs with quantum anomalies, 2nd law of thermodynamics implies that currents flow along magnetic fields and along vorticity
(see also Vilenkin's work in 1980s,
explicitly realized in QFTs with gravity dual, see Erdmenger et al.)

Son&Surowka,
PRL 103 (2009) 191601

- In QCD, this implies $J_V^\mu \propto C \mu_A B^\mu$

$$J_A^\mu \propto C \mu_V B^\mu$$

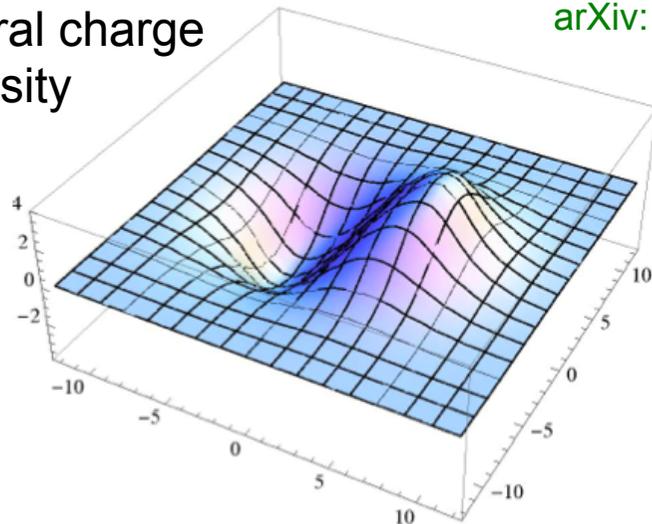
- Heavy ion collisions have strong but transient magnetic field, $eB \sim O(m_\pi^2)$ and finite vector potential.

This realizes the condition for an axial current, that will generate an axial chemical potential, that will separate electric charge along B-field

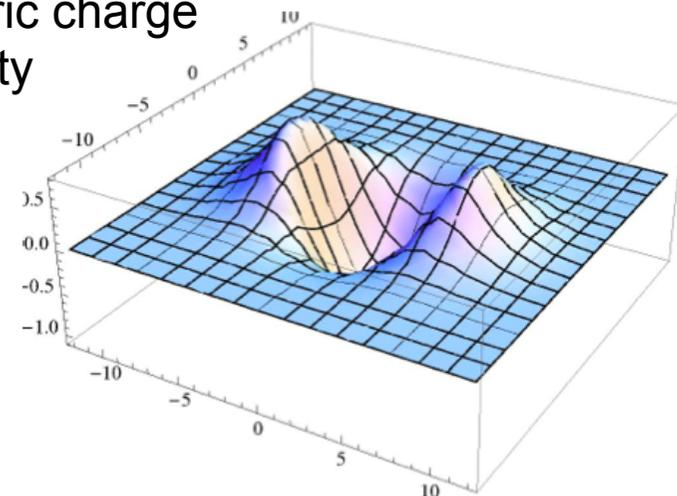
- Can we see this chiral magnetic wave in heavy ion collisions?

Y. Burnier, D. Kharzeev, ..
arXiv:1103.1307

Chiral charge density



Electric charge density





Πάντα ρεῖ

End

Back-ups

Viscous fluid dynamics

Characterizes dissipative corrections in gradient expansion

Ideal fluid:

$$N_i^\mu = n_i u^\mu + \bar{n}_i \quad (4n \text{ comp.})$$

(n comp.)

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p \Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu} \quad (10 \text{ comp.})$$

(5 comp.)

To close equation of motion, supplement conservation laws and eos

$$\partial_\mu N_i^\mu \equiv 0 \quad (n \text{ constraints})$$

$$\partial_\mu T^{\mu\nu} \equiv 0 \quad (4 \text{ constraints})$$

$$p = p(\varepsilon, n) \quad (1 \text{ constraint})$$

by point-wise validity of 2nd law of thermodynamics

$$T \partial_\mu S^\mu(x) \geq 0 \quad S^\mu = s u^\mu + \beta q^\mu + O(\nabla^2)$$

The resulting Israel-Stewart relativistic fluid dynamics depends in general on

relaxation times

and

transport coefficients.

$$(\varepsilon + p) D u^\mu = \nabla^\mu p - \Delta_\nu^\mu \nabla^\sigma \Pi^{\nu\sigma} + \Pi^{\mu\nu} D u_\nu$$

$$D \varepsilon = -(\varepsilon + p) \nabla_\mu u^\mu + \frac{1}{2} \Pi^{\mu\nu} \langle \nabla_\nu u_\mu \rangle$$

$$\tau_\pi \Delta_\alpha^\mu \Delta_\beta^\nu D \Pi^{\alpha\beta} + \Pi^{\mu\nu} = \eta \langle \nabla^\mu u^\nu \rangle - 2\tau_\pi \Pi^{\alpha(\mu} \omega_{\alpha}^{\nu)}$$

Odd harmonics dominate central collisions

In the most central 0-5% events,

$$v_3 \geq v_2$$

Fluctuations in initial conditions dominate flow measurements

