Interplay of NP and OPE predictions for optimized observables at high-q^2 in exclusive $b \to s\,\ell^+\ell^-$ decays

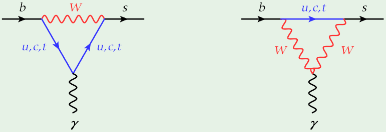
Christoph Bobeth
TU Munich – Universe Cluster

Workshop on the physics reach of rare and exclusive B decays

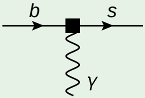
University of Sussex, Brighton

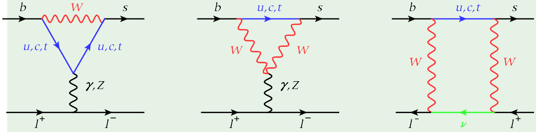
EFT (Effective Field Theory) in the SM (Standard Model) for . . .

$b \to s + \gamma$ and $b \to s + \ell^+\ell^-$

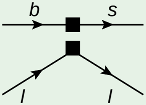


$\longrightarrow \quad C_7^{\gamma} \quad \times$





$\longrightarrow \quad C_{9,10}^{\ell\bar{\ell}} \quad \times$



$\mathcal{O}_7^{\gamma} = m_b[\bar{s}\,\sigma^{\mu\nu} P_R\, b]F_{\mu\nu} \qquad \mathcal{O}_{9,10}^{\ell\bar{\ell}} = [\bar{s}\,\gamma^{\mu} P_L\, b][\bar{\ell}\,\gamma_{\mu}(1,\,\gamma_5)\,\ell]$

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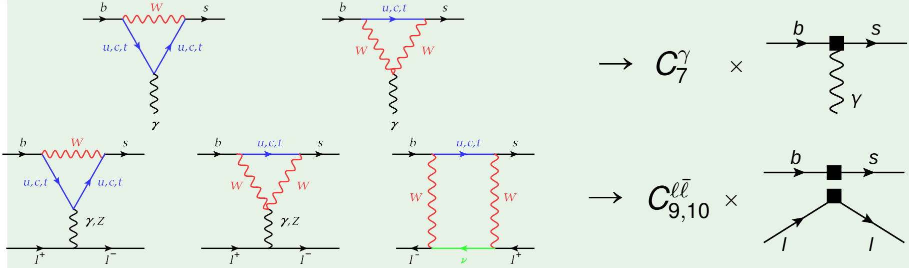

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and

- current-current op's $b \to s + Q\overline{Q}$, $(Q = u, c)$
- QCD penguin op's $b \to s + q\overline{q}$, $(q = u, d, s, c, b)$
- chromo-magnetic dipole $b \to s + gluon$

More $b \to s + (\gamma, \ell^+\ell^-)$ operators beyond the SM . . .

. . . frequently considered in model-(in)dependent searches

SM' = χ-flipped SM analogues

$$\mathcal{O}_{7'}^{\gamma} = \frac{e}{(4\pi)^2} m_b [\bar{s}\,\sigma_{\mu\nu} P_L\, b] F^{\mu\nu}, \qquad \mathcal{O}_{9',10'}^{\ell\ell} = \frac{\alpha_e}{4\pi} [\bar{s}\,\gamma^\mu P_R\, b][\bar{\ell}\,(\gamma^\mu, \gamma^\mu\gamma_5)\,\ell]$$

S + P = scalar + pseudoscalar

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new Dirac-structures beyond SM:

- SM' : right-handed currents
- S + P : higgs-exchange & box-type diagrams
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$B \to K^* [\to K\pi] + \ell^+\ell^-$:

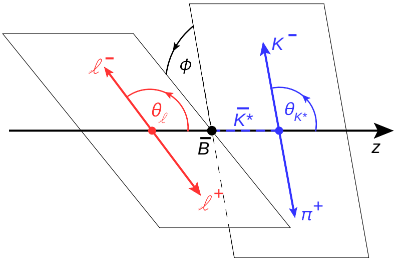
4-body decay with intermediate on-shell K^* (vector)

1) $q^2 = m^2_{\ell\bar{\ell}} = (p_\ell + p_{\bar{\ell}})^2 = (p_B - p_{K*})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_B, \vec{p}_\ell)$ in $(\ell\bar{\ell})$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle(\vec{p}_B, \vec{p}_K)$ in $(K\pi)$ – c.m. system

4) $\phi \angle(\vec{p}_K \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



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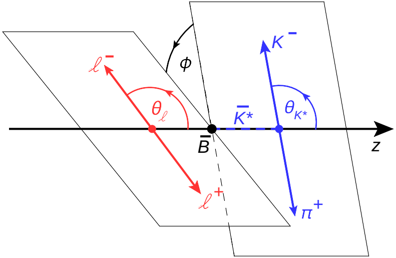


Twelve $J_i(q^2)$ = "Angular Observables"

$$\frac{32\pi}{9} \frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = J_{1s} \sin^2\theta_K + J_{1c} \cos^2\theta_K + (J_{2s} \sin^2\theta_K + J_{2c} \cos^2\theta_K) \cos 2\theta_\ell$$

$$+ J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$

$$+ (J_{6s} \sin^2\theta_K + J_{6c} \cos^2\theta_K) \cos\theta_\ell + J_7 \sin 2\theta_K \sin\theta_\ell \sin\phi$$

$$+ J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$

Which operators contribute to which J_i ?

[CB/Hiller/van Dyk in progress]

J_i	$\text{SM}^{(\prime)}$, $\text{SM}\times\text{SM}'$	S	P	$\text{SM}^{(\prime)}\times$(S,P)	T,T5	$\text{SM}^{(\prime)}\times$(T,T5)	(S,P)$\times$(T,T5)
$1s$	1	–	–	–	1	m_ℓ	–
$1c$	1	1	1	m_ℓ/m_b	1	m_ℓ ($\text{SM}^{(\prime)}\times$T5)	–
$2s$	1	–	–	–	1	–	–
$2c$	1	–	–	–	1	–	–
3	1	–	–	–	1	–	–
4	1	–	–	–	1	–	–
5	1	–	–	m_ℓ	–	m_ℓ	S$\times$T5, P$\times$T
$6s$	1	–	–	–	–	m_ℓ	–
$6c$	–	–	–	m_ℓ	–	m_ℓ ($10^{(\prime)}\times$T)	S$\times$T5, P$\times$T
7	1	–	–	m_ℓ	–	m_ℓ	P$\times$T5, S$\times$T
8	1	–	–	–	T$\times$T5	–	–
9	1	–	–	–	T$\times$T5	–	–

[Krüger/Matias hep-ph/0502060], [Altmannshofer et al. arXiv:0811.1214v5], [Alok et al. arXiv:1008.2367, CB/Hiller/van Dyk]

–	=	no contribution
1	=	order one contribution
m_ℓ	=	kinematic suppression by lepton mass: $m_\ell/\sqrt{q^2}$

Naive factorization & narrow width approximation of $K^* \to K\pi$

At high-q^2 ?

1) Naive factorization is leading dim 3 term in local OPE a la

    a) Grinstein/Pirjol hep-ph/0404250

   and/or

    b) Beylich/Buchalla/Feldmann arXiv:1101.5118

   NLO QCD corrections are known, included via $C_{7,9} \to C_{7,9}^{\text{eff}}$

    Seidel hep-ph/0403185

    Greub/Pilipp/Schupbach arXiv:0810.4077

2) at high-q^2 $B \to K^* \ell^+ \ell^-$ not much affected by scalar K_0^*

    Becirevic/Tayduganov arXiv:1207.4004

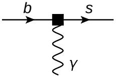
   right choice of distributions avoids mixing of S and P-waves

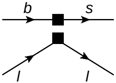
    Matias arXiv:1209.1525

Optimized observables in $B \to K^*(\to K\pi)\ell^+\ell^-$ @ high-q^2

Hadronic amplitude $B \to K^*(\to K\pi)\,\ell^+\ell^-$

neglecting 4-quark operators

$\mathcal{M} = \langle K\pi |\; C_7 \times$ 

$+ \; C_{9,10} \times$ 

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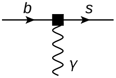

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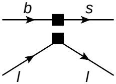

$|B\rangle$

$\mathcal{M}$ may expressed in terms of transversity amplitudes ($m_\ell = 0$)

... using narrow width approximation & intermediate K^* on-shell

$\Rightarrow$ "just" requires $B \to K^*$ form factors V, $A_{1,2}$, $T_{1,2,3}$ in K^*-transversity amp's:

$$A_\perp^{L,R} \sim \sqrt{2\,\lambda}\left[(C_9 \mp C_{10})\frac{V}{M_B + M_{K*}} + \frac{2\,m_b}{q^2} C_7\, T_1\right],$$

$$A_\parallel^{L,R} \sim -\sqrt{2}\,(M_B^2 - M_{K*}^2)\left[(C_9 \mp C_{10})\frac{A_1}{M_B - M_{K*}} + \frac{2\,m_b}{q^2} C_7\, T_2\right],$$

$$A_0^{L,R} \sim -\frac{1}{2\,M_{K*}\sqrt{q^2}}\left\{(C_9 \mp C_{10})\left[\ldots A_1 + \ldots A_2\right] + 2\,m_b C_7\left[\ldots T_2 + \ldots T_3\right]\right\}$$

Optimized observables in $B \to K^*(\to K\pi)\ell^+\ell^-$ @ high-q^2

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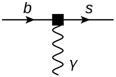

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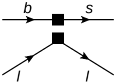

$$|B\rangle$$

Form factor relations upto $\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$: [Isgur/Wise PLB232 (1989) 113, PLB237 (1990) 527]

$$T_1 = V + \mathcal{O}(\lambda)\,, \qquad T_2 = A_1 + \mathcal{O}(\lambda)\,, \qquad T_3 = A_2\,\frac{M_B^2}{q^2} + \mathcal{O}(\lambda)$$

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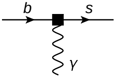
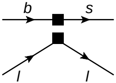
$\mathcal{M} = \langle K\pi|\; C_7 \times$ 

$+\; C_{9,10} \times$ 

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Trans amp's factorize → 1 short-distance coeff $C^{L,R}$ (for A_i^L and one for A_i^R)

FF symmetry breaking OPE $C_7^{\rm SM} \approx -0.3,\; C_9^{\rm SM} \approx 4.2,\; C_{10}^{\rm SM} \approx -4.2$

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda) + \mathcal{O}\left(\lambda^2\right), \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2 m_b^2}{q^2} C_7,$$

and 3 form factors (FF) f_i ($i = \perp, \|, 0$) "helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249]

$$f_\perp = \frac{\sqrt{2\hat\lambda}}{1 + \hat M_{K*}} V, \qquad f_\| = \sqrt{2}\,(1 + \hat M_{K*})\, A_1, \qquad f_0 = \frac{(1 - \hat s - \hat M_{K*}^2)(1 + \hat M_{K*})^2 A_1 - \hat\lambda\, A_2}{2\,\hat M_{K*}\,(1 + \hat M_{K*})\sqrt{\hat s}}$$

Observables in SM basis . . .

[CB/Hiller/van Dyk arXiv:1006.5013, 1105.0376]

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duality violating contributions

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. . . and $J_{7,8,9} = 0$

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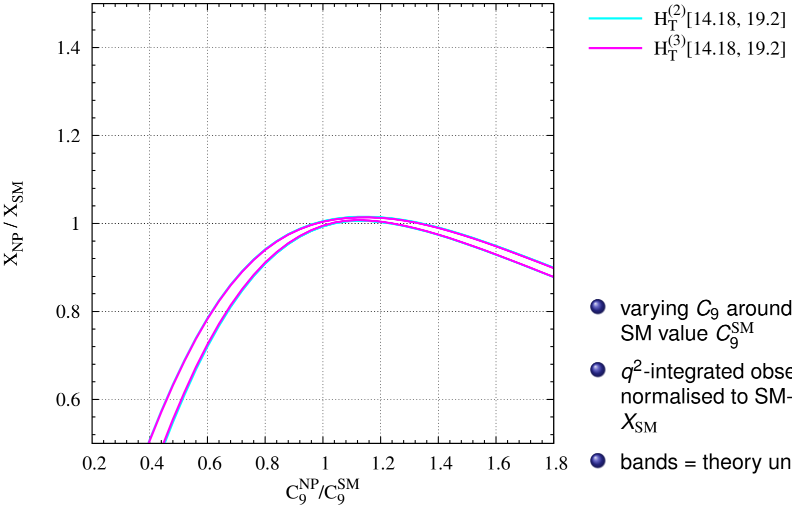
"short-distance free" → measure form factors $f_{0,\parallel,\perp}$ (SM-operator basis only)

$$\frac{f_0}{f_\parallel} = \frac{\sqrt{2} J_5}{J_{6s}} = \frac{-J_{2c}}{\sqrt{2} J_4} = \frac{\sqrt{2} J_4}{2J_{2s} - J_3} = \sqrt{\frac{-J_{2c}}{2J_{2s} - J_3}} = \frac{\sqrt{2} J_8}{-J_9},$$

$$\frac{f_\perp}{f_\parallel} = \sqrt{\frac{2J_{2s} + J_3}{2J_{2s} - J_3}} = \frac{\sqrt{-J_{2c}\,(2J_{2s} + J_3)}}{\sqrt{2} J_4}, \qquad \frac{f_0}{f_\perp} = \sqrt{\frac{-J_{2c}}{2J_{2s} + J_3}}$$

Sensitivity of $H_T^{(2,3)}$ – example: real C_9



- varying C_9 around SM value C_9^{SM}
- q^2-integrated observables X_{NP} normalised to SM-prediction X_{SM}
- bands = theory uncertainties

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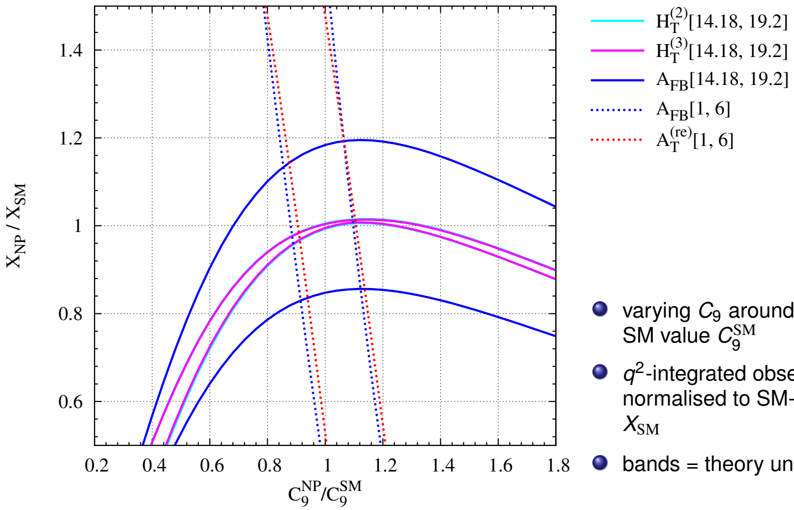

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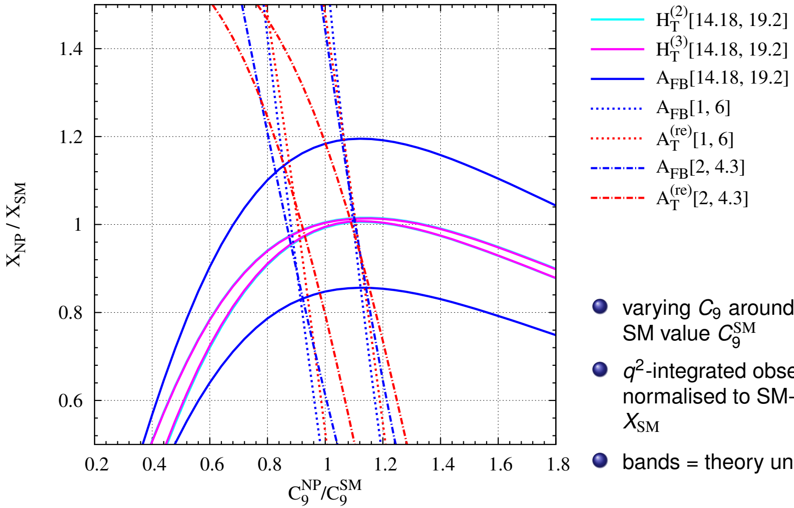

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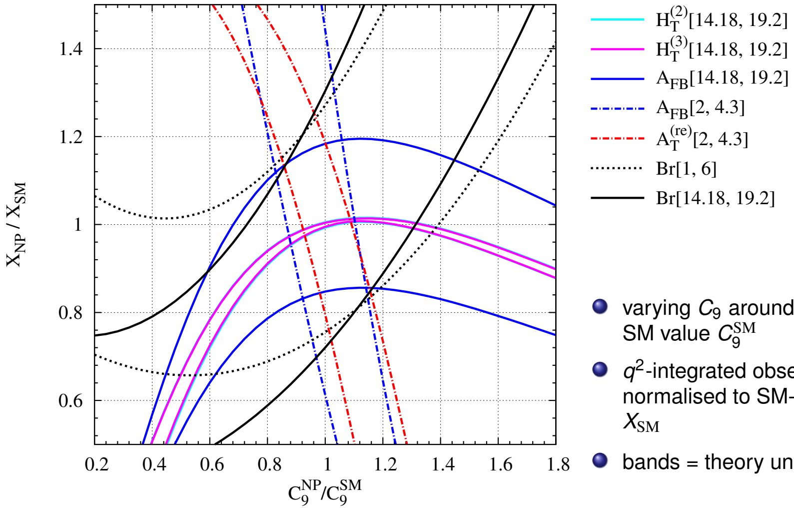

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What will remain of

- $|H_T^{(1)}| = 1$ as test of OPE
- $H_T^{(2,3)}$ are “long-distance free” and $H_T^{(2)} = H_T^{(3)}$ as test of OPE
- $J_{7,8,9} = 0$
- “short-distance free” ratios

$$f_0/f_{\|}, \quad f_0/f_{\perp}, \quad f_{\perp}/f_{\|}, \quad F_L, \quad A_T^{(2,3)}$$

when including BSM operators ???

SM' = $\mathcal{O}_{7',9',10'}$ @ low recoil

[work in progress CB/Hiller/van Dyk]

transversity amplitudes : $A^{L,R}_{0,\parallel} = -C^{L,R}_{-}\, f_{0,\parallel}, \qquad A^{L,R}_{\perp} = +C^{L,R}_{+}\, f_{\perp}$

with short-distance coefficients $C^{L,R} \to C^{L,R}_{\pm}$

$$C^{L,R}_{-} = \left[(C_9^{\text{eff}} - C_{9'}^{\text{eff}}) + \kappa \frac{2 m_b^2}{q^2} (C_7^{\text{eff}} - C_{7'}^{\text{eff}}) \right] \mp (C_{10} - C_{10'}),$$

$$C^{L,R}_{+} = \left[(C_9^{\text{eff}} + C_{9'}^{\text{eff}}) + \kappa \frac{2 m_b^2}{q^2} (C_7^{\text{eff}} + C_{7'}^{\text{eff}}) \right] \mp (C_{10} + C_{10'})$$

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$$C^{L,R}_{-} = \left[(C^{\text{eff}}_9 - C^{\text{eff}}_{9'}) + \kappa \frac{2m_b^2}{q^2} (C^{\text{eff}}_7 - C^{\text{eff}}_{7'}) \right] \mp (C_{10} - C_{10'}),$$

$$C^{L,R}_{+} = \left[(C^{\text{eff}}_9 + C^{\text{eff}}_{9'}) + \kappa \frac{2m_b^2}{q^2} (C^{\text{eff}}_7 + C^{\text{eff}}_{7'}) \right] \mp (C_{10} + C_{10'})$$

Now the angular observables J_i ($m_\ell = 0$) read

$$\frac{4}{3}(2J_{2s} + J_3) = 2\,\rho_1^+ f_\perp^2, \qquad \frac{4\sqrt{2}}{3} J_4 = 2\,\rho_1^- f_0 f_\parallel, \qquad J_7 = 0,$$

$$\frac{4}{3}(2J_{2s} - J_3) = 2\,\rho_1^- f_\parallel^2, \qquad \frac{2\sqrt{2}}{3} J_5 = 4\,\mathrm{Re}(\rho_2) f_0 f_\perp, \qquad \frac{4\sqrt{2}}{3} J_8 = 4\,\mathrm{Im}(\rho_2) f_0 f_\perp,$$

$$-\frac{4}{3} J_{2c} = 2\,\rho_1^- f_0^2, \qquad \frac{2}{3} J_{6s} = 4\,\mathrm{Re}(\rho_2) f_\parallel f_\perp, \qquad -\frac{4}{3} J_9 = 4\,\mathrm{Im}(\rho_2) f_\parallel f_\perp$$

where ρ_1 and ρ_2 have to be generalised

$$\rho_1^\pm = \frac{1}{2}\left(|C^R_\pm|^2 + |C^L_\pm|^2 \right), \qquad \rho_2 = \frac{1}{4}\left(C^R_+ C^{R*}_- - C^L_- C^{L*}_+ \right)$$

SM' = $\mathcal{O}_{7',9',10'}$ @ low recoil

- extension to $\rho_1 \to \rho_1^{\pm}$
- still have: $H_T^{(1)} = \text{sgn}(f_0) \quad \Rightarrow$ deviations test OPE
- $J_7 = 0$, but $J_{8,9} \neq 0$
- generalisation: $H_T^{(2)} = H_T^{(3)} = \dfrac{2\,\text{Re}(\rho_2)}{\sqrt{\rho_1^- \cdot \rho_1^+}}$
- 2 new FF-free ratios

$$H_T^{(4)} = \frac{\sqrt{2}J_8}{\sqrt{-J_{2c}\,(2J_{2s} + J_3)}} = \frac{2\,\text{Im}(\rho_2)}{\sqrt{\rho_1^- \cdot \rho_1^+}}, \qquad H_T^{(5)} = \frac{-J_9}{\sqrt{(2J_{2s})^2 - J_3^2}} = \frac{2\,\text{Im}(\rho_2)}{\sqrt{\rho_1^- \cdot \rho_1^+}}$$

- $a_{\text{CP}}^{(1)} \to a_{\text{CP}}^{(1,\pm)}$ and $a_{\text{CP}}^{(2)} \to a_{\text{CP}}^{(2,\pm)}$
- generalisation of $a_{\text{CP}}^{(3)}$ and additional

$$a_{\text{CP}}^{(3)} = \frac{2\,\text{Re}(\rho_2 - \bar{\rho}_2)}{\sqrt{(\rho_1^+ - \bar{\rho}_1^+)\cdot(\rho_1^- - \bar{\rho}_1^-)}}, \qquad a_{\text{CP}}^{(4)} = \frac{2\,\text{Im}(\rho_2 - \bar{\rho}_2)}{\sqrt{(\rho_1^+ - \bar{\rho}_1^+)\cdot(\rho_1^- - \bar{\rho}_1^-)}}$$

- less “short-distance free” ratios: only $f_0/f_{||}$ remains

Scalar (S + P) operators @ low recoil . . . and no tensor op's

- New form factor A_0 needed !!!
- No contribution to $J_{1s,\,2s,\,2c,\,3,\,4,\,6s,\,8,\,9}$

$\Rightarrow$ still $\quad \left|H_T^{(1)}\right| = 1, \quad J_{1s} = 3\,J_{2s}, \quad H_T^{(4)} = H_T^{(5)}$

- unsuppressed contributions to J_{1c}

$$J_{1c} = -J_{2c} + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right) \qquad \Rightarrow \qquad J_{1c} = -J_{2c} + \ldots (C_{S,P} - C'_{S,P}) A_0 + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right)$$

since $F_L = (J_{1c} - J_{2c}/3)/\Gamma$, relation does not hold:

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} \neq \frac{3}{4} F_L \sin^2\theta_\ell + \frac{3}{8} F_T (1 + \cos^2\theta_\ell) + A_{\rm FB}\cos\theta_\ell$$

- (SM + SM') $\times$ S to $J_{5,\,6c,\,7}$ suppressed by $\sim m_\ell/\sqrt{q^2}$

$\Rightarrow H_T^{(2)}$ modified, only $\quad H_T^{(2)} + \mathcal{O}(m_\ell/\sqrt{q^2}) \approx H_T^{(3)}$

$\Rightarrow J_{6c} \neq 0$ modified

$\Rightarrow J_7 \neq 0$ if CPV beyond SM, since $J_7 \sim \mathrm{Im}[\ldots(C_S - C'_S)]$

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Scalar (S + P) operators @ low recoil ... and no tensor op's

- New form factor A_0 needed !!!
- No contribution to $J_{1s,\,2s,\,2c,\,3,\,4,\,6s,\,8,\,9}$

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$$J_{1c} = -J_{2c} + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right) \quad \Rightarrow \quad J_{1c} = -J_{2c} + \ldots (C_{S,P} - C'_{S,P})A_0 + \mathcal{O}\left(\frac{m_\ell^2}{q^2}\right)$$

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$\Rightarrow H_T^{(2)}$ modified, only $\quad H_T^{(2)} + \mathcal{O}(m_\ell/\sqrt{q^2}) \approx H_T^{(3)}$
$\Rightarrow J_{6c} \neq 0$ modified
$\Rightarrow J_7 \neq 0$ if CPV beyond SM, since $J_7 \sim \mathrm{Im}[\ldots(C_S - C'_S)]$

Tensor (T + T5) operators @ low recoil

- 2 new short-distance coefficients

$$\rho_1^T \equiv 16\,\kappa^2 \frac{M_B^2}{q^2}\left(|C_T|^2 + |C_{T5}|^2\right), \qquad \rho_2^T \equiv 16\,\kappa^2 \frac{M_B^2}{q^2}\, C_T C_{T5}^*$$

- not $m_\ell/\sqrt{q^2}$ suppressed contributions to

  $\Rightarrow J_{1s,\,1c,\,2s,\,2c,\,3,\,4,\,8,\,9}$

  $\Rightarrow$ (S $\times$ T5) and (P $\times$ T) in $J_{5,\,6c}$ and (S $\times$ T) and (P $\times$ T) in J_7

- $\sim m_\ell/\sqrt{q^2}$-suppressed interference (SM + SM') $\times$ (T + T5) in $J_{1s,\,1c,\,5,\,6s,\,6c,\,7}$

- now $|H_T^{(1)}| = 1 + \mathcal{O}(M_{K*}^2/M_B^2)$

$$H_T^{(1)} \approx 1 + \frac{M_{K*}^2}{M_B^2} \times F(\rho_1^-, \rho_1^T) + \mathcal{O}\left(\frac{M_{K*}^4}{M_B^4}\right)$$

In BSM scenarios without tensor operators, deviations $|H_T^{(1)}| \neq 1$ can signal large long-distance effects

Summary

New Physics scenarios and which observables remain free of hadronic input

Scenario	$H_T^{(1)}$	$H_T^{(2)}$	$H_T^{(3)}$	$H_T^{(4)}$	$H_T^{(5)}$
SM	✓	✓	✓	—	—
SM × S × P	✓	A_0	✓	—	—
SM × T	✓	✓	✓	—	—
SM × SM'	✓	✓	✓	✓	✓
SM × SM' × S × P × T	✓	A_0	✓	✓	✓

✓ at most corrections of order $\alpha_s \times \lambda$ and $C_7/C_9 \times \lambda$

A_0 breaking through terms involving the corresponding $B \to K^*$ form factor

— vanish in the considered scenario

Summary

SM + SM'

- $|H_T^{(1)}| = 1, \quad H_T^{(2)} = H_T^{(3)}, \quad H_T^{(4)} = H_T^{(5)}, \quad J_7 = 0$
- however, only $f_0/f_{||}$ as "short-distance free" ratio

SM + SM' + (S,P)

- $|H_T^{(1)}| = 1, \quad H_T^{(2)} + \mathcal{O}(m_\ell/\sqrt{q^2}) = H_T^{(3)}, \quad H_T^{(4)} = H_T^{(5)}, \quad J_7 \sim m_\ell/\sqrt{q^2}$
- $B_s \to \mu^+\mu^-$ constrains directly $(C_{S,P} - C_{S',P'})$, which enter also $B \to K^*\ell^+\ell^-$ combination $(C_{S,P} + C_{S',P'})$ is constraint by $B \to K\ell^+\ell^-$

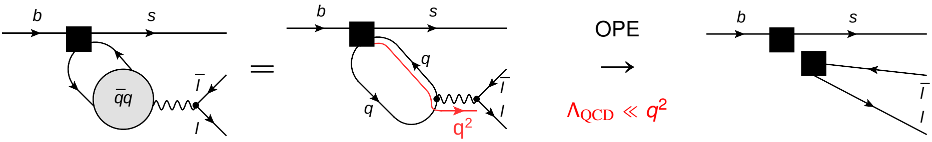
SM + SM' + (S,P) + (T,T5)

- $|H_T^{(1)}| = 1 + \mathcal{O}(M_{K*}^2/M_B^2) \times \rho_1^- \rho_1^T$
- $H_T^{(2)} \neq H_T^{(3)}, \quad H_T^{(4)} \neq H_T^{(5)}$

– Backup Slides –

High-q^2 = Low Recoil

Hard momentum transfer $(q^2 \sim M_B^2)$ through $(\bar{q}q) \to \bar{\ell}\ell$ allows local OPE



$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi^2}{q^2} i \int d^4x \, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{L}^{\text{eff}}(0), j_\mu^{\text{em}}(x)\} | \bar{B} \rangle \, [\bar{\ell}\gamma^\mu \ell]$$

$$= \left(\sum_a \mathcal{C}_{3a} \mathcal{Q}_{3a}^\mu + \sum_b \mathcal{C}_{5b} \mathcal{Q}_{5b}^\mu + \sum_c \mathcal{C}_{6c} \mathcal{Q}_{6c}^\mu + \mathcal{O}(dim > 6) \right) [\bar{\ell}\gamma_\mu \ell]$$

Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann arXiv:1101.5118

Leading $dim = 3$ operators: $\langle \bar{K}^* | \mathcal{Q}_{3,a} | \bar{B} \rangle \sim$ usual $B \to K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$

$$\mathcal{Q}_{3,1}^\mu = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) [\bar{s}\,\gamma_\nu (1 - \gamma_5)\, b] \quad \to \quad C_9 \to C_9^{\text{eff}}, \quad (V, A_{1,2})$$

$$\mathcal{Q}_{3,2}^\mu = \frac{i m_b}{q^2} q_\nu \, [\bar{s}\, \sigma_{\nu\mu} (1 + \gamma_5)\, b] \quad \to \quad C_7 \to C_7^{\text{eff}}, \quad (T_{1,2,3})$$

High-q^2 = Low Recoil

[Beylich/Buchalla/Feldmann arXiv:1101.5118]

$dim = 3$ α_s matching corrections are also known

$m_s \neq 0$ 2 additional $dim = 3$ operators, suppressed with $\alpha_s m_s / m_b \sim 0.5\,\%$,
NO new form factors

$dim = 4$ absent

$dim = 5$ suppressed by $(\Lambda_{\rm QCD}/m_b)^2 \sim 2\,\%$,
explicite estimate @ $q^2 = 15$ GeV^2: $< 1\%$

$dim = 6$ suppressed by $(\Lambda_{\rm QCD}/m_b)^3 \sim 0.2\,\%$ and small QCD-penguin's: $C_{3,4,5,6}$
spectator quark effects: from weak annihilation

beyond OPE duality violating effects

- based on Shifman model for c-quark correlator + fit to recent BES data
- $\pm 2\,\%$ for integrated rate $q^2 > 15$ GeV^2

$\Rightarrow$ OPE of exclusive $B \to K^{(*)} \ell^+ \ell^-$ predicts small sub-leading contributions !!!

BUT, still missing $B \to K^{(*)}$ form factors @ high-q^2
for predictions of angular observables J_i

High-q^2: OPE + HQET

Framework developed by Grinstein/Pirjol hep-ph/0404250

1) OPE in $\Lambda_{\rm QCD}/Q$ with $Q = \{m_b, \sqrt{q^2}\}$ + matching on HQET + expansion in m_c

$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi}{q^2} \sum_{i=1}^{6} \mathcal{C}_i(\mu)\, \mathcal{T}_\alpha^{(i)}(q^2, \mu)\, [\bar{\ell}\gamma^\alpha \ell]$$

$$\mathcal{T}_\alpha^{(i)}(q^2, \mu) = i \int d^4x\, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{O}_i(0), j_\alpha^{\rm em}(x)\} | \bar{B}\rangle$$

$$= \sum_{k \geqslant -2} \sum_j C_{i,j}^{(k)} \langle \mathcal{Q}_{j,\alpha}^{(k)} \rangle$$

$\mathcal{Q}_{j,\alpha}^{(k)}$	power	$\mathcal{O}(\alpha_s)$
$\mathcal{Q}_{1,2}^{(-2)}$	1	$\alpha_s^0(Q)$
$\mathcal{Q}_{1-5}^{(-1)}$	$\Lambda_{\rm QCD}/Q$	$\alpha_s^1(Q)$
$\mathcal{Q}_{1,2}^{(0)}$	m_c^2/Q^2	$\alpha_s^0(Q)$
$\mathcal{Q}_{j>3}^{(0)}$	${\Lambda_{\rm QCD}}^2/Q^2$	$\alpha_s^0(Q)$
$\mathcal{Q}_i^{(2)}$	m_c^4/Q^4	$\alpha_s^0(Q)$

included,
unc. estimate by naive pwr cont.

2) HQET FF-relations at sub-leading order + α_s corrections in leading order

$$T_1(q^2) = \kappa\, V(q^2), \qquad T_2(q^2) = \kappa\, A_1(q^2), \qquad T_3(q^2) = \kappa\, A_2(q^2)\frac{M_B^2}{q^2},$$

$$\kappa = \left(1 + \frac{2D_0^{(v)}(\mu)}{C_0^{(v)}(\mu)}\right)\frac{m_b(\mu)}{M_B}$$

can express everything in terms of QCD FF's $V, A_{1,2}$ @ $\mathcal{O}(\alpha_s\Lambda_{\rm QCD}/Q)$!!!

High-q^2: OPE + HQET – Transversity Amplitudes

$$A_\perp^{L,R} = +\left[C^{L,R} + \tilde{r}_a\right] f_\perp\,, \qquad A_\parallel^{L,R} = -\left[C^{L,R} + \tilde{r}_b\right] f_\parallel\,,$$

$$A_0^{L,R} = -C^{L,R} f_0 - N M_B \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 \tilde{r}_b A_1 - \hat{\lambda}\,\tilde{r}_c A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

⇒ Universal short-distance coefficients: $C^{L,R} = C_9^{\rm eff} + \kappa \frac{2 m_b M_B}{q^2} C_7^{\rm eff} \mp C_{10}$

(SM: $C_9 \sim +4$, $C_{10} \sim -4$, $C_7 \sim -0.3$)

known structure of sub-leading corrections [Grinstein/Pirjol hep-ph/0404250]

$$\tilde{r}_i \sim \pm \frac{\Lambda_{\rm QCD}}{m_b} \left(C_7^{\rm eff} + \alpha_s(\mu) e^{i\delta_i}\right)\,, \qquad i = a, b, c$$

form factors ("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K*}} V, \qquad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K*})\,A_1, \qquad f_0 = \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

Angular observables @ Low Recoil using FF relations

[CB/Hiller/van Dyk arXiv:1006.5013]

$$\frac{4}{3}(2\,J_{2s} + J_3) = 2\,\rho_1\,f_\perp^2, \qquad -\frac{4}{3}J_{2c} = 2\,\rho_1\,f_0^2, \qquad \frac{2\sqrt{2}}{3}J_5 = 4\,\rho_2\,f_0 f_\perp,$$

$$\frac{4}{3}(2\,J_{2s} - J_3) = 2\,\rho_1\,f_\parallel^2, \qquad \frac{4\sqrt{2}}{3}\,J_4 = 2\,\rho_1\,f_0 f_\parallel, \qquad \frac{2}{3}J_{6s} = 4\,\rho_2\,f_\parallel f_\perp,$$

$$J_7 = J_8 = J_9 = 0, \qquad f_{\perp,\parallel,0} = \text{form factors}$$

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$$J_7 = J_8 = J_9 = 0,$$

$f_{\perp,\parallel,0}$ = form factors

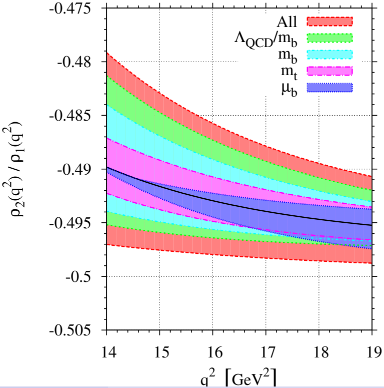
ρ_1 and ρ_2 are largely μ_b-scale independent

$$\rho_1(q^2) \equiv \left| C_9^{\rm eff} + \kappa \frac{2m_b^2}{q^2} C_7^{\rm eff} \right|^2 + |C_{10}|^2,$$

$$\rho_2(q^2) \equiv \mathrm{Re}\left[\left(C_9^{\rm eff} + \kappa \frac{2m_b^2}{q^2} C_7^{\rm eff}\right) C_{10}^*\right]$$

$\kappa(\mu_b)$ radiative QCD-correction to matching of FF relations between QCD and HQET

$\Rightarrow$ accounts for μ_b-dependence of tensor form factors $T_{1,2,3}$



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$$J_7 = J_8 = J_9 = 0, \qquad f_{\perp,\parallel,0} = \text{form factors}$$

$$\frac{d\Gamma}{dq^2} = 2\,\rho_1 \times (f_0^2 + f_\perp^2 + f_\parallel^2), \qquad A_{\rm FB} = 3\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp f_\parallel}{(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

$$F_L = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}, \qquad A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}, \qquad A_T^{(3)} = \frac{f_\parallel}{f_\perp}, \qquad A_T^{(4)} = 2\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp}{f_\parallel}$$

at low recoil: F_L, $A_T^{(2)}$, $A_T^{(3)}$ are short-distance independent, contrary to large recoil

$\Rightarrow$ could be used to fit form factor shape

Angular observables @ Low Recoil using FF relations [CB/Hiller/van Dyk arXiv:1006.5013]

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$$J_7 = J_8 = J_9 = 0,$$

$f_{\perp,\parallel,0}$ = form factors

$$\frac{d\Gamma}{dq^2} = 2\,\rho_1 \times (f_0^2 + f_\perp^2 + f_\parallel^2), \qquad A_{\rm FB} = 3\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp f_\parallel}{(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

$$F_L = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}, \qquad A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}, \qquad A_T^{(3)} = \frac{f_\parallel}{f_\perp}, \qquad A_T^{(4)} = 2\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp}{f_\parallel}$$

at low recoil: F_L, $A_T^{(2)}$, $A_T^{(3)}$ are short-distance independent, contrary to large recoil

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All relations valid up to sub-leading corrections in $C_7/C_9 \times \Lambda_{\rm QCD}/m_b$ due to FF relations.
(Later: OPE of 4-quark contributions yield also additional $(\Lambda_{\rm QCD}/m_b)^2$)