# Particle Dipole Moments: Most Sensitive Tests of the Standard Model

Gerald Gabrielse
Leverett Professor of Physics, Harvard University

1. Electron magnetic moment — Harvard
   → Most precisely measured property of an elementary particle
   → Most precise prediction of the Standard Model
2. Compare to the muon magnetic moment
   → It is possible to trap particles from accelerators to make extremely precise measurements — TRAP, ATRAP
   → Can the muon magnetic moment be measured in a small trap?
3. Does the electron also have an electric dipole moment? — ACME

Download papers: http://gabrielse.physics.harvard.edu — NSF, AFOSR

Gabrielse

# Electron Spin Magnetic Moment

magnetic moment $\vec{\mu} = -\frac{g}{2}\mu_B \frac{\vec{S}}{\hbar/2}$ ← angular momentum

↑ Bohr magneton $\frac{e\hbar}{2m}$

$\mu/\mu_B = -g/2$ magnetic moment in Bohr magnetons for spin 1/2

$\mu/\mu_B = -g/2 = -1/2$ mechanical model with identical charge and mass distribution

$\mu/\mu_B = -g/2 = -1$ spin for simple Dirac point particle

$\mu/\mu_B = -g/2 = -1.001\,159\ \ldots$ simplest Dirac spin, plus QED

(if electron g/2 is different → electron has substructure)

Gabrielse

# Three Programs to Measure Electron g

| U. Michigan | U. Washington | Harvard |
|---|---|---|
| beam of electrons | one electron | one electron |
| spins precess with respect to cyclotron motion | | quantum jump spectroscopy of quantum levels |
| keV | 4.2 K | 100 mK |
| Crane, Rich, … | Dehmelt, Van Dyck | |

cylindrical Penning trap

inhibit spontaneous emission

measure cavity shifts

self-excited oscillator

# Measuring the Electron Magnetic Moment to 3 Parts in $10^{13}$

# Need Good Students and Stable Funding

Elise Novitski
Joshua Dorr
Shannon Fogwell Hogerheide
David Hanneke
Brian Odom,
Brian D'Urso,
Steve Peil,
Dafna Enzer,
Kamal Abdullah
Ching-hua Tseng
Joseph Tan

20 years
8 theses

**N$F**

# Need New Ideas Needed → quantum homemade atom

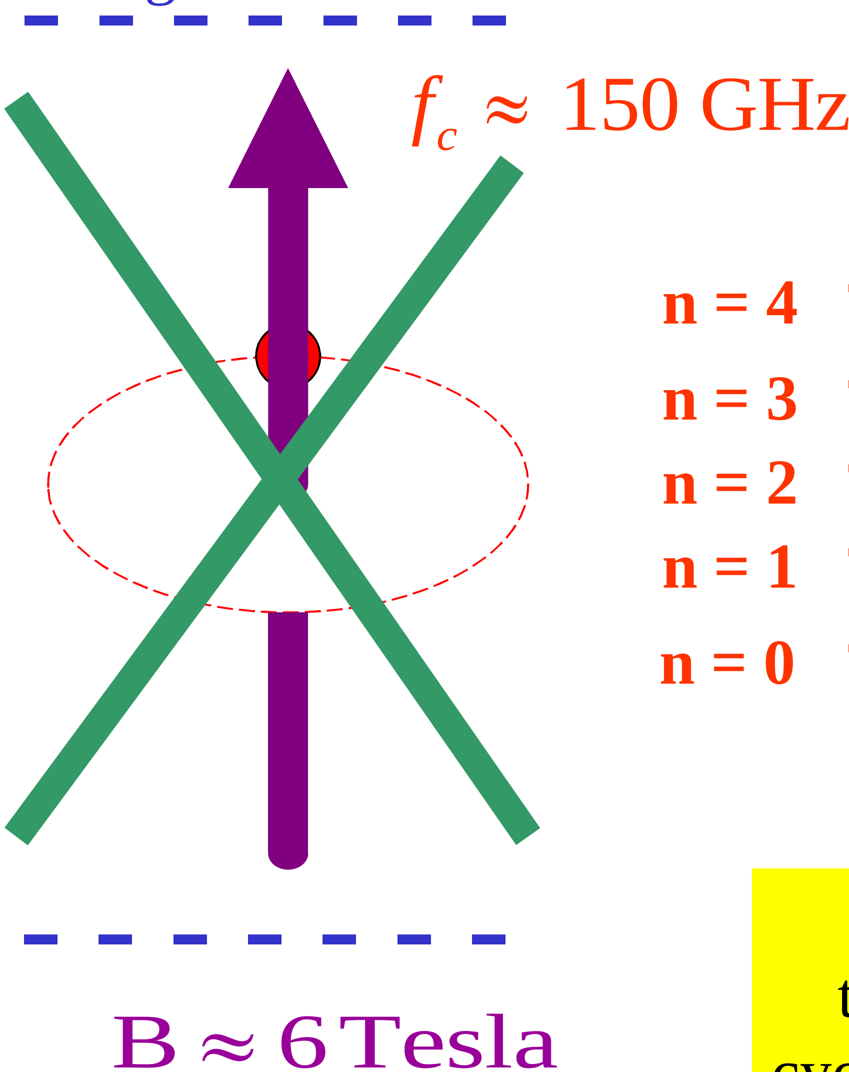
Van Dyck, Schwinberg, Dehemelt did a good job in 1987
Phys. Rev. Lett. **59**, 26 (1987)
(spent some years trying to improve but …)

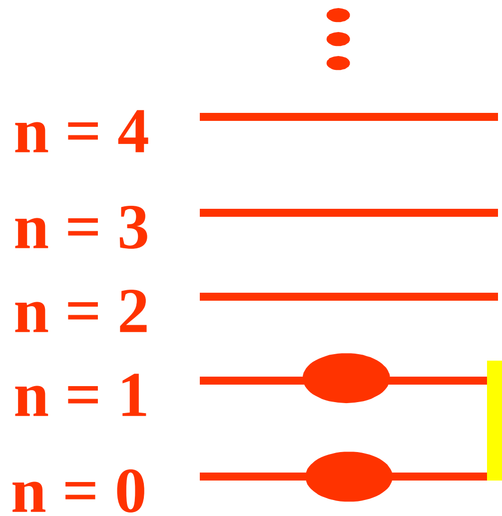
Takes time to develop new ideas and methods
needed to measure with 2.8 parts in $10^{13}$ uncertainty

first measurement with these methods

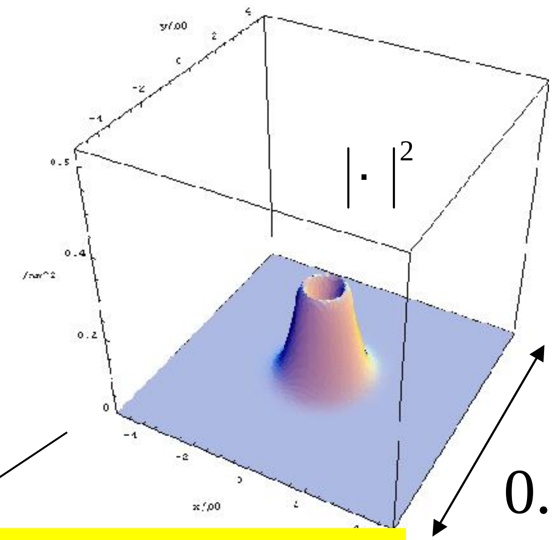
- One-electron quantum cyclotron
- Resolve lowest cyclotron states as well as spin
- Quantum jump spectroscopy of spin and cyclotron motions
- Cavity-controlled spontaneous emission
- Radiation field controlled by cylindrical trap cavity
- Cooling away of blackbody photons
- Synchronized electrons identify cavity radiation modes
- Trap without nuclear paramagnetism
- One-particle self-excited oscillator

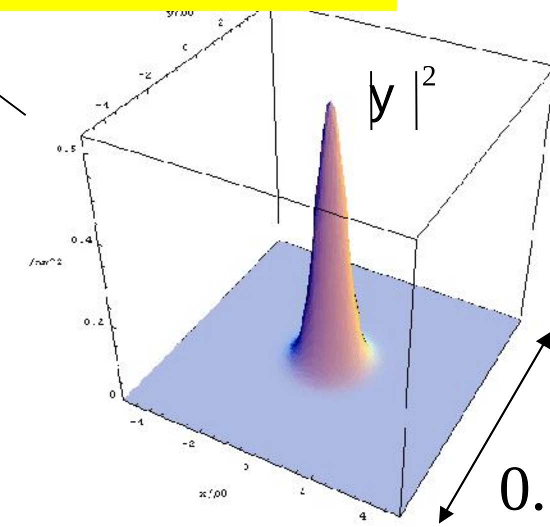
# One Electron Quantum Cyclotron




$h^3{}_c = 7.2$ kelvin

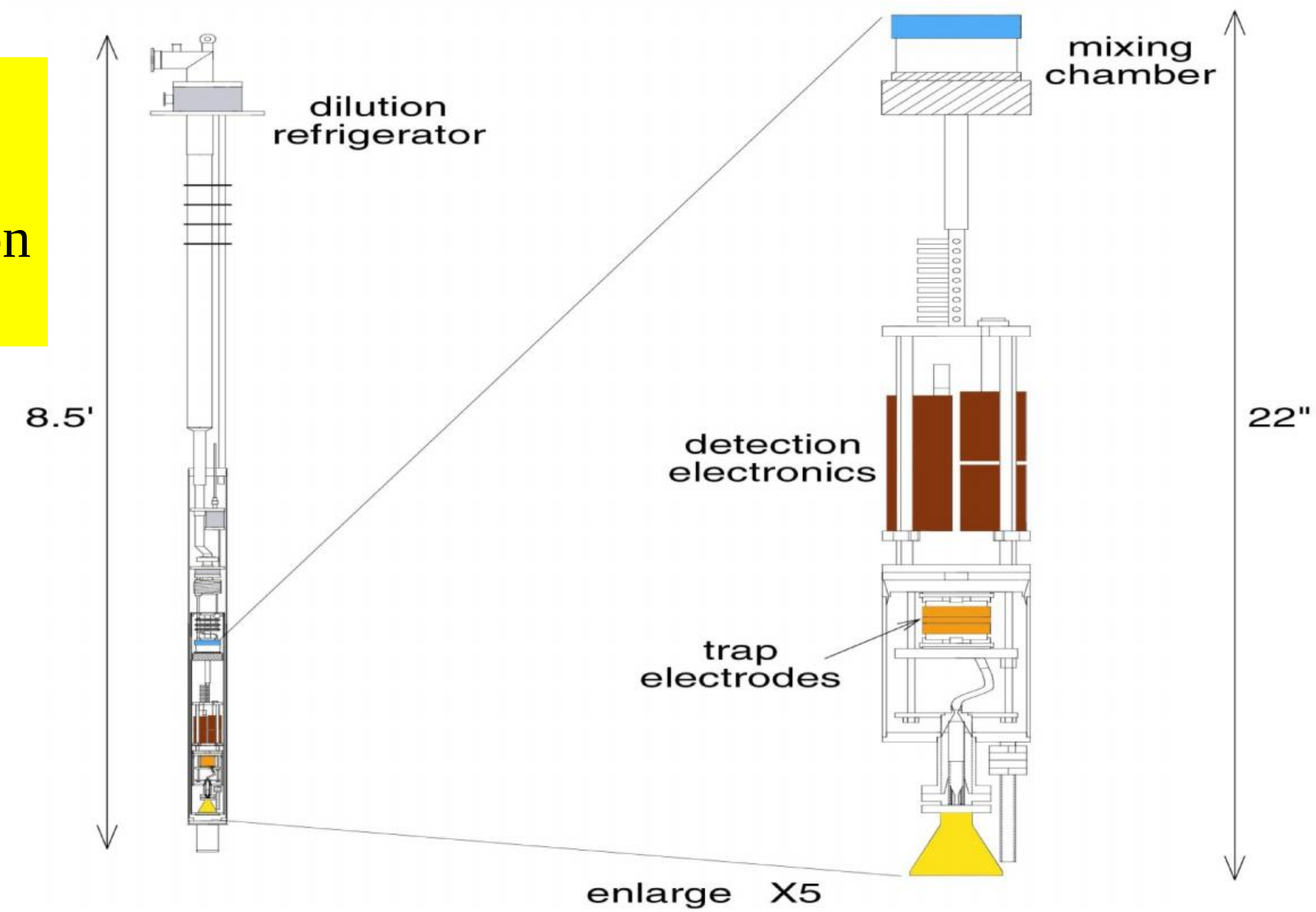



Need low temperature cyclotron motion
T << 7.2 K

# First Penning Trap Below 4 K → 70 mK

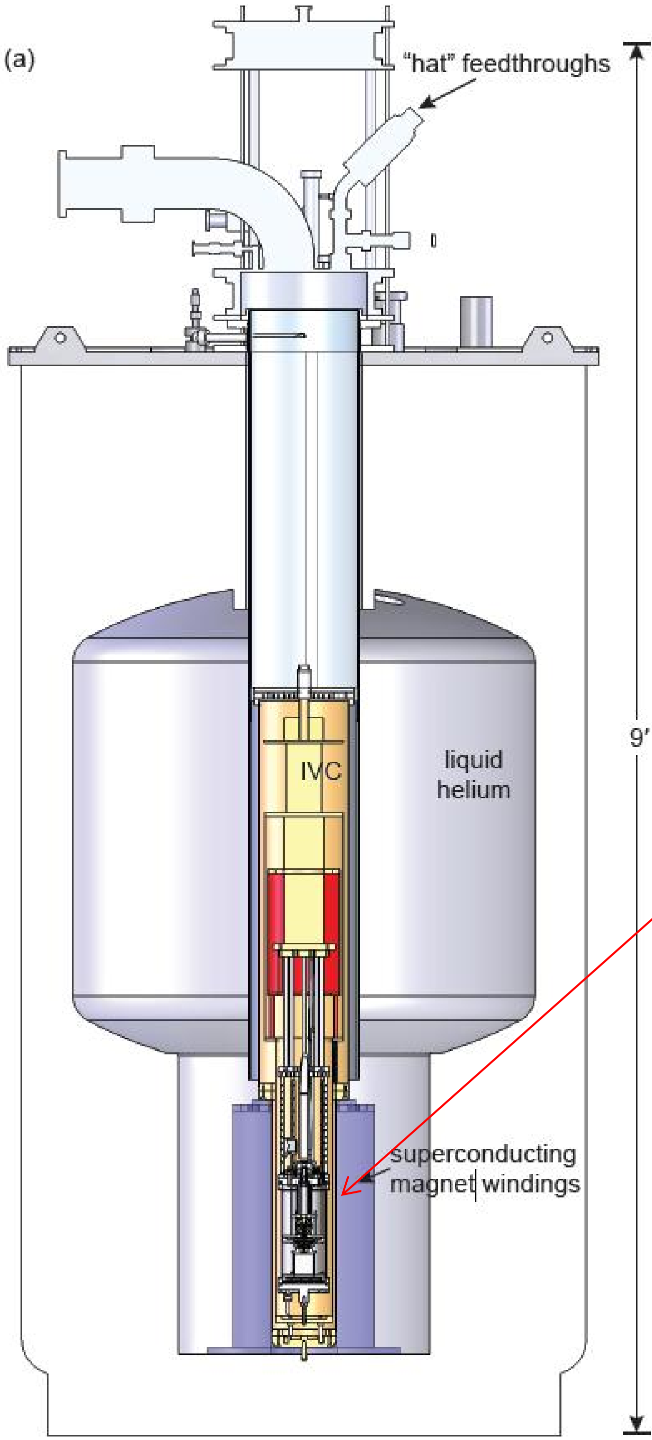
Need low temperature cyclotron motion T << 7.2 K



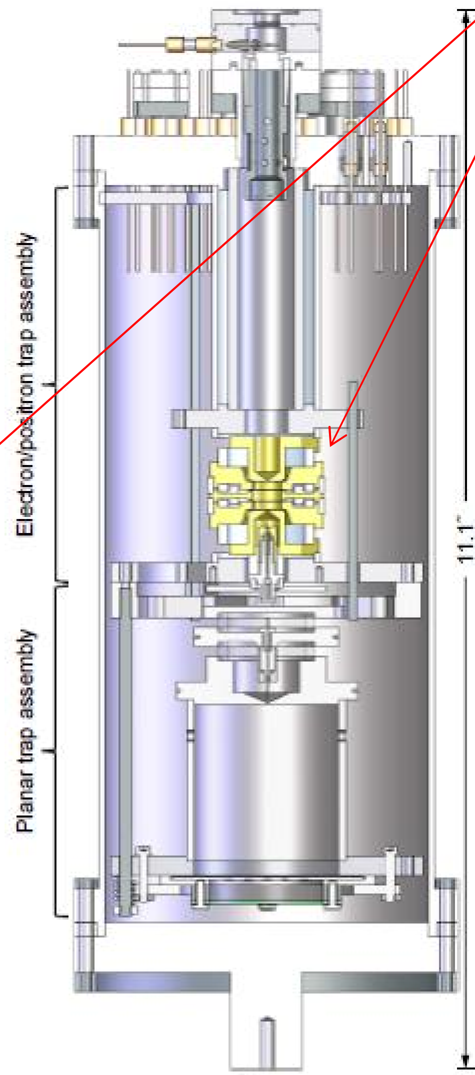

David Hanneke G.G.
FRAGILE
joining the dance
StreetFest

Gabrielse

# New Positron and Electron Apparatus



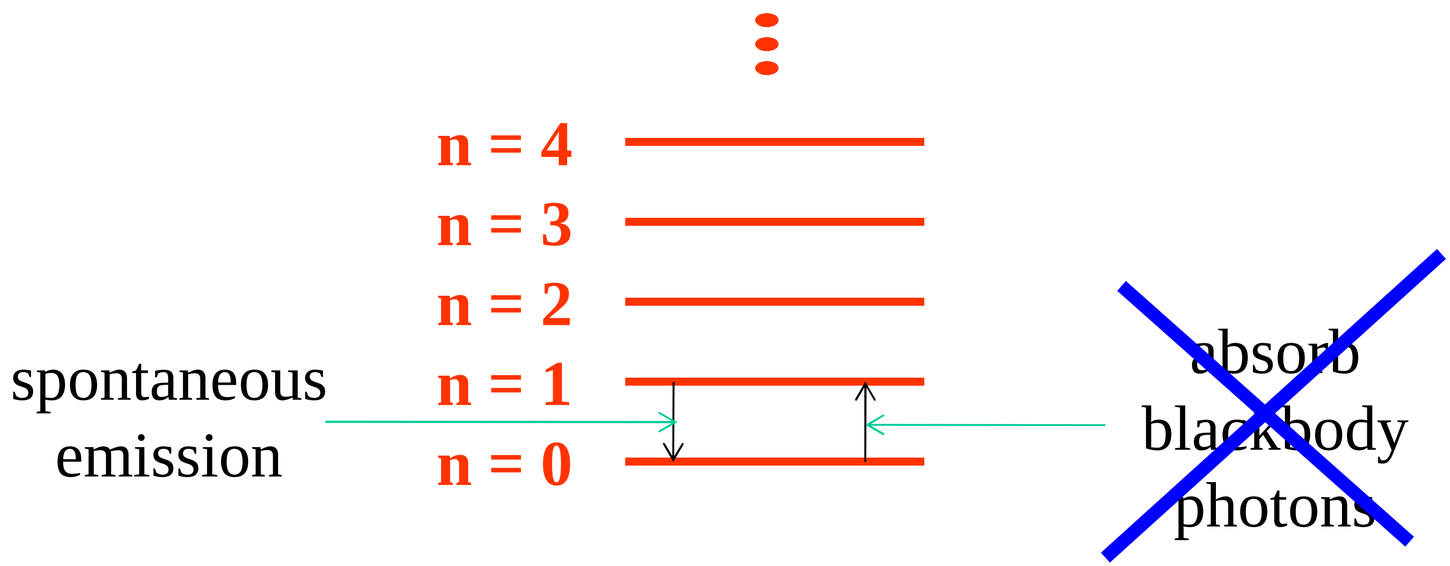

electron magnetic
moment trap




Gabrielse

# Electron Cyclotron Motion Comes Into Thermal Equilibrium

T = 100 mK << 7.2 K → ground state always
Prob = 0.99999…

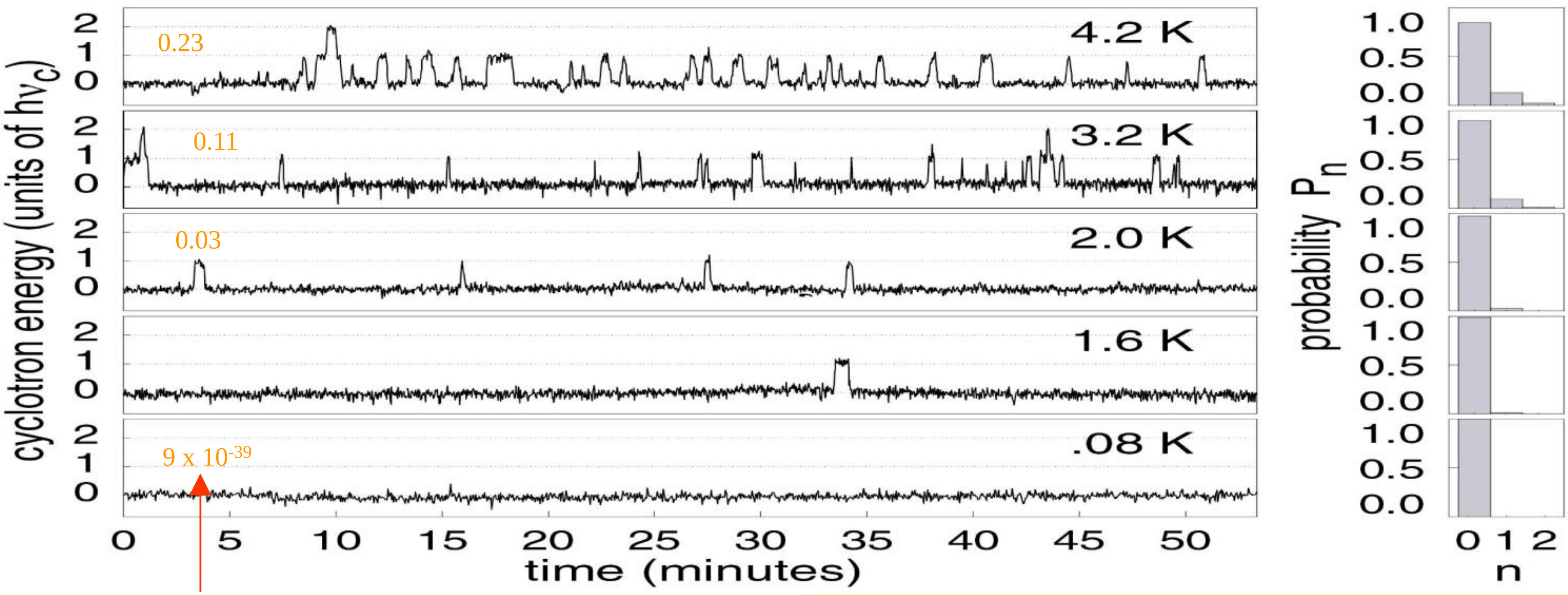



cold
~~hot~~
cavity

electron “temperature” – describes its energy distribution on average

# Electron in Cyclotron Ground State

QND Measurement of Cyclotron Energy vs. Time




average number of blackbody photons in the cavity

**On a short time scale**

→ in one Fock state or another

**Averaged over hours**

→ in a thermal state

S. Peil and G. Gabrielse, Phys. Rev. Lett. **83**, 1287 (1999).

# Electron has a Cyclotron Temperature

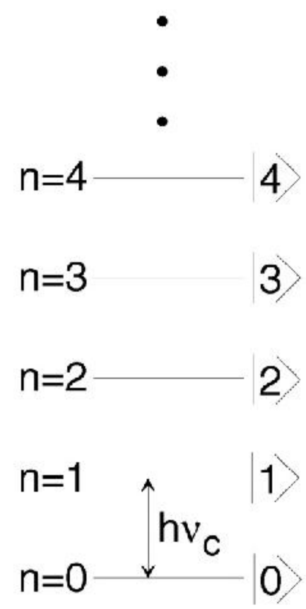
At any instant in time the electron does <u>not</u> have a temperature

→ it is either in one energy eigenstate or another

Averaged over time, electron cyclotron motion has a temperature

→ Boltzmann probability describes the probability of it occupying the various energy levels

$$P_n \sim e^{-\frac{(n+1/2)\hbar\mu_c}{kT}}$$



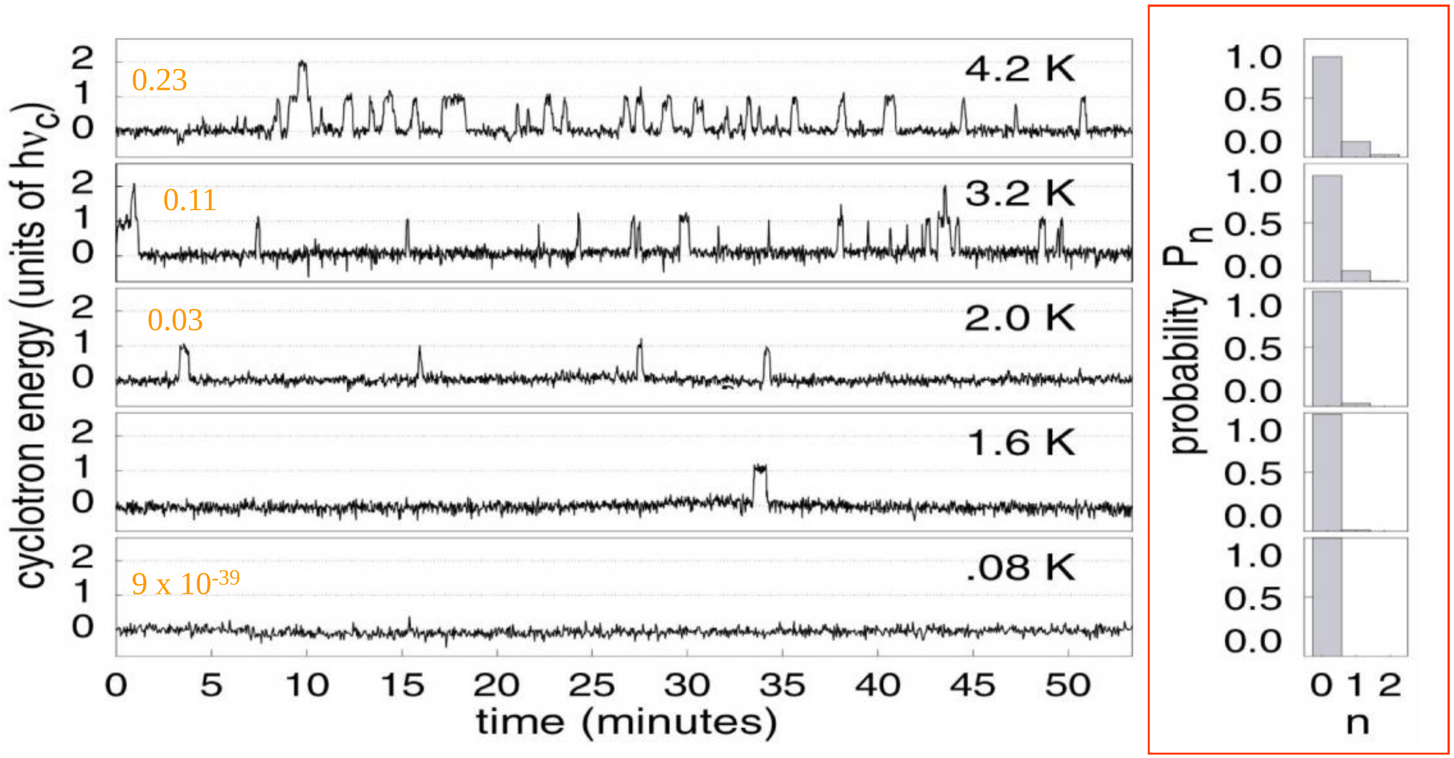

For low enough temperature, temperature no longer matters

→ electron stays in its ground state → no fluctuations

# Measure Electron's Cyclotron Temperature

Temperature relates to time average of electron cyclotron energy

→ measure the probability to find electron in each fock state




Fit to Boltzmann distribution:

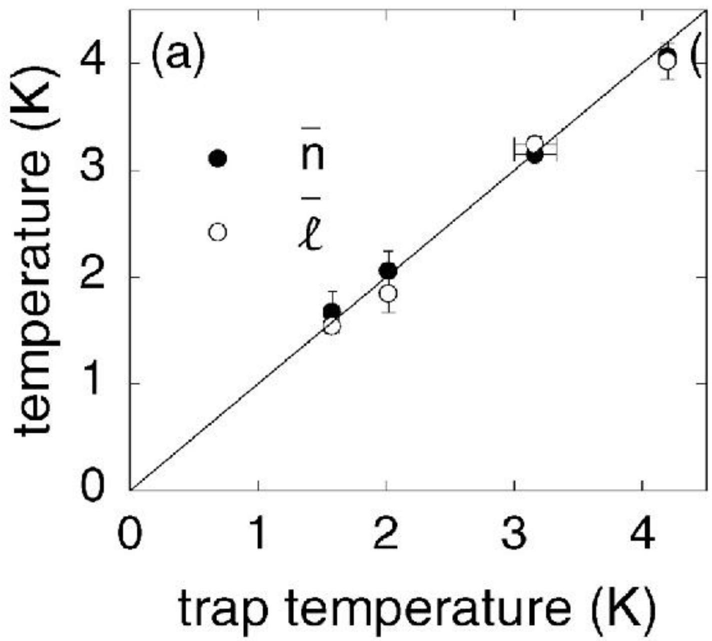
→ extract cyclotron temperature

$$P_n \sim e^{-\frac{(n+1/2)\hbar\mu_c}{kT}}$$

# Electron Cyclotron Temp = Trap Temp

Two methods used to measure electron cyclotron's temperature

- from <n>
- from measured rates



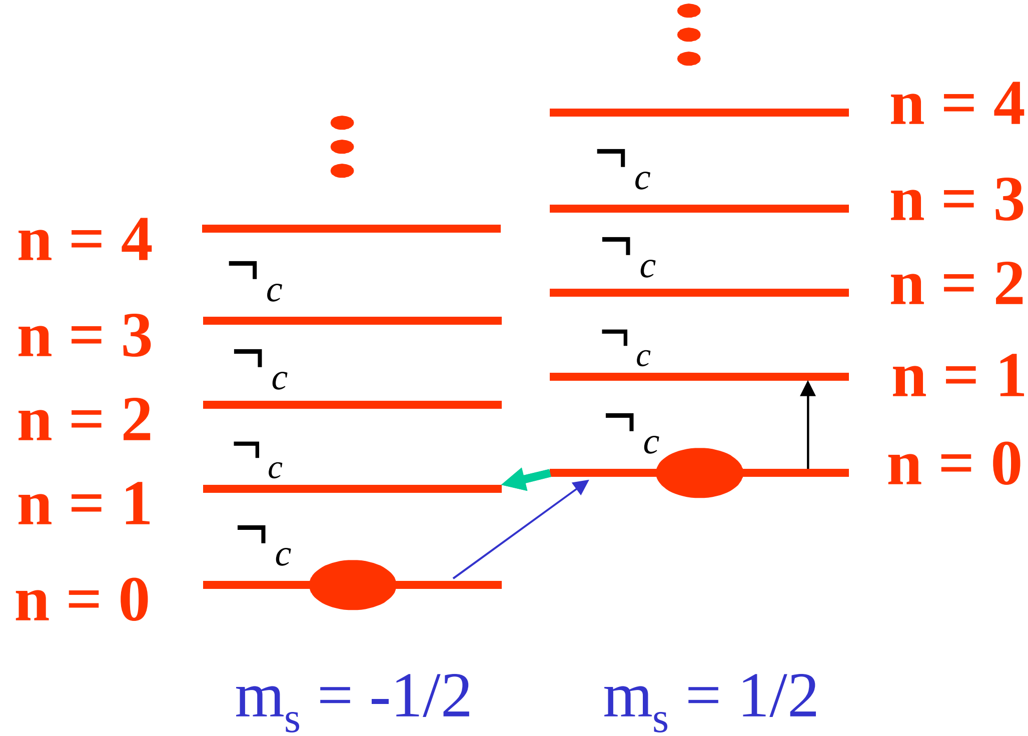

Measured with temperature sensor

Gabrielse

# Basic Idea of the Fully-Quantum Measurement

Cyclotron frequency:

$$\neg_c = \frac{1}{2®}\frac{eB}{m}$$




Spin frequency:

$$\neg_s = \frac{g}{2}\neg_c$$

Measure a ratio of frequencies:

$$\frac{g}{2} = \frac{\neg_s}{\neg_c} = 1 + \frac{\neg_s - \neg_c}{\neg_c}$$

$$\approx 10^{-3}$$

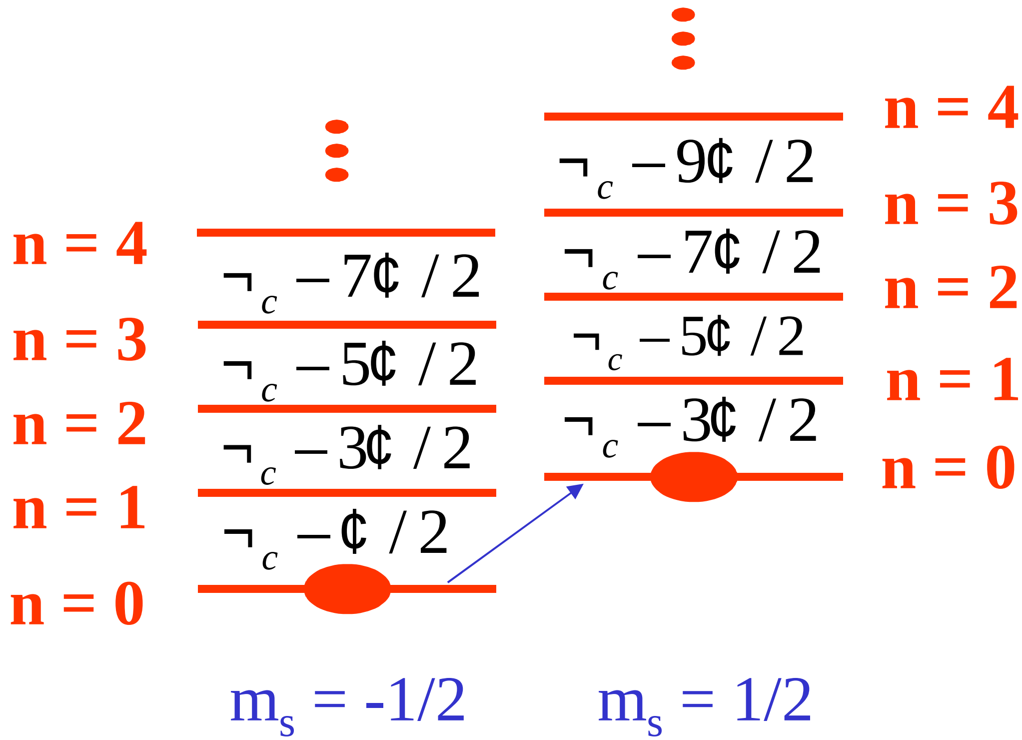
B in free space

- almost nothing can be measured better than a frequency
- the magnetic field cancels out (self-magnetometer)

# Special Relativity Shift the Energy Levels d

Cyclotron frequency:

$$2®¬_c = \frac{eB}{m}$$




Spin frequency:

$$¬_s = \frac{g}{2}¬_c$$

Not a huge relativistic shift,
but important at our accuracy

$$\frac{¢}{¬_c} = \frac{h¬_c}{mc^2} \approx 10^{-9}$$

Solution: Simply correct for δ if we fully resolve the levels

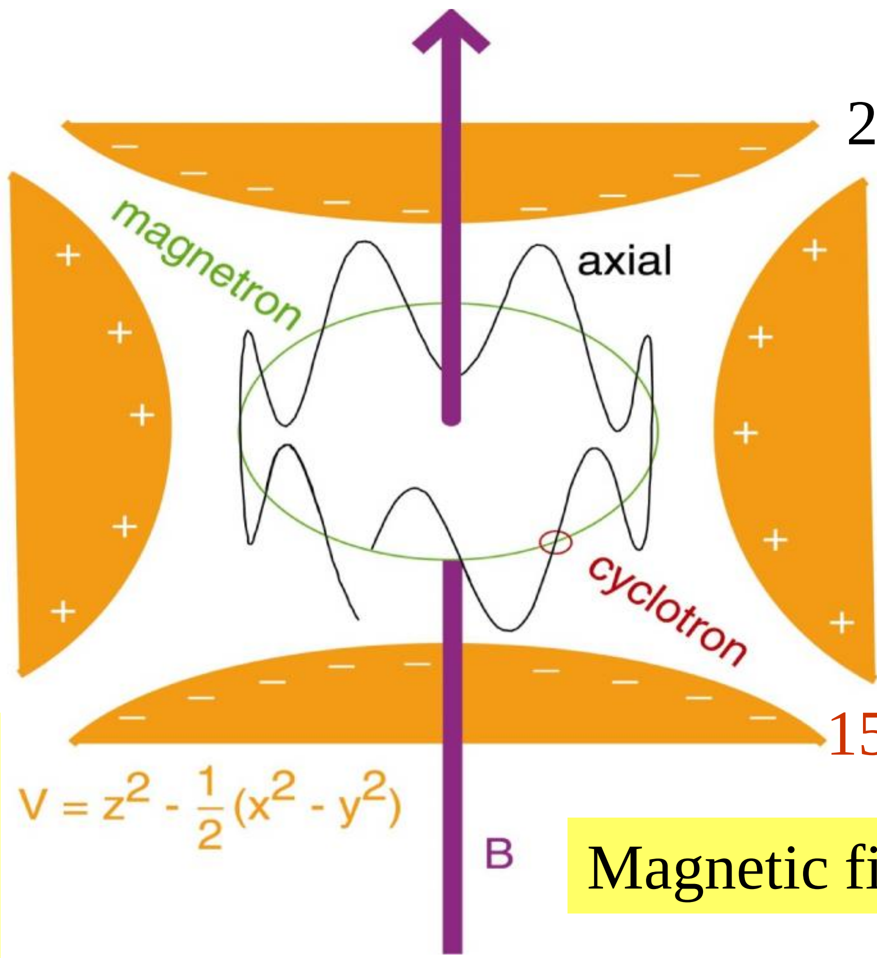
(superposition of cyclotron levels would be a big problem)

# One Electron in a Penning Trap

- very small accelerator
- designer atom

cool 12 kHz

200 MHz detect




153 GHz

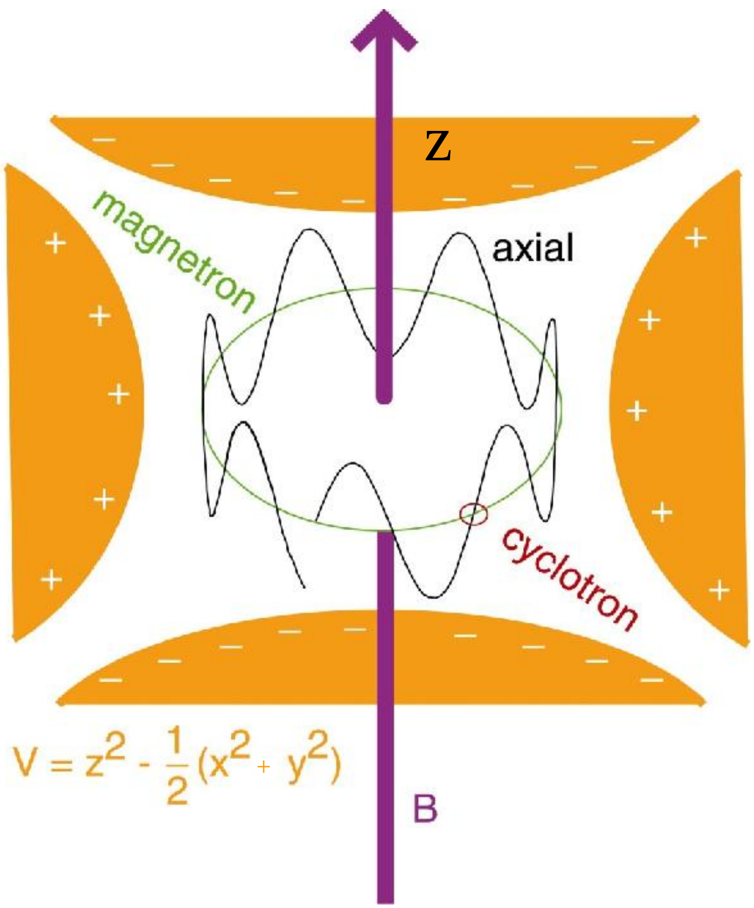
need to measure for g/2

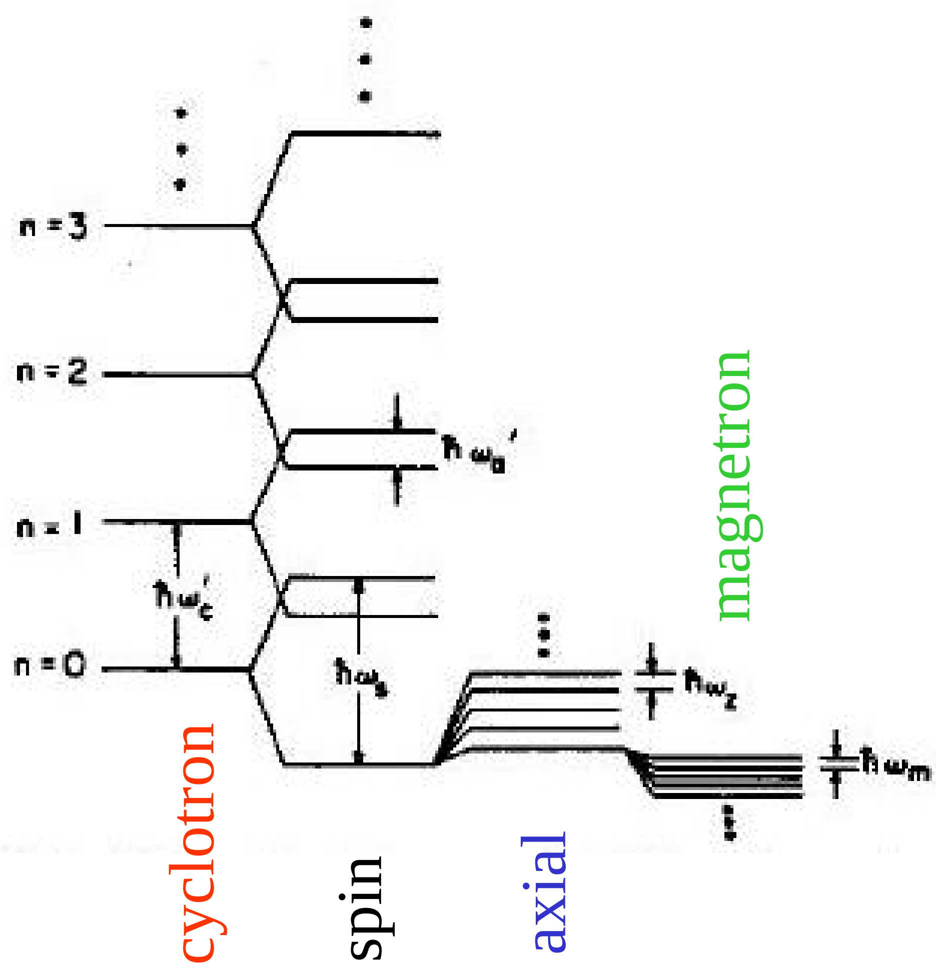
Electrostatic quadrupole potential

Magnetic field

Gabrielse

# All Motions in a Penning Trap are Really Quantum Mechanical (also true for a large accelerator)

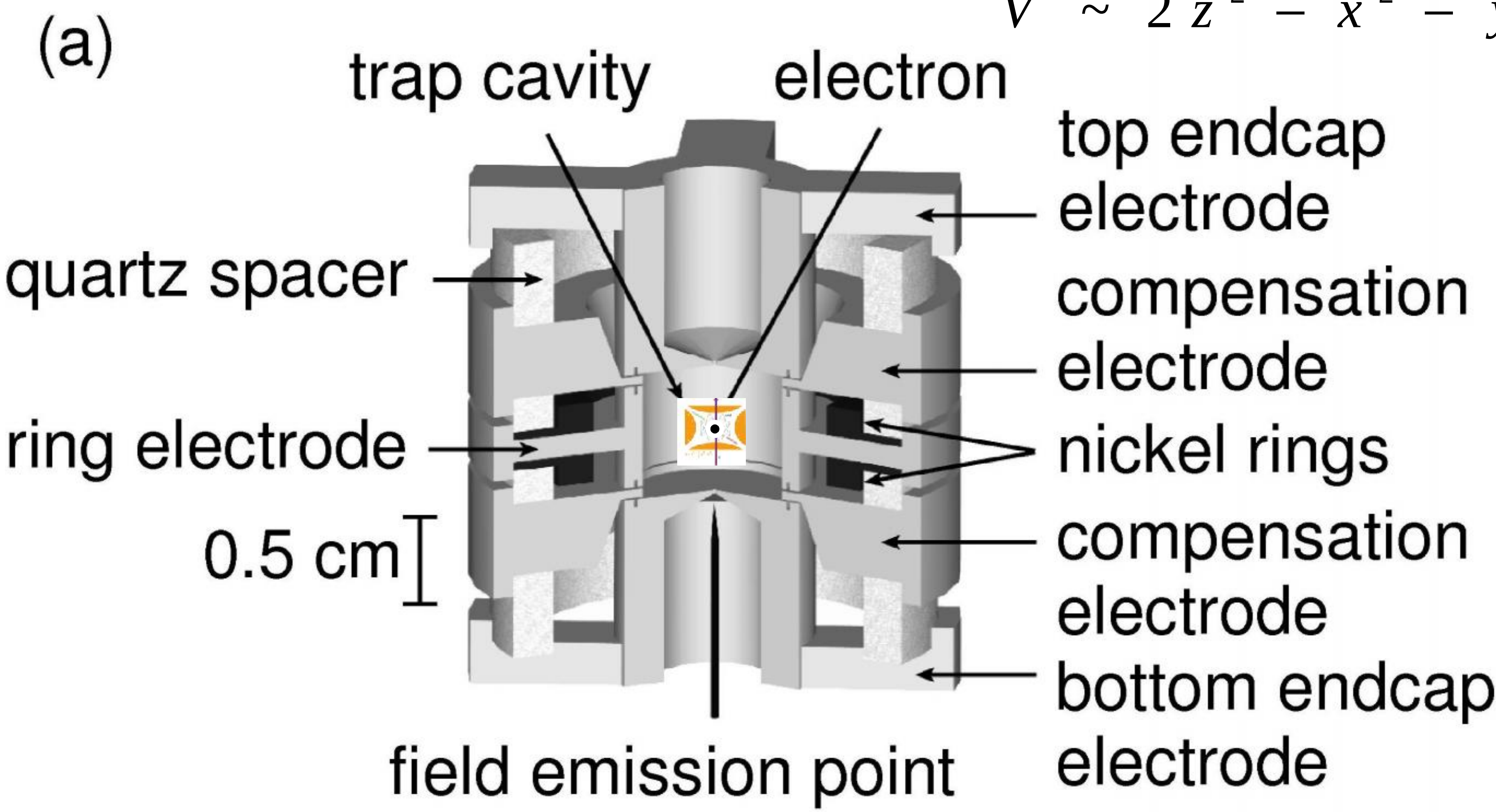



When is QM needed?

# Cylindrical Penning Trap

$$V \sim 2z^2 - x^2 - y^2$$




- Electrostatic quadrupole potential → good near trap center
- Control the radiation field → inhibit spontaneous emission by 200x

(Invented for this purpose: G.G. and F. C. MacKintosh; Int. J. Mass Spec. Ion Proc. ***57***, 1 (1984)

# Frequencies Shift

**B in Free Space**

$$\nu_c = \frac{eB}{m}$$

$$\nu_s = \frac{g}{2}\nu_c$$

→

**Perfect Electrostatic Quadrupole Trap**

$$\nu_c' < \nu_c$$

$$\nu_z \ll \nu_c'$$

$$\nu_m \ll \nu_z$$

$$\nu_s = \frac{g}{2}\nu_c$$

→

**Imperfect Trap**
- **tilted B**
- **harmonic distortions to V**

$$\bar{\nu}_c$$

$$\bar{\nu}_z$$

$$\bar{\nu}_m$$

$$\nu_s = \frac{g}{2}\nu_c$$

Problem: $\frac{g}{2} = \frac{\nu_s}{\nu_c}$ ← not a measurable eigenfrequency in an imperfect Penning trap

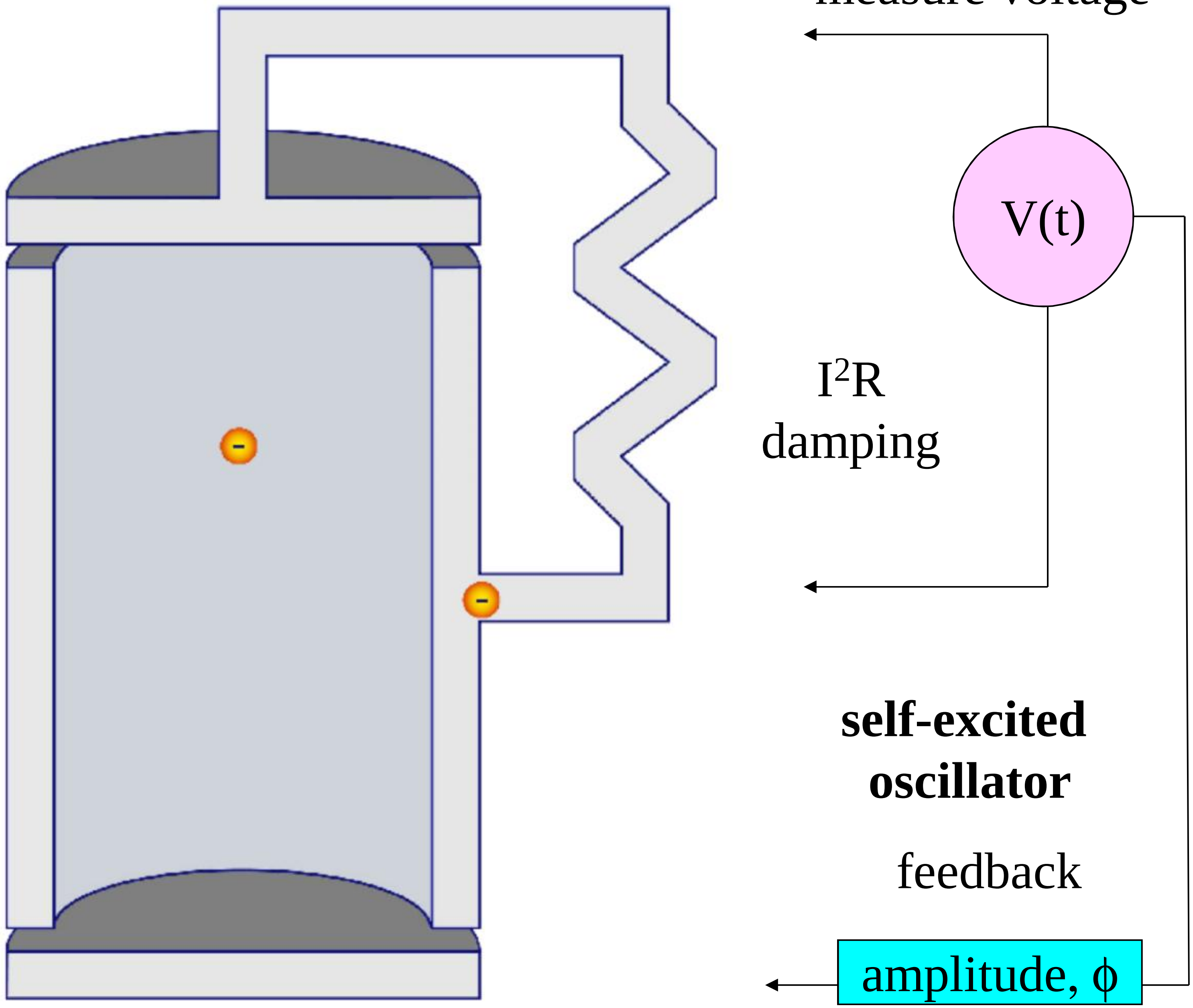
Solution: Brown-Gabrielse invariance theorem

$$\nu_c = \sqrt{(\bar{\nu}_c)^2 + (\bar{\nu}_z)^2 + (\bar{\nu}_m)^2}$$

Gabrielse

# Detecting and Damping Axial Motion

Axial motio
200 MHz
of
trapped
electron



Gabrielse

# Brown-Gabrielse Invariance Theorem

$$\neg_c = \sqrt{(\bar{\neg}_c)^2 + (\bar{\neg}_z)^2 + (\bar{\neg}_m)^2}$$

**Leading Imperfections of a Reasonable Trap**

- **tilted B**
- **harmonic distortions to V**

Eg. ~ xy/d

- Does not protect from magnetic field gradients
- Does not protect from higher order (smaller) imperfections in the electrostatic potential

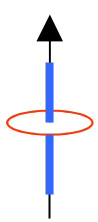
  e.g. ~ $(z/d)^4$

Gabrielse

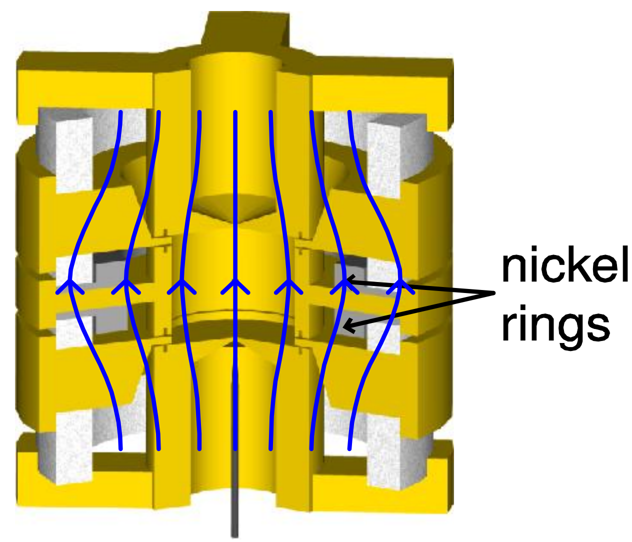
# Detecting the Cyclotron Motion

| | | |
|---|---|---|
| **cyclotron frequency** | $\nu_c$ **= 150 GHz** | **too high to detect directly** |
| **axial frequency** | $\nu_z$ **= 200 MHz** | **relatively easy to detect** |

**Couple the axial frequency $\nu_z$ to the cyclotron energy.**




**Small measurable shift in $\nu_z$ indicates a change in cyclotron energy.**

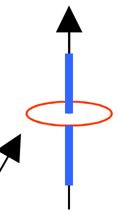



$$B_z = B_0 + B_2 z^2$$

Gabrielse

# Couple Axial Motion and Cyclotron Motion

**Add a "magnetic bottle" to uniform B**

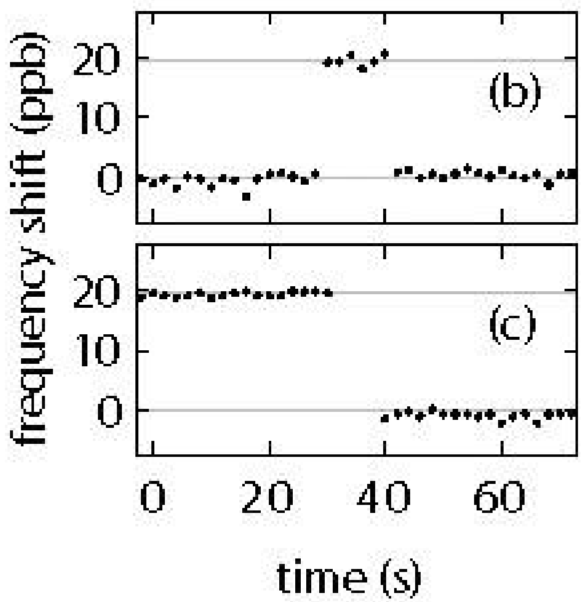



$$\Delta\vec{B} = B_2[(z^2 - \circ^2/2)\hat{z} - z\vec{\circ}]$$

$$H = \frac{1}{2} m \mu_z^2 z^2 - \text{«} B_2 z^2$$

**change in m**
**changes effective $w_z$**

**n=3**
**n=2**
**n=1**
**n=0**

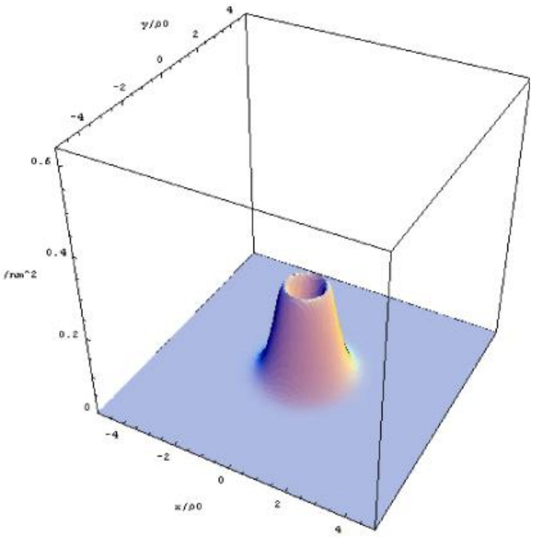



spin flip
is also a change in m

Gabrielse

# QND Detection of One-Quantum Transitions

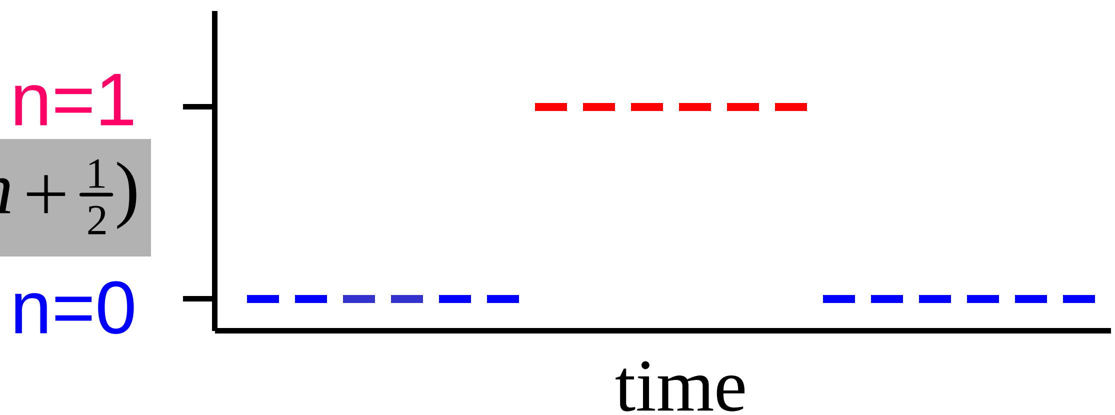
one-electron self-excited oscillator



$$\Delta\vec{B} = B_2\, z^2 \quad \rightarrow \quad H = \frac{1}{2} m \mu_z{}^2 z^2 - \text{«} B_2 z^2$$

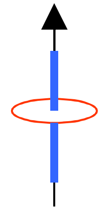
| n=0 cyclotron ground state | n=1 cyclotron excited state | n=0 cyclotron ground state |
|---|---|---|

$$freq = E_{cyclotron} = hf_c(n + \tfrac{1}{2})$$



Gabrielse

# Quantum Non-demolition Measurement



B

$$H = H_{cyclotron} + H_{axial} + H_{coupling}$$

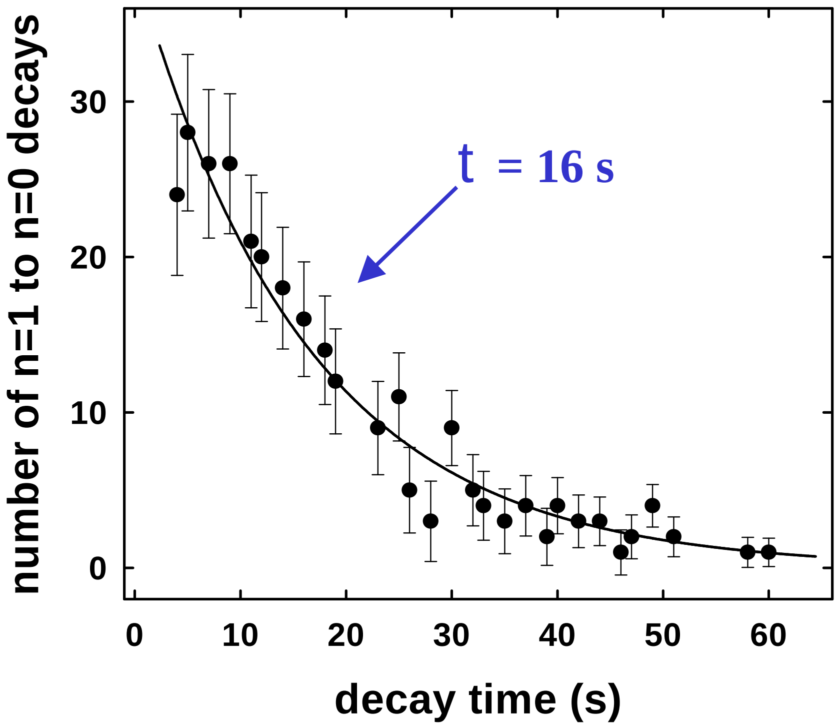
$$[\, H_{cyclotron}, H_{coupling} \,] = 0$$

QND condition

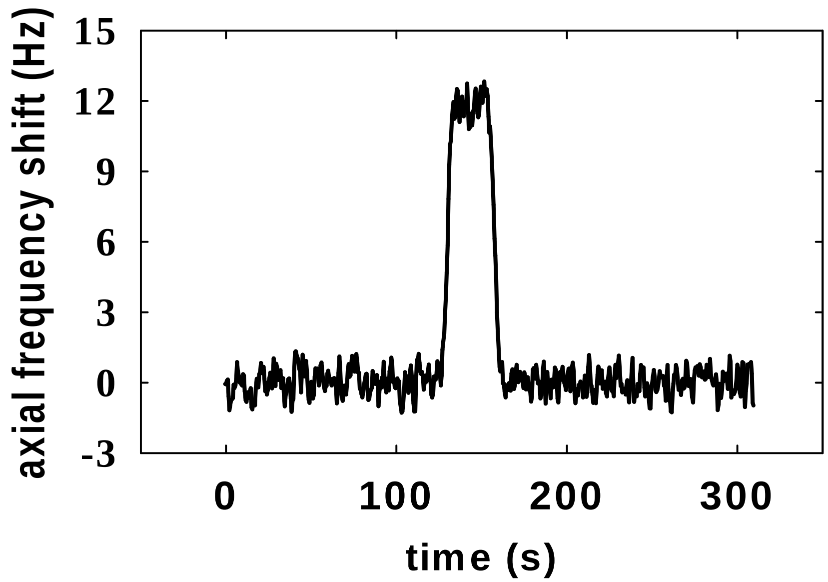
**QND:** Subsequent time evolution of cyclotron motion is not altered by additional QND measurements

# Inhibited Spontaneous Emission

Application of Cavity QED



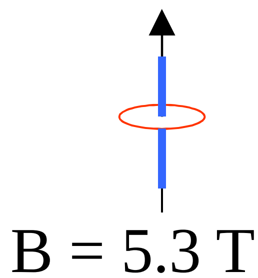

excite,
measure time in excited state



# Cavity-Inhibited Spontaneous Emission

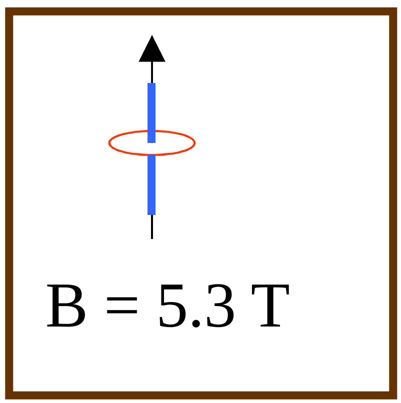
Purcell
Kleppner
Gabrielse and Dehmelt

**Free Space**



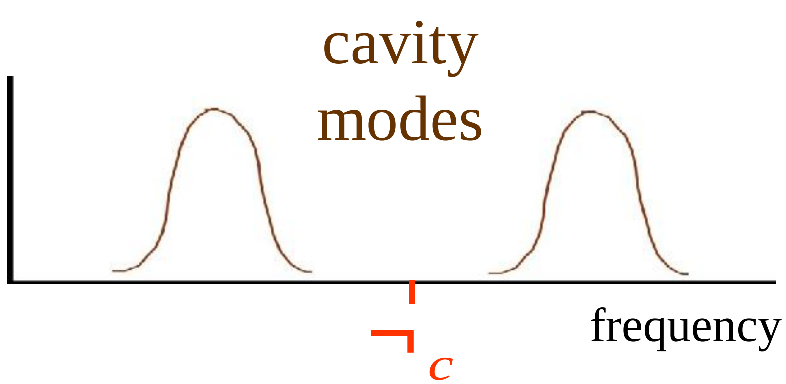

$¥ = \frac{1}{75\ ms}$

**Within Trap Cavity**



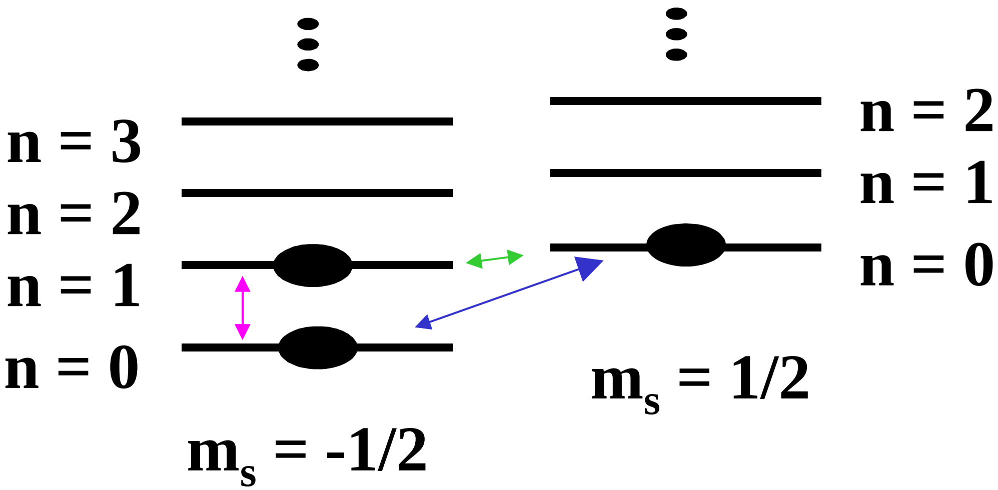

$¥ = \frac{1}{16\ \text{sec}}$

**Inhibited By 210!**




Inhibition gives the averaging time needed to resolve a one-quantum transition

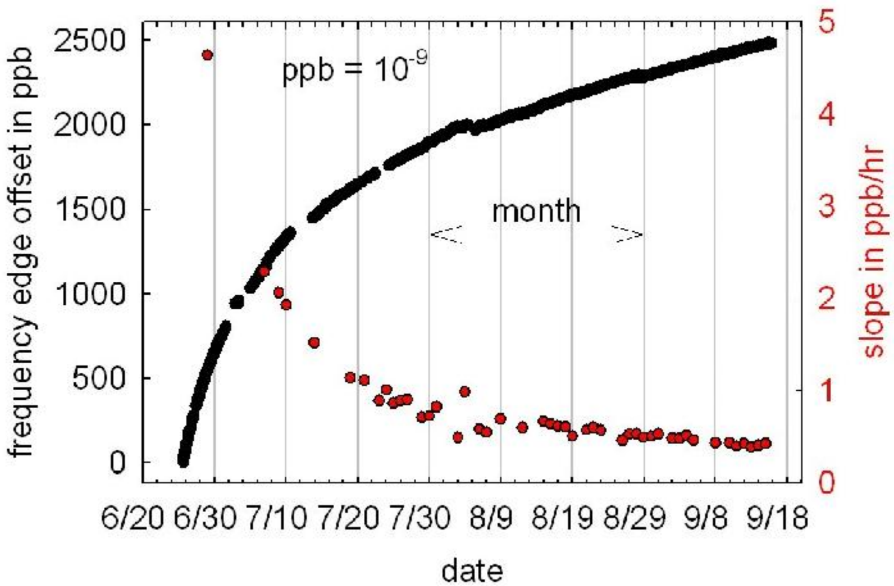
# Big Challenge: Magnetic Field Stability




Magnetic field cancels out

$$\frac{g}{2} = \frac{\mu_s}{\mu_c} = 1 + \frac{\mu_a}{\mu_c}$$

But: problem when B drifts during the measurement




Magnetic field take ~ month to stabilize

# Self-Shielding Solenoid Helps a Lot

Flux conservation → Field conservation

Reduces field fluctuations by about a factor > 150

**United States Patent** [19]

Gabrielse et al.

[11] Patent Number: 4,974,113

[45] Date of Patent: Nov. 27, 1990

[54] SHIELDING SUPERCONDUCTING SOLENOIDS

[75] Inventors: Gerald S. Gabrielse, Lexington, Mass.; Joseph N. Tan, Cebu City, Philippines

[73] Assignee: President and Fellows of Harvard College, Cambridge, Mass.

[21] Appl. No.: 168928

[22] Filed: Mar. 16, 1988

[56] References Cited

U.S. PATENT DOCUMENTS

FOREIGN PATENT DOCUMENTS

OTHER PUBLICATIONS

Sci. Instrum. 51 (4), Apr. 1987, 1987 American Institute of Physics, pp. 638–631.

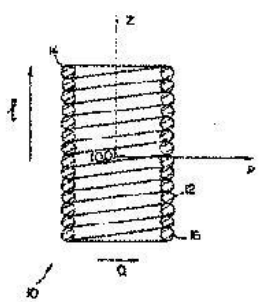
Primary Examiner—L. T. Hix

Assistant Examiner—David M. Gray

Attorney, Agent, or Firm—Fish & Richardson

[57] ABSTRACT

A self-shielding system of closed superconducting circuits shields a specific volume from changes in an external magnetic field in which the circuits are located; the configuration of circuits is chosen so that induced currents in the circuits, arising from magnetic flux conservation for each closed circuit, tend to cancel any change in the external magnetic field. In another aspect, a single closed self-shielding superconducting circuit comprised of more than two circular loops connected in series shields a specific volume from changes in an external magnetic field in which the circuit is located; the configuration of the circuit is chosen so that induced currents in the circuit, arising from magnetic flux conservation for the circuit, tends to cancel any change in the external magnetic field.




U.S. Patent Nov. 27, 1990 Sheet 6 of 8 4,974,113

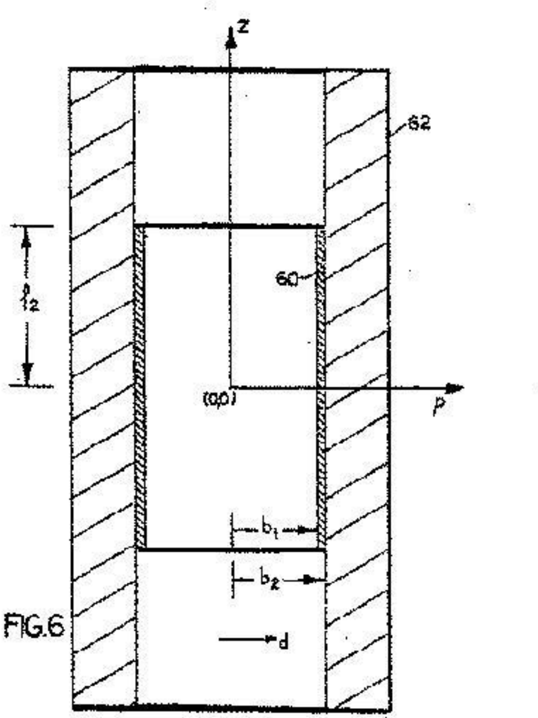


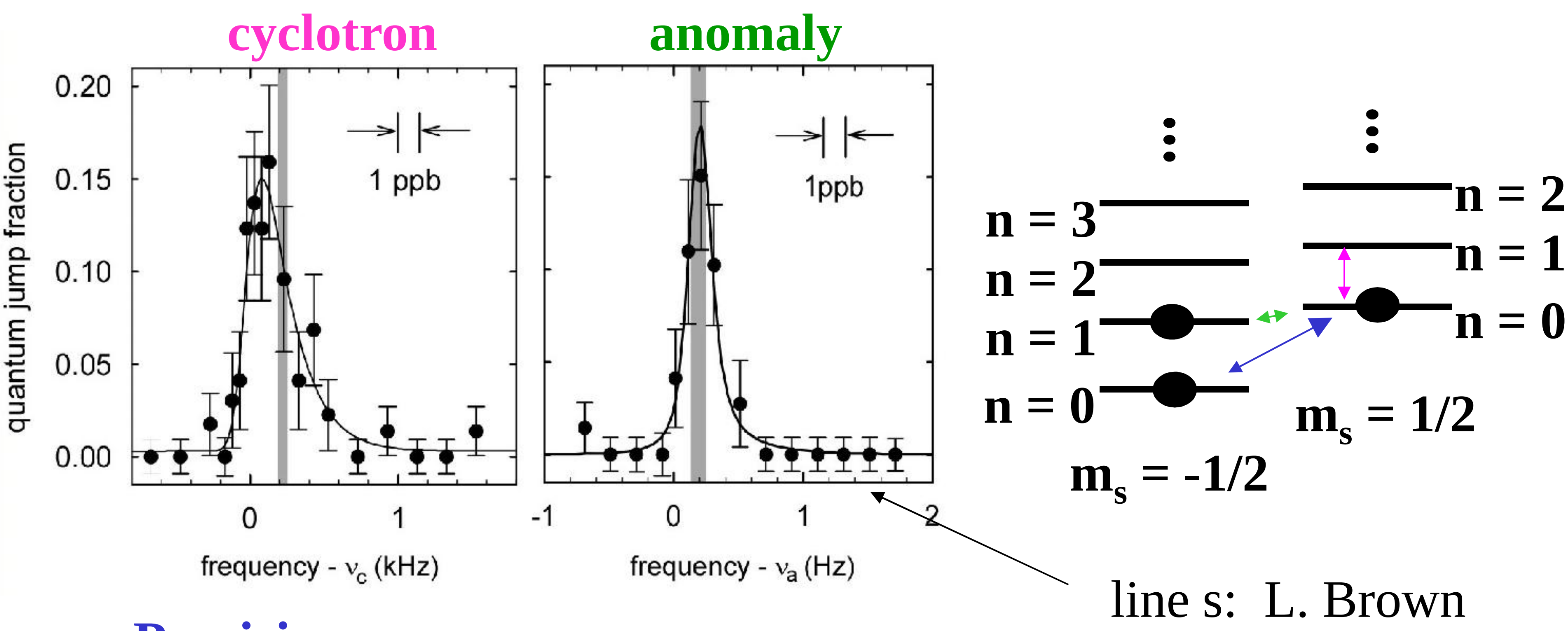

FIG.6

“Self-shielding Superconducting Solenoid Systems”,
G. Gabrielse and J. Tan, J. Appl. Phys. **63**, 5143 (1988)

# Measured Line Shapes for g-value Measurement

## It all comes together:

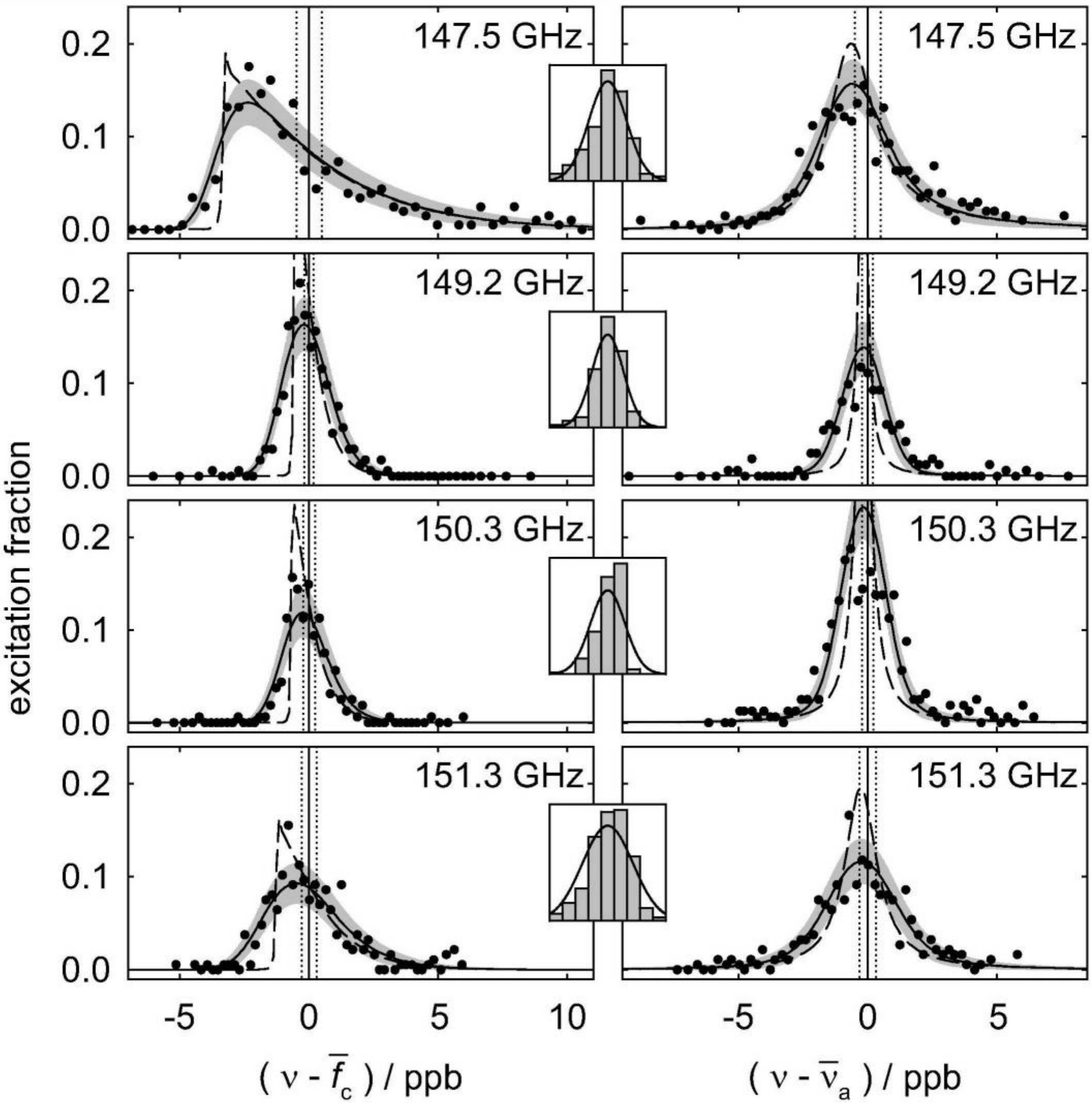
- Low temperature, and high frequency make narrow line shapes
- A highly stable field allows us to map these lines




## Precision:

Sub-ppb line splitting (i.e. sub-ppb precision of a *g*-2 measurement) is now "easy" after years of work

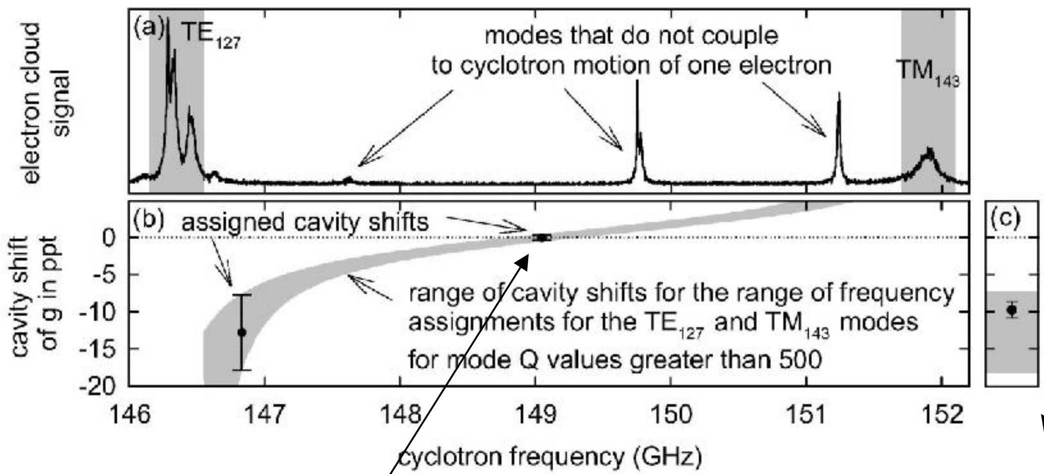
Gabrielse

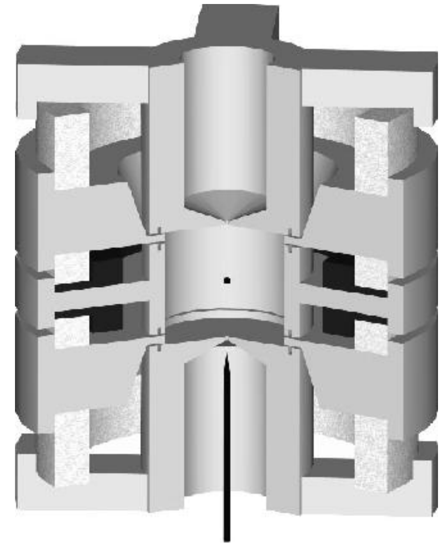


excitation fraction
0.0
0.1
0.2
147.5 GHz
149.2 GHz
150.3 GHz
151.3 GHz
-5
0
5
10
( ν - $\bar{f}_c$ ) / ppb
( ν - $\bar{\nu}_a$ ) / ppb

# Cavity modes and Magnetic Moment Error

use synchronization of electrons to get cavity modes

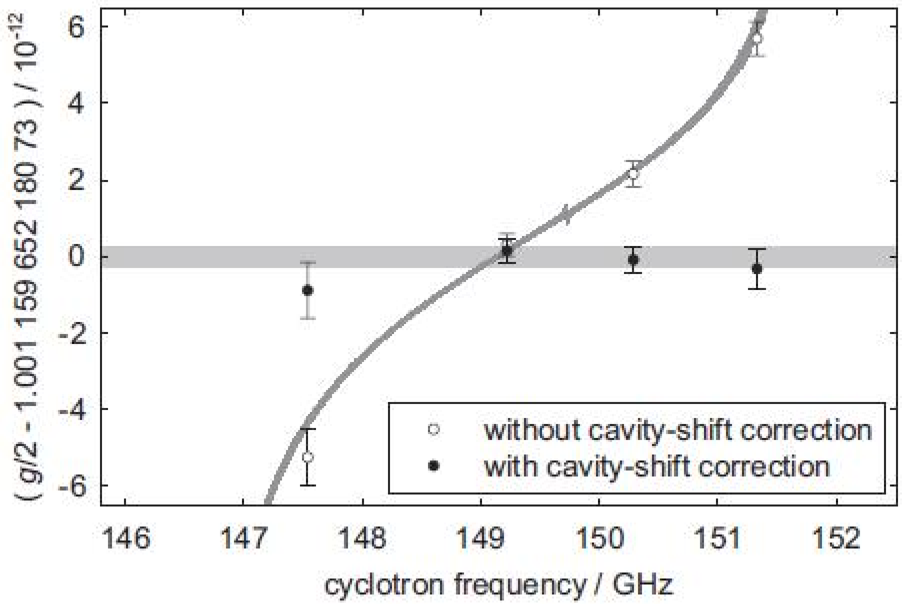


Operating between modes of cylindrical trap where shift from two cavity modes cancels approximately

first measured cavity shift of g

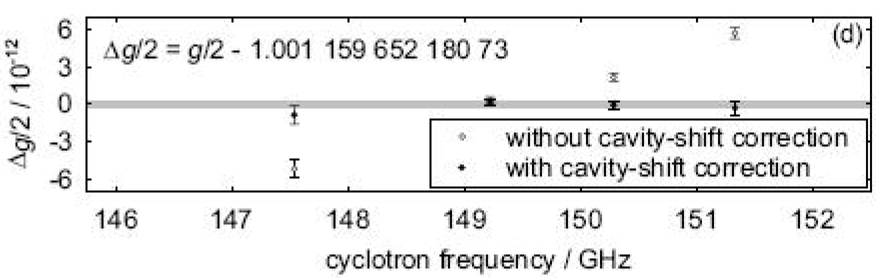
# Cavity Shifts

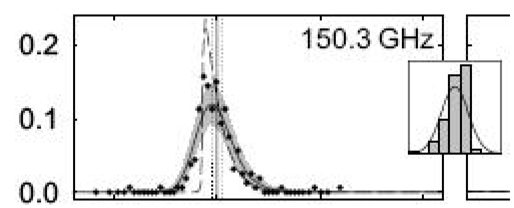



→ Related: Does the cavity affect the spin frequency?

# Shifts and Uncertainties g (in ppt = $10^{-12}$ )

| $\bar{f}_c$ | 147.5 GHz | 149.2 GHz | 150.3 GHz | 151.3 GHz |
|---|---|---|---|---|
| $g/2$ raw | -5.24 (0.39) | 0.31 (0.17) | 2.17 (0.17) | 5.70 (0.24) |
| Cav. shift | 4.36 (0.13) | -0.16 (0.06) | -2.25 (0.07) | -6.02 (0.28) |
| Lineshape | | | | |
| correlated | (0.24) | (0.24) | (0.24) | (0.24) |
| uncorrelated | (0.56) | (0.00) | (0.15) | (0.30) |
| $g/2$ | -0.88 (0.73) | 0.15 (0.30) | -0.08 (0.34) | -0.32 (0.53) |




cavity shifts not a problem

lineshape broadening

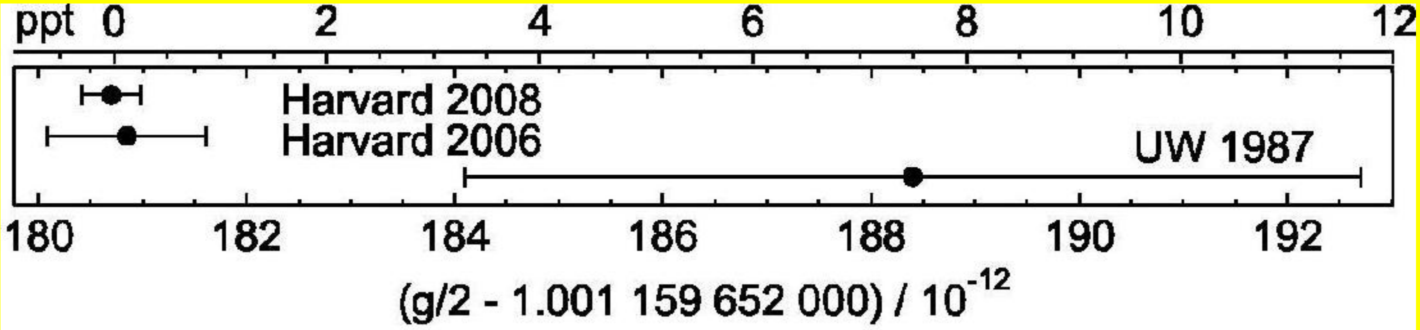
Most precisely measured property of an elementary particle



# Electron Magnetic Moment Measured to 3 x $10^{-13}$

$$\mu/\mu_B = -g/2 = -1.001\,159\,652\,180\,73\,(28) \quad [0.28\ \text{ppt}].$$

$2.8\times10^{-13}$




(improved measurement is underway)


"New Measurement of the Electron Magnetic Moment and the Fine Structure Constant"
D. Hanneke, S. Fogwell and G. Gabrielse,
Phys. Rev. Lett. **100**, 120801 (2008) and arXiv:0801.1134v1 [physics.atom-ph].

"Cavity Control of a Single-Electron Quantum Cyclotron: Measuring the Electron Magnetic Moment"
D. Hanneke, S. Fogwell Hoogerheide and G. Gabrielse,
Phys. Rev. A **83**, 052122 (2011).

# Most Precise Prediction of the Standard Model is the Electron Magnetic Moment

Gabrielse

# Standard Model of Particle Physics Prediction

$$-\frac{«}{«_B} = \frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a_{\text{hadronic}} + a_{\text{weak}}$$

$$\sim = \frac{1}{4®£_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}$$

fine structure constant

$$C_2 = 0.500\,000\,000\,000\,00\ (\text{exact})$$
$$C_4 = -\,0.328\,478\,444\,002\,55\,(33)$$
$$C_6 = 1.181\,234\,016\,815\,(11)$$
$$C_8 = -\,1.909\,7\,(20)$$
$$C_{10} = 9.16\,(0.57).$$

essentially exact

$$a_e^{\text{hadronic}} = 1.677(16) \times 10^{-12}$$

# Lowest Orders are Calculated Analytically (eg. $C_4$)

$$C_4 = A_1^{(4)} + A_2^{(4)}(\frac{m_e}{m_\mu}) + A_2^{(4)}(\frac{m_e}{m_\tau}).$$

$$A_1^{(4)} = \frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4}\zeta(3) - \frac{\pi^2}{2}\ln(2)$$
$$= -0.328\,478\,965\,579\,193\ \ldots$$

7 Feynman diagrams

$$A_2^{(4)}(1/x) = -\frac{25}{36} - \frac{\ln(x)}{3} + x^2[4 + 3\ln(x)] + \frac{x}{2}(1 - 5x^2) \times \left[\frac{\pi^2}{2} - \ln(x)\ln(\frac{1-x}{1+x}) - \mathrm{Li}_2(\mathrm{x}) + \mathrm{Li}_2(-\mathrm{x})\right] + x^4\left[\frac{\pi^2}{3} - 2\ln(x)\ln(\frac{1}{x} - x) - \mathrm{Li}_2(\mathrm{x}^2)\right].$$

1 Feynman diagram


A. Petermann. Fourth Order Magnetic Moment of the Electron. *Helv. Phys. Acta*, 30:407–408, 1957.
C. M Sommerfield. Magnetic Dipole Moment of the Electron. *Phys. Rev.*, 107:328–329, 1957.
C. M. Sommerfield. The Magnetic Moment of the Electron. *Ann. Phys. (N.Y.)*, 5:26–57, 1958.


$\zeta(s)$ is the Riemann zeta function

Zeta[s] in Mathematica

polylogarithm $\mathrm{Li_n(x)} = \sum_{k=1}^{\infty} x^k/k^n$

Log[n,x] in Mathematica


H. H. Elend, *Phys. Rev. Lett.* **20**, 682, (1966). **21**, 720(E) (1966).
M. Passera, *Phys. Rev. D.* **75**, 013002, (2007).

# Lowest Orders are Calculated Analytically (eg. $C_6$)

$$C_k = A_1^{(k)} + A_2^{(k)}\left(\frac{m_e}{m_\mu}\right) + A_2^{(k)}\left(\frac{m_e}{m_\tau}\right) + A_3^{(k)}\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right).$$

$$\begin{aligned} A_1^{(6)} &= \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) - \frac{239}{2160}\pi^4 + \frac{28259}{5184} \\ &+ \frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln(2) + \frac{17101}{810}\pi^2 \\ &+ \frac{100}{3}\left[\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{\ln^4(2)}{24} - \frac{\pi^2\ln^2(2)}{24}\right] \\ &= 1.181\ 241\ 456\ 587\ \ldots. \end{aligned}$$

72 Feynman diagrams

$A_2^{(6)}$ Too many terms to write down

48 Feynman diagrams

S. Laporta, *Nuovo Cim. A.* **106A**, 675 – 683, (1993).
S. Laporta and E. Remiddi, *Phys. Lett. B.* **301**, 440 – 446, (1993).

S. Laporta and E. Remiddi, *Phys. Lett. B.* **379**, 283, (1996).

Numerical check: $A_1^{(6)} = 1.181\ 259\ (40)$ [44].

T. Kinoshita, *Phys. Rev. Lett.* **75**, 4728, (1995).

# $C_8$ and $C_{10}$ are Calculated Numerically

$C_8 = A_1^{(8)} = -1.9144\ (35)$ 891 Feynman diagrams

Estimate till last year: $C_{10} = 0.0\,(4.6)$

Complete calculation completed only very recently

$C_{10} = 9.16\ (0.57)$ 12696 Feynman diagrams

Uncertainties come from numerical integration error.
Depends upon available comptuter time.

T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio. xxx. *Prog. Theor. Exp. Phys.*, 01A:207, 2012.

# Basking in the Reflected Glow of Theorists

$$\frac{g}{2} = 1 + C_2\left(\frac{\sim}{\text{®}}\right)$$

$$+ C_4\left(\frac{\sim}{\text{®}}\right)^2$$

$$+ C_6\left(\frac{\sim}{\text{®}}\right)^3$$

$$+ C_8\left(\frac{\sim}{\text{®}}\right)^4$$

$$+ C_{10}\left(\frac{\sim}{\text{®}}\right)^5$$

$$+ \ldots \text{¢}\, a$$

Remiddi Kinoshita G.G

(also Laporta) (also Nio)

2004

# The Standard Model Predicts the Electron Magnetic Moment

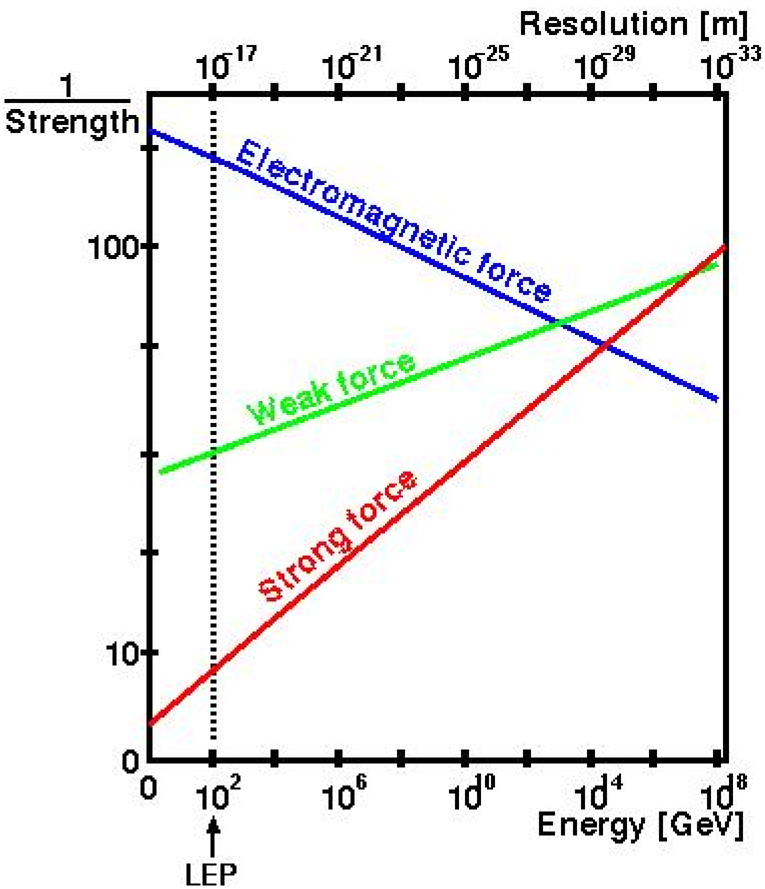
Most Precise Prediction

in terms of the fine structure constant

$$\sim = \frac{1}{4\text{®£}_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}$$

# Fine Structure Constant

$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}$$




- Strength of the electromagnetic interaction (in the low energy limit)
- Important component of our system of fundamental constants
- Increased importance for new mass standard
- Energy scale for atoms

$$\text{Binding energy: } \mathrm{E} \sim \alpha^2 mc^2$$

$$\text{Fine structure: } \Delta\mathrm{E} \sim \alpha^4 mc^2$$

$$\text{Lamb shift: } \Delta\mathrm{E} \sim \alpha^5 mc^2$$

$$\text{Hyperfine structure: } \Delta\mathrm{E} \sim \frac{m}{M}\alpha^4 mc^2$$

Feynman: every theorist should put 137 on their office wall

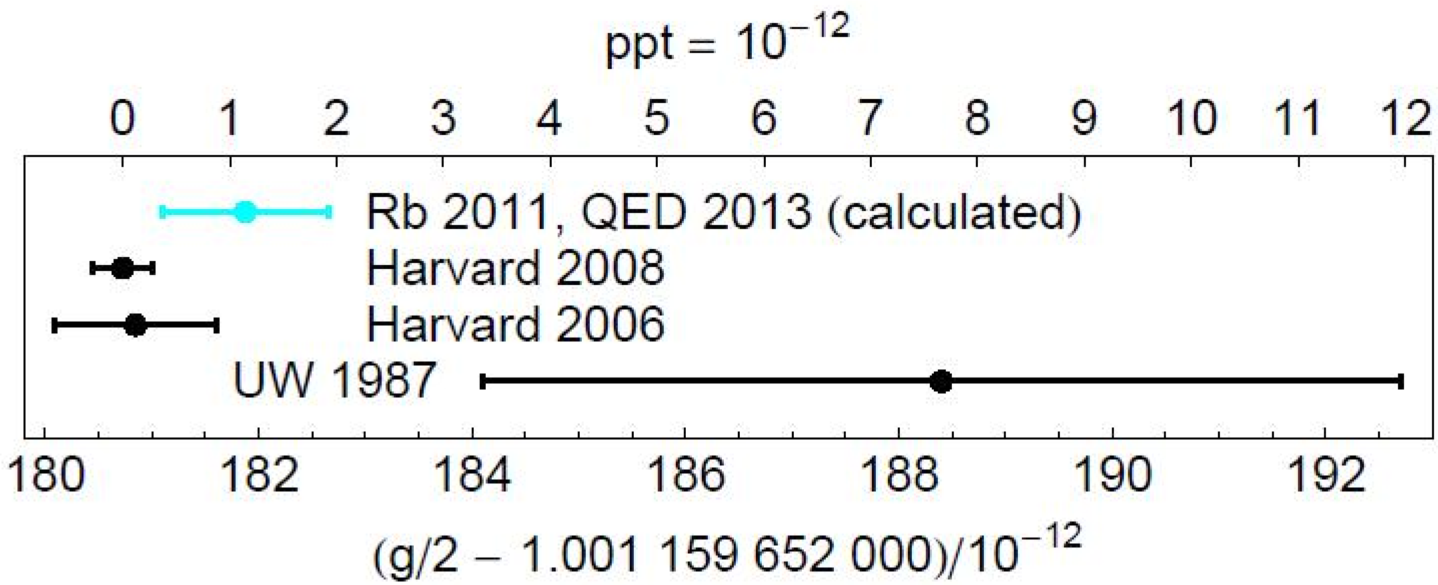
Gabrielse

# (Greatest?) Triumph of the Standard Model

Measured: $\mu/\mu_B = -g/2 = -1.001\,159\,652\,180\,73\,(28) \quad [0.28 \text{ ppt}].$

"Calculated": $\mu/\mu_B = -g/2 = -1.001\ 159\ 652\ 181\ 88\ (78)\ [0.77\ \text{ppt}]$

(Uncertainty from measured fine structure constant)




$$\frac{\mu - \mu(SM)}{\mu} = 0.000\,000\,000\,000\,15\ (82)\ [0.82\ \text{ppt}],$$

$$= 1.5\ (0.8) \times 10^{-12}\ [0.8\ \text{ppt}].$$

# From Freeman Dyson – One Inventor of QED

Dear Jerry,

... I love your way of doing experiments, and I am happy to congratulate you for this latest triumph. Thank you for sending the two papers.

Your statement, that QED is tested far more stringently than its inventors could ever have envisioned, is correct. As one of the inventors, I remember that we thought of QED in 1949 as a temporary and jerry-built structure, with mathematical inconsistencies and renormalized infinities swept under the rug. We did not expect it to last more than ten years before some more solidly built theory would replace it. We expected and hoped that some new experiments would reveal discrepancies that would point the way to a better theory. And now, 57 years have gone by and that ramshackle structure still stands. The theorists … have kept pace with your experiments, pushing their calculations to higher accuracy than we ever imagined. And you still did not find the discrepancy that we hoped for. To me it remains perpetually amazing that Nature dances to the tune that we scribbled so carelessly 57 years ago. And it is amazing that you can measure her dance to one part per trillion and find her still following our beat.

With congratulations and good wishes for more such beautiful experiments, yours ever, Freeman.

# Assume Standard Model Prediction is Correct, Deduce Alpha from Electron g/2

$$-\frac{«}{«_B} = \frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a_{\text{hadronic}} + a_{\text{weak}}$$

$$C_2 = 0.500\,000\,000\,000\,00 \text{ (exact)}$$
$$C_4 = -\,0.328\,478\,444\,002\,55\,(33)$$
$$C_6 = 1.181\,234\,016\,815\,(11)$$
$$C_8 = -\,1.909\,7\,(20)$$
$$C_{10} = 9.16\,(0.57).$$

essentially exact ($C_4$, $C_6$)

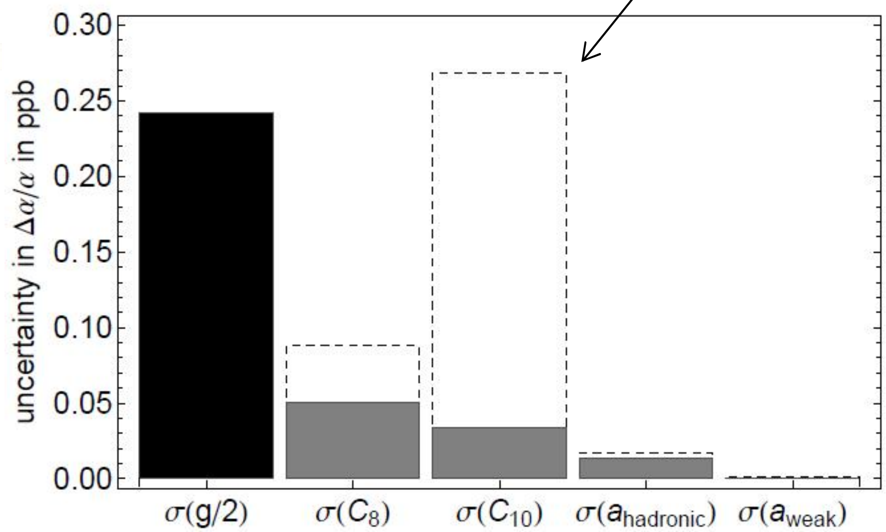
$$a_e^{\text{hadronic}} = 1.677(16) \times 10^{-12}$$

# Most Precise Determination of the Fine Structure Constant (g/2 + QED)

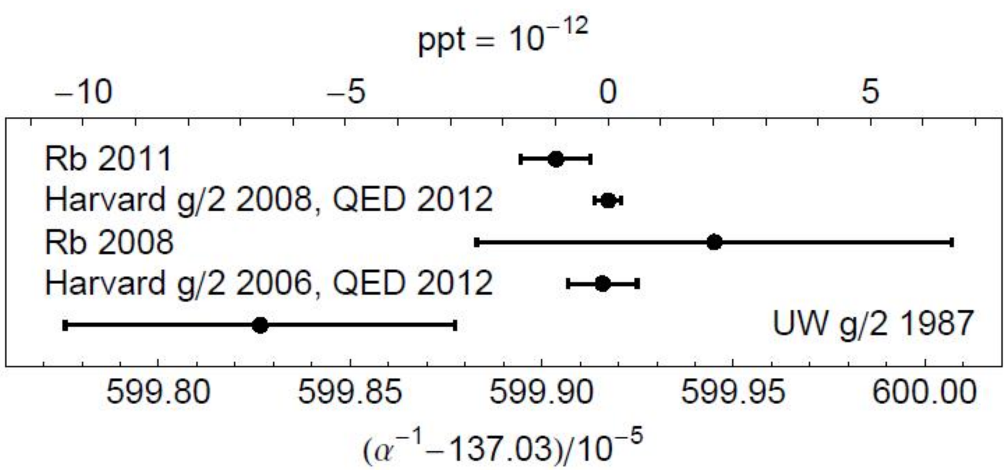
exp`t theory

$$\alpha^{-1} = 137.035\,999\,173\,(33)\,(8) \quad [0.24\text{ ppb}]\,[0.06\text{ ppb}]$$
$$= 137.035\,999\,173\,(34) \quad [0.25\text{ ppb}],$$

before 2012




Determinations of the fine structure constant



# Test for Physics Beyond the Standard Model

$$-\frac{\text{«}}{\text{«}_B}=\frac{g}{2}=1+a_{QED}(\sim)+\text{¢}\,a_{SM:Hadronic+Weak}+\text{¢}\,a_{New\,Physics}$$

# Does the electron have internal structure?

S. J. Brodsky and S. D. Drell. Anomalous Magnetic Moment and Limits on Fermion Substructure. *Phys. Rev. D*, 22:2236 – 2243, 1980.

$$m^* = total\ mass\ of\ particles\ bound\ together\ to\ form\ electron$$

$R<5\times10^{-19}\ m$ $\quad m^*>\frac{m}{\sqrt{\text{¢}\,a}}=360\ GeV/c^2$ $\quad$ limited by the uncertainty in independent α value

$R<2\times10^{-19}\ m$ $\quad m^*>\frac{m}{\sqrt{\text{¢}\,a}}=1\ TeV/c^2$ $\quad$ if our uncertainty was the only limit

**Not bad for an experiment done at 100 mK, but LEP does better**

$R<2\times10^{-20}\ m$ $\quad m^*>10.3\ TeV/c^2$ $\quad$ LEP contact interaction limit

> 20000 electron masses of binding energy

# Compare to Muon

more massive version
of the electron

Gabrielse

# Test for Physics Beyond the Standard Model

$$-\frac{\text{«}}{\text{«}_B} = \frac{g}{2} = 1 + a_{QED}(\sim) + \text{¢}\, a_{SM:Hadronic+Weak} + \text{¢}\, a_{New\,Physics}$$

measure
to a very high
precision

calculate these
to a very high
precision

look for a
disagreement

expected to be 40000 times
larger for a muon
compared to an electron

$$\approx \left(\frac{m_{\text{«}}}{m_e}\right)^2$$

Measure electron moment → test predictions of the standard model
Measure muon moment → look for physics beyond the Standard Model

# Compare to Muon Magnetic Moment

PHYSICAL REVIEW D 73, 072003 (2006)

## Final report of the E821 muon anomalous magnetic moment measurement at BNL

G. W. Bennett,[2] B. Bousquet,[10] H. N. Brown,[2] G. Bunce,[2] R. M. Carey,[1] P. Cushman,[10] G. T. Danby,[2] P. T. Debevec,[8] M. Deile,[13] H. Deng,[13] W. Deninger,[8] S. K. Dhawan,[13] V. P. Druzhinin,[3] L. Duong,[10] E. Efstathiadis,[1] F. J. M. Farley,[13] G. V. Fedotovich,[3] S. Giron,[10] F. E. Gray,[8] D. Grigoriev,[3] M. Grosse-Perdekamp,[13] A. Grossmann,[7] M. F. Hare,[1] D. W. Hertzog,[8] X. Huang,[1] V. W. Hughes,[13,*] M. Iwasaki,[12] K. Jungmann,[6,7] D. Kawall,[13] M. Kawamura,[12] B. I. Khazin,[3] J. Kindem,[10] F. Krienen,[1] I. Kronkvist,[10] A. Lam,[1] R. Larsen,[2] Y. Y. Lee,[2] I. Logashenko,[1,3] R. McNabb,[10,8] W. Meng,[2] J. Mi,[2] J. P. Miller,[1] Y. Mizumachi,[11] W. M. Morse,[2] D. Nikas,[2] C. J. G. Onderwater,[8,6] Y. Orlov,[4] C. S. Özben,[2,8] J. M. Paley,[1] Q. Peng,[1] C. C. Polly,[8] J. Pretz,[13] R. Prigl,[2] G. zu Putlitz,[7] T. Qian,[10] S. I. Redin,[3,13] O. Rind,[1] B. L. Roberts,[1] N. Ryskulov,[3] S. Sedykh,[8] Y. K. Semertzidis,[2] P. Shagin,[10] Yu. M. Shatunov,[3] E. P. Sichtermann,[13] E. Solodov,[3] M. Sossong,[8] A. Steinmetz,[13] L. R. Sulak,[1] C. Timmermans,[10] A. Trofimov,[1] D. Urner,[8] P. von Walter,[7] D. Warburton,[2] D. Winn,[5] A. Yamamoto,[9] and D. Zimmerman[10]

(Muon $(g-2)$ Collaboration)

[1]*Department of Physics, Boston University, Boston, Massachusetts 02215, USA*
[2]*Brookhaven National Laboratory, Upton, New York 11973, USA*
[3]*Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia*
[4]*Newman Laboratory, Cornell University, Ithaca, New York 14853, USA*
[5]*Fairfield University, Fairfield, Connecticut 06430, USA*
[6]*Kernfysisch Versneller Instituut, Rijksuniversiteit Groningen, NL-9747 AA, Groningen, The Netherlands*
[7]*Physikalisches Institut der Universität Heidelberg, 69120 Heidelberg, Germany*
[8]*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[9]*KEK, High Energy Accelerator Research Organization, Tsukuba, Ibaraki 305-0801, Japan*
[10]*Department of Physics, University of Minnesota, Minneapolis, Minnesota 55455, USA*
[11]*Science University of Tokyo, Tokyo, 153-8902, Japan*
[12]*Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo, 152-8551, Japan*
[13]*Department of Physics, Yale University, New Haven, Connecticut 06520, USA*
(Received 26 January 2006; published 7 April 2006)

We present the final report from a series of precision measurements of the muon anomalous magnetic moment, $a_\mu = (g-2)/2$. The details of the experimental method, apparatus, data taking, and analysis are summarized. Data obtained at Brookhaven National Laboratory, using nearly equal samples of positive and negative muons, were used to deduce $a_\mu(\text{Expt}) = 11659208.0(5.4)(3.3) \times 10^{-10}$, where the statistical and systematic uncertainties are given, respectively. The combined uncertainty of 0.54 ppm represents a 14-fold improvement compared to previous measurements at CERN. The standard model value for $a_\mu$ includes contributions from virtual QED, weak, and hadronic processes. While the QED processes account for most of the anomaly, the largest theoretical uncertainty, $\approx 0.55$ ppm, is associated with first-order hadronic vacuum polarization. Present standard model evaluations, based on $e^+e^-$ hadronic cross sections, lie 2.2–2.7 standard deviations below the experimental result.

- More people
- Much bigger budget
- 2500 times less precise
- >2.2 to 2.7 std. dev. disagreement with theory
- Moving from BNL to Fermilab

Hint of break down of the Standard Model???????

Gabrielse

# Could We Check the 2.5 s Disagreement between Muon g Measurement and "Calculation"?

$$-\frac{\text{«}}{\text{«}_B} = \frac{g}{2} = 1 + a_{QED}(\sim) + \text{¢}\, a_{SM:Hadronic+Weak} + \text{¢}\, a_{New\,Physics}$$

$(m_\mu/m_e)^2 \sim 40{,}000$ ← muon more sensitive to "new physics"

$\div 2{,}500$ ← how much more accurately we measure

$\div\ 2$ ← 2σ disagreement is now seen

→ If we can reduce the electron g uncertainty by 8 times more should be able to have the precision to see the 2σ effect (or not)

Also need:
- QED and SM calculations slightly improved
- Independent measurement of α improved by factor of 20

These are large numbers → hard to imagine that this will happen ~~quickly~~

# Can One Measure the Muon Magnetic Moment Using a Trap (Rather than a Storage Ring)?

Particles can be trapped from an accelerator

Extremely precise measurements can be made with these
Trapped particles

→ compare antiproton and proton to 9 parts in $10^{11}$

→ 680-fold improved measured of antiproton magnetic moment

Some challenges

Gabrielse

# Can One Measure the Muon Magnetic Moment Using a Trap (Rather than a Storage Ring)?

Electron and antiproton: live for > months in a trap
Short muon lifetime is a big challenge: 2.2 microseconds

Want big many oscillations before decay
→ big product: (magnetic field) x lifetime

| | | | |
|---|---|---|---|
| Storage ring (BNL): | 1.4 Tesla | 64 «s | 3 GeV |
| In a trap: | 41 Tesla | 2.2 «s | at rest |

Need a serious collaborator

Other considerations:
- Statistics and signal-to-noise:
- Systematics:
- Budget and collaborators

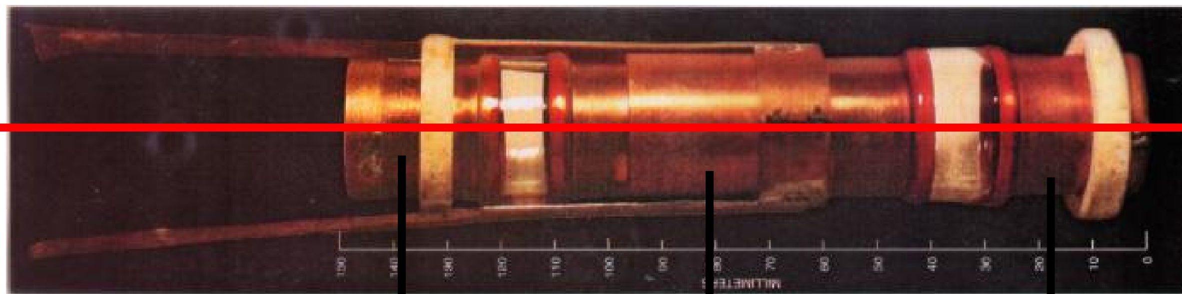
# Particles from an Accelerator Can be Trapped (27 Years Ago We First Trapped Antiprotons)

TRAP Collaboration
at CERN's LEAR

1 cm

magnetic
field

21 MeV
antiprotons



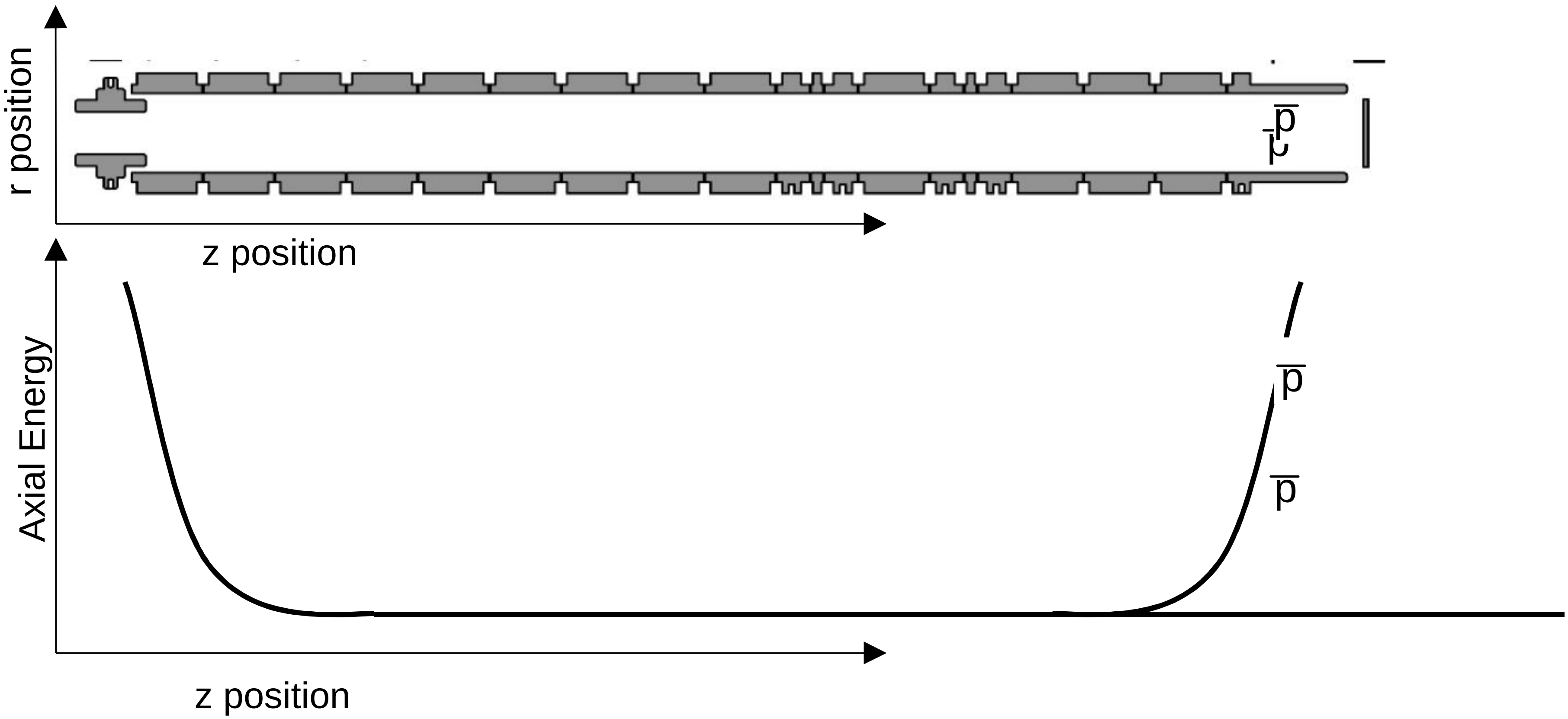
– + –

$10^{-10}$
energy
reduction

- **Slow antiprotons in matter**
- **Capture antiprotons in flight**
- **Electron cooling → 4.2 K**
- **5 x $10^{-17}$ Torr**

Now used by 5 collaborations
at the CERN AD
ATRAP, ALPHA, ASACUSA,
AEGIS, BASE

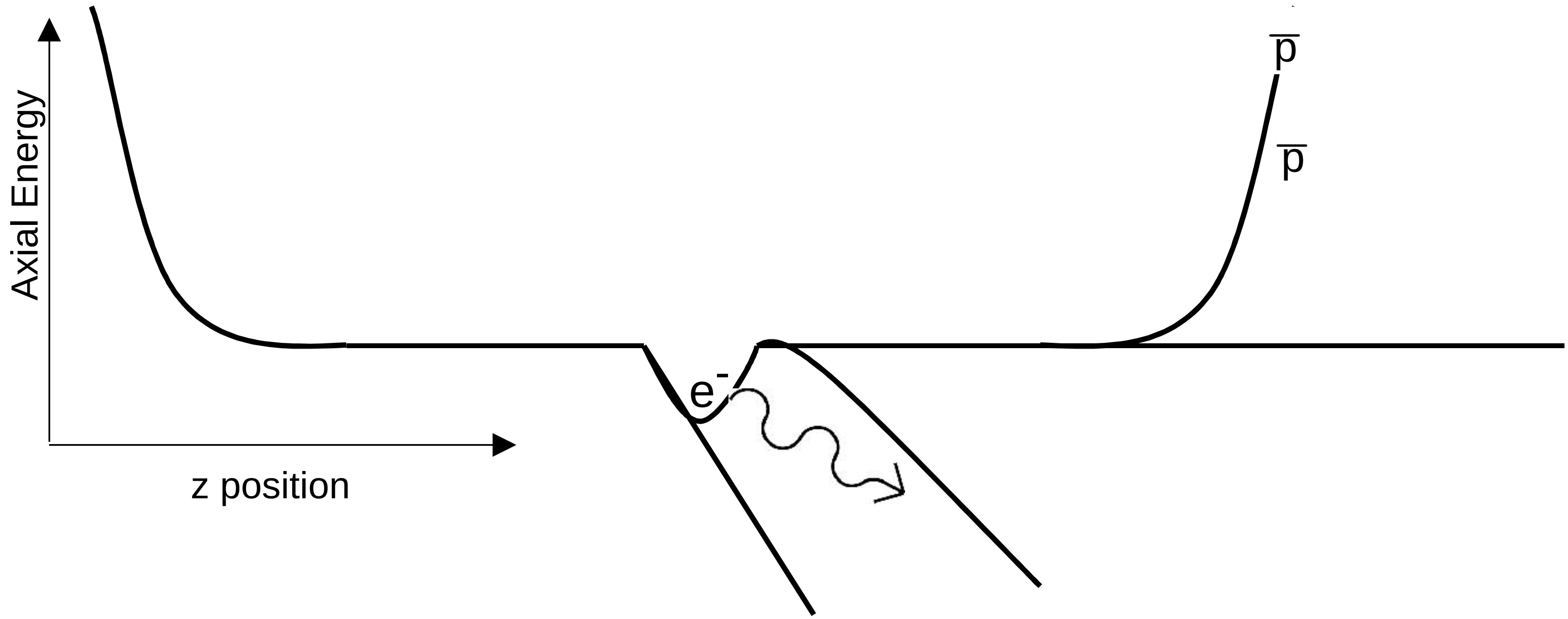
# Antiproton Capture – the Movie




**"First Capture of Antiprotons in a Penning Trap: A KeV Source",**
G. Gabrielse, X. Fei, K. Helmerson, S.L. Rolston, R. Tjoelker, T.A. Trainor, H. Kalinowsky, J. Haas, and W. Kells;
Phys. Rev. Lett. 57, 2504 (1986).

# Electron-Cooling of Antiprotons – in a Trap

- Antiprotons cool via collisions with electrons
- Electrons radiate away excess energy



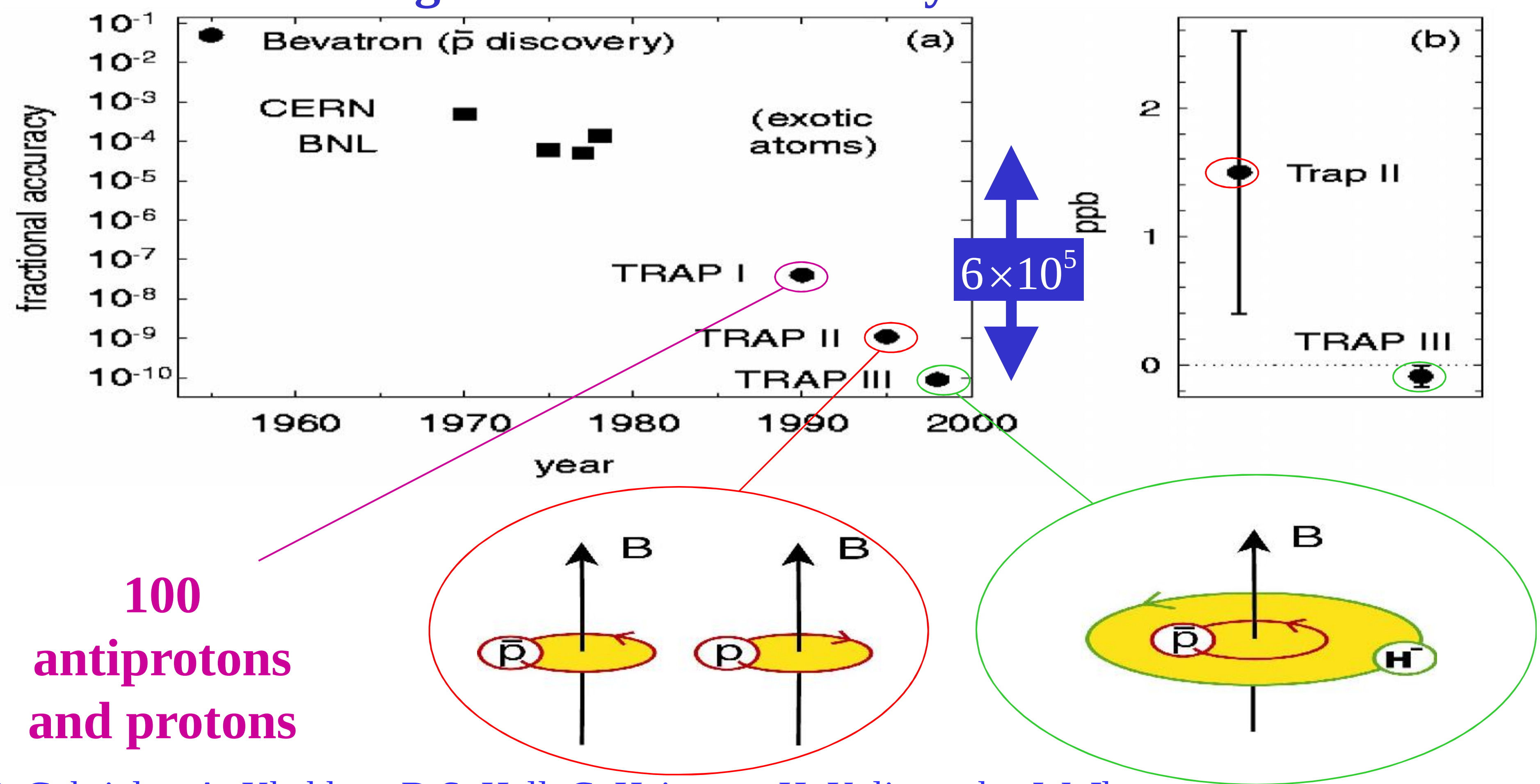

**"Cooling and Slowing of Trapped Antiprotons Below 100 meV",**
G. Gabrielse, X. Fei, L.A. Orozco, R. Tjoelker, J. Haas, H. Kalinowsky, T.A. Trainor, W. Kells;
Phys. Rev. Lett. 63, 1360 (1989).

# We Improved the Comparison of Antiproton and Proton by ~$10^6$

$$\frac{q/m \ \text{(antiproton)}}{q/m \ \text{(proton)}} = -0.99999999991(9)$$

$$9\times10^{-11} = 90\,\text{ppt}$$

**most stringent CPT test with baryons**




**100 antiprotons and protons**

G. Gabrielse, A. Khabbaz, D.S. Hall, C. Heimann, H. Kalinowsky, W. Jhe;
Phys. Rev. Lett. **82**, 3198 (1999).

# Gravity is the Same for Proton and Antiproton to at lease a precision of 1 part per million

Gravitational red shift for a clock: $\Delta\mu / \mu = g\, h / c^2$

→ Antimatter and matter clocks run at different rates if g is different for antimatter and matter

$$\frac{\Delta\mu_c}{\mu_c} = 3(© - 1)\frac{U}{c^2}$$

Hughes and Holzscheiter, Phys. Rev. Lett. 66, 854 (1991).

for tensor gravity (would be 1 for scalar gravity)

grav. pot. energy difference between empty flat space time and inside of hypercluster of galaxies

Experiment: TRAP Collaboration, Phys. Rev. Lett. 82, 3198 (1999).

$$\frac{\Delta\mu_c}{\mu_{\partial c}} < 10^{-10} \quad --> \quad © = 1 \pm\ (< 10^{-6})$$

# First One-Particle Measurement of the Antiproton Magnetic moment

$$\mu_{\overline{p}}/\mu_N = -2.792\,845 \pm 0.000\,012 \qquad [4.4\ \text{ppm}].$$

680 times lower than previous

$$\mu_{\overline{p}}/\mu_p = -\,1.000\,000\ \pm 0.000\,005 \qquad [5.0\ \text{ppm}]$$

$$\mu_{\overline{p}}/\mu_p = -\,0.999\,999\,2 \pm 0.000\,004\,4 \quad [4.4\ \text{ppm}].$$

| Resonance | Source | ppm |
|---|---|---|
| spin | resonance frequency | 2.7 |
| spin | magnetron broadening | 1.3 |
| cyclotron | resonance frequency | 3.2 |
| cyclotron | magnetron broadening | 0.7 |
| total | | 4.4 |

TABLE I. Significant uncertainties in ppm.

# 680 – Fold Improved Precision

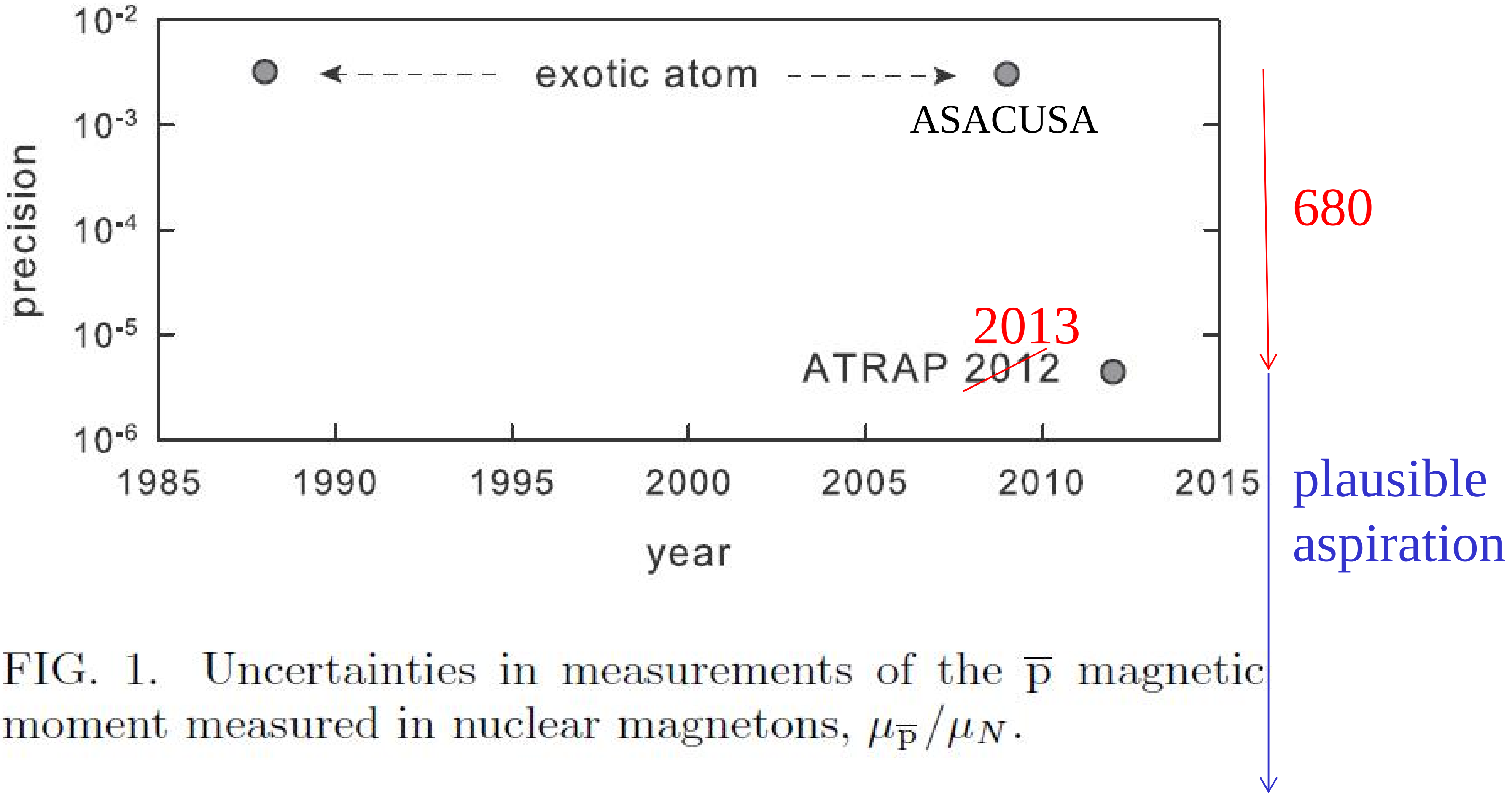
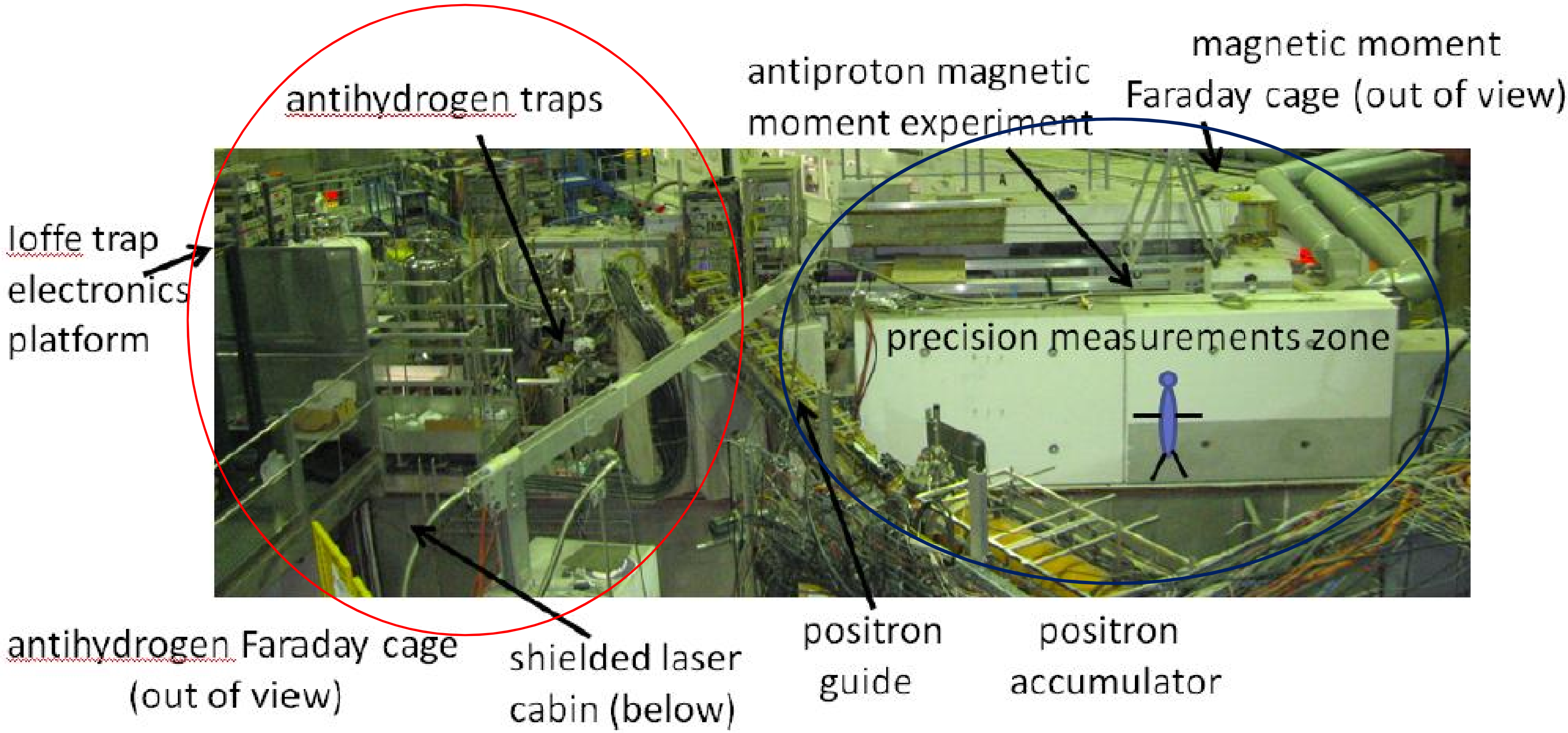



FIG. 1. Uncertainties in measurements of the $\overline{\mathrm{p}}$ magnetic moment measured in nuclear magnetons, $\mu_{\overline{\mathrm{p}}}/\mu_N$.

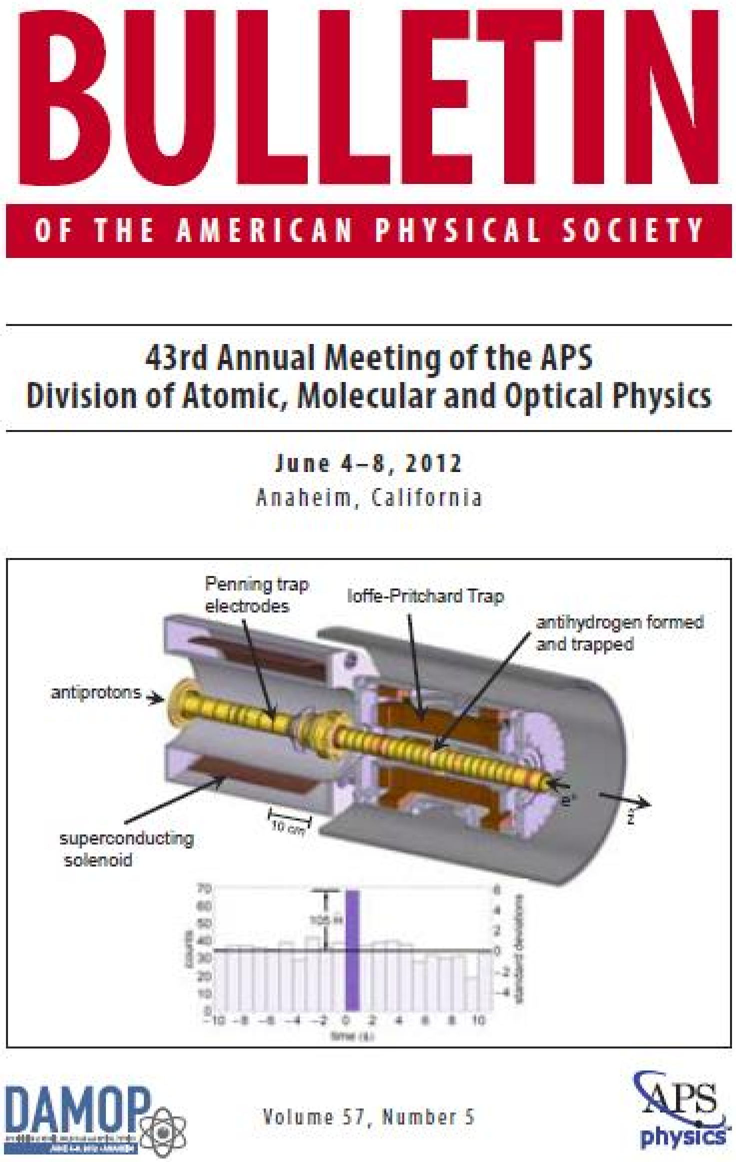
ATRAP, Phys. Rev. Lett. (2013).

# Simultaneous Antihydrogen Experiments and Precision Measurements




ATRAP Experimental Area

# Trapped Antihydrogen in Its Ground State




Used larger antiproton and positron plasmas

→ Much more trapped antihydrogen per trial

→ still not nearly enough

5 +/- 1 ground state atoms simultaneously trapped

ATRAP, "Trapped Antihydrogen in Its Ground State", Phys. Rev. Lett. **108**, 113002 (2012)

# Does the Electron Also Have an Electric Dipole Moments

# Particle EDM Requires Both P and T Violation

Magnetic moment:

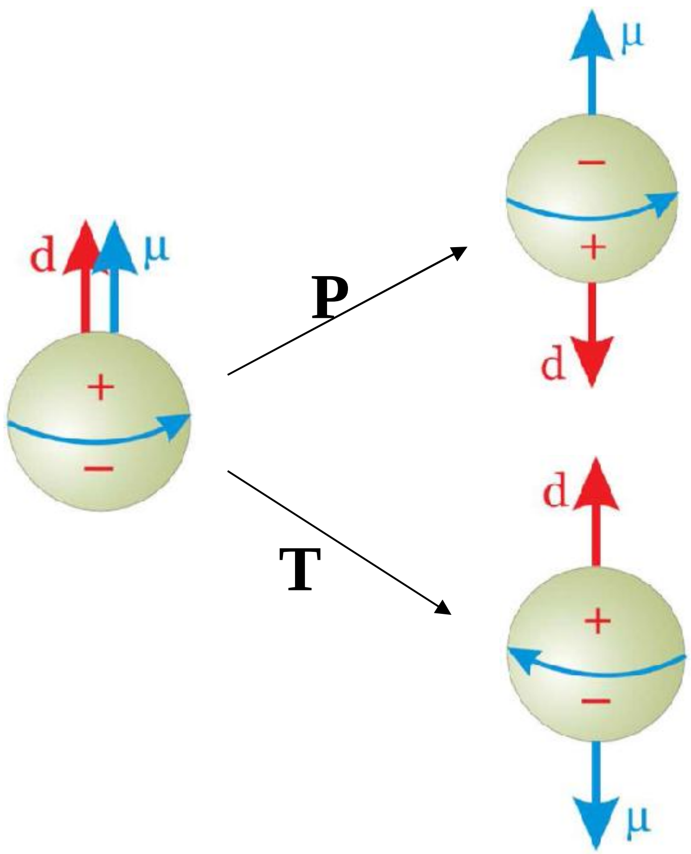
$$\vec{\mu} = -\mu \frac{\vec{S}}{\hbar / 2}$$

(exists and well-measured)

Electric dipole Moment:

$$\vec{d} = -d \frac{\vec{S}}{\hbar / 2}$$

(d is extremely small)




If reality is invariant under parity transformations **P**

→ **d = 0**

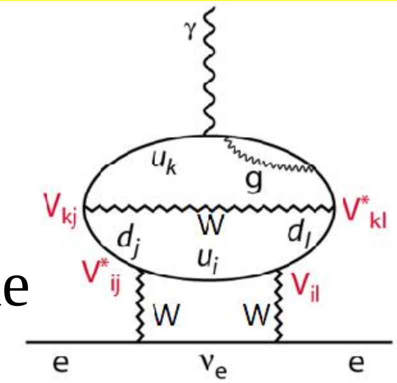
If reality is invariant under time reversal transformations **T**

→ **d = 0**

# Standard Model of Particle Physics → Currently Predicts a Non-zero Electron EDM

Standard model: d ~ $10^{-38}$ e-cm

Too small to measure by orders of magnitude
best measurement: d ~ 2 x $10^{-27}$ e-cm




four-loop level in perturbation theory

M. Pospelov and I. B. Khriplovich, "Electric dipole moment of the W boson and the electron in the Kobayashi-Maskawa model," Sov. J. Nucl. Phys. **53**, 638–640 (1991).

Weak interaction couples quark pairs (generations) $\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} t \\ b' \end{pmatrix}$

CKM matrix relates to d, s, b quarks
(Cabibbo-Kabayashi-Maskawa matrix)

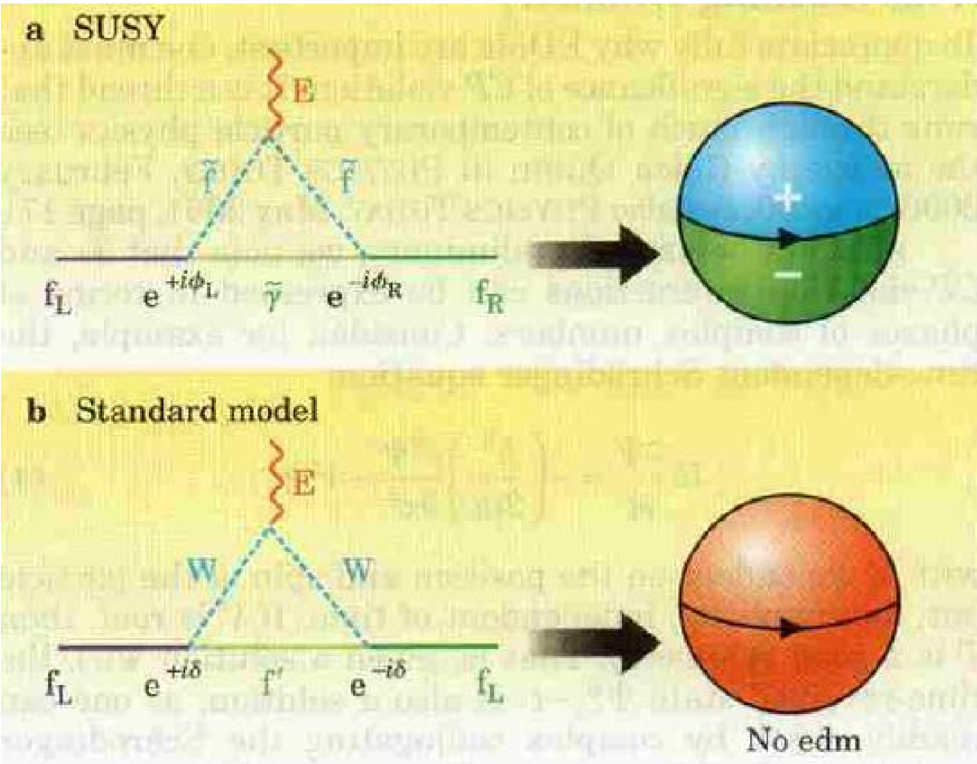
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

almost the unit matrix

$$\begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

# Extensions to the Standard Model → Measureable Electron EDM

An example




Low order contribution → larger moment

Low order contribution → vanishes

From Fortson, Sandars and Barr, Physics Today, 33 (June 2003)

# Does the Electron Also Have an Electric Dipole Moment?

Magnetic moment: $\vec{\mu} = -\mu \dfrac{\vec{S}}{\hbar/2}$

(exists and well-measured)

Electric dipole moment: $\vec{d} = -d \dfrac{\vec{S}}{\hbar/2}$

(d is extremely small)

# No Electron EDM Detected so Far

Commins limit (2002)

$$|d_e| \leq 1.6 \times 10^{-27} e \text{ cm}$$

Tl

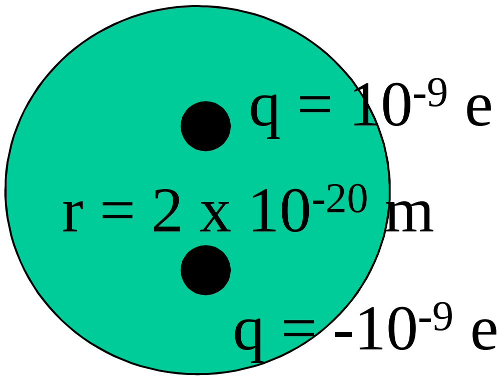
Regan, Commins, Schmidt, DeMille,
Phys. Rev. Lett. **88**, 071805 (2002)

Imperial College (2011)

$$|d_e| < 10.5 \times 10^{-28} e\,\text{cm}$$

YbF

Hudson, Kara, Smallman, Sauer, Tarbutt, Hinds,
Nature **473**, 493 (2011)



Gabrielse

# No Particle EDM Has Yet Been Detected

Electron EDM limit $|d_e| < 10.5 \times 10^{-28} e\,\text{cm}$

Hudson, Kara, Smallman, Sauer, Tarbutt, Hinds,
Nature **473**, 493 (2011)

Neutron EDM limit $|d_n| < 2.9 \times 10^{-26} e\ \text{cm}$

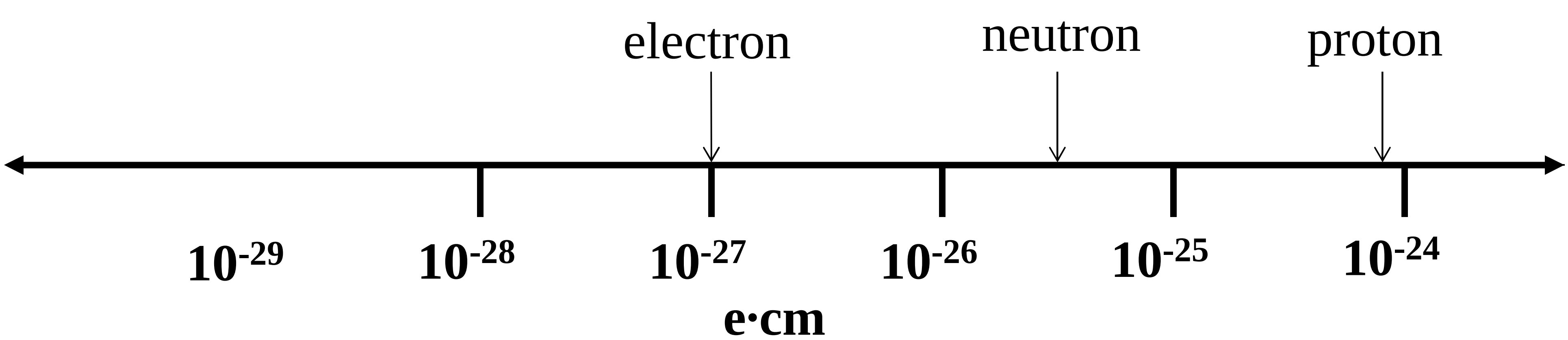
IIL Grenoble,
PRL **97**, 131801 (2006)

Proton EDM limit $|d_p| < 7.9 \times 10^{-25}\ e\,\text{cm}$

Heckel, Fortson, …
PRL **102**, 101601 (2009)

from $|d(^{199}\text{Hg})| < 3.1 \times 10^{-29}\ e\,\text{cm}$

also sets $|d_n| < 5.8 \times 10^{-26}\ e\,\text{cm}$



# Measuring a Particle EDM (Basic Idea)

**Magnetic moment:** $\vec{\mu} = -\mu \dfrac{\vec{S}}{\hbar/2}$

(exists and well-measured)

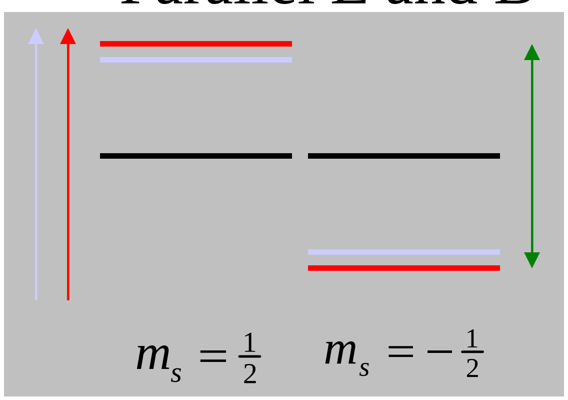
**Electric dipole moment:** $\vec{d} = -d \dfrac{\vec{S}}{\hbar/2}$

(d is extremely small)

**External fields gives energy shifts:** $\Delta H = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E} \quad = \quad (\mu\, B + d\, E) \dfrac{S_z}{\hbar/2}$

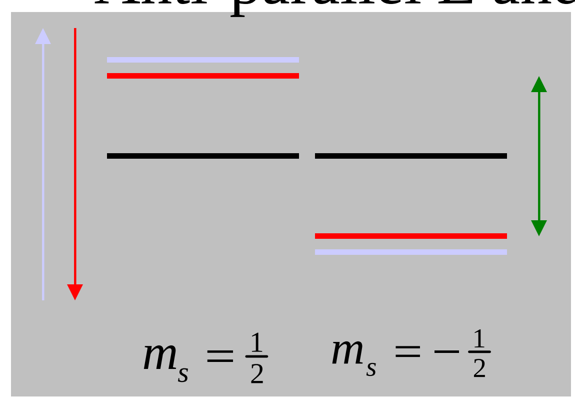
$\vec{B} = B\,\hat{z} \qquad \vec{E} = E\,\hat{z}$

Parallel E and B




$\Delta E(B,E) = 2(\mu\, B + d\, E)$

Anti-parallel E and B




$\Delta E(B,E) = 2(\mu\, B - d\, E)$

$$\Delta E(B,E) - \Delta E(B,-E) = 4\,d\,E$$

# Cannot Use Electric Field Directly on an Electron or Proton

Simple E and B can be used for neutron EDM measurement
(neutron has magnetic moment but no net charge)

Electric field would accelerate an electron out of the apparatus

Electron EDM are done within atoms and molecules
(first molecular ion measurement is now being attempted)

# Schiff Theorem – for Electron in an Atom or Molecule

Schiff (1963) – no atomic or molecular EDM (i.e. linear Stark effect)

- from electron edm
- nonrelativistic quantum mechanics limit

Sandars (1965) – can get atomic or molecular EDM (i.e. linear Stark effect)

- from electron edm
- relativistic quantum mechanics
- get significant enhancement (D >> d) for large Z

Commins, Jackson, DeMille (2007) – intuitive explanation for escape from Schiff

→ Lorentz contraction of the electron EDM viewed in lab frame

Schiff, Phys. Rev. Lett. **132**, 2194 (1963);

Sandars, Phys. Rev. Lett. **14**, 194 (1965); *ibid* **22**, 290 (1966).

Commins, Jackson, DeMille, Am. J. Phys. **75**, 532 (2007).

# Advanced Cold-Molecule Electron EDM

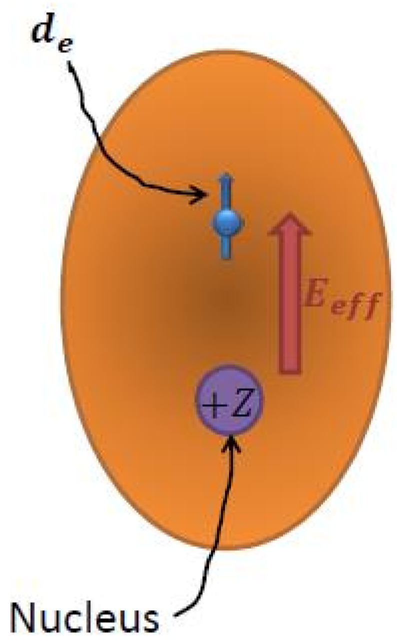
ACME

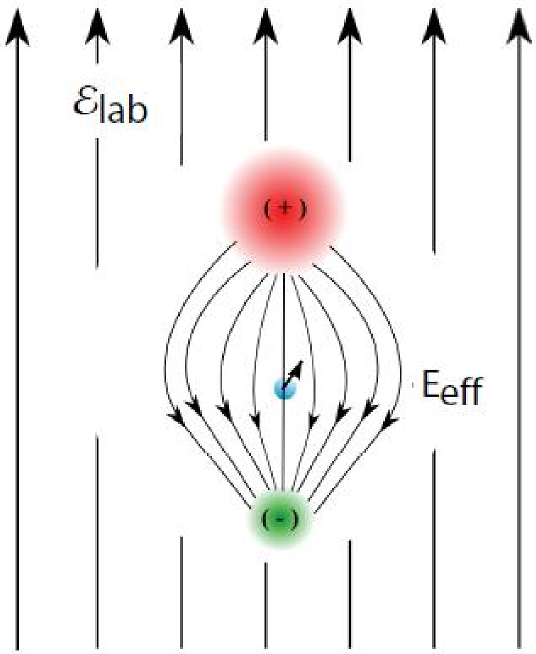
| **Harvard University** | **Yale University** |
| --- | --- |
| John Doyle Group<br>Gerald Gabrielse Group | David DeMille Group |

Nearing publication of a new result for the electron EDM

Funding from NSF

# Why Use a Molecule?
# → To Make Largest Possible Electric Field on Electron




**Tl atom (best EDM limit till YbF)**

$E_{lab} = 123$ kV/cm → $E_{eff} = 72$ MV/cm

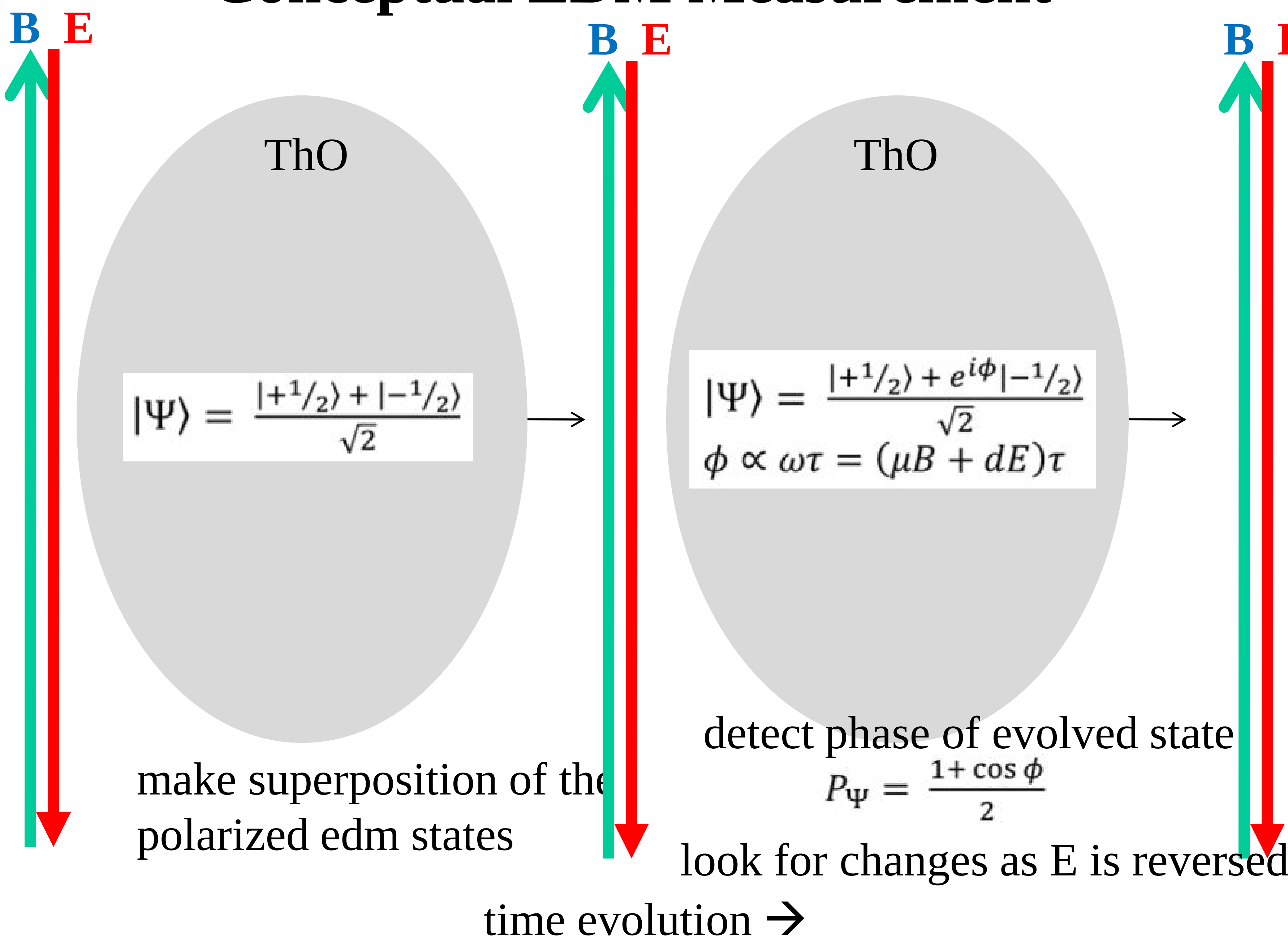
**ThO molecule**

$E_{lab} = 100$ V/cm → $E_{eff} = 100$ GV/cm

Molecule can be more easily polarized
using nearby energy levels with opposite parity
(not generally available in atoms)

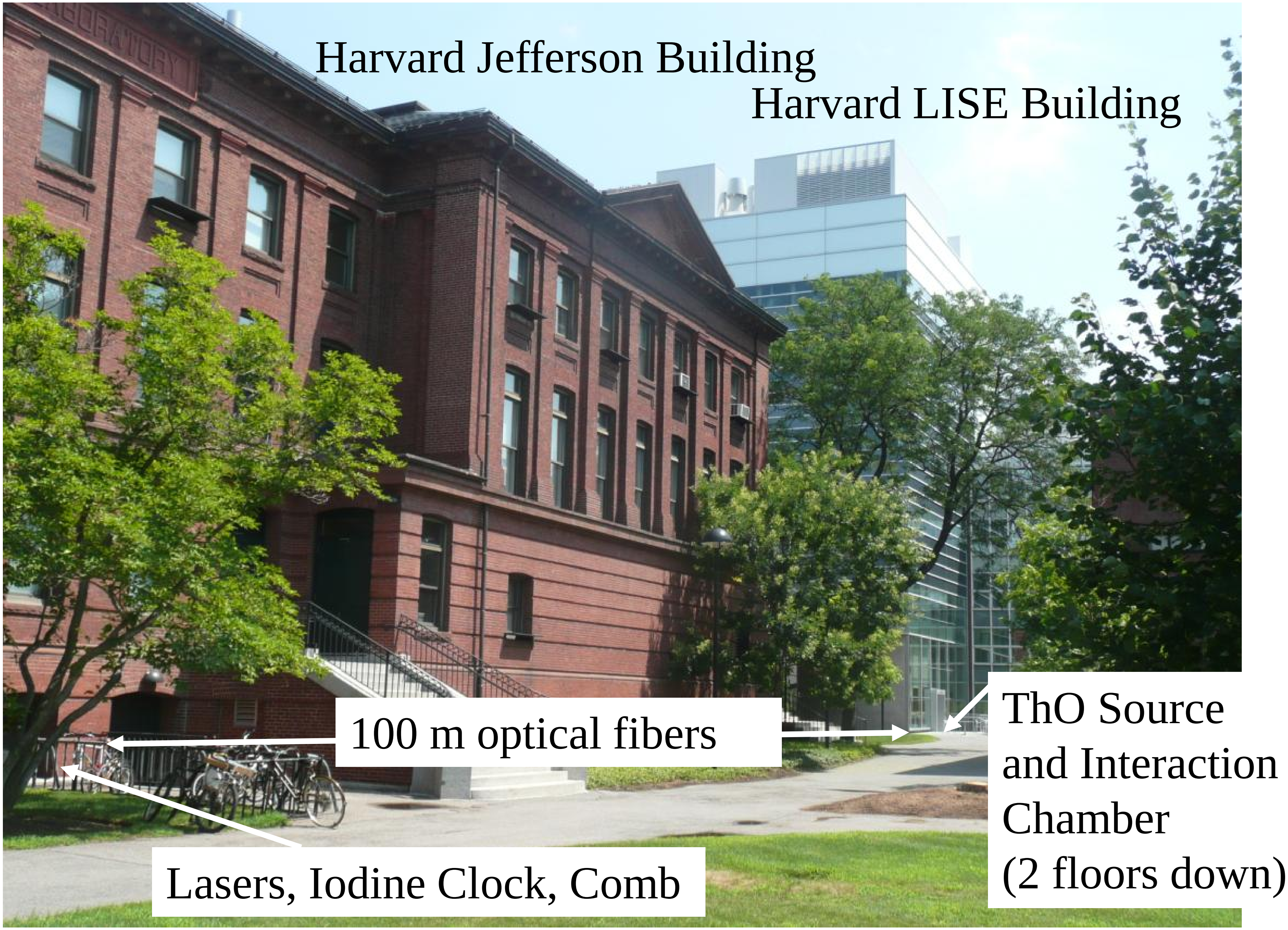
Gabrielse

# Conceptual EDM Measurement



Gabrielse

# Experiment in Two Labs – 100 Meters Separated

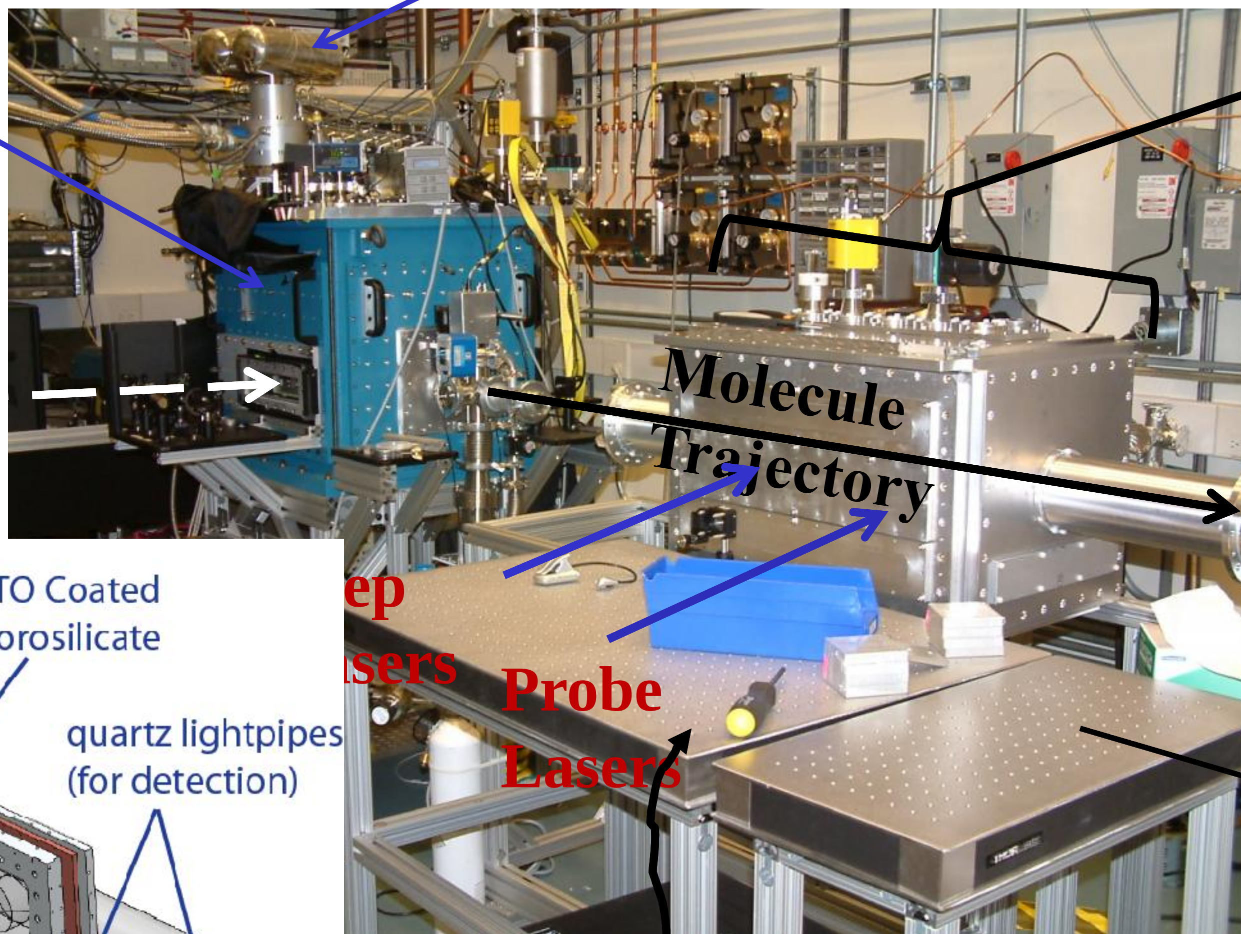


# Apparatus in Lab 1

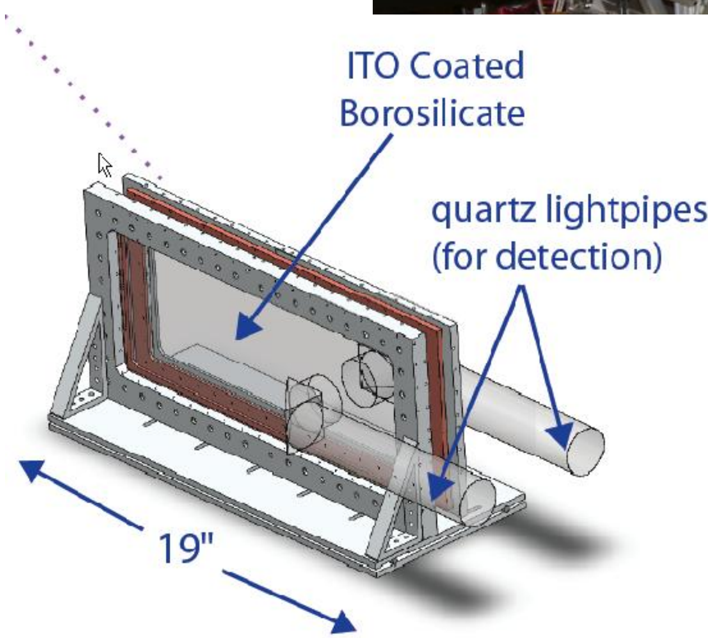
Pulse Tube Cooler

Molecu
lar
Beam
Source

Pulsed
YAG




“Interacti
on
Region”:
E-field
plates
inside, B-
field
shields
and coils
outside




Lasers
100m
away

laser fiber docks
to be located

Gabrielse

# Statistical Comparison of ACME and Imperial

Statistical sensitivity:

$$\delta d_e = \frac{1}{2 E_{eff}} \frac{\hbar}{\tau \sqrt{\dot{N} T}}$$

internal electric field ($E_{eff}$); coherence time ($\tau$); counting rate ($\dot{N}$); integration time ($T$)

7 x 1.7 x 2 = 24

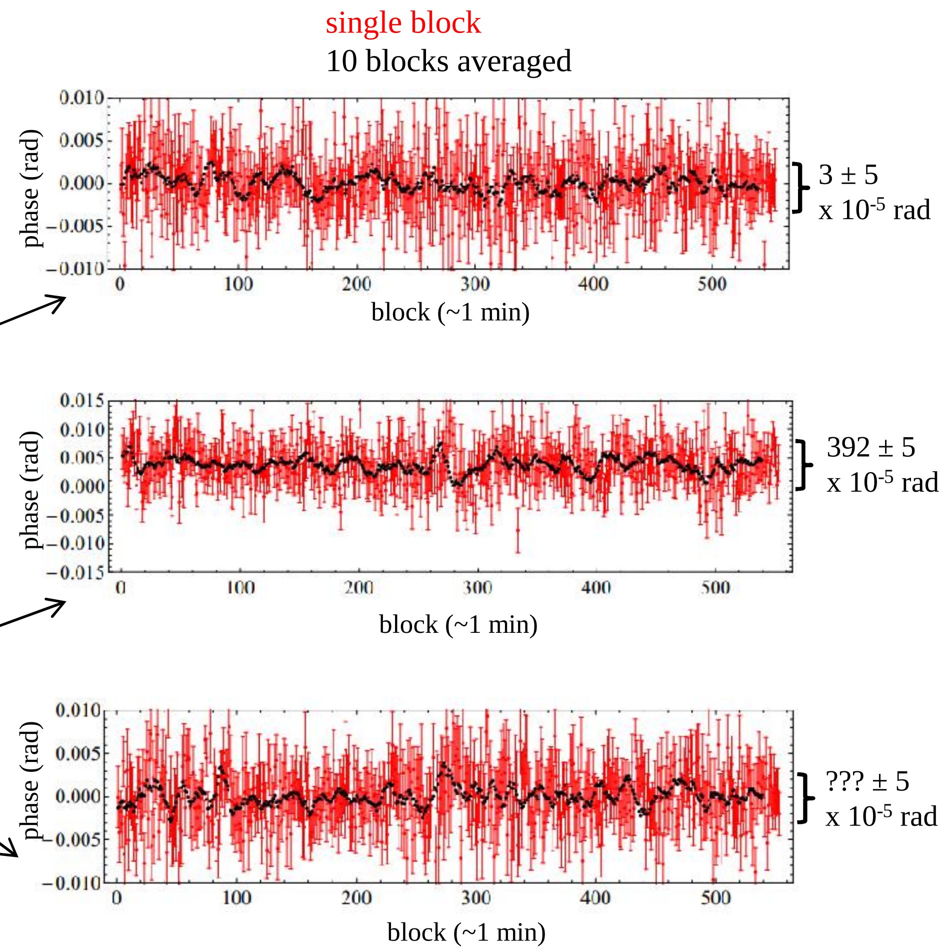
| | ACME ThO | Imperial YbF | |
|---|---|---|---|
| Effective E field | 100 GV/cm | 14 GV/cm | 7 |
| Coherence time | 1.1 ms | 0.65 ms | 1.7 |
| Photons/second* | 1000 x 50 =50,000 | 500 x 25 =12,500 | $4^{1/2}$ |
| Precision in same time: | 1 | 24 | |
| Time for same precision | 1 | $(24)^2 \sim 600$ | |

*Our molecule source is much more intense, allowing us to use a metastable state rather than the ground state (as needed in ThO)

# Total Phase Equation:

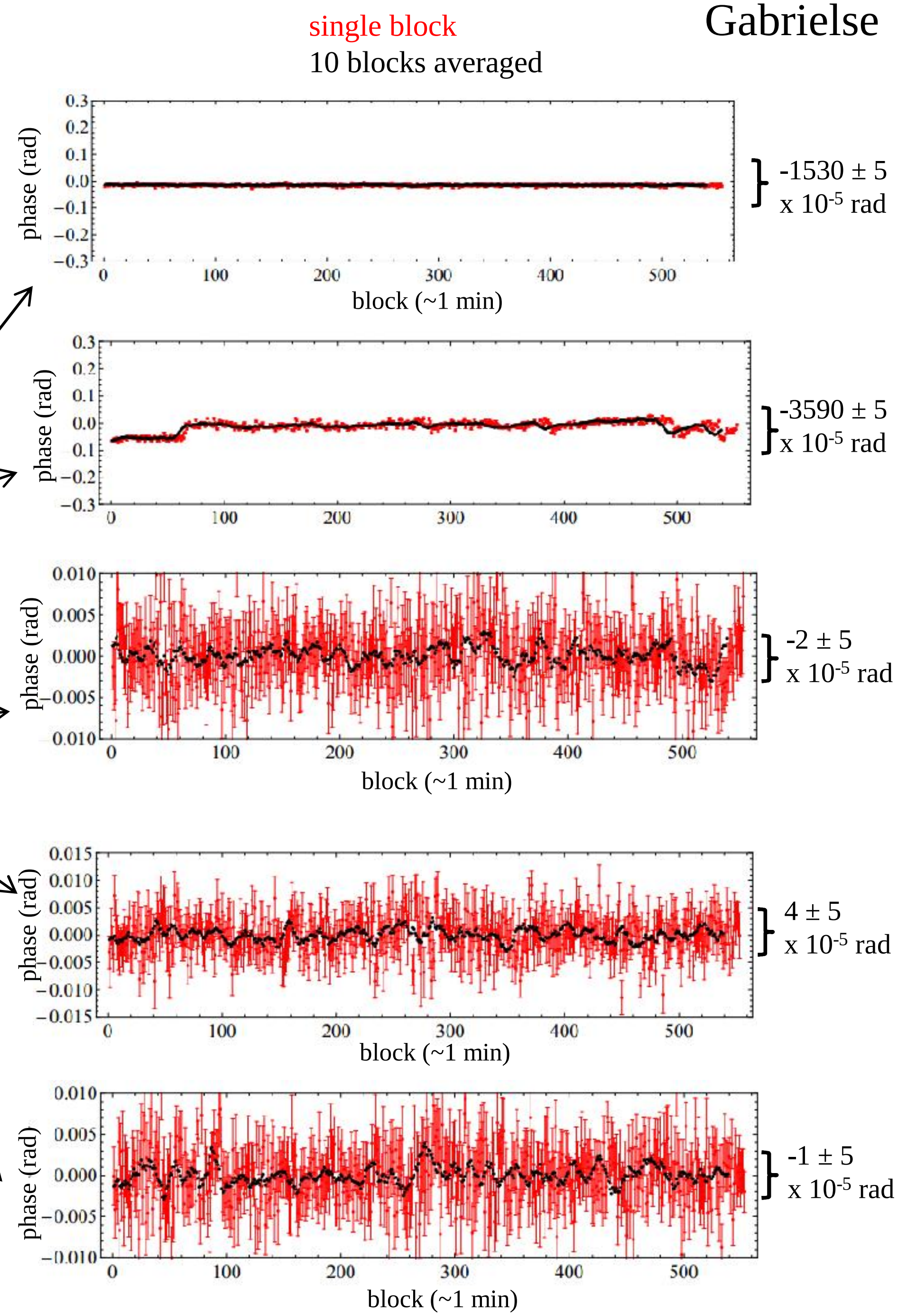
$$\phi(N,E,B)$$
$$= \left(g + \Delta g\,|\hat{N}|\right)\left(B_0 + B_{nr}|\hat{B}| + B_{leak}|\hat{E}|\right)\mu_H \tau + d_e\, E_{eff}|\hat{E}||\hat{N}|$$

| Parity sum (***N*•*E*•*B***) | **Derived quantities** |
|---|---|
| + + + | $B_{nr}$ $g$ m ŧ $\theta_{nr}$ |
| + + - | $B_0$ $g$ m t |
| + - + | $B_{leak}$ $g$ m t |
| + - - | *0* |
| - + + | $B_{nr}$ $\Delta g$ m t |
| - + - | $B_0$ $\Delta g$ m t |
| - - + | $d_e$ $E_{eff}$ t |
| - - - | $B_0$ $\eta$ $E_{nr}$ m t |

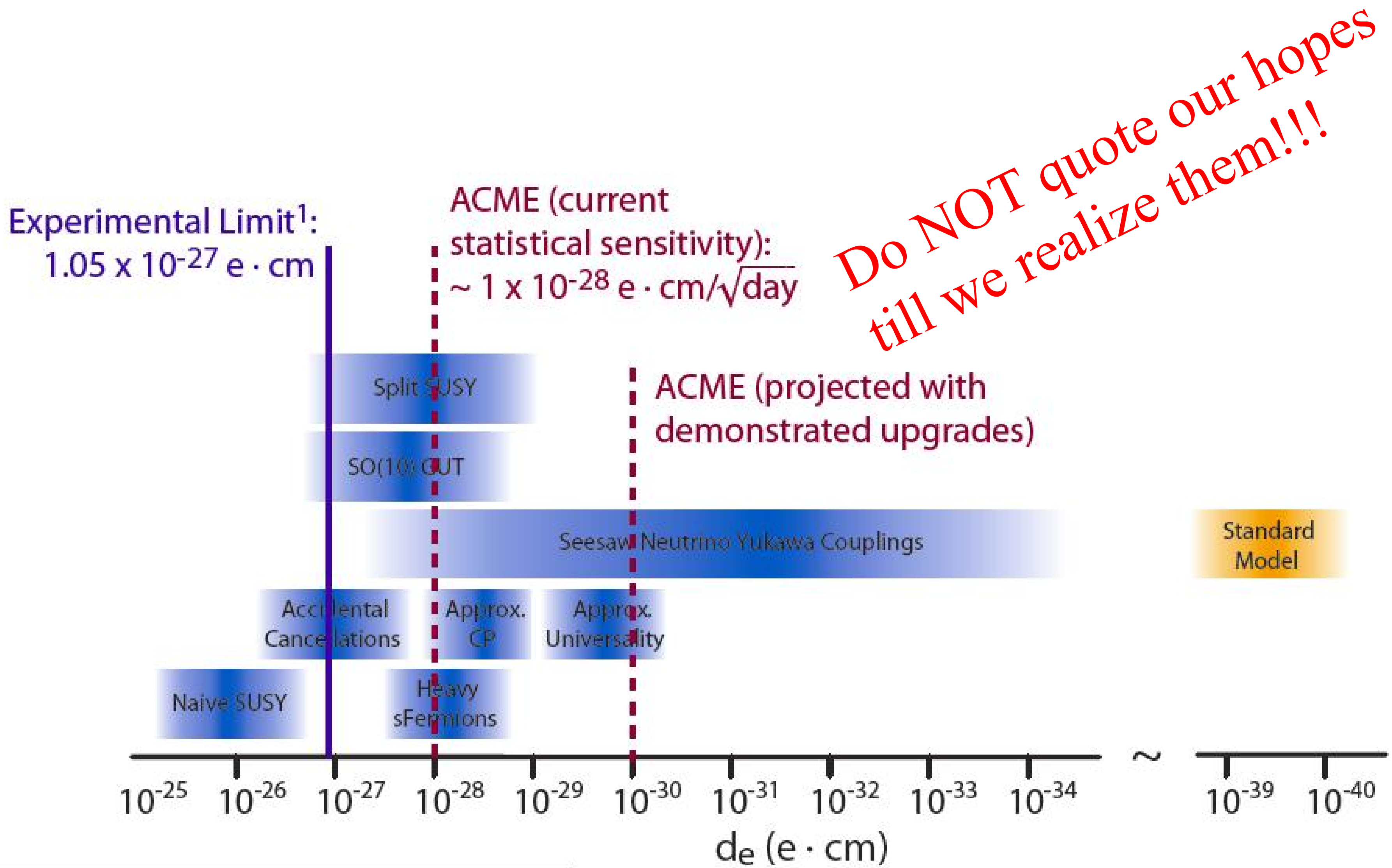


Gabrielse

| Parity sum (*N*•*E*•*B*) | Derived quantities |
|---|---|
| + + + | $B_{nr}\, g$ m t $\theta_{nr}$ |
| + + - | $B_0\, g$ m t |
| + - + | $B_{leak}\, g$ m t |
| + - - | $0$ |
| - + + | $B_{nr}\, \Delta g$ m t |
| - + - | $B_0\, \Delta g$ m t |
| - - + | $d_e\, E_{eff}$ t |
| - - - | $B_0\ \eta\ E_{nr}$ m t |



# New Electron EDM Measurement Close




[1] J. J. Hudson et al, Nature 473, 493-496 (2011)

# Summary

Electron magnetic moment measured to 3 parts in $10^{13}$

-- most precisely measured property of an elementary particle

-- Standard Model predicts this value nearly as precisely

-- Arguable the most stringent test of the Standard Model and its greatest triumph

-- We are preparing a new experiment → higher precision
→ positron and electron

Does the electron also have an electric dipole moment?

-- not detected yet

-- much more precise new measurement soon to be released

-- either the electron edm will be soon be discovered or the extensions to the Standard Model will require more fine tuning

## Summary

# How Does One Measure g to 7.6 Parts in $10^{13}$?

# → Use New Methods

first measurement with these new methods

- One-electron quantum cyclotron
- Resolve lowest cyclotron as well as spin states
- Quantum jump spectroscopy of lowest quantum states
- Cavity-controlled spontaneous emission
- Radiation field controlled by cylindrical trap cavity
- Cooling away of blackbody photons
- Synchronized electrons probe cavity radiation modes
- Trap without nuclear paramagnetism
- One-particle self-excited oscillator

# We Intend to do Better

**Stay Tuned – The methods have just been made to work all together relatively recently**

- With time we can utilize them better
- Some new ideas are being tried (e.g. cavity-sideband cooling)
- Lowering uncertainty by factor of 10

**Spin-off Experiments**

- Antiproton magnetic moment → million-fold improvement is coming?
- Compare positron and electron g-values to make best test of CPT for leptons
- Muon? Worth looking into is enough <keV muons are available