Thermal widths of non-relativistic particles

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1. Non-relativistic EFTs at finite T

Scales

We consider systems where a heavy particle of mass M is in a thermal bath at a temperature T such that

$$M \gg T$$

Examples are:

- heavy particles in the early universe (like heavy Majorana neutrinos), the thermal bath is made of a plasma of SM particles;
- heavy quarkonium produced at heavy-ion colliders, the thermal bath is made of a plasma of light quarks and gluons;
- ...

Non-relativistic EFTs

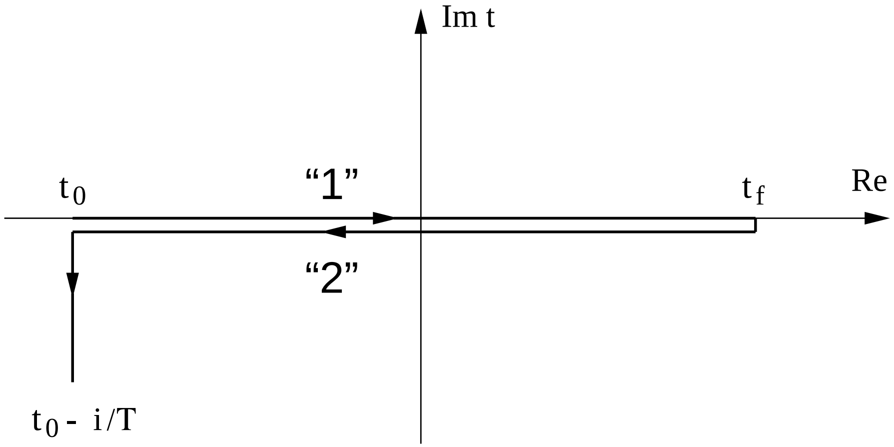
The hierarchy $M \gg T$ (and M greater than any other energy scale in the system) allows describing at low energy the heavy particle, H, in terms of a non-relativistic EFT:

$$\begin{aligned} \mathcal{L} &= H^\dagger i D_0 H + \text{higher dimension operators suppressed in } 1/M \\ &\quad + \mathcal{L}_{\text{light fields}} \end{aligned}$$

- The Lagrangian has been written in a reference frame where the heavy particle is at rest up to fluctuation of order T or smaller.
- In the heavy-particle sector the Lagrangian is organized as an expansion in $1/M$. Contributions of higher-order operators to physical observables are suppressed by powers of T/M.
- The Lagrangian $\mathcal{L}$ may be computed at $T = 0$, i.e. the Wilson coefficients encoding the high-energy modes may be computed in vacuum.

Real-time formalism

Temperature is introduced via the partition function. In real-time formalism the contour of the partition function is modified to allow for real time:



In real time, the degrees of freedom double ("1" and "2"), however, the advantages are

- the framework becomes very close to the one for $T = 0$ EFTs;
- in the heavy-particle sector, the second degrees of freedom, labeled "2", decouple from the physical degrees of freedom, labeled "1".

This usually leads to a simpler treatment with respect to alternative calculations in imaginary time formalism.

Real-time gauge boson propagator

- Gauge boson propagator (in Coulomb gauge):

$$\mathbf{D}_{00}^{(0)}(\vec{k}) = \frac{i}{\vec{k}^2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbf{D}_{ij}^{(0)}(k) = \left(\delta_{ij} - \frac{k^i k^j}{\vec{k}^2}\right)\left\{\begin{pmatrix} \dfrac{i}{k^2 + i\epsilon} & \theta(-k^0)\,2\pi\delta(k^2) \\ \theta(k^0)\,2\pi\delta(k^2) & -\dfrac{i}{k^2 - i\epsilon} \end{pmatrix} + 2\pi\delta(k^2)\,n_{\mathrm{B}}(|k^0|)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right\}$$

where

$$n_{\mathrm{B}}(k^0) = \frac{1}{e^{k^0/T} - 1}$$

Real-time heavy-particle propagator

- The free heavy-particle propagator is proportional to

$$\mathbf{S}^{(0)}(p) = \begin{pmatrix} \dfrac{i}{p^0 + i\epsilon} & 0 \\ 2\pi\delta(p^0) & \dfrac{-i}{p^0 - i\epsilon} \end{pmatrix}$$

Since $[\mathbf{S}^{(0)}(p)]_{12} = 0$, the static quark fields labeled "2" never enter in any physical amplitude, i.e. any amplitude that has the physical fields, labeled "1", as initial and final states.

These properties hold also for interacting heavy particle(s): interactions do not change the nature ("1" or "2") of the interacting fields.

◦ Brambilla Ghiglieri Vairo Petreczky PRD 78 (2008) 014017

Weak coupling

We will consider heavy particles interacting weakly with a weakly coupled plasma:

- a heavy Majorana neutrino in the primordial universe that interacts weakly with a plasma of massless SM particles;
- a $\Upsilon(1S)$ formed in heavy-ion collisions of sufficiently high energy that is a Coulombic bound state interacting with a weakly coupled quark-gluon plasma.

2. Heavy Majorana neutrinos

A model for neutrino oscillation and thermal leptogenesis

We consider a heavy Majorana neutrino ψ of mass $M \gg M_W$ coupled to the SM only through a Higgs-lepton vertex:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}\,\bar{\psi}\,i\partial\!\!\!/\,\psi - \frac{M}{2}\,\bar{\psi}\psi - F_f\,\bar{L}_f\tilde{\phi}P_R\psi - F_f^*\,\bar{\psi}P_L\tilde{\phi}^\dagger L_f\;,$$

This extension of the SM provides a model for neutrino mass generation through the seesaw mechanism. It also provides a model of baryogenesis through leptogenesis.

The production of a net lepton asymmetry starts when the (lightest) sterile neutrino decouples from the plasma. This happens when $T \sim M$. During the universe expansion, the sterile neutrino continues to decay in the regime $T < M$. For $T < M$ the recombination process is almost absent and a net lepton asymmetry is generated.

- `Minkowski PLB 67 (1977) 421`

  `Gell-Mann Ramond Slansky CPC 790927 (1979) 315, ...`

  `Fukugita Yanagida PLB 174 (1986) 45`

  `Luty PRD 45 (1992) 455, ...`

The non-relativistic Majorana neutrino EFT

We consider the temperature regime

$$M \gg T \gg M_W$$

At an energy scale smaller than M and comparable with T, the low-energy modes of the Majorana neutrino are described by a field N whose effective interaction with the SM particles is described by the EFT:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{N}}$$

where

$$\mathcal{L}_{\text{N}} = \bar{N}\left(i\partial_0 - \frac{i\Gamma_{T=0}}{2}\right)N + \frac{\mathcal{L}^{(1)}}{M} + \frac{\mathcal{L}^{(2)}}{M^2} + \mathcal{O}\left(\frac{1}{M^3}\right)$$

Higher-order operators are suppressed by powers of $1/M$.

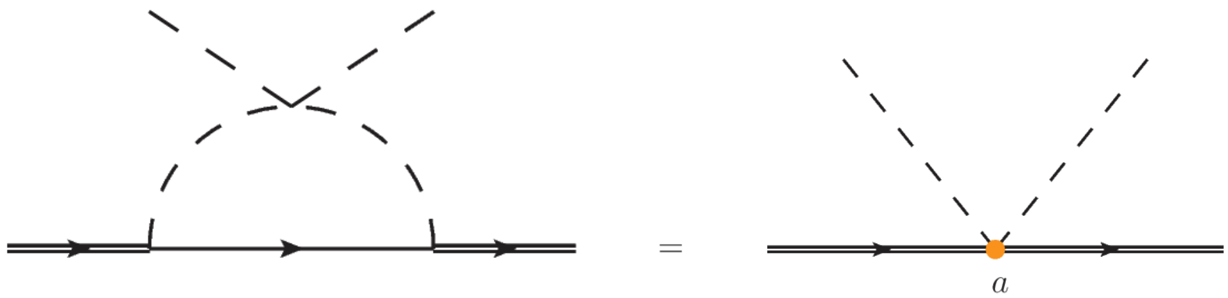
- Biondini Brambilla Escobedo Vairo arXiv::1307.7680

Dimension 5 operator

The leading operator describing the low-energy effective interaction of the Majorana neutrino with the SM particles is the dimension 5 operator

$$\mathcal{L}^{(1)} = a\, \bar{N} N\, \phi^\dagger \phi\, .$$

It is a neutrino-two Higgs vertex, fixed at one loop by the matching condition

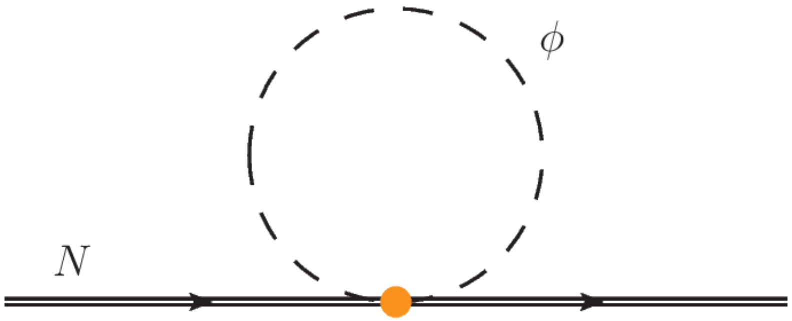


The Wilson coefficient a develops an imaginary part, which is

$$\mathrm{Im}\, a = -\frac{3}{8\pi} |F|^2 \lambda$$

The leading-order Majorana neutrino thermal width

The imaginary part of the coefficient a is the main responsible for the emergence of a heavy neutrino thermal width induced by the interaction with the plasma of SM (Higgs) particles, through the neutrino-Higgs tadpole diagram:



The thermal width is

$$\Gamma = 2\frac{\mathrm{Im}\, a}{M}\langle\phi^\dagger(0)\phi(0)\rangle_T = -\frac{|F|^2 M}{8\pi}\lambda\left(\frac{T}{M}\right)^2$$

- Salvio Lodone Strumia JHEP 1108 (2011) 116
  Laine Schröder JHEP 1202 (2012) 068

3. Heavy quarkonia

Scales

Quarkonium being a composite system is characterized by several energy scales, these in turn may be sensitive to thermodynamical scales smaller than the temperature:

- the scales of a non-relativistic bound state
  (v is the relative heavy-quark velocity; $v \sim \alpha_{\rm s}$ for a Coulombic bound state):
  M (mass),
  Mv (momentum transfer, inverse distance),
  Mv^2 (kinetic energy, binding energy, potential V), ...

- the thermodynamical scales:
  πT (temperature),
  m_D (Debye mass, i.e. screening of the chromoelectric interactions), ...

The non-relativistic scales are hierarchically ordered: $M \gg Mv \gg Mv^2$
We assume this to be also the case for the thermodynamical scales: $\pi T \gg m_D$

$\Upsilon(1S)$ scales

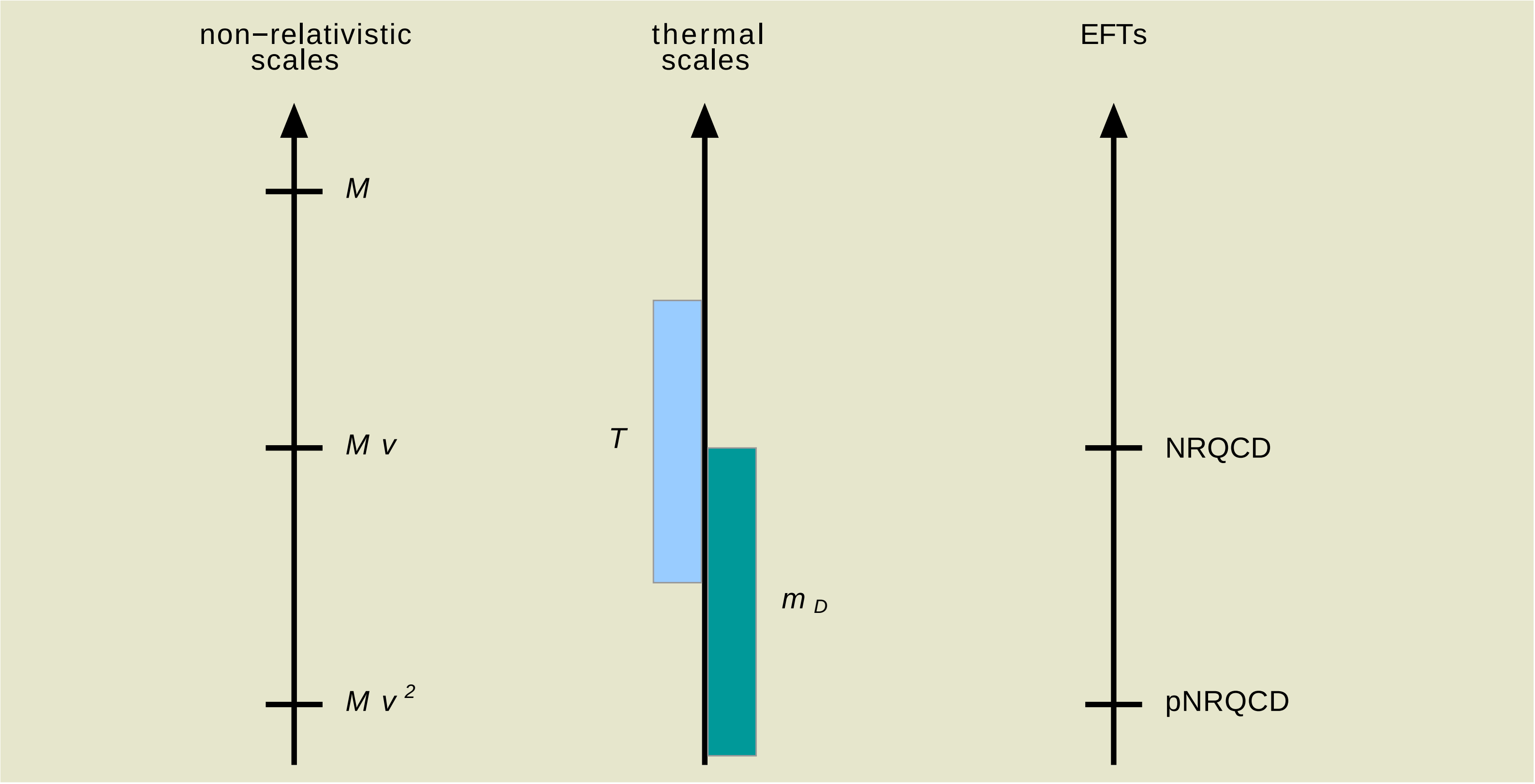
The bottomonium ground state produced in the QCD medium of heavy-ion collisions at the LHC may possibly realize the hierarchy:

$$M_b \approx 5 \text{ GeV } > M_b\alpha_s \approx 1.5 \text{ GeV } > \pi T \approx 1 \text{ GeV } > M_b\alpha_s^2 \approx 0.5 \text{ GeV } \sim m_D \gtrsim \Lambda_{\rm QCD}$$

- Brambilla Escobedo Ghiglieri Soto Vairo JHEP 1009 (2010) 038
  Vairo AIP CP 1317 (2011) 241

Non-relativistic EFTs of QCD

The existence of a hierarchy of energy scales calls for a description of the system (quarkonium at rest in a thermal bath) in terms of a hierarchy of EFTs.



For larger temperatures the quarkonium does not form.

NRQCD

NRQCD is obtained by integrating out modes associated with the scale M and possibly with thermal scales larger than Mv.

- The Lagrangian is organized as an expansion in $1/M$:

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2M} + \dots \right) \psi + \chi^\dagger \left(iD_0 - \frac{\mathbf{D}^2}{2M} + \dots \right) \chi + \dots$$
$$+\mathcal{L}_{\text{light}}$$

ψ (χ) is the field that annihilates (creates) the (anti)fermion.

◦ Caswell Lepage PLB 167 (1986) 437
Bodwin Braaten Lepage PRD 51 (1995) 1125

pNRQCD

pNRQCD is obtained by integrating out modes associated with the scale Mv and possibly with thermal scales larger than Mv^2.

- The degrees of freedom of pNRQCD are quark-antiquark states (color singlet S, color octet O), low energy gluons and light quarks propagating in the medium.
- The Lagrangian is organized as an expansion in $1/M$ and r:

$$\mathcal{L} = \int d^3r \, \mathrm{Tr} \left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{M} - V_s + \ldots \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{M} - V_o + \ldots \right) \mathrm{O} \right\}$$
$$\mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \, \mathrm{S} + \mathrm{H.c.} \right\} + \frac{1}{2} \mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \, \mathrm{O} + \mathrm{c.c.} \right\} + \cdots$$
$$+\mathcal{L}_{\mathrm{light}}$$

- At leading order in r, the singlet S satisfies a Schrödinger equation.
The explicit form of the potential depends on the version of pNRQCD.

- Pineda Soto NP PS 64 (1998) 428
Brambilla Pineda Soto Vairo NPB 566 (2000) 275, RMP 77 (2005) 1423

Dissociation mechanisms at LO

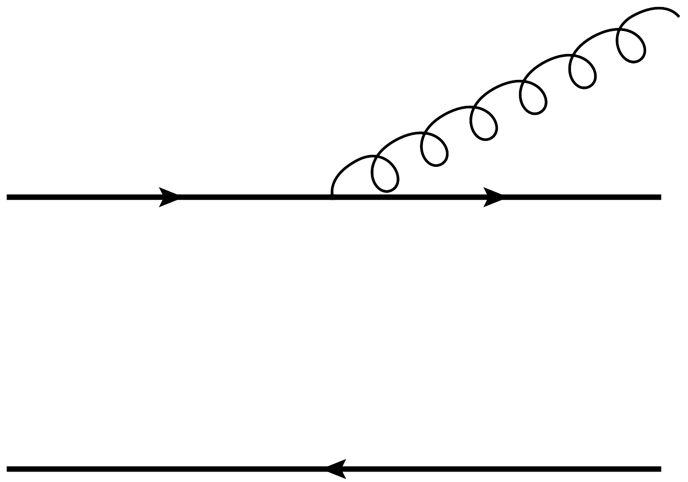
Two distinct dissociation mechanisms may be identified at leading order:

- gluodissociation,
  which is the dominant mechanism for $Mv^2 \gg m_D$;
- dissociation by inelastic parton scattering,
  which is the dominant mechanism for $Mv^2 \ll m_D$.

Beyond leading order the two mechanisms are intertwined and distinguishing between them becomes unphysical, whereas the physical quantity is the total decay width.

Gludissociation

Gludissociation is the dissociation of quarkonium by absorption of a gluon from the medium.

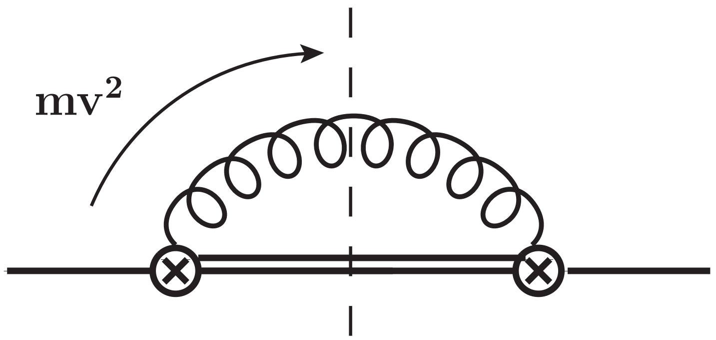


- The exchanged gluon is lightlike or timelike.
- The process happens when the gluon has an energy of order Mv^2.

○ `Kharzeev Satz PLB 334 (1994) 155`
`Xu Kharzeev Satz Wang PRC 53 (1996) 3051`

Gluodissociation

From the optical theorem, the gluodissociation width follows from cutting the gluon propagator in the following pNRQCD diagram



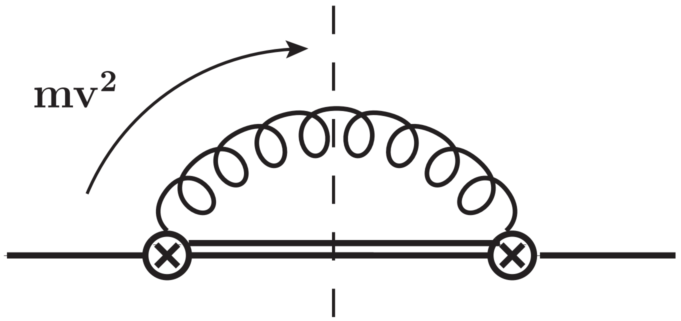
For a quarkonium at rest with respect to the medium, the width has the form

$$\Gamma_{nl} = \int_{q_{\min}} \frac{d^3q}{(2\pi)^3}\, n_{\rm B}(q)\, \sigma^{nl}_{\rm gluo}(q)\,.$$

- $\sigma^{nl}_{\rm gluo}$ is the in-vacuum cross section $(Q\overline{Q})_{nl} \to Q + \overline{Q} + g$.
- Gluodissociation is also known as singlet-to-octet break up.

○ Brambilla Ghiglieri Vairo Petreczky PRD 78 (2008) 014017

$1S$ gluodissociation at LO



The LO gluodissociation cross section for $1S$ Coulombic states is

$$\sigma^{1S}_{\text{gluo LO}}(q) = \frac{\alpha_s C_F}{3} 2^{10}\pi^2\rho(\rho+2)^2 \frac{E_1^4}{Mq^5}\left(t(q)^2+\rho^2\right)\frac{\exp\left(\frac{4\rho}{t(q)}\arctan\left(t(q)\right)\right)}{e^{\frac{2\pi\rho}{t(q)}}-1}$$

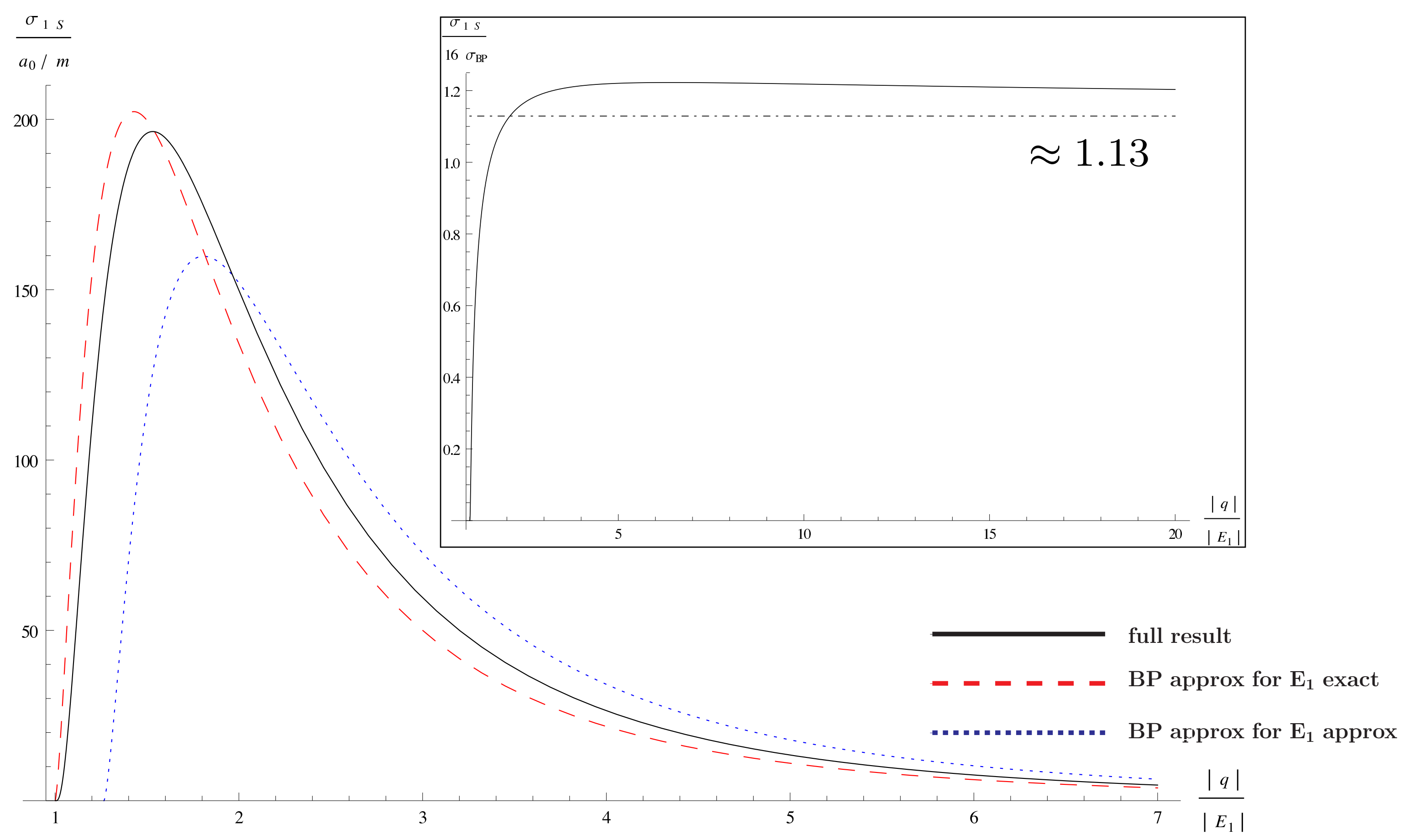
where $\rho \equiv 1/(N_c^2-1)$, $t(q) \equiv \sqrt{q/|E_1|-1}$ and $E_1 = -MC_F^2\alpha_s^2/4$.

- Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116
  Brezinski Wolschin PLB 707 (2012) 534

The Bhanot–Peskin approximation corresponds to the large N_c limit, i.e. to neglecting final state interactions (the rescattering of a $Q\overline{Q}$ pair in a color octet configuration).

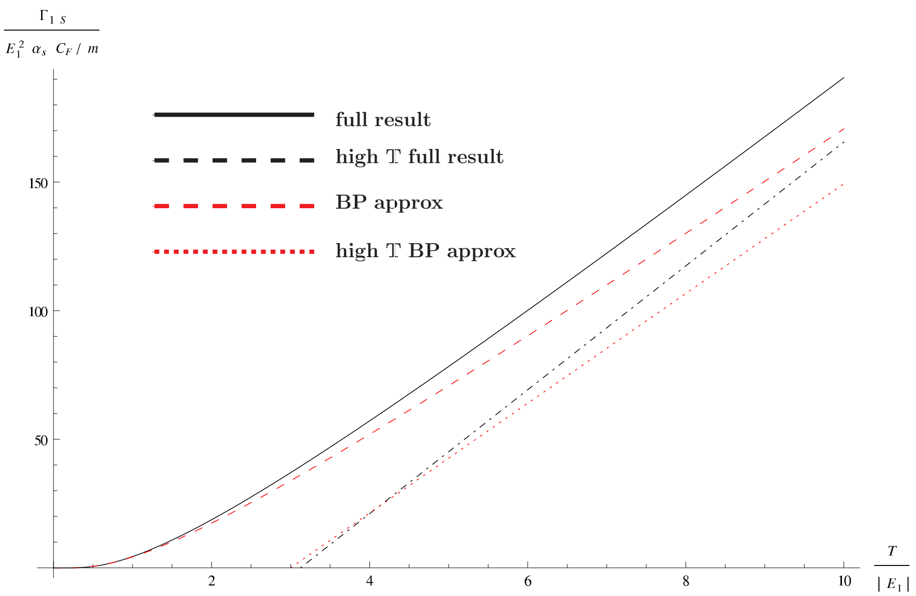
- Peskin NPB 156 (1979) 365, Bhanot Peskin NPB 156 (1979) 391

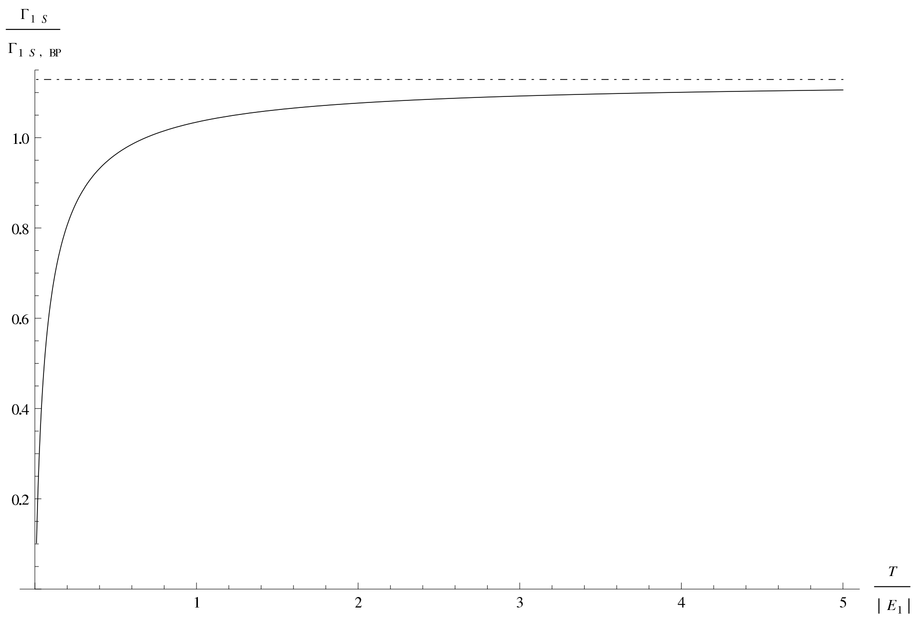
Bhanot–Peskin cross section vs full cross section



- Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116

Bhanot–Peskin width vs full width

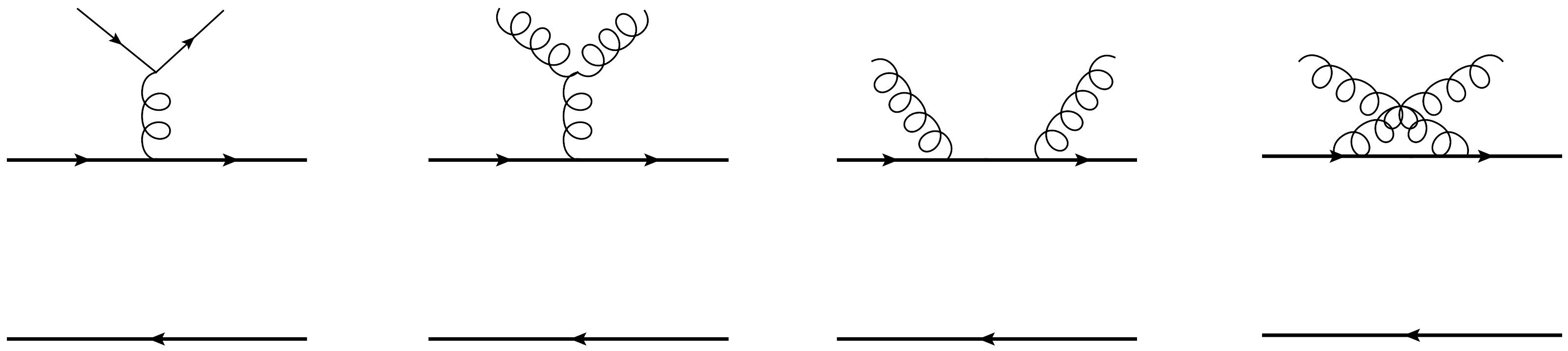




- Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116

Dissociation by inelastic parton scattering

Dissociation by inelastic parton scattering is the dissociation of quarkonium by scattering with gluons and light-quarks in the medium.



○ `Grandchamp Rapp PLB 523 (2001) 60, NPA 709 (2002) 415`

- The exchanged gluon is spacelike.
- In Coulomb gauge, external thermal gluons are transverse.
- In the NRQCD power counting, each external transverse gluon is suppressed by T/M.

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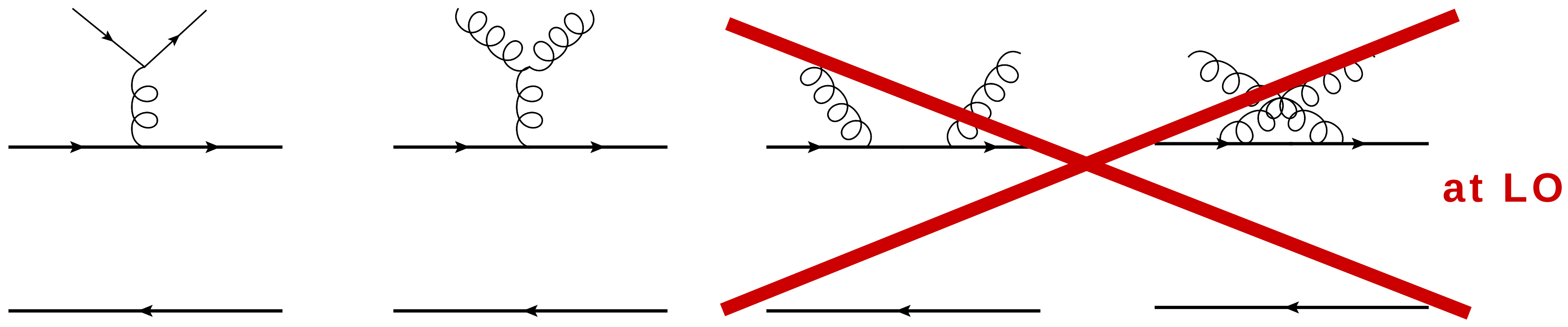

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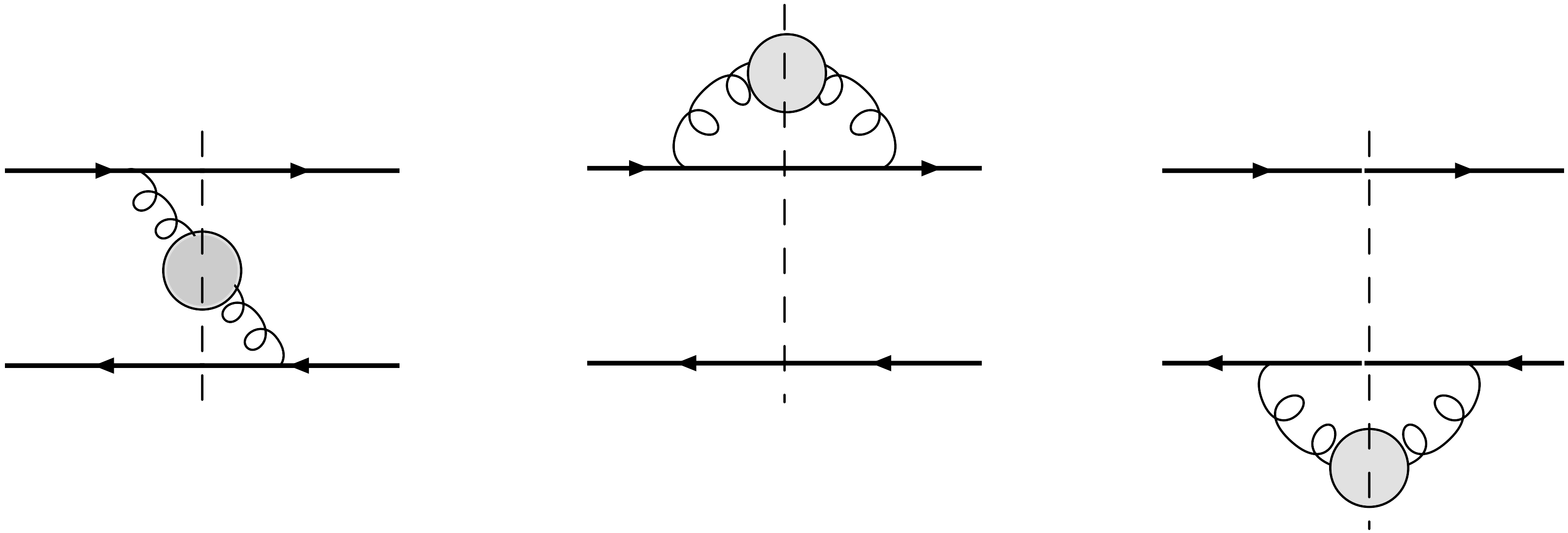
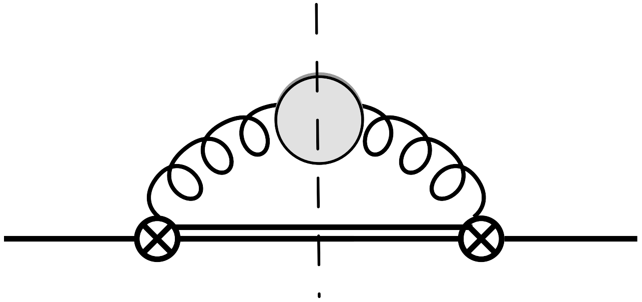
Dissociation by inelastic parton scattering

From the optical theorem, the thermal width follows from cutting the gluon self-energy in the following NRQCD diagrams (momentum of the gluon $\gtrsim Mv$)



and/or pNRQCD diagram (momentum of the gluon $\ll Mv$)



- Dissociation by inelastic parton scattering is also known as Landau damping.

Dissociation by inelastic parton scattering

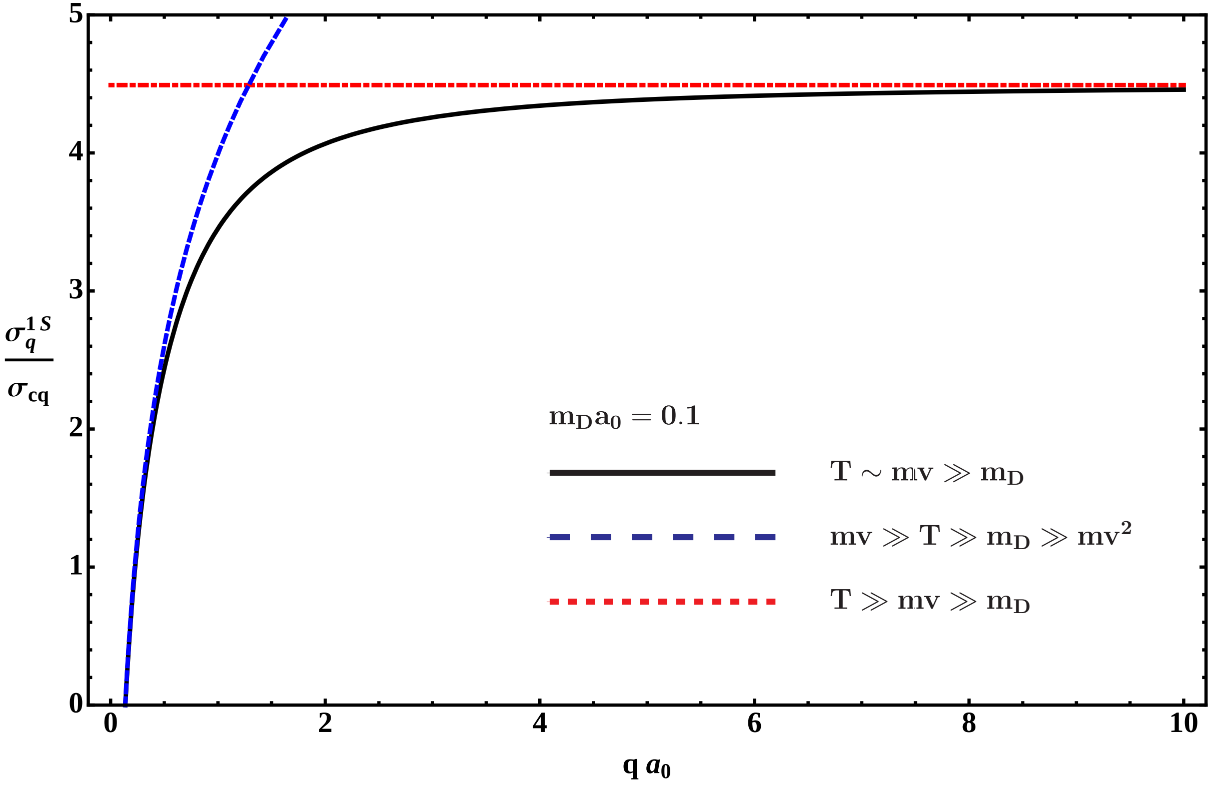
For a quarkonium at rest with respect to the medium, the thermal width has the form

$$\Gamma_{nl} = \sum_p \int_{q_{\min}} \frac{d^3q}{(2\pi)^3} \, f_p(q) \, [1 \pm f_p(q)] \, \sigma_p^{nl}(q)$$

where the sum runs over the different incoming light partons and $f_g = n_{\rm B}$ or $f_q = n_{\rm F}$.

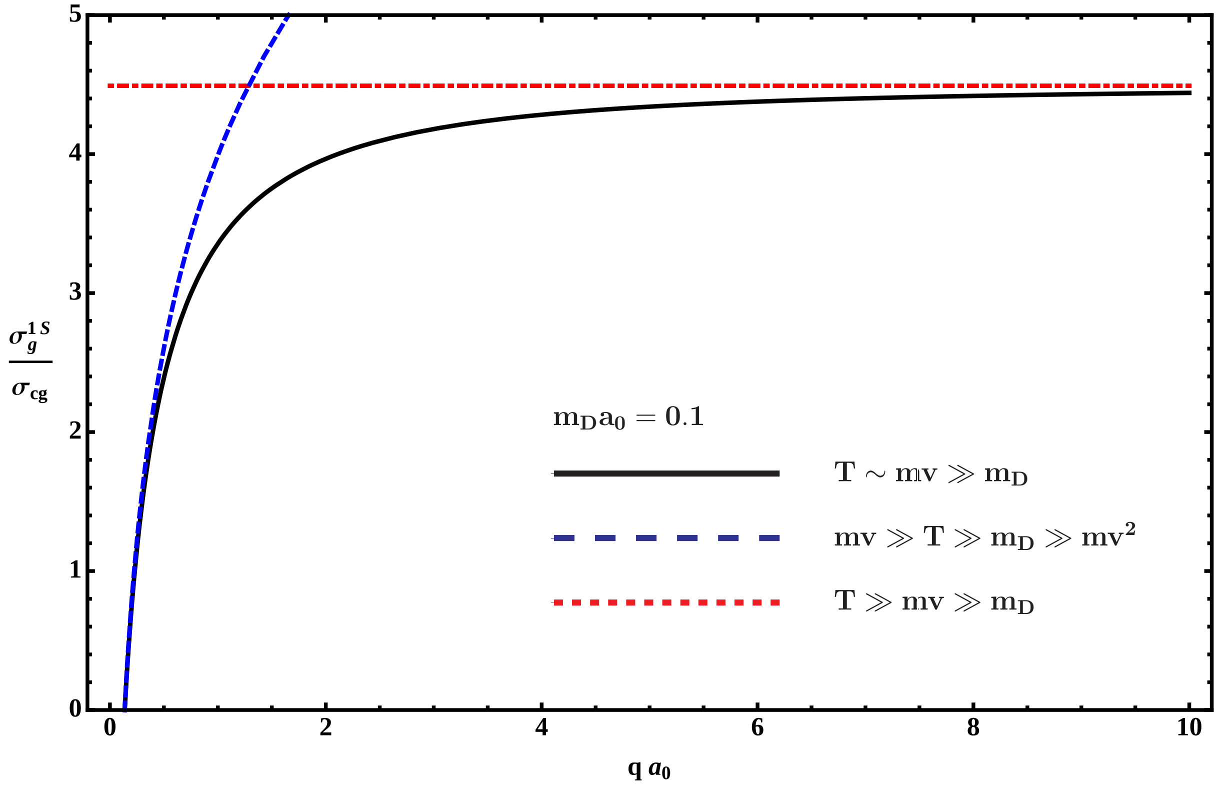
- σ_p^{nl} is the in-medium cross section $(Q\overline{Q})_{nl} + p \to Q + \overline{Q} + p$.
- The convolution formula correctly accounts for Pauli blocking in the fermionic case (minus sign).
- The formula differs from the gluodissociation formula.
- The formula differs from the one used so far in the literature, which has been inspired by the gluodissociation formula.
  - `Grandchamp Rapp PLB 523 (2001)`
    `Park Kim Song Lee Wong PRC 76 (2007) 044907, ...`

Dissociation by quark inelastic scattering



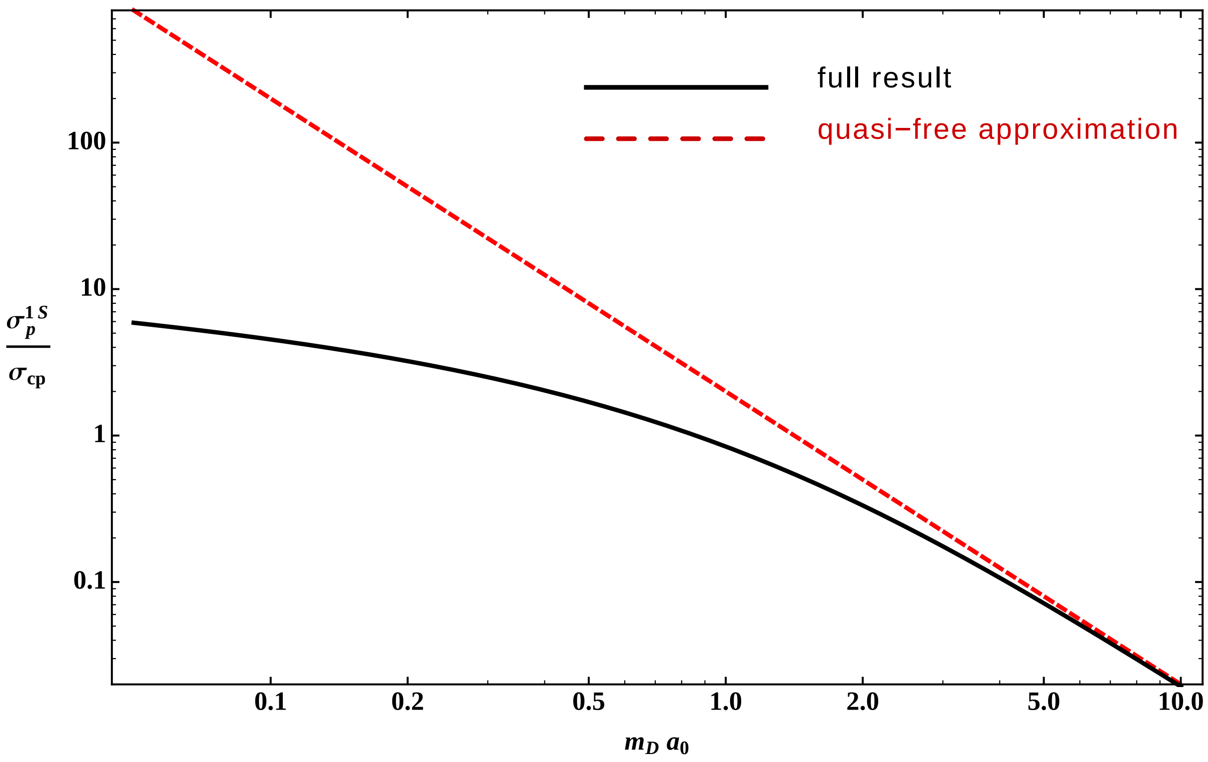
- Brambilla Escobedo Ghiglieri Vairo JHEP 05 (2013) 130

Dissociation by gluon inelastic scattering



- Brambilla Escobedo Ghiglieri Vairo JHEP 05 (2013) 130

Quasi-free approximation vs full result



Conclusions

In a framework that makes close contact with modern effective field theories for non relativistic particles at zero temperature, one can compute the thermal width of non-relativistic particles in a thermal bath in a systematic way.

In the situation $M \gg T$ one may organize the computation in two steps and compute the physics at the scale M as in vacuum. If other scales are larger than T, then also the physics of those scales may be computed as in vacuum. We have illustrated this on the examples of

- a heavy Majorana neutrino in the early universe plasma;
- a heavy quarkonium in a quark-gluon plasma.