Undulator Spin Polarimetry for the ”precursor” storage ring eEDM experiment

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The EDM storage ring experiment(s)

We plan a storage ring based experiment (100 M$ in 10 years) to measure with an accuracy of 10^{-29} e-cm the electric dipole moment (EDM) of the proton. In a second "precursor" storage ring experiment we plan to measure the electric dipole moment of the electron. The eEDM ring will be smaller and much less expensive than the pEDM ring and might be constructed first.

In the pEDM we will use polarized protons at the "magic" (see next) momentum of 0.7 Gev/c. In the precursor eEDM, polarized electrons at the "magic" momentum of 15 MeV/c. The EDM will be measured by spin polarimetry.

The rings will be strictly electrostatic (no magnets). At the magic momentum, in absence of magnetic fields, the spin direction will remain "frozen" in its direction at injection (longitudinal) and the EDM will be measured as proportional to a small vertical component of the spin that will gradually appear. Interested people may check another parallel talk to this Conference.

The magic condition

Spin dynamics is governed by the covariant T-BMT equation

$$\frac{d\mathbf{s}}{dt} = -\frac{q}{m\gamma}\mathbf{f} \times \mathbf{s}, \tag{1}$$

where $\mathbf{s}$ is a real 3-dimensional spin vector of a 1/2-spin particle, and $\mathbf{f}$ is a function of the position and the momentum of the particle and of the electric and magnetic field encountered by the particle along its trajectory. In a pure electrostatic ring, $\mathbf{f}$ reduces to

$$\mathbf{f} = \left(a\gamma - \frac{\gamma}{\gamma^2 - 1}\right)\frac{\mathbf{E} \times \mathbf{v}}{c^2}, \tag{2}$$

with a the spin anomaly. At the magic momentum $pc = mc^2/\sqrt{a}$ it is exactly $\mathbf{f} = \mathbf{0}$ and the spin remains frozen in its injection (longitudinal) direction.

For the electron it is $mc^2 \approx$ 0.51 MeV and $a = (g-2)/2 \approx$ 1.1597 10^{-3}, so the magic momentum is pc =15 MeV.

Spin Polarimetry for the eEDM

Spin polarimetry for the pEDM will be performed by nuclear means. This is not practical for 15 MeV polarized electrons. Among electron spin polarimetry methods, spin polarimetry by undulator synchrotron radiation seems promising[1][2][3][4][5][6]:

1. Radiation will be detected and measured by optical means, using a spectroscope with suitable polarization sensitive detector and probably a lock-in amplifiers. A measurement that should be very "clean" and accurate.

2. Undulator synchrotron radiation shows a line spectrum. Spectral lines from the oscillating electron charge and lines from the oscillating electron magnetic moment have opposite polarization.

3. in a conventional magnetic ring, one would use a magnetic undulator, while in an all-electric bend ring one might wanted use an all-electric undulator

Radiation from an accelerated charge

The radiation power from an accelerated charged particle, per solid angle and in the time interval Δt is given by the Poynting formula

$$4\pi \frac{dW}{d\Omega} = \frac{\mu_0}{4\pi} e^2 c \int_{\Delta t} A(t)^2 dt \qquad (3)$$

with $\mathbf{A}^{(q)}$ the vector potential, in a direction defined by **n** from the source

$$\mathbf{A^{(q)}(t)} = \frac{\mathbf{n} \times \left[(\mathbf{n} - \vec{\beta}) \times \frac{d\vec{\beta}}{dt} \right]}{\left[\mathbf{1} - \mathbf{n} \cdot \vec{\beta} \right]^{\mathbf{2}}} \qquad (4)$$

with $\vec{\beta} = \vec{v}/c$ the reduced velocity, and t the retarded time, *i.e* the time at the point of observation.

Radiation from accelerated momentum

If the accelerated particle has a magnetic moment $\vec{\mu}$ proportional to its spin (1/2) $\vec{s}$ in the particle rest frame it is

$$\vec{\mu} = \frac{ge}{mc}\frac{1}{2}\hbar\,\mathbf{s} \tag{5}$$

In the LAB the spin transforms as

$$\mathbf{S} = \mathbf{s} + \frac{\gamma^2}{1+\gamma}(\vec{\beta}\cdot\mathbf{s})\vec{\beta} \tag{6}$$

and the accelerated momentum produces in the LAB radiation with vector potential[6]

$$\mathbf{A}^{(\mu)}(\mathbf{t}) = \frac{1}{D}\frac{d}{dt}\left[\frac{\mathbf{n}\times\left[\mathbf{S}+\mathbf{n}\times(\vec{\beta}\times\mathbf{S}\right]}{D}\right],\ \mathbf{D} = (\mathbf{1}-\vec{\beta}\cdot\mathbf{n}) \tag{7}$$

Total radiation: charge + momentum

The total radiation vector potential is

$$\mathbf{A}^{(\mathrm{t})}(\mathbf{t}) = \mathbf{A}^{(\mathrm{q})}(\mathbf{t}) + \eta \mathbf{A}^{(\mu)}, \ \eta = \frac{1}{4}\left(\frac{g\gamma}{mc^2}\right)\hbar\omega. \tag{8}$$

The contribution from the spin is proportional to $(g\gamma/mc^2)$.

Perform the time derivative in Eq.(7) and obtain, after some work, an expression of $\mathbf{A}^{(\mu)}$ as a sum of one term proportional to **s** and one proportional to $\dot{\mathbf{s}}$

$$\mathbf{A}^{(\mu)} = \mathbf{F_1}\,\mathbf{s} + \mathbf{F_2}\,\dot{\mathbf{s}}. \tag{9}$$

F_1 and F_2 contain both $\vec{\beta}$ and $\dot{\vec{\beta}}$ that will be calculated by the Lorentz equation of motion in an electrostatic undulator (that will not magically act on the spin) and **s** and $\dot{\mathbf{s}}$, calculated by the T-BMT equation, respectively.

$$\text{Lorentz}: \ \frac{d\mathbf{p}}{dt} = e\mathbf{E}, \quad \text{T}-\text{BMT}: \ \frac{d\mathbf{s}}{dt} = \mathbf{f}\times\mathbf{s} \tag{10}$$

Electrostatic undulator

Let's pass the electron beam through an electrostatic undulator as schematically shown in Fig.1 (with N=2 periods)

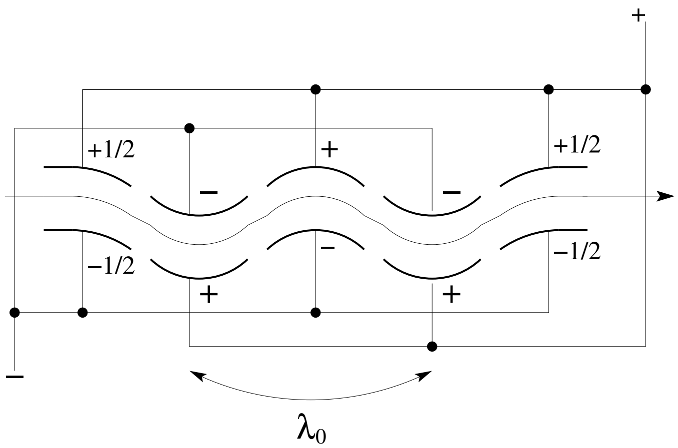

Fig.1 - electrostatic undulator

Here all components of the spin will remain longitudinal frozen from injection and containing a possible emerging vertical component due to the EDM (the aim of the experiment).

Polarized p, σ components of $A^{(q)}$ and $A^{(\mu)}$

In an electric undulator, the equation of motion are

$$\frac{d\vec{\beta}}{dt} = \vec{\beta} \times \vec{\Omega}, \text{ with } \Omega = \frac{e\mathbf{E}}{\beta^2 c^2 m \gamma} \tag{11}$$

In far field (Fraunhofer) the radiation is calculated along **n**. Components of $\mathbf{A}^{(q)}$ polarized $\parallel$ to the horizontal plane and $\perp$ to **n**, and $\mathbf{A}^{(\mu)}$ $\perp$ to **n** and to $\mathbf{A}^{(q)}$ are

$$\mathbf{A}_p = \vec{P} \cdot \vec{\Omega}/D^3, \quad \mathbf{A}_\sigma = \vec{\Sigma} \cdot \vec{\Omega}/D^3. \tag{12}$$

with $\vec{P}$ and $\vec{\Sigma}$ some functions.

Spectrum of undulator radiation

The spectrum is obtained by Fourier transform of the vector potential of Eq.(8). In an undulator with N periods (lines of width 1/N) the electric field can be written as

$$\Omega_x = \Omega_0 \sin \omega_0 t, \quad \omega_0 = \frac{2\pi c}{\lambda_0}, \tag{13}$$

with λ_0 the undulator period. The component of the vector potential, spin independent and spin dependent are, respectively, with different polarization and only odd harmonics on the axis

$$\begin{cases} A_p(t) = (1-\gamma^2\theta_b^2)\Omega \\ A_\sigma(t) = -(1+\gamma^2\theta_b^2)(C_1\theta_b s_y + s_z)\omega \end{cases}, \tag{14}$$

$\theta_b = (k/\gamma)\sin \omega_0 t$ is the instantaneous angle of the trajectory, with the undulator parameter $k = e/(2\pi m_e c)E/(\beta c)\lambda_0$.

Spin dependent and spin independent components have opposite polarization. Therefore:

spin state can be measured

Competitors

1. Finite emittance of the beam, that produces a contribution to A_σ on axis. However this only appears in the *even* spectral harmonics. The ratio is

$$\frac{A_\sigma^{(s)}}{A_\sigma^{(t)}} = \frac{2\eta}{\gamma}\sqrt{\frac{\beta_x}{\epsilon_x}} \tag{15}$$

   with η defined in Eq.(8). This may show that it is convenient to keep the beam rather wide in x.

2. undulator Field Imperfections, since a small $\delta\Omega_x$ field residual component with the same periodicity will produce an unwanted A_σ on axis

3. Chromaticity: electrons in the beam are not all magic, which produces widening of the spectral lines.

λ and Intensity of the undulator radiation

In a transverse electrostatic undulator with: period λ_0, no. of periods N and vertical field E_0. Angles of the unit vector **n** along the radiation are θ, azimuth and ϕ, latitude.

Wavelength λ at harmonic n and photon flux at $n = 1$ in a bandwith $\Delta\omega/\omega$ are

$$
\begin{aligned}
&\text{(a) } \lambda_n = \lambda_0 \frac{1 + \frac{1}{2}k^2 + \gamma^2(\theta^2 + \phi^2)}{2n\gamma^2} \\
&\text{(b) } n_\nu = \pi N \alpha \frac{I}{e} \frac{\Delta\omega}{\omega} k^2, \;\; \alpha \approx \frac{1}{137}\text{(fine struct. constant)}
\end{aligned}
\tag{16}
$$

I is the current in the electron beam and k is in Eq.(**??**). Eq.(a) above shows that the width of the spectral line is $\approx 1/(n\gamma^2)$. (b) shows that the intensity of the line is proportional to E^2.

Example - rounded values

constants

particle	=	electron
rest mass-energy mc^2 [MeV]	=	0.511
magnetic anomaly a	=	1.1597e-3
fine structure constant α	=	1/137 = 7.299e-3

beam and undulator

ring radius	=	5 m
magic momentum pc [MeV]	=	15.054
electric field E	=	750 KV/m
und.period λ_0	=	10 cm
ρ in the undulator	=	20 m
number of periods N	=	20
undulator parameter k	=	0.0233
beam undulation amplitude	=	62 μm
wavelength λ	=	57.92 μm
$\Delta\omega/\omega$	=	1.e-6
beam current I	=	1 μA
photons/sec n_ν	$\approx$	3 e12

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