The deconfinement phase transition in the Hamiltonian approach to Yang–Mills theory in Coulomb gauge

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in collaboration with:
D. Campagnari and J. Heffner

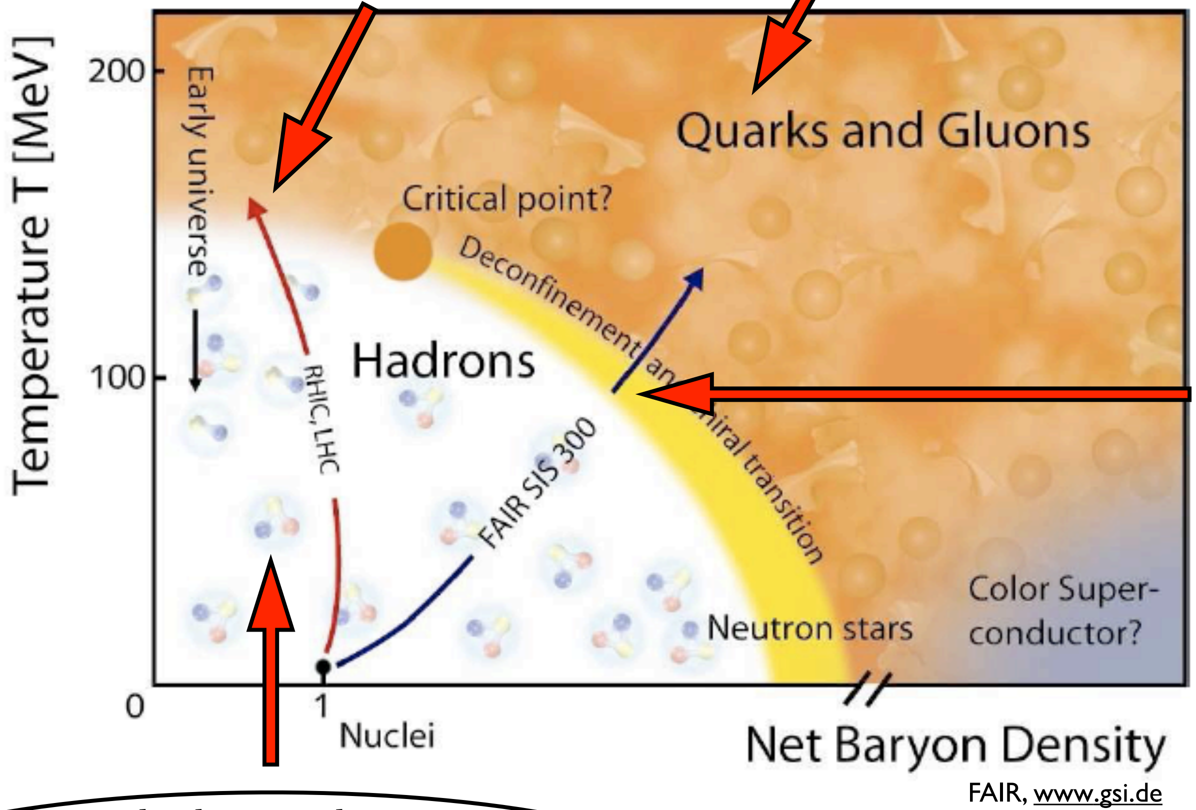
Phase diagram of QCD

Strongly correlated quark-gluon-plasma

'RHIC serves the perfect fluid'

massless quarks (chiral symmetry)

deconfinement



quarkyonic:

confinement & chiral symmetry?

hadronic phase

confinement & chiral symmetry breaking

Outline

- introduction
- Hamiltonian approach to YMT
- T=0 results
- $T \neq 0$: grand canonical ensemble
- Polyakov loop potential
- conclusions

Hamiltonian approach to Yang-Mills theory

Weyl gauge: $A_0^a(x) = 0$ cartesian coordinates $A_i^a(x)$

momenta $\Pi_i^a(x) = \delta S / \delta \dot{A}_i^a(x) = E_i^a(x)$

$$H = \frac{1}{2}\int d^3x(\Pi^2(x) + B^2(x))$$

$$\Pi_k^a(x) = \delta / i\delta A_k^a(x)$$

YM Schrödinger equation $H\Psi[A] = E\Psi[A]$

Gauss law $D\Pi\Psi = 0$ *gauge invariant wave functionals:* $\Psi[A]$

more convenient: gauge fixing
explicit resolution of Gauss´law $\partial A = 0$

Hamiltonian approach to YMT in Coulomb gauge $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi^{\perp} J \Pi^{\perp} + B^2) + H_C$$

$$\Pi = \delta / i\delta A$$

$$J(A^{\perp}) = Det(-D\partial) \qquad D = \partial + gA$$

Christ and Lee

$$H_C = \tfrac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1} J\rho$$

color charge density: $\rho = -A^{\perp}\Pi^{\perp}$

$$\langle \Phi | \ldots | \Psi \rangle = \int_{\Lambda} DAJ(A)\Phi^*(A)\ldots\Psi(A)$$

$$H\Psi[A] = E\Psi[A]$$

Hamiltonian approach to YMT in Coulomb gauge $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi^{\perp} J \Pi^{\perp} + B^2) + H_C + H_F \qquad \Pi = \delta / i\delta A$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D = \partial + gA$$

$$H_F = \int \Psi^{\dagger}(\alpha \bullet (p + gA) + \beta m_0)\Psi$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1} J\rho$$

color charge density: $\rho = -A^{\perp}\Pi^{\perp} + \Psi^{\dagger}\Psi$

$$\langle \Phi | \ldots | \Psi \rangle = \int_{\Lambda} DAJ(A)\Phi^{*}(A)\ldots\Psi(A)$$

$$H\Psi[A] = E\Psi[A]$$

Perturbation theory

D. Campagnari, H. R. & A. Weber, Phys. Rev D(2009)

- Rayleigh-Schrödinger PT

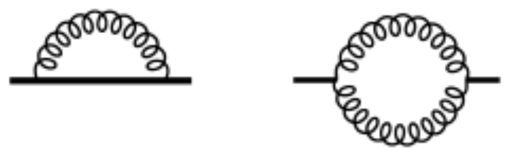
$$\widetilde{H} = H_0 + g\widetilde{H}_1 + g^2\widetilde{H}_2 + \mathcal{O}(g^3)\,,$$

$$\left[|0\rangle + g\,|0\rangle^{(1)} + g^2\,|0\rangle^{(2)} + \mathcal{O}(g^3)\right].$$

  - vacuum (QED)

$$\langle A|0\rangle = \mathcal{N}\exp\left\{-\frac{1}{2}\int đk\, A^a_\sigma(\mathbf{k})\,|\mathbf{k}|\,A^a_\sigma(-\mathbf{k})\right\}.$$

- ß-function



$$v(\mathbf{k}) = 1 + g^2\frac{N_c}{(4\pi)^{2-\varepsilon}}\left\{\frac{11}{3}\left[\frac{1}{\varepsilon} - \gamma - \ln\frac{\mathbf{k}^2}{\mu^2}\right] + \frac{31}{9} + \mathcal{O}(\varepsilon)\right\}.$$

$$\beta(g) = \frac{\partial g}{\partial \ln\mu} = \frac{1}{(4\pi)^2}\beta_0\, g^3 + \mathcal{O}(g^5)\,,$$

$$\beta_0 = -\frac{11}{3}N_C$$

Variational approach

- trial ansatz

C.Feuchter & H. R. PRD70(2004)

$$\Psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-D\partial)}} \exp\left[-\tfrac{1}{2}\int dx\, dy\, A(x)\omega(x,y)A(y)\right]$$

gluon propagator $\langle A(x)A(y)\rangle = (2\omega(x,y))^{-1}$

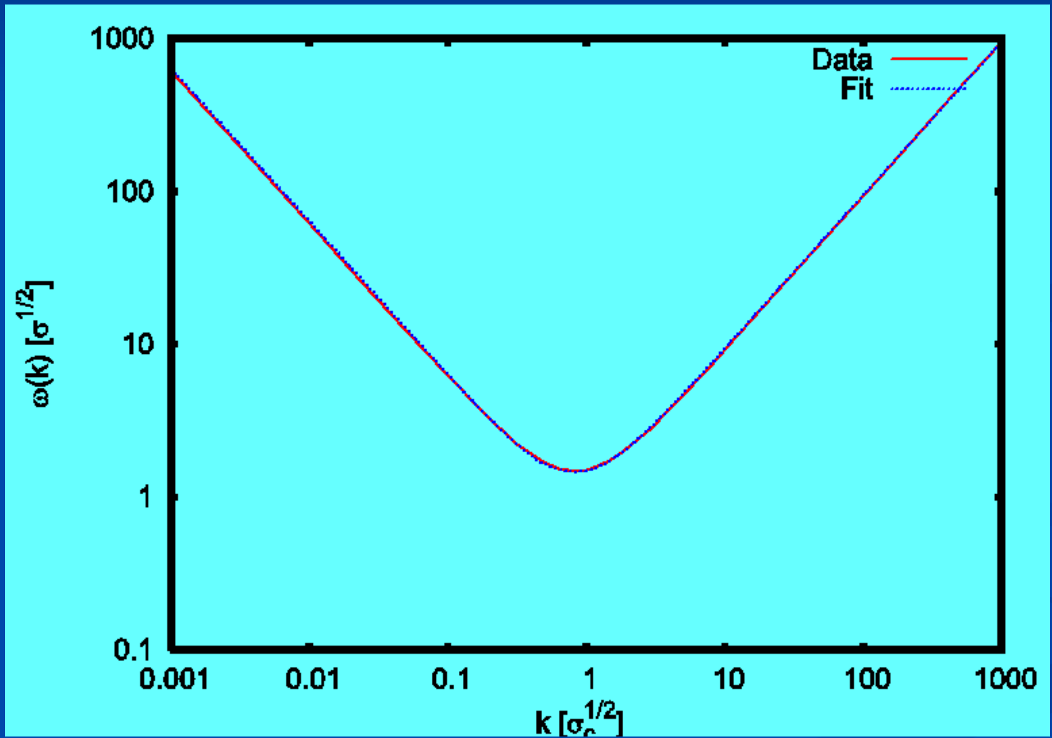
variational kernel $\omega(x,x')$ determined from

$$\langle\Psi|H|\Psi\rangle \to \min$$

Numerical results

gluon energy

D. Epple, H. R. and W.Schleifenbaum, PRD 75 (2007)



$$IR: \quad \omega(k) \sim 1/k \qquad UV: \quad \omega(k) \sim k$$

Static gluon propagator in D=3+1

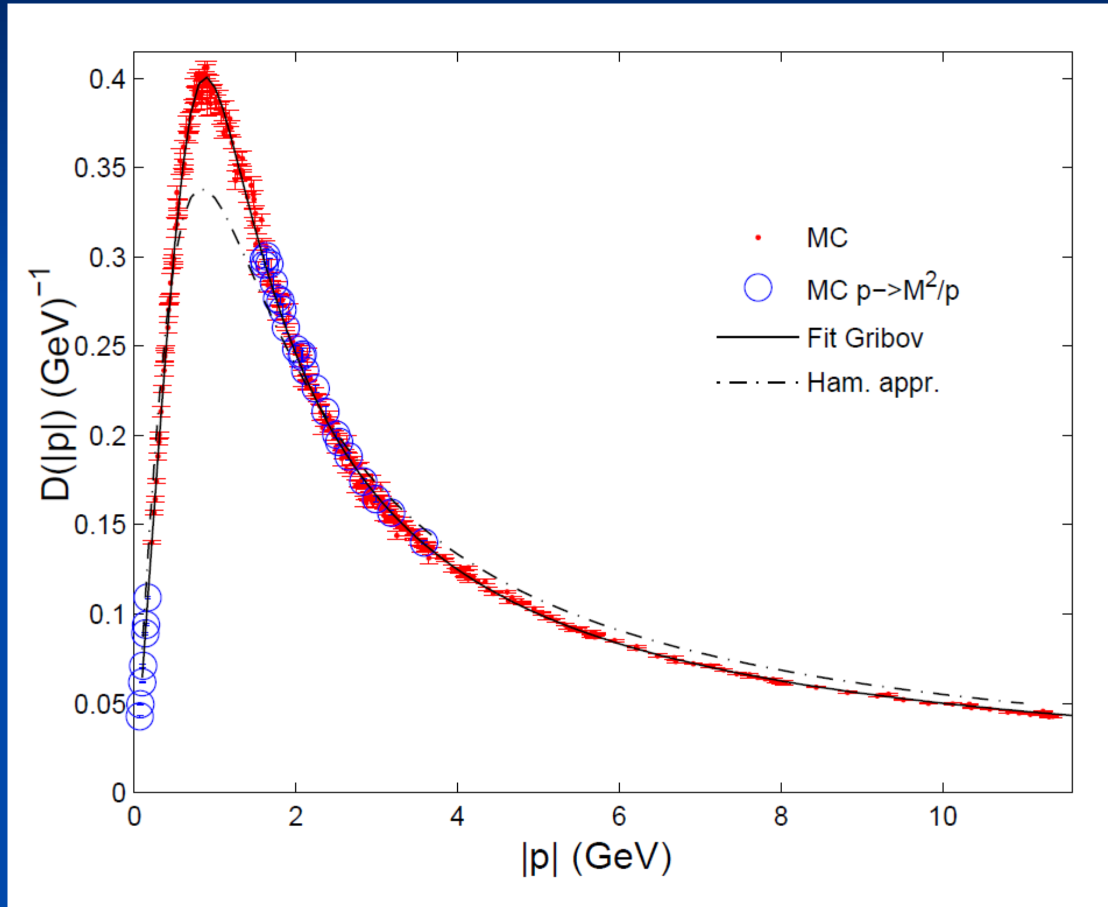
$$D(k) = (2\omega(k))^{-1}$$

Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

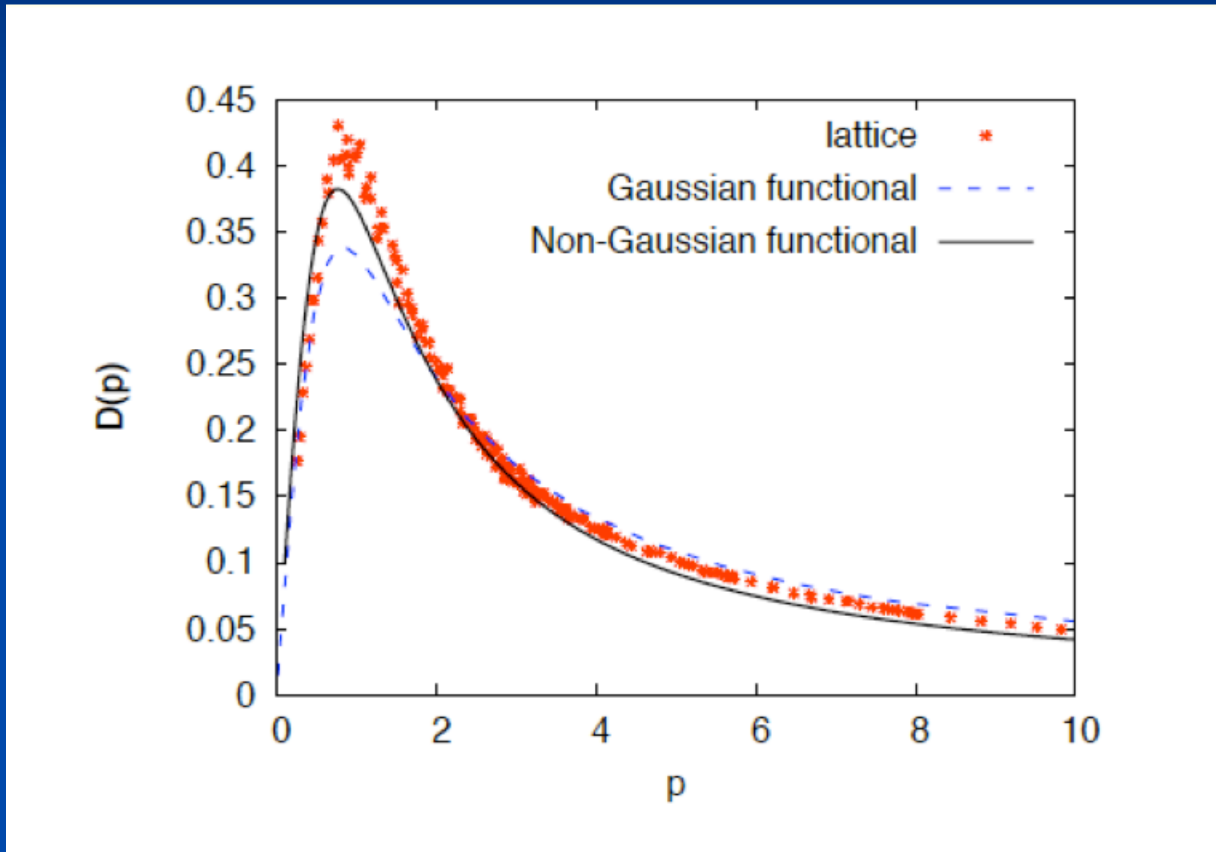
$$M = 0.88 GeV$$

missing strength in
mid momentum regime:
missing gluon loop



G. Burgio, M.Quandt , H.R., **PRL102(2009)**

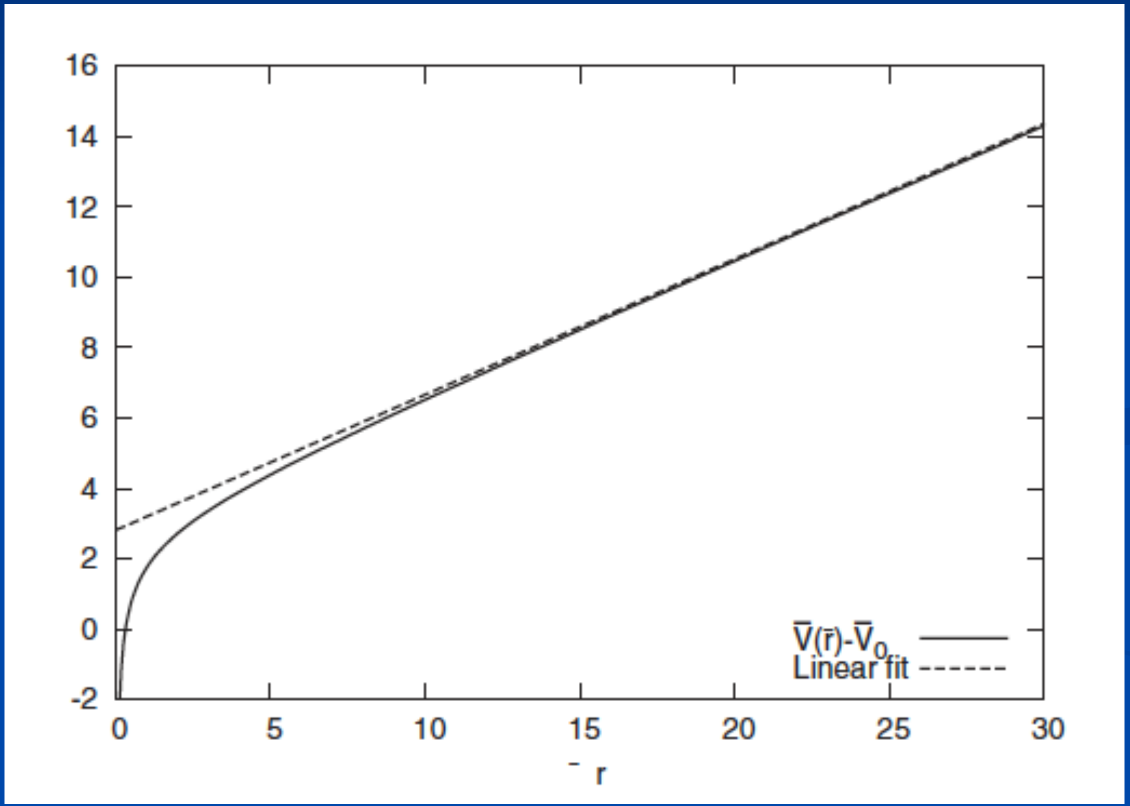
non-Gausssian wave functional



D. Campagnari & H.R, Phys.Rev.D82(2010)1125029

Static Coulomb potential

$$V(|x-y|) = g^2 \langle \langle x | (-D\partial)^{-1} (-\partial^2)(-D\partial)^{-1} | y \rangle \rangle$$



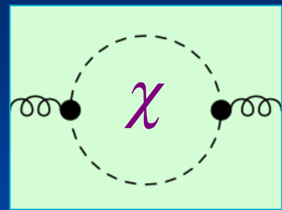
D. Epple, H. Reinhardt
W.Schleifenbaum,
PRD 75 (2007)

$$V(R) = \xrightarrow[R\to\infty]{} \sigma_C R, \qquad \text{lattice} \;\; \sigma_C = 2...3\sigma_W$$

$$V(R) = \xrightarrow[R\to 0]{} \sim 1/R$$

equations of motion

$$\omega^2(k) = k^2 + \chi^2(k)$$



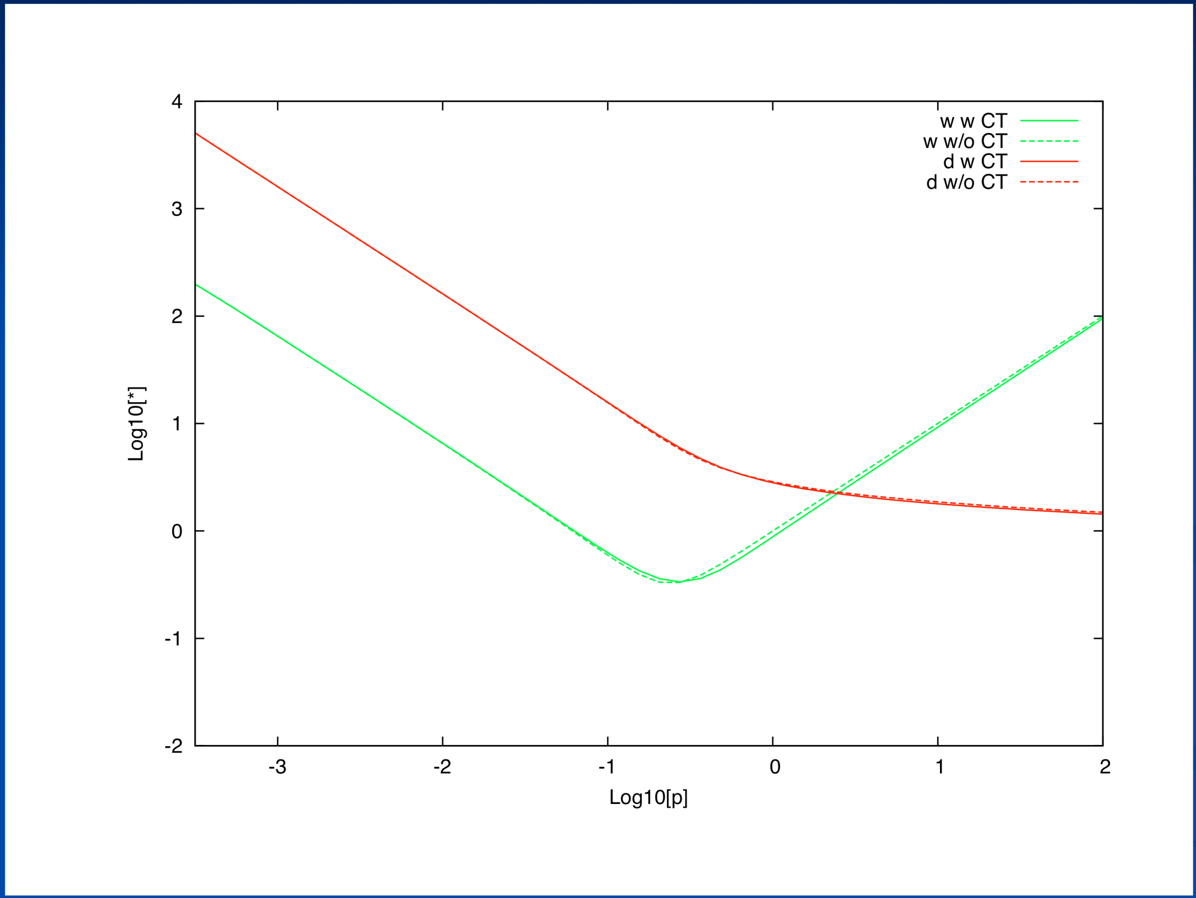
ghost propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d(\Delta)/(-\Delta)$$

d ghost form factor

Dyson-Schwinger equation

T=0 soliutions



The color dielectric function of the QCD vacuum

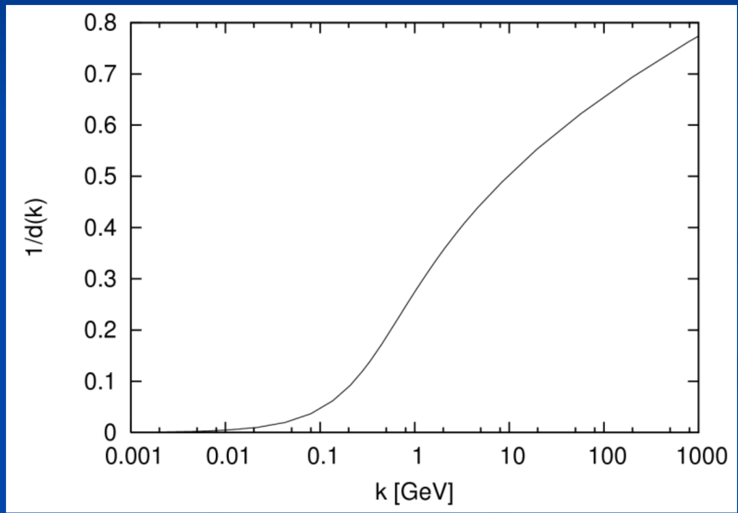
- ghost propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d / (-\Delta)$$

- dielectric „constant“

$$\varepsilon = d^{-1}$$

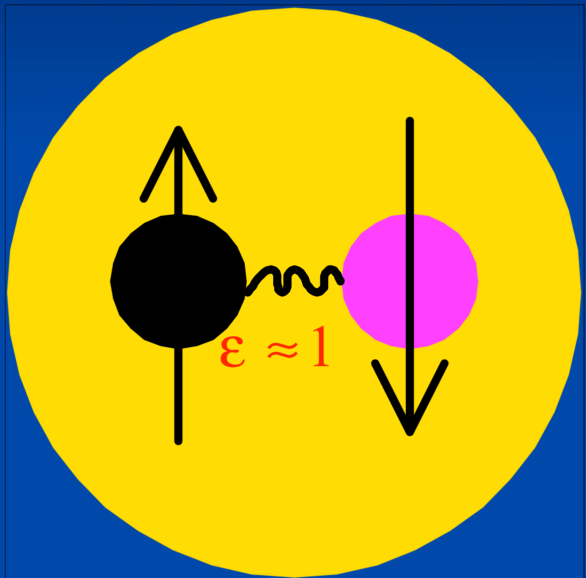
H.R. PRL101 (2008)



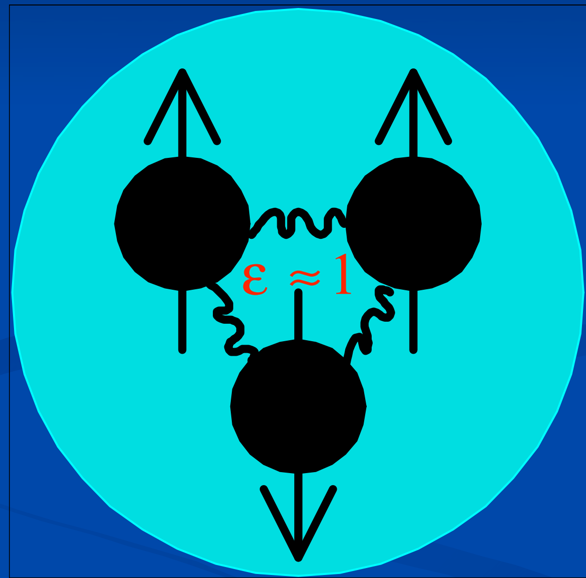
- horizon condition:
  - : $d^{-1}(k=0)=0$ $\varepsilon(k=0)=0$
- QCD vacuum: perfect color dia-electricum
  - dual superconductor: Meißner effect

$\varepsilon(k)<1$ anti-screening

$$D = \varepsilon E \qquad \partial D = \rho_{free}$$



$\varepsilon = 0$



no free color charges in the vacuum: confinement

Hamiltonian approach to YMT at finite T

- grand canonical ensemble with $\mu = 0$
- minimization of the free energy

Reinhardt, Campagnari, Szczepaniak, PR84(2011)045006
Heffner, Reinhardt, Campagnari, Phys. Rev D85(2012)125029

equations of motion

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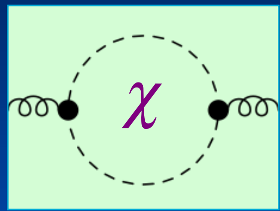

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d ghost form factor

Dyson-Schwinger equation

Infrared analysis

gluon energy $\omega(p) = A / p^{\alpha}$ ghost form factor $d(p) = B / p^{\beta}$

$T = 0$ *sum rule* $\alpha = 2\beta + 2 - d$

$d = 3$ $\beta = 1.0(0.99)$ $\beta = 0.796(0.79)$

$d = 2$ $\beta = 0.5$

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at arbitrarily finite T infrared analysis=impossible! $n(k) = [\exp(\beta\omega(k)) - 1]^{-1}$

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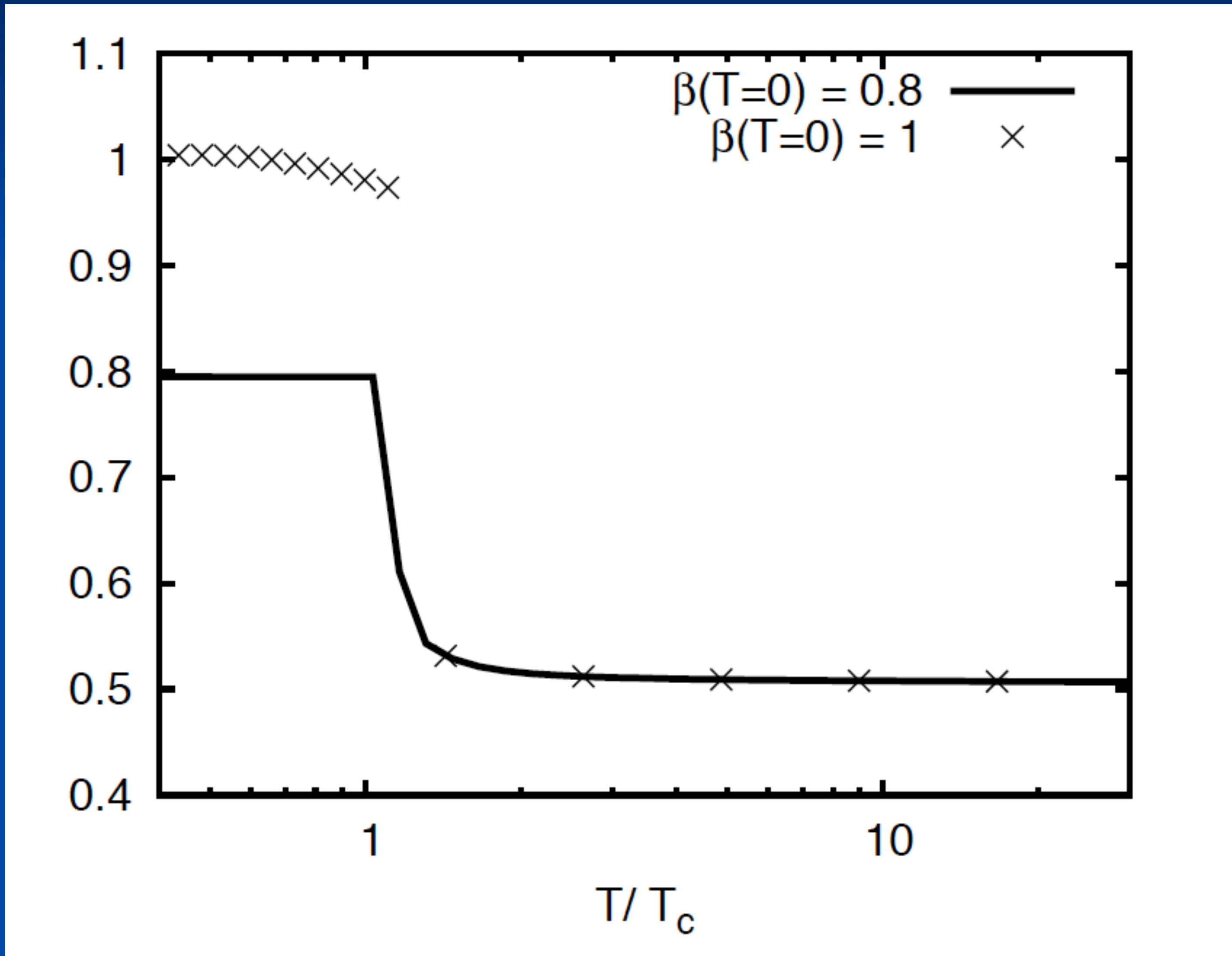
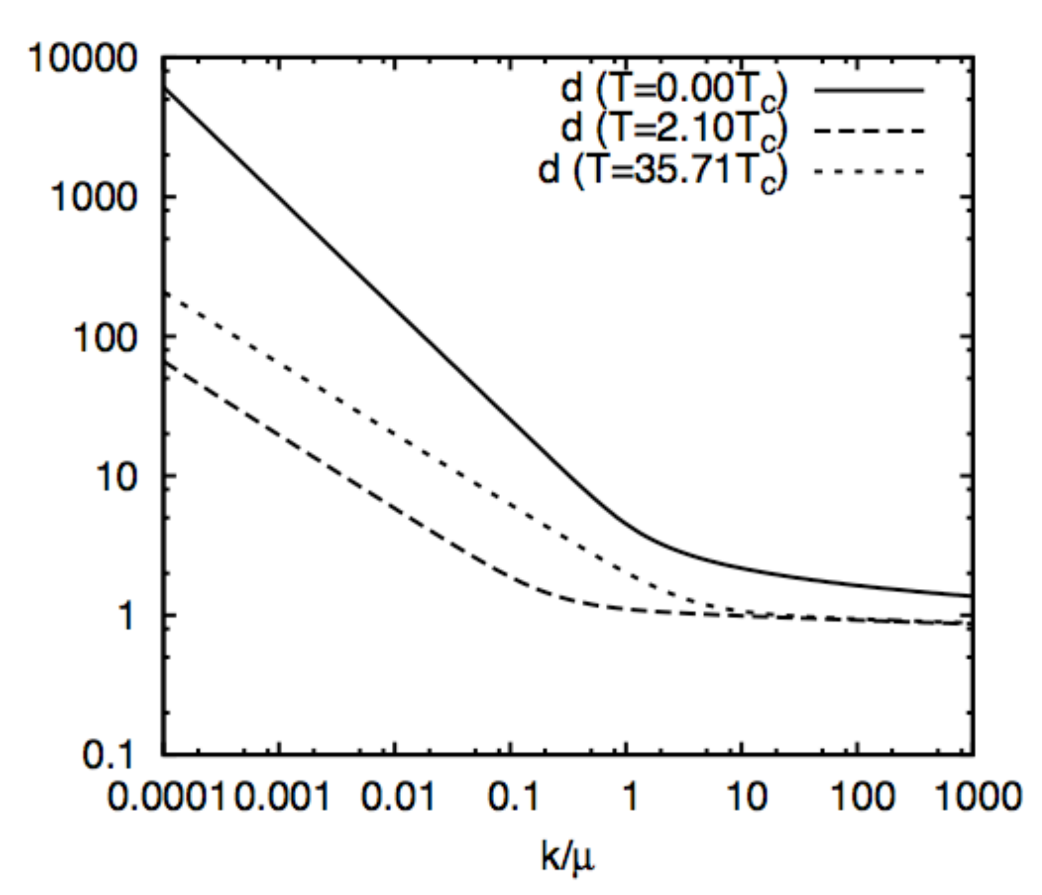
sum rule $\alpha = 2\beta + 2 - d$

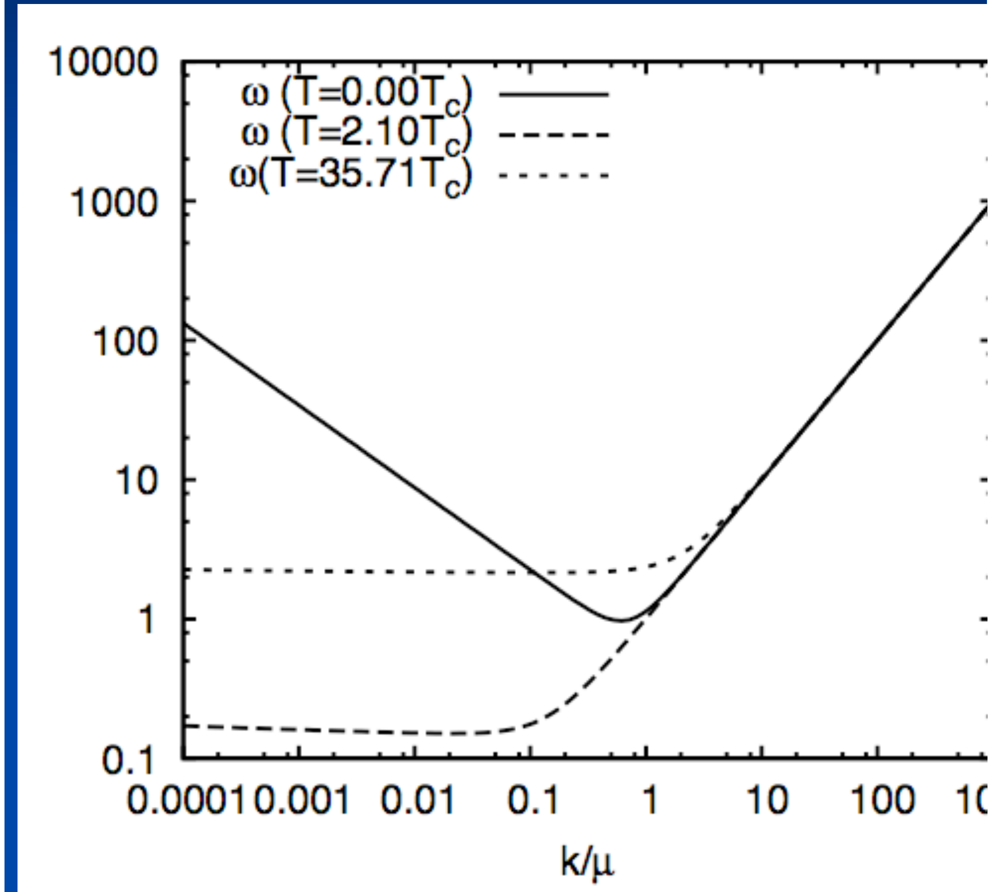
$d = 3$ $\beta = 0.5$ $\alpha = 0$

IR-exponent of ghost



numerical results





Critical temperature

$input: \quad scale$

$SU(2)-lattice: \qquad \omega(k)=\sqrt{k^2+M^4/k^2}$

$Gribov\ mass\ M=880\,MeV$

$\Rightarrow critical\ temperature\ T_C=275......290\,MeV$

$SU(2)-lattice: \qquad T_C=295\,MeV$

alternative way to determine the critical temperature:

Polyakov loop

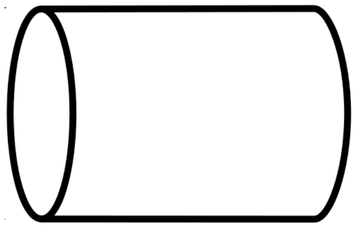
see also talk by K.Redlich

Polyakov loop

- YMT at finite temperature T :compact Euclidean time

$$P[A_0](\vec{x}) = \tfrac{1}{d_r} tr P \exp\left[i\int_0^L dx_0 A_0(x_0,\vec{x}) \right]$$

$$T^{-1} = L$$



- order parameter for confinement: $\langle P[A_0](\vec{x})\rangle \sim \exp\left[-F_\infty(\vec{x})L\right]$
  - conf. phase: center symmetry $\langle P[A_0](\vec{x})\rangle = 0$
  - deconf. phase: center symmetry-broken $\langle P[A_0](\vec{x})\rangle \neq 0$
- Polyakov gauge $\partial_0 A_0 = 0, \quad A_0 = diagonal$ $\quad SU(2): \; P[A_0](\vec{x}) = \cos(\frac{A_0(\vec{x})L}{2})$
  - fundamental modular region $0 < A_0 L/2 < \pi$ $\quad P[A_0] - unique\ function\ of\ A_0$
  - alternative order parameters: $\langle P[A_0](\vec{x})\rangle \quad P[\langle A_0(\vec{x})\rangle] \quad \langle A_0(\vec{x})\rangle$

- *F.Marhauser and J. M. Pawlowski, arXiv:0812.11144*
- *J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684(2010)262*

Effective potential of the order parameter for confinement

- background field calculation $a_0 = \langle A_0(\vec{x}) \rangle - \text{const, diagonal (Polyakov gauge)}$
- effective potential $e[a_0] \to \min \qquad \Rightarrow a_0 = \bar{a}_0$
- order parameter $\langle P[A_0] \rangle \approx P[\bar{a}_0]$

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- 1-loop perturbation theory

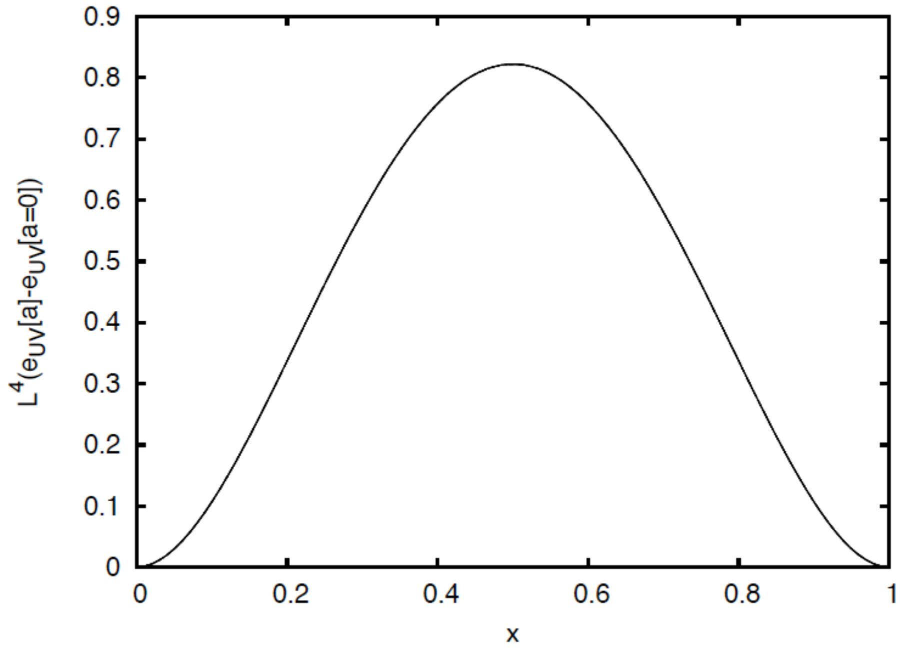
$$e_{PT}[a_0 = x2\pi / L]$$

Gross, Pisarski, Yaffe, Rev.Mod.Pys.53(1981)

N.Weiss, Phys.Rev.D24(1981)

$$P[\bar{a}_0 = 0] = 1$$

deconfined phase



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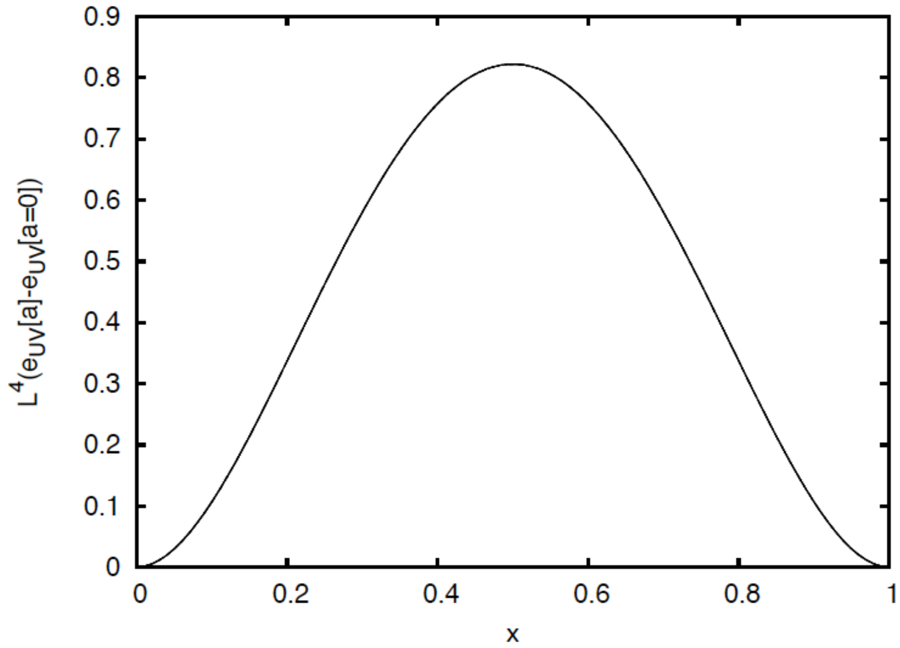


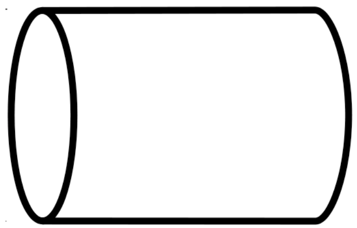
aim of this talk: non-perturbative evaluation of $e[a_0]$ in the Hamiltonian approach

Polyakov loop potential in the Hamiltonian approach

- Hamiltonian approach assumes Weyl gauge $A_0 = 0$

- O(4)-invariance

- compactify (instead of time) one spatial axis to a circle of circumference L and interprete L^{-1} as temperature

L 

- compactify $x_3 -$ axis $\vec{a} = a\vec{e}_3$

- YMT at finite length in a constant, color diagonal background field a

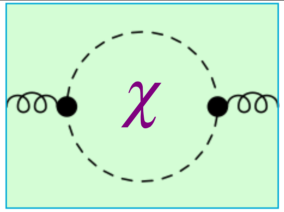
- calculate the effective potential $e[a]$

H. R. & J. Heffner, Phys. Lett.B718(2012)672
PRD(in press)arXiv:1304.2980

The effective potential

- energy density

$$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_{\perp}(\omega(p^{\sigma})-\chi(p^{\sigma}))$$



- background field $p^{\sigma}=p_{\perp}+(p_n-\sigma\cdot a)e_3 \qquad p_n=2\pi n/L \qquad \sigma-root$

- periodicity $e(a,L)=e(a+\mu_k/L,L) \qquad \exp(i\mu_k)=z_k\in Z(N)$

ghost loop χ arises from the FP determinant in the kinetic energy

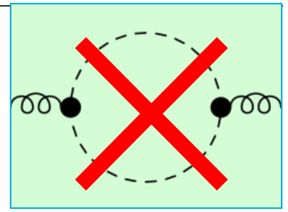
- input:

$\omega(p),\chi(p)$ from the variational calculation in Coulomb gauge at T=0

C. Feuchter & H. Reinhardt,Phys. Rev.D71(2005)
D. Epple, H. Reinhardt, W. Schleifenbaum, Phys. Rev.D75(2007)

The effective potential

- energy density $$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_{\perp}(\omega(p^{\sigma})-\chi(p^{\sigma}))$$



- background field $p^{\sigma}=p_{\perp}+(p_n-\sigma\cdot a)e_3 \quad p_n=2\pi n/L \quad \sigma - roots$
- periodicity $e(a,L)=e(a+\mu_k/L,L) \quad \exp(i\mu_k)=z_k\in Z(N)$
- neglect ghost loop $\chi(p)=0$ $$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_{\perp}\omega(p^{\sigma})$$
  - quasi-gluon gas
- limiting cases
  - UV: $\omega_{UV}(p)=p$
  - IR: $\omega_{IR}(p)=M^2/p$
  - Gribov: $\omega(p)=\sqrt{(p^2+M^4/p^2)}\approx\omega_{IR}(p)+\omega_{UV}(p)$

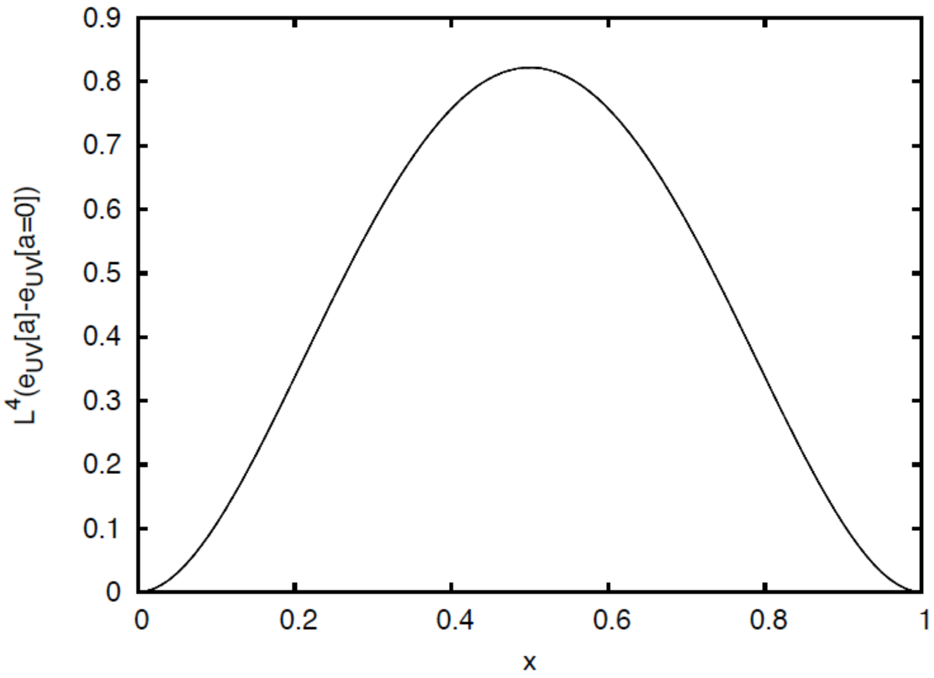
The UV-effective potential

$$\chi(p) = 0 \qquad \omega(p) = p \qquad e(a,L) = \sum_\sigma \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp (\omega(p^\sigma) - \chi(p^\sigma))$$

$$e(a,L) = \frac{8}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{\sin^2(naL/2)}{n^4}$$

$$= \frac{4\pi^2}{3L^4} \Big(\underbrace{\frac{aL}{2\pi}}_{x} \Big)^2 \left[\frac{aL}{2\pi} - 1 \right]^2$$

N.Weiss 1-loop PT



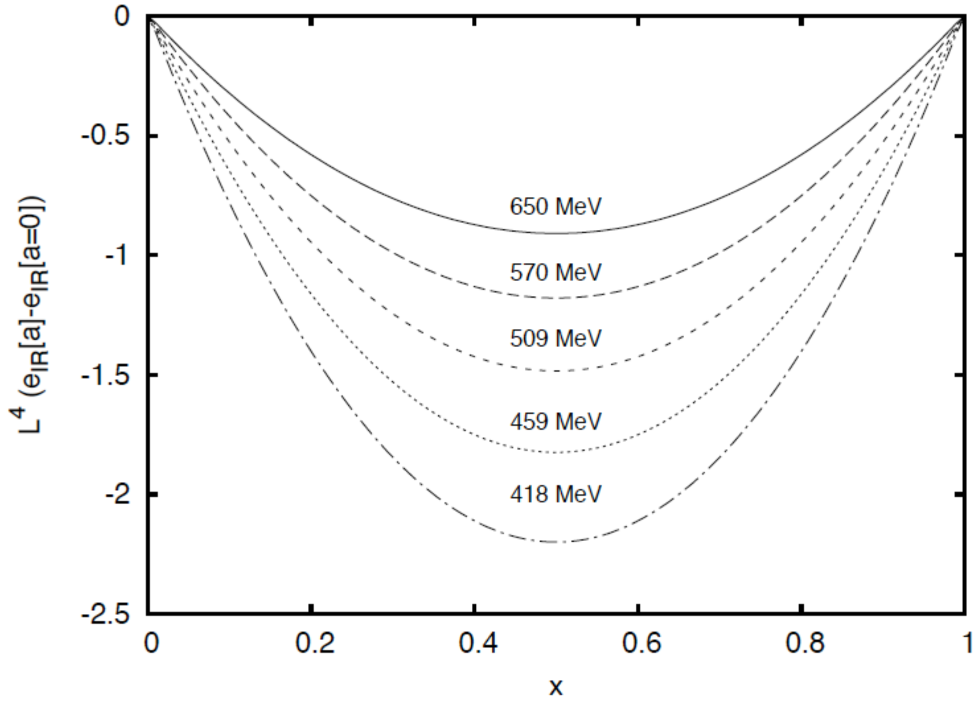
$$Polyakov-loop \qquad \langle P \rangle \simeq P[a_{\min} = 0] = 1 \qquad deconfining \quad phase$$

The IR-effective potential

$\chi(p) = 0 \qquad \omega(p) = M^2 / p$

$$e(a,L) = \sum_\sigma \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp (\omega(p^\sigma) - \chi(p^\sigma))$$

$$e_{IR}(a,L) = -\frac{4M^2}{\pi^2 L^2} \sum_{n=1}^{\infty} \frac{\sin^2(naL/2)}{n^2} = \frac{2M^2}{L^2} \underbrace{\left(\frac{aL}{2\pi}\right)}_{x} \left[\frac{aL}{2\pi} - 1\right]$$



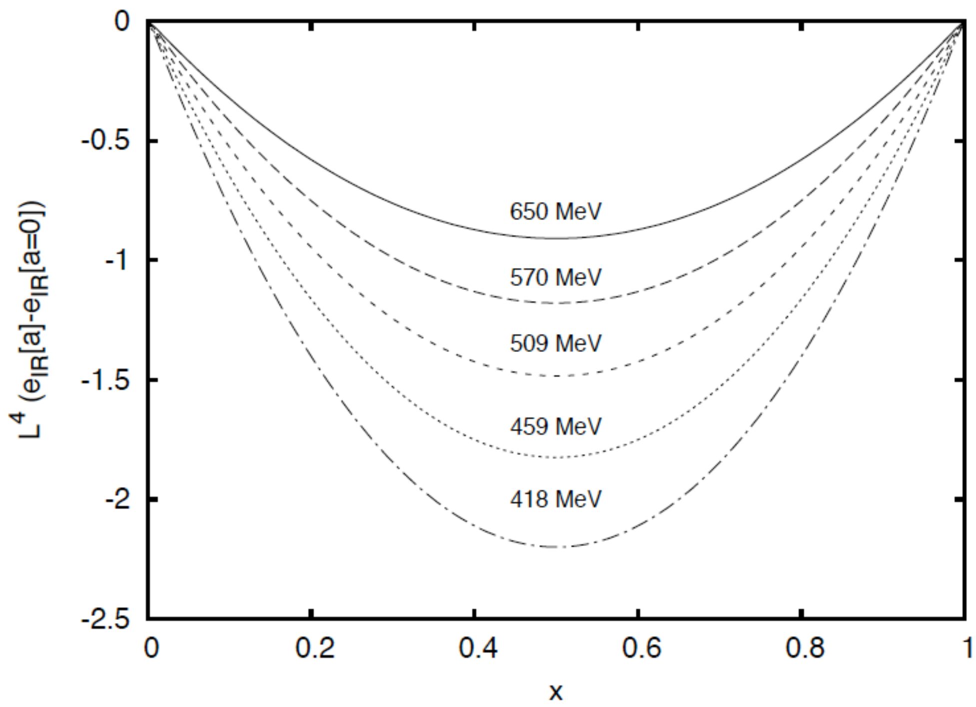
$Polyakov - loop \qquad \langle P \rangle \simeq P[a_{\min} = \pi / L] = 0 \qquad confining \quad phase$

The IR-effective potential

$\chi(p) = 0$ $\qquad$ $\omega(p) = M^2 / p$ $\qquad$ $e(a,L) = \sum_\sigma \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp (\omega(p^\sigma) - \chi(p^\sigma))$

$$e_{IR}(a,L) = -\frac{4M^2}{\pi^2 L^2} \sum_{n=1}^{\infty} \frac{\sin^2(naL/2)}{n^2}$$

$$= \frac{2M^2}{L^2} \underbrace{\left(\frac{aL}{2\pi}\right)}_{x} \left[\frac{aL}{2\pi} - 1\right]$$



$$Polyakov - loop \qquad \langle P \rangle \simeq P[a_{\min} = \pi / L] = 0 \qquad confining \quad phase$$

deconfinement phase transition results from the interplay between the confining IR-potential and deconfining UV-potential

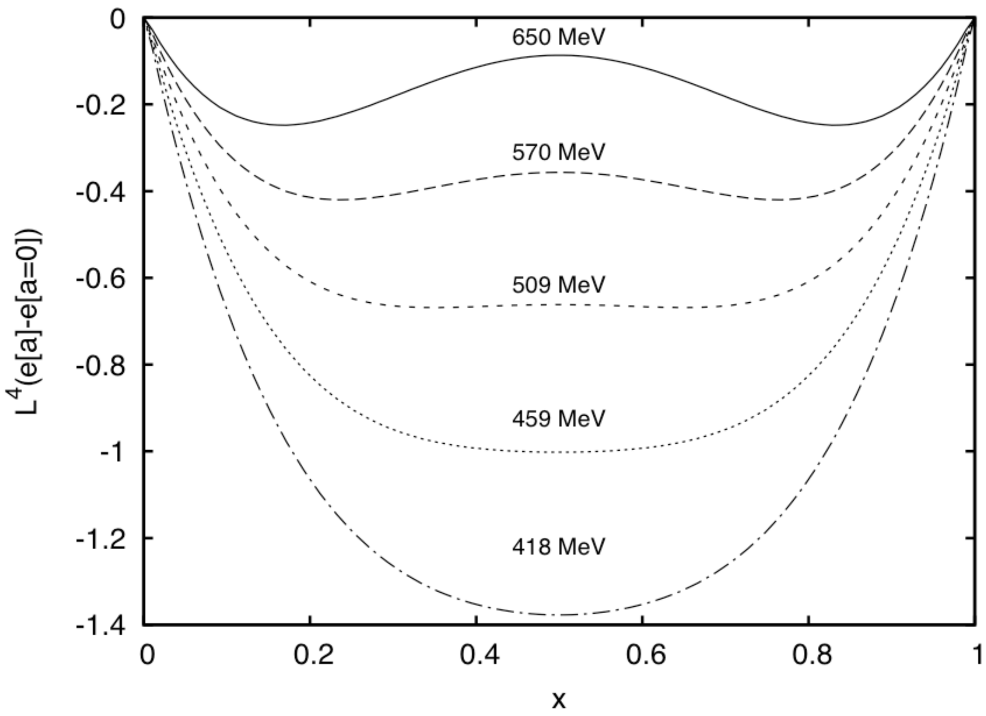
The IR+UV effective potential:

$\chi(p) = 0$ $\omega(p) = p + M^2 / p$ $e(a, L) = e_{UV}(a, L) + e_{IR}(a, L)$

phase transition

critical temperature:

$T_C = \sqrt{3} M / \pi$



$lattice : M \simeq 880\, MeV \quad \Rightarrow \quad T_C \simeq 485\, MeV$

The IR+UV effective potential:

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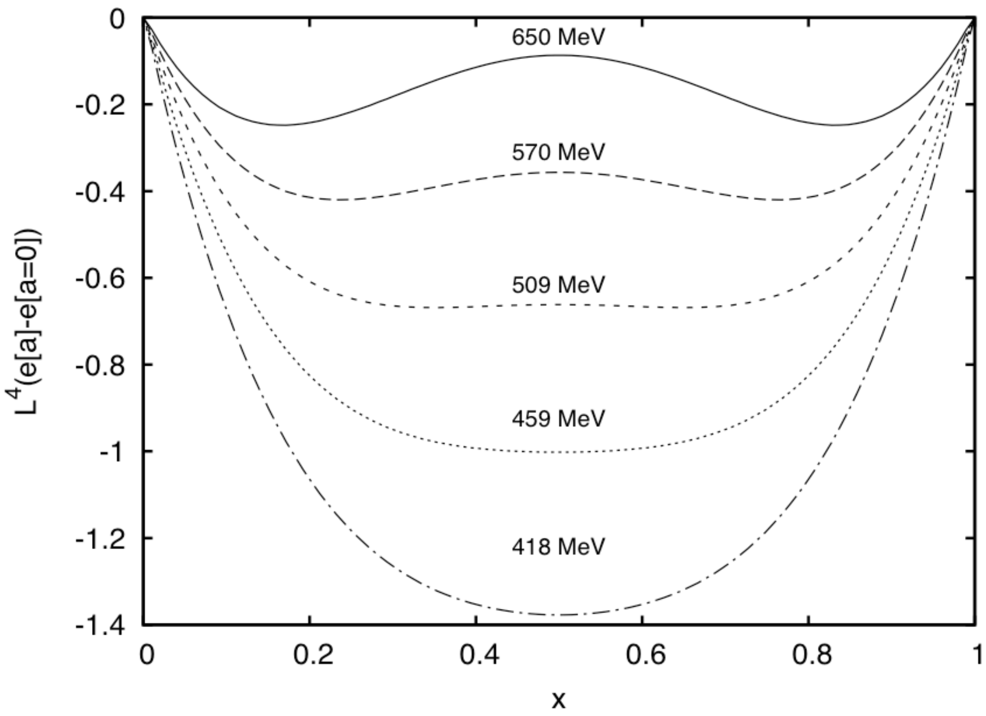
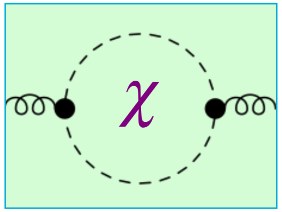


$lattice : M \simeq 880 MeV \Rightarrow T_C \simeq 485 MeV$

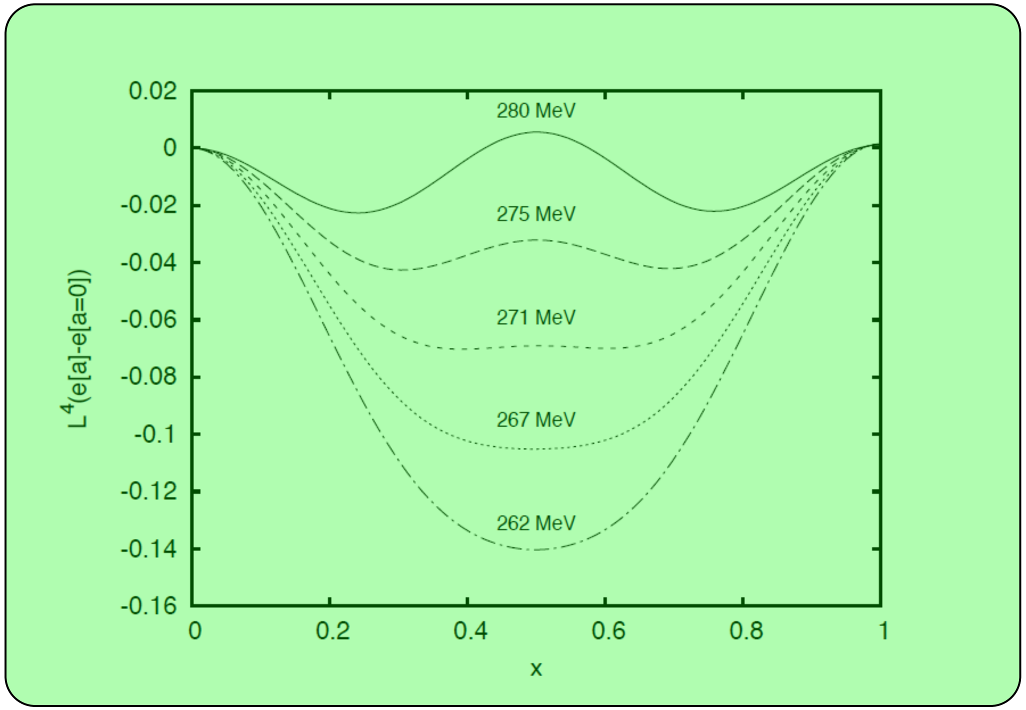
$\chi(p) = 0$ $\omega(p) = \sqrt{p^2 + M^4 / p^2}$ $T_C \simeq 432 MeV$

The full effective potential

$$e(a,L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_{\perp} (\omega(p^{\sigma}) - \chi(p^{\sigma}))$$



variational calculation in Coulomb gauge



SU(2)

critical temperature:

$$T_C \simeq 270\,MeV$$

The effective potential for SU(3)

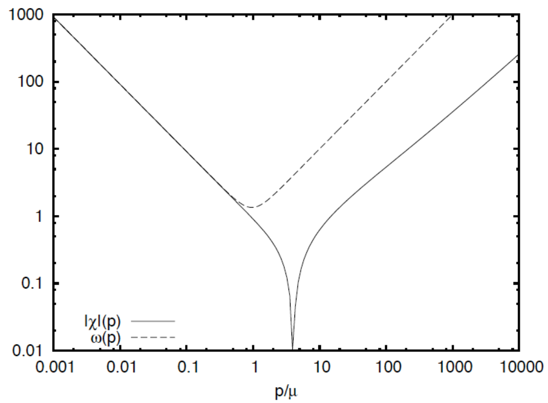
SU(3)-algebra consists of 3 SU(2)-subalgebras characterized by the 3 non-zero positive rooots

$$\sigma = (1,0), \quad \left(\frac{1}{2}, \frac{1}{2}\sqrt{3}\right), \quad \left(\frac{1}{2}, -\frac{1}{2}\sqrt{3}\right)$$

$$e_{SU(3)}[a] = \sum_{\sigma>0} e_{SU(2)(\sigma)}[a]$$

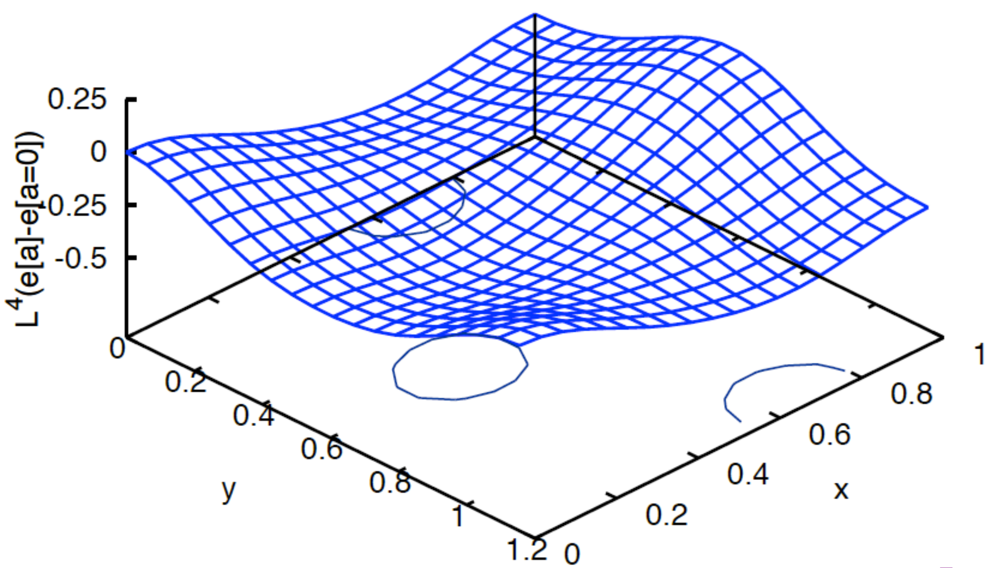
The full effective potential for SU(3)

$$e(a, L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_{\perp} (\omega(p^{\sigma}) - \chi(p^{\sigma}))$$

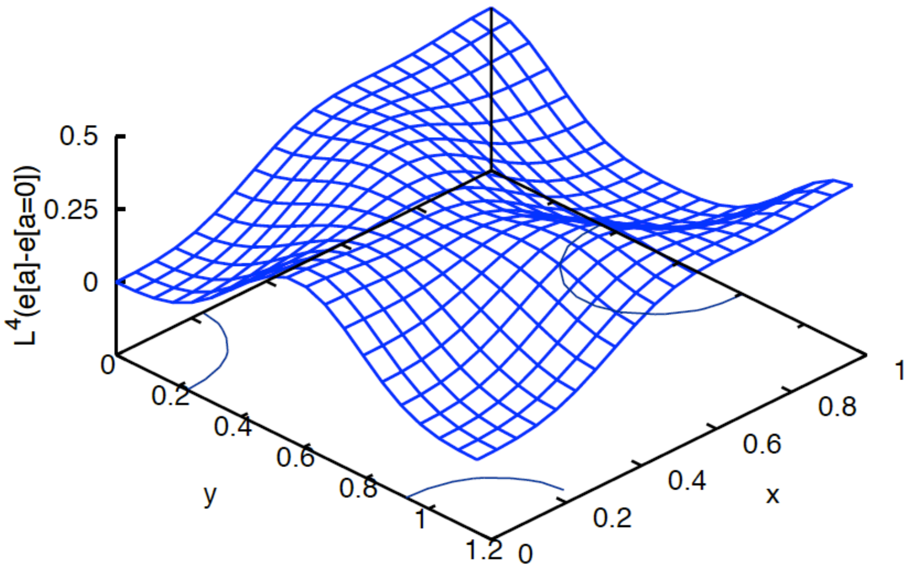


variational calculation in Coulomb gauge

$T < T_C$

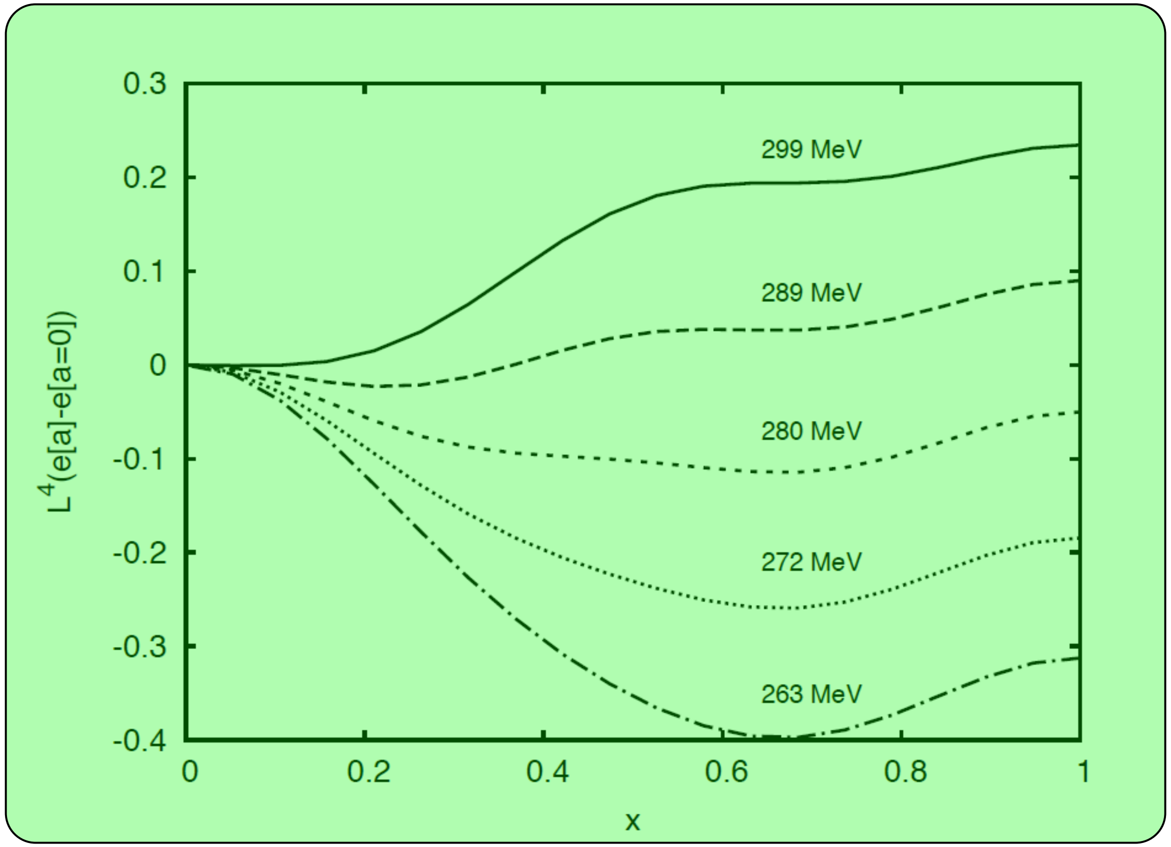


$T > T_C$



$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi}$

Polyakov loop potential for SU(3)

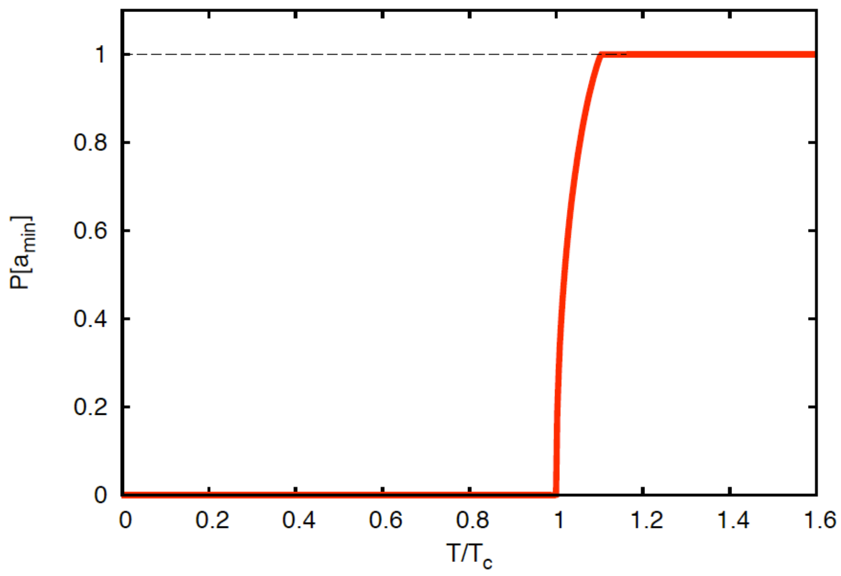


$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi} = 0$$

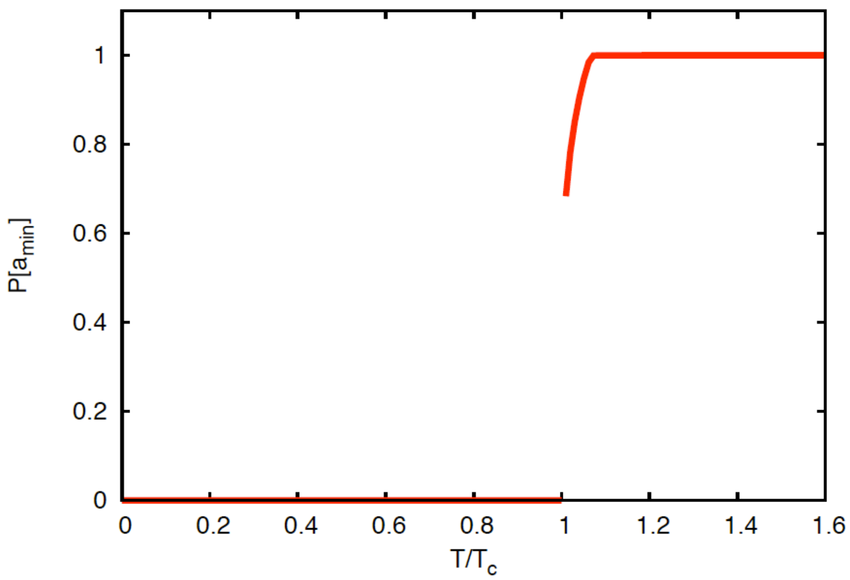
$$input: \quad M = 880 MeV$$

$$T_C = 283 MeV$$

The Polyakov loop



SU(2)



SU(3)

critical temperature

$lattice:$ $T_C^{SU(2)} = 295\,MeV$ $T_C^{SU(3)} = 270\,MeV$

$this\ work:$ $T_C^{SU(2)} = 267\,MeV$ $T_C^{SU(3)} = 277\,MeV$

$FRG(Fister\ \&\ Pawlowski):$ $T_C^{SU(2)} = 230\,MeV$ $T_C^{SU(3)} = 275\,MeV$

Conclusions

- Hamiltonian approach to to Yang-Mills theory
- confinement of quarks and gluons
- deconfinement phase transition
  - critical temperature: $T_C^{SU(3)} \simeq 277\,MeV$
  - SU(2): 2.order
  - SU(3): 1.order

- extension to full QCD
- spontaneous breaking of chiral symmetry
  - M.Pak & H.R. Phys.Lett.B707(2012)566
- external magnetic field
- finite baryon density

Thanks for your attention