Estimate of Neutron Star masses through the Field Correlator Method Equation of State.

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Outline

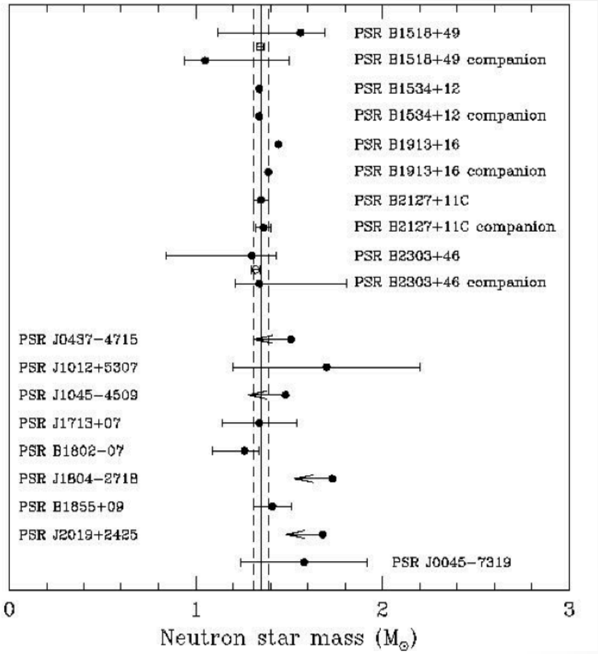
① Motivations : Experimental data of Neutron Stars (NS) masses as a test of the Equation of State (EoS) of nuclear matter and quark matter at high baryon density and zero temperature. Challenging recent finding of heavy NS.

② Survey of the nuclear matter EoS ond of the Field Correlator Method (FCM) adopted to describe the quark matter phase.

③ Determination of the maximum mass of a stable NS according to the specific EoS considered and on the consequent constraints that can be put on the parameters that characterize the FCM.

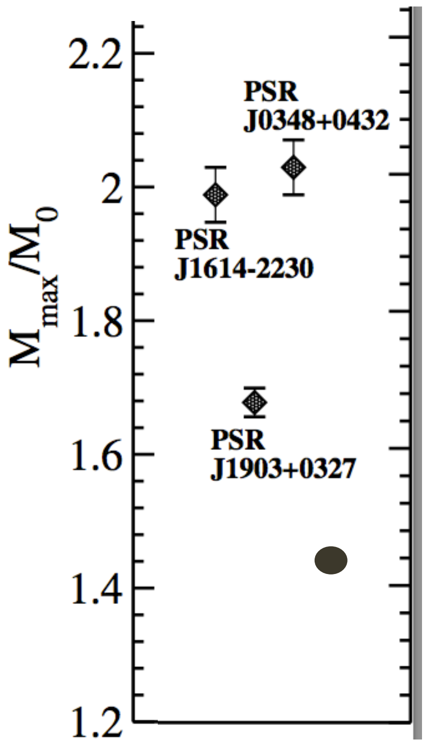
G.F. Burgio, V. Greco, S. Plumari, and D. Zappalà, arXiv:1307.3055[hep-ph]

OBSERVATIONAL DATA



Accurate determination of NS mass

Recent data on heavy NS around 2 $M_\odot$ are an excellent test for the microscopic description of nuclear and quark matter at high ρ_B and $T \approx 0$.



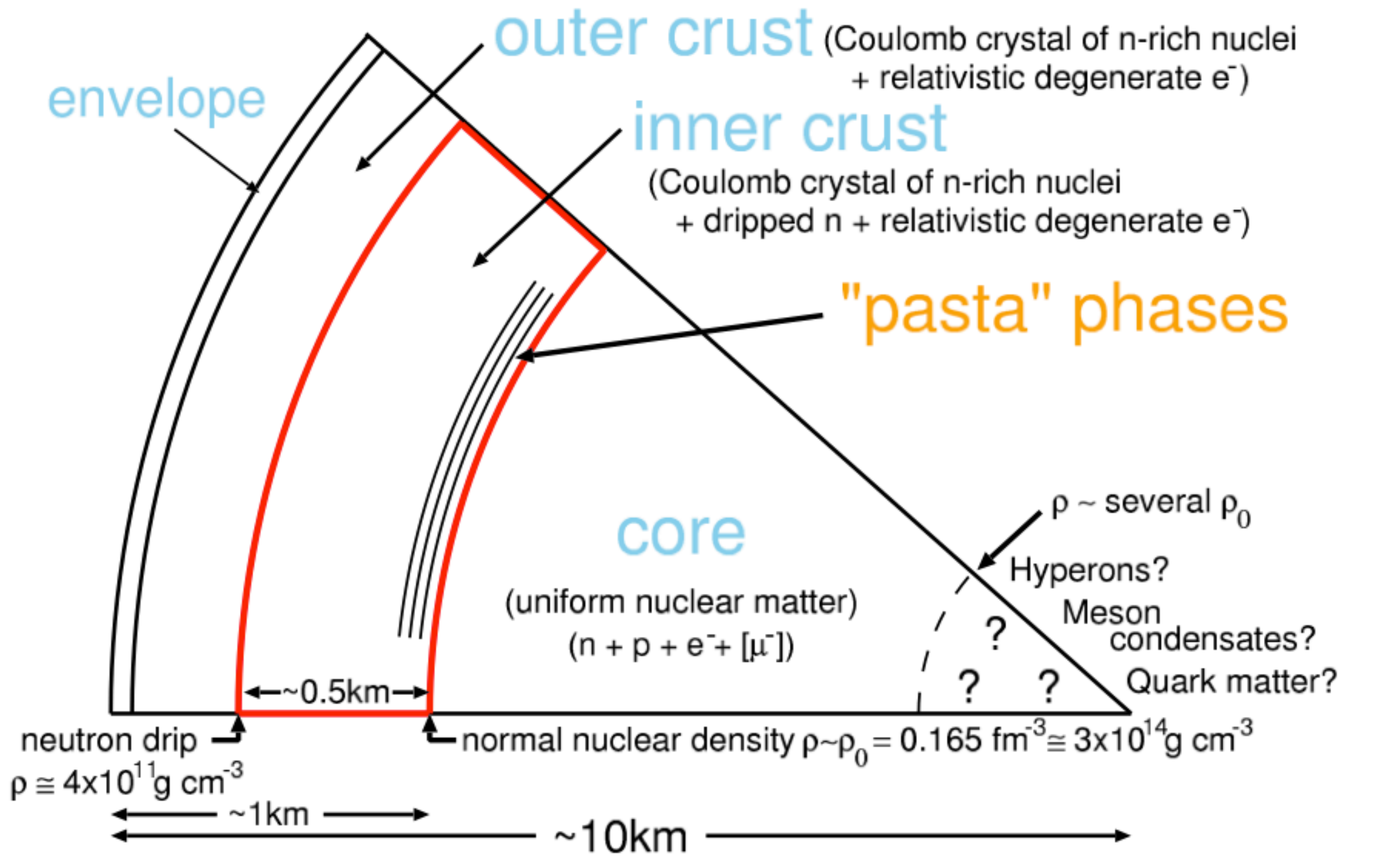
2.01 ± 0.04 PSR J0348+0432 (´13)

1.97 ± 0.04 PSR J1614 -2230 (´10)

1.67 ± 0.02 PSR J1903+0327 (´11)

1.441 ± 0.003 PSR B1913+16 (´82)

NS interior



Inner layers of the NS contain states of matter with higher density.

EoS of various phases are necessary to describe the NS structure.

Tolman-Oppenheimer-Volkov differential equations

$$\frac{dP(r)}{dr} = -\frac{Gm(r)\epsilon(r)}{r^2}\,\frac{[1+\frac{P(r)}{\epsilon(r)}][1+\frac{4\pi r^3 P(r)}{m(r)}]}{1-\frac{2Gm(r)}{r}}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r)$$

Relativistic corrections

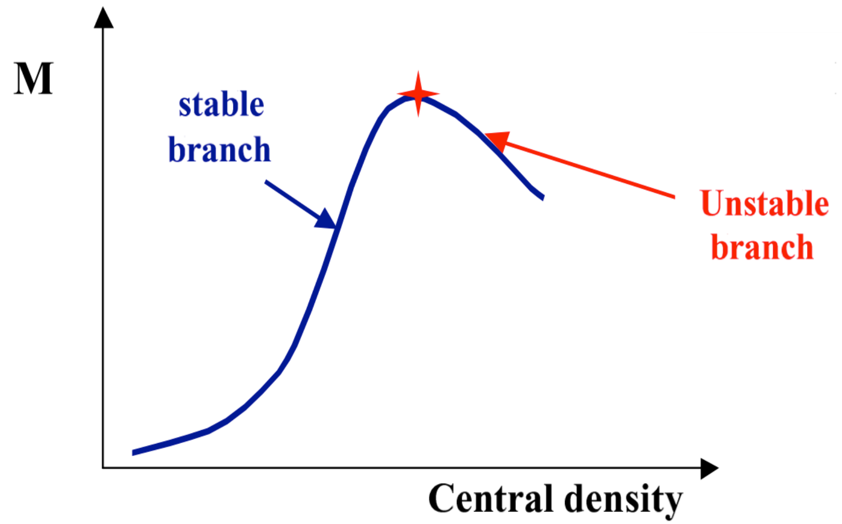
GM/c^2R	
10^{-6}	sun
10^{-4}	w. Dwarf
10^{-1}	neutron st.
1	black hole

Pressure ***P***, mass ***m*** and total energy density ***ε*** functions of the radial distance ***r***. Boundary conditions are fixed by ***m(0)=0 , ε(0) =ε***$_o$

Integration ends at ***r=R***, radius of the star, such that ***P(R)=0*** and the NS mass is ***M=m(R)***

Essential role played by the **EoS** that relates ***P(r)*** to ***ε(r)***

Expected behavior : NS mass vs. central density ε_o



A decreasing mass signals an instability :
the star collapses into a black hole

Maximum predicted NS mass to be compared with observational data

NS matter neutrally charged and in mechanical and chemical equilibrium :

Neutrality, N_B conservation and β-equilibrium put constraints on species densities and chemical potentials leaving only one free parameter: the baryon chemical potential μ_B

The equilibrium between hadronic and quark phase is realized by comparing the granpotential $\Omega = -PV$ in the two phases and adopting the Maxwell construction at the transition point.

Other quantites are related to P by standard thermodynamical relations (at $T \approx 0$ for NS)

$$dP = \sigma dT + \rho d\mu$$

$$\epsilon = \frac{E}{V} = T\sigma + \mu\rho - P$$

$$\sigma = \frac{S}{V} = \left(\frac{\partial P}{\partial T}\right)_\mu$$

$$\rho = \frac{N}{V} = \left(\frac{\partial P}{\partial \mu}\right)_T$$

MISSING INGREDIENT :

Equation of State (EoS) for nuclear and quark matter.

NUCLEAR MATTER:

-- Selfconsistent Brueckner- Hartree- Fock (BHF) approximation scheme .

-- The corresponding relativistic version : Dirac- Brueckner- Hartree- Fock (DBHF) scheme .

QUARK (AND GLUON) MATTER:

-- Field Correlator Method (FCM) : observables computed from QCD correlators.

HADRONIC EoS

In the **BHF** selfconsistent scheme :

$$G(\rho;\omega) = v + v \sum_{k_a k_b} \frac{|k_a k_b\rangle Q \langle k_a k_b|}{\omega - e(k_a) - e(k_b)} G(\rho;\omega)$$

$$U(k;\rho) = \sum_{k' \leq k_F} \langle k k' | G(\rho; e(k) + e(k')) | k k' \rangle_a$$

$$e(k) = e(k;\rho) = \frac{\hbar^2}{2m} k^2 + U(k;\rho)$$

Coupled eqs for the 2-body scattering matrix ***G*** and
the 1-particle potential ***U*.**
e(k) is the single particle energy and
υ is the bare interaction (**Argonne** υ_{18} chosen here) .

The energy per nucleon is :

$$\frac{E}{A}(\rho) = \frac{3}{5} \frac{\hbar^2 k_F^2}{2m} + D_2$$

$$D_2 = \frac{1}{2A} \sum_{k,k' \leq k_F} \langle k k' | G(\rho; e(k) + e(k')) | k k' \rangle_a$$

3-body forces (TBF) included according to the phenomenological Urbana Model.

(Effective density dependent interaction obtained by averaging on the third particle position).

Necessary to reproduce phenomenological data :

--- nuclear matter saturation density ρ_0 =.17 fm^{-3}

--- average energy per nucleon E/A= -16 MeV

M. Baldo, International Review of Nuclear Physics, Vol. 8 (World Scientific, Singapore, 1999).

M. Baldo, I. Bombaci, and G. F. Burgio, Astron. Astrophys. 328, (1997) 274.

X. R. Zhou, G. F. Burgio, U. Lombardo, H.-J. Schulze, and W. Zuo, Phys. Rev. C69, (2004) 018801.

W. D. Myers and W. J. Swiatecki, Nucl. Phys. A601, 141 (1996); Phys. Rev. C57, 3020 (1998).

RELATIVISTIC HADRONIC EoS

BHF selfconsistent scheme reconsidered in the relativistic formalism leads to the DBHF EoS .

In the DBHF EoS the repulsive interaction is dominant w.r.t. to the BHF EoS which implies a stiffer behavior at large density ($\rho > 0.3$ fm^{-3}) .

T. Gross-Boelting, C. Fuchs, and A. Faessler, Nucl. Phys. A 648, 105 (1999).

G. E. Brown, W. Weise, G. Baym, and J. Speth, Comm. Nucl. Part. Phys. 17, 39 (1987).

G. Taranto, M. Baldo and G. F. Burgio, Phys. Rev. C 87, 045803 (2013).

➔ Comparison of these two EoS in the hadronic phase.

The Field Correlator Method (FCM)

A. Di Giacomo, H.G. Dosch, V.I.Shevchenko and Y.A. Simonov, Phys. Rep 372,(2002) 319

QCD dynamics described in terms of gauge invariant, quadratic in the field, C-Electric and C-Magnetic Correlators D^E $D^E{}_1$ D^M $D^M{}_1$

$$g^2\left\langle \hat{tr}_f[E_i(x)\Phi(x,y)E_k(x)\Phi(y,x)]\right\rangle = \delta_{ik}[D^E + D_1^E + u_4^2\frac{\partial D_1^E}{\partial u_4^2}] + u_i u_k \frac{\partial D_1^E}{\partial u^2}$$

$$g^2\left\langle \hat{tr}_f[H_i(x)\Phi(x,y)H_k(x)\Phi(y,x)]\right\rangle = \delta_{ik}[D^H + D_1^H + u_4^2\frac{\partial D_1^H}{\partial u_4^2}] - u_i u_k \frac{\partial D_1^H}{\partial u^2}$$

where the Correlators are function of $u = x - y$

and Φ is the parallel transporter :

$$\Phi(x,y) = P\exp\left[ig\int_x^y A_\mu dx^\mu\right]$$

FCM generalized at finite T and μ_B in Single Line Appr.

Yu.A. Simonov, and M.A. Trusov, JETP Lett. 85 (2007) 598 ; Phys. Lett.B650 (2007) 36.
A. V. Nefediev, Yu.A. Simonov and M.A. Trusov, Int. J. Mod. Phys. E18 (2009) 549.

The quark pressure : ($\nu = m_q/T$)

$$P_q/T^4 = \frac{1}{\pi^2}\left[\phi_\nu\left(\frac{\mu_q - V_1/2}{T}\right) + \phi_\nu\left(-\frac{\mu_q + V_1/2}{T}\right)\right]$$

with $$\phi_\nu(a) = \int_0^\infty du \frac{u^4}{\sqrt{u^2+\nu^2}} \frac{1}{(\exp[\sqrt{u^2+\nu^2} - a] + 1)}$$

The gluon pressure :

$$P_g/T^4 = \frac{8}{3\pi^2}\int_0^\infty d\chi \chi^3 \frac{1}{\exp(\chi + \frac{9V_1}{8T}) - 1}$$

$$P_{qg} = P_g + \sum_{j=u,d,s} P_q^j + \Delta\epsilon_{\rm vac}$$

with $$\Delta\epsilon_{\rm vac} \approx -\frac{(11 - \frac{2}{3}N_f)}{32}\frac{G_2}{2}$$

V_1 : large distance $q\bar{q}$ potential

$$V_1 = \int_0^{1/T} d\tau (1 - \tau T)\int_0^\infty d\chi \chi D_1^E(\sqrt{\chi^2+\tau^2})$$

Gluon condensate

$$G_2 = (\alpha_s/\pi) \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle$$

$$G_2 = 0.012 \pm 0.006 \text{ GeV}^4 \quad (T=\mu_B=0)$$

No lattice simulations at finite μ_B and $T \approx 0$, then no definite indications on $V_1(\mu_B)$ and $G_2(\mu_B)$.

Previous studies of neutron star masses performed by taking constant V_1 and G_2 .

M. Baldo, G.F. Burgio, P. Castorina, S. Plumari, and D. Zappalà, Phys. Rev. D78, (2008) 063009.

I. Bombaci, and D. Logoteta, arXiv:1212.5907[astro-ph.SR].

However it is reasonable to expect some screening effect due to the finite density which should lower the effective value of $V_1(\mu_B)$ and $G_2(\mu_B)$ with respect to the $\mu_B = 0$ case.

G.F. Burgio, V. Greco, S. Plumari, and D. Zappala`, arXiv:1307.3055[hep-ph] .

Indications on $G_2(\mu_B)$ in nuclear matter

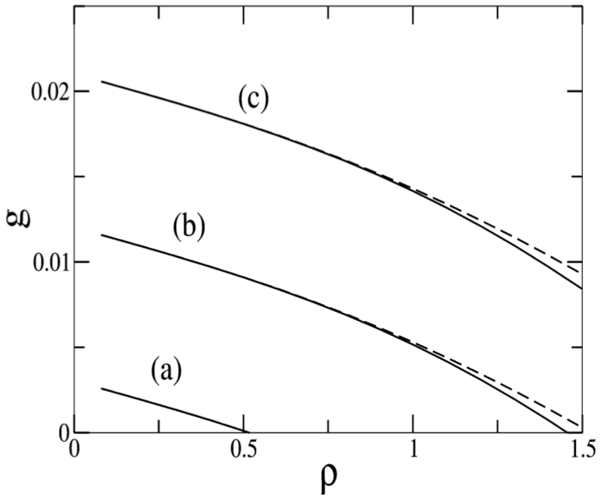
The first approximation of the expectation value of a (spin independent) operator in nuclear matter is obtained by treating the medium as a Fermi gas of nucleons :

$$< M \,|\, A \,|\, M > \; = \; < 0 \,|\, A \,|\, 0 > \; + \rho_B < N \,|\, A \;\,|\, N >$$

For A=G_2 the last term is computed relating G_2 to the trace anomaly.

$$< M \,|\, G_2 \,|\, M > \; = \; < 0 \,|\, G_2 \,|\, 0 > \; - 8\, \rho_B\, m_N/9 + O(m_{u,d,s}) + O(\rho_B{}^2)$$

T. D. Cohen, R. J. Furnstahl and D. K. Griegel, Phys. Rev. C45 (1992) 1881.
E. G. Drukarev, M. G. Ryskin and V. A. Sadovnikova, Prog. Part. Nucl. Phys. 47 (2001) 73.



The decrease is substantially linear even including higher order corrections.

This calculation indicates a critical density that corresponds to a vanishing condensate, unless new physics shows up.

Prediction on $G_2(\mu_B)$ from two-color N_c=2 gauge theory.

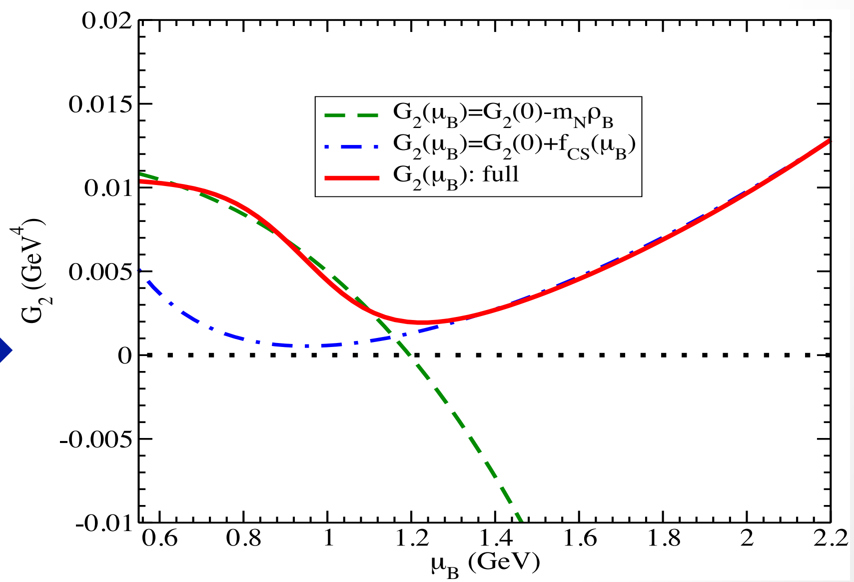
Effective description by a chiral lagrangian . From the computation of its energy-momentum tensor the gluon condensate is

$$f_{CS}(\mu) = G_2(\mu) - G_2(0) = 4 f_\pi^2 (\mu^2 - M^2)\left(1 - \frac{M^2}{\mu^2}\right)$$

Pair formation induces a minimum in $G_2(\mu_B)$

M. A. Metlitski and A. R. Zhitnitsky, Nucl. Phys. B731 (2005) 309.
A. R. Zhitnitsky, AIP Conf.Proc. 892 (2007) 518. ArXiv:hep-ph/0701065.

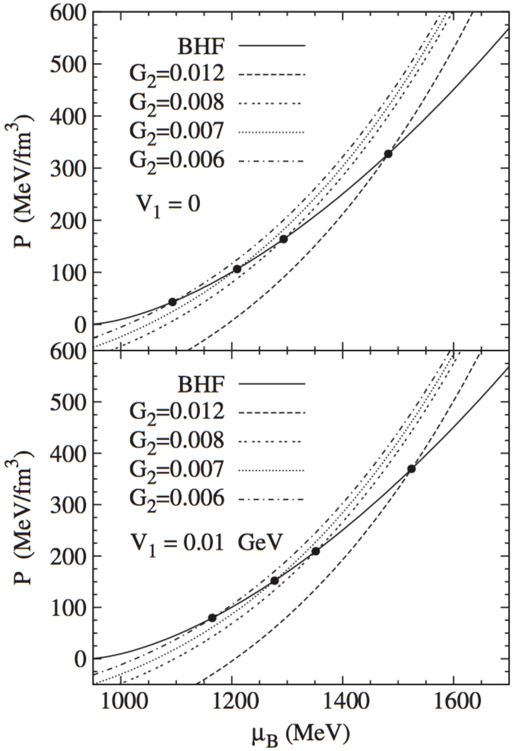
By following the suggestion that the same effects occurs in full N_C=3 QCD, the ansatz on $G_2(\mu_B)$ is given by ➔



Transition between two phases in thermal, chemical and mechanical equilibrium ➔ Comparison of granpotentials $-P(\mu_B)$

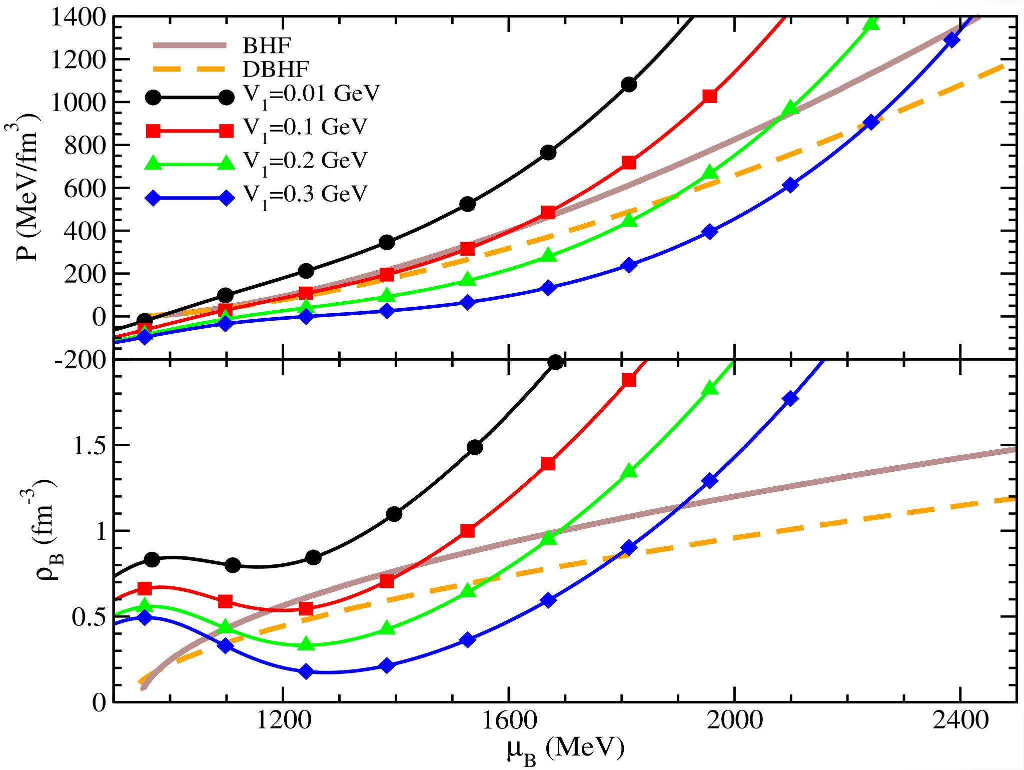
BHF

G_2 and $V_1 \approx 0$ constant



BHF & DBHF

$G_2(\mu_B)$ and V_1 constant



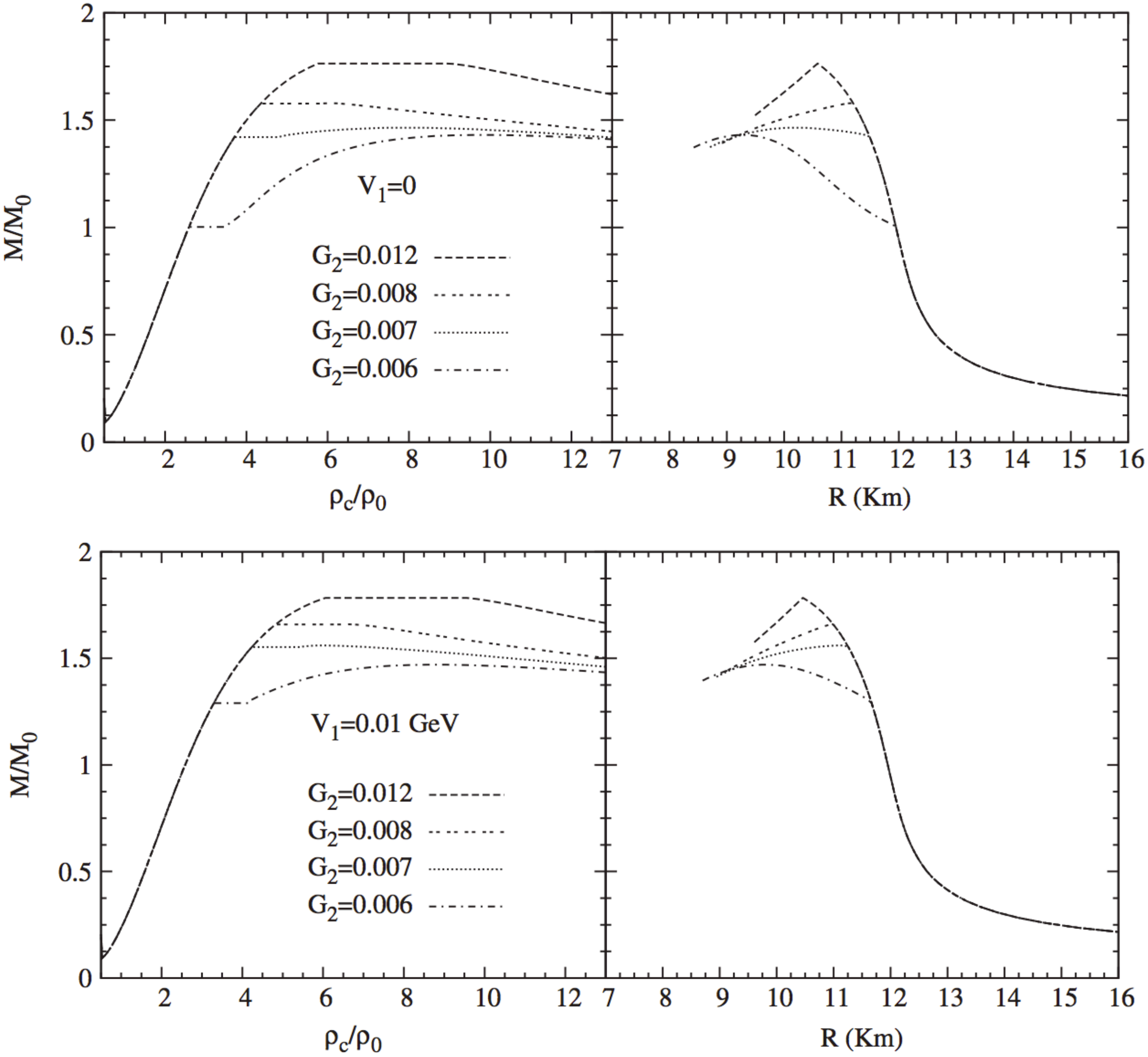
The transition point grows with increasing G_2 and V_1

$G_2(\mu_B)$ induces deformations in the curves

Very large P and μ_B reached at V_1 = 0.3 GeV

Output of TOV eqs. for constant G_2 and V_1 with BHF EoS

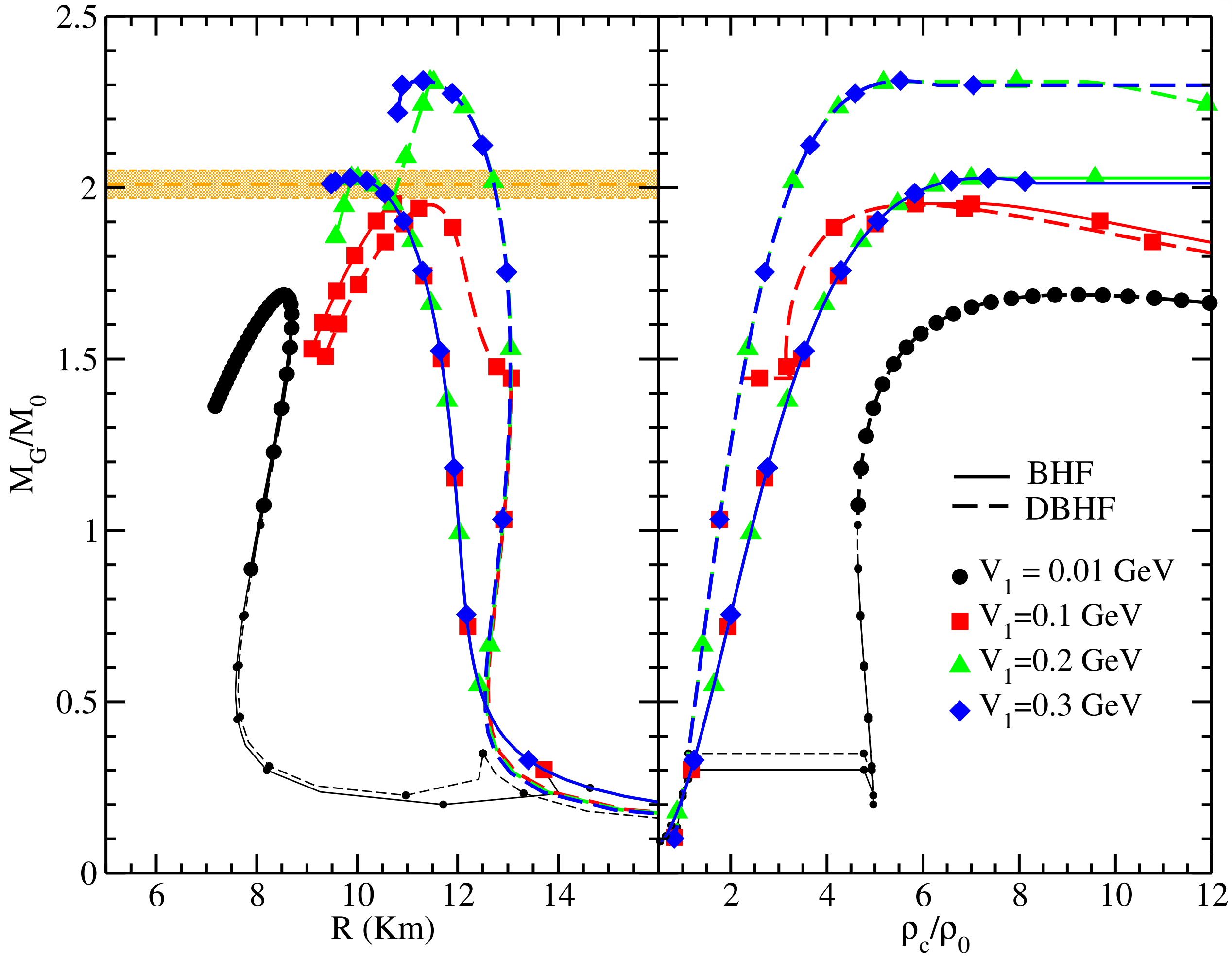
M. Baldo, et al., Phys. Rev. D78, (2008) 063009.



Configurations with $dM / d\rho_c < 0$ give unstable stars

M grows with G_2 and V_1 but pure quark phase tends to disappear and only stars with a mixed phase core survive.

The case with μ_B – dependent G_2



1) V_1 = 0.01 GeV excluded.
2) V_1 =0.1 GeV gives a stable quark phase and M marginally compatible with the data for both hadronic EoS.
3) V_1 =0.2 GeV gives mixed phase . DBHF admits heavier NS masses.
4) V_1 =0.3 GeV hadronic content only.

Final considerations

The range 0.1 GeV ÷ 0.3 GeV for the potential V_1 gives sufficiently heavy masses with quark matter in the NS core (pure or mixed phase). Heavier masses reached with the DBHF hadronic EoS.

V_1 much smaller than $V_1(T_c)$ which, according to the FCM, corresponds to $V_1(T=\mu_B=0) = 0.8 \div 0.9$ GeV .

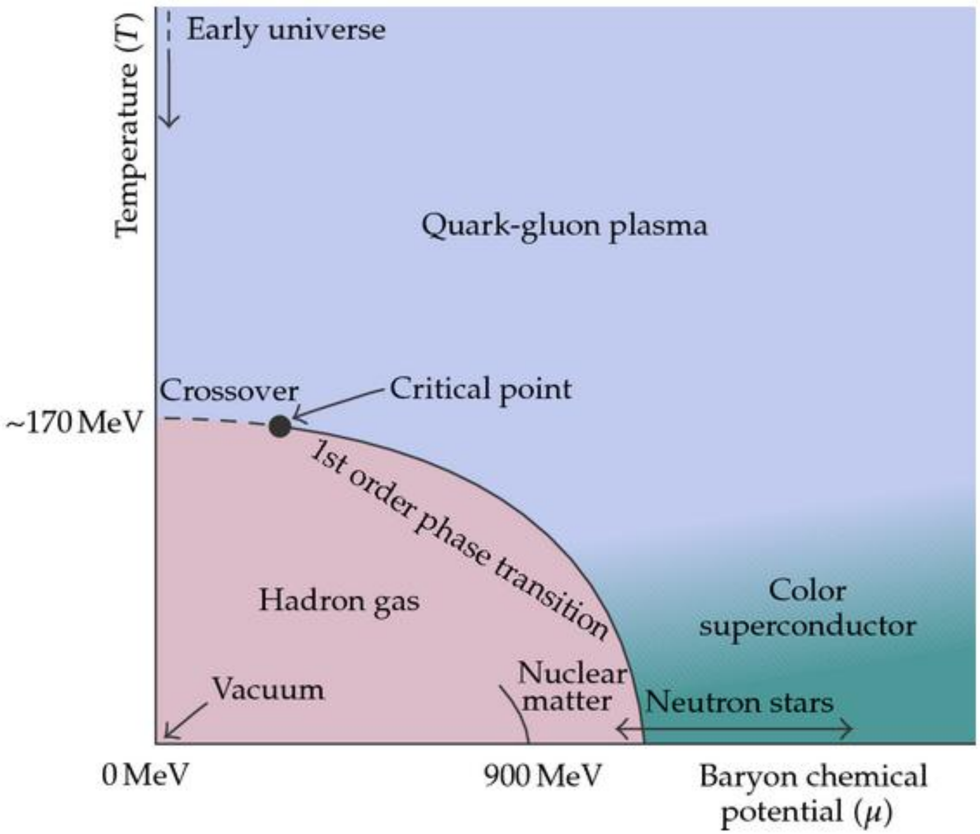
Possible explanation of this strong reduction of the potential :

The q q interactions at high density become more relevant than $\bar{q}$ q and V_1 should be effectively replaced by V_3 and V_6 .

This is supported by their relative weights V_1 [1] , V_3 [1/2] , V_6 [-1/4] that indicate $V_{eff}(\mu_B)=1/4\ V_1(T=\mu_B=0)$ as suggested by our findings

ADDITIONAL SLIDES

QCD PHASE DIAGRAM



Compact stars located in the region around the phase transitions
with large μ_B and $T \approx 0$,
far from the Heavy Ion Collision experiments region.

The pressure gets contribution from various species :

$$P = P_B + P_l$$

$$P_B = \rho^2 \frac{d(\epsilon_B/\rho)}{d\rho} \qquad P_l = \rho^2 \frac{d(\epsilon_l/\rho)}{d\rho}$$

Equilibrium in star matter :

Neutrons, protons and leptons must be in β-equilibrium,

with conserved baryon number and zero electric charge:

$$\mu_n = \mu_p + \mu_{e^-}$$

$$x_p = x_{e^-} + x_{\mu^-}$$

$$1 = x_n + x_p$$

$$x_i = \rho_i/\rho$$

i-species density fraction and ρ is the baryon density

Electrons and muons = relativistic free gas .

Thermodynamical variables expressed in terms of ρ_B

$D^E{}_1(x)$ related to the large distance quark-antiquark potential

$$V_1 = \int_0^{1/T} d\tau (1 - \tau T) \int_0^{\infty} d\chi \chi D_1^E(\sqrt{\chi^2 + \tau^2})$$

and at large x, $D^E{}_1(x)$ is parametrized as

$$D_1^E(x) = D_1^E(0) \mathrm{e}^{-|x|/\lambda}$$

where the QCD correlation lenght is taken λ =0.34 fm.

The C-Electric part is normalized in terms of the gluon condensate G_2 as

$$D^E(0) + D_1^E(0) = \frac{\pi^2}{18} G_2$$

and $G_2 = (\alpha_s/\pi) < G^a{}_{\mu\nu} G^a{}_{\mu\nu} >$ is estimated from QCD sum rules :

$$G_2 = 0.012 \pm 0.006 \ \mathrm{GeV}^4$$

$D^C(x)$, $D^C_1(x)$ contain perturbative and nonperturbative components with the latter relevant at large x.

$D^E(x)$ is responsible for confinement through its relation with the string tension :

$$\sigma^E = (1/2) \int D^E(x) d^2x$$

In the confined phase $D^E(x)$ is finite.

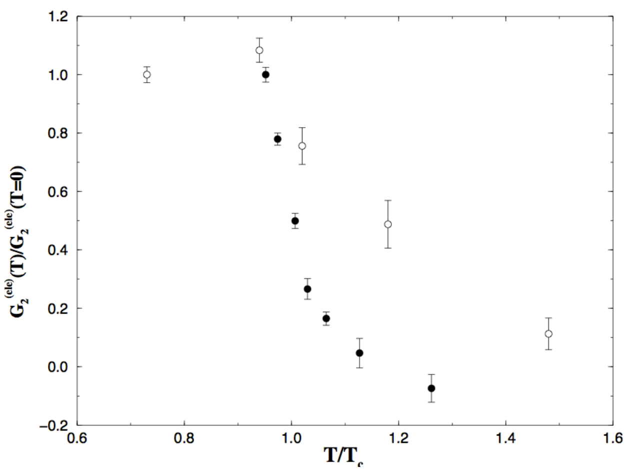
In the deconfined phase it vanishes with σ^E, while the other correlators remain finite in both phases.

T dependence of G_2

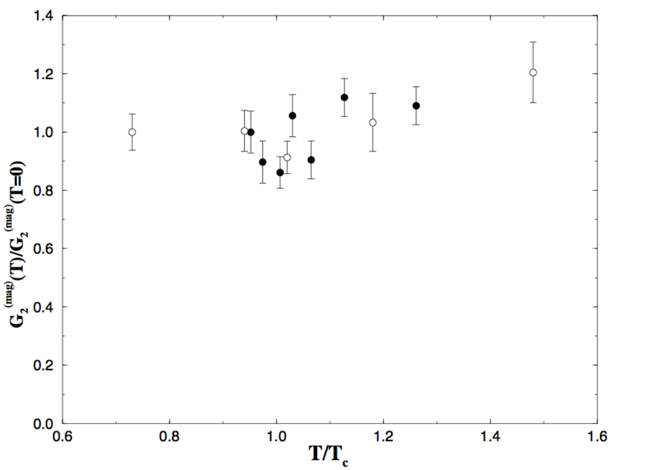
At $\mu_B = 0$ lattice simulations provide these indications for $G_2\,(T)$

$G_2\,(T=0) = G_E\,(T=0) + G_M\,(T=0)$ and $G_E\,(T=0) \approx G_M\,(T=0) \approx G_2\,(T=0)\,/2$

(E)



(M)



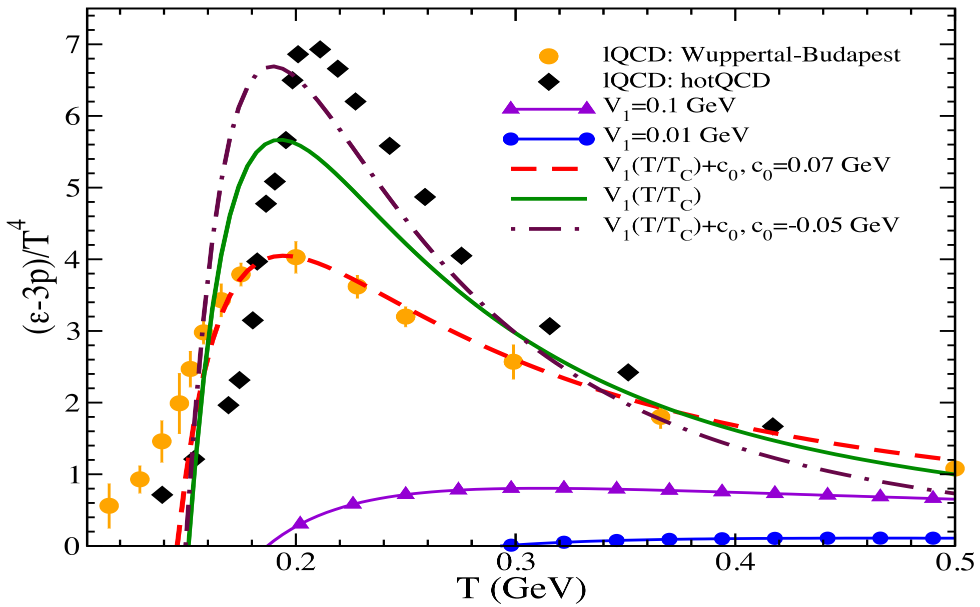
$T<T_C$	$T>T_C$
$G_E\,(T) \approx G_M\,(T) \approx G_2(T=0)/2$	$G_E\,(T) \approx 0$, $G_M\,(T) \approx G_2(T=0)/2$
$G_2\,(T) \approx G_2(T=0)$	$G_2\,(T) \approx G_2(T=0)\,/2$

Indications at finite T and $\mu_B = 0$

Lattice → change for $T>T_C$: $G_2(T) \approx G_2(T=0)/2$.

M. D'Elia, A. Di Giacomo,E. Meggiolaro, Phys. Lett.B408 (1997) 315 ;Phys. Rev.D67(2003)114504.

Estimate of V_1 from the interaction measure $\Delta(T) = (\varepsilon - 3p)/T^4$



(Wuppertal-Budapest)
S. Borsanyi et al., JHEP 1011 (2010) 077.

(hotQCD)
A. Bazavov et al., Phys. Rev. D 80 (2009) 014504.

Small constant V_1 (**0.01 GeV** —— , **0.1 GeV** ——) highly disfavoured.

$$V_1(T) = V_0 + 0.175 \left(1.35 \frac{T}{T_c} - 1\right)^{-1} \text{ GeV}$$

— · — ($V_1(T_C)$ = **0.45 GeV**) in agreement with '09 hotQCD data.

- - - ($V_1(T_C)$ = **0.57 GeV**) in agreement with W.- B. data.

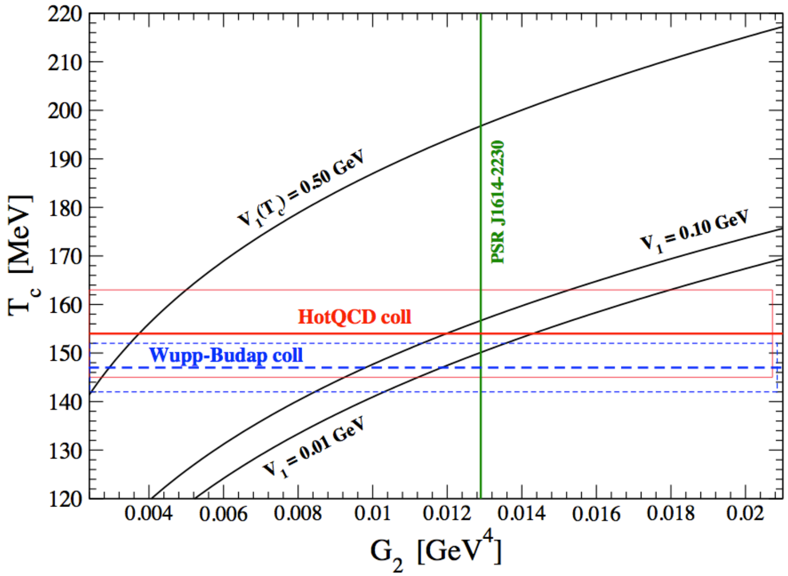
The analysis on a large set of data for $T>T_C$ suggests larger $V_1(T_C)$ w.r.t. those obtained from the determination of T_C.

FCM predicts

$$T_c = \frac{a_0 G_2^{1/4}}{2}\left(1+\sqrt{1+\frac{V_1(T_c)}{2a_0 G_2^{1/4}}}\right)$$

$$a_0 = (3\pi^2/768)^{1/4}$$

Bombaci



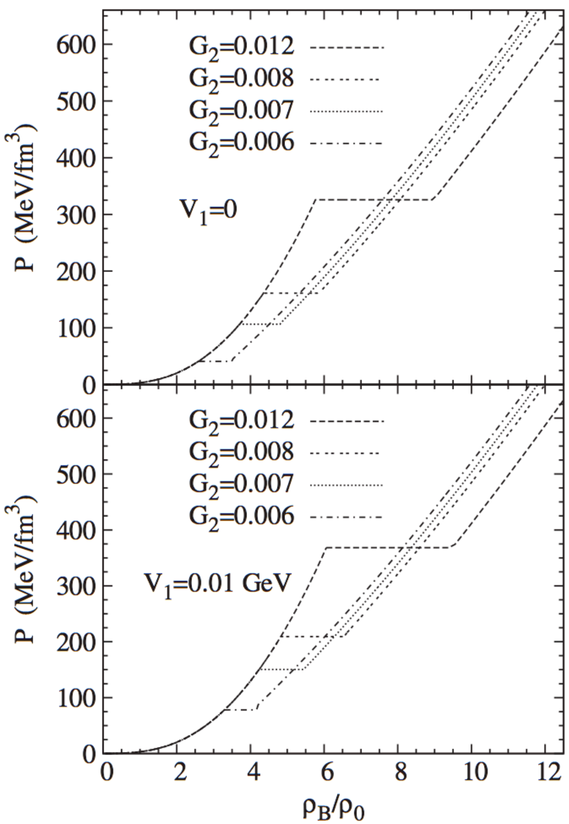
However :

1) Only the lattice determination of T_C is involved.
2) Theoretical prediction of T_C affected by approximations.

Discontinuity at the transition (first order transition) : plateaus in $P(\rho_B)$ (Maxwell construction)

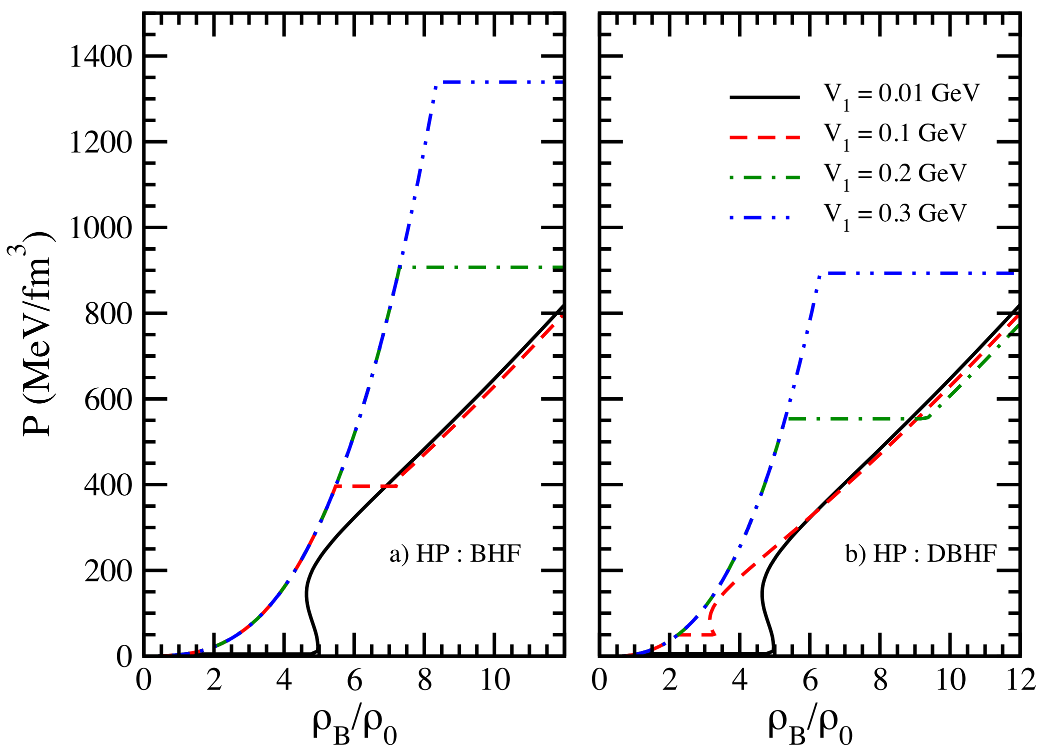
G_2 and $V_1 \approx 0$ constant

BHF



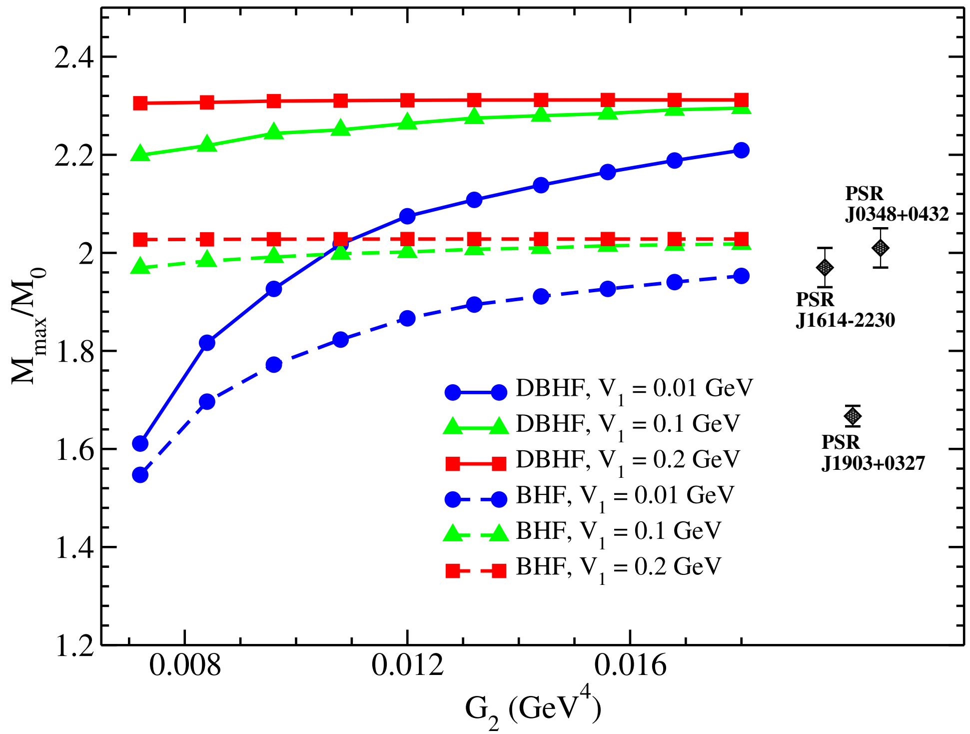
$G_2(\mu_B)$ and V_1 constant

BHF DBHF



S-shape region for V_1 =0.01 GeV gives unstable stars

Extending the range of V_1 with constant G_2



1) BHF EoS marginally in agreement with the data. DBHF EoS predicts larger ***M.***

2) V_1 =0.01 GeV requires G_2 > 0.012 GeV^4 .

3) Data suggest larger values of V_1 .

4) At V_1 =0.2 GeV M is practically independent of G_2 .
   The Quark Eos becomes irrelevant.