

Relativistic Quark-Model Description of Low-Energy Baryons

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Low-energy

QCD

RCQM

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

Gravitational FF

Vertex FFs

πNN , $\pi N\Delta$

Summary

To be able to describe/understand

- in a consistent manner
- on the microscopic level
- in accordance with the properties of low-energy QCD

such phenomena like

- ▶ **hadron spectra:** $\pi, \pi^*, \rho, \rho^*, \dots; N, N^*, \Delta, \Delta^*, \Lambda, \Lambda^*, \dots$
- ▶ **hadron structure:** $r_E, \mu, g_A; G_E, G_M, G_A, G_P, \dots$
i.e. electroweak form factors etc.
- ▶ **resonance excitations:** $\gamma N \rightarrow N^*, e^- N \rightarrow N^*, \dots$
- ▶ **resonance decays:**
 $\rho \rightarrow \pi\pi, \omega \rightarrow \pi\pi\pi, N^* \rightarrow N\pi, \Delta \rightarrow N\pi, \Lambda^* \rightarrow KN, \dots$
- ▶ **meson-baryon interactions:** $\pi - N, K - N, \dots$
- ▶ **hyperon-hyperon interactions:** $N - N, N - Y, \dots$
etc. etc.

Low-Energy QCD / Relevant Degrees of Freedom

Relativistic Constituent-Quark Model (RCQM)

Baryon Spectroscopy

Light, strange, charm, bottom

Baryon Structure

Nucleon e.m. form factors - Flavor analysis

Baryon electromagnetic form factors

Nucleon and baryon axial form factors / charges

Nucleon gravitational form factors

Meson-Baryon Interaction Vertices

πNN and $\pi N\Delta$ vertex form factors

Summary and Conclusions

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- ▶ **Relativistic** (i.e. Poincaré-invariant) **framework**
 - ▶ **Hamiltonian** theory with a **finite number** of d.o.f.
 - ▶ based on relativistically **invariant mass operator**
 - ▶ solvable with advanced **few-body methods** (differential and / or integral equations)
- ▶ Observe **low-energy QCD dynamics**
 - ▶ **confinement**
 - ▶ **spontaneous chiral-symmetry breaking** ($SB_{\chi S}$)
- ▶ The **Relativistic Constituent Quark Model** (RCQM)
 - ▶ is a **model**
 - ▶ is not a (fundamental) **dynamical theory**
 - ▶ has a **limited range** of validity (\sim non-perturbative regime of QCD)

Low-energy QCD of N_f flavors is characterized by:

- spontaneous breaking of chiral symmetry ($SB_{\chi}S$):

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

→ appearance of $(N_f^2 - 1)$ **Goldstone bosons** $\vec{\phi}$

→ generation of quasiparticles with dynamical mass,
i.e. **constituent quarks** ψ

- thus (effective) interaction Lagrangian:

$$\mathcal{L}_{\text{int}} \sim ig\bar{\psi}\gamma_5\vec{\lambda}^f \cdot \vec{\phi}\psi$$

A. Manohar and H. Georgi: Nucl. Phys. B 234 (1984) 189

E.V. Shuryak: Phys. Rep. **115**, 151 (1984)

L.Ya. Glozman and D.O. Riska: Phys. Rep. **268**, 263 (1996)

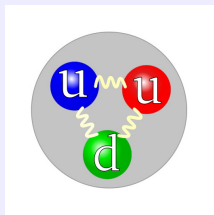
see also:

S. Weinberg: Phys. Rev. Lett. **105**, 261601 (2010)

E.g., Baryons

Baryons are considered as colorless bound states of **three constituent quarks**.

Here the proton:



- ▶ 'Constituent' quarks are quasiparticles NOT the original QCD degrees of freedom (i.e. the 'current' quarks).
- ▶ 'Constituent' quarks are confined and interact via hyperfine interactions associated with $SB_\chi S$, i.e. Goldstone-boson exchange.

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Relativistic quantum mechanics (RQM)

i.e. **quantum theory** respecting **Poincaré invariance**

(theory on a Hilbert space \mathcal{H} corresponding to a finite number of particles, not a field theory)

Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

Eigenvalue equations

$$\hat{M} |P, J, \Sigma\rangle = M |P, J, \Sigma\rangle \quad , \quad \hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$$

$$\hat{P}^\mu |P, J, \Sigma\rangle = P^\mu |P, J, \Sigma\rangle \quad , \quad \hat{P}^\mu = \hat{M} \hat{V}^\mu$$

Interacting mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

$$\hat{M}_{free} = \sqrt{\hat{H}_{free}^2 - \hat{\vec{P}}_{free}^2}$$

$$\hat{M}_{int}^{rest\ frame} = \sum_{i < j}^3 \hat{V}_{ij} = \sum_{i < j}^3 [\hat{V}_{ij}^{conf} + \hat{V}_{ij}^{hf}]$$

fulfilling the **Poincaré algebra**

$$\begin{aligned} [\hat{P}_i, \hat{P}_j] &= 0, & [\hat{J}_i, \hat{H}] &= 0, & [\hat{P}_i, \hat{H}] &= 0, \\ [\hat{K}_i, \hat{H}] &= -i\hat{P}_i & [\hat{J}_i, \hat{J}_j] &= i\epsilon_{ijk}\hat{J}_k & [\hat{J}_i, \hat{K}_j] &= i\epsilon_{ijk}\hat{K}_k, \\ [\hat{J}_i, \hat{P}_j] &= i\epsilon_{ijk}\hat{P}_k, & [\hat{K}_i, \hat{K}_j] &= -i\epsilon_{ijk}\hat{J}_k, & [\hat{K}_i, \hat{P}_j] &= -i\delta_{ij}\hat{H} \end{aligned}$$

\hat{H}, \hat{P}_i ... time and space translations,

\hat{J}_i ... rotations, \hat{K}_i ... Lorentz boosts

Phenomenologically, baryons with 5 flavors: u, d, s, c, b

$$\Rightarrow H_{free} = \sum_{i=1}^3 \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V^{conf}(\vec{r}_{ij}) = B + C r_{ij}$$

$$V^{hf}(\vec{r}_{ij}) = \left[V_{24}(\vec{r}_{ij}) \sum_{f=1}^{24} \lambda_i^f \lambda_j^f + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- ▶ i.e., for $N_f = 5$, we have the exchange of a **24-plet** plus a **singlet** of Goldstone bosons.

L.Ya. Glozman, W. Plessas, K. Varga, and R.F. Wagenbrunn: Phys. Rev. D **58**, 094030 (1998)

J.P. Day, K.-S. Choi, and W. Plessas: arXiv:1205.6918

Universal GBE RCQM Parametrization

$$V^{conf}(\vec{r}_{ij}) = B + C r_{ij}$$

$$V_{\beta}(\vec{r}_{ij}) = \frac{g_{\beta}^2}{4\pi} \frac{1}{12m_i m_j} \left\{ \mu_{\beta}^2 \frac{e^{-\mu_{\beta} r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right\}$$

$$= \frac{g_{\beta}^2}{4\pi} \frac{1}{12m_i m_j} \left\{ \mu_{\beta}^2 \frac{e^{-\mu_{\beta} r_{ij}}}{r_{ij}} - \Lambda_{\beta}^2 \frac{e^{-\Lambda_{\beta} r_{ij}}}{r_{ij}} \right\}$$

$$B = -402 \text{ MeV}, \quad C = 2.33 \text{ fm}^{-2}$$

$$\beta = 24 : \quad \frac{g_{24}^2}{4\pi} = 0.7, \quad \mu_{24} = \mu_{\pi} = 139 \text{ MeV}, \quad \Lambda_{24} = 700.5 \text{ MeV}$$

$$\beta = 0 : \quad \left(\frac{g_0}{g_{24}} \right)^2 = 1.5, \quad \mu_0 = \mu_{\eta'} = 958 \text{ MeV}, \quad \Lambda_0 = 1484 \text{ MeV}$$

$$m_u = m_d = 340 \text{ MeV}, \quad m_s = 480 \text{ MeV},$$

$$m_c = 1675 \text{ MeV}, \quad m_b = 5055 \text{ MeV}$$

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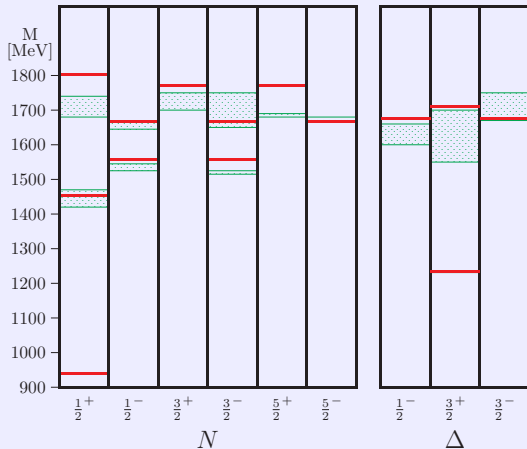
Summary

Baryon **Excitation Spectra**

and

Mass-Operator **Eigenstates**

Light Baryon Spectra



red Universal GBE RCQM

green PDG 2012 (experiment)

Low-energy QCD

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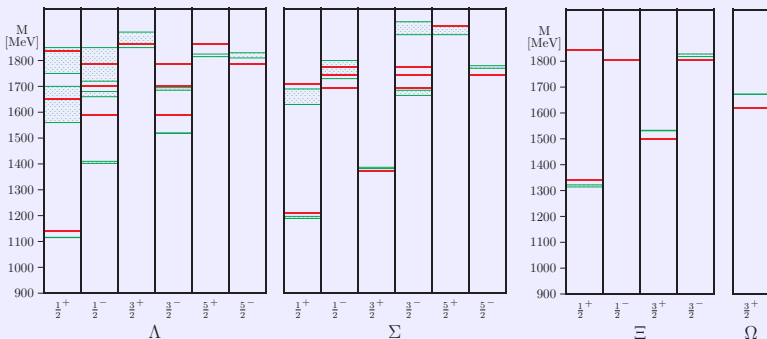
Vertex FFs

$\pi NN, \pi N\Delta$

Summary

Strange Baryon Spectra

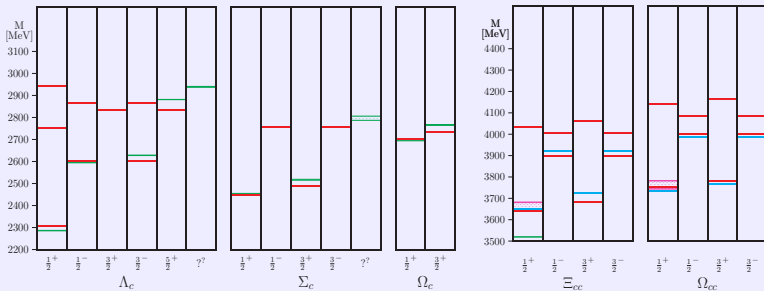
- Low-energy QCD
- RCQM
- Spectroscopy
 - Light, strange, charm, bottom
- Structure
 - Nucleon E.m.
 - Baryon E.m.
 - Axial FFs
 - Gravitational FF
- Vertex FFs
 - πNN , $\pi N\Delta$
- Summary



red Universal GBE RCQM

green PDG 2012 (experiment)

Charm Baryon Spectra



red Universal GBE RCQM

green Single charm (left panel): PDG 2010 (experiment)

green Double charm (right panel):

M. Mattson et al.: Phys. Rev. Lett. 89 (2002) 112001 (SELEX experiment)

cyan S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

magenta L. Liu et al.: Phys. Rev. D 81 (2010) 094505 (lattice QCD)

Low-energy QCD

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Axial FFs

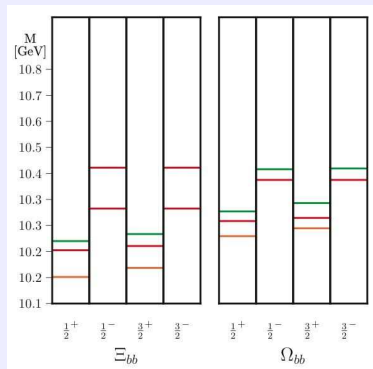
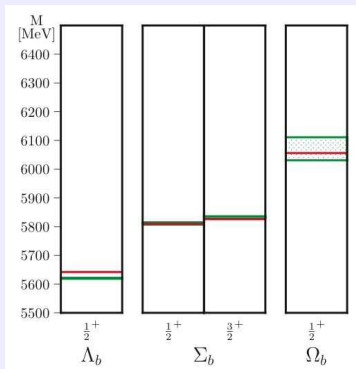
Gravitational FF

Vertex FFs

πNN , $\pi N\Delta$

Summary

Bottom Baryon Spectra



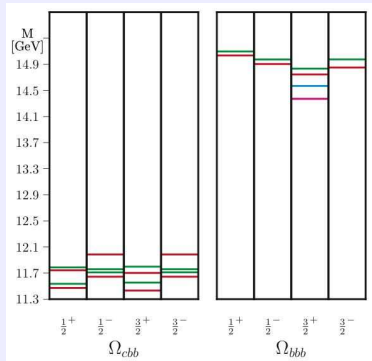
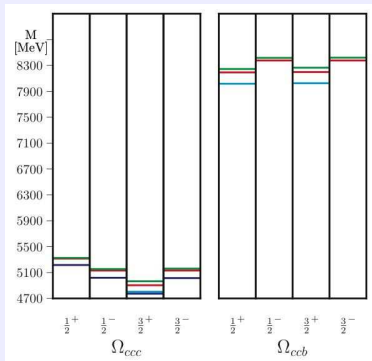
red Universal GBE RCQM

green left panel: PDG 2010 (experiment)

green right panel: W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817
(nonrelativistic one-gluon-exchange CQM)

orange D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko: Phys. Rev. D 66 (2002) 014008 (RCQM)

Triple-Heavy Baryon Spectra



red Universal GBE RCQM

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817
(nonrelativistic one-gluon-exchange CQM)

blue S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

cyan A.P. Martyntenko: Phys. Lett. B 663 (2008) 317 (RCQM)

magenta S. Meinel: Phys. Rev. D 82 (2010) 114502 (lattice QCD)

Influence of Light-Heavy Q-Q Interaction



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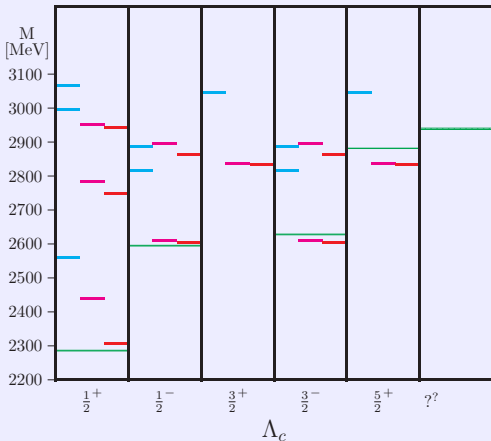
Axial FFs

Gravitational FF

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Summary



leftmost cyan levels

confinement only

middle magenta levels

including only light-light GBE

rightmost red levels

including full GBE RCQM

Mass operator eigenstates

$$\hat{M} |P, J, \Sigma, T, M_T\rangle = M |P, J, \Sigma, T, M_T\rangle$$

represented in configuration space

$$\langle \vec{\xi}, \vec{\eta} | P, J, \Sigma, T, M_T \rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi}, \vec{\eta})$$

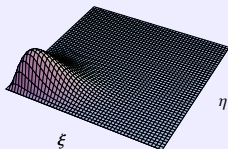
with $\vec{\xi}$ and $\vec{\eta}$ the usual Jacobi coordinates.

Picture the baryon wave functions through
spatial probability density distributions

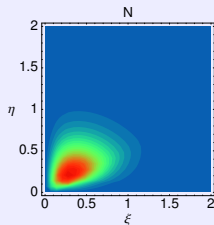
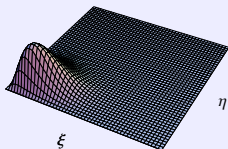
$$\rho(\xi, \eta) = \xi^2 \eta^2 \int d\Omega_\xi d\Omega_\eta \Psi_{PJ\Sigma TM_T}^*(\xi, \Omega_\xi, \eta, \Omega_\eta) \Psi_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)$$

Pictures of Baryons (rest frame)

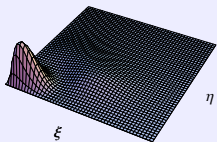
N GBE CQM



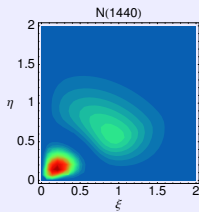
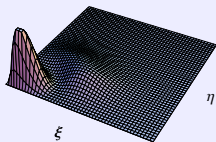
N OGE CQM



N(1440) GBE CQM

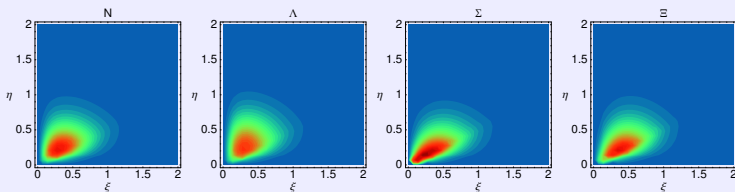


N(1440) OGE CQM

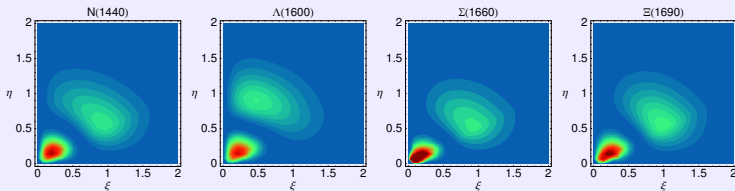


Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ octet baryon ground states $N(939)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$:



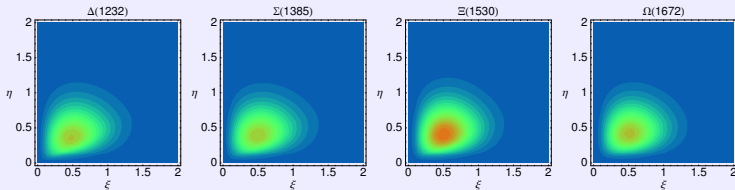
$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ octet baryon states $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(1690)$:



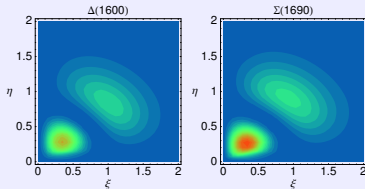
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the Σ_{3^+} decuplet baryon states $\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, $\Omega(1672)$:



$\rho(\xi, \eta)$ for the Σ_{3^+} decuplet baryon states $\Delta(1600)$, $\Sigma(1690)$:



	multiplet	$(LS)J^P$				
Low-energy QCD	octet	$(0 \frac{1}{2} \frac{1}{2})^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$
	octet	$(0 \frac{1}{2} \frac{1}{2})^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\Xi(1690)^{100}$
RCQM	octet	$(0 \frac{1}{2} \frac{1}{2})^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$	
	octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\Sigma(1560)^{94}$	
Spectroscopy Light, strange, charm, bottom	octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\Sigma(1620)^{100}$	
	octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$
Structure Nucleon E.m. Baryon E.m. Axial FFs	octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1700)^{100}$		$\Sigma(1940)^{100}$	
	octet	$(1 \frac{1}{2} \frac{1}{2})^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\Xi(1950)^{100}$
Gravitational FF	decuplet	$(0 \frac{3}{2} \frac{3}{2})^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
	decuplet	$(0 \frac{3}{2} \frac{3}{2})^+$	$\Delta(1600)^{100}$	$\Sigma(1690)^{99}$		
Vertex FFs $\pi NN, \pi N\Delta$	decuplet	$(1 \frac{1}{2} \frac{1}{2})^-$	$\Delta(1620)^{100}$	$\Sigma(1750)^{94}$		
	decuplet	$(1 \frac{1}{2} \frac{1}{2})^-$	$\Delta(1700)^{100}$			
Summary	singlet	$(1 \frac{1}{2} \frac{1}{2})^-$	$\Lambda(1405)^{71}$			
	singlet	$(1 \frac{1}{2} \frac{1}{2})^-$	$\Lambda(1520)^{71}$			
	singlet	$(0 \frac{1}{2} \frac{1}{2})^+$	$\Lambda(1810)^{92}$			

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77**, 114002 (2008)

See also the PDG: Phys. Rev. D **86**, 010001 (2012)

Various Baryon Reactions

Matrix elements of a transition operator \hat{O} between baryon eigenstates $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T'_3, Y' | \hat{O} | P, J, \Sigma, T, T_3, Y \rangle$$

$\hat{O} \dots \hat{J}_{\text{em}}^\mu \rightarrow$ electromagnetic FF's

$\dots \hat{A}_{\text{axial}}^\mu \rightarrow$ axial FF's

$\dots \hat{S} \rightarrow$ scalar FF

$\dots \hat{\Theta}^{\mu\nu} \rightarrow$ gravitational/tensor FF's

$\dots \hat{D}_\lambda^\mu \rightarrow$ hadronic decays

To be calculated from microscopic three-quark ME's

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3; f'_{i_1}, f'_{i_2}, f'_{i_3} | \hat{O} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3} \rangle$$

\uparrow
boosted 3-body states

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Electroweak Structure of the Nucleons / Baryons

- ▶ **Electromagnetic** nucleon form factors

$$G_E^p, G_M^p; G_E^n, G_M^n$$

- ▶ **Electric radii** and **magnetic moments**

$$r_E^p, \mu^p; r_E^n, \mu^n; r_E^Y, \mu^Y, \text{ for } Y = \Delta, \Lambda, \Sigma, \Xi, \Omega$$

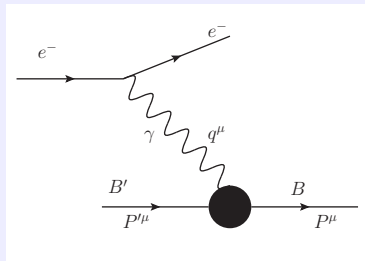
- ▶ **Axial** nucleon form factors

$$G_A, G_P$$

- ▶ **Axial charges** of N , N^* and Δ , Σ , Ξ , Σ^* , Ξ^*

$$g_A$$

Elastic electron scattering:



Invariant form factors:

$$F_{\Sigma'\Sigma}^\nu(Q^2) = \langle P', J, \Sigma', T, M_T | \hat{J}_{\text{em}}^\nu | P, J, \Sigma, T, M_T \rangle$$

$$\text{with } Q^2 = -q^2; \quad q^\mu = P^\mu - P'^\mu$$

Transition Matrix Elements in Point Form

Incoming baryon state: $|V, M, J, \Sigma\rangle \hat{=} |P, J, \Sigma\rangle$
 Outgoing baryon state: $|V', M', J', \Sigma'\rangle \hat{=} |P', J', \Sigma'\rangle$
 Transition operator: $\hat{O} = \hat{J}_{em}^\mu$

$$\begin{aligned}
 & \langle V', M', J', \Sigma' | \hat{J}_{em}^\mu | V, M, J, \Sigma \rangle = \\
 & = \frac{2}{MM'} \sum_{\sigma_i \sigma'_i} \sum_{\mu_i \mu'_i} \int d^3 \vec{k}_2 d^3 \vec{k}_3 d^3 \vec{k}'_2 d^3 \vec{k}'_3 \\
 & \times \sqrt{\frac{(\sum_i \omega'_i)^3}{\prod_i 2\omega'_i}} \prod_{\sigma'_i} D_{\sigma'_i \mu'_i}^{* \frac{1}{2}} \{R_W [k'_i; B(V')]\} \Psi_{M' J' \Sigma'}^* (\vec{k}'_1, \vec{k}'_2, \vec{k}'_3; \mu'_1, \mu'_2, \mu'_3) \\
 & \times \langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{J}_{rd}^\mu | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \\
 & \times \sqrt{\frac{(\sum_i \omega_i)^3}{\prod_i 2\omega_i}} \prod_{\sigma_i} D_{\sigma_i \mu_i}^{\frac{1}{2}} \{R_W [k_i; B(V)]\} \Psi_{MJ\Sigma} (\vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3) \\
 & \times 2MV_0 \delta^3 (M\vec{V} - M'\vec{V}' - \vec{q})
 \end{aligned}$$

where $p_i = B_c(V)k_i$, $p'_i = B_c(V')k'_i$, and $\omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$

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Spin- $\frac{1}{2}$ baryons:

$$G_E^B(Q^2) = \frac{1}{2M} F_{\frac{1}{2}\frac{1}{2}}^{\nu=0}(Q^2)$$

$$G_M^B(Q^2) = \frac{1}{Q} F_{\frac{1}{2}-\frac{1}{2}}^{\nu=1}(Q^2)$$

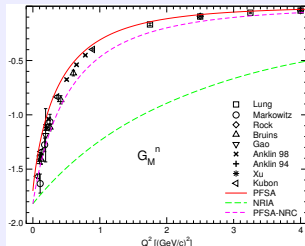
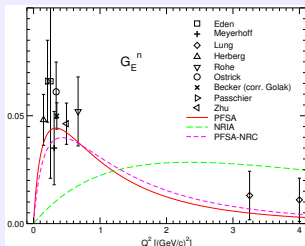
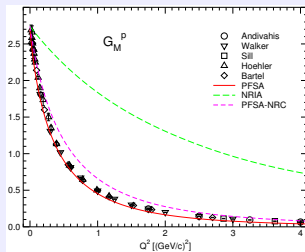
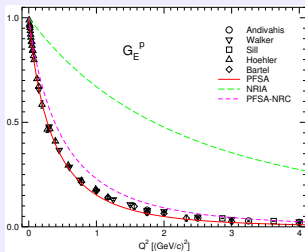
Spin- $\frac{3}{2}$ baryons:

$$G_E^B(Q^2) = \frac{1}{4M} [F_{\frac{1}{2}\frac{1}{2}}^{\nu=0}(Q^2) + F_{\frac{3}{2}\frac{3}{2}}^{\nu=0}(Q^2)]$$

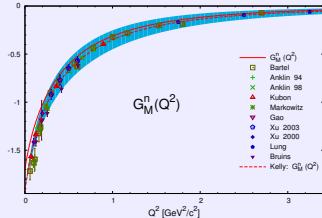
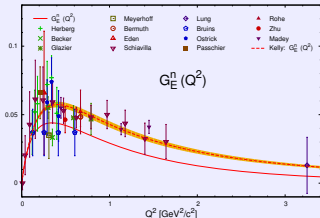
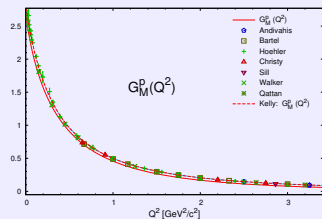
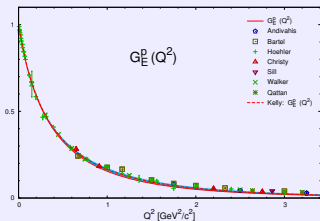
$$G_M^B(Q^2) = \frac{3}{5Q} [F_{\frac{1}{2}-\frac{1}{2}}^{\nu=1}(Q^2) + \sqrt{3} F_{\frac{3}{2}\frac{1}{2}}^{\nu=1}(Q^2)]$$

Electromagnetic Nucleon Form Factors

Covariant predictions of the GBE CQM:



Covariant predictions of the GBE CQM:



R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

M. Rohrmoser: Diploma Thesis, Univ. of Graz, 2013

Nucleon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

Baryon	GBE PFSM	Experiment
p	0.82	0.7692 ± 0.0123 ¹⁾
		0.70870 ± 0.00113 ²⁾
n	-0.13	-0.1161 ± 0.0022

¹⁾ CODATA value (PDG)

²⁾ Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Low-energy
QCD

RCQM

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

Gravitational FF

Vertex FFs

πNN , $\pi N\Delta$

Summary

Nucleon r_E^2 and μ – Nonrelativistic !!!

Electric radii r_E^2 [fm²]

Baryon	GBE PFSM	GBE NR1A	Experiment
p	0.82	0.10	0.7692 ± 0.0123 ¹⁾ 0.70870 ± 0.00113 ²⁾
n	-0.13	-0.01	-0.1161 ± 0.0022

¹⁾ CODATA value (PDG)

²⁾ Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	GBE NR1A	Experiment
p	2.70	2.74	2.792847356
n	-1.70	-1.82	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

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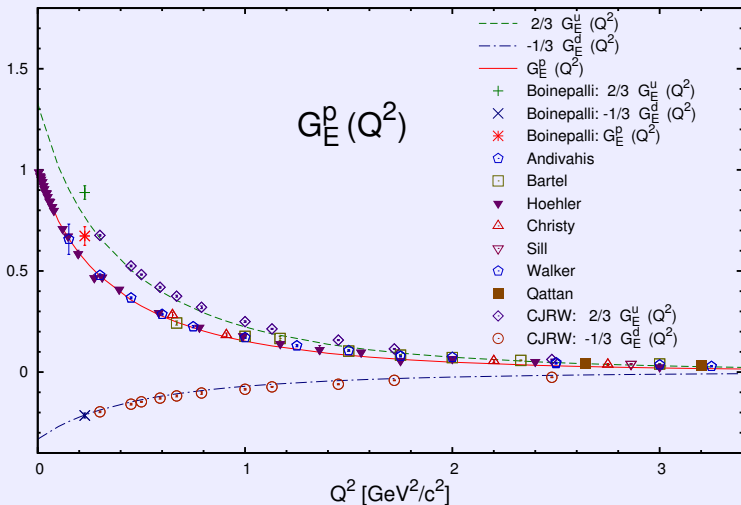
πNN , $\pi N\Delta$

Summary

Nucleons N

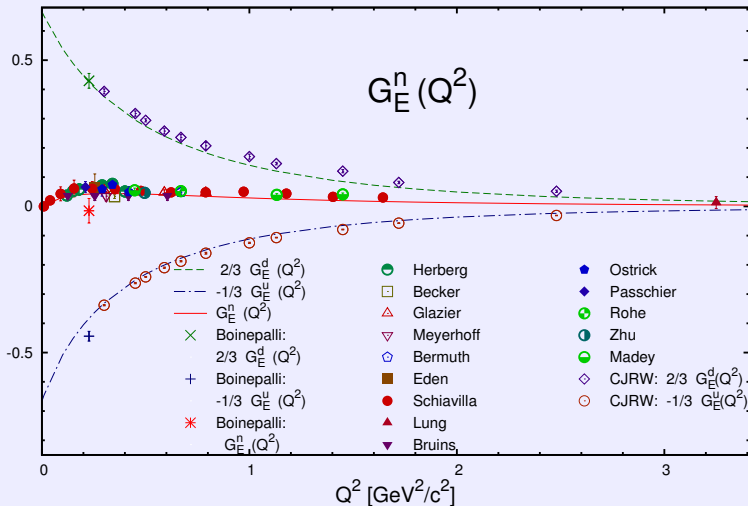
Proton Electric Form Factor

$$G_E^p = \frac{2}{3} G_E^u - \frac{1}{3} G_E^d$$



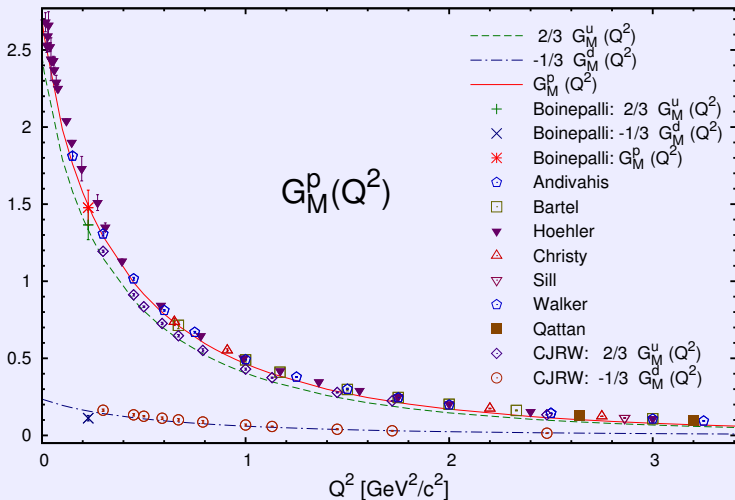
Neutron Electric Form Factor

$$G_E^n = \frac{2}{3} G_E^d - \frac{1}{3} G_E^u$$



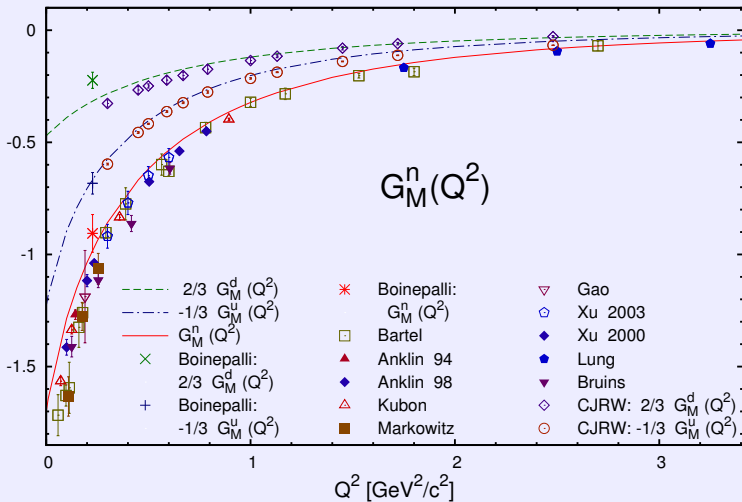
Proton Magnetic Form Factor

$$G_M^p = \frac{2}{3} G_M^u - \frac{1}{3} G_M^d$$

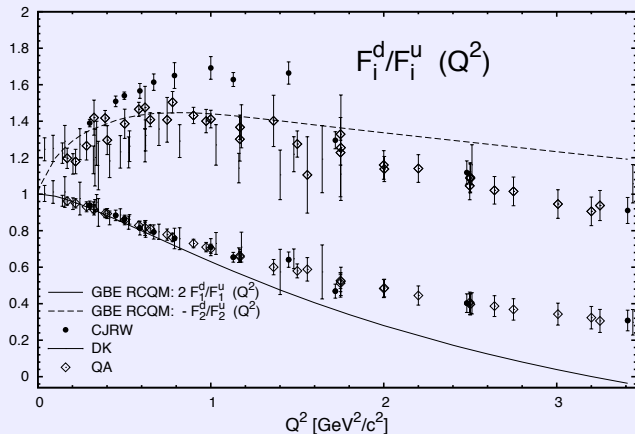


Neutron Magnetic Form Factor

$$G_M^n = \frac{2}{3} G_M^d - \frac{1}{3} G_M^u$$



Ratios F_i^d/F_i^u of Flavor Contr. to F_1 and F_2



GBE RCQM prediction: M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Phenomenology:

● G. D. Cates et al.: Phys. Rev. Lett. **106**, 252003 (2011)

□ M. Diehl and P. Kroll: arXiv:1302.4604

◇ I.A. Qattan and J. Arrington: Phys. Rev. C **86**, 065210 (2012)

Conclusions from Nucleon Flavor Analysis

- ▶ **Flavor analysis of nucleon e.m. form factors** in a relativistically invariant framework (point form).
- ▶ The **GBE RCQM** predicts flavor contributions in reasonable agreement with **experimental data**.
- ▶ The GBE RCQM relies on $\{QQQ\}$ degrees of freedom only; no explicit $\{QQQQ\bar{Q}\}$ etc.
- ▶ No explicit **meson-cloud effects** are included.
- ▶ No **strangeness content** in the nucleon for the low momentum transfers considered here.
- ▶ With respect to F_2^d/F_2^u three different phenomenological analyses give **distinct answers**.
- ▶ Details:
M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665
W. Plessas: Mod. Phys. Lett. A **28**, 136022 (2013)

Low-energy

QCD

RCQM

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

Gravitational FF

Vertex FFs

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πNN , $\pi N\Delta$

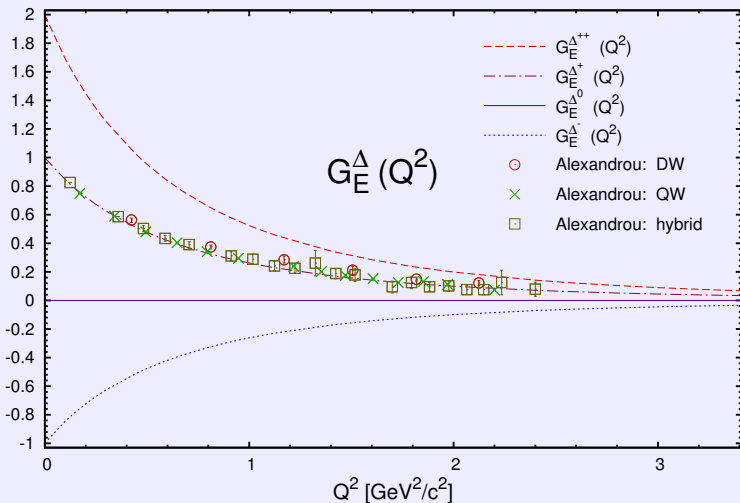
Summary

$$\Delta$$

$$\Lambda, \Sigma, \Xi$$

$$\Sigma^*, \Xi^*, \Omega$$

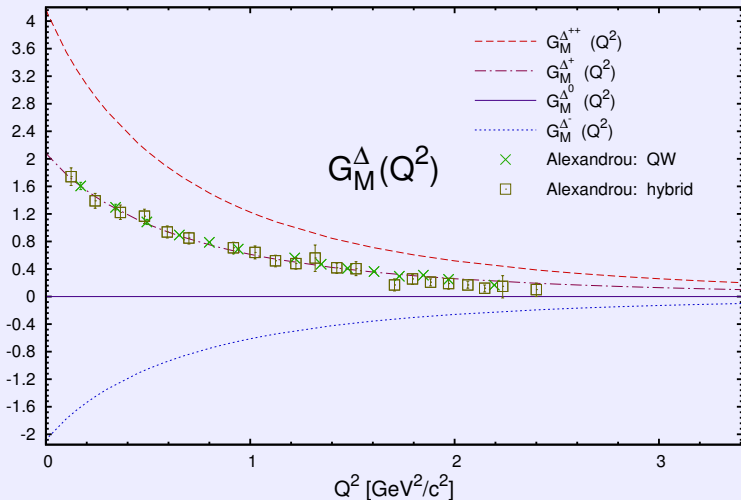
Electric Δ Form Factors



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

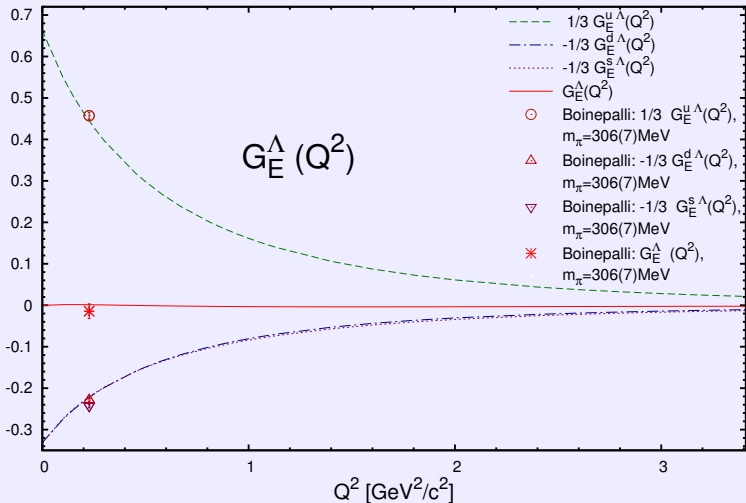
Magnetic Δ Form Factors



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

Octet $\Lambda(uds)$ Electric Form Factor



Low-energy QCD

RCQM

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Nucleon E.m.

Baryon E.m.

Axial FFs

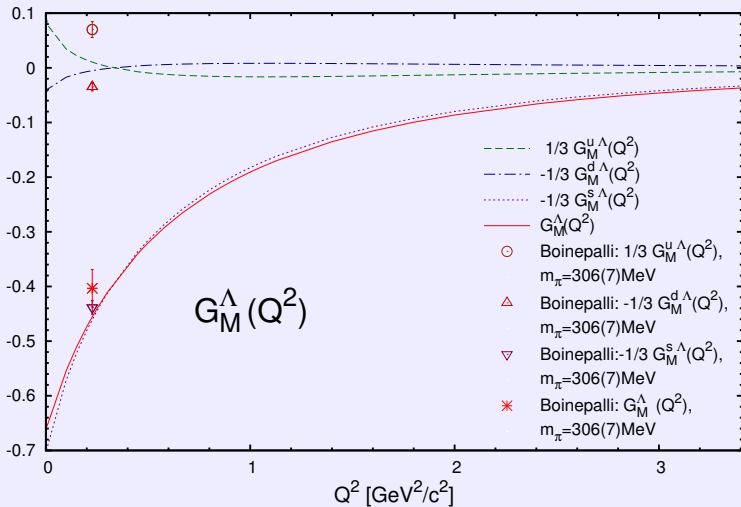
Gravitational FF

Vertex FFs

πNN , $\pi N\Delta$

Summary

Octet $\Lambda(uds)$ Magnetic Form Factor



Low-energy
QCD

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Nucleon E.m.

Baryon E.m.

Axial FFs

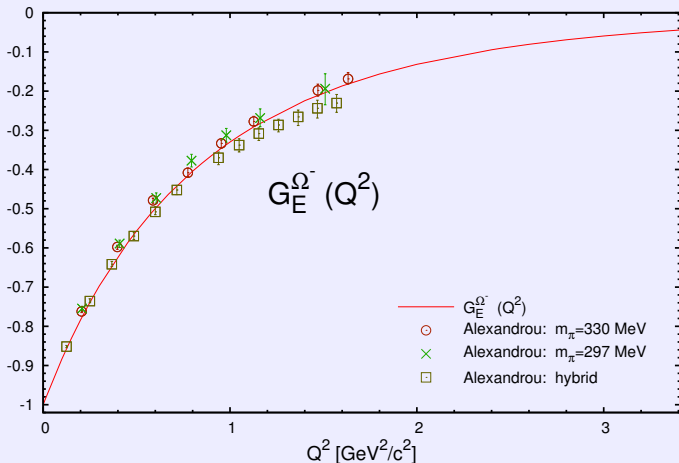
Gravitational FF

Vertex FFs

πNN , $\pi N\Delta$

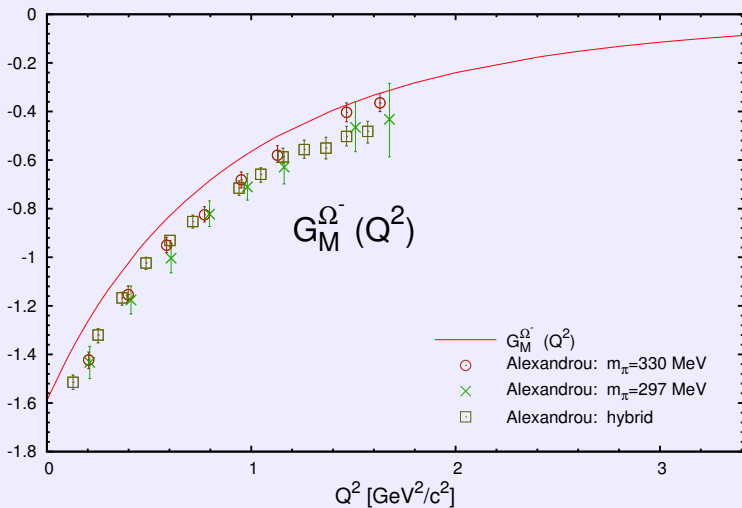
Summary

Decuplet Ω^- (sss) Electric Form Factor



Lattice-QCD: C. Alexandrou et al.: Phys. Rev. D **82** (2010) 034504

Decuplet Ω^- (sss) Magnetic Form Factor



Baryon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm²]

Baryon	GBE PFSM	Experiment
p	0.82	0.7692 ± 0.0123
n	-0.13	-0.1161 ± 0.0022
Σ^-	0.72	$0.61 \pm 0.12 \pm 0.09$

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427
Λ	-0.64	-0.613 ± 0.004
Σ^+	2.38	2.458 ± 0.010
Σ^-	-0.93	-1.160 ± 0.025
Ξ^0	-1.25	-1.250 ± 0.014
Ξ^-	-0.70	-0.6507 ± 0.0025
Δ^+	2.08	$2.7_{-1.3}^{+1.0} \pm 1.5 \pm 3$
Δ^{++}	4.17	3.7 - 7.5
Ω^-	-1.59	-2.020 ± 0.05



- Low-energy QCD
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- Vertex FFs
- $\pi NN, \pi N\Delta$
- Summary

Axial **Charges** and Axial **Form Factors**

of

N Ground State and **N^*** Resonances

as well as

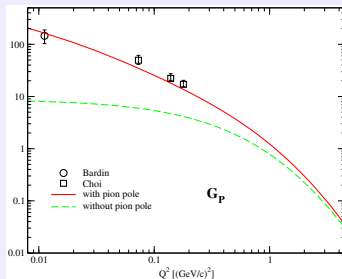
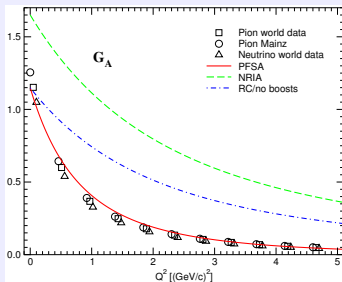
$\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

Axial Nucleon Form Factors



- Low-energy QCD
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- Gravitational FF
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- Summary

Covariant predictions of the GBE RCQM:



$$g_A^{GBE} = 1.15 \quad \text{vs.}$$

$$g_A^{exp} = 1.2695 \pm 0.0029$$

L.Ya. Glozman, M. Radici, R.F. Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B **516**, 183 (2001)

Axial Charges of N and N^* Resonances

State	J^P	EGBE	Lattice QCD	GN	NR
N(939)	$\frac{1}{2}^+$	1.15	1.23~1.26	1.66	1.65
N(1440)	$\frac{1}{2}^+$	1.16	?	1.66	1.61
N(1535)	$\frac{1}{2}^-$	0.02	~0.00	-0.11	-0.20
N(1710)	$\frac{1}{2}^+$	0.35	?	0.33	0.42
N(1650)	$\frac{1}{2}^-$	0.51	~0.55	0.55	0.64

EGBE **E**xtended **G**BE RCQM covariant result

Lattice **L**attice **Q**CD calculations by LHPC Collaboration and Takahashi-Kunihiro (Kyoto)

GN **G**lozman-**N**efediev $SU(6) \times O(3)$ nonrelativistic QM

NR **N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

Low-energy QCD

RCQM

Spectroscopy

Light, strange, charm, bottom

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Nucleon E.m.

Baryon E.m.

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Gravitational FF

Vertex FFs

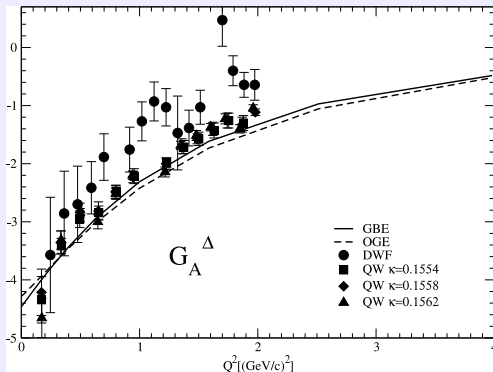
πNN , $\pi N\Delta$

Summary

Axial Charges of N and N^* Resonances

State	J^P	EGBE		psGBE		OGE	
		Mass	g_A	Mass	g_A	Mass	g_A
N(939)	$\frac{1}{2}^+$	939	1.15	939	1.15	939	1.11
N(1520)	$\frac{3}{2}^-$	1524	-0.64	1519	-0.21	1520	-0.15
N(1440)	$\frac{1}{2}^+$	1464	1.16	1459	1.13	1578	1.10
N(1535)	$\frac{1}{2}^-$	1498	0.02	1519	0.09	1520	0.13
N(1680)	$\frac{5}{2}^+$	1689	0.89	1728	0.83	1858	0.70
N(1675)	$\frac{5}{2}^-$	1676	0.84	1647	0.83	1690	0.80
N(1710)	$\frac{1}{2}^+$	1757	0.35	1776	0.37	1860	0.32
N(1650)	$\frac{1}{2}^-$	1581	0.51	1647	0.46	1690	0.44
N(1720)	$\frac{3}{2}^+$	1746	0.35	1728	0.34	1858	0.25
N(1700)	$\frac{3}{2}^-$	1608	-0.10	1647	-0.50	1690	-0.47

Covariant predictions of the GBE and OGE RCQMs:



Ki-Seok Choi: PhD Thesis, Graz, 2011

(Lattice QCD data from C. Alexandrou et al., PoS LATTICE2010, 141 (2010))

Axial Charges of $\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

	J^P	Exp	EGBE	LO	EOT	JT	NR
Low-energy QCD	$N \quad \frac{1}{2}^+$	1.2695	1.15	1.18	1.314	1.18	1.65
RCQM	$\Sigma \quad \frac{1}{2}^+$	-	0.65	0.636	0.686	0.73	0.93
Spectroscopy	$\Xi \quad \frac{1}{2}^+$	-	-0.21	-0.277	-0.299	-0.23	-0.32
Structure	$\Delta \quad \frac{3}{2}^+$	-	-4.48	-	-	~ -4.5	-6.00
Nucleon E.m.	$\Sigma^* \quad \frac{3}{2}^+$	-	-1.06	-	-	-	-1.41
Baryon E.m.	$\Xi^* \quad \frac{3}{2}^+$	-	-0.75	-	-	-	-1.00
Axial FFs							
Gravitational FF							

EGBE **E**xtended **G**BE RCQM covariant result
LO **L**in and **O**rginos lattice-QCD calculation
EOT **E**rkol, **O**ka, and **T**akahashi lattice-QCD calculation
JT **J**iang and **T**iburzi χ PT calculation
NR **N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. D **82**, 014007 (2010)

Low-energy QCD
 RCQM
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 Light, strange, charm, bottom
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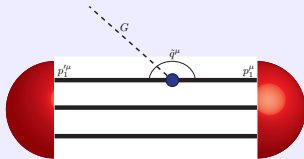
Vertex FFs

πNN , $\pi N\Delta$

Summary

Gravitational Form Factors of the Nucleon

Gravitational Form Factors

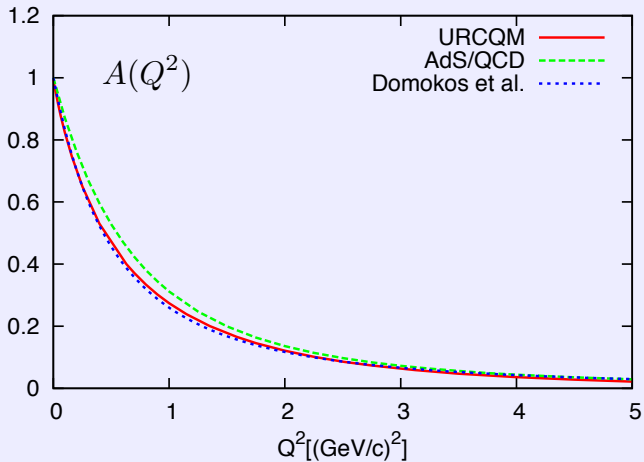


Invariant ME of **energy-momentum tensor** $\hat{\Theta}^{\mu\nu}$:

$$\langle P' J \Sigma' | \hat{\Theta}^{\mu\nu} | P J \Sigma \rangle = \bar{U}(P') \left[\gamma^{(\mu} \bar{P}^{\nu)} A(Q^2) + \frac{i}{2M} \bar{P}^{(\mu} \sigma^{\nu)} B(Q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} C(Q^2) \right] U(P)$$

$$A(Q^2) \sim \langle P' J \Sigma' | \Theta^{00} | P J \Sigma \rangle$$

Nucleon Gravitational Form Factor $A(Q^2)$



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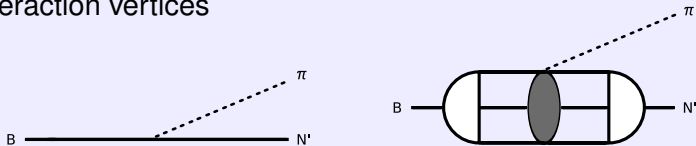
Microscopic Description

of

Meson-Baryon Interaction Vertices

Meson-Baryon Interaction Vertices

Interaction vertices



$$F_{i \rightarrow f} = (2\pi)^4 \langle f | \mathcal{L}_I(0) | i \rangle \equiv \langle V', M', J', \Sigma' | \hat{D}_{\text{rd}}^\pi | V, M, J, \Sigma \rangle$$

where

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{D}_{\text{rd}}^\pi | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle =$$

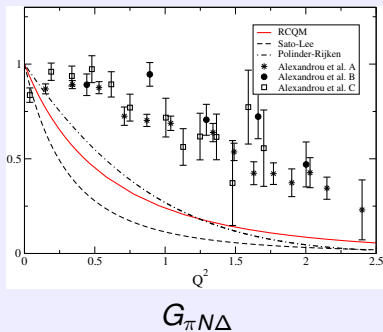
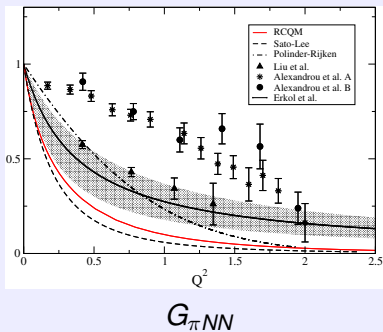
$$3\mathcal{N}_S \frac{ig_{qqm}}{2m_1 (2\pi)^{\frac{3}{2}}} \bar{u}(p'_1, \sigma'_1) \gamma_5 \gamma_\mu \lambda_m u(p_1, \sigma_1) \tilde{q}^\mu 2p_{20} \delta(\vec{p}_2 - \vec{p}'_2) 2p_{30} \delta(\vec{p}_3 - \vec{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3}$$

and

$$G_{\pi NN}(Q^2) = \frac{1}{f_{\pi NN}} \frac{m_\pi \sqrt{2\pi}}{\sqrt{2M_N}} \frac{\sqrt{E'_N + M'_N}}{E'_N + M'_N + \omega} \frac{F_{i \rightarrow f}}{Q_z}$$

$$G_{\pi N\Delta}(Q^2) = -\frac{1}{f_{\pi N\Delta}} \frac{3\sqrt{2\pi}}{2} \frac{m_\pi}{\sqrt{E'_N + M'_N} \sqrt{2M_\Delta}} \frac{F_{i \rightarrow f}}{Q_z}$$

πNN and $\pi N\Delta$ Interaction Vertices



T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Form-Factor Parametrizations

$$G(\vec{q}^2) = \frac{1}{1 + \left(\frac{\vec{q}}{\Lambda_1}\right)^2 + \left(\frac{\vec{q}}{\Lambda_2}\right)^4}$$

$$G(Q^2) = \frac{1}{1 + \left(\frac{Q}{\Lambda}\right)^2}$$

		RCQM	SL	PR		LIU	ERK	ALX
Structure	$\frac{f_N^2}{4\pi}$	0.0691	0.08	0.075		0.0649	0.0481	0.0412
	$N \Lambda_1$	0.451	0.453	0.940	Λ	0.747	0.614	1.65
	Λ_2	0.931	0.641	1.102		-	-	-
Vertex FFs	$\frac{f_\Delta^2}{4\pi}$	0.188	0.334	0.478				
	$\Delta \Lambda_1$	0.594	0.458	0.853				
	Λ_2	0.998	0.648	1.014				

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Summary and Conclusions

- ▶ Surprisingly **good agreement** of predictions by GBE RCQM with experimental data (wherever such data are available)
- ▶ **Small deviations** left in some observables, such as electric radii and magnetic moments
- ▶ Surprisingly **good agreement** of predictions by GBE RCQM with lattice-QCD results
- ▶ Most important symmetries of GBE RCQM:
 - ▶ **SB χ S**
 - ▶ **Lorentz invariance**
 - ▶ **time-reversal invariance**
 - ▶ **current conservation**
- ▶ The **non-relativistic** quark model **does not work** in any instance

Graz

K. Berger, J.P. Day, K.-S. Choi, L. Glozman, T. Melde,
M. Rohrmoser, R.C. Schardmüller, B. Sengl,
R.F. Wagenbrunn

(Theoretical Physics, University of Graz)

Pavia

S. Boffi and M. Radici

(INFN, Sezione di Pavia)

Padova

L. Canton

(INFN, Sezione di Padova)

Iowa City

W. Klink

(Department of Physics, University of Iowa, USA)

Low-energy
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Baryon E.m.

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Summary

Thank you very much
for
your attention!