



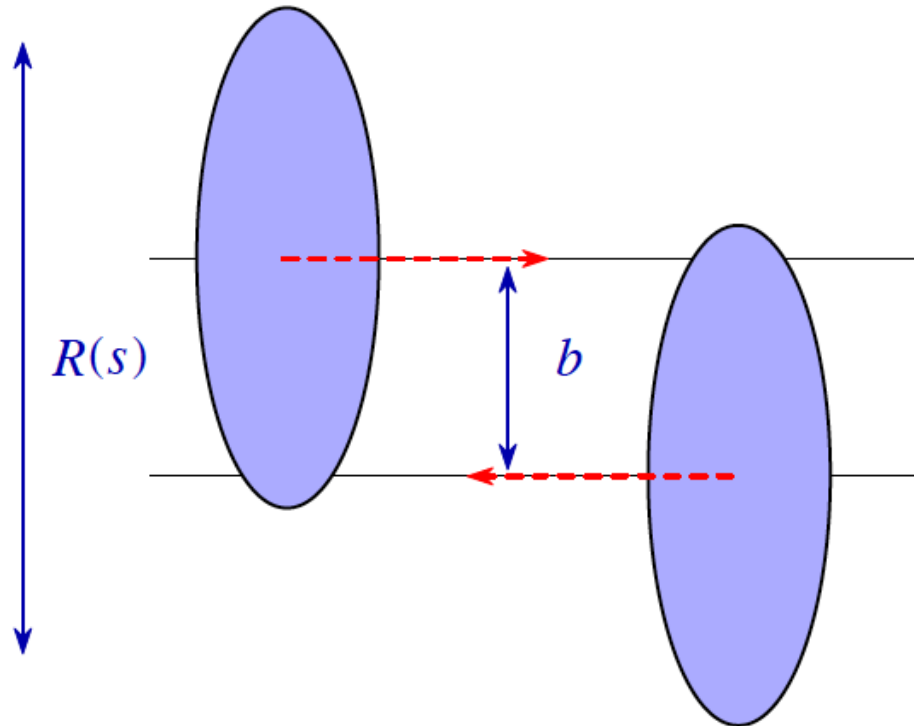
# Geometrical scaling in high energy collisions and its breaking

Michał Praszalowicz  
Jagellonian University  
Kraków, Poland

# „Geometrical Scaling”

J. Dias de Deus, Nucl. Phys. B 59 (1973) 231;  
A.J. Buras, J. Dias de Deus, Nucl.Phys. B 71 (1974) 481;  
J. Dias de Deus, P. Kroll, J. Phys. G 9 (1983) L81;  
J. Dias de Deus, Acta Phys. Polon. B 6 (1975) 613.

$$A(b,s) = A(b/R(s))$$





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*Geometric scaling for the total  $\gamma^*$  p cross-section in the low  $x$  region.*

A.M. Stasto, K. J. Golec-Biernat, J. Kwiecinski PRL 86 (2001) 596-599

M. P. and T. Stebel JHEP (2013) 1303 090, 1304 169

$$\sigma_{\gamma^*p} \sim \frac{F_2(x, Q^2)}{Q^2} = \sigma_0 \mathcal{F} \left( \frac{Q^2}{Q_{\text{sat}}^2(x)} \right)$$



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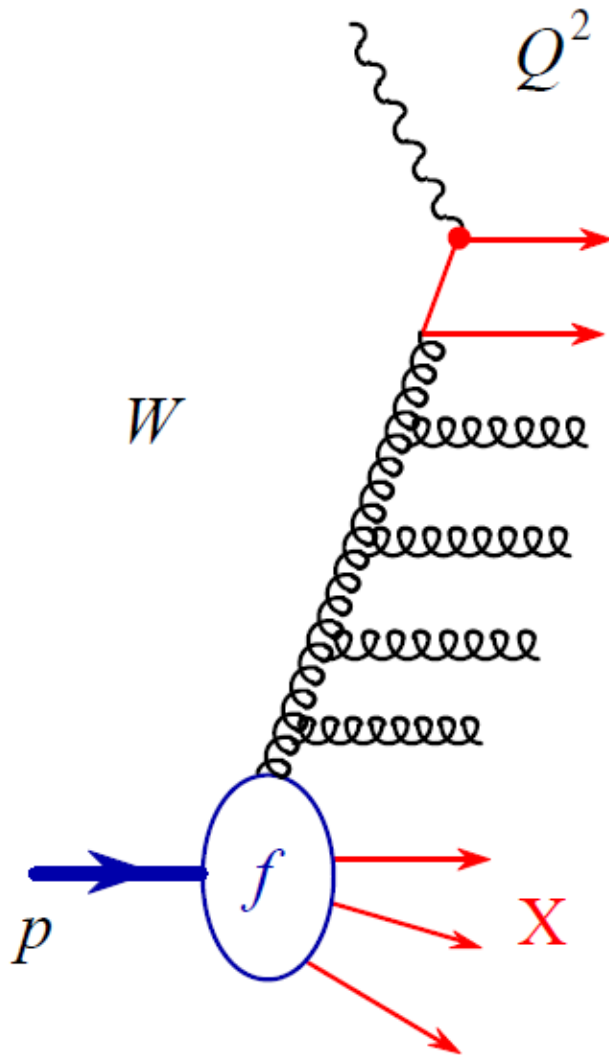
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L. McLerran, M. P. Acta Phys.Polon.B41:1917,2010, B42:99,2011  
M. P. Phys.Rev.Lett.106:142002,2011, Acta Phys.Pol. B42 (2011) 1557-1566  
Phys.Rev. D87 (2013) 071502(R)  
L. McLerran, M.P. and B. Schenke, arXiv:1306.2350 [hep-ph] (Nucl. Phys. A)

$$\frac{dN_{\text{ch}}}{d\eta dp_{\text{T}}^2}(s, p_{\text{T}}) = \frac{1}{Q_0^2} \mathcal{F} \left( \frac{p_{\text{T}}^2}{Q_{\text{sat}}^2(s)} \right)$$

# Geometrical scaling in DIS at low $x$



$$dP \sim \frac{\alpha_s C_R}{\pi^2} \frac{d^2 k_T}{k_T^2} \frac{d\xi}{\xi}$$

Resumations:

$$\int \frac{d^2 k_T}{k_T^2} \rightarrow \ln Q^2$$

$$\sum \alpha_s^n \ln^n Q^2 \rightarrow \text{DGLAP}$$

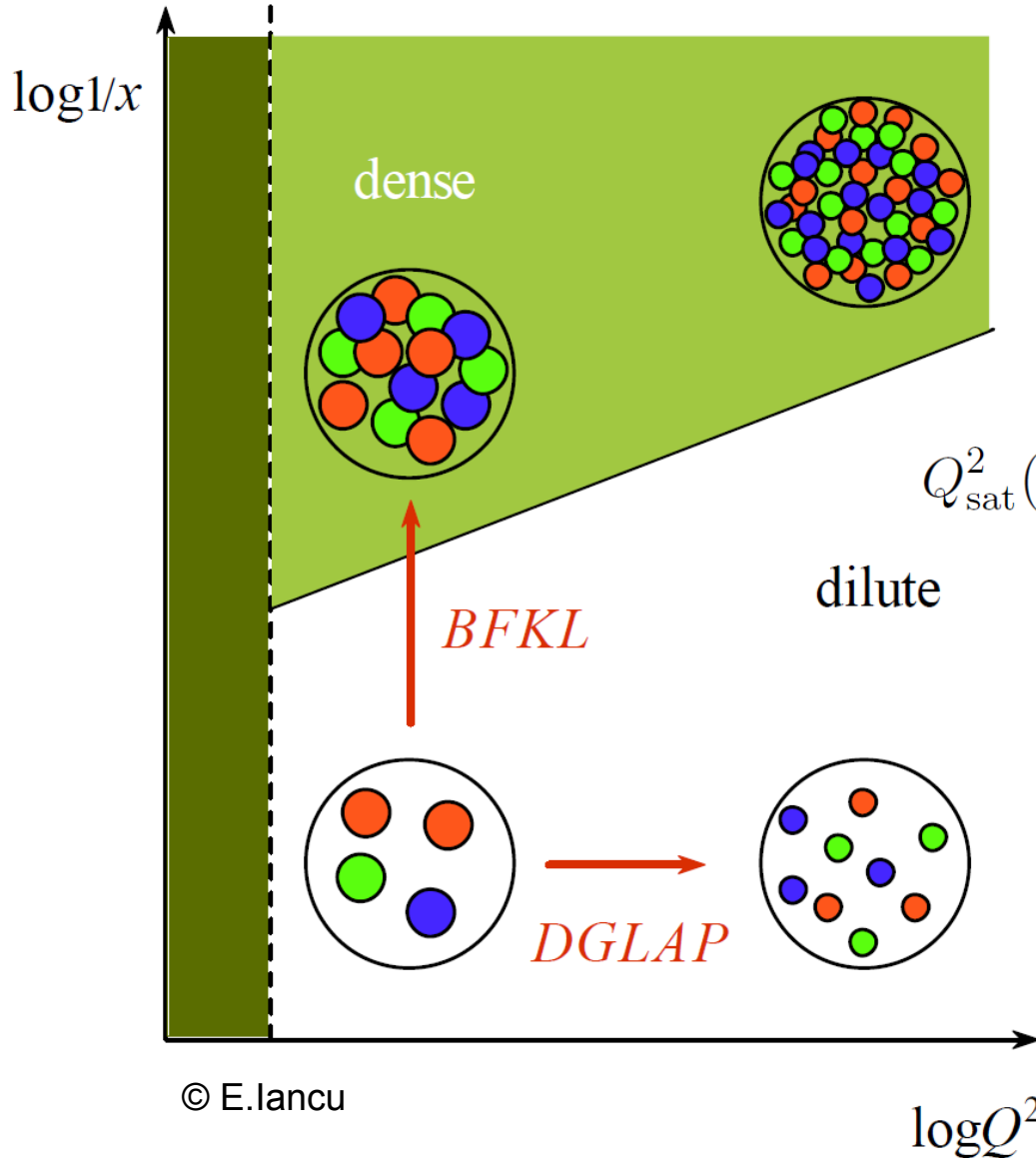
$$\int \frac{d\xi}{\xi} \rightarrow \ln W$$

$$\sum \alpha_s^n \ln^n W \rightarrow \text{BFKL}$$



# Saturation

small  $x$   
large  $W$



large  $x$   
small  $W$



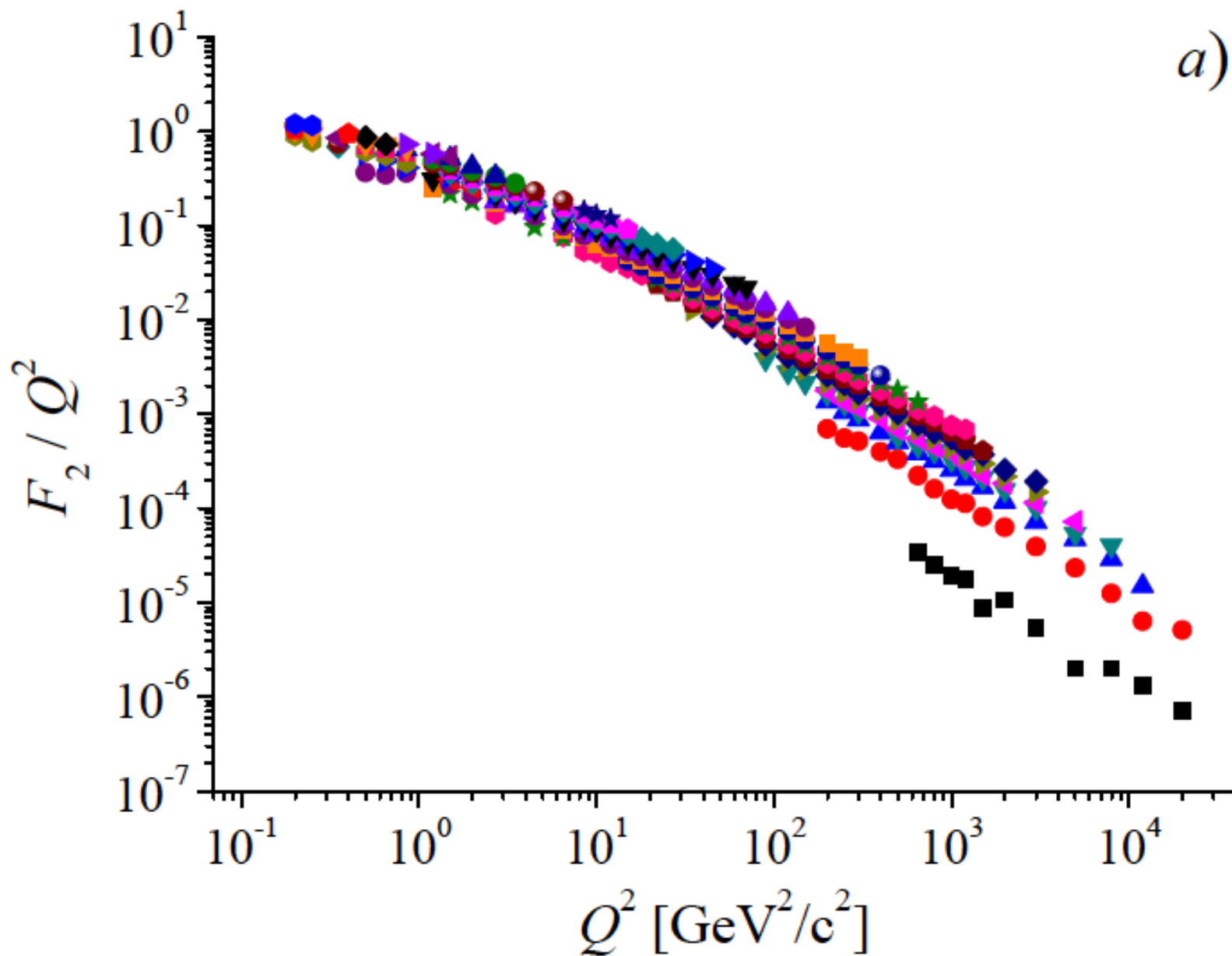
# Geometrical Scaling in DIS

$$\sigma_{\gamma^*p}(x, Q^2) = \sigma_{\gamma^*p} \left( \frac{Q^2}{Q_{\text{sat}}^2(x)} \right)$$

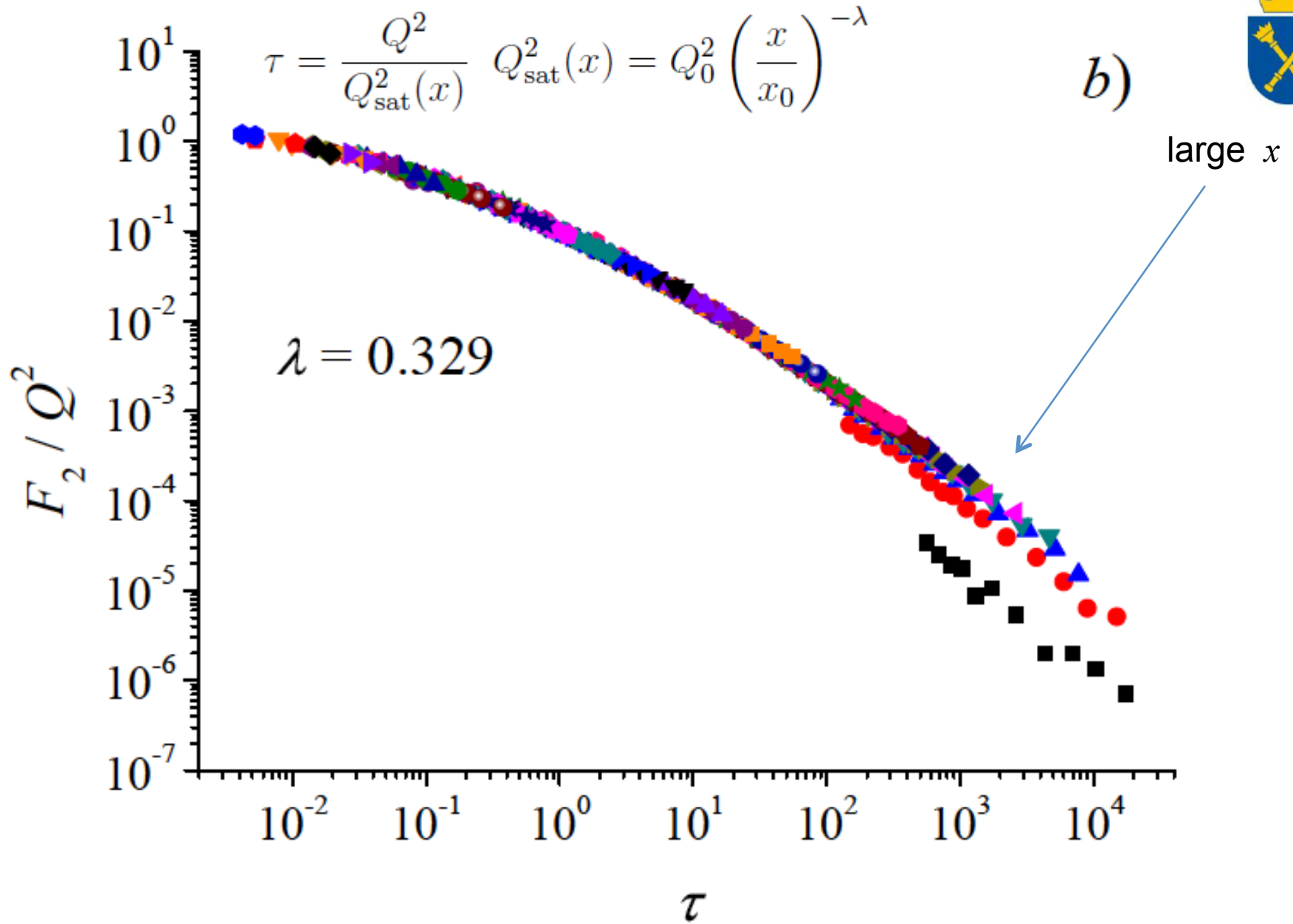
Combined HERA data 2009 for  $e^+$



a)









# Domain of GS in DIS

$$\lambda = 0.329 \pm 0.005$$

up to  $x = 0.08$  (!)

# Saturated gluonic matter at the LHC



„Old”, conventional physics with a new tool:

# Saturated gluonic matter at the LHC

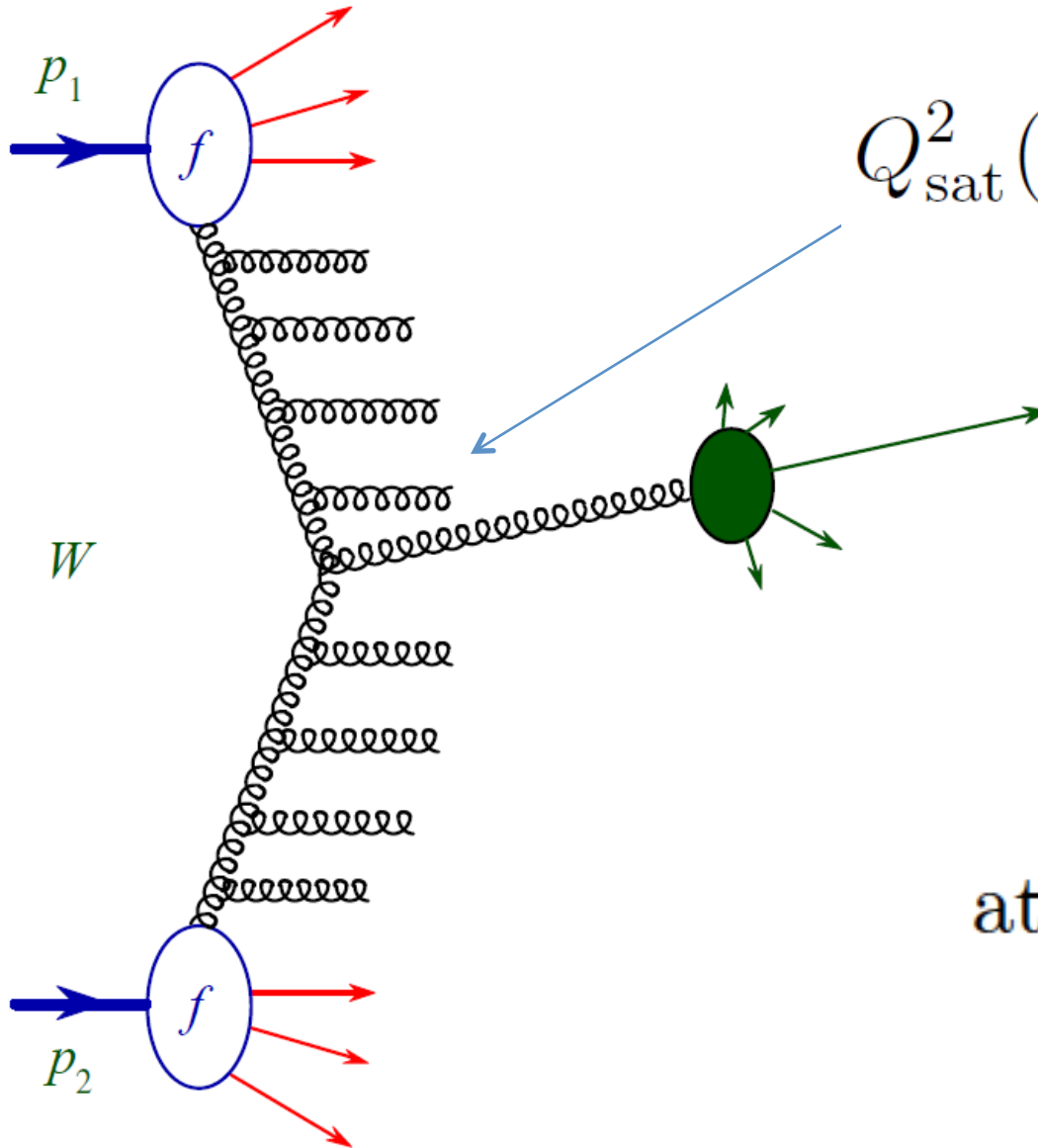


„Old”, conventional physics with a new tool:

$$Q_{\text{sat}}^2(x) = Q_0^2 \left( \frac{x}{x_0} \right)^{-\lambda}$$



# p-p at the LHC



$$Q_{\text{sat}}^2(x) = Q_0^2 \left( \frac{x}{x_0} \right)^{-\lambda}$$

$$x = \frac{p_T}{W} e^{\pm y}$$

at the LHC  $y \sim 0$

# Geometrical scaling of $p_T$ distributions

$$\frac{dN_{\text{ch}}}{dy dp_T^2}(s, p_T) = \frac{1}{Q_0^2} F(\tau)$$

multiplicity distribution  
is a universal function  
of scaling variable  $\tau$

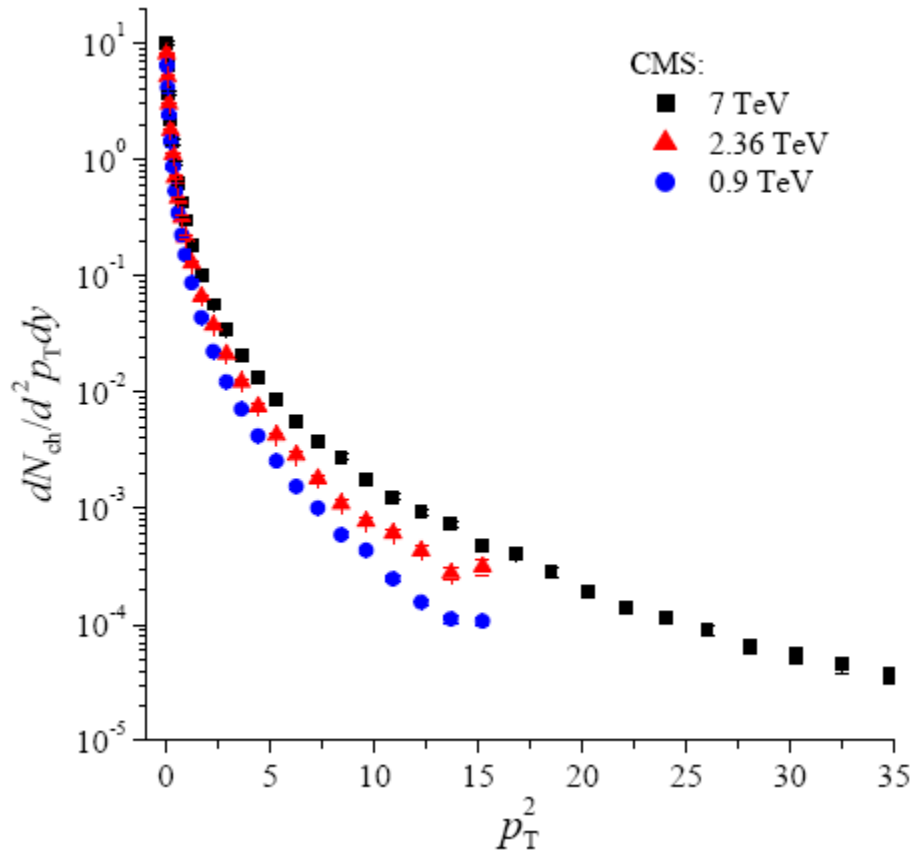
$$Q_{\text{sat}}^2(x) = Q_0^2 \left( \frac{1}{x_0} \frac{p_T}{\sqrt{s}} \right)^{-\lambda}$$



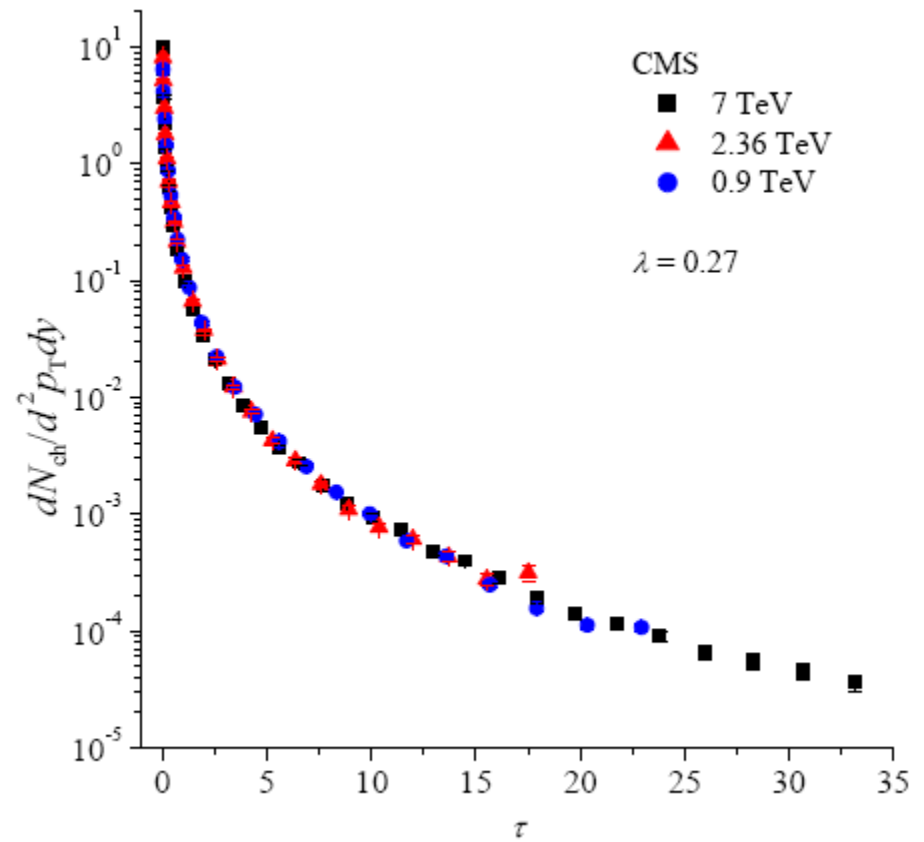
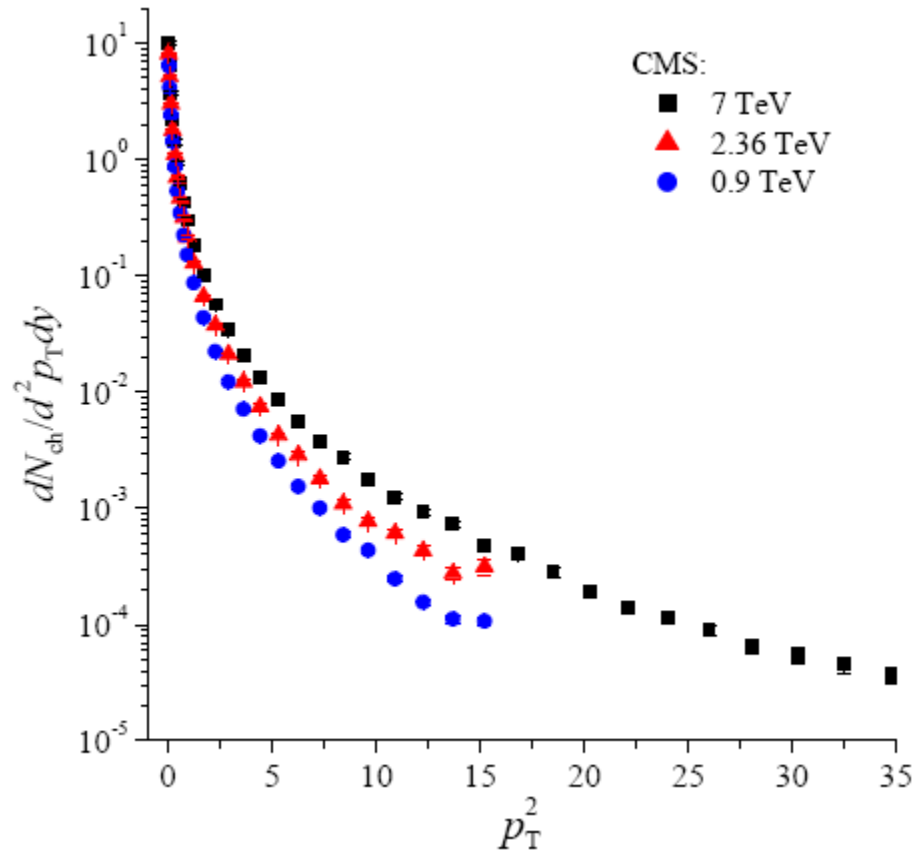
$$\tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left( \frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^\lambda$$

note that for  $\lambda = 0$  scaling variable  $\tau = p_T^2$

# Geometrical scaling of $p_T$ distributions



# Geometrical scaling of $p_T$ distributions

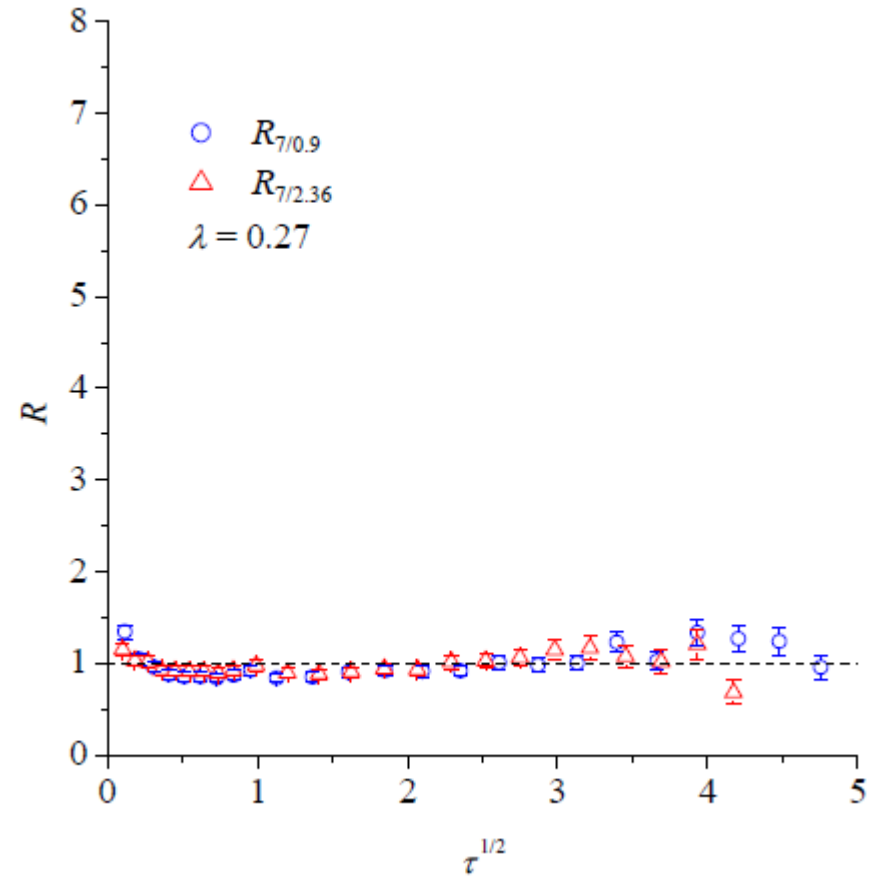
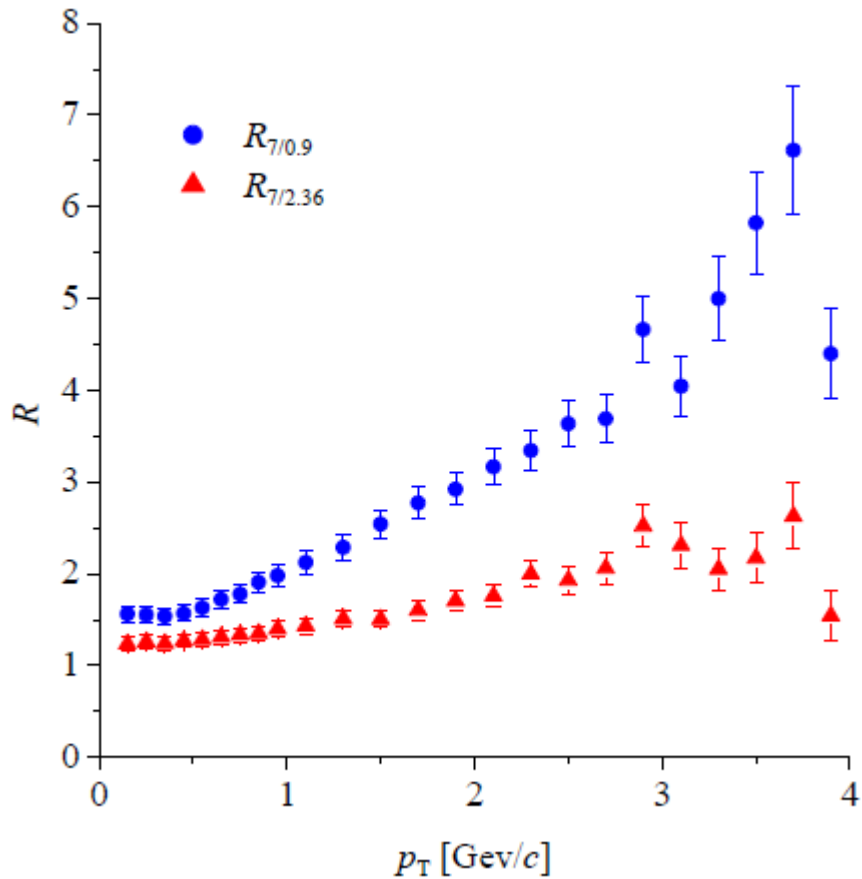






# Ratios of $p_T$ spectra

quality of GS can be examined by looking at the ratios:



small increase with  $\tau$

# Geometrical scaling of $p_T$ distributions

$$\frac{dN_{\text{ch}}}{dy dp_T^2} = \frac{1}{Q_0^2} F(\tau) \quad \longrightarrow \quad \frac{dN_{\text{ch}}}{dy} = \int \frac{dp_T^2}{Q_0^2} F(\tau)$$

$$\tau = \frac{p_T^2}{Q_0^2} \left( \frac{p_T}{W} \right)^{\lambda/2}$$

$$W \sim \sqrt{s}$$

# Geometrical scaling of $p_T$ distributions

$$\frac{dN_{\text{ch}}}{dy dp_T^2} = \frac{1}{Q_0^2} F(\tau) \quad \longrightarrow \quad \frac{dN_{\text{ch}}}{dy} = \int \frac{dp_T^2}{Q_0^2} F(\tau)$$

$$\tau = \frac{p_T^2}{Q_0^2} \left( \frac{p_T}{W} \right)^{\lambda/2} \quad \text{integral over } d\tau \text{ is energy independent}$$

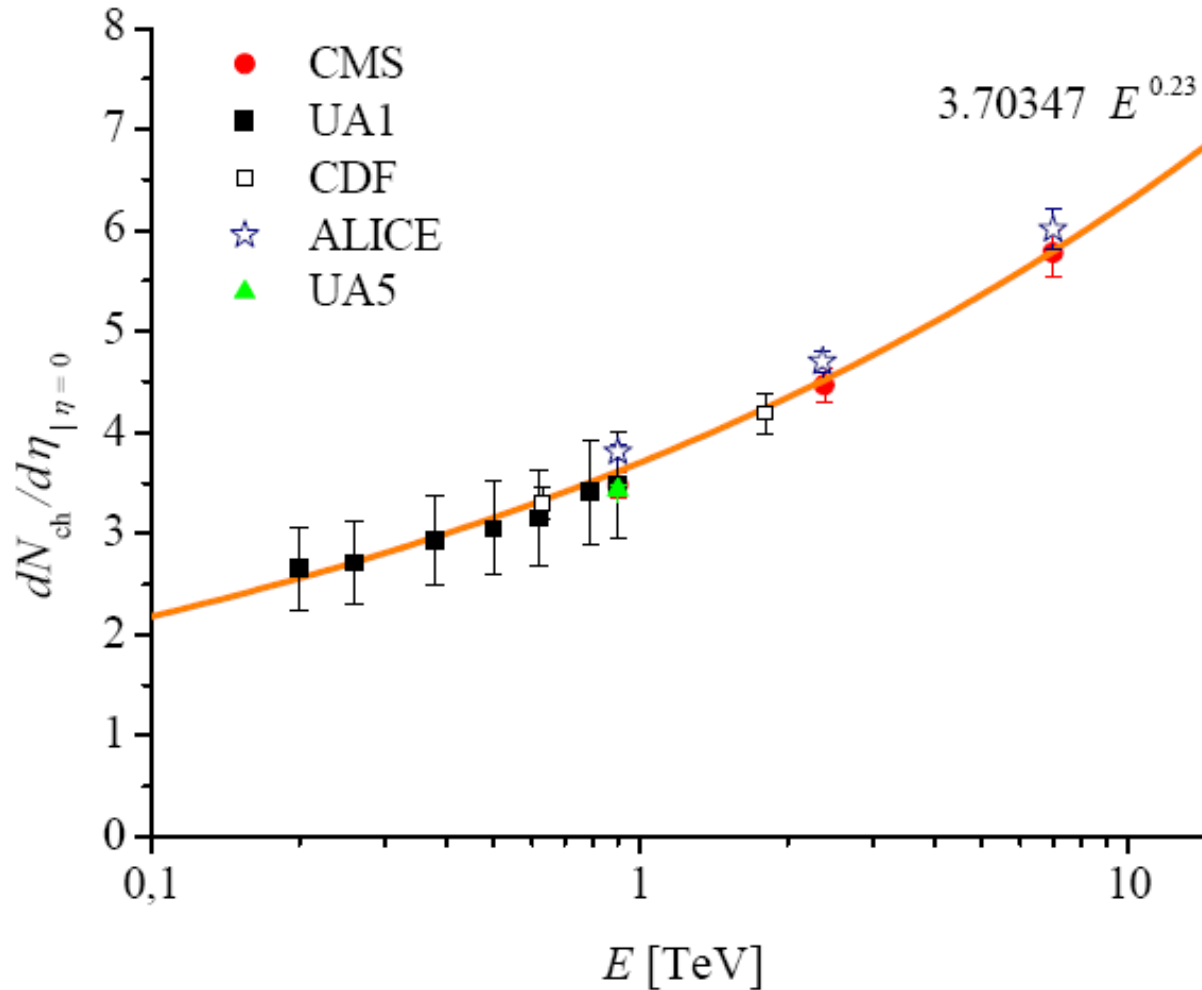
$$W \sim \sqrt{s} \quad \frac{dp_T^2}{Q_0^2} = \frac{2}{2 + \lambda} \left( \frac{W}{Q_0} \right)^{\frac{2\lambda}{2+\lambda}} \tau^{-\frac{\lambda}{2+\lambda}} d\tau$$

effective growth  
of multiplicity is  
slower than  $\lambda$

$$\lambda_{\text{eff}} = \frac{2\lambda}{2 + \lambda} < \lambda$$

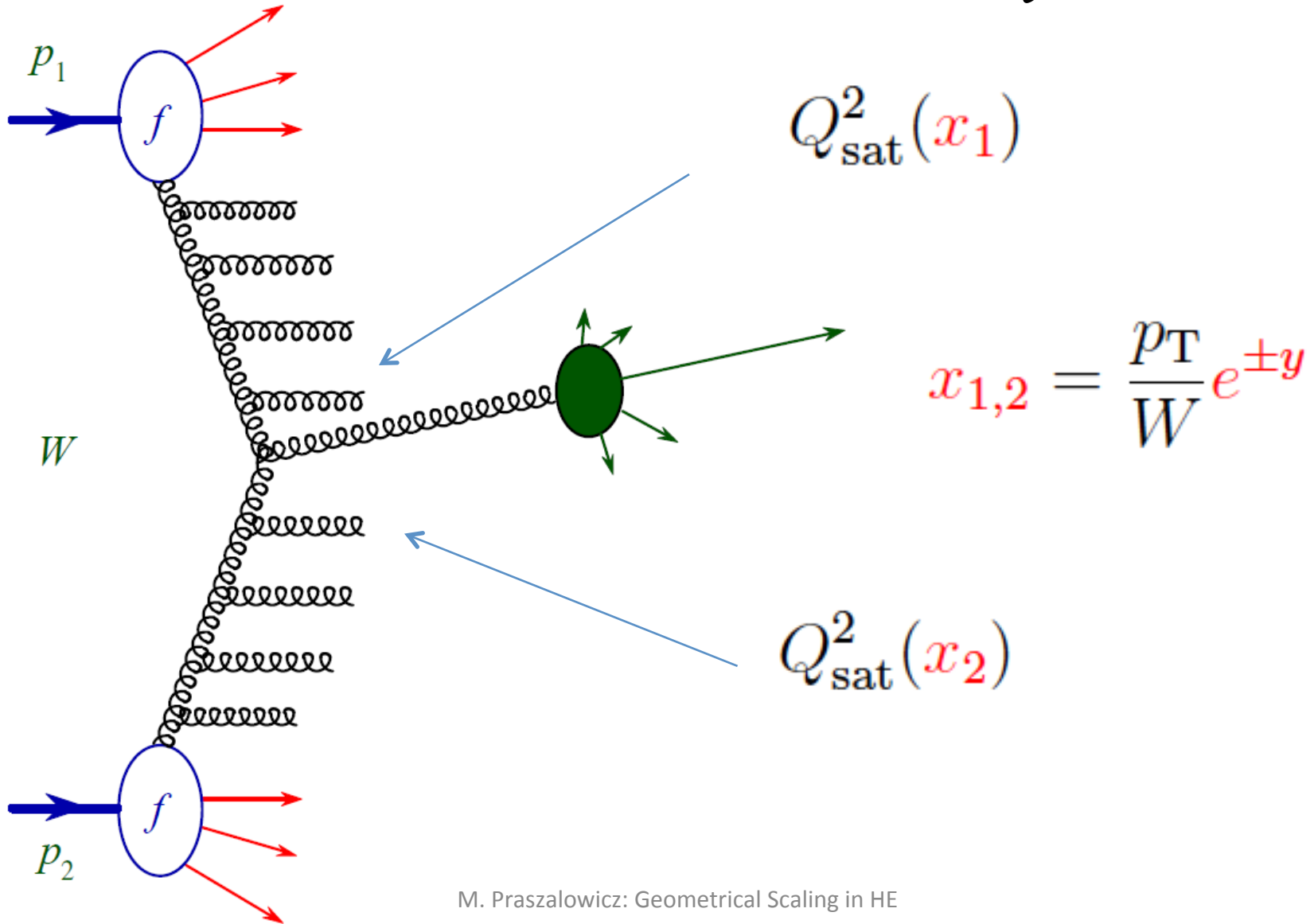


# Multiplicity from GS





# p-p at forward rapidity $y > 0$





# Kinematical range of GS in pp

$$x_1 < x_{\max}$$



$$p_{T\max}(W, y) < x_{\max} W e^{-y}$$



# Kinematical range of GS in pp

$$x_1 < x_{\max}$$



$$p_{T\max}(W, y) < x_{\max} W e^{-y}$$

transverse momentum should be larger than some nonperturbative scale  $\Lambda$

$$p_{T\min} > \Lambda$$



# NA61 Shine data

9th Polish Workshop on Relativistic Heavy-Ion Collisions  
"From p-p to p-Pb and Pb-Pb collisions"

24-25 November 2012 *Collegium Maius, Jagiellonian University*  
Poland (timezone)

Hadron spectra: p+p vs. Pb+Pb at the SPS energies

Szymon Puławski  
for NA61/SHINE Collaboration

University of Silesia, Katowice

- Data analyzed:

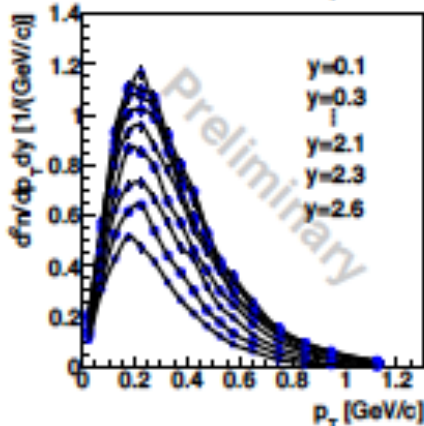
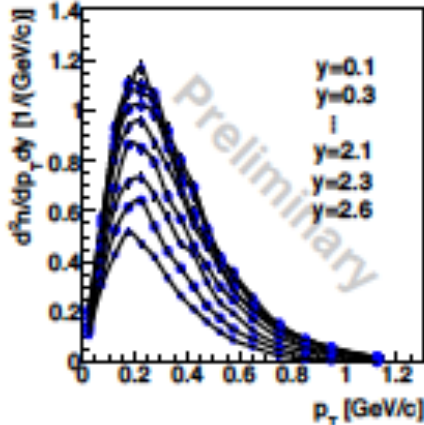
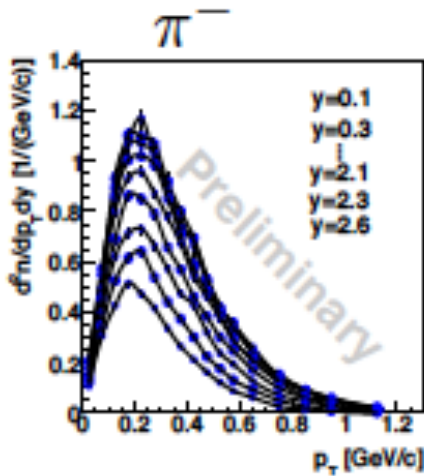
- p+p @ 20 GeV/c ( $\sqrt{s} = 6.2$  GeV):  $1.3 \cdot 10^6$  events
- p+p @ 31 GeV/c ( $\sqrt{s} = 7.7$  GeV):  $3.1 \cdot 10^6$  events
- p+p @ 40 GeV/c ( $\sqrt{s} = 8.8$  GeV):  $5.2 \cdot 10^6$  events
- p+p @ 80 GeV/c ( $\sqrt{s} = 12.3$  GeV):  $4.3 \cdot 10^6$  events
- p+p @ 158 GeV/c ( $\sqrt{s} = 17.3$  GeV):  $3.5 \cdot 10^6$  events

M. Praszalowicz: Geometrical Scaling in HE

40 GeV/c

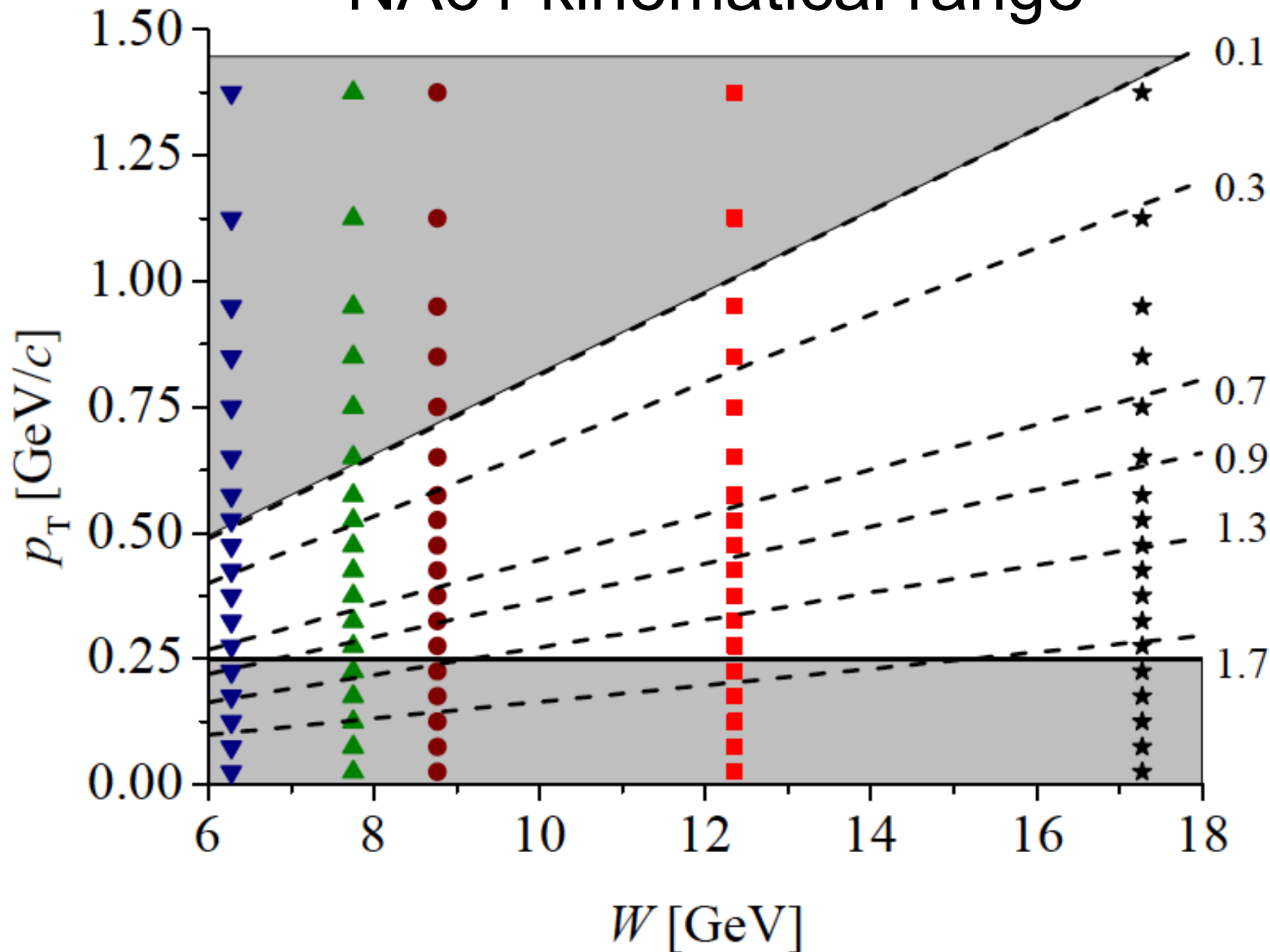
80 GeV/c

158 GeV/c

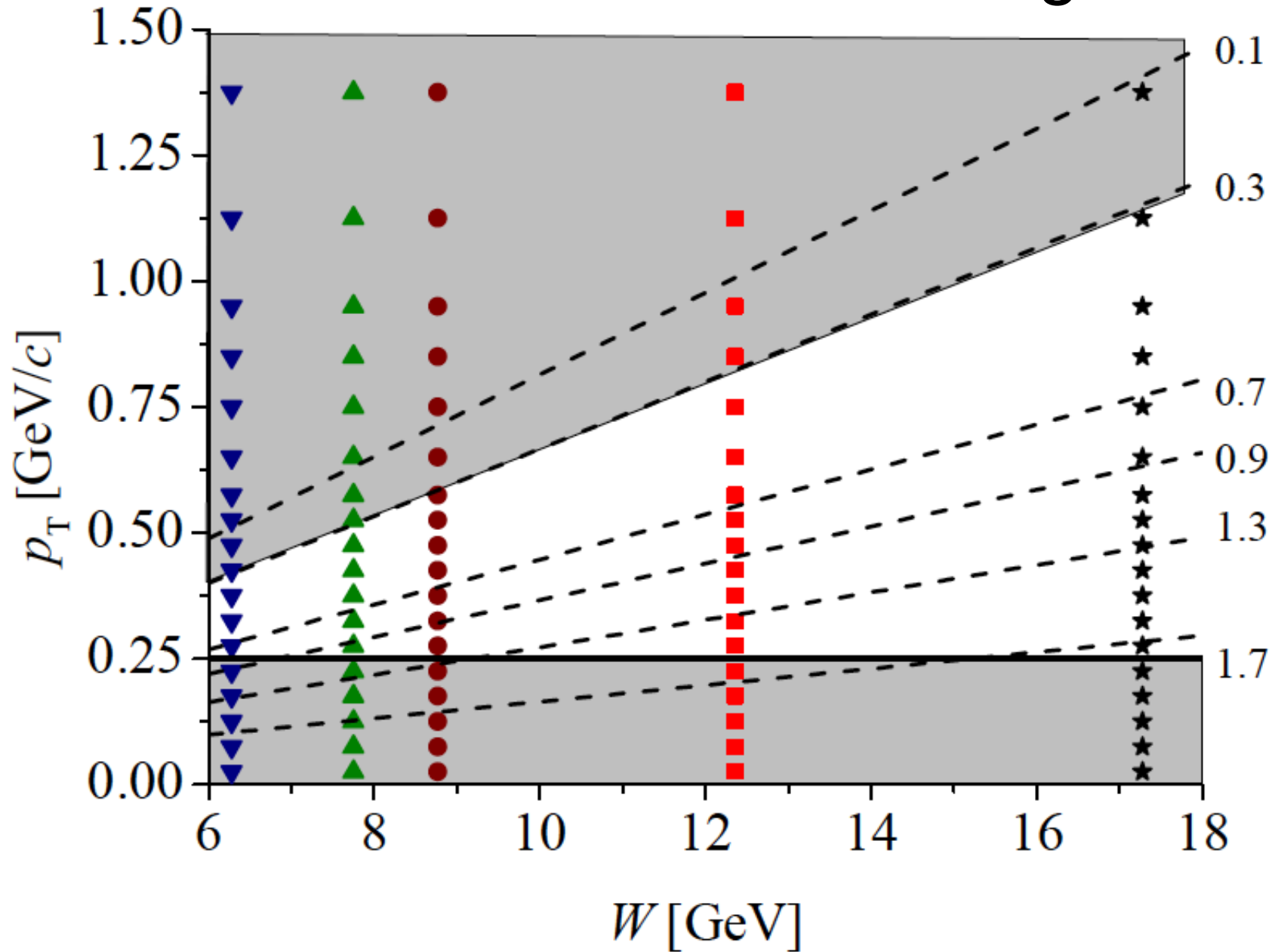




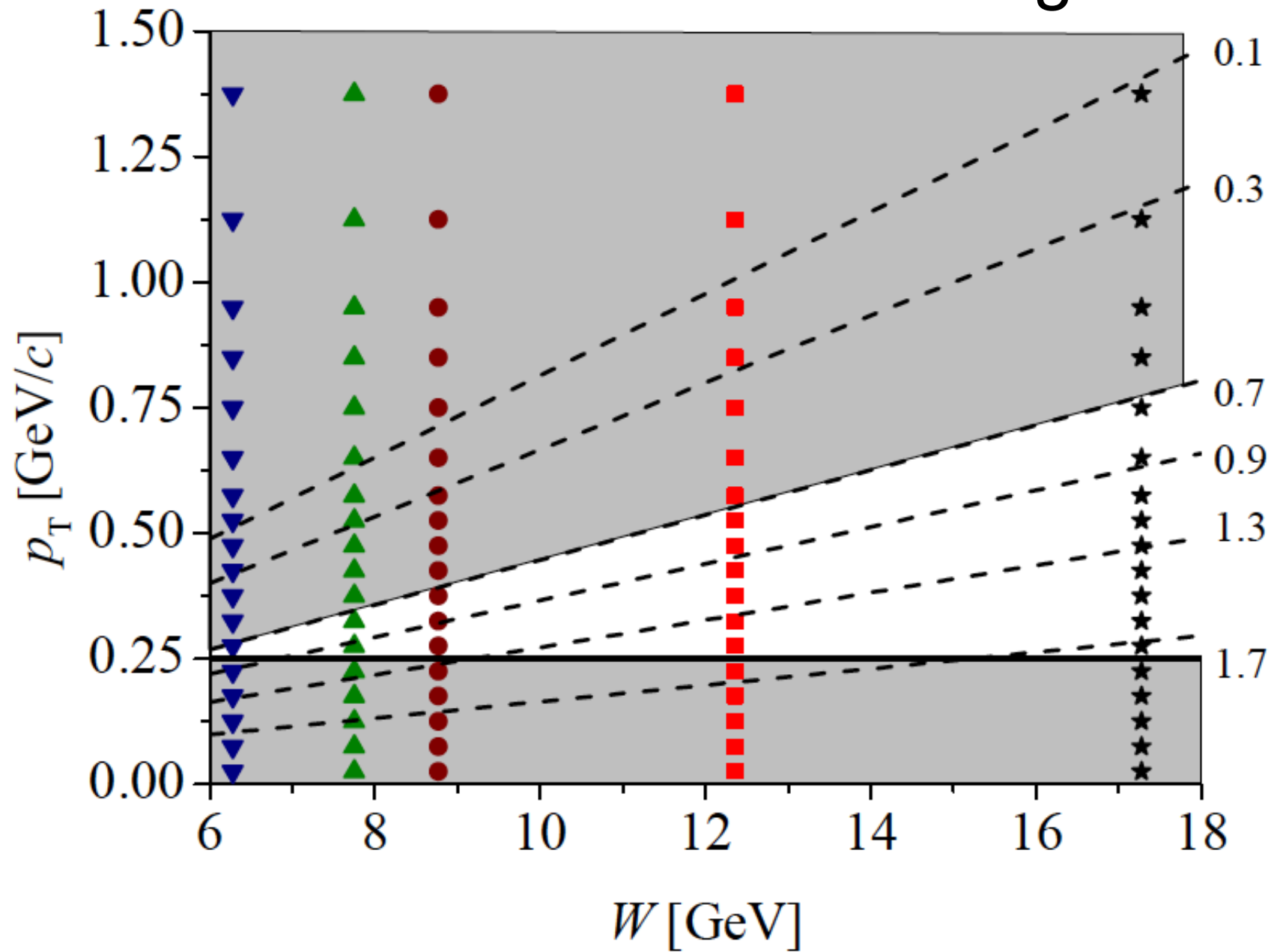
# NA61 kinematical range



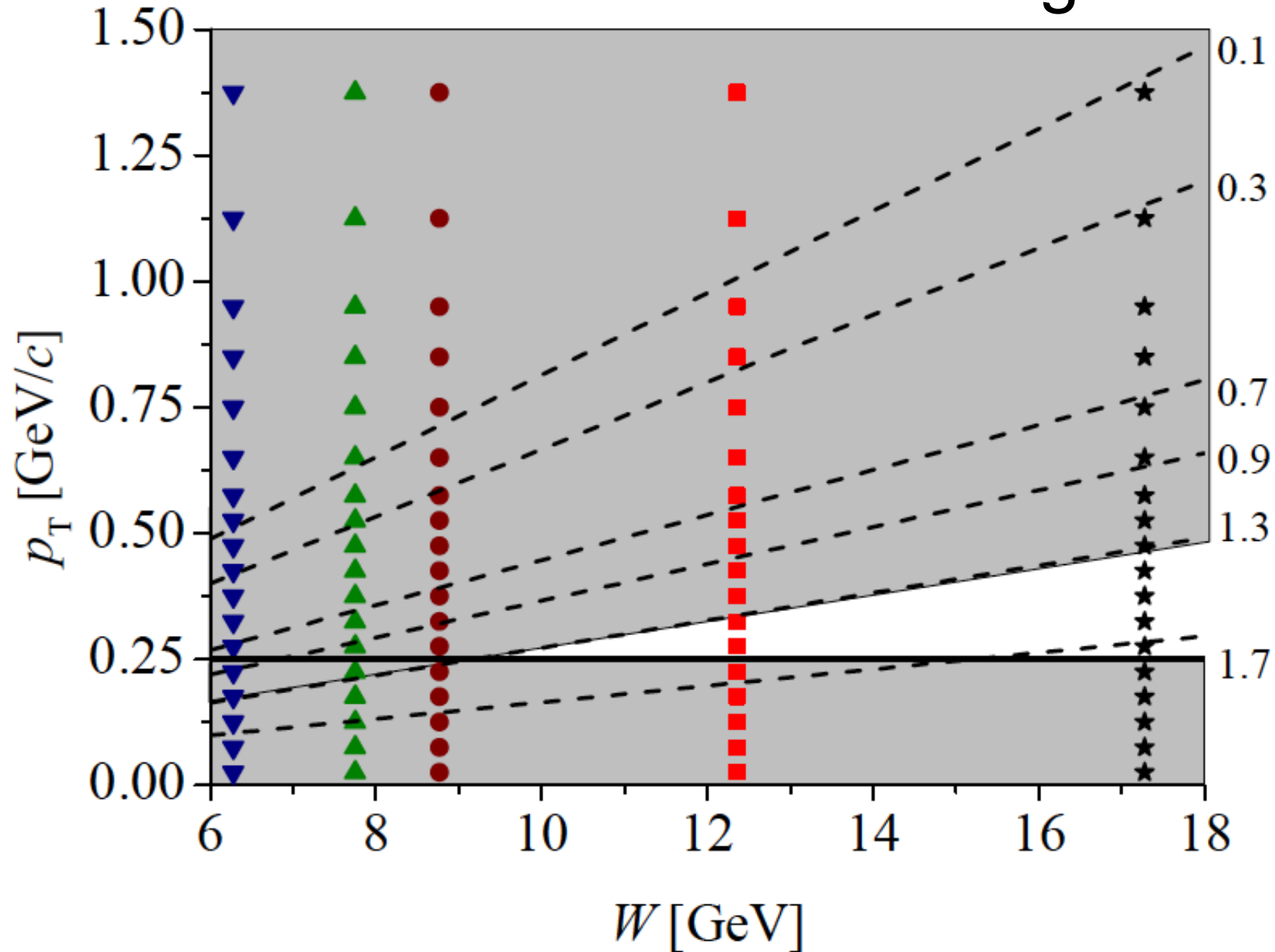
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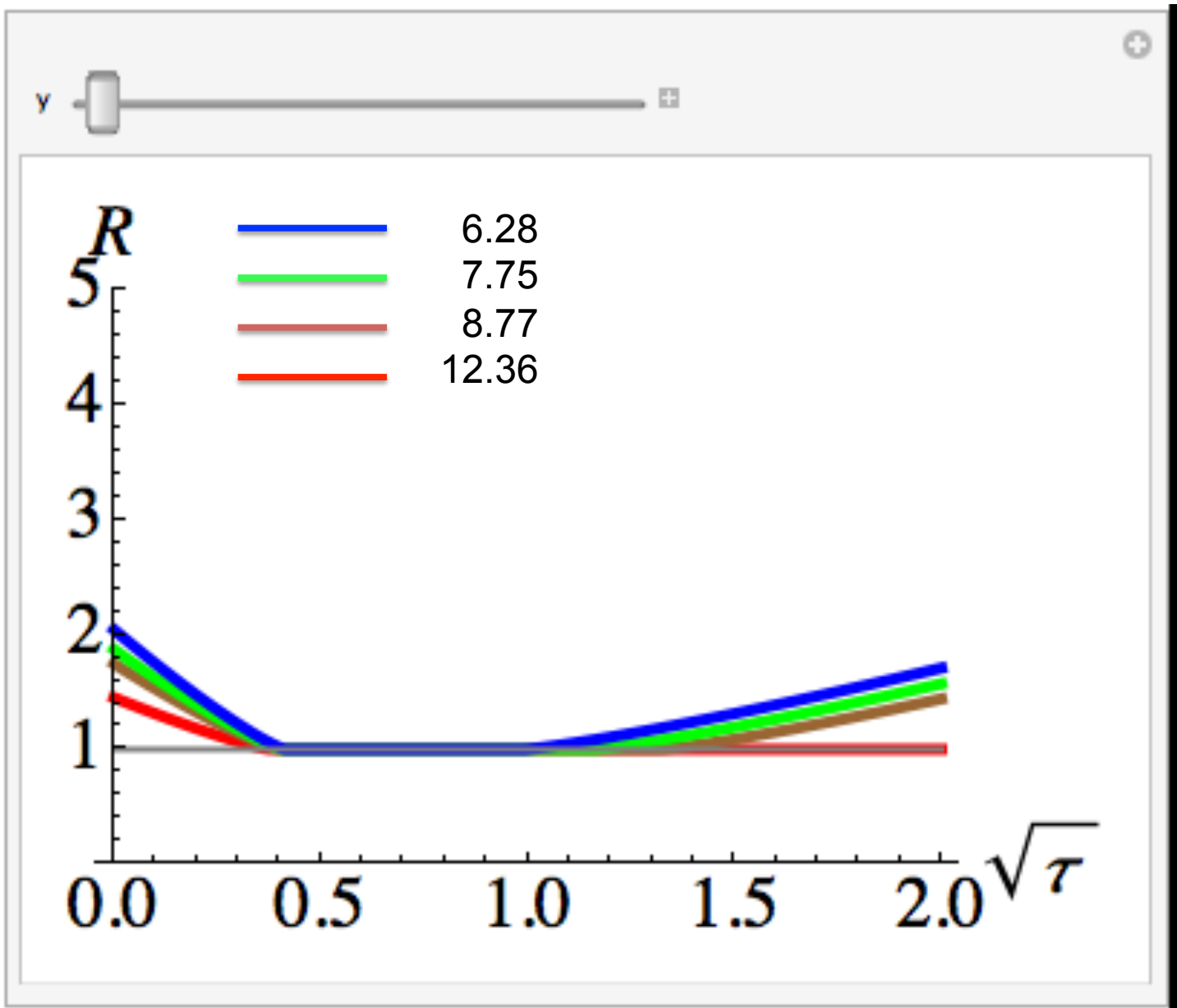


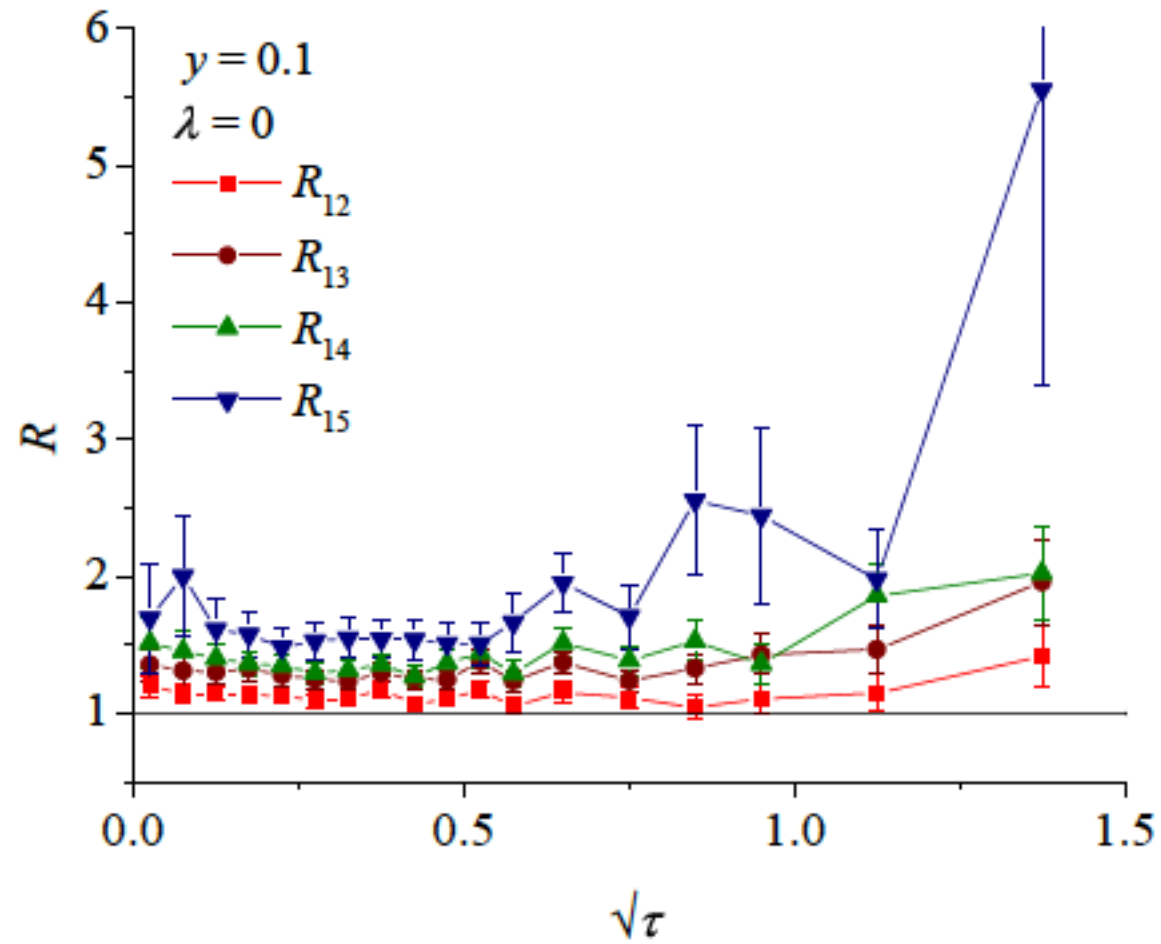
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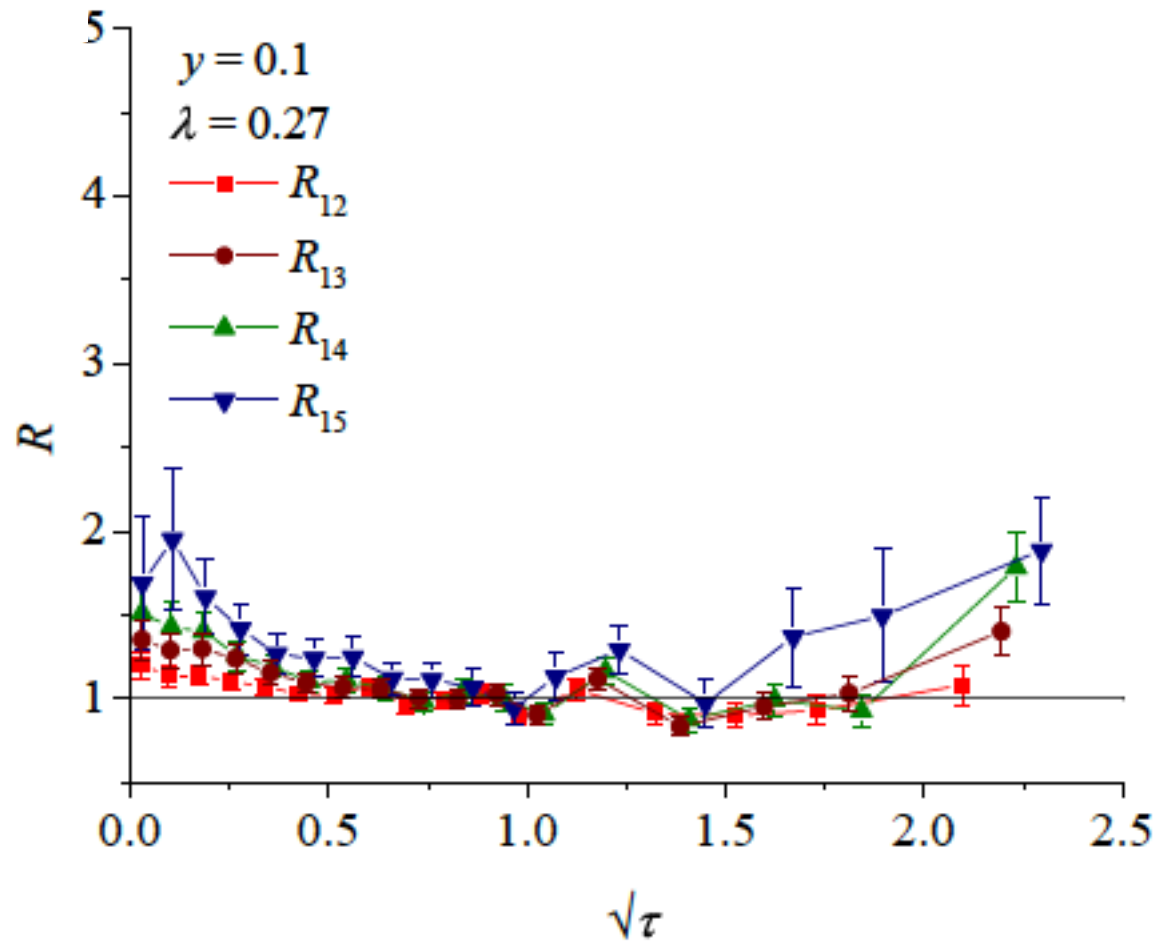
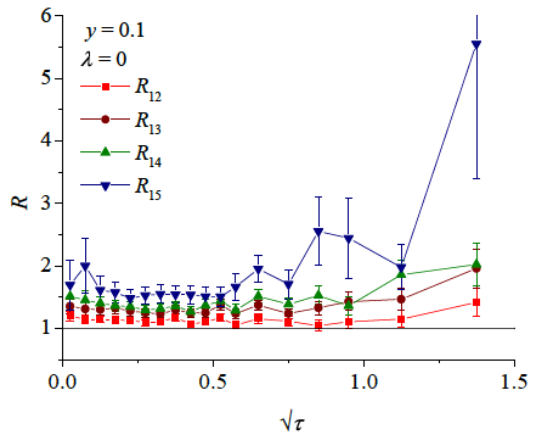


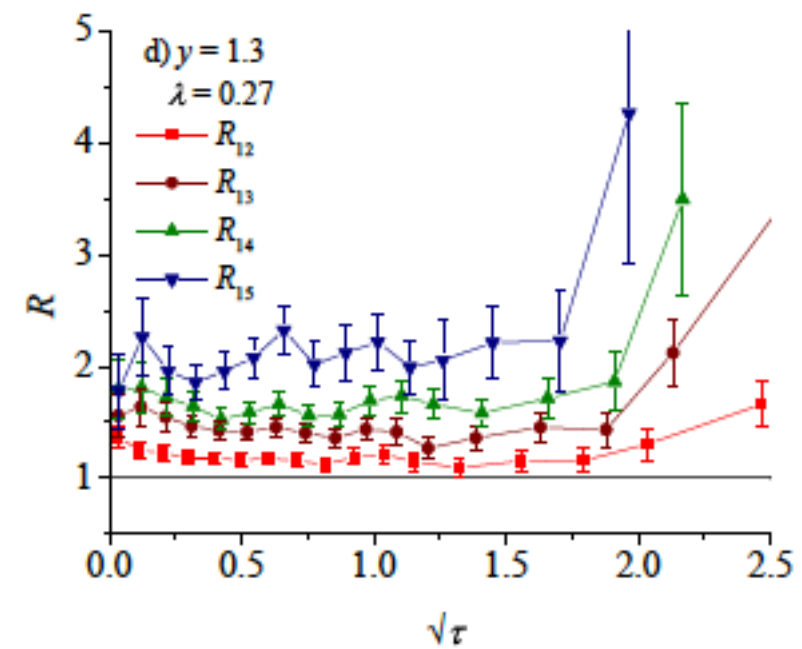
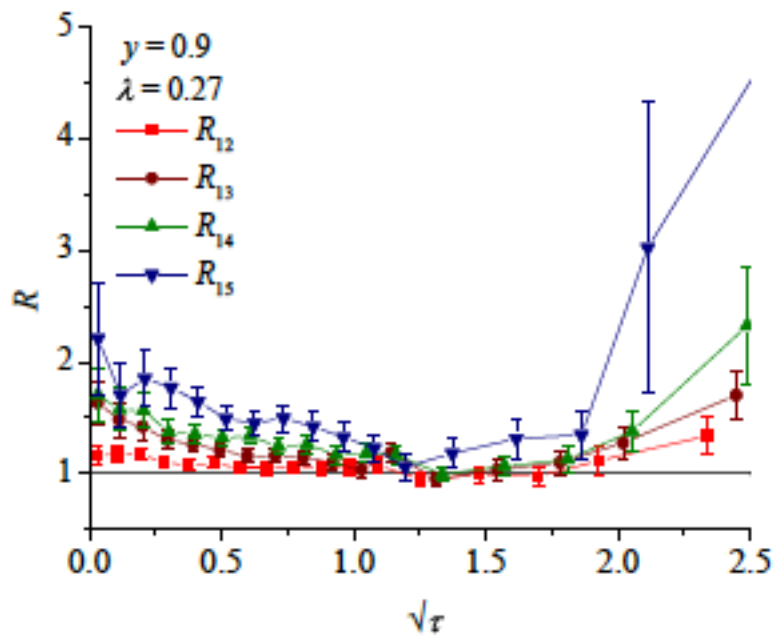
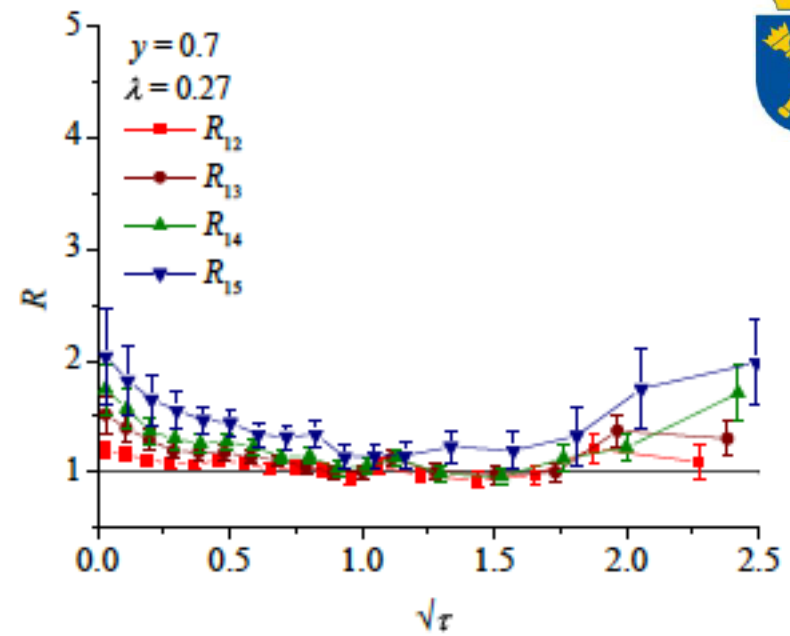
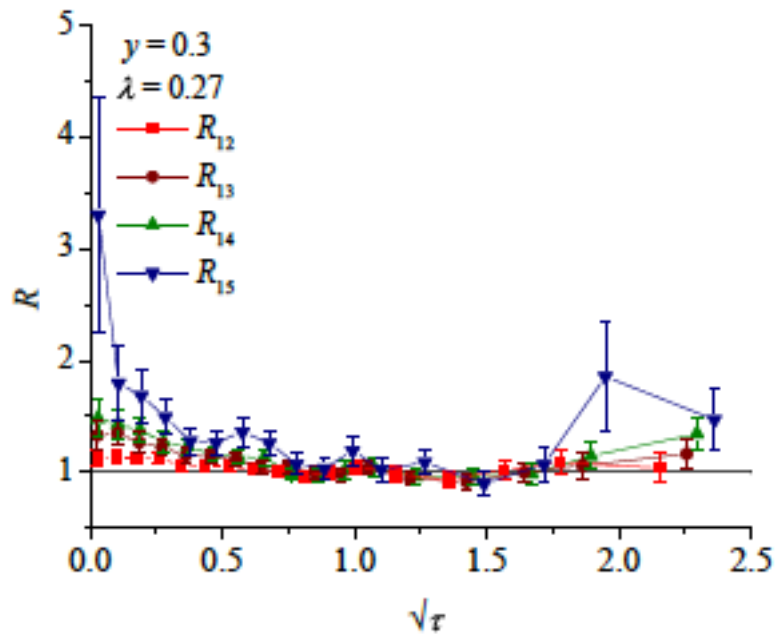
# NA61 kinematical range









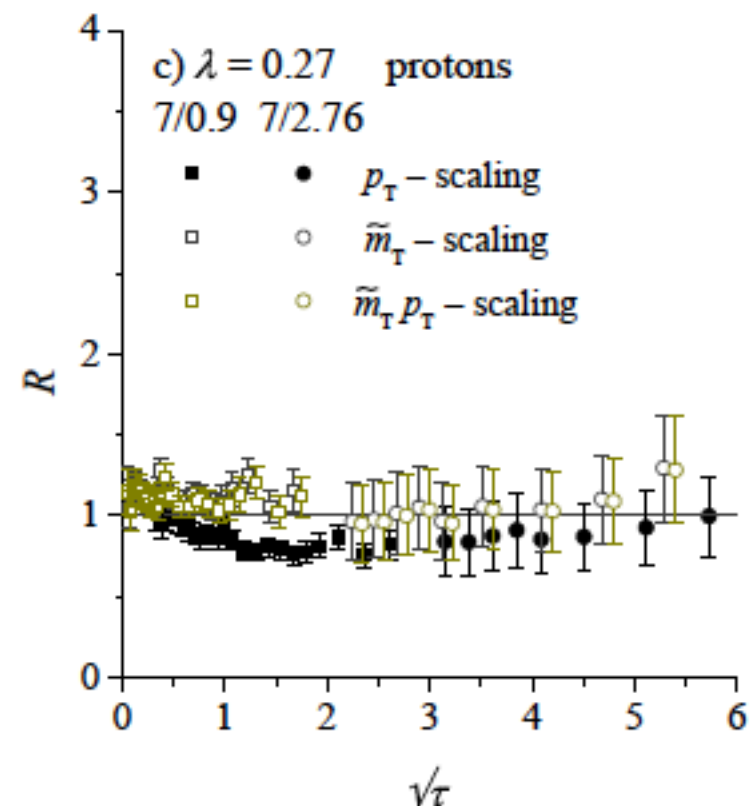
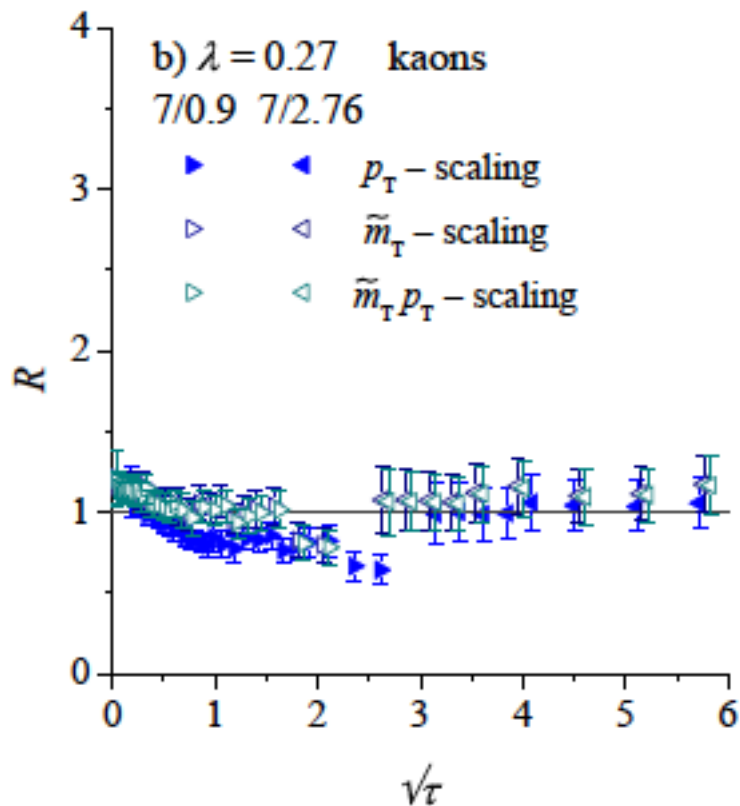






# GS for identified particles

$$\tau_{p_T} = \frac{p_T^2}{Q_S^2} = \frac{p_T^2}{Q_0^2} \left( \frac{p_T}{W} \right)^\lambda \quad \longrightarrow \quad \tau_{\tilde{m}_T} = \frac{\tilde{m}_T^2}{Q_0^2} \left( \frac{\tilde{m}_T}{W} \right)^\lambda$$





# Summary

- GS in DIS works for rather high Bjorken  $x$
- GS works also for charge particles in pp
- GSV is found for  $y \neq 0$  in agreement with experimental data
- GS for identified particles in  $m_T - m$

Universal shape of GS

connection with Tsallis-like parametrization

relation to unintegrated PDFs

GS in HI

A dependence on the saturation scale

centrality dependence

why pp  $\lambda$  is different than in DIS?

quantitative analysis is needed

**Why does this work?**



# Summary

- GS in DIS works for rather high Bjorken  $x$
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relation to unintegrated gluon

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why pp  $\lambda$  is different than in DIS?

quantitative analysis is needed

# Backup Slides





# GS in HI: $A$ dependence

$$\text{transverse parton size } s \sim \frac{\pi}{Q^2}$$

$$\text{cross section } \sigma \sim \alpha(Q^2) \frac{\pi}{Q^2}$$

$$\text{nucleus transverse size } S_A \sim \pi R_A^2$$

$$\text{critical \# partons} \sim \frac{S_A}{\sigma} \sim \frac{Q^2 R_A^2}{\alpha(Q^2)}$$

Saturation starts when # of partons in the nucleus  $N_A$  is equal to the critical #  $S_A/\sigma$

$$N_A \sim \frac{S_A}{\sigma} \implies Q_{\text{sat}}^2 \sim \alpha(Q^2) \frac{N_A}{R_A^2} \sim \frac{A}{A^{2/3}} \sim A^{1/3}$$

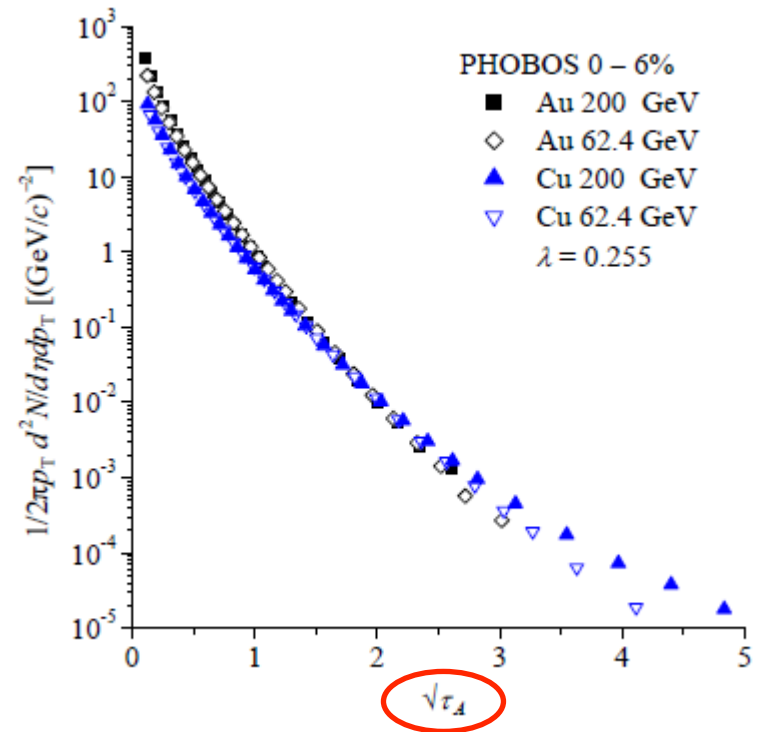
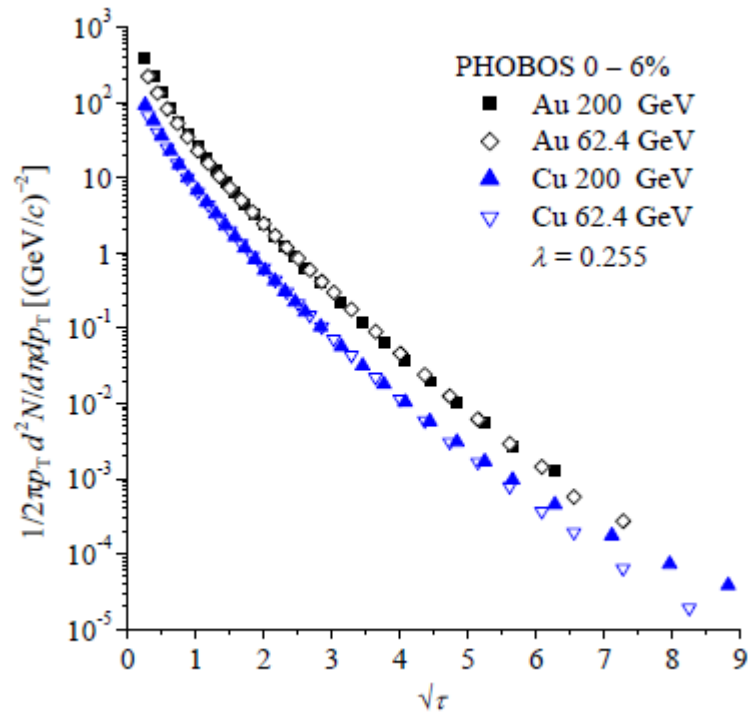
$$Q_{A \text{ sat}}^2 \sim A^{1/3} Q_{\text{sat}}^2$$



# GS in HI: A dependence

B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **94** (2005) 082304 [arXiv:nucl-ex/0405003].

B. Alver *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **96** (2006) 212301 [arXiv:nucl-ex/0512016].

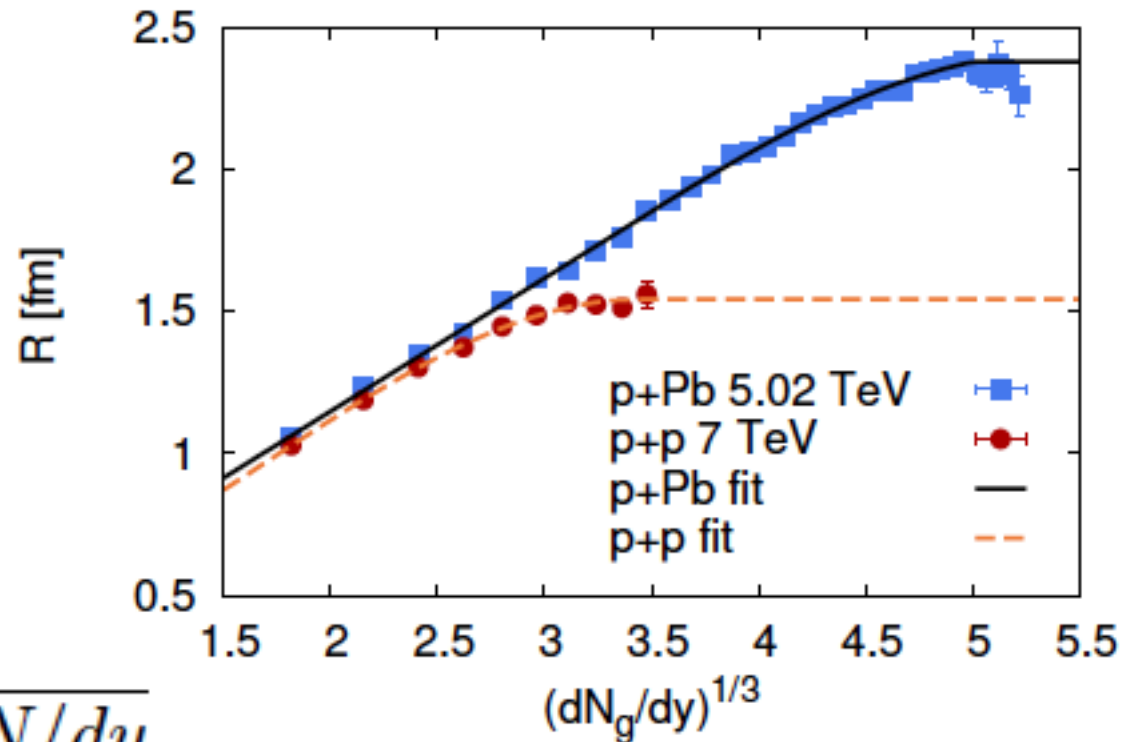




# GS in pA

A. Bzdak, B. Schenke, P. Tribedy and R. Venugopalan, arXiv:1304.3403

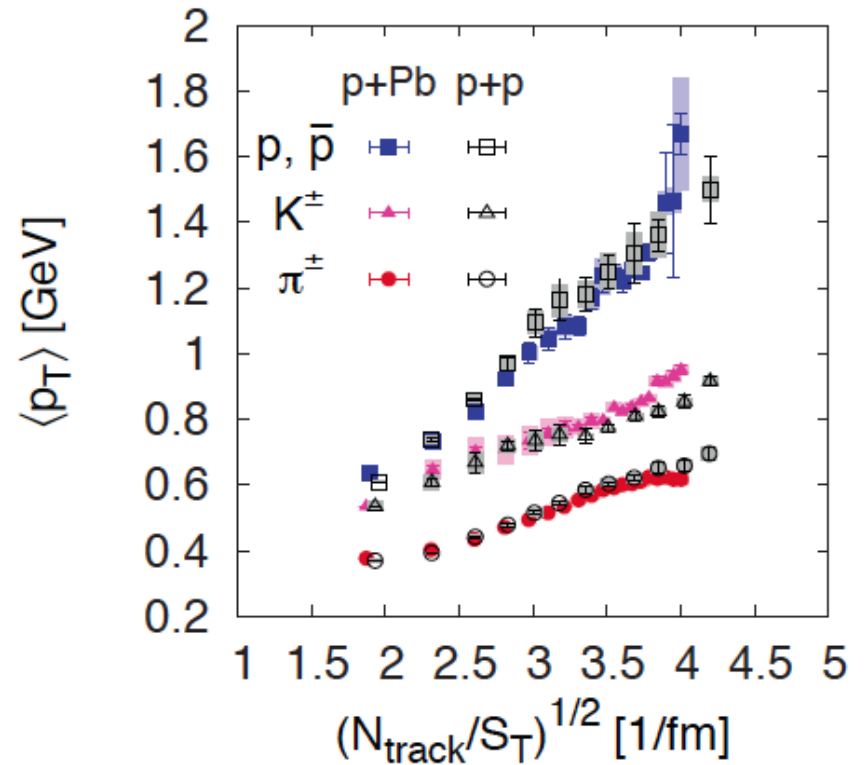
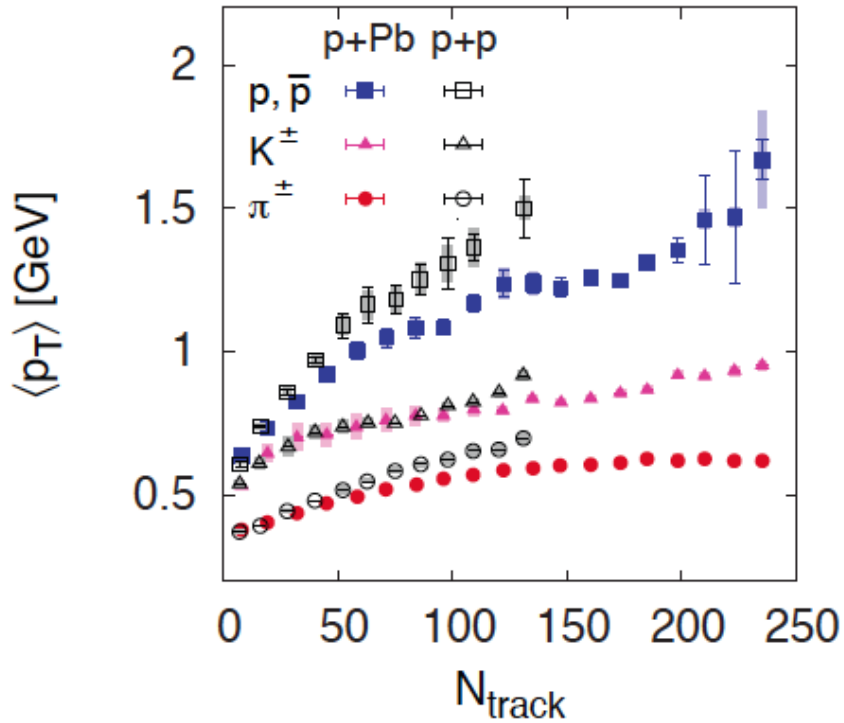
$$\bar{Q}_{\text{sat}}^2 \sim \frac{dN}{dy} \frac{1}{S_T}$$



$$\langle p_T \rangle = A + B \sqrt{dN/dy}$$



# GS in pA



L. McLerran, M.P. and B. Schenke, arXiv:1306.2350 [hep-ph] (Nucl. Phys. A)