A journey in the Extra Dimensions Landscape

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This lecture is for non-experts. The aim is to give a quick view on some ideas or tools in phenomenological studies of extra-dimensions.

Proper references will be included in the proceedings.

Thanks to my collaborators on this subject:
I. Antoniadis, M. Quiros, E. Accomando and S. Davidson

Plan

- Flat extra-dimensions: KK states, localization, thick branes.
- A glimpse into some warp and more general geometries
- Experimental bounds for flat extra-dimensions.
- A very short comment on SUSY breaking

One may ask ...

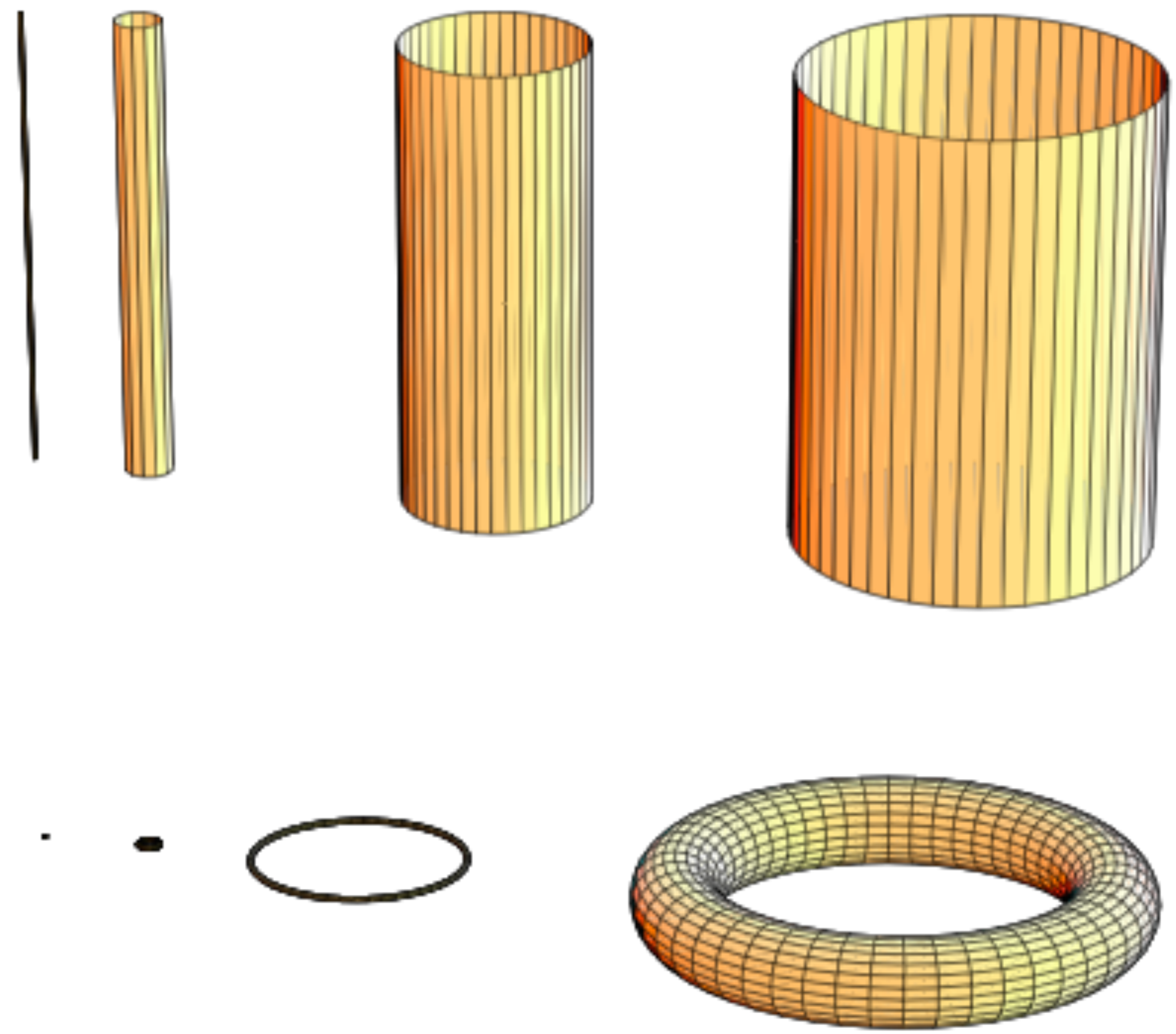
- If it is possible that our world has more dimensions than those we are aware of?

- If so, why don't we see the other dimensions? Is there a way to detect them?

1.We do not have a good theoretical reason for d=4

2. Extra-dimensions are important ingredients of theories for quantum gravity (supergravity, string theory, ...).

Hidden dimensions: D= ... ???



One hidden dimension

Two hidden dimensions

At high energies, space might have more dimensions

Now ...

1. Extra-dimensions can be just tools: AdS/CFT to study QCD, some auxiliary mathematical tool.

2. Extra-dimensions can be "real" and one could observe the effects due to states propagating in them.

   Knowledge of actual experimental bounds is necessary in order to figure if there is some hope for an experimental investigation in a "near future".

The answer to the last question depends strongly on the details of:

1. their geometrical realization

2. and which states populate them.

Extra dimensions can be characterized by:

- Continuous, discrete, fractal ...
- Number
- Compact or Non-compact
- Shape
- Which states propagate inside

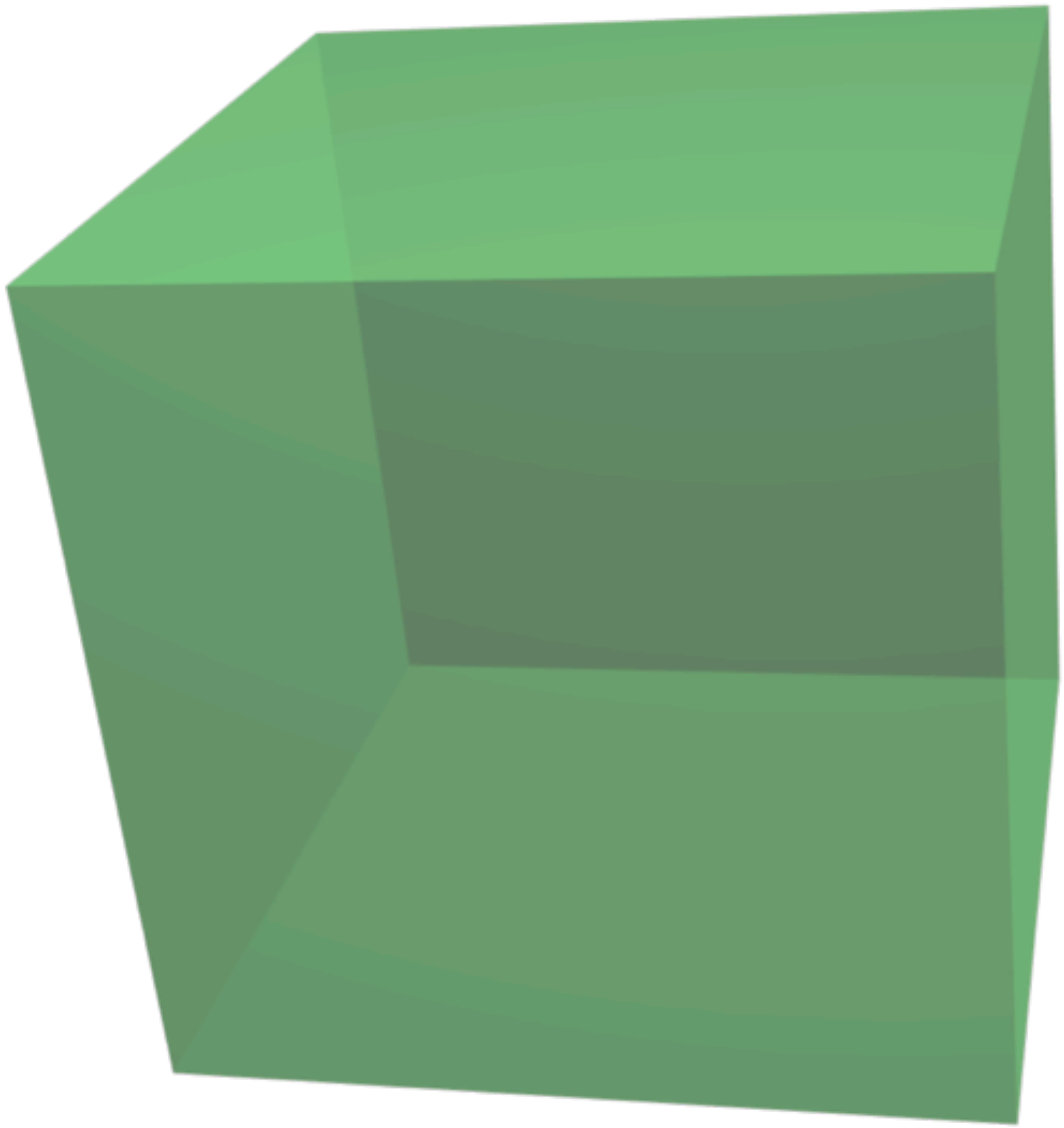
The answer to the last question depends strongly on the details of:

1. their geometrical realization

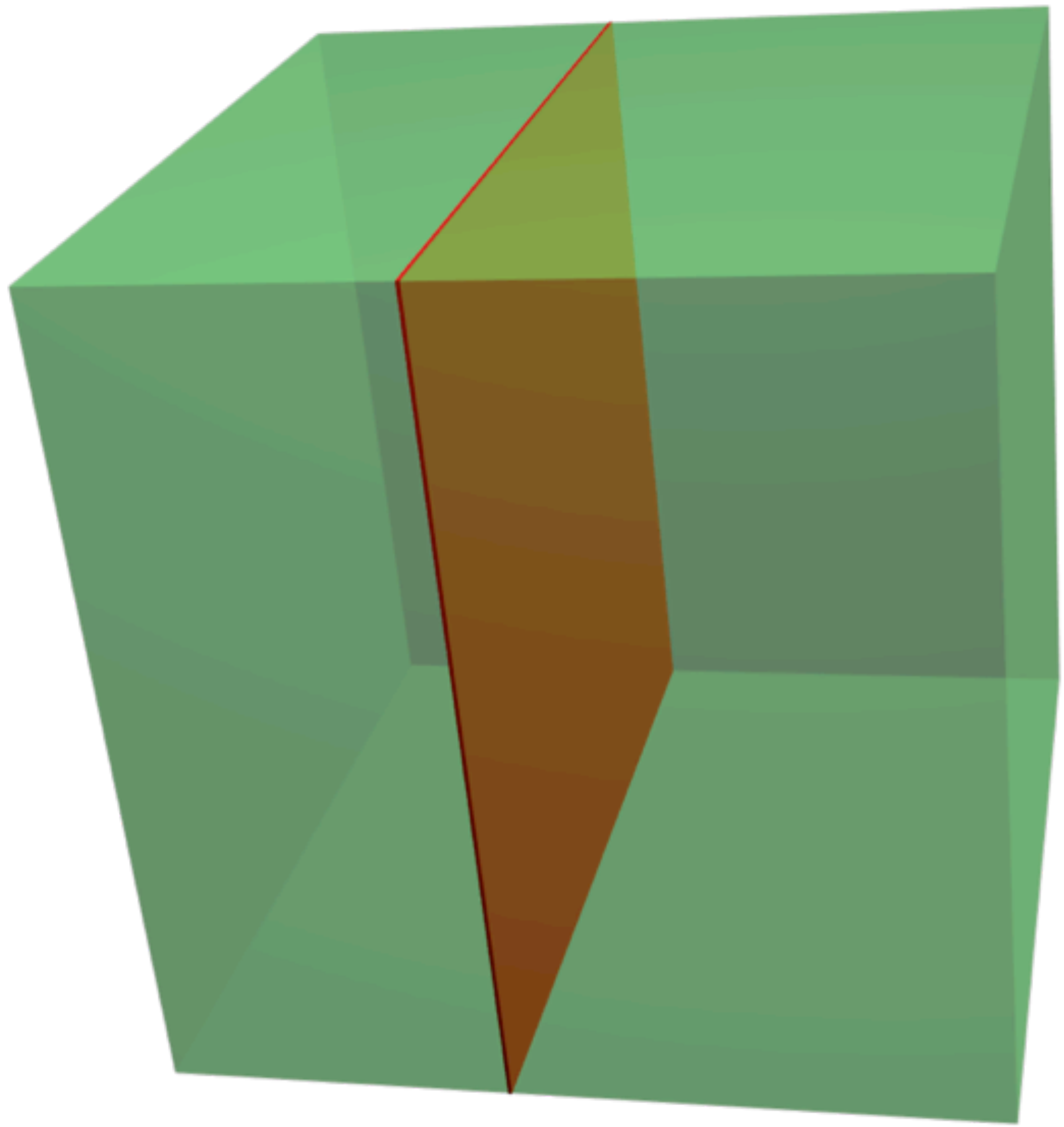
2. and which states populate them.

We "learn from examples": we look at a few simple realizations, and many features can be carried over to more complicated ones.

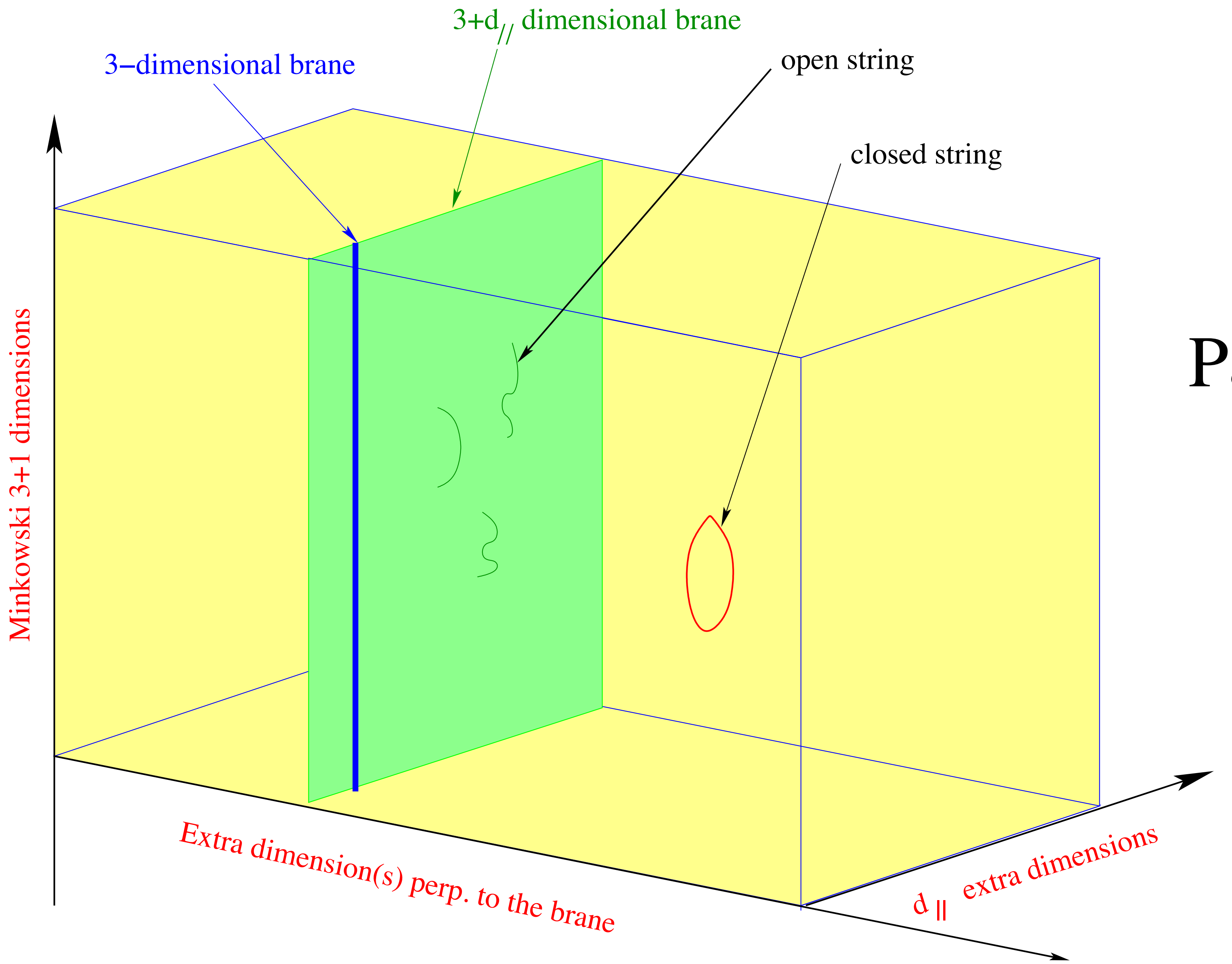
Space-time



Minkowski space as a slice



Our world is a slice in the slice



A 3−brane inside (intersecting) a 3+d //dimensional brane

Parallel / orthogonal dimensions

The simplest Extra-Dimensions

Extra dimensions as a flat box

Extra-dimensions of size $2\pi R_i$

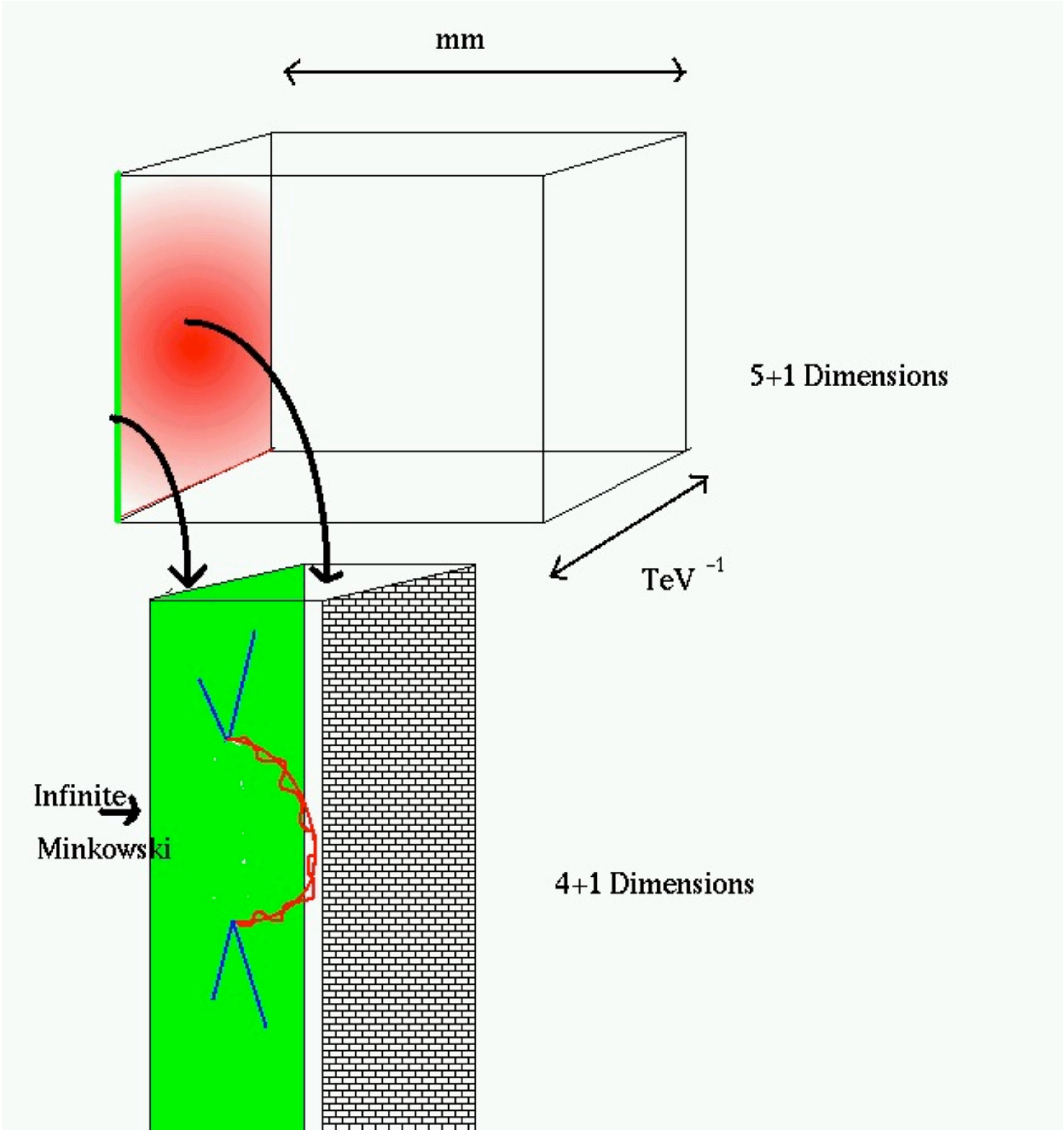
Periodic wave functions correspond to states with quantized momenta along the extra dimensions:

$$\vec{P} = \{p_0, p_1, p_2, p_3, \frac{n_1}{R_1}, \cdots, \frac{n_D}{R_D}\}$$

From the 4D point of view, they appear as massive KK states, with (squared) mass:

$$M_{KK}^2 \equiv M_{\vec{n}}^2 = m_0^2 + \frac{n_1^2}{R_1^2} + \frac{n_2^2}{R_2^2} + \cdots + \frac{n_D^2}{R_D^2},$$

Particles can travel to the “bulk” of the extra-dimensions



Scales of the system

Four energy scales can play a major role:

1. The energy scale of the process: E
2. Extra-dimensions at scale: $\dfrac{1}{R}$
3. particles sub-structure at scale: $M_s = l_s^{-1}$
4. Quantum gravitational effects at scale: $\dfrac{M_s}{g^2}$

Because: $$\frac{1}{R} < M_s < \frac{M_s}{g^2}$$

we shall concentrate on the effects of KK states.

Note:

•The case of $1/R \sim M_s$ the extra degrees of freedom might not have an interpretation as classical geometry dimensions.

•The (string) states at the scale M_s play an important role because they "pollute" dimension 6 and 8 operators induced by other sources. Important to keep in mind when extracting bounds!

The Planck scale

- Using Gauss theorem in higher dimension for the gravitational force, we can derive that the Planck scale is related to the higher dimension fundamental scale:

$$M_p^2 \sim R^d M_s^{d+2}$$

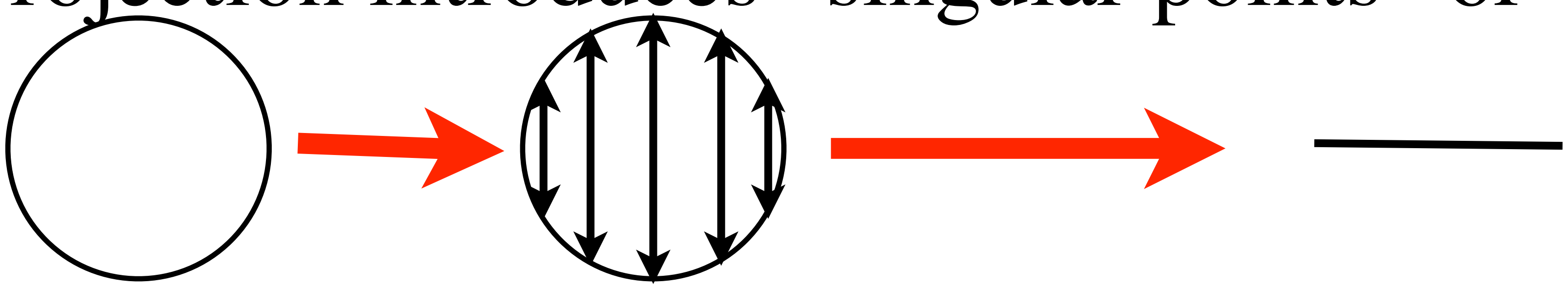
Note that M_p is large if

1. M_s is large
2. or the size R of extra-dimensions is large and the fundamental scale M_s is low: LARGE extra-dimensions

Projection

Getting chiral 4D fermions from 5D requires projections.

Projection introduces “singular points” or “boundaries”.



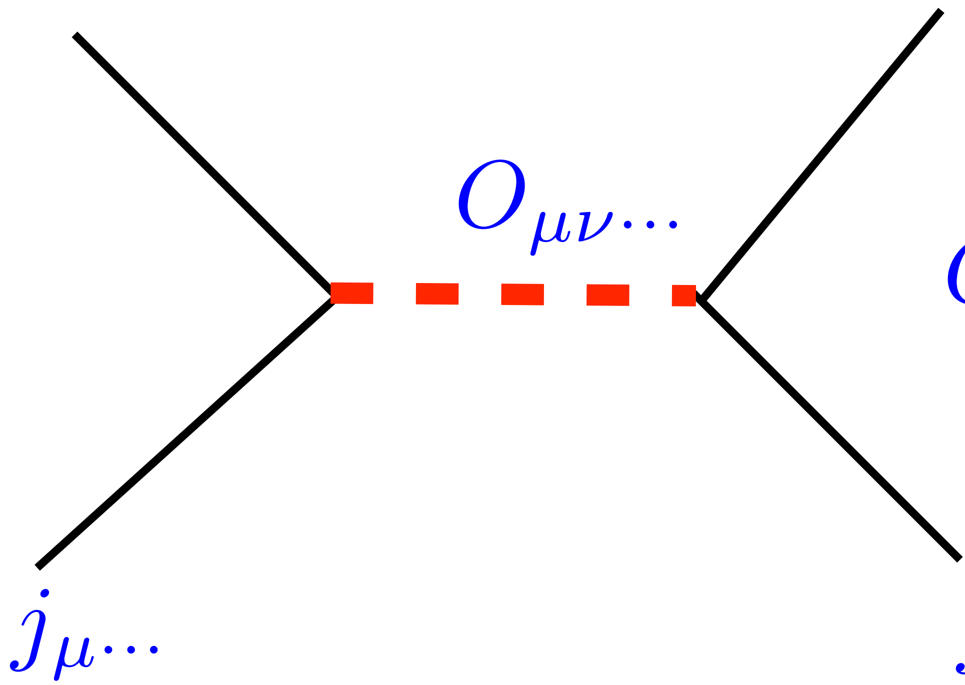
circle Projection Segment

Example: the segment as a $\mathbf{Z}_2$ orbifold projection on the circle.

The segment has boundaries

- Generally, degrees of freedom localized on them.
- Localized states on these boundaries make the "brane-worlds".
- Translation invariance is broken by the boundaries.
- => Non-conservation of momenta along the extra dimensions as KK states interact with localized ones.
- Processes with emission or absorption of a single KK state.

Exchange of virtual heavy KK states



$$O_{\mu\nu\cdots} = \sum_{\vec{n}} \frac{\Pi^{\mu\nu\cdots}}{s - M_{\vec{n}}^2}$$

$$M_{\vec{n}}^2 = m_0^2 + \frac{n_1^2}{R_1^2} + \frac{n_2^2}{R_2^2} + \cdots + \frac{n_D^2}{R_D^2} \, ,$$

The sum diverges for $D \geq 2$ if $\Pi^{\mu\nu\cdots}$ independent of $\vec{n}$

(Hard cut-off on $\vec{n}$ is not the smart way to go)

Amplitudes are OK because the branes are thick!

For example, localized fermions are felt by bulk (KK) gauge bosons as a Gaussian distributions of charges. the corresponding higher dimensional current:

$$j_\mu(x,\vec{y}) = e^{-\frac{\vec{y}^2}{2\sigma^2}} j_\mu(x)$$

where $\vec{y}$ represent the extra-dimensions coordinates.

The "brane effective width" is $\sigma \sim l_s$

Effective coupling of KK states $A^\mu_{\vec{n}}$ with j_μ

exponentially damped (example $\mathbf{Z}_2$ orbifolds):

$$g_{\vec{n}} = \sqrt{2} \sum_{\vec{n}} e^{-\ln\delta \sum_i \frac{n_i^2 l_s^2}{2R_i^2}} g_0$$

4D couplings from Higher Dimensions

Higher Dim states $\Phi_i(x,\vec{y}) = c_i(\vec{y})\phi_i(x)$ have couplings:

$$Y_{ijk}\int d^4x d\vec{y}\ \Phi_i\Phi_j\Phi_k = Y_{ijk}\left[\int d\vec{y} c_i c_j c_k(\vec{y})\right]\int d^4x\phi_i(x)\phi_j(x)\phi_k(x)$$

=> the 4D induced coupling is given by the wave function overlap

$$\tilde{Y}_{ijk} = Y_{ijk}\int d\vec{y} c_i c_j c_k(\vec{y})$$

For localized states: $c_i(\vec{y}) \sim e^{-a(\vec{y}-\vec{y}_0)^2}$

Creating hierarchy of couplings is an easy game!

But be careful to dangerous flavor changing operators

Not only states but also couplings can appear localized

(Well known in the studies of the Casimir energies)
On the boundaries, all counter-terms allowed by symmetries of the system will appear.

Radiative corrections can induce both corrections to the bulk and localized couplings. Example:

$$\left(\frac{\delta(\vec{y})}{g_0^2} + \frac{1}{g^2}\right)\mathrm{trF}_{\mu\nu}\mathrm{F}^{\mu\nu}$$

Correction to gauge couplings come both from the localized states and from the bulk states. These corrections modify the shape of wave functions of bulk states.

Life goes off the boundaries

Also, localized mass terms are possible. Example:

$$\int d^4x d\vec{y}\ (\delta(\vec{y})m_0^2 + m^2)\phi^2(x,\vec{y})$$

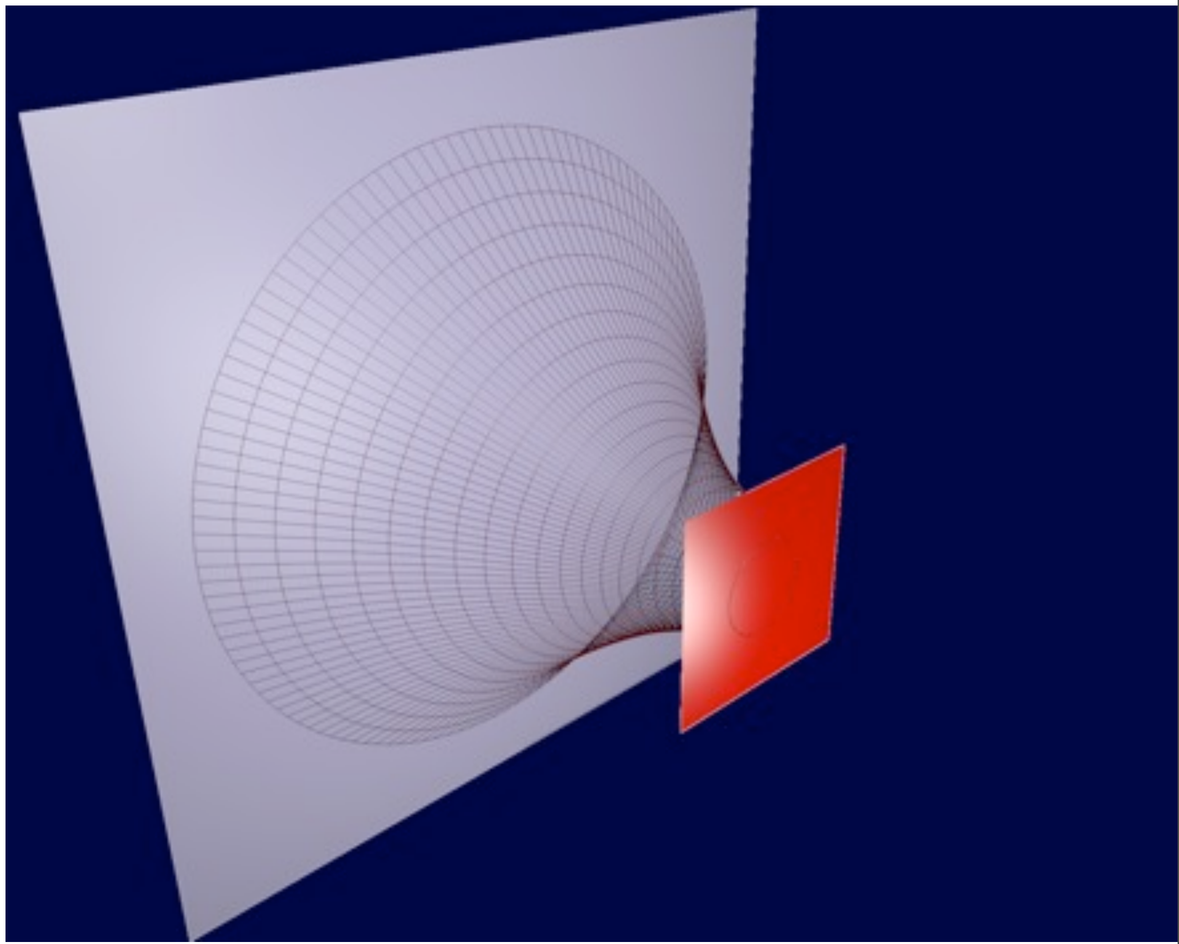
Minimization of the potential tells you that these localized masses will push the wave function out-of the brane.

Useful tool to engineer weaker bulk-boundary couplings.

More fancy spaces

Metric of The 4D space-time depends on the 5th coordinate:

$$ds^2 = a(z)(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$



Example Randall-Sundrum:

$$a(z) = \frac{1}{z^2} \quad \text{slice of AdS.}$$

$$m_n \simeq (n + c)\pi k e^{-\pi k R}$$

Example of the Soft Wall:

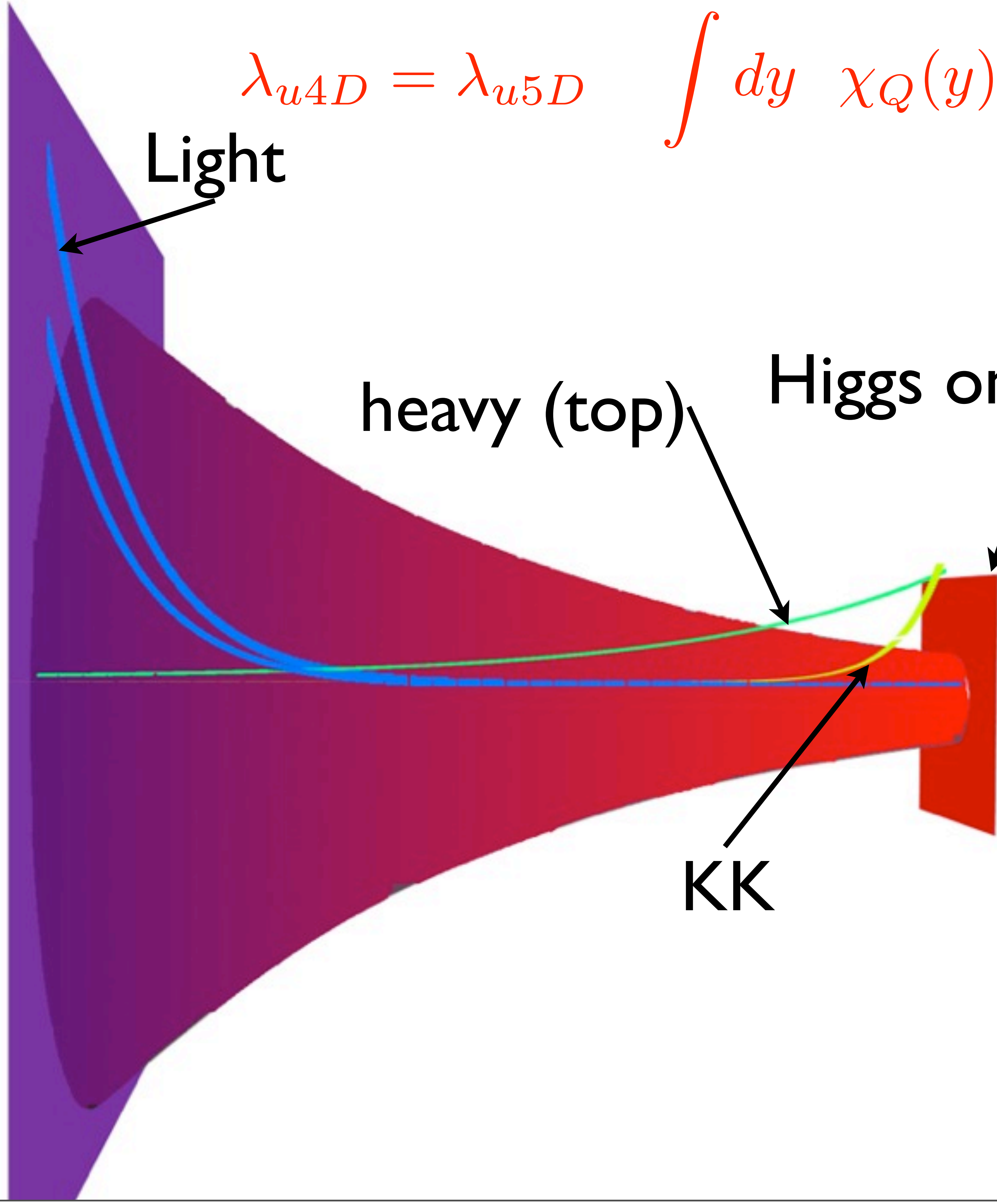
$$a(z) = -g_{00} = \frac{1}{z^2} e^{-bz^2}$$

$$m_n \sim \sqrt{n}$$

The energy scale redshifts with gravitational potential $\sqrt{-g_{00}}$

5D wave functions overlap

$$\lambda_{u4D} = \lambda_{u5D} \quad \int dy \;\; \chi_Q(y)\, \chi_H(y)\, \chi_u(y)$$



- RS geometry allows to build models using AdS/CFT insights.
- The 5th dimension plays the role of the RGE trajectory.
- KK of graviton strongly coupled to the SM.
- Modification of the warp factor close to the infrared (SM) allows to play with the shape of wave functions and their overlap.

More fun?

Consider a space with a metric:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - dr^2 - e^{-\frac{4}{D-1} W(r)} d\Omega_{D-1}$$

It is possible to show that the propagation equation in the internal space takes the simple form of one-dimensional Schrödinger equation with the potential:

$$V(r) = -W''(r) + W'^2(r) + A\, e^{\frac{4}{D-1} W(r)}$$

Playing with the form of $W(r)$ allows to engineer:

1. A discrete spectrum of KK states but different forms for the spacing of the eigenvalues. Getting the second KK close to the first one makes it more likely to be experimentally found (signature of dimensions)

2. Continuum

3. Continuum with a mass gap: at some energy scale one starts to produce wave packets of 5D states. (Non-compact finite volume dimensions, eigen-states non-normalizable, no 4D Lagrangian description)

Experimental bounds and searches (flat dimensions)

ADD, non collider searches

- Table top experiments excludes modification of gravity (size of extra-dimension) at scales larger than about 30 microns.

ADD: non-collider bounds

- The flux of light in the sky tells us that there are not many KK gravitons produced earlier in history that are decaying now to photons.
- Supernovae cooling is not modified by many light KK gravitons taking energy away.

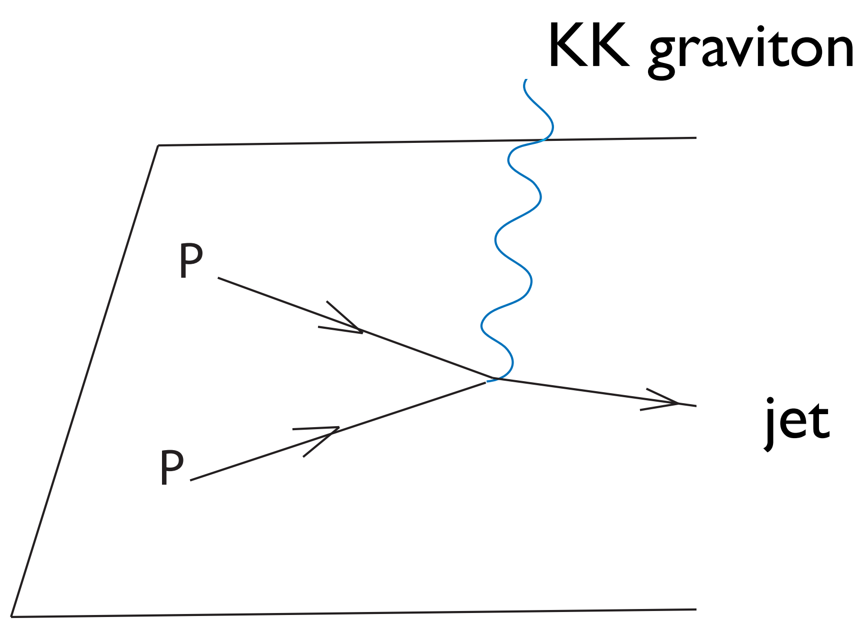
	$R_\perp(n=2)$	$R_\perp(n=4)$	$R_\perp(n=6)$
Present non-collider bounds			
SN1987A	3×10^{-4}	1×10^{-8}	6×10^{-10}
COMPTEL	5×10^{-5}	-	-

For D=2, neutron star shining and heating gives a stronger bound of 0.00016 microns (Hannestad and Raffelt)

ADD: LHC bounds

KK gravitons can be created in brane states scattering and lead to two types of signals:

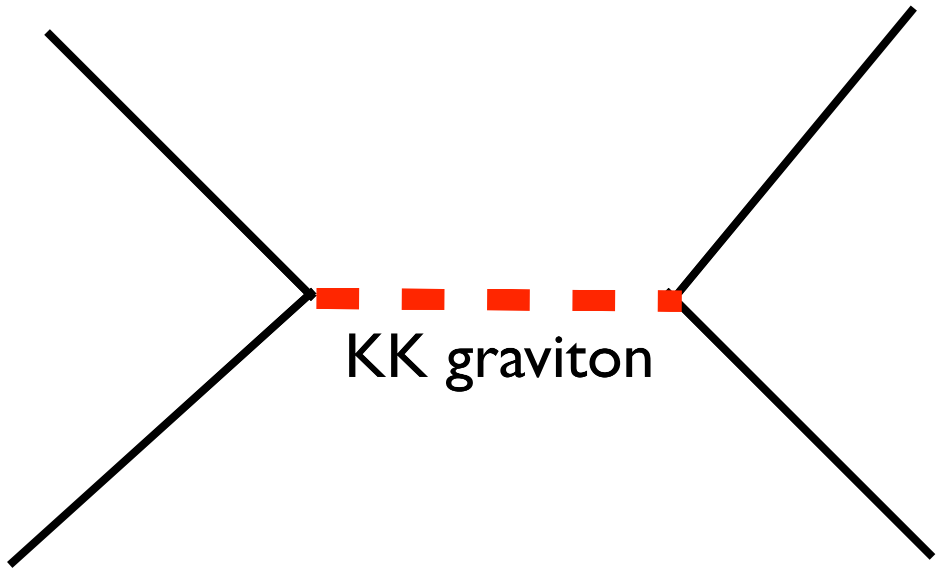
- Emission to the bulk: Monojet with missing energy.



For D=2 to 6, one gets bounds on M_D of around 2.6 TeV to 2.9 TeV.

ADD: LHC bounds

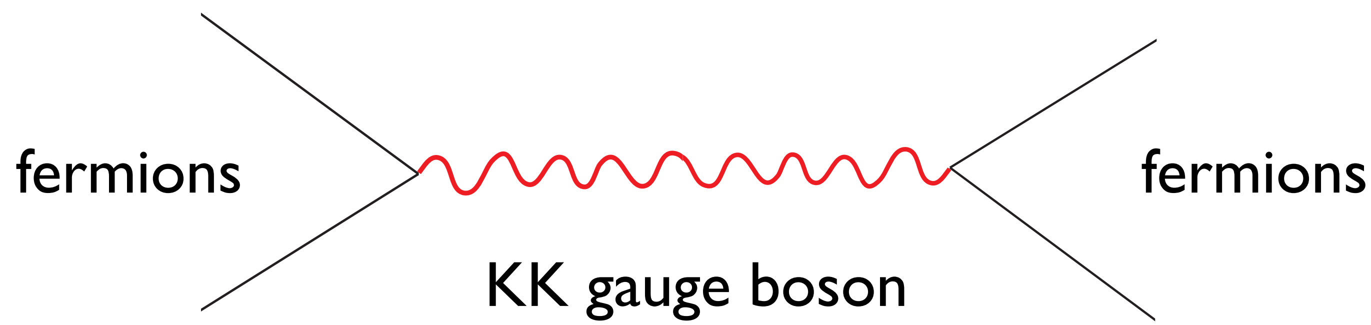
- Virtual exchange of spin 2 states leading to higher dimension operators. Polluted by other contributions, they do not allow to extract robust bounds.

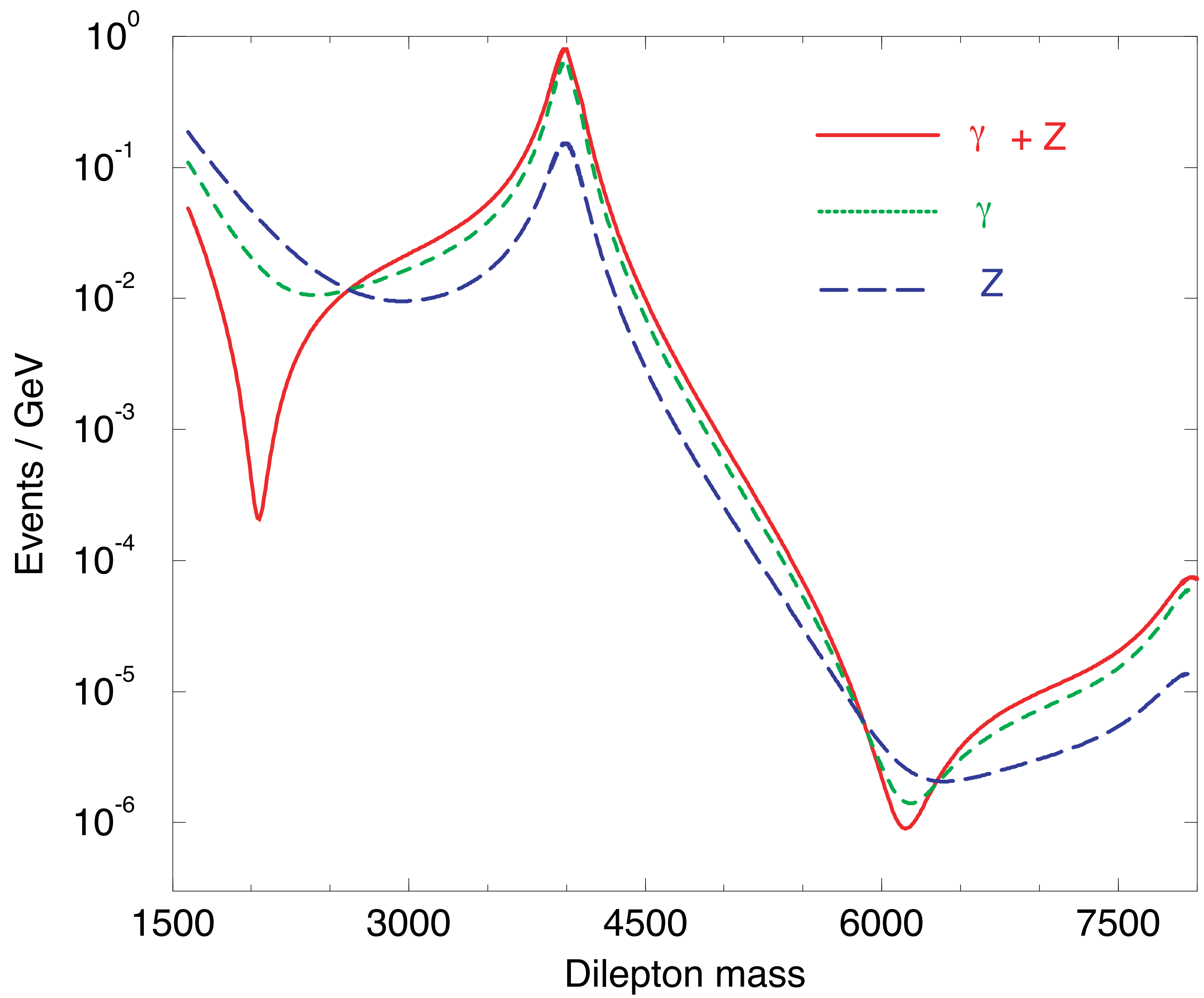


- To take with care: Present fits give bounds between 5 TeV (D=3) and 3.3 TeV (D=7) (CMS@8TeV)

KK excitations of gauge bosons

- Bounds depend on “what propagates” in the extra dimension
- Gauge bosons are in the bulk and matter on the boundary I. Antoniadis, K.B, M. Quiros ’94:



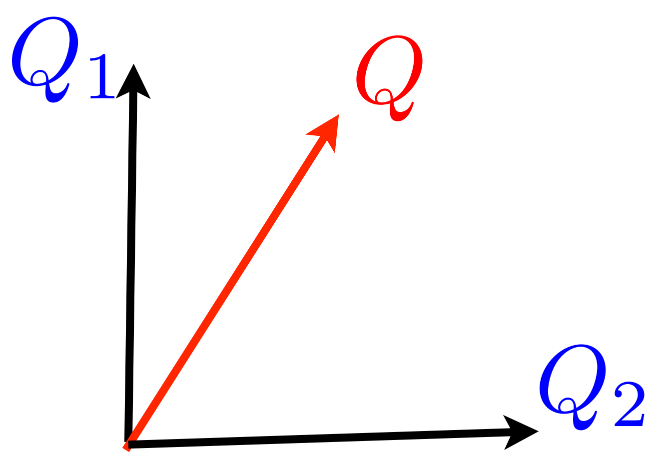


Signal for 4 TeV KK

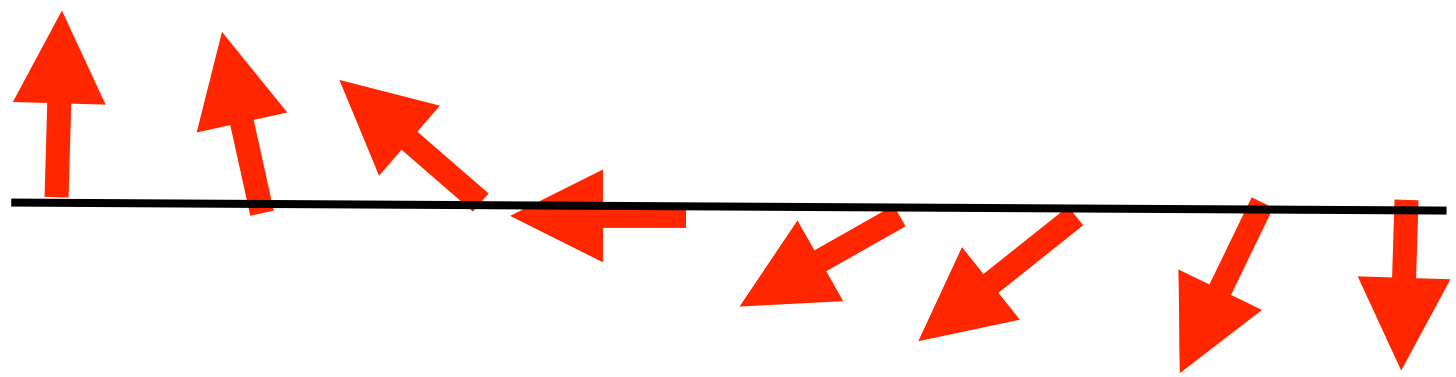
Absence of resonances at LHC puts bounds on KK of Z bosons and gluons of order 2 TeV (7 TeV data), but LEP EW data gives a bound of 3.6 TeV.

SUSY breaking in extra-dimensions

- In SUSY theories, a brane breaks half of the bulk supersymmetries.
- Example N=2 SUSY and branes break half to N=1.
- Think of the direction of the preserved SUSY as a "spin"



- In the extra-dimension, branes located at different points may point in different directions



- SUSY is preserved if all the "spins" are parallel
- A bulk field, the gravitino wave function, interpolates between different SUSY direction.
- No zero mode for the gravitino: SUSY is broken

- SUSY breaking order parameter is : $m_{3/2} = \alpha/R$

  Two limits:

- $m_{3/2}R \ll 1$ for $\not{\text{SUSY}}$ can use the usual 4D description
- $m_{3/2}R \sim 1$ $\not{\text{SUSY}}$ has higher dimensional description

  Both possibilities are still allowed.

Conclusions

- Experimental bounds depend on the geometry and content of extra-dimensions.
- Inherited from the top-down studies (string theory), examples (flat, AdS slice, ...) are motivated by simplicity and allow first exploration.
- More exotic internal spaces can be considered.

Conclusions

- Search for effects of extra-dimensions at LHC.

- Even if they are not experimentally observed, extra-dimensions have became a useful tool for theorists and phenomenologists exploring new models.