Loop quantum gravity, twisted geometries and twistors

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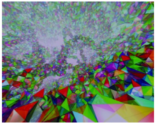
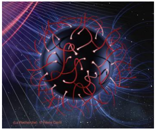
Loop quantum gravity

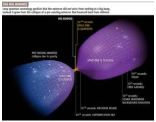
- LQG is a background-independent quantization of general relativity
  The metric tensor is turned into an operator acting on a (kinematical) Hilbert space whose states are Wilson loops of the gravitational connection



- Main results and applications
  - discrete spectra of geometric operators
  - physical cut-off at the Planck scale:
    no trans-Planckian dofs, UV finiteness
  - local Lorentz invariance preserved
  - black hole entropy from microscopic counting
  - singularity resolution, cosmological models and big bounce



- Many research directions
  - understanding the quantum dynamics
  - recovering general relativity in the semiclassical limit
    some positive evidence, more work to do
  - computing quantum corrections, renormalize IR divergences
  - contact with EFT and perturbative scattering processes
  - matter coupling, ...



Main difficulty: Quanta are exotic
different language: QFT $\longrightarrow$ General covariant QFT, TQFT with infinite dofs

Outline

Brief introduction to LQG

From loop quantum gravity to twisted geometries

Twistors and LQG

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Twistors and LQG

A case for background independence

Usual quantization scheme for general relativity:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Spin-2 massless particle, 2 dofs per point
Non-renormalizable $\Rightarrow$ useful as effective field theory, not fundamental

Could the reason be the brutal split into background + perturbations?

Background-independent approaches:

- quantize the full metric tensor, $g \mapsto \hat{g}$
- $\hat{g}$ acting on a suitably defined Hilbert space

To identify the Hilbert space, we look at the canonical analysis of GR

A case for background independence

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Other background-independent approaches:

- Causal dynamical triangulations
- Quantum Regge calculus
- Causal sets

LQG is the one more rooted in a canonical/phase space formalism with explicit geometric spectra

GR phase space: ADM vs Ashtekar variables

Old approach (Wheeler-De Witt, '70s):

- phase space of GR described by ADM variables: 3d intrinsic and extrinsic geometry

$$\{g_{ab}(x), K^{cd}(y)\} = \delta^{cd}_{(ab)}\delta^{(3)}(x,y), \qquad K \sim \dot{g}$$

- attempt quantization as

$$\Psi[g_{ab}], \qquad \hat{g} = g, \qquad \hat{K} = -i\hbar\frac{\delta}{\delta g}$$

- Some interesting results, but many problems!
  not even scalar product well defined
  minisuperspace models can be built, no singularity resolution
  e.g. Hartle-Hawking-Herzog '00s

Key development (Ashtekar '86, Ashtekar-Barbero '90)

- new formulation of the theory's phase space as that of a SU(2) gauge theory

$$\{A^i_a, E^b_j\} = \delta^i_j\delta^b_a\delta^{(3)}(x,y)$$

- simply a canonical transformation from ADM variables: $(g_{\mu\nu} = e^I_\mu e^J_\nu \eta_{IJ})$

$$(g_{ab}, K^{ab}) \Rightarrow (E^a_i = ee^a_i, A^i_a = \Gamma^i_a(E) + \gamma K^i_a)$$

  truly a family thereof: γ Barbero-Immirzi parameter

Action principle

The canonical transformation is related to a different action principle for general relativity, where the connection is taken as an independent variable

- $g_{\mu\nu} \mapsto (g_{\mu\nu}, \Gamma^{\rho}_{\mu\nu})$
- *Three* lowest dim. operators:

$$\frac{\Lambda}{G}\sqrt{-g}, \quad \frac{1}{G}\sqrt{-g}g^{\mu\nu}R_{\mu\nu}(\Gamma), \quad \frac{1}{\gamma G}\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}(\Gamma)$$

- γ Immirzi parameter: classically irrelevant in the absence of torsion

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- In tetrad formalism (Einstein-Cartan-Holst action)

$$S\,(e,\omega) = 2\Lambda\int e - \int \mathrm{Tr}[e\wedge e\wedge \star F(\omega)] - \frac{1}{\gamma}\int \mathrm{Tr}[e\wedge e\wedge F(\omega)]$$

Remark on the local gauge group: the full Lorentz group is redundant, can be reduced reduced phase space described by an $\mathrm{SU}(2)$ connection $A = \Gamma(E) + \gamma K, \quad \gamma \in \mathbb{R}$

More on the Immirzi parameter

- canonical transformation from ADM variables $(E^a_i, A^i_a = \Gamma^i_a(E) + \gamma K^i_a)$
- becomes classically relevant when sources of torsion are present:

$$\Gamma = \Gamma(g) + C(\psi), \qquad \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}(\Gamma) \sim C^2 \neq 0$$

- e.g. extra 4-fermions effective interaction, A. Perez and C. Rovelli '07, Minic et al. '08

$$c_1 A^2 + c_2 V^2 + c_3 A \cdot V \qquad \text{(axial and vector currents)}$$

  can lead to CP-odd effects (for non-minimal coupling)
- compatible with supersymmetry (requires an analogue extra term in the RS action)
- perturbative running (Holst action non-renormalizable) $\beta_\gamma(\tilde{G}) = \frac{4}{3\pi}\gamma\tilde{G}$
  D. Benedetti and S, JHEP'11
- LQG non-perturbative quantization

Area gap in LQG: $A_{\min} = \frac{\sqrt{3}}{2}\gamma\ell_P^2$

Dynamics in phase space: Dirac's algorithm

- Phase space of an $\mathrm{SU}(2)$ gauge theory, $P = T^*A, \qquad \{A, E\} = \delta$
- Three sets of constraints:

1.	Gauss law	$G_i = D_a E_i^a = 0$
2.	Spatial diffeomorphisms	$H^a = 0$
3.	Hamiltonian constraint	$H = 0$

- Dirac's algorithm:

$$\boxed{P \xrightarrow{G=0} P_0 \xrightarrow{H^\mu = 0} P_{\text{phys}}}$$

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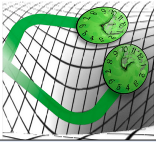
- Hamiltonian is just a linear combination of these constraints:
  Fully constrained system!
  Dynamics encoded in the constraints as a consequence of general covariance

Where is evolution in time?

There is no such thing: there is no universal time in GR, so no sense of general evolution in time. The physical time is singled out by the solution itself

Rovelli, Partial observables



e.g. Muller-Peters-Chu, Nature '10
NIST, Science Express '13

The problem of time

No absolute time variable identifiable from the start:
You need to solve the dynamics before you can talk about time: only on the physical phase space solution of $H^\mu = 0$ there is a physical evolution, whose explicit form will depend on which variable is chosen as "clock"

No general solution since we do not know the general solution of Einstein's equations

In many physical applications, this "problem of time" is solved by the presence of a clear privileged time:

- cosmology with a perfect fluid $\longrightarrow$ cosmic time as a clock
- asymptotic flatness $\longrightarrow$ asymptotic Poincaré symmetry

GR can be profitably used without being stuck by this more fundamental issue

Same story at the quantum level, manifest in the Wheeler-De Witt equation

$$i\partial_t \Psi = \hat{H}\Psi \ \longrightarrow \ \hat{H}\Psi = 0$$

More on the Ashtekar phase space: the holonomy-flux algebra

- to avoid use of background-dependent Gaussian measures, $\{A, E\} = \delta$ is smeared along *distributional* test fields **along a graph** Γ:

	smearing	A on 1d paths	E on 2d surfaces
(A, E)	$\longrightarrow$	$g = \mathcal{P}\exp\int_l A$	$X = \int_{l*}(gE)$

- on each link $(g, X) \in \mathrm{SU}(2) \times \mathbb{R}^3 \cong T^*\mathrm{SU}(2)$ (cf. LGT)
- holonomy-flux algebra $\{g^A{}_B, g^C{}_D\} = 0, \quad \{X_i, g\} = \tau_i g, \quad \{X_i, X_j\} = \epsilon_{ijk} X_k$
- graph phase space

$$P_\Gamma = T^*\mathrm{SU}(2)^L$$

- gauge invariance : $G_n = \sum_{l \in n} X_l = 0$, generates $\mathrm{SU}(2)$ transf. at the nodes

$$S_\Gamma = T^*\mathrm{SU}(2)^L // \mathrm{SU}(2)^N$$

- gauge invariant states: Wilson loops

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- graph phase space quantization

$$P_\Gamma = T^*\mathrm{SU}(2)^L \quad \rightarrow \quad L_2[SU(2)^L] \quad \text{(Wigner matrices)}$$

- gauge invariance : $G_n = \sum_{l \in n} X_l = 0$, generates $\mathrm{SU}(2)$ transf. at the nodes

$$S_\Gamma = T^*\mathrm{SU}(2)^L /\!/ \mathrm{SU}(2)^N \quad \rightarrow \quad \mathcal{H}_\Gamma = L_2[SU(2)^L / SU(2)^N]$$

- gauge invariant states: Wilson loops

Quantization

- Kinematical Hilbert space of a $\mathrm{SU}(2)$ lattice gauge theory

$$\mathcal{H}_\Gamma = L_2[\mathrm{SU}(2)^L], \qquad \mathcal{H} := \lim_{\Gamma\mapsto\infty} \mathcal{H}_\Gamma = L_2[\overline{\mathcal{A}}, d\mu_{\mathrm{AL}}]$$

  back-indep measure induced by the Haar measure graph by graph,

- Three sets of constraints:

1. Gauss law		$\hat{G}_i = \widehat{D_a E^a_i} = 0$
2. Spatial diffeomorphisms		$\hat{H}^a = 0$
3. Hamiltonian constraint		$\hat{H} = 0$

- Dirac's algorithm:

$$\boxed{\mathcal{H} \xrightarrow{\hat{G}=0} \mathcal{H}_0 \xrightarrow{\hat{H}^\mu = 0} \mathcal{H}_{\mathrm{phys}}}$$

- Schrödinger equation reduces to Wheeler-De Witt equation $\hat{H}\psi = 0$

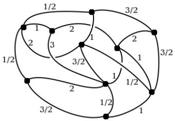
State of the art:

- $\mathcal{H}_0$ well defined, complete basis known, scalar product, operator spectra explicit
- $\hat{H}$ well defined but ambiguites present!
  Dynamics mostly studied using a path-integral-like framework: *spin foam formalism*

Gauge-invariant Hilbert space: Spin networks and quantum geometry

$\mathcal{H}_0$ is spanned by Wilson loops, but not independent observables!
(Mandelstam identities)

orthogonal basis: reduce the Wilson loops using $\mathrm{SU}(2)$'s recoupling theory
spin network states $\quad |\Gamma, j_l, i_n\rangle$
(• graph Γ; • spin j_l on each link; • an intertwiner i_n on each node)



diagonalizes geometric operators such as surface areas

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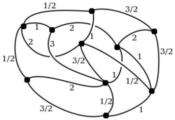


Quantum geometry

geometric operators turn out to have discrete spectra
with minimal excitations proportional to the Planck length

- spins j_l are quantum numbers for areas of surfaces dual to links

  quanta of area $\qquad A(\Sigma) = \gamma\hbar G \sum_{l\in\Sigma} \sqrt{j_l(j_l+1)}$

- intertwiners i_n are quantum numbers for volumes of regions dual to nodes

  quanta of volumes $\qquad V(R) = (\gamma\hbar G)^{3/2} \sum_{n\in R} f(j_e, i_n)$

Quantum geometry

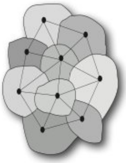
Operators $\mathcal{O}(\hat{E}, \hat{A})$

- spins $\mapsto$ quanta of area
- intertwiners $\mapsto$ quanta of volumes
- Non-commutativity of certain metric observables, e.g. angles between surfaces

Three aspects of quantum geometry:

- discrete eigenvalues
- non-commutativity
- graph structure

QFT	LQG
$\mathcal{F} = \bigoplus_n \mathcal{H}_n$	$\mathcal{H} = \bigoplus_\Gamma \mathcal{H}_\Gamma$
$\vert n, p_i, h_i\rangle \to$ quanta of fields	$\vert\Gamma, j_e, i_v\rangle \to$ quanta of space



Dynamics

The constraints can be imposed through a path integral

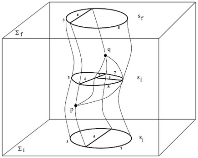
$$K[g_1, g_2] = \int_{g_2}^{g_1} \mathcal{D}g\, e^{iS[g]}, \qquad \hat{H} K[g_1, g_2] = 0$$

$$\begin{array}{ccc} \mathcal{H}_0 & \ni & |s\rangle \\ \downarrow & \hat{H} = 0 & \downarrow \\ \mathcal{H}_{phys} & \ni & |s\rangle_{phys} \end{array}$$

Projector

$$\langle s|s'\rangle_{phys} := \langle s|\hat{P}|s'\rangle \qquad \hat{P} \sim \text{“} \int d\lambda e^{i\lambda H} \text{”}$$

$$= \sum_{\sigma | \partial\sigma = s \cup s'} A_\sigma(s, s')$$



σ: spin foam: a 2-complex decorated with group irreps

Analogy with electromagnetism

Propagation kernel for Maxwell's theory: (or linearized YM)

$$\begin{aligned} K[A_1, A_2, T] &= \int_{\substack{A_1\\A_2}} \mathcal{D}A\, e^{iS[A]}, \qquad \hat{G}K[A_1,A_2,T]=0 \\ &= N(T)\exp\{\frac{i}{2}\int d^3p\, \frac{p}{\sin pT}\, [(|A_1^T|^2+|A_2^T|^2)\cos pT - 2A_1^T\cdot A_2^T]\} \\ &= \sum\!\!\!\!\!\!\int_{n,s_i,p_i} \exp\{-i\sum_n E_n T\}\bar{\Psi}_{n,s_i,p_i}[A_1]\Psi_{n,s_i,p_i}[A_2] \end{aligned}$$

- naturally decomposes into gauge-invariant energy eigenstates
- $T_E \mapsto -\infty$ projects on vacuum state
- same true in linearized gravity, T asymptotic time Mattei-Rovelli-Testa-S '05

Situation quite different in a general covariant field theory!

$$K[g] = \int_g \mathcal{D}g\, e^{iS[g]}, \qquad \hat{H}K[g]=0$$

- Physical states have zero energy
- No meaning of T as a coordinate:
  it is the value of $g_{\mu\nu}$ along the timelike boundary to determine the elapsed time

$\Rightarrow$ general boundary formalism Oeckl '04, Rovelli '05

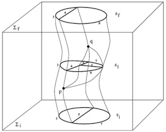
Spin foam amplitudes

In LQG,

$$K[g] = \int_g \mathcal{D}g \, e^{iS[g]}, \qquad \hat{H} K[g] = 0$$

becomes a sum of histories of spin networks: a *spin foam*

a 2-complex defined by vertices, edges and faces labelled by $\mathrm{SU}(2)$ irreps



$$\langle W | s \rangle_{phys} = \sum_{\sigma | \partial\sigma = s} \sum_{\{j_f\}} \prod_f \hat{A}(j_f) \prod_v A_v(j_f)$$

a sum over histories of spin networks, providing amplitudes to them

$$A_\sigma = \int \prod_l dg_l \prod_f A(g_l) = \sum_{\{j_f\}} \prod_f A(j_f) \prod_v A_v(j_f)$$

- same structure of the partition function of LGT (character expansion)
  $\Big(A_f = (2j_f + 1) \exp\{-\beta S_f[j_f, a]\}, \quad A_v \text{ fixed by gauge invariance} \Big)$
- but no background metric structure, no fixed lattice spacing
  a priori no continuum limit to take, ∞ dofs recovered by summing over all graphs

Fixing the weights: EPRL model

- Can be derived in a number of different ways, based on a definition of GR as a constrained topological theory
- Defined in $3+1$ dimensions, Lorentzian signature
- Typical case: 2-complex a simplicial manifold made up of 4-simplices
- Partition function (Engle-Pereira-Rovelli-Livine '09, Freidel-Krasnov '09, Livine-S '09)

$$Z_\Delta = \sum_{j_f} \prod_f (2j_f + 1) \prod_v A_v^{EPRL}(j_f, \gamma)$$

Key result in support of its validity: Barrett et al. '10

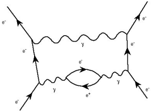
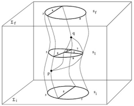
$$A_v^{EPRL}(j_f, \gamma) \quad \underset{j \mapsto \infty}{\longrightarrow} \quad \exp\{iS_{Regge}\}$$

$\Rightarrow$ sum over histories of spin networks weighted by exponentials of discretized GR

No Planck scale built in: $S_{Regge} = \sum_{t \in \Delta} \gamma j_t \theta_t = \frac{1}{\ell_{\mathrm{P}}^2} \sum_{f \in v} A_f \theta_f$
(From the canonical quantum theory, $A_f = \ell_{\mathrm{P}}^2 \gamma j_f$)

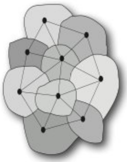
Seems a bit magic: How does the geometry emerges from just spins and Lorentz irreps?

QFT vs LQG

	kinematics	dynamics
QFT:	$\lvert n, p_i, h_i \rangle$	
	quanta: momenta, helicities, etc.	Feynman diagrams
	observables n: # of quantum particles	perturbative expansion degree of the graph $\Downarrow$ order of approximation desired
LQG:	$\lvert \Gamma, j_l, i_n \rangle$	
	quanta: areas and volumes	spin foams
	a fuzzy discrete geometry	histories of fuzzy geometries

Fuzzy discrete geometries

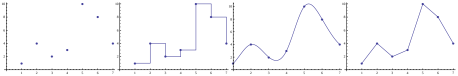
What is exactly this quantum geometry?



$$\mathcal{H} = \bigoplus_{\Gamma} \mathcal{H}_\Gamma, \qquad |\Gamma, j_e, i_v\rangle$$

- Consider a single graph Γ, and the associated Hilbert space $\mathcal{H}_\Gamma$.
- This truncation captures only a finite number of degrees of freedom of the theory, thus (semiclassical) states in $\mathcal{H}_\Gamma$ do not represent smooth geometries.
- Can they represent a *discrete* geometry, approximation of a smooth one on Γ?

Can we interpret $\mathcal{H}_\Gamma = \bigoplus_{j_e} \left[\bigotimes_v \mathcal{H}_v \right]$ as the quantization of a space of discrete geometries?



- A natural guess is Regge calculus, a lattice version of GR adapted to triangulations
- As it turns out, this is too rigid to capture the degrees of freedom of spin networks: the correct answer is a generalization of Regge calculus, called **twisted geometries**
- Covariantly described by **twistors**

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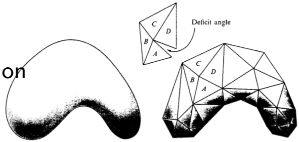
Twistors and LQG

Preamble: Regge calculus



- Spacetime approximated with a simplicial triangulation

- Each 4-simplex locally flat: flat metric described uniquely by the edge lengths

$$g_{\mu\nu} \mapsto \ell_e$$

- Curvature: a *deficit angle* associated with the triangles

$$\epsilon_t(\ell_e) = 2\pi - \sum_{\sigma \in t} \theta_t^\sigma(\ell_e)$$

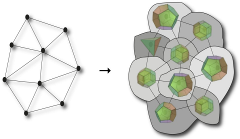
- Dynamics: Regge action

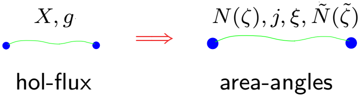
$$S_R[\ell_e] = \sum_t A_t(\ell_e)\epsilon_t(\ell_e) \ \sim \ \int \sqrt{g}R$$

- 3d boundary geometry: edges lengths and dihedral angles among tetrahedra

LQG and twisted geometries

On a fixed graph $\mathcal{H}_\Gamma = L_2[\mathrm{SU}(2)^L]$ $\quad P_\Gamma = T^*\mathrm{SU}(2)^L$, holonomies and fluxes
represents a truncation of the theory to a finite number of degrees of freedom
These can be interpreted as discrete geometries, called **twisted geometries** Freidel and S '10





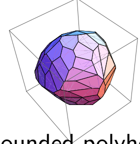
Each classical holonomy-flux configuration on a fixed graph can be visualized as a collection of adjacent polyhedra with extrinsic curvature between them

- consider a lattice adapted to the graph: a cellular decomposition dual to Γ
- the holonomy-flux variables can be mapped to a set of data (a "twisted geometry") describing the 3-cells as piecewise flat polyhedra, glued together on each link
- this structure gives rise to a notion of discrete 3-geometry, with both intrinsic and extrinsic non-trivial curvatures, similar but more general than Regge calculus

Two results:

1. polyhedra from fluxes
2. covariant description using twistors

Minkowski theorem



X_l fluxes on Γ, Gauss law around each node: $G_n = \sum_{l \in n} X_l = 0$

F (non-coplanar) closed normals identify a *unique* flat, convex and bounded polyhedron, with areas gives by their norms, and dihedral angles given by their scalar product

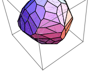
- Explicit reconstruction procedure: $X_l \mapsto$ edge lengths, volume, adjacency matrix
  Lasserre '83, E. Bianchi, P. Doná and S, '10
- Interesting open mathematical problems (i.e. analytic formula for the volume, etc.)
- the space of shapes $\mathcal{S}_F$ given by varying the unit normals keeping the areas fixed is a symplectic manifold of dimensions $2(F-3)$
  Kapovich-Millson
- quantizing this space one gets precisely the space of $\mathrm{SU}(2)$ invariants!

$$\mathcal{S}_F \qquad \longrightarrow \qquad \mathrm{Inv}\Big[\otimes_l V^{j_l}\Big]$$

$$X_l \cdot X_{l'} \qquad \longrightarrow \qquad J_l \cdot J_{l'}$$

An $\mathrm{SU}(2)$-invariant state (an "intertwiner") **is** a fuzzy polyhedron!

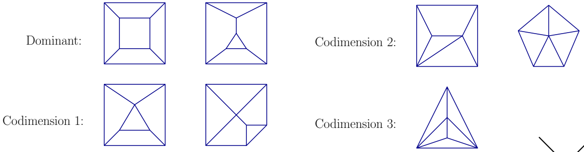
Geometry of polyhedra



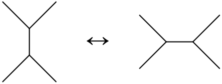
Explicit reconstruction procedure: $X_l \mapsto$ edge lengths, volume, adjacency matrix

For $F > 4$ there are many different combinatorial structures, or *classes*

$F = 6$

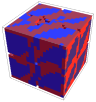


- The classes are all connected by 2-2 Pachner moves (they are all tessellations of the 2-sphere)



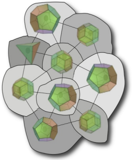
It is the configuration of normals to determine the class

- The phase space $\mathcal{S}_F$ can be mapped in regions corresponding to different classes.
  – *Dominant classes* have all 3-valent vertices.
  [maximal n. of vertices, $V = 2(F-2)$, and edges, $E = 3(F-2)$]
  – *Subdominant classes* are special configurations with lesser edges and vertices, and span measure zero subspaces.
  [lowest-dimensional class for maximal number of triangular faces]



3d slice of $\mathcal{S}_6$, cuboids blue

Geometry on the graph

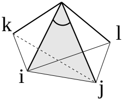


Each classical holonomy-flux configuration
on a fixed graph describes
a collection of polyhedra and their embedding

- each polyhedron is locally flat: curvature emerges at the faces, as in Regge calculus
- extrinsic geometry encoded in the parallel transport g between adjacent polyhedra

> the structure defines a piecewise discrete metric, in general **discontinuous**:
> two neighbouring polyhedra share a face with same area but *different shape*

- shape-matching conditions can be written explicitly
  Dittrich and S '08
  This subcase describes ordinary Regge calculus



kinematical LQG (twisted geometries)	$\xrightarrow{\text{shape-matchings}}$	Regge geometries

Outline

Brief introduction to LQG

From loop quantum gravity to twisted geometries

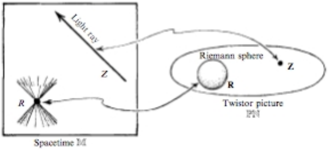
Twistors and LQG

Twistor space

$$Z^\alpha = \begin{pmatrix} \omega^A \\ \bar\pi_{\dot A} \end{pmatrix} \in \mathbb{T} := \mathbb{C}^2 \times \bar{\mathbb{C}}^{2*}, \qquad s = \tfrac{1}{2}\bar Z_\alpha Z^\alpha = \mathrm{Re}(\pi\omega), \qquad \Theta_{\mathbb{T}} = i\pi_A d\omega^A + cc$$

Geometric interpretation via the incidence relation, $\omega^A = iX^{A\dot A}\bar\pi_{\dot A}$,

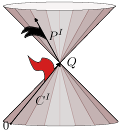
(Recall the isomorphism $X^I = \frac{i}{\sqrt{2}}\sigma^I_{A\dot A}X^{A\dot A}, \qquad \sigma^I$



- $X \in M$ iff twistor is *null*, $s = 0$

  Incidence relation is solved by

$$X^{A\dot A} = -\frac{1}{\pi\omega}\, i\omega^A\bar\omega^{\dot A} + b\, i\pi^A\bar\pi^{\dot A}, \qquad b \in \mathbb{R}.$$



  a null ray in the direction of the null-pole of π^A,
  going through a point along the null-pole of ω^A

Twistor space

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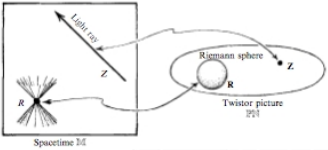

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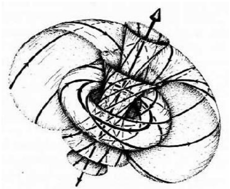
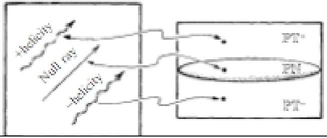


- non-null twistors describe congruences of light rays



descriptions unchanged by a complex rescalings
Complex projective twistor space $\qquad \mathbb{PT} \cong \mathbb{CP}^3$

Twistors and the Lorentz algebra

- twistor space $\mathbb{T}$ carries a rep. of $\mathrm{SL}(2,\mathbb{C})$ and $\mathrm{SU}(2,2)$
  $\pi\omega$ Lorentz invariant, helicity $s = \mathrm{Re}(\pi\omega)$ conformal invariant
  $\Rightarrow$ physical picture as a massless particle with a certain spin and momentum

$$\Pi^{AB} = \frac{1}{2}(L+iK)^{AB} = \frac{1}{2}\omega^{(A}\pi^{B)}, \quad U^{A}{}_{B}\omega^{B} = e^{\{\alpha\cdot L+\beta\cdot K,\cdot\}}\omega^{A}, \quad \{\pi_A,\omega^B\} = \delta_A^B$$

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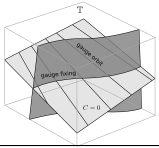
- both Lorentz generators and holonomies can be expressed as simple functions on a space of two twistors, provided they have the same complex helicity: $\pi\omega = \tilde{\pi}\tilde{\omega}$
  Dupuis, Freidel, Livine and S '12, Wieland '12, Wieland and S '13

$$C = \pi\omega - \tilde{\pi}\tilde{\omega} = 0,$$
$$\mathbb{C}^8//C \cong T^*SL(2,\mathbb{C})$$



$$\boxed{T^*\mathrm{SL}(2,\mathbb{C}) \text{ is a symplectic submanifold of } \mathbb{T}^2}$$

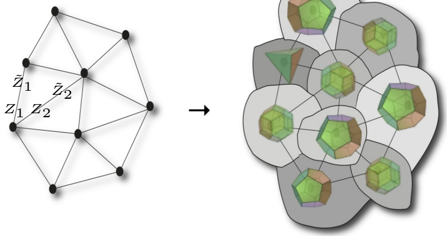
- $\mathbb{PT}$ vs. $\mathbb{T}$: the scale of the twistor determines the value of the Casimir of the algebra
  $\Rightarrow$ the area of spin foam faces
- no infinity twistor needed, conformal invariance broken enforcing equal dilatations, $\mathrm{Im}(\pi\omega) = \mathrm{Im}(\tilde{\pi}\tilde{\omega})$

Twistors and discrete geometries

Twistor space comes with a natural flat metric, $\eta_{IJ} = \epsilon_{AB}\epsilon_{\dot{A}\dot{B}}$

How to describe generally curved manifolds? Penrose: local twistors

Different strategy: extract the geometry from the twistors themselves, motivated by the algebraic structure of loop quantum gravity and the Regge idea of getting a curved metric gluing together patches of flat space



Collection of twistors on a graph of equal complex helicities

Impose further conditions:

- local Lorentz invariance on the nodes
- simplicity constraints to identify left and right metric structures $\mathfrak{sl}(2,\mathbb{C}) \cong \mathfrak{su}(2) \oplus \mathfrak{su}(2)$

One obtains a collection of polyhedra with extrinsic geometry: a twisted geometry

Quantization

S and Wieland '12

- Quantize initial twistorial phase space, **à la Schrödinger**:

$$[\hat{\pi}_A, \hat{\omega}^B] = -i\hbar\delta_A^B, \qquad f(\omega) \in L^2(\mathbb{C}^2, d^4\omega) \qquad \left(\hat{\omega} = \omega,\ \hat{\pi} = -i\hbar\frac{\partial}{\partial\omega}\right)$$

- basis of hom. functions, carrying *unitary* ∞-dim. Lorentz irreps $(\rho \in \mathbb{R}, k \in \mathbb{N}/2)$
- impose constraints, obtain a basis equivalent to LQG usual spin networks (characterised by $\rho = \gamma k$)

$$G^{(j)}_{m\tilde{m}}(\omega, \tilde{\pi}) := f^{(\gamma j, j)}_{jm}(\omega) f^{(\gamma j, j)}_{j\tilde{m}}(\tilde{\pi}), \qquad f^{(\gamma j, j)}_{jm}(\omega) = \|\omega\|^{2(i\gamma j - j - 1)} \langle j, m | j, \omega\rangle_{\text{Perelomov}}$$

- LQG transition amplitudes rewritten as integrals in twistor space

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Equivalent to Penrose's holomorphic quantization??

- reformulate QLG transition amplitudes as holomorphic integrals
- improved evaluation of quantum corrections
- new bridge to S-matrix theory and perturbative scattering amplitudes
- relate the googly problem of twistor theory to the problem of reality conditions in the original complex Ashtekar variables

Key distinction: twistors as asymptotic states vs local twistors

on a fixed graph, we reconstructed a discrete curved spacetime à la Regge, gluing together patches of flat spacetime

Is there a continuum description of Ashtekar variables in terms of twistors?

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Quantum corrections and continuum limit

$$\begin{array}{ccc} & \Gamma \text{ semiclassical limit} & \\ \text{EPRL spin foam} & \longrightarrow & \text{Regge action} \\ & & \\ & \text{continuum limit} & \downarrow \\ & & \\ & & \text{general relativity} \end{array}$$

Quantum corrections and continuum limit

$$\begin{array}{ccc} & \Gamma\text{ semiclassical limit} & \\ \text{EPRL spin foam} & \longrightarrow & \text{Regge action} \\ \downarrow & \text{continuum limit} & \downarrow \\ \Gamma \mapsto \infty & \longrightarrow ?? & \text{general relativity} \\ & \text{semiclassical limit} & \end{array}$$

Quantum radiative corrections can spoil the Γ semiclassical limit

Can introduce IR divergences and require renormalization
How to tame the infinite summation
⇒ group field theories and tensor models Rivasseau,Oriti,Gurau,...

Hard to compute: include complicated summations
Lack in general a clear geometric meaning
⇒ I expect the twistorial description to lead the way to future developments ...

Quantum corrections and continuum limit

	Γ semiclassical limit	
EPRL spin foam	$\longrightarrow$	Regge action
$\downarrow$	continuum limit	$\downarrow$
$\Gamma \mapsto \infty$	$\longrightarrow$??	general relativity
	semiclassical limit	

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Hard to compute: include complicated summations
Lack in general a clear geometric meaning
$\Rightarrow$ I expect the twistorial description to lead the way
to future developments . . .
of course surprises may be lurking beyond a corner . . .

What has the theory delivered so far?

- A compelling description of the possible fundamental quanta of the gravitational field
- Loop Quantum Cosmology Ashtekar,Bojowald,Sing,Agullo,Pawlowski,...
  Singularity resolution, big bounce
- Black hole physics Rovelli, Krasnov, Ashtekar, Mena, Perez, Bianchi, ...
  Derivation of entropy counting microstates
- Summing over the graphs, IR divergence and continuum limit
  GFT and tensor models Rivasseau, Oriti, Gurau, Freidel, Rovelli, ...
- Possible phenomenology from modified dispersion relations and non-local effects
  DSR/relative locality Amelino-Camelia, Freidel, Smolin, Kowalski, ...

Loops'13, Perimeter Institute, July 2013
http://pirsa.org/C13029

A message from this talk:

- We can parametrize LQG in terms of twistors
- Geometric meaning of graph expansion clarified,
  identification of a subsector corresponding to Regge geometries
- New tools to study quantum corrections and graph refinement/continuum limit
- Possible new bridge to twistor theory and S-matrix scattering amplitudes