

Unconstrained Hamiltonian formulation of low energy QCD¹

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- H.-P. P., *SU(2) Yang-Mills quantum mechanics of spatially constant fields*, Phys. Lett. B **648** (2007) 97-106.
- H.-P. P., *Expansion of the Yang-Mills Hamiltonian in spatial derivatives and glueball spectrum*, Phys. Lett. B **685** (2010) 353-364.
- H.-P. P., *SU(2) Dirac-Yang-Mills quantum mechanics of spatially constant quark and gluon fields*, Phys. Lett. B **700** (2011) 265-276.
- H.-P. P., *Unconstrained Hamiltonian formulation of low energy SU(3) Yang-Mills quantum theory*, arXiv: 1205.2237v1 [hep-th] (2012).
- H.-P. P., *QCD in terms of gauge invariant dynamical variables*, PoS (Confinement X) (2013) 071, arXiv: 1303.3763 v1 [hep-th] (2013).

- Gribov ambiguity (1978) → Attempt of an **exact resolution of the non-Abelian Gauss-laws** to have an QCD Hamiltonian at low energy (a.o. Jackiw+Goldstone, Faddeev, T.D.Lee, t'Hooft)
- Unconstrained Hamiltonian of 2-color QCD
- Derivative expansion
- Spectrum: fermions, Lorentz-inv and renormalisation in the IR
- Extension to SU(3)

Aim: Alternative nonperturbative formulation of QCD

The QCD action

$$\begin{aligned} \mathcal{S}[A, \psi, \bar{\psi}] &:= \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi} (i\gamma^\mu D_\mu - m) \psi \right] \\ F_{\mu\nu}^a &:= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf_{abc} A_\mu^b A_\nu^c, \quad a = 1, \dots, 8 \\ D_\mu &:= \partial_\mu - igA_\mu^a \tau_a / 2 \end{aligned}$$

is invariant under the $SU(3)$ gauge transformations $U[\omega(x)] \equiv \exp(i\omega_a \tau_a / 2)$

$$\begin{aligned} \psi^\omega(x) &= U[\omega(x)] \psi(x) \\ A_{a\mu}^\omega(x) \tau_a / 2 &= U[\omega(x)] \left(A_{a\mu}(x) \tau_a / 2 + \frac{i}{g} \partial_\mu \right) U^{-1}[\omega(x)] \end{aligned}$$

chromoelectric : $E_i^a \equiv F_{i0}^a$ and chromomagnetic $B_i^a \equiv \frac{1}{2} \epsilon_{ijk} F_{jk}^a$

$\Pi_{ai} = -E_{ai}$ momenta can. conj. to the spatial $A_{ai} \rightarrow$ canonical Hamiltonian

$$\begin{aligned} H_C &= \int d^3x \left[\frac{1}{2} E_{ai}^2 + \frac{1}{2} B_{ai}^2(A) - g A_{ai} j_{ia}(\psi) + \bar{\psi} (\gamma_i \partial_i + m) \psi \right. \\ &\quad \left. - g A_{a0} (D_i(A)_{ab} E_{bi} - \rho_a(\psi)) \right] \end{aligned}$$

with the covariant derivative $D_i(A)_{ab} \equiv \delta_{ab} \partial_i - gf_{abc} A_{ci}$

Exploit the **time dependence of the gauge transformations** to put

$$A_{a0} = 0, \quad a = 1, \dots, 8 \quad (\text{Weyl gauge})$$

The dynamical variables A_{ai} , $-E_{ai}$, $\psi_{\alpha r}$ and $\psi_{\alpha r}^*$ are quantized in the Schrödinger functional approach imposing the equal-time (anti-)CR, e.g. $-E_{ai} = -i\partial/\partial A_{ai}$.

The physical states Φ satisfy

$$H\Phi = \int d^3x \left[\frac{1}{2} E_{ai}^2 + \frac{1}{2} B_{ai}^2[A] - A_{ai} j_{ia}(\psi) + \bar{\psi} (\gamma_i \partial_i + m) \psi \right] \Phi = E\Phi,$$

$$G_a(x)\Phi = [D_i(A)_{ab} E_{bi} - \rho_a(\psi)] \Phi = 0, \quad a = 1, \dots, 8.$$

The Gauss law operators G_a are the generators of the residual **time independent gauge transformations**, satisfying $[G_a, H] = 0$ and $[G_a, G_b] = i f_{abc} G_c$.

Angular momentum operators $[J_i, H] = 0$

$$J_i = \int d^3x \left[-\epsilon_{ijk} A_{aj} E_{ak} + \Sigma_i(\psi) + \text{orbital parts} \right], \quad i = 1, 2, 3,$$

The matrix element of an operator O is given in the **Cartesian** form

$$\langle \Phi' | O | \Phi \rangle \propto \int dA \, d\bar{\psi} \, d\psi \, \Phi'^*(A, \bar{\psi}, \psi) O \Phi(A, \bar{\psi}, \psi).$$

For $SU(3)$ Yang-Mills QM of spat.const.gluon fields: P. Weisz and V. Ziemann (1986)

Point trafo to new set of adapted coordinates,

$A_{ai}, \psi_\alpha \rightarrow$ 3 gauge angles q_j , the pos. definite symmetric 3×3 matrix S , and new ψ'_β

$$A_{ai}(q, S) = O_{ak}(q) S_{ki} - \frac{1}{2g} \epsilon_{abc} \left(O(q) \partial_i O^T(q) \right)_{bc}, \quad \psi_\alpha(q, \psi') = U_{\alpha\beta}(q) \psi'_\beta$$

orthog. $O(q)$ and unitary $U(q)$ related via $O_{ab}(q) = \frac{1}{2} \text{Tr} (U^{-1}(q) \tau_a U(q) \tau_b)$.

Generalisation of the polar decomposition of A and corresponds to (A. Khvedelidze and H.-P. P. 1999)

$$\chi_i(A) = \epsilon_{ijk} A_{jk} = 0 \quad (\text{"symmetric gauge"}).$$

Preserving the CCR \rightarrow old canonical momenta in terms of the new variables

$$-E_{ai}(q, S, p, P) = O_{ak}(q) \left[P_{ki} + \epsilon_{kil} {}^*D_{ls}^{-1}(S) \left(\Omega_{sj}^{-1}(q) p_j + \rho_s(\psi') + D_n(S)_{sm} P_{mn} \right) \right]$$

$$\Rightarrow \quad G_a \Phi \equiv O_{ak}(q) \Omega_{ki}^{-1}(q) p_i \Phi = 0 \quad \Leftrightarrow \quad \frac{\delta}{\delta q_i} \Phi = 0 \quad (\text{Abelianisation})$$

$$\text{Ang. mom. op.} \quad J_i = \int d^3x \left[-2\epsilon_{ijk} S_{mj} P_{mk} + \Sigma_i(\psi') + \rho_i(\psi') + \text{orbital parts} \right]$$

\rightarrow S colorless spin 0,2 glueball field, ψ' colorless reduced quark fields of spin 0,1
 Reduction: Color \rightarrow Spin (unusual spin-statistics relation specific to SU(2) !)

The correctly ordered physical quantum Hamiltonian (Christ and Lee 1980) in terms of the physical variables $S_{ik}(\mathbf{x})$ and the can. conj. $P_{ik}(\mathbf{x}) \equiv -i\delta/\delta S_{ik}(\mathbf{x})$ reads

$$H(S, P) = \frac{1}{2} \mathcal{J}^{-1} \int d^3 \mathbf{x} P_{ai} \mathcal{J} P_{ai} + \frac{1}{2} \int d^3 \mathbf{x} \left[B_{ai}^2(S) - S_{ai} j_{ia}(\psi') + \bar{\psi}' (\gamma_i \partial_i + m) \psi' \right] \\ - \mathcal{J}^{-1} \int d^3 \mathbf{x} \int d^3 \mathbf{y} \left\{ \left(D_i(S)_{ma} P_{im} + \rho_a(\psi') \right) (\mathbf{x}) \mathcal{J} \right. \\ \left. \langle \mathbf{x} a | {}^*D^{-2}(S) | \mathbf{y} b \rangle \left(D_j(S)_{bn} P_{nj} + \rho_b(\psi') \right) (\mathbf{y}) \right\}$$

with the FP operator

$${}^*D_{kl}(S) \equiv \epsilon_{kmi} D_i(S)_{ml} = \epsilon_{kli} \partial_i - g(S_{kl} - \delta_{kl} \text{tr} S)$$

and the Jacobian $\mathcal{J} \equiv \det |{}^*D|$

The matrix element of a physical operator O is given by

$$\langle \Psi' | O | \Psi \rangle \propto \int_{S \text{ pos.def.}} \int_{\bar{\psi}', \psi'} \prod_{\mathbf{x}} \left[dS(\mathbf{x}) d\bar{\psi}'(\mathbf{x}) d\psi'(\mathbf{x}) \right] \mathcal{J} \Psi'^* [S, \bar{\psi}', \psi'] O \Psi [S, \bar{\psi}', \psi']$$

The inverse of the FP operator and hence the physical Hamiltonian can be expanded in the number of spatial derivatives \equiv expansion in $\lambda = g^{-2/3}$

Introduce UV cutoff a : infinite spatial lattice of granulas $G(\mathbf{n}, a)$ at $\mathbf{x} = a\mathbf{n}$ ($\mathbf{n} \in Z^3$) and averaged variables

$$S(\mathbf{n}) := \frac{1}{a^3} \int_{G(\mathbf{n}, a)} d\mathbf{x} S(\mathbf{x})$$

and the discretised spatial derivatives.

Expansion of the Hamiltonian in $\lambda = g^{-2/3}$

$$H = \frac{g^{2/3}}{a} \left[\mathcal{H}_0 + \lambda \sum_{\alpha} \mathcal{V}_{\alpha}^{(\partial)} + \lambda^2 \left(\sum_{\beta} \mathcal{V}_{\beta}^{(\Delta)} + \sum_{\gamma} \mathcal{V}_{\gamma}^{(\partial\partial \neq \Delta)} \right) + \mathcal{O}(\lambda^3) \right]$$

The "free" Hamiltonian

$$\mathcal{H}_0 = \sum_{\mathbf{n}} \mathcal{H}_0^{QM}(\mathbf{n})$$

is the sum of the Hamiltonians of $SU(2)$ -Yang-Mills quantum mechanics of constant fields in each box (which include the local gluon-gluon interactions!).

The interaction terms

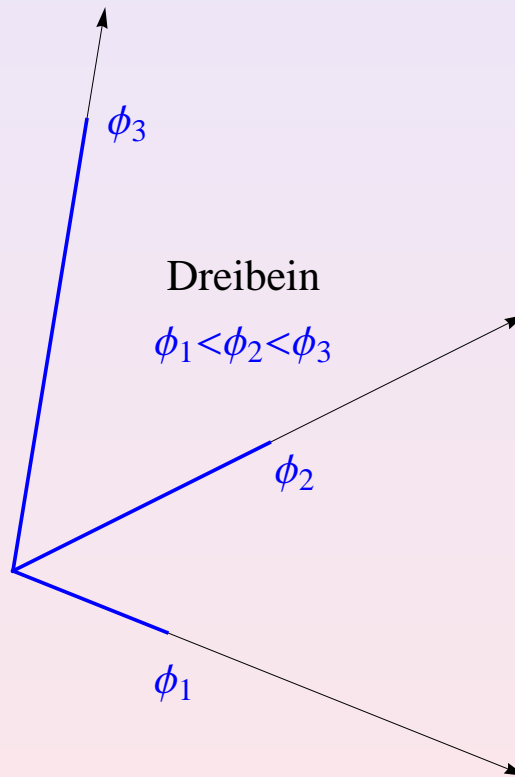
$$\mathcal{V}^{(\partial)}, \mathcal{V}^{(\Delta)}, \dots$$

lead to interactions between the granulas (non-local terms).

Intrinsic System of symmetric tensor S

Intrinsic system:
$$S = R^T(\alpha, \beta, \gamma) \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \end{pmatrix} R(\alpha, \beta, \gamma)$$

Jacobian:
$$\sin(\beta) (\phi_3 - \phi_1)(\phi_3 - \phi_2)(\phi_2 - \phi_1)$$



→ local part of magn.pot.
$$B^2 = \phi_2^2 \phi_3^2 + \phi_3^2 \phi_1^2 + \phi_1^2 \phi_2^2$$

$B^2 = 0$: " $\phi_1 = \phi_2 = 0$, ϕ_3 arbitrary" (0-valleys)

Derivative Expansion (1): Zeroth order Hamiltonian

Zeroth order Hamiltonian

$$H = \frac{g^{2/3}}{V^{1/3}} \left[\mathcal{H}^G + \mathcal{H}^D + \mathcal{H}^C \right] + \frac{1}{2} m \left[\left(\tilde{u}_L^{(0)\dagger} \tilde{v}_R^{(0)} + \sum_{i=1}^3 \tilde{u}_L^{(i)\dagger} \tilde{v}_R^{(i)} \right) + h.c. \right]$$

$$\mathcal{H}^G := \frac{1}{2} \sum_{ijk}^{\text{cyclic}} \left(-\frac{\partial^2}{\partial \phi_i^2} - \frac{2}{\phi_i^2 - \phi_j^2} \left(\phi_i \frac{\partial}{\partial \phi_i} - \phi_j \frac{\partial}{\partial \phi_j} \right) + (\xi_i - \tilde{J}_i^Q)^2 \frac{\phi_j^2 + \phi_k^2}{(\phi_j^2 - \phi_k^2)^2} + \phi_j^2 \phi_k^2 \right),$$

$$\mathcal{H}^D := \frac{1}{2} (\phi_1 + \phi_2 + \phi_3) \left(\tilde{N}_L^{(0)} - \tilde{N}_R^{(0)} \right) + \frac{1}{2} \sum_{ijk}^{\text{cyclic}} (\phi_i - (\phi_j + \phi_k)) \left(\tilde{N}_L^{(i)} - \tilde{N}_R^{(i)} \right),$$

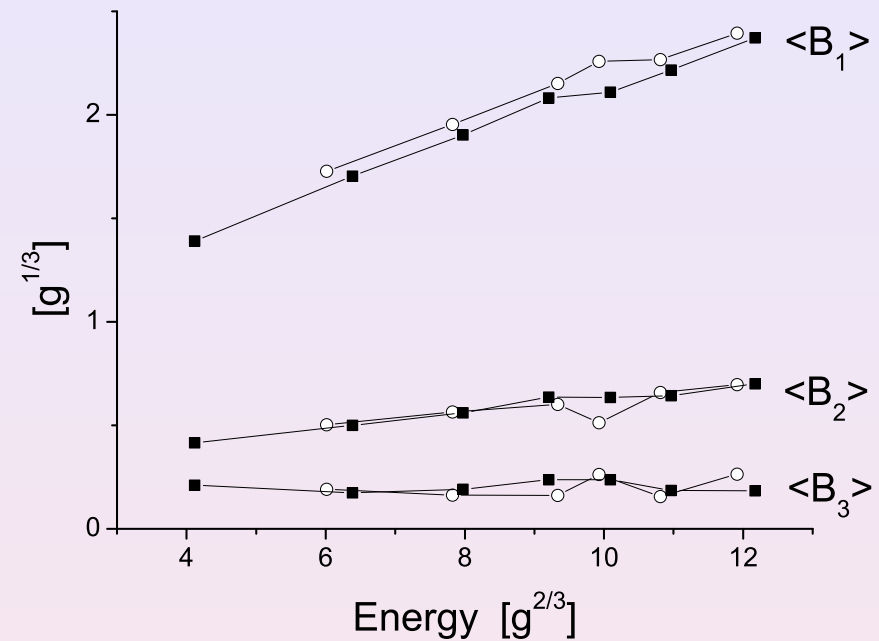
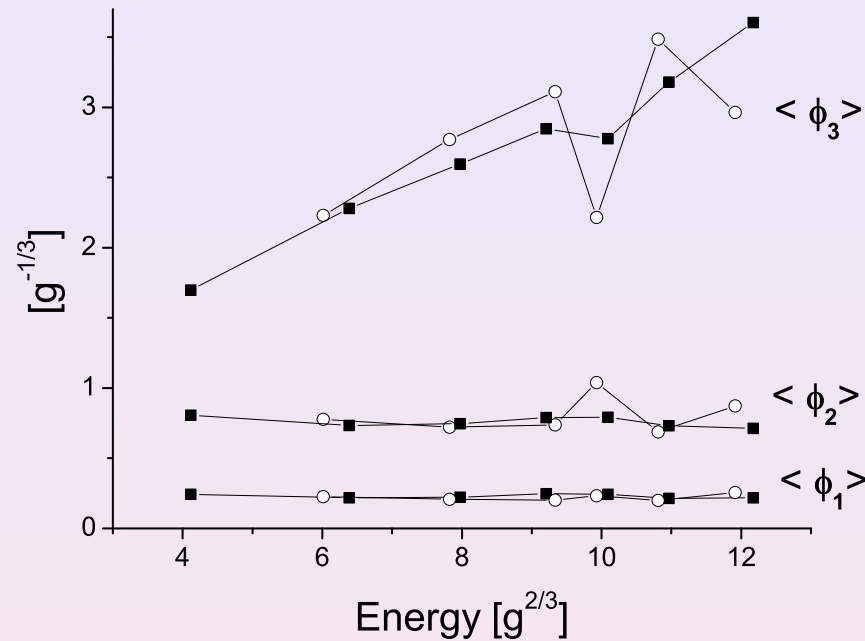
$$\mathcal{H}^C := \sum_{ijk}^{\text{cyclic}} \frac{\tilde{\rho}_i (\xi_i - \tilde{J}_i^Q + \tilde{\rho}_i)}{(\phi_j + \phi_k)^2},$$

and the total spin $J_i = R_{ij}(\chi) \xi_j$, $[J_i, H] = 0$

in terms of the intrinsic spin ξ_i satisfying $[J_i, \xi_j] = 0$ and $[\xi_i, \xi_j] = -i\epsilon_{ijk} \xi_k$

The matrix elements become

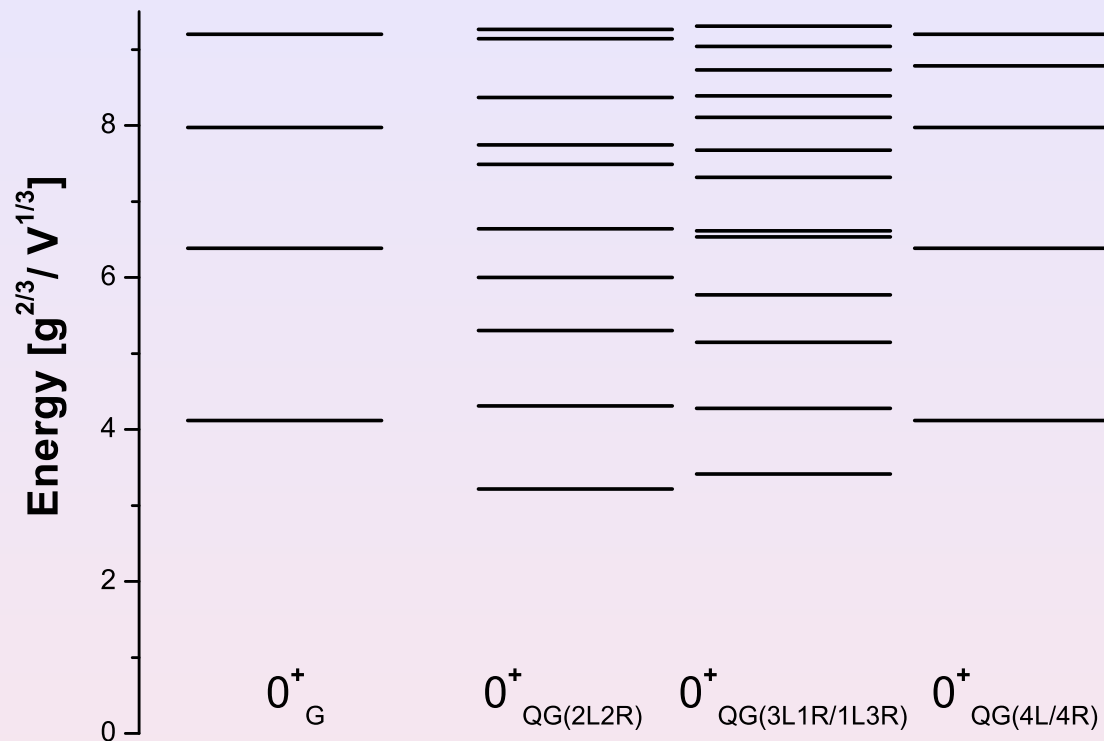
$$\langle \Phi_1 | \mathcal{O} | \Phi_2 \rangle = \int d\alpha \sin \beta d\beta d\gamma \int_{0 < \phi_1 < \phi_2 < \phi_3} d\phi_1 d\phi_2 d\phi_3 (\phi_1^2 - \phi_2^2)(\phi_2^2 - \phi_3^2)(\phi_3^2 - \phi_1^2) \int d\bar{\psi}' d\psi' \Phi_1^* \mathcal{O} \Phi_2.$$



$\langle \phi_3 \rangle$ is raising with increasing excitation, whereas $\langle \phi_1 \rangle$ and $\langle \phi_2 \rangle$ are practically constant. $\langle B_3 \rangle$ is practically constant with increasing excitation, whereas $\langle B_1 \rangle$ and $\langle B_2 \rangle$ are raising.

H.-P. P., Phys. Lett. B **648** (2007) 97-106.

0^+ energy spectrum for the pure-gluon and the quark-gluon cases



The energy of the **quark-gluon groundstate** 3.22 (5.63, -2.43, 0.02) is lower than that of the **pure-gluon groundstate** 4.11

The energy for the **sigma-antisigma excitation** is lower than that for the lowest **glueball excitation** (about one fifth)

H.-P. P., Phys. Lett. B **700** (2011) 265-276.

1st and 2nd order pert. theory in $\lambda = g^{-2/3}$ give the result (for the (+) b.c.)

$$E_{\text{vac}}^+ = \mathcal{N} \left[4.1167 + 29.894\lambda^2 + \mathcal{O}(\lambda^3) \right] \frac{g^{2/3}}{a}$$

for the energy of the interacting glueball vacuum, and

$$E_1^{(0)+}(k) - E_{\text{vac}}^+ = \left[2.270 + 13.511\lambda^2 + \mathcal{O}(\lambda^3) \right] \frac{g^{2/3}}{a} + 0.488 \frac{a}{g^{2/3}} k^2 + \mathcal{O}((a^2 k^2)^2)$$

for the energy spectrum of the interacting spin-0 glueball.

Lorentz invariance : $E = \sqrt{M^2 + k^2} \simeq M + \frac{1}{2M} k^2$

→ Consider $J = L + S$ states:

H.-P. P., Phys. Lett. B **685** (2010) 353-364.

Consider the physical mass

$$M = \frac{g_0^{2/3}}{a} \left[\mu + c g_0^{-4/3} \right]$$

Demanding its independence of box size a , one obtains

$$\gamma(g_0) \equiv a \frac{d}{da} g_0(a) = \frac{3}{2} g_0 \frac{\mu + c g_0^{-4/3}}{\mu - c g_0^{-4/3}}$$

vanishes for $g_0 = 0$ (pert. fixed point) or $g_0^{4/3} = -c/\mu$ (IR fixed point, if $c < 0$)

My (incomplete) result $c_1^{(0)}/\mu_1^{(0)} = 5.95(1.34)$ suggests, that no IR fixed points exist.

$$\text{for } c > 0 : \quad g_0^{2/3}(Ma) = \frac{Ma}{2\mu} + \sqrt{\left(\frac{Ma}{2\mu}\right)^2 - \frac{c}{\mu}}, \quad a > a_c := 2\sqrt{c\mu}/M$$

critical coupling $g_0^2|_c = 14.52$ (1.55) and

$$\text{for } M \sim 1.6 \text{ GeV} : \quad a_c \sim 1.4 \text{ fm} \text{ (0.9 fm)} .$$

Use idea of *minimal embedding* of $su(2)$ in $su(3)$ by Kihlberg + Marnelius (1982)

$$\begin{aligned}
 \tau_1 := \lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} & \tau_2 := -\lambda_5 &= \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} & \tau_3 := \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
 \tau_4 := \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} & \tau_5 := \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & \tau_6 := \lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
 \tau_7 := \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \tau_8 := \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}
 \end{aligned}$$

The corresponding non-trivial non-vanishing structure constants $[\frac{\tau_a}{2}, \frac{\tau_b}{2}] = ic_{abc} \frac{\tau_c}{2}$, have at least one index $\in \{1, 2, 3\}$

"symmetric gauge" for SU(3): $\chi_a(A) = \sum_{b=1}^8 \sum_{i=1}^3 c_{abi} A_{bi} = 0, \quad a = 1, \dots, 8$

H.-P. P., arXiv: 1205.2237 [hep-th] (2012);

H.-P. P., PoS (Confinement X) (2013) 071, arXiv: 1303.3763 [hep-th] (2013).

Carrying out the coordinate transformation (generalized polar decomposition)

$$A_{ak} \left(q_1, \dots, q_8, \widehat{S} \right) = O_{a\hat{a}}(q) \widehat{S}_{\hat{a}k} - \frac{1}{2g} c_{abc} \left(O(q) \partial_k O^T(q) \right)_{bc},$$

$$\psi_\alpha \left(q_1, \dots, q_8, \psi^{RS} \right) = U_{\alpha\hat{\beta}}(q) \psi_{\hat{\beta}}^{RS}$$

$$\widehat{S}_{\hat{a}k} \equiv \begin{pmatrix} S_{ik} \\ \overline{S}_{Ak} \end{pmatrix} = \begin{pmatrix} S_{ik} \text{ pos. def.} \\ \hline W_0 & X_3 - W_3 & X_2 + W_2 \\ X_3 + W_3 & W_0 & X_1 - W_1 \\ X_2 - W_2 & X_1 + W_1 & W_0 \\ -\frac{\sqrt{3}}{2} Y_1 - \frac{1}{2} W_1 & \frac{\sqrt{3}}{2} Y_2 - \frac{1}{2} W_2 & W_3 \\ -\frac{\sqrt{3}}{2} W_1 - \frac{1}{2} Y_1 & \frac{\sqrt{3}}{2} W_2 - \frac{1}{2} Y_2 & Y_3 \end{pmatrix}, \quad c_{\hat{a}\hat{b}\hat{c}} \widehat{S}_{\hat{b}\hat{c}} = 0$$

exists and unique : $\widehat{S}_{\hat{a}i} \widehat{S}_{\hat{a}j} = A_{ai} A_{aj}$ (6) $d_{\hat{a}\hat{b}\hat{c}} \widehat{S}_{\hat{a}i} \widehat{S}_{\hat{b}j} \widehat{S}_{\hat{c}k} = d_{abc} A_{ai} A_{bj} A_{ck}$ (10)

reduced gluons (glueballs): Spin 0,1,2,3 reduced quarks: Spin 3/2 Rarita-Schwinger

Reduction: Color \rightarrow Spin, consequ. for Spin-Physics? $\Delta^{++}(3/2) : (3/2, +1/2, -1/2)?$

Rotate into the intrinsic frame of submatrix S representing the embedded $su(2)$

$$\hat{S} = \begin{pmatrix} S \\ \hline \bar{S} \end{pmatrix} = \begin{pmatrix} R(\alpha, \beta, \gamma) & 0 \\ \hline 0 & D^{(2)}(\alpha, \beta, \gamma) \end{pmatrix} \begin{pmatrix} \text{diag}(\phi_1, \phi_2, \phi_3) \\ \hline \bar{S}(X_i \rightarrow x_i \\ Y_i \rightarrow y_i \\ W_i \rightarrow w_i) \end{pmatrix} \begin{pmatrix} R^T(\alpha, \beta, \gamma) \end{pmatrix}$$

The magnetic potential V_{magn} has the zero-energy valleys ("constant Abelian fields")

$$B^2 = 0 \quad : \quad \phi_3 \text{ and } y_3 \text{ arbitrary} \quad \wedge \quad \text{all others zero}$$

At the bottom of the valleys the string-interaction becomes diagonal

$$\mathcal{H}_{\text{diag}}^D = \frac{1}{2} \tilde{\psi}_L^{(1, \frac{1}{2})\dagger} [(\phi_3 \lambda_3 + y_3 \lambda_8) \otimes \sigma_3] \tilde{\psi}_L^{(1, \frac{1}{2})} - \frac{1}{2} \tilde{\psi}_R^{(\frac{1}{2}, 1)\dagger} [\sigma_3 \otimes (\phi_3 \lambda_3 + y_3 \lambda_8)] \tilde{\psi}_R^{(\frac{1}{2}, 1)}$$

Symmetric gauge for SU(3): 1 spatial dimension

In one spatial dimension the symmetric gauge for SU(3) reduces to (\sim t'Hooft gauge)

$$A^{(1d)} = \begin{pmatrix} 0 & 0 & A_{13} \\ 0 & 0 & A_{23} \\ 0 & 0 & A_{33} \\ 0 & 0 & A_{43} \\ 0 & 0 & A_{53} \\ 0 & 0 & A_{63} \\ 0 & 0 & A_{73} \\ 0 & 0 & A_{83} \end{pmatrix} \rightarrow S^{(1d)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \phi_3 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \quad (\sim A_3^a \lambda_a = U(\phi_3 \lambda_3 + y_3 \lambda_8) U^+)$$

which consistently reduces the above equs. for S for given A_3 to

$$\phi_3^2 + y_3^2 = A_{a3} A_{a3} \quad \wedge \quad \phi_3^2 y_3 - 3 y_3^3 = d_{abc} A_{a3} A_{b3} A_{c3}$$

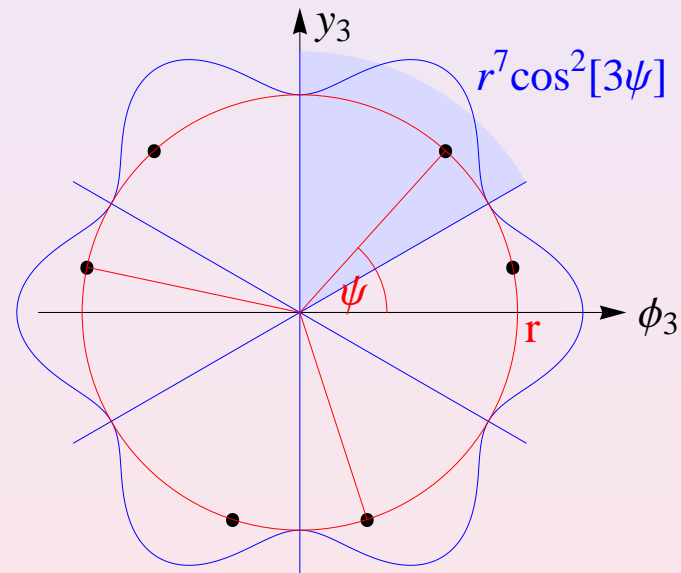
$$\phi_3 = r \cos[\psi] \quad y_3 = r \sin[\psi]$$

$$r^2 = A_3^a A_3^a$$

$$r^3 \sin[3\psi] = \sqrt{3} d_{abc} A_3^a A_3^b A_3^c$$

$$\text{FP-det} \rightarrow r^7 \cos^2[3\psi]$$

\rightarrow 6 Weyl-chambers



with 6 sol. separated by 0-lines of the FP-det $\phi_3^2 (\phi_3^2 - 3y_3^2)^2$ ("Gribov-horizons"). Exactly 1 sol. exists in the "fundamental domain" (blue area) and we can replace

$$\int_{-\infty}^{+\infty} \prod_{a=1}^8 dA_{a3} \rightarrow \int_0^{\infty} d\phi_3 \int_{\phi_3/\sqrt{3}}^{\infty} dy_3 \phi_3^2 (\phi_3^2 - 3y_3^2)^2 \propto \int_0^{\infty} r^7 dr \int_{\pi/6}^{\pi/2} d\psi \cos^2(3\psi)$$

Symmetric gauge for SU(3): 2 and 3 spatial dimensions

For two spatial dimensions, one can show that (putting $W_1 \equiv X_1, W_2 \equiv -X_2$)

$$A^{(2d)} = \begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & 0 \\ A_{41} & A_{42} & 0 \\ A_{51} & A_{52} & 0 \\ A_{61} & A_{62} & 0 \\ A_{71} & A_{72} & 0 \\ A_{81} & A_{82} & 0 \end{pmatrix} \rightarrow \widehat{S}_{\text{intr}}^{(2d)} = \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & 0 \\ \hline 0 & x_3 & 0 \\ x_3 & 0 & 0 \\ 2x_2 & 2x_1 & 0 \\ -\frac{\sqrt{3}}{2}y_1 - \frac{1}{2}x_1 & \frac{\sqrt{3}}{2}y_2 + \frac{1}{2}x_2 & 0 \\ -\frac{\sqrt{3}}{2}y_1 + \frac{1}{2}x_1 & -\frac{\sqrt{3}}{2}y_2 + \frac{1}{2}x_2 & 0 \end{pmatrix}$$

consistently reduces the above eqs. for S to a system of 7 eqs. for 8 physical fields (incl. rot.-angle γ), which, adding as an 8th eq. $(d_{\hat{a}\hat{b}\hat{c}}\widehat{S}_{\hat{b}1}\widehat{S}_{\hat{c}2})^2 = (d_{abc}A_{b1}A_{c2})^2$, can be solved numerically for randomly generated $A^{(2d)}$, again yielding solutions separated by horizons. Restricting to a fundamental domain

$$\int_{-\infty}^{+\infty} \prod_{a,b=1}^8 dA_{a1}dA_{b2} \rightarrow \int_0^{2\pi} d\gamma \int_{0 < \phi_1 < \phi_2 < \infty} d\phi_1 d\phi_2 (\phi_2 - \phi_1) \int_{R_1(\phi_1, \phi_2)} dx_1 dx_2 dx_3 \int_{R_2(x_1, x_2, x_3, \phi_1, \phi_2)} dy_1 dy_2 \mathcal{J}$$

Due to the difficulty of the FP-determinant, I have, however, not yet succeeded in a satisfactory description of the regions R_1 and R_2 .

For 3 dimensions, I have found several solutions of the S-equations numerically for a randomly generated A , but to write the corresponding unconstrained integral over a fundamental domain is a difficult, but I think solvable, future task.

- Using a canonical transformation of the dynamical variables, which Abelianises the non-Abelian Gauss-law constraints to be implemented, a reformulation of QCD in terms of gauge invariant dynamical variables can be achieved.
- Using minimal embedding, the symmetric gauge $\epsilon_{ijk}A_{jk} = 0$ for $SU(2)$ can be generalized to the corresponding $SU(3)$ symmetric gauge $c_{abi}A_{bi} = 0$.
- The exact implementation of the Gauss laws reduces the colored spin-1 gluons and spin-1/2 quarks to unconstrained colorless spin-0, spin-1, spin-2 and spin-3 glueball fields and colorless Rarita-Schwinger fields respectively.
- The obtained physical Hamiltonian admits a systematic strong-coupling expansion in powers of $\lambda = g^{-2/3}$, equiv to an expansion in spatial derivatives.
- leading-order term \longrightarrow non-interacting hybrid-glueballs,
Dirac-Yang-Mills QM of spatially constant fields \longrightarrow low-lying mass spectrum
- higher-order terms in $\lambda \longrightarrow$ interactions between the hybrid-glueballs
perturbation theory in $\lambda \longrightarrow$ e.g. energy-momentum relation of glueballs
 \longrightarrow study of Lorentz invariance and renormalisation in the IR.
- conversion of color to spin \longrightarrow new insights into low energy Spin-Physics
- Gauge reduced approach is difficult (due to the complicated Jacobian), but possible and direct. It should be a useful alternative to lattice calculations.
- The investigation can be extended to flux-tubes (string-tension).