

# ***HEXAGONAL FLOW AS INTERPLAY OF ELLIPTIC AND TRIANGULAR FLOWS***

***E. Zabrodin***

**in collaboration with**

***L. Bravina, H. Brusheim Johansson, G. Eyyubova,  
V. Korotkikh, I. Lokhtin, L. Malinina, S. Petrushanko,  
and A. Snigirev***

**University of Oslo and Moscow State University**

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***Kolumbari, Crete, Greece 31.08.2013***

# OUTLINE

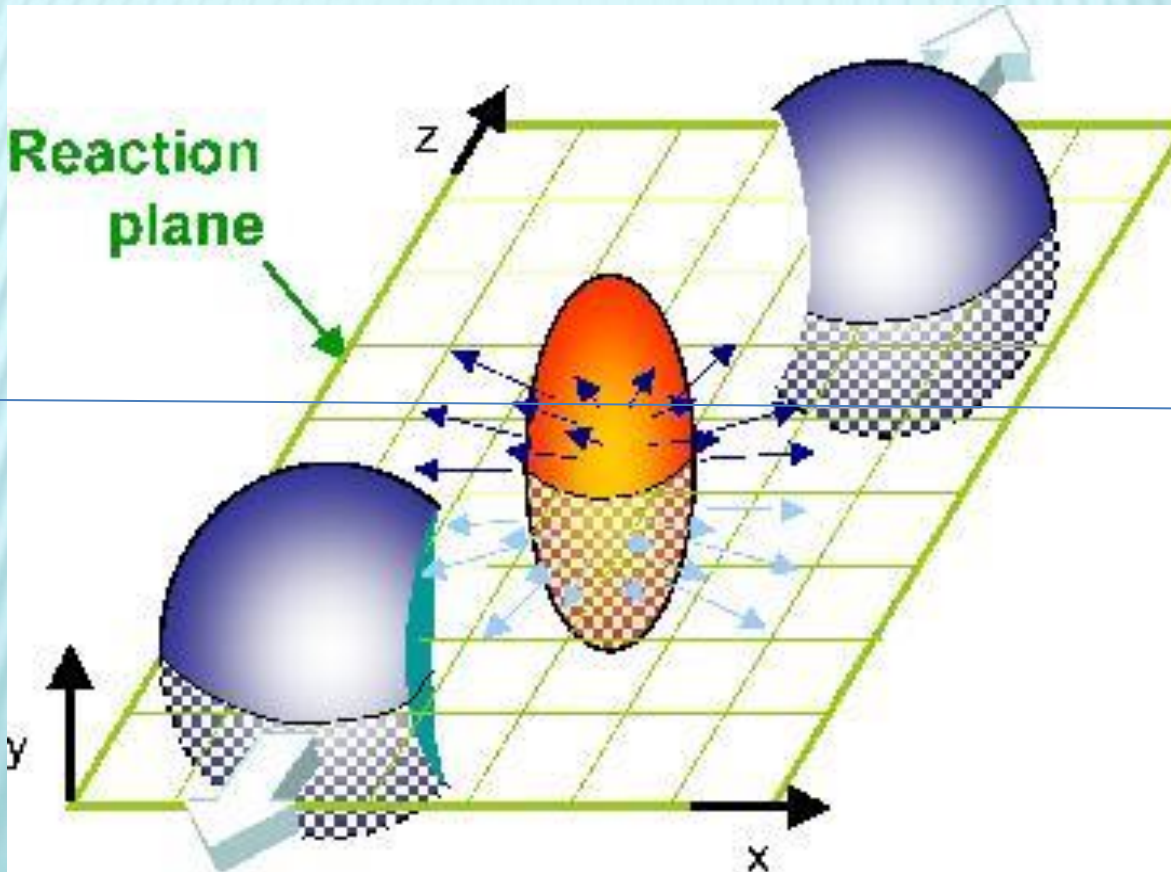
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- I. **Description of anisotropic flow in relativistic heavy ion collisions:**
  - (a) **elliptic and triangular flows**
  - (b) **initial fluctuations and higher harmonics**
- II. **HYDJET++ model (hydro + jets)**
- III. **Jets influence on NCQ-scaling and  $v_4/(v_2)^2$  ratio at RHIC and LHC**
- IV. **Hexagonal flow  $v_6$**

# ANISOTROPIC FLOW

S.Voloshin and Y.Zhang, Z.Phys.C70 (1996) 665

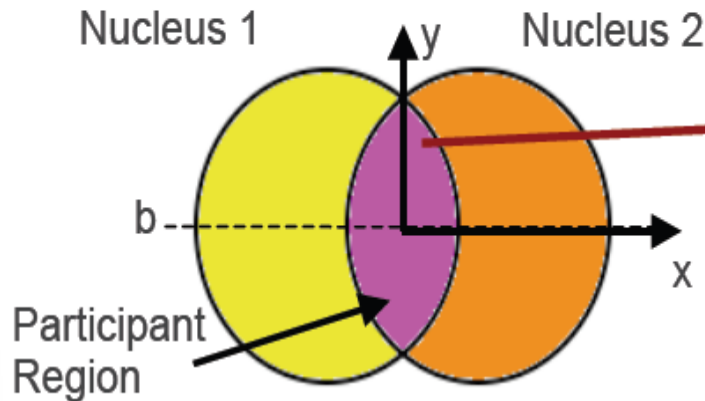
$$\frac{dN}{d\varphi} = \frac{1}{2\pi} \left( 1 + \sum_{n=1}^{\infty} 2v_n(p_t) \cos[n(\varphi - \psi_r)] \right)$$



# ELLIPTIC FLOW

B. Alver, talk at Rencontres lons Lourds (2010)

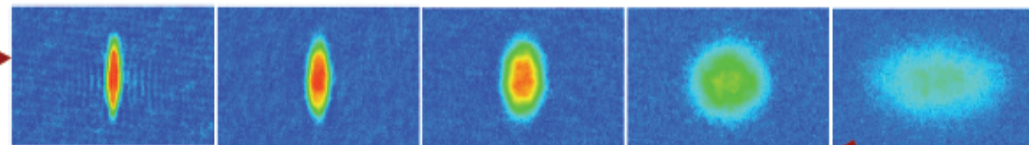
Initial anisotropy



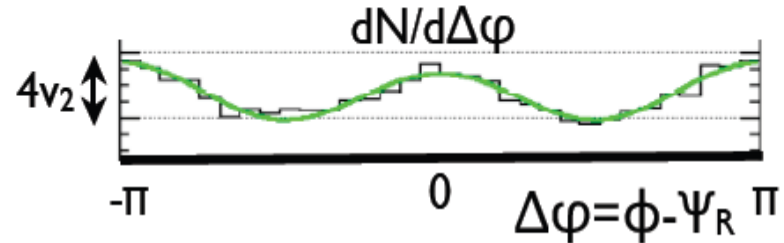
$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left( 1 + \sum 2v_n \cos(n(\phi - \psi_R)) \right)$$

$$v_2 = \langle \cos(2(\phi - \psi_R)) \rangle \propto \varepsilon$$

Pressure driven expansion

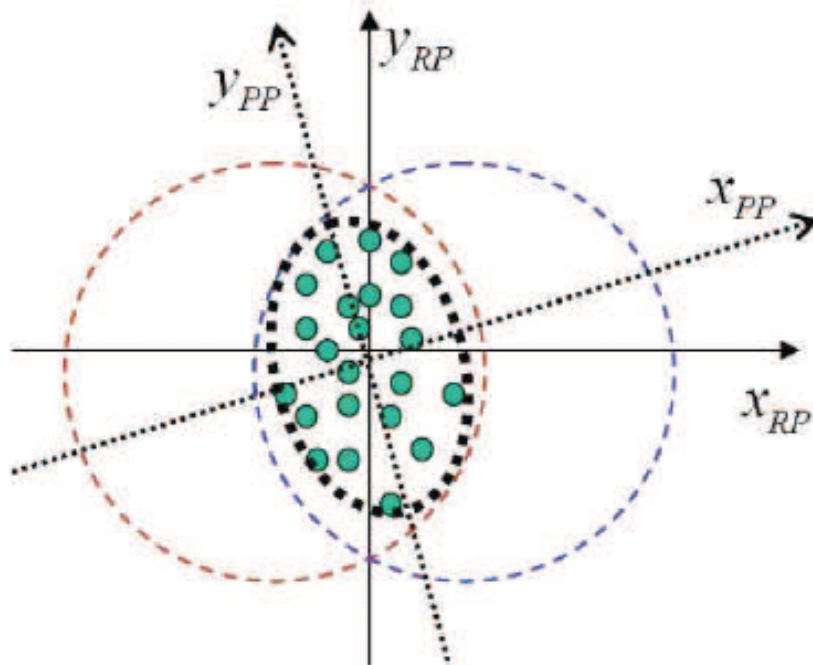


Final anisotropy



Elliptic flow is quantified by the second Fourier coefficient ( $v_2$ ) of the observed particle distribution

# Eccentricity fluctuations



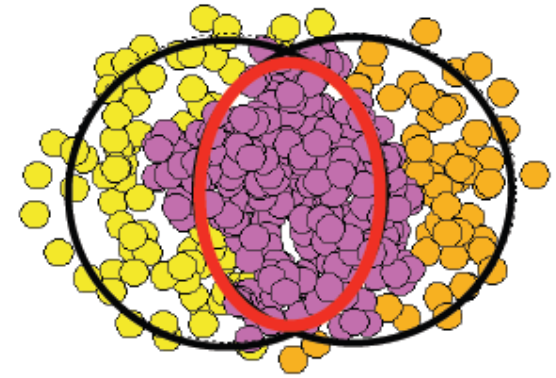
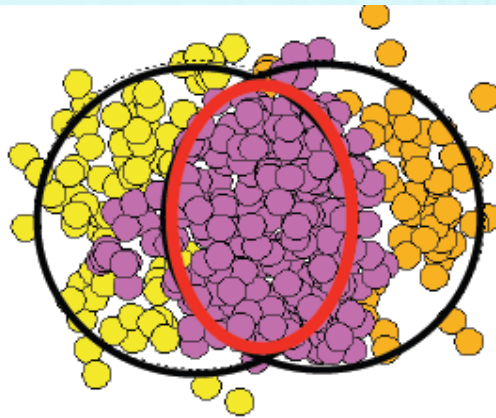
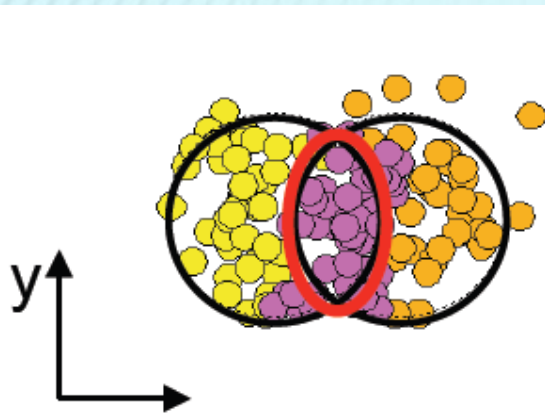
Depending on where the participant nucleons are located within the nucleus at the time of the collision, the actual shape of the overlap area may vary: the **orientation and eccentricity** of the ellipse defined by participants fluctuates.

Assuming that  $v_2$  scales like the eccentricity, **eccentricity fluctuations** translate into  **$v_2$  fluctuations**

Eccentricity fluctuation can be computed in MC Glauber model or derived from experiment by comparing different methods for flow calculation.

# ECCENTRICITY

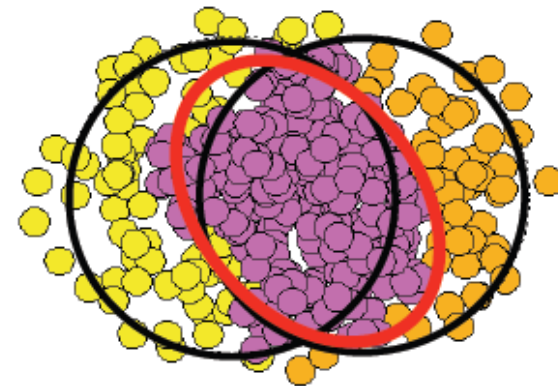
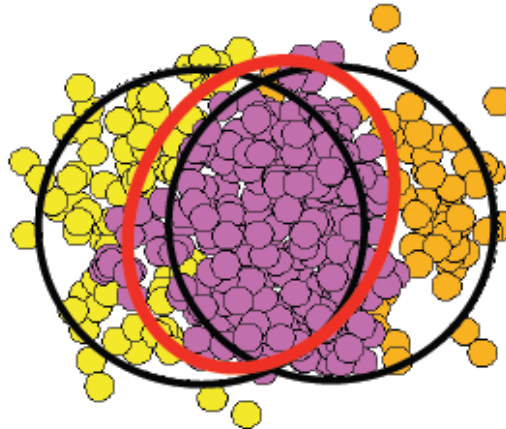
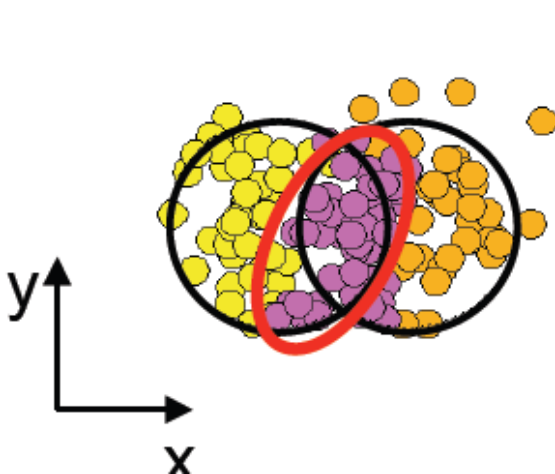
## STANDARD



$$\epsilon_{\text{RP}} = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$$

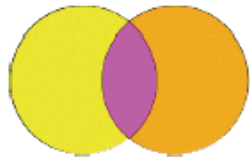
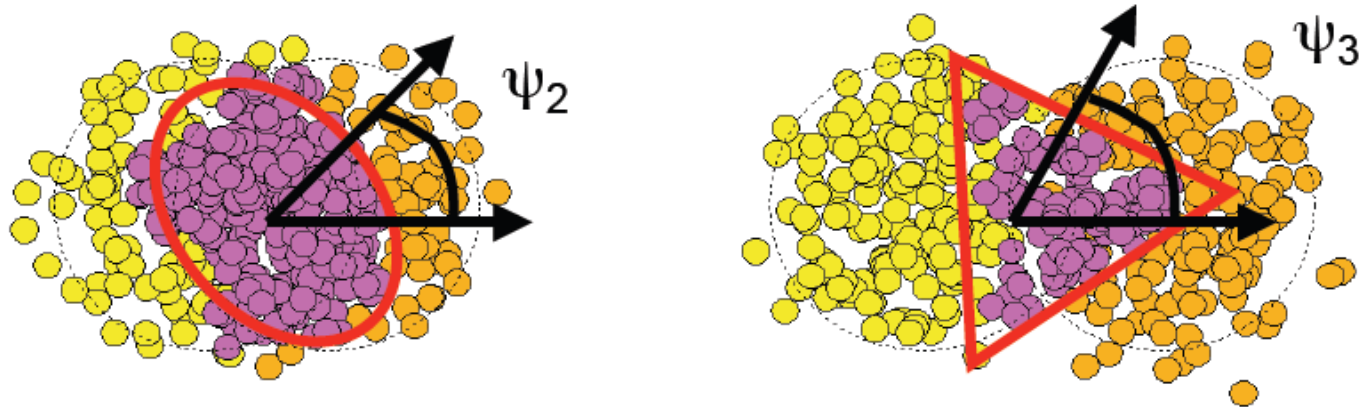
## PARTICIPANT

$$\epsilon_{\text{part}} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$



# TRIANGULAR FLOW

B. Alver and G.Roland, PRC 81 (2010) 054905



$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left( 1 + \sum 2v_n \cos(n(\phi - \psi_R)) \right)$$

$$v_2 = \langle \cos(2(\phi - \psi_R)) \rangle$$

$$v_3 = 0$$



$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left( 1 + \sum 2v_n \cos(n(\phi - \psi_n)) \right)$$

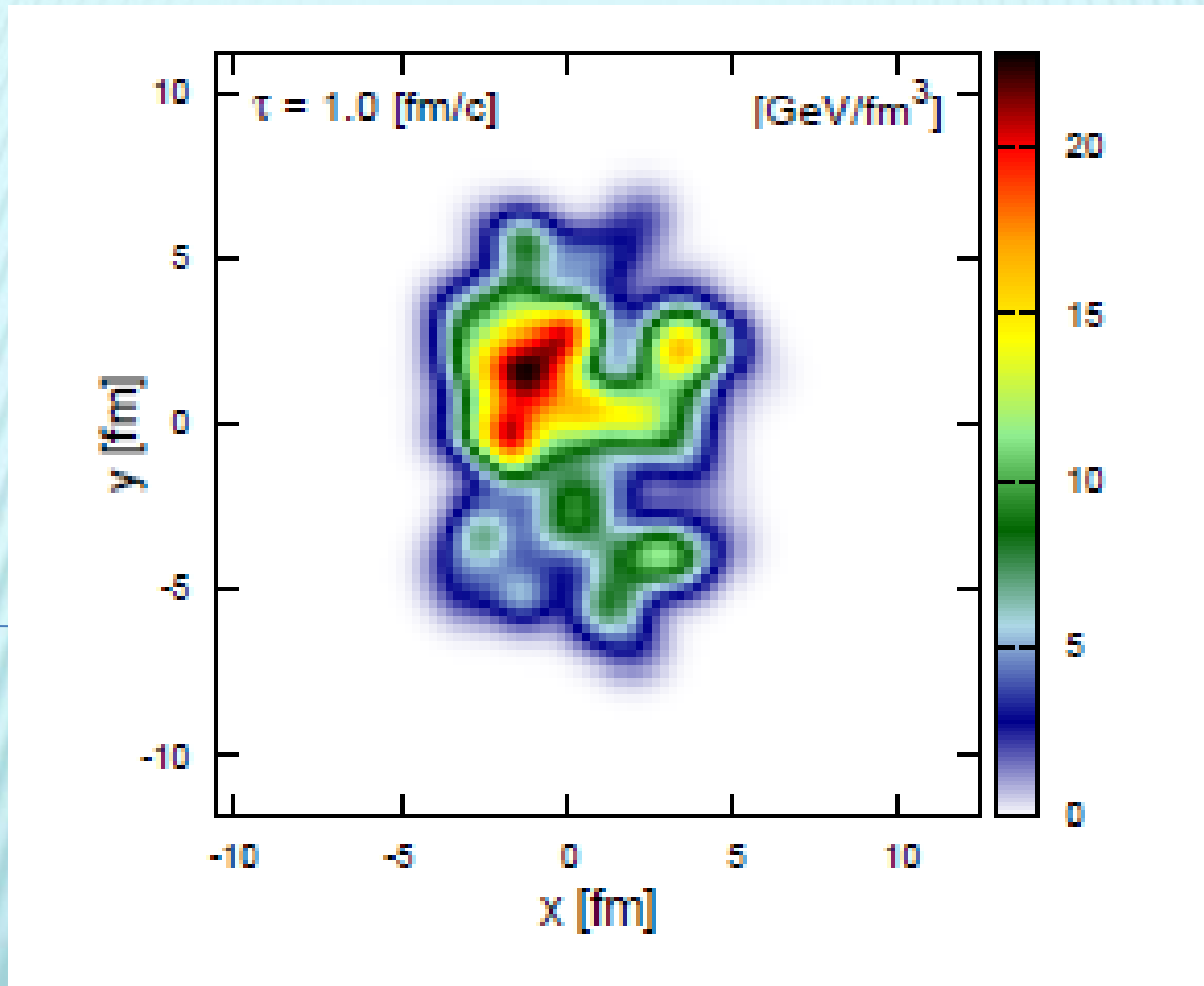
$$v_2 = \langle \cos(2(\phi - \psi_2)) \rangle$$

$$v_3 = \langle \cos(3(\phi - \psi_3)) \rangle$$

The triangular initial shape leads to triangular hydrodynamic flow

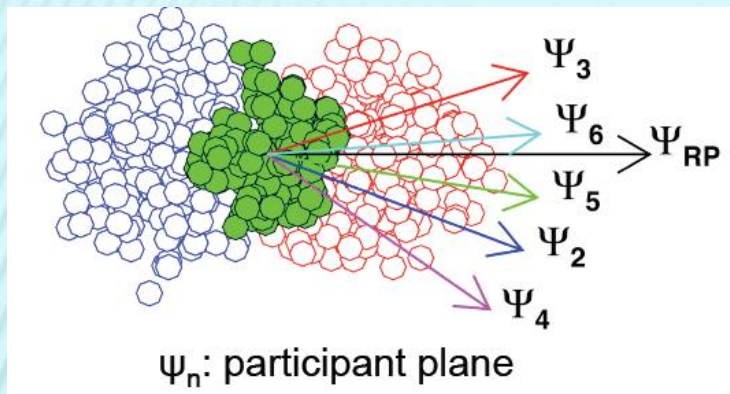
# INITIAL-STATE FLUCTUATIONS (example)

W.-L. Qian et al., arXiv: 1305.4673 [hep-ph]

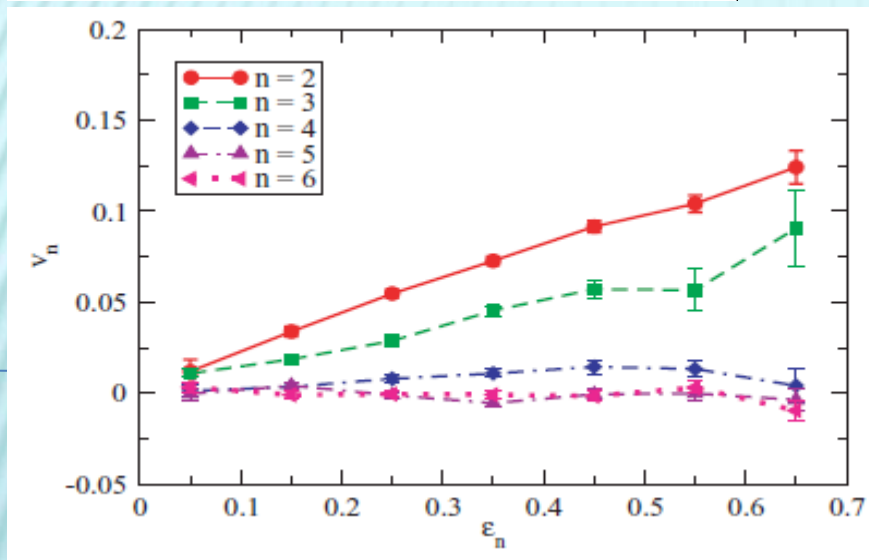
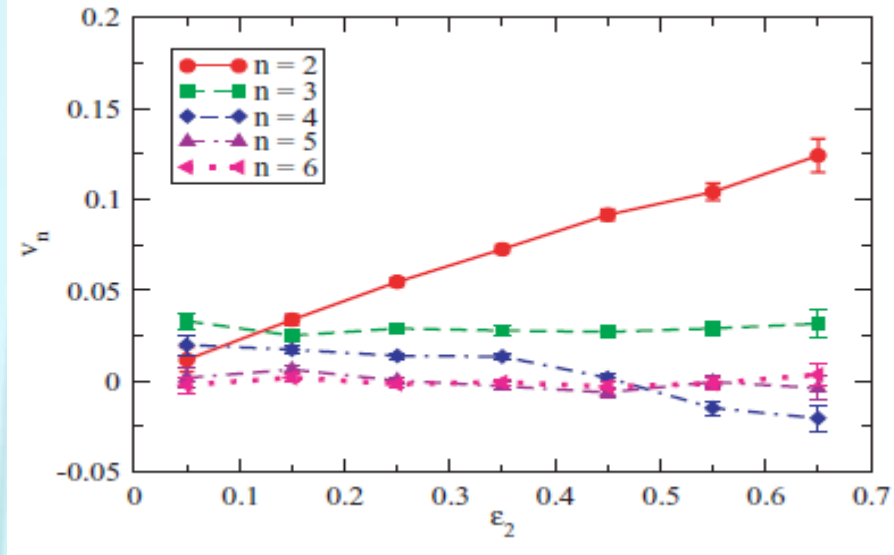


Energy distribution of a random NeXuS event

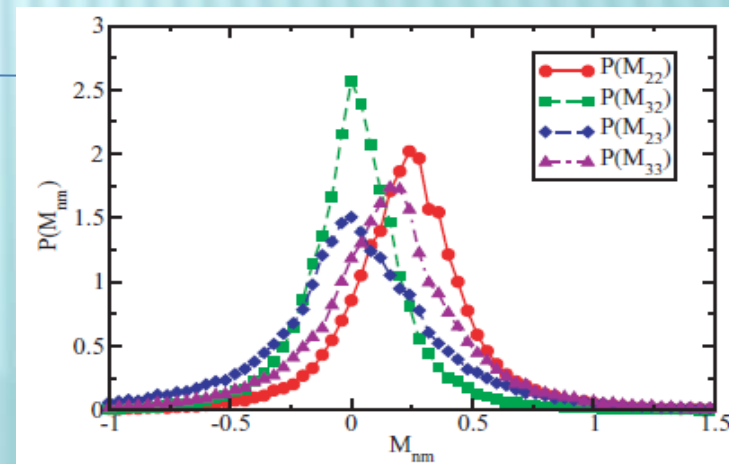
# CROSS-TALK BETWEEN FLOW HARMONICS



G.-Y. Qin et al, PRC 82 (2010) 064903



$$\begin{pmatrix} v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} M_{22} & M_{23} \\ M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} \epsilon_2 \\ \epsilon_3 \end{pmatrix}$$



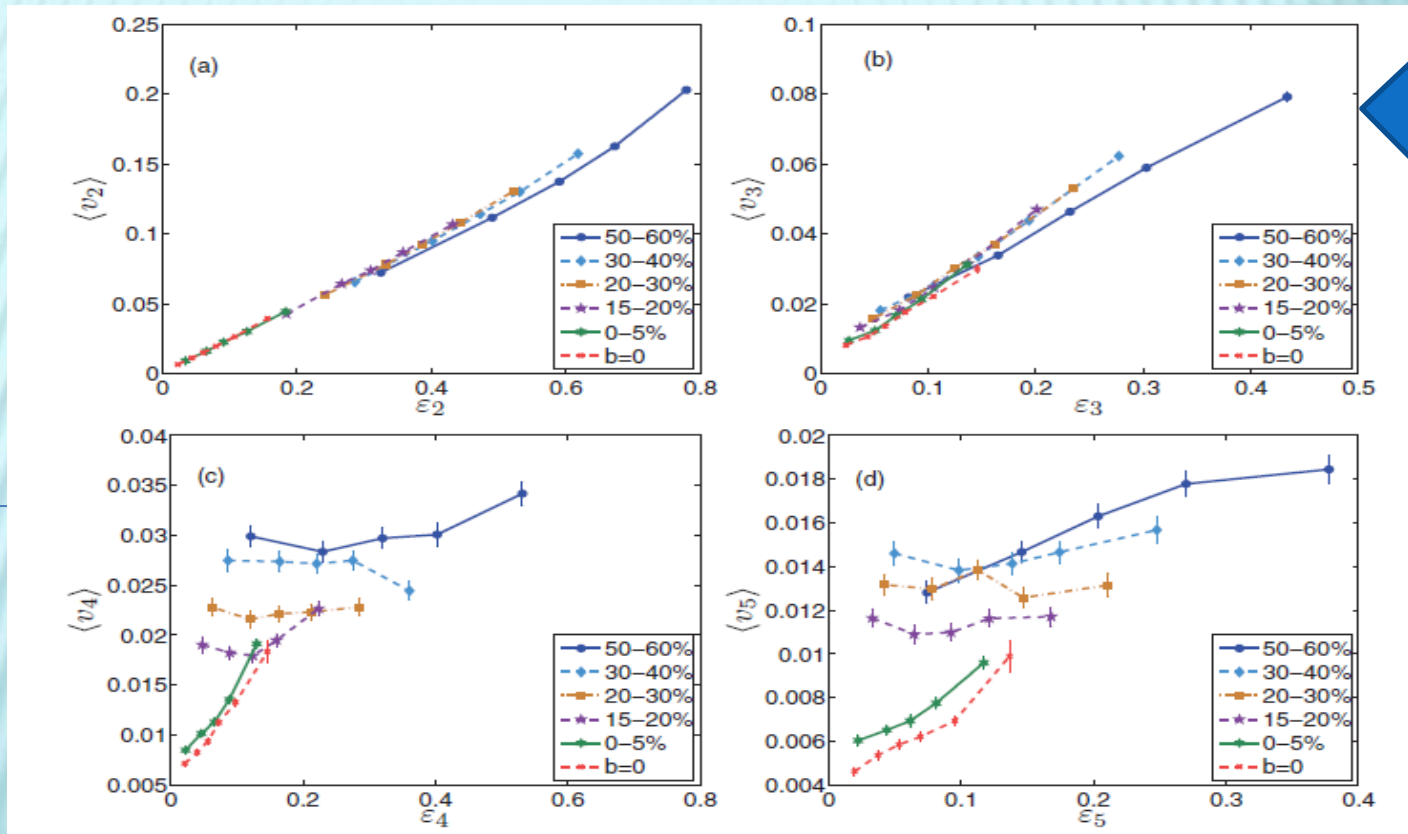
Only the first few flow harmonics of final-state hadrons survive after hydrodynamic evolution

# CROSS-TALK BETWEEN FLOW HARMONICS

Z. Qiu and U. Heinz, PRC 84 (2011) 024911

$$\varepsilon_n e^{in\psi_n^{\text{EP}}} = \frac{\int dx dy r^2 e^{in\phi} e(x, y)}{\int dx dy r^2 e(x, y)}$$

$$v_n(y, p_T) e^{in\psi_n^{\text{EP}}(y, p_T)} = \frac{\int d\phi_p e^{in\phi_p} \frac{dN}{dy p_T dp_T d\phi_p}}{\frac{dN}{dy p_T dp_T}}$$

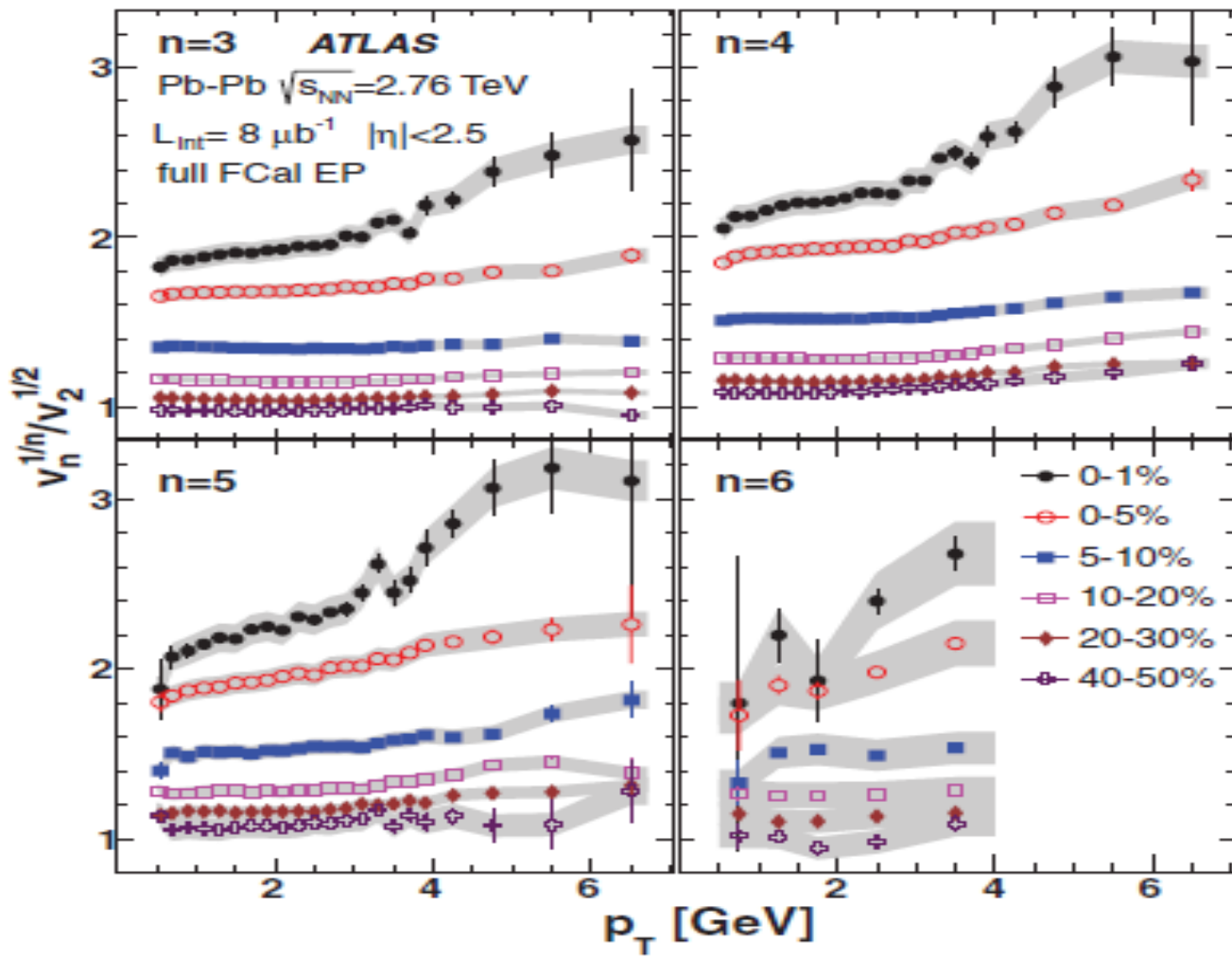


- (1) The basic response of  $v_2$  and  $v_3$  to eccentricities is approx. **linear**
- (2) Higher flow coefficients show **poor correlation** with the eccentricities of the same order

# SCALING OF HIGHER ORDER FLOW HARMONICS

J.-Y. Ollitrault :

$$V_n^{1/n} \propto V_2^{1/2}$$



# HYDJET++ event generator

I.Lokhtin, L.Malinina, S.Petrushanko, A.Snigirev, I.Arsene, K.Tywoniuk,  
Comp. Phys. Commun.180 (2009) 779-799 (arXiv:0809.2708[hep-ph])

- The soft part of HYDJET++ event represents the "thermal" hadronic state.
  - ✓ multiplicities are determined assuming thermal equilibrium
  - ✓ hadrons are produced on the hypersurface represented by a parameterization of relativistic hydrodynamics with given freeze-out conditions
  - ✓ chemical and kinetic freeze-outs are separated
  - ✓ decays of hadronic resonances are taken into account (360 particles from **SHARE** data table) with "home-made" decayer

*the model reproduces soft hadroproduction features at RHIC and LHC (particle spectra, elliptic flow, HBT)*

- The hard, multi-partonic part of HYDJET++ event is identical to the hard part of Fortran written HYDJET (PYTHIA6.4xx + PYQUEN1.5) => **now PYTHIA Pro-Q20 tune !!** PYQUEN event generator is used for simulation of rescattering, radiative and collisional energy loss of hard partons in expanding quark-gluon plasma created in ultrarelativistic heavy ion AA collisions. HYDJET++ includes nuclear shadowing correction for parton distributions (important at LHC!) Impact-parameter dependent parameterization of *nuclear shadowing* (K.Tywoniuk, I.Arsene, L.Bravina, A.Kaidalov and E.Zabrodin, Phys. Lett. B 657 (2007) 170)

## Model parameters.

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1. Thermodynamic parameters at chemical freeze-out:  $T_{ch}$  ,  $\{\mu_B, \mu_S, \mu_Q\}$
  2. If thermal freeze-out is considered:  $T_{th}$  ,  $\mu\pi$ -normalisation constant
  3. Volume parameters:  $T, \Delta T, R$ 
    1.  $\rho_{max}$  -maximal transverse flow rapidity for Bjorken-like parametrization
  5.  $\eta_{max}^y$  -maximal space-time longitudinal rapidity which determines the rapidity interval  $[-\eta_{max}, \eta_{max}]$  in the collision center-of-mass system.
  6. Impact parameter range: minimal  $b_{min}$  and maximal  $b_{max}$  impact parameters
  7. Flow anisotropy parameters  $\delta(b), \epsilon(b)$
- 

### PYTHIA+PYQUEN obligatory parameters

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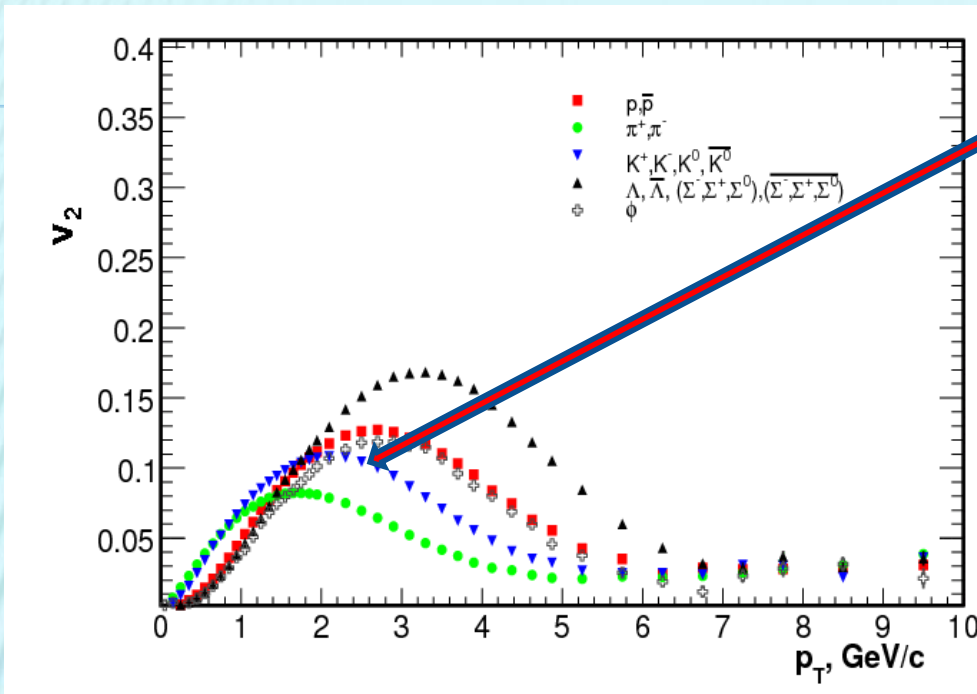
9. Beam and target nuclear atomic weight  $A$
10.  $\sqrt{s_{NN}}$  -c.m.s. energy per nucleon pair (PYTHIA initialization at given energy)
11. **ptmin** – minimal pt of parton-parton scattering in PYTHIA event (ckin(3) in /pysubs/)
12. **nhsel** flag to include jet production in hydro-type event:  
  
0 - jet production off (pure FASTMC event),  
1 - jet production on, jet quenching off (FASTMC+njet\*PYTHIA events),  
2 - jet production & jet quenching on (FASTMC+njet\*PYQUEN events),  
3 - jet production on, jet quenching off, FASTMC off (njet\*PYTHIA events),  
4 - jet production & jet quenching on, FASTMC off (njet\*PYQUEN events);
13. **ishad** flag to switch on/off nuclear shadowing

# Parameters of energy loss model in PYQUEN

(default, but can be changed from the default values by the user)

1. **T0** - initial temperature of quark-gluon plasma for central Pb+Pb collisions at mid-rapidity (initial temperature for other centralities and atomic numbers will be calculated automatically)  
at LHC: **T0=1 GeV**, at RHIC(200 AGeV) **T0=0.300 GeV**
2. **tau0** - proper time of quark-gluon plasma formation  
at LHC: **tau0=0.1 fm/c**, at RHIC(200 AGeV) **tau0=0.4 fm/c**
3. **nf** - number of active quark flavours in quark-gluon plasma (nf=0, 1, 2 or 3) at LHC: **nf=0**, at RHIC(200 AGeV) **nf=2**
4. **ienglu** - flag to fix type of medium-induced partonic energy loss (ienglu=0 - radiative and collisional loss, ienglu=1 - radiative loss only, ienglu=2 - collisional loss only, default value is ienglu=0);  
**ianglu** - flag to fix type of angular distribution of emitted gluons (ianglu=0 - small-angular, ianglu=1 - wide-angular, ianglu=2 - collinear, default value is ianglu=0).  
**ienglu=0**

# $V_2$ in HYDJET++ for different particles (centrality 30%)



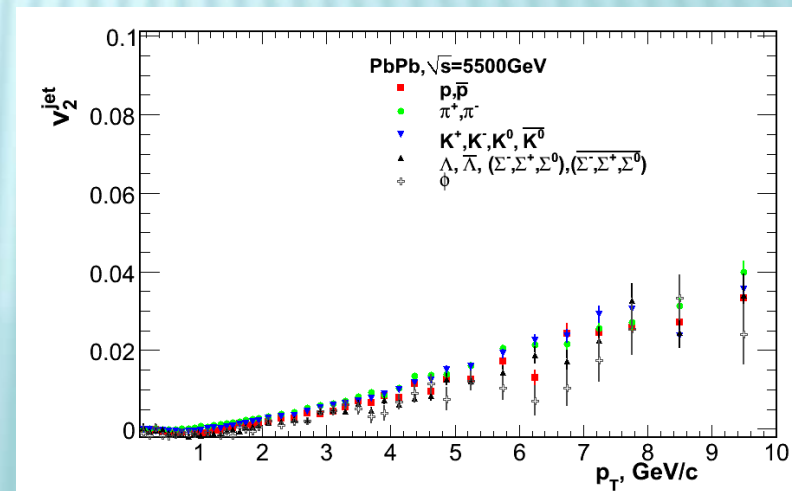
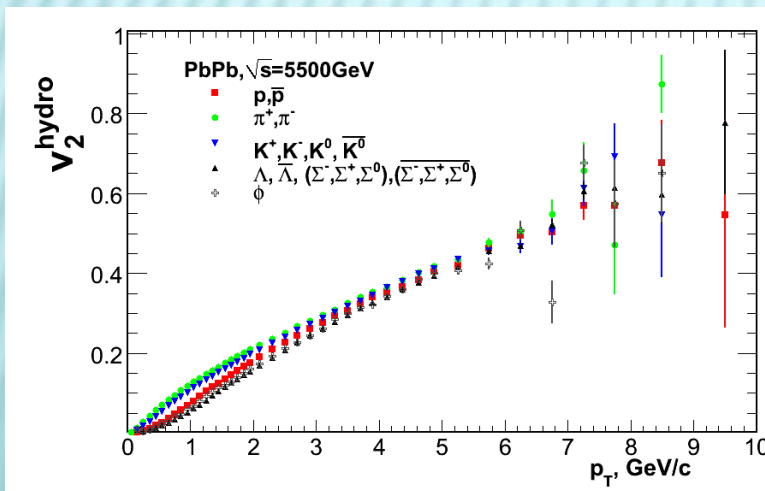
Mass ordering in soft  $p_T$  regions then breaks.

Why?

Hydrodynamics gives mass ordering of  $v_2$ .  
The model possesses crossing of baryon and meson branches.

*Hydrodynamics*

*Jet part + quenching*



Interplay of hydrodynamics and jets

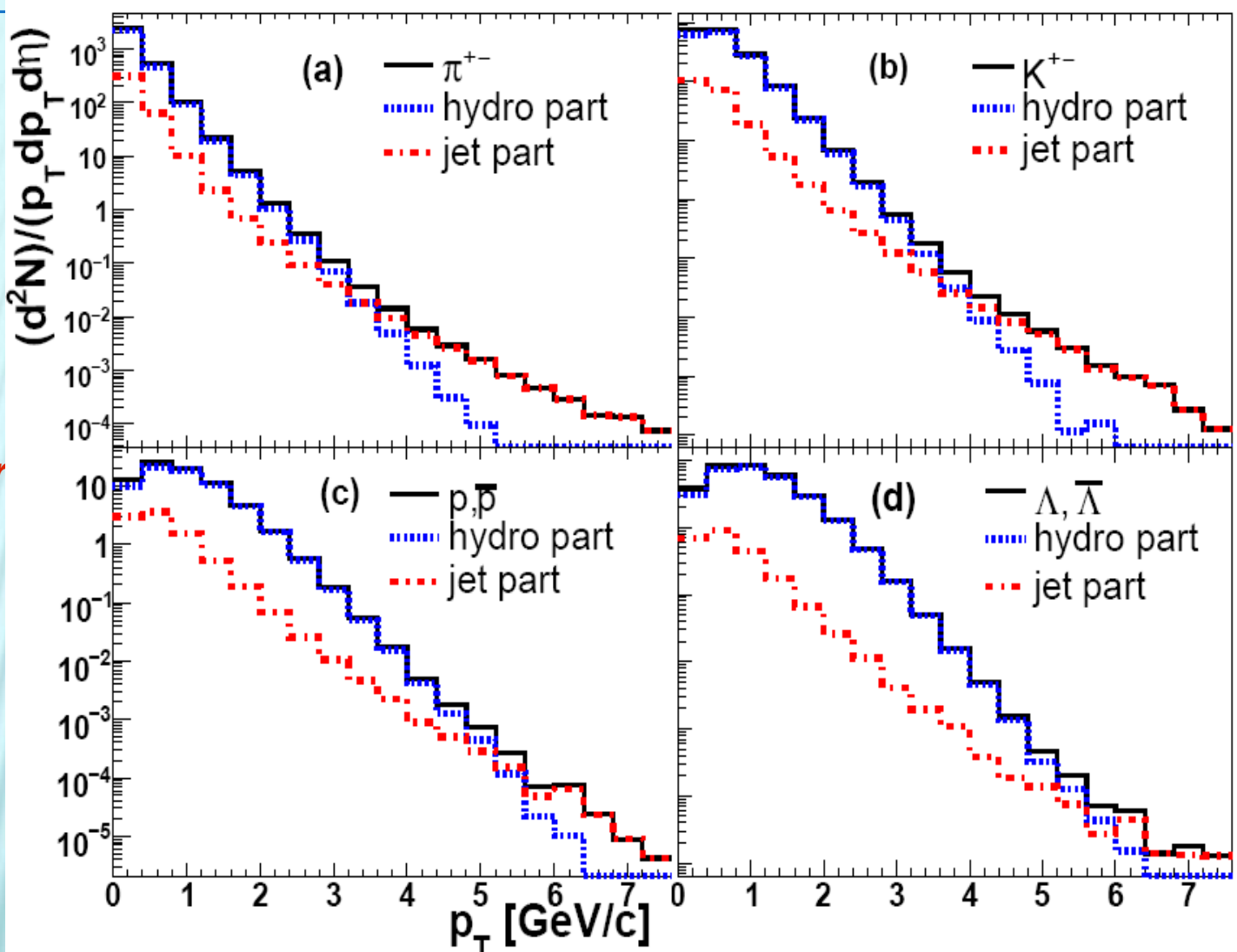
# The $p_T$ spectra of $\pi$ , $K$ , $p$ , $\Lambda$ with HYDJET++ model, $\sqrt{s}=200\text{GeV}$

The slope for the hydro part depends strongly on mass:

- the heavier the particle -- the harder the spectrum



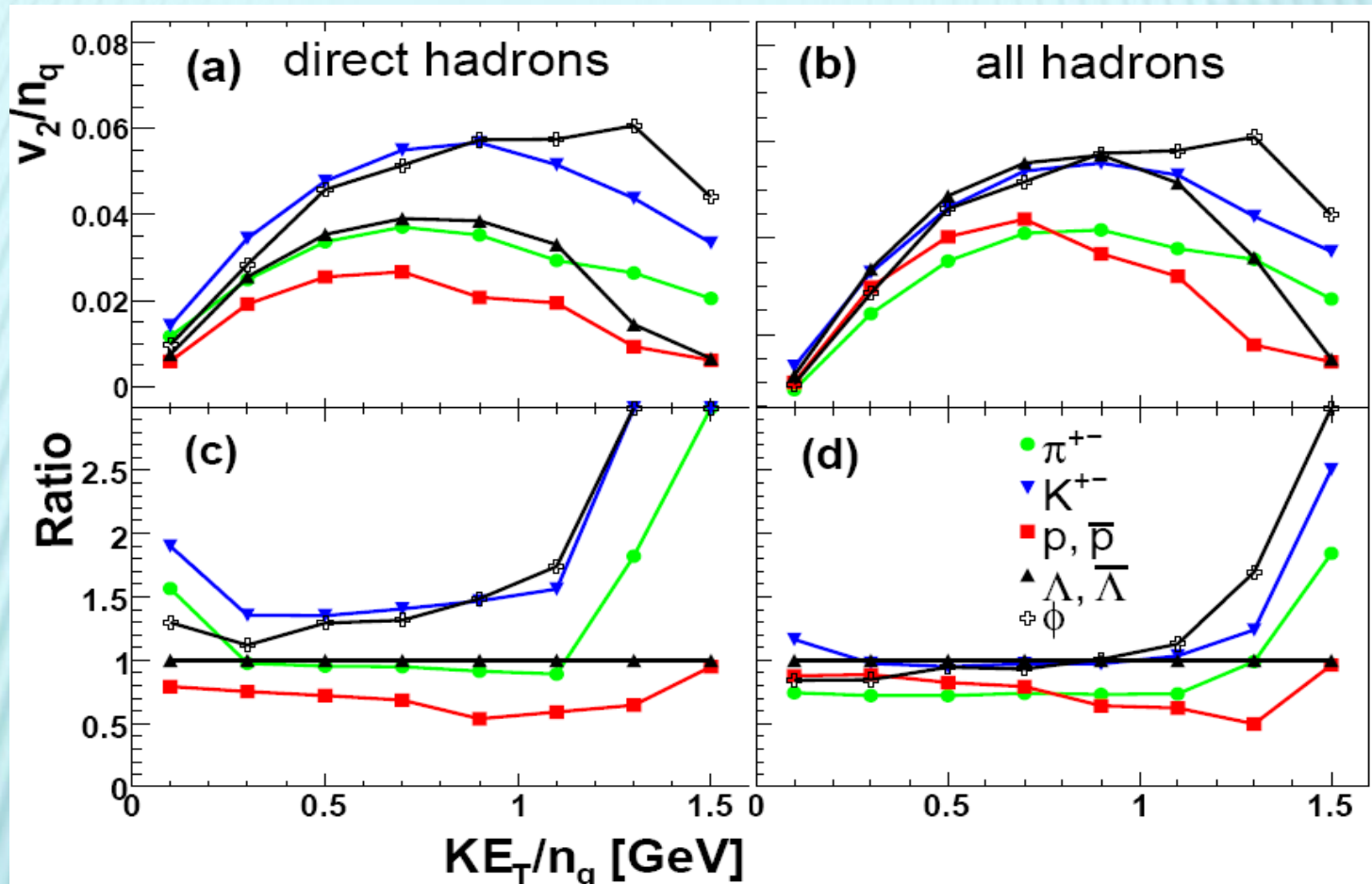
The hydro part dies out earlier for light particles than for heavy ones



# NCQ scaling at LHC

No scaling for  
direct particles

Appearance of the approximate  
scaling for all particles



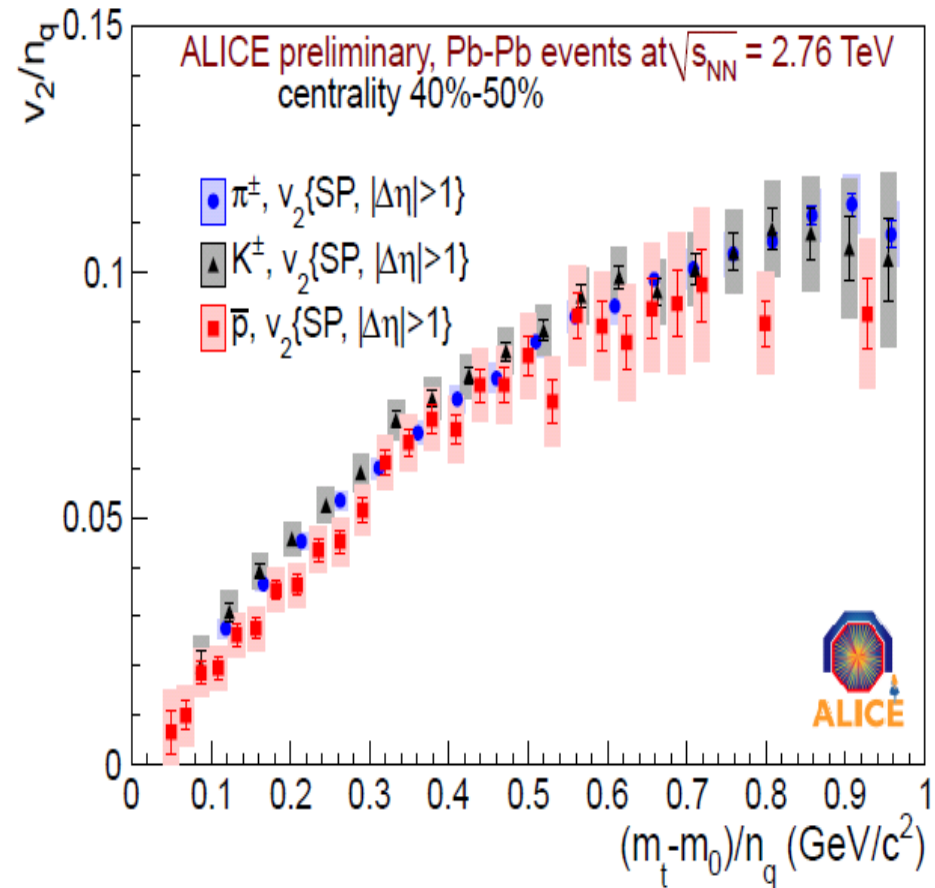
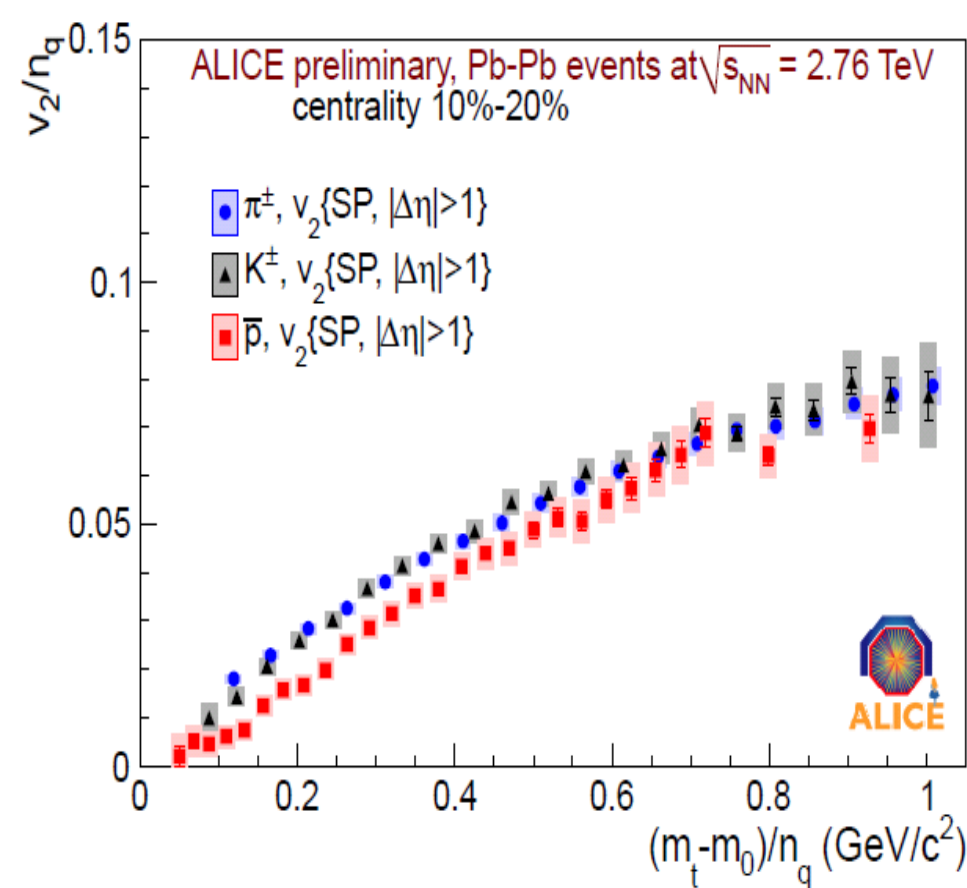
**LHC: NCQ scaling will be only approximate (prediction, 2009)**

# Experimental results (LHC)

ALICE Collaboration, M. Krzewicki et al., JPG 38 (2011) 124047

## Semi-central collisions

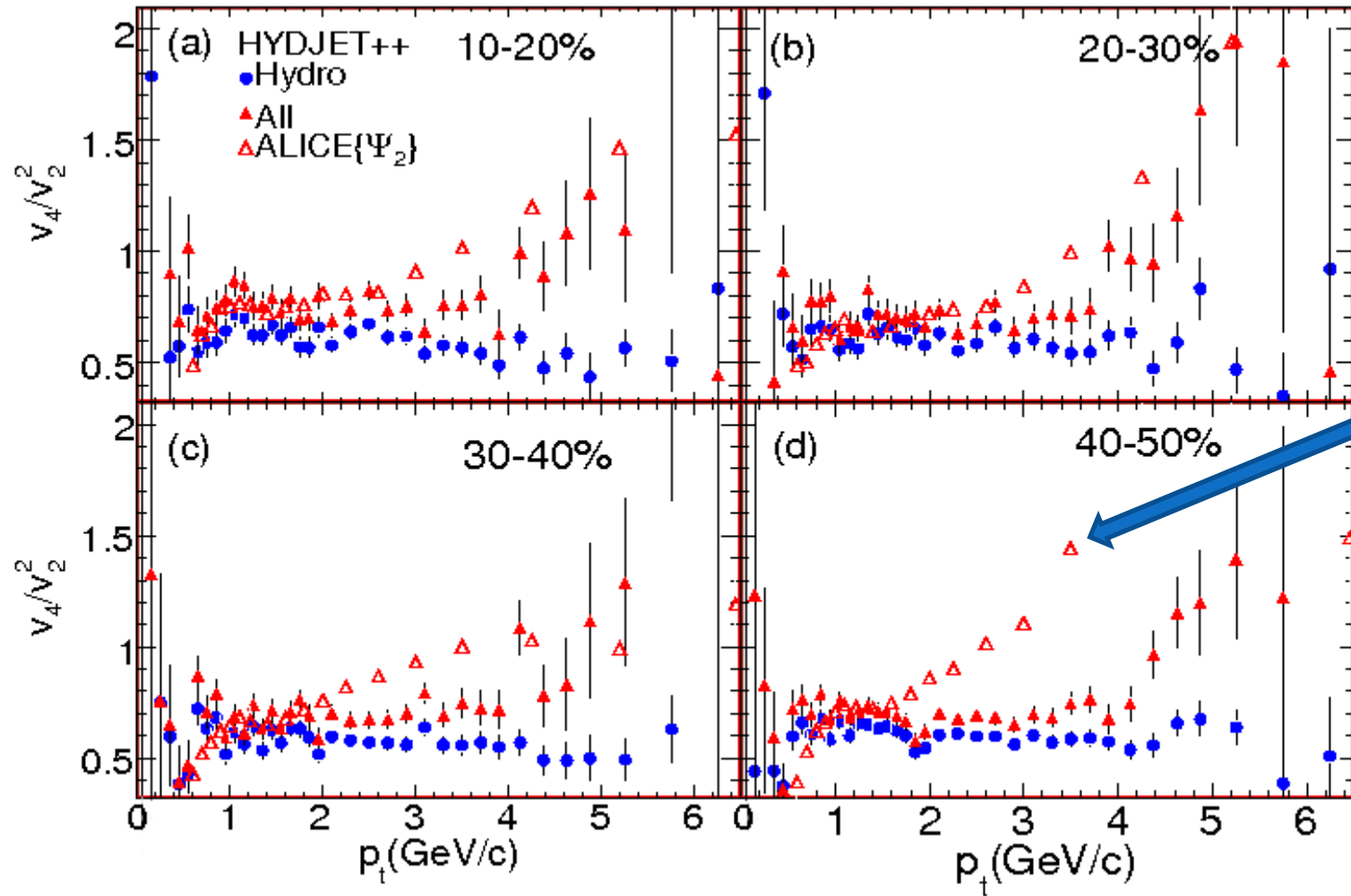
## Semi-peripheral collisions



The NCQ scaling is indeed only approximate (2011)

# HYDJET++ RESULTS FOR LHC

L.Bravina et al., PRC 87 (2013) 034901

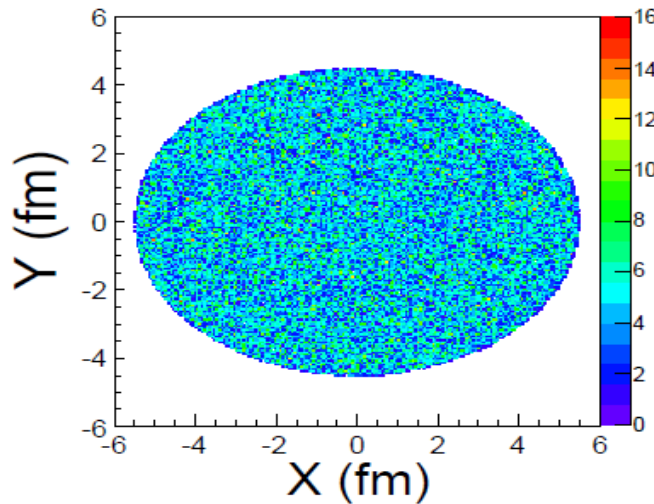


Jets !

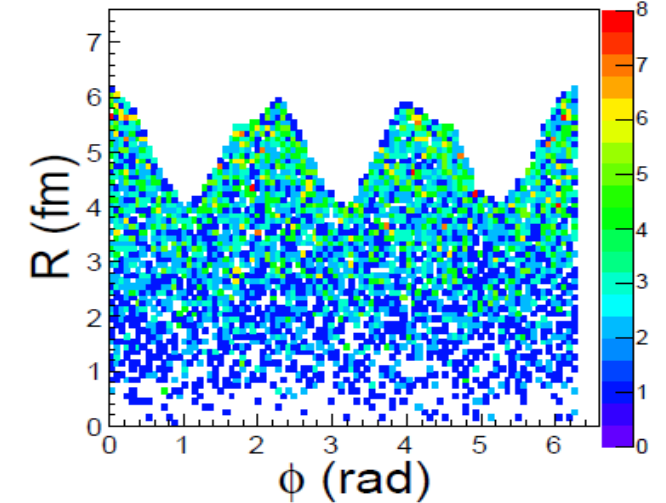
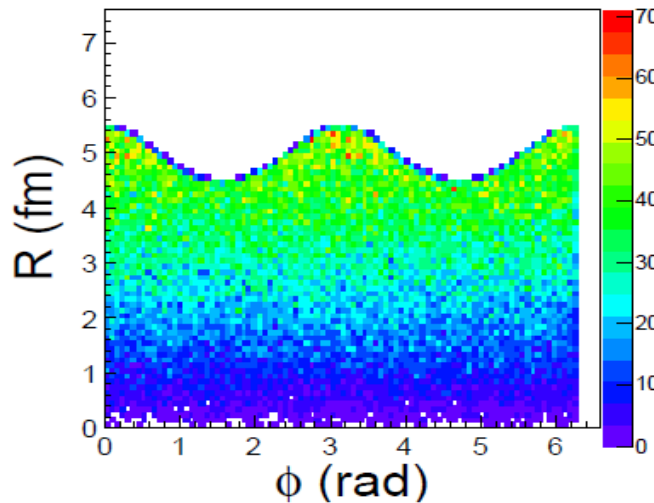
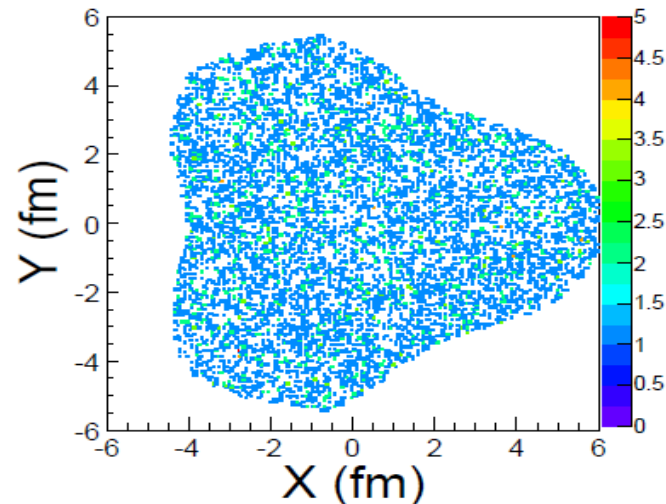
**Jets increase the ratio and lead to rise of high- $p_T$  tail**

# GENERATION OF TRIANGULAR FLOW

V2



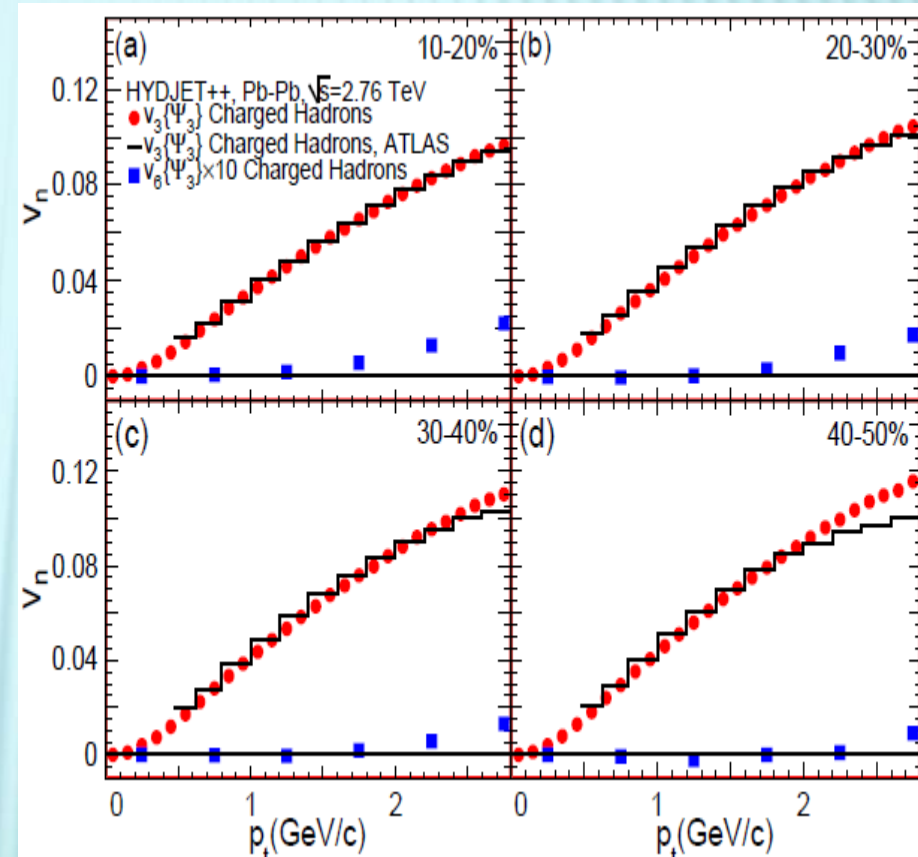
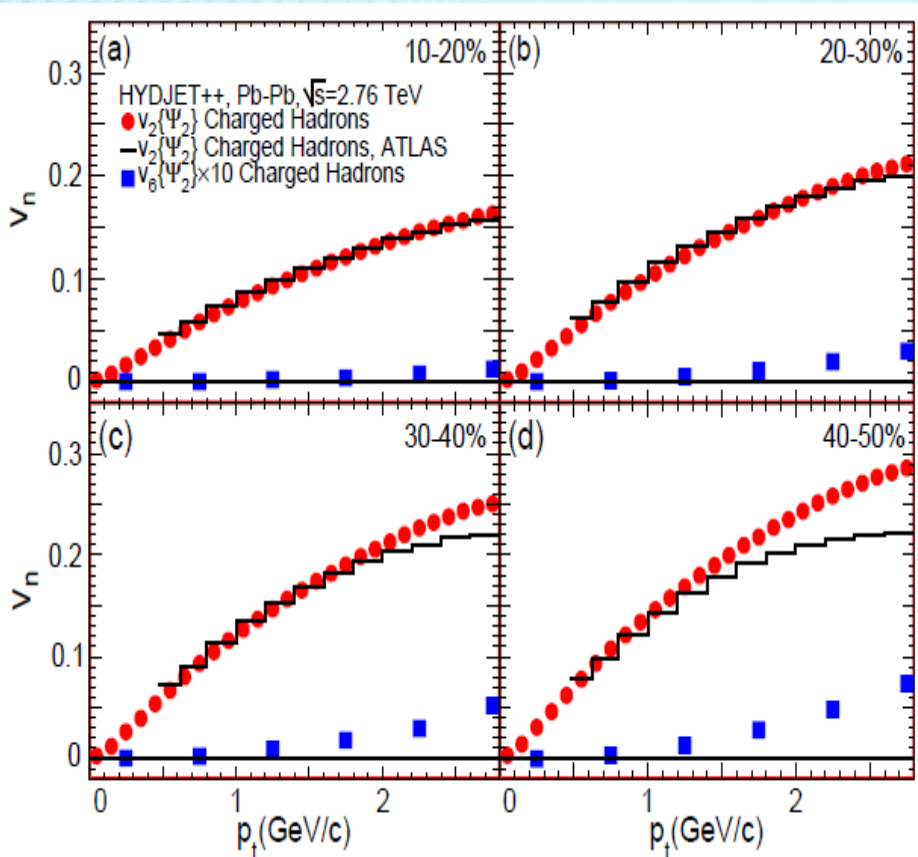
V3



Hint (Teaney, Ollitrault, ...):  $V_5 \propto V_2 V_3$      $V_6 \propto \alpha V_2^3 + \beta V_3^2$

# HEXAGONAL FLOW IN HYDJET++ AT LHC

L.Bravina et al., to be submitted

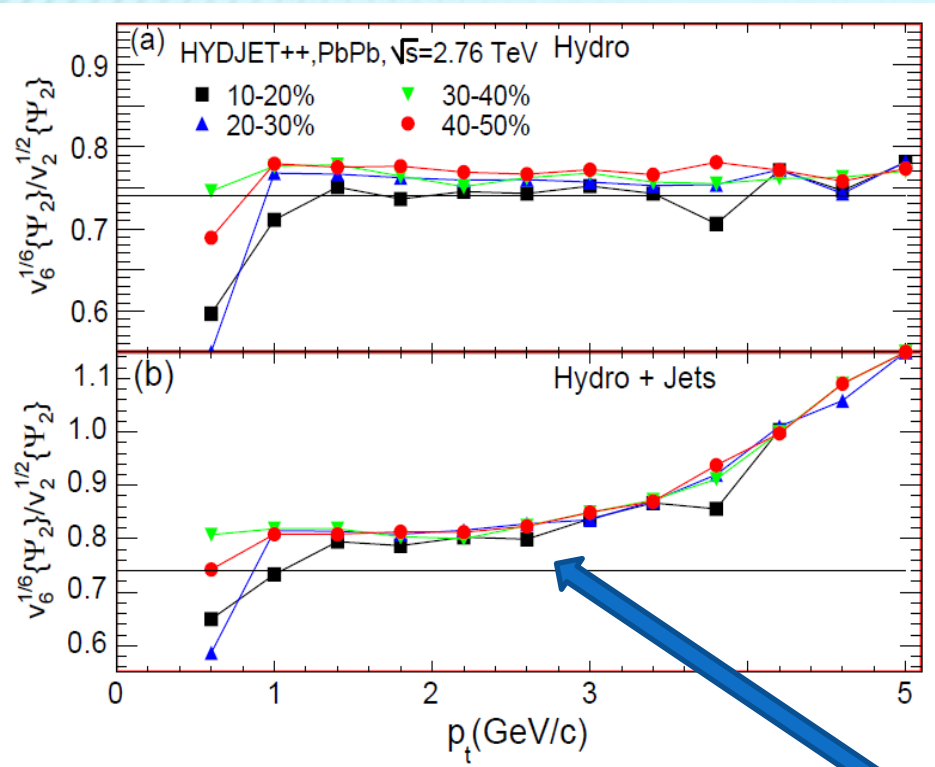


- (1)  $V_6$  is weak
- (2) Its high- $p_t$  tail increases with rising  $p_T$

# Hexagonal flow:

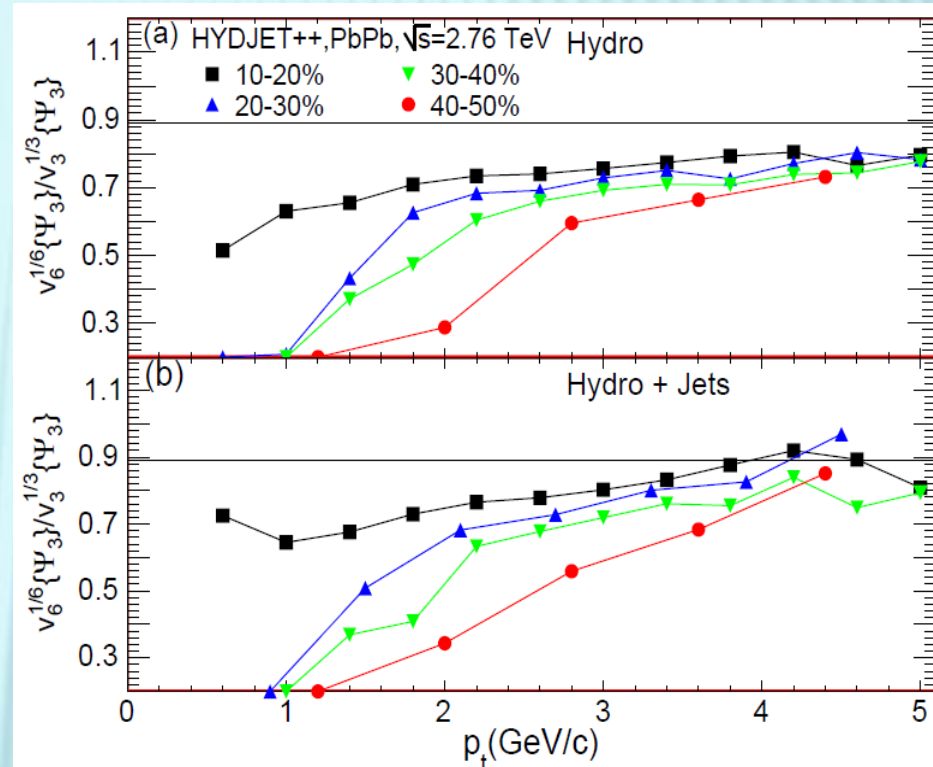
$$V_6 \propto \alpha V_2^3 + \beta V_3^2$$

L.Bravina et al., to be submitted



$\Psi_2$

Scaling?

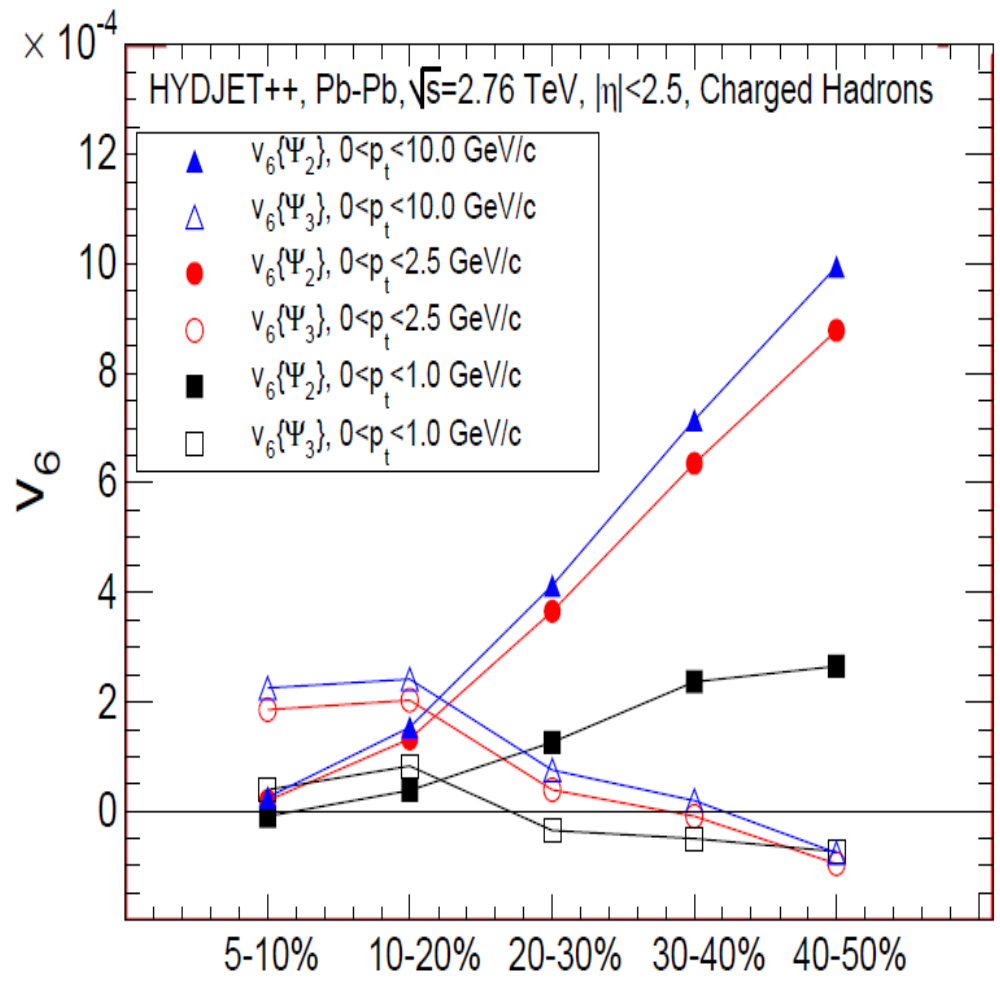
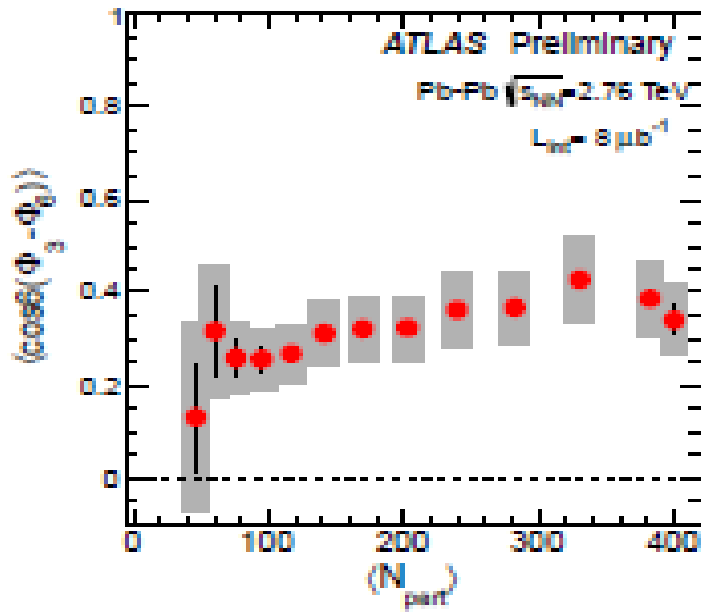
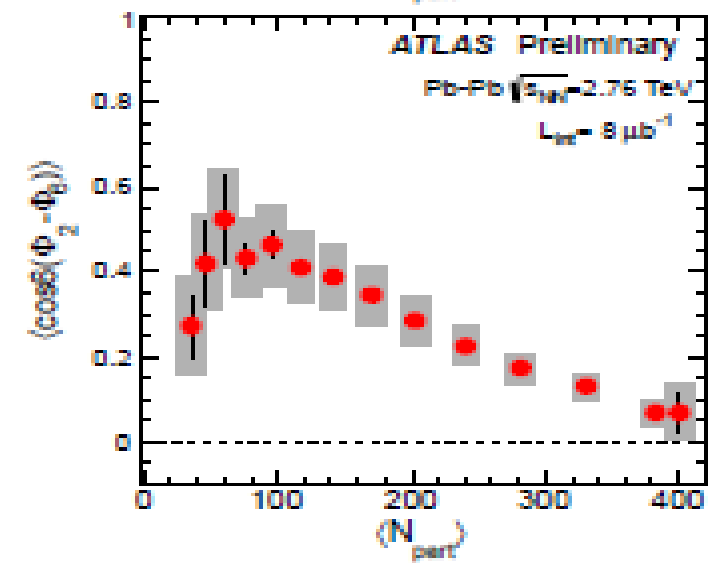


$\Psi_3$

It would be interesting to study  $V_6(\Psi_2)$  and  $V_6(\Psi_3)$  in experiment

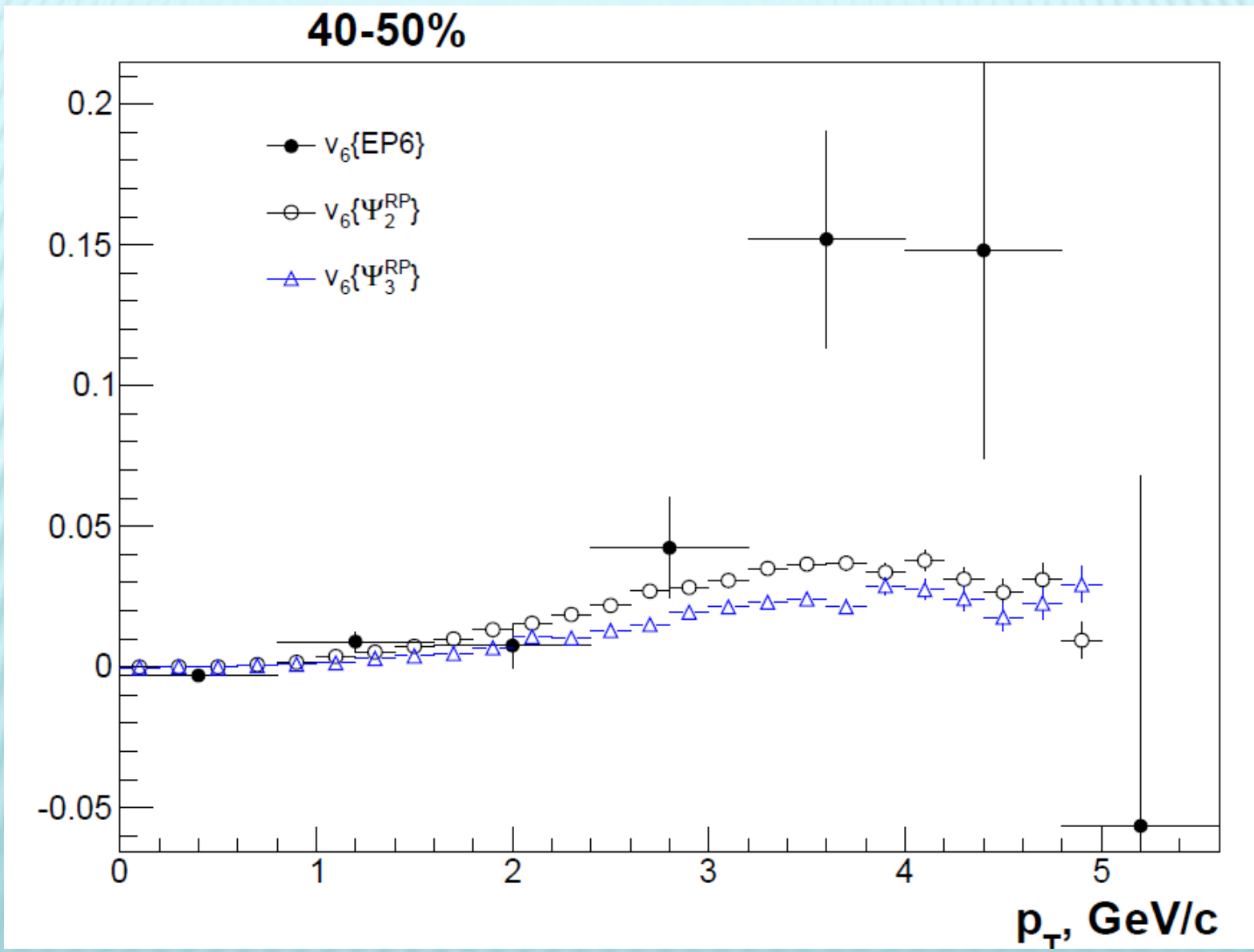
# Hexagonal flow: centrality dependence

ATLAS-CONF-2012-049



In line with exp. data

# Extraction of $V_6$ (Event Plane method)



# CONCLUSIONS

The HYDJET++ model allows to investigate flow of hydro and jet parts separately, to look at reconstruction of pure hydro flow and its modification due to jet part.

**Non-linearities between eccentricities and flow harmonics of n-th order ( $n > 3$ ) can be understood in terms of elliptic and triangular flow contributions**

**Scaling of  $v_6^{1/6} \{\Psi_2\} / v_2^{1/2} \{\Psi_2\}$  is observed for centralities up to 50%**

**Jets increase this ratio by 10-15% and lead to rise of the high-pT tail**

**Centrality dependences  $v_6(v_2)$  and  $v_6(v_3)$  are in line with the behavior of the plane correlators  $\langle \cos\{6(\Psi_2 - \Psi_6)\} \rangle$  and  $\langle \cos\{6(\Psi_3 - \Psi_6)\} \rangle$  measured experimentally**

**Difficult to get genuine information about higher eccentricities !**

**Many thanks to organizers!**



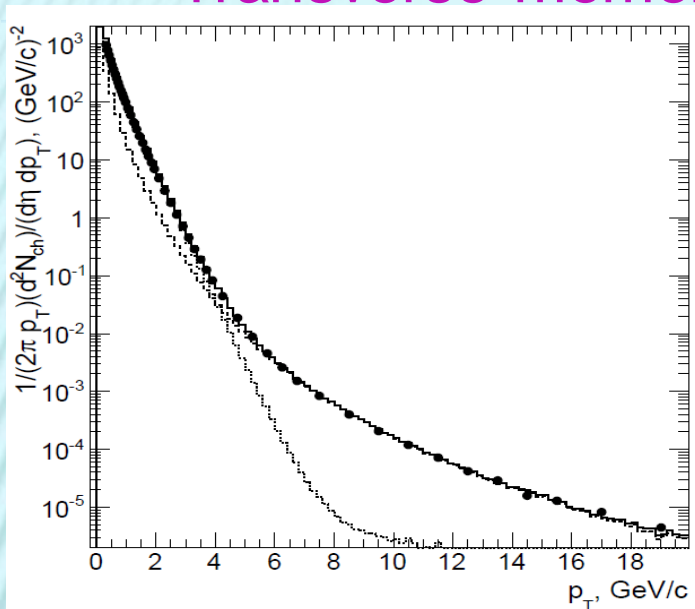
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# Back-up Slides

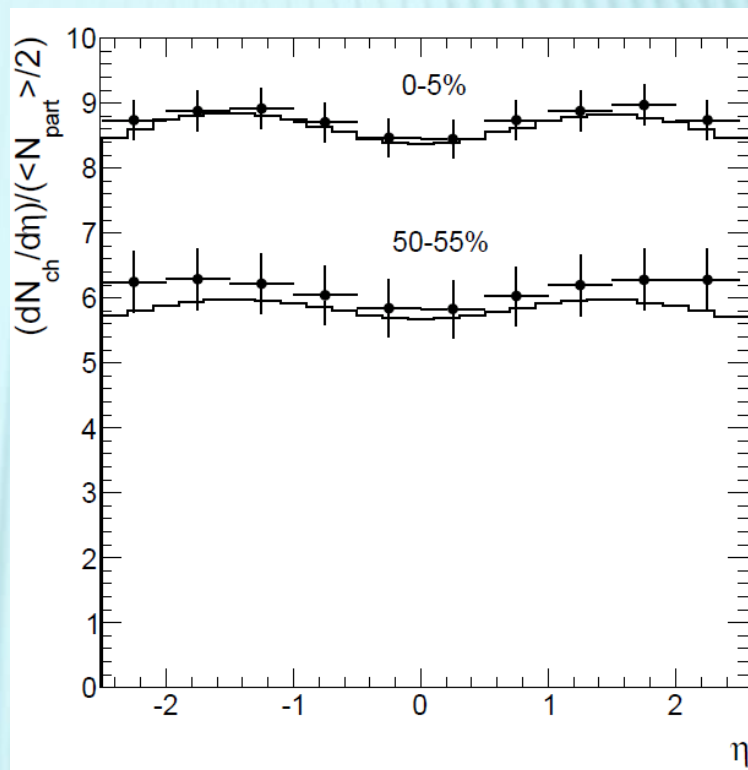
# LHC DATA VS. HYDJET++ MODEL

Transverse momentum

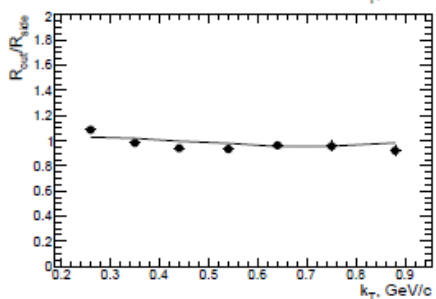
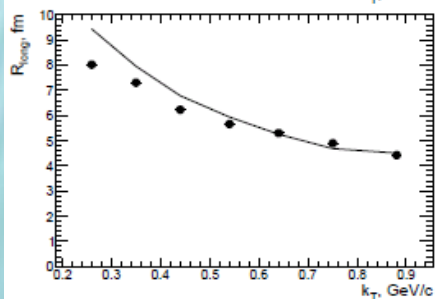
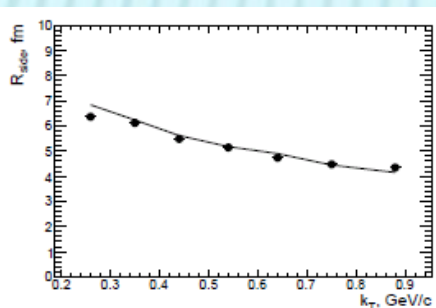
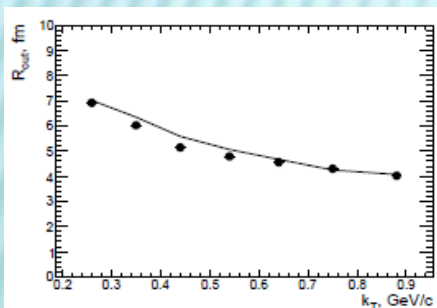
Pb+Pb @ 2.76 ATeV



Rapidity



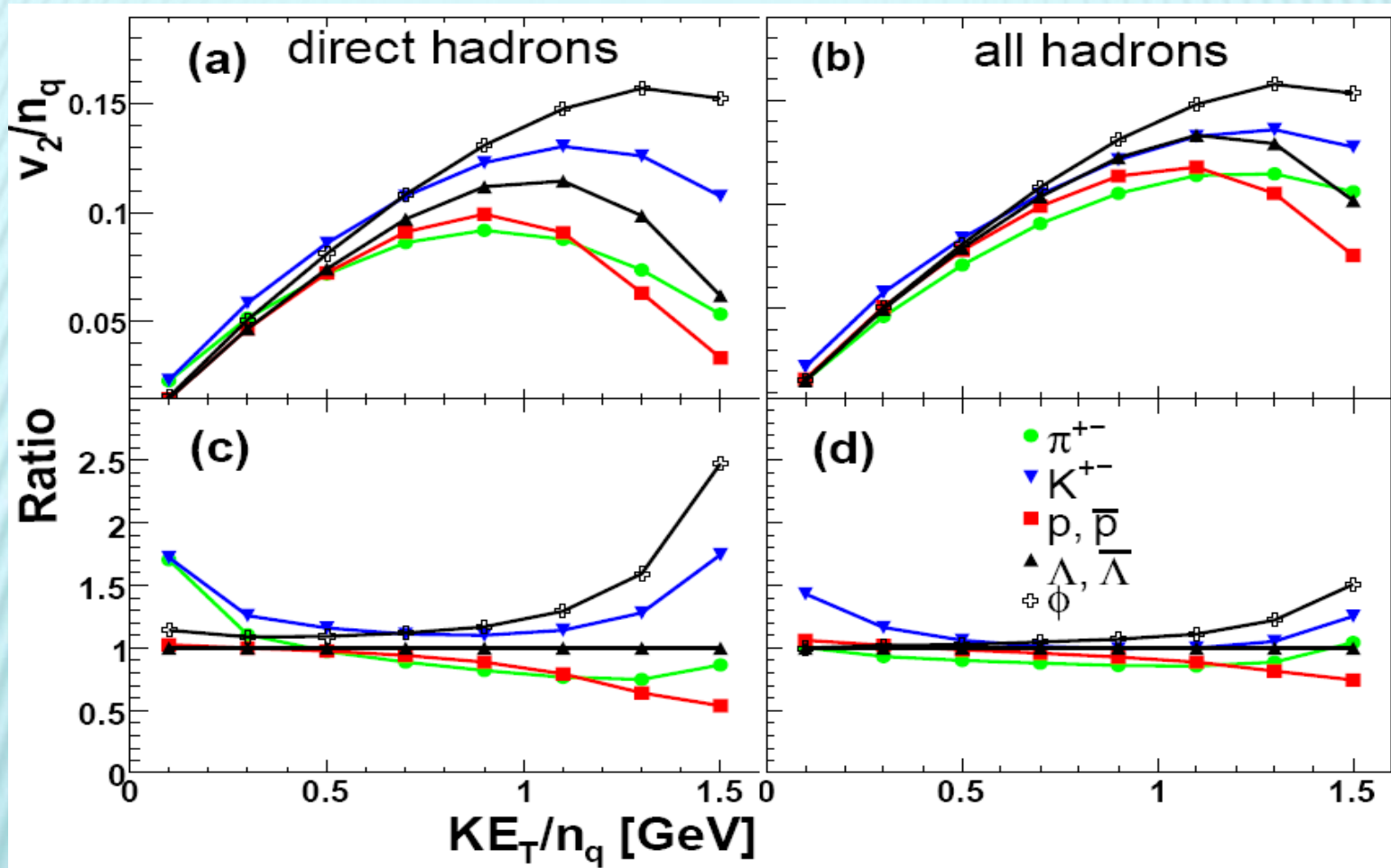
I. Lokhtin et al., Eur. Phys. J C72 (2012) 2045



Correlation radii (femtoscropy)

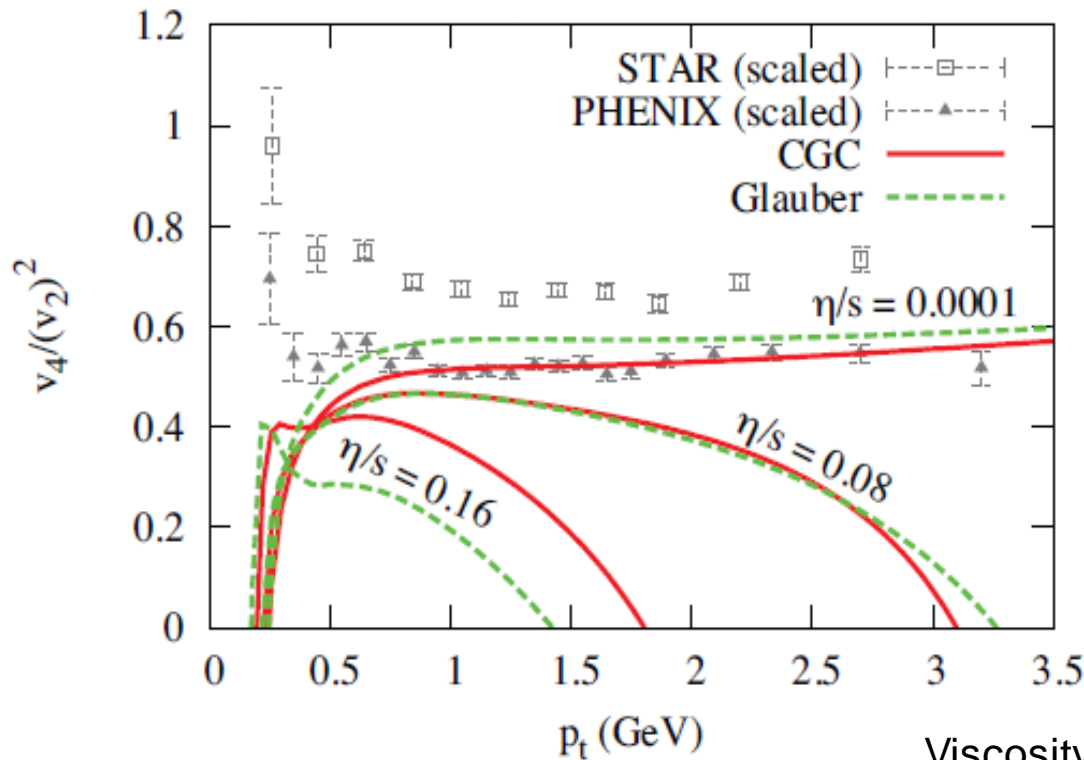
# Number-of-constituent-quark scaling at RHIC

Direct particles: scaling is not good. All particles:  $KE_T/n_q$  scaling



One of the explanations of  $KE_T/n_q$  scaling is partonic origin of the elliptic flow. *However, final state effects (such as resonance decays and jets) may also lead to appearance of the scaling*

# Effects of initial profile and viscosity



Initial profile has little effect although eccentricities differ.

results strongly depend on viscosity

Viscosity lowers  $v_4/(v_2)^2$  for a realistic  $T_f$

# Why $\varepsilon$ fluctuations change $v_4/v_2^2$

Experimentally, no direct measure of  $v_2$  and  $v_4$

$v_2$  and  $v_4$  are measured via azimuthal correlations

$$v_2 \text{ from } \langle \cos(2\phi_1 - 2\phi_2) \rangle = \langle (v_2)^2 \rangle$$

$$v_4 \text{ from } \langle \cos(4\phi_1 - 2\phi_2 - 2\phi_3) \rangle = \langle v_4 (v_2)^2 \rangle$$

Experimentally measured

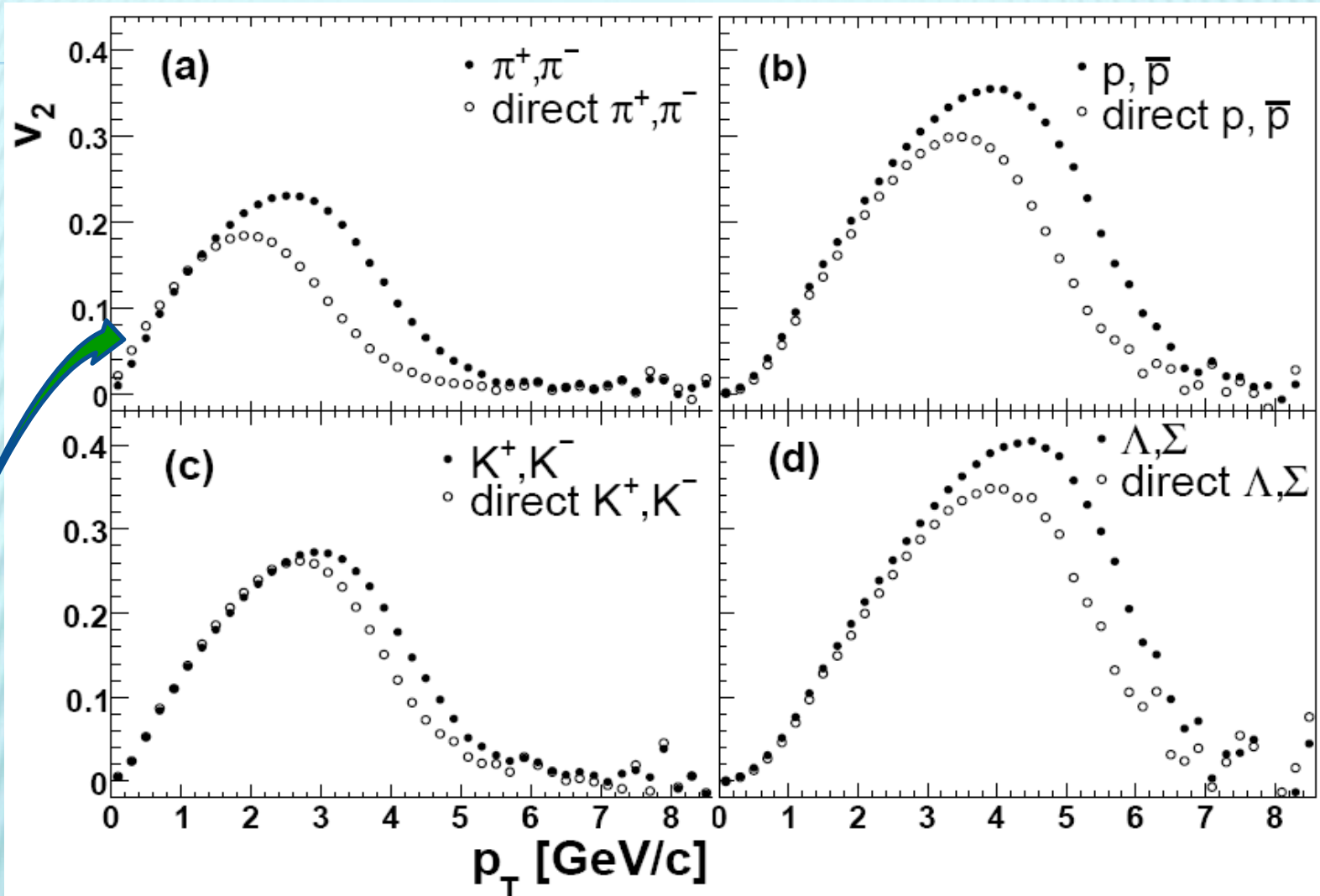
$$\frac{v_4}{v_2^2} = \frac{\langle v_4 (v_2)^2 \rangle}{\langle (v_2)^2 \rangle^2} = \frac{1}{2} \frac{\langle (v_2)^4 \rangle}{\langle (v_2)^2 \rangle^2} > \frac{1}{2}$$

fluctuations

hydro

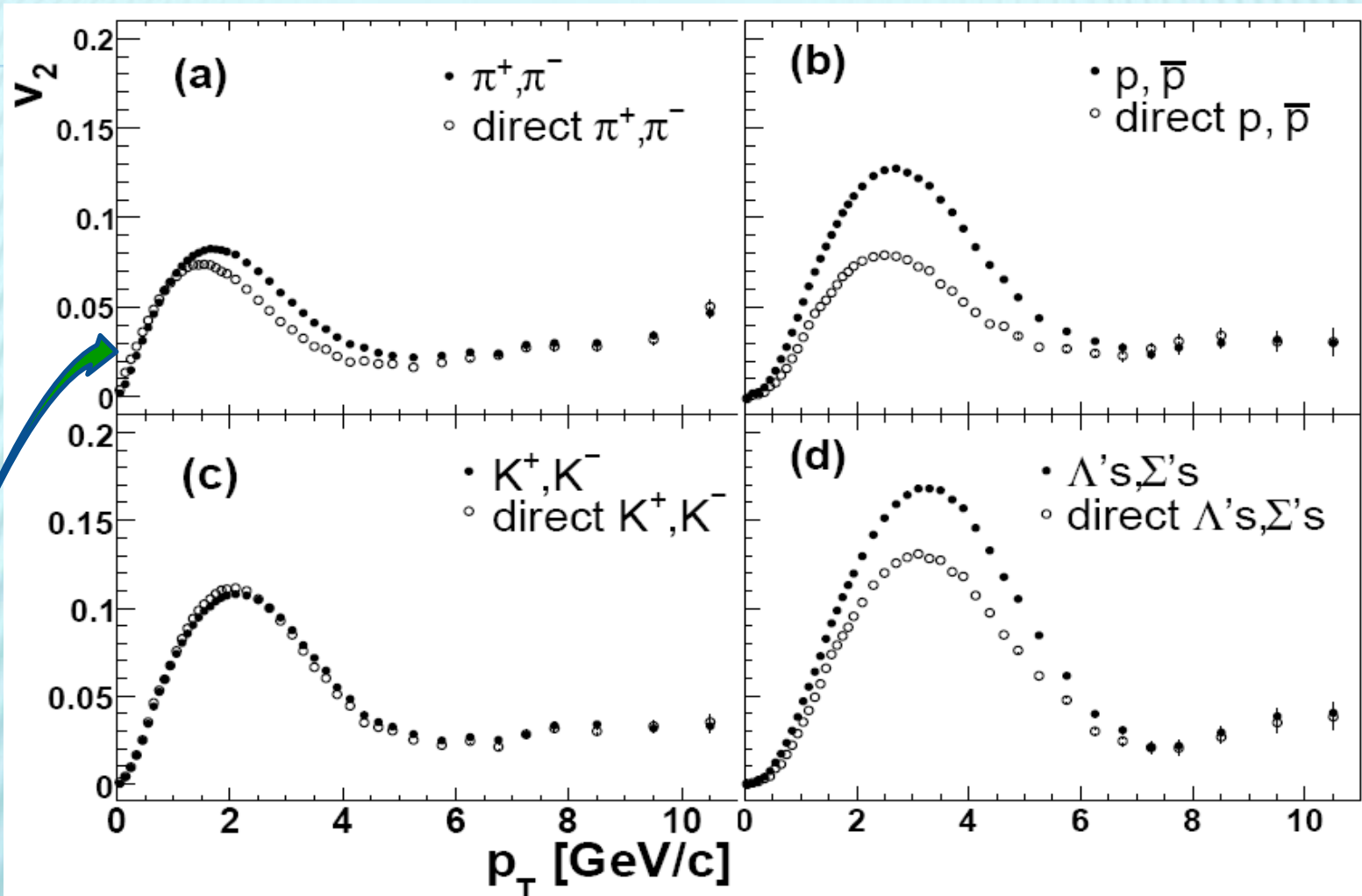
Similar results obtained using Event Plane method

# Influence of resonance decays for different type of particles at RHIC



**Pions and kaons:** the resulting flow is weaker at low-pt and larger at high-pt  
**Baryons:** the resulting flow is stronger than the flow of direct particles

# Influence of resonance decays for different type of particles at LHC



**Pions:** the resulting flow is **weaker** at low-pt and **larger** at high-pt

**Kaons:** both flows almost coincide

**Baryons:** the resulting flow is **stronger** than the flow of direct particles

# Methods for $v_2$ calculation

## (1) Event plane method

$$v_2^{obs}\{EP\} = \langle \cos 2(\varphi_i - \Psi_2) \rangle$$

$\Psi_2$  is the calculated reaction plane angle:  $\tan n \psi_n = \frac{\sum_i \omega_i \sin n \varphi_i}{\sum_i \omega_i \cos n \varphi_i}$ ,  $n \geq 1$ ,  $0 \leq \psi_n < 2\pi/n$

$$v_2\{EP\} = \frac{v_2^{obs}\{EP\}}{R} = \frac{v_2^{obs}\{EP\}}{\langle \cos 2(\Psi_2 - \Psi_R) \rangle}$$

## (2) Two particle correlation method

$$v_2\{2\} = \sqrt{\langle \cos 2(\varphi_i - \varphi_j) \rangle}$$

## (3) Lee-Yang zero method

$$G(ir) = \langle e^{irQ} \rangle, Q = \sum \cos(2\varphi)$$

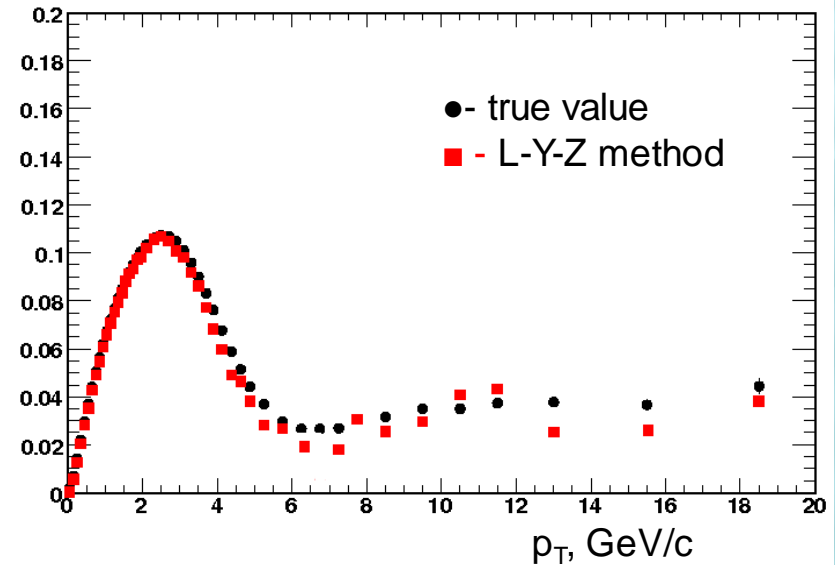
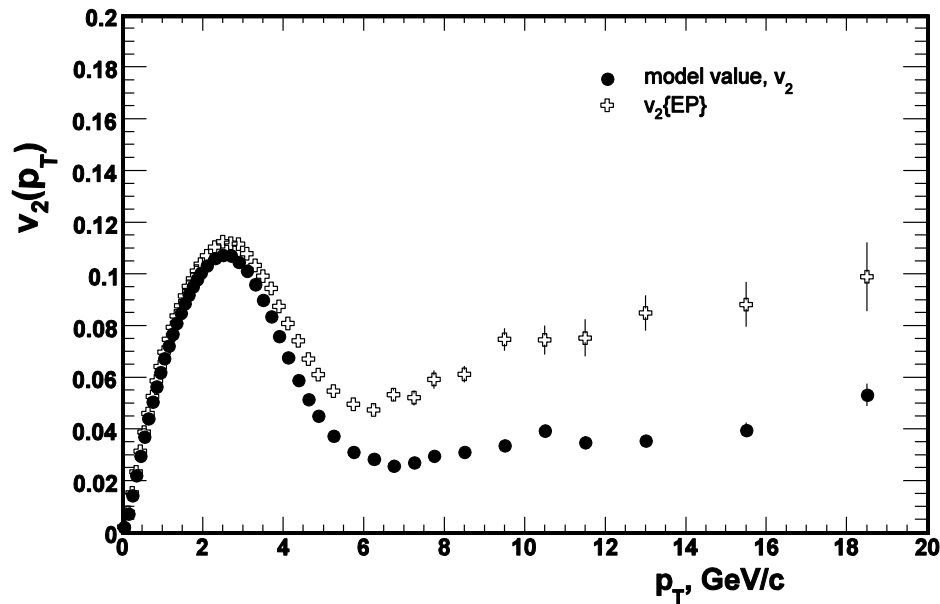
Integral  $v_2$  is connected with the first minimum  $r_0$  of the module of the  $G(ir)$ :

$$v_2 = \frac{j_0}{Nr_0}$$

Differential flow is calculated by the formula: 
$$\frac{v_2(p_T)}{Nv_2} = \text{Re} \left( \frac{\langle \cos(2\varphi) e^{ir_0 Q} \rangle}{\langle Q e^{ir_0 Q} \rangle} \right)$$

# Comparison of Event Plane and Lee-Yang zeroes methods ( $c=30\%$ )

## EventPlane method



## Lee-Yang zeroes Method

Event Plane method overestimates  $v_2$  at high  $p_T$  due to non-flow correlation (mostly because of jets).