Impact of $B \to K_0^* \ell^+ \ell^-$ on the New Physics search in $B \to K^* \ell^+ \ell^-$ decay

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Outline

- A problem is often ignored in the literature is the contamination of the angular distribution of $B \to K^*(\to K\pi)\ell^+\ell^-$ by the events coming from $B \to K_0^*(\to K\pi)\ell^+\ell^-$. This effect was recently studied in the experimental analysis of e.g. $D \to K^*\mu\bar{\nu}_\mu$, and was shown to be important.
- For the transverse asymmetries this is not a problem because the product of the $K_0^* \to K\pi$ decay is in its S-wave and cannot make any impact on the $B \to K^*(\to K\pi)\ell^+\ell^-$ transverse amplitudes.
- However, in the extraction of transverse asymmetries from the angular distributions the unwanted $(K\pi)_S$ from $B \to K_0^*\ell^+\ell^-$ are troublesome and result in an error that is q^2-dependent and can be large.

Full angular distribution

Full angular distributions in $B \to (K\pi)_{K^*+K_0^*}\ell^+\ell^-$ is given as [Becirevic&AT('12), arXiv:1207.4004]:

$$\frac{d^5\Gamma}{dq^2 dm_{K\pi}^2 d\cos\theta_\ell d\cos\theta_K d\phi} =$$

$$J_1^c(q^2, m_{K\pi}^2, \theta_K) + 2J_1^s(q^2, m_{K\pi}^2, \theta_K)$$

$$+ [J_2^c(q^2, m_{K\pi}^2, \theta_K) + 2J_2^s(q^2, m_{K\pi}^2, \theta_K)] \cos 2\theta_\ell$$

$$+ 2J_3(q^2, m_{K\pi}^2, \theta_K) \sin^2\theta_\ell \cos 2\phi$$

$$+ 2\sqrt{2}J_4(q^2, m_{K\pi}^2, \theta_K) \sin(2\theta_\ell) \cos\phi$$

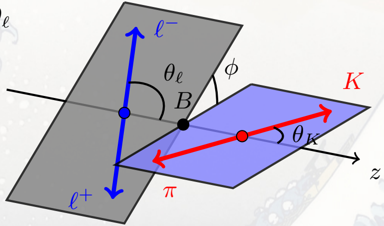
$$+ 2\sqrt{2}J_5(q^2, m_{K\pi}^2, \theta_K) \sin\theta_\ell \cos\phi$$

$$+ 2J_6(q^2, m_{K\pi}^2, \theta_K) \cos\theta_\ell$$

$$+ 2\sqrt{2}J_7(q^2, m_{K\pi}^2, \theta_K) \sin\theta_\ell \sin\phi$$

$$+ 2\sqrt{2}J_8(q^2, m_{K\pi}^2, \theta_K) \sin 2\theta_\ell \sin\phi$$

$$+ 2J_9(q^2, m_{K\pi}^2, \theta_K) \sin^2\theta_\ell \sin 2\phi$$



$J^{(s)}_{1,2,3,6,9}(q^2, m_{K\pi}^2, \theta_K) \propto \mathcal{I}^{(s)}_{1,2,3,6,9}(q^2) \sin^2\theta_K$ - are the functions of $\mathcal{M}_{\|,\perp}$ only $\Rightarrow$ NO $(K\pi)_S$ contribution from $B \to K_0^*\ell^+\ell^-$ + have small hadronic uncertainties !

In ref. [Matias('12), arXiv:1209.1525] it was demonstrated that using the *full* folded distributions one can avoid the problem of the S-wave contribution and extract the clean observables,

$$P_1(q^2) = \frac{\mathcal{I}_3(q^2)}{2\mathcal{I}_2^s(q^2)} \quad \Leftrightarrow \mathcal{A}_T^{(2)}(q^2)$$

$$P_2(q^2) = \beta_\ell \frac{\mathcal{I}_6(q^2)}{8\mathcal{I}_2^s(q^2)} \quad \Leftrightarrow \mathcal{A}_T^{(\mathrm{re})}(q^2)$$

$$P_3(q^2) = -\frac{\mathcal{I}_9(q^2)}{4\mathcal{I}_2^s(q^2)} \quad \Leftrightarrow \mathcal{A}_T^{(\mathrm{im})}(q^2)$$

in a way completely free from this pollution and in the exact lepton mass case.

- The folding exploits the angular symmetries of the distribution and reduce the number of coefficients. E.g. the initial number of $\mathcal{I}_i$'s, $10(K^*) + 8(K_0^*)$, can be reduced to $7(K^*) + 4(K_0^*)$ when using the "folded" angle $\hat{\phi} \in [0, \pi]$ ($\phi \leftrightarrow \phi + \pi$ when $\phi < 0$).

It is claimed that the deviation between the exact massive and massless predictions for the observables, integrated within $q^2 \in [1, 6]$ GeV^2, is small [Matias('12), arXiv:1209.1525].

We keep $m_\ell \neq 0$ since the lowest bins are the least ambiguous to test the NP. Moreover, in order to consistently combine $B \to K^* e^+ e^-$ and $B \to K^* \mu^+ \mu^-$ decays at low q^2, the lepton mass effect should be taken into account.

$$\mathcal{A}_T^{(2)}(q^2) = \frac{4\mathcal{I}_3(q^2)}{3\mathcal{I}_1^s(q^2) - \mathcal{I}_2^s(q^2)} \xrightarrow[m_\ell \to 0]{} \frac{\mathcal{I}_3(q^2)}{2\mathcal{I}_2^s(q^2)} \xrightarrow[q^2 \to 0]{} \frac{2\mathcal{R}e[C_{7\gamma} C_{7\gamma}'^*]}{|C_{7\gamma}|^2 + |C_{7\gamma}'|^2}$$

$$\mathcal{A}_T^{(\mathrm{im})}(q^2) = \frac{4\mathcal{I}_9(q^2)}{3\mathcal{I}_1^s(q^2) - \mathcal{I}_2^s(q^2)} \xrightarrow[m_\ell \to 0]{} \frac{\mathcal{I}_9(q^2)}{2\mathcal{I}_2^s(q^2)} \xrightarrow[q^2 \to 0]{} \frac{2\mathcal{I}m[C_{7\gamma} C_{7\gamma}'^*]}{|C_{7\gamma}|^2 + |C_{7\gamma}'|^2}$$

$$\mathcal{A}_T^{(\mathrm{re})}(q^2) = \frac{\beta_\ell \mathcal{I}_6(q^2)}{3\mathcal{I}_1^s(q^2) - \mathcal{I}_2^s(q^2)} \xrightarrow[m_\ell \to 0]{} \frac{\mathcal{I}_6(q^2)}{8\mathcal{I}_2^s(q^2)} \Longleftrightarrow \mathcal{A}_{\mathrm{FB}}(q^2)$$

- A common denominator is chosen for convenience.
- **If the data sample is so large** that all the coefficient functions $\mathcal{I}_i^{(s)}(q^2)$ can be reliably extracted from the full angular distribution, one can get the denominator unaffected by $(K\pi)_S$.
- If we study the distributions in ϕ, θ_ℓ and θ_K **separately**, then the denominator cannot be extracted without picking up the events coming from $B \to K_0^* \ell^+ \ell^-$.

How to extract the numerators of $\mathcal{A}_T^{(2,\mathrm{im},\mathrm{re})}(q^2)$?

The numerators in $\mathcal{A}_T^{(2,\mathrm{im},\mathrm{re})}(q^2)$ are *not* plagued by $B \to K_0^* \ell^+ \ell^-$ and can be extracted from the ϕ and θ_ℓ distributions,

$$\frac{d^2\Gamma}{dq^2 d\phi} = a_\phi(q^2) + b_\phi^c(q^2)\cos\phi + b_\phi^s(q^2)\sin\phi + \boxed{c_\phi^c(q^2)\cos 2\phi} + \boxed{c_\phi^s(q^2)\sin 2\phi}$$

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = a_{\theta_\ell}(q^2) + \boxed{b_{\theta_\ell}(q^2)\cos\theta_\ell} + c_{\theta_\ell}(q^2)\cos^2\theta_\ell$$

where the coefficients of our interest must be identified as

$$c_\phi^{c,s}(q^2) = \frac{4}{3\pi}\mathcal{I}_{3,9}(q^2)\int |BW_{K^*}|^2 dm_{K\pi}^2$$

$$b_{\theta_\ell}(q^2) = 2\mathcal{I}_6(q^2)\int |BW_{K^*}|^2 dm_{K\pi}^2$$

The other coefficients contain the K_0^* contribution as well as the longitudinal and "time-like" amplitudes of K^*, which involves the $B \to K^*$ form factors $A_{2,0}(q^2)$ and $T_3(q^2)$ which have *large uncertainties*.

How to extract the denominator of $\mathcal{A}_T^{(2,\mathrm{im},\mathrm{re})}(q^2)$?

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_K} = \boxed{a_{\theta_K}(q^2)} + b_{\theta_K}(q^2)\cos\theta_K + c_{\theta_K}(q^2)\cos^2\theta_K$$

with

$$a_{\theta_K}(q^2) = \frac{1}{8}\Big\{\left[3\mathcal{I}_1^{c\prime}(q^2) - \mathcal{I}_2^{c\prime}(q^2)\right]\int |BW_{K_0^*}|^2 dm_{K\pi}^2 + 3\left[3\mathcal{I}_1^{s}(q^2) - \mathcal{I}_2^{s}(q^2)\right]\int |BW_{K^*}|^2 dm_{K\pi}^2\Big\}$$

$$b_{\theta_K}(q^2) = \frac{\sqrt{3}}{4}\int \mathcal{R}e\left[\left(3\mathcal{I}_1^{c\prime\prime}(q^2) - \mathcal{I}_2^{c\prime\prime}(q^2)\right)BW_{K_0^*}BW_{K^*}^\dagger\right]dm_{K\pi}^2$$

$$c_{\theta_K}(q^2) = \frac{3}{8}\Big\{3\mathcal{I}_1^{c}(q^2) - \mathcal{I}_2^{c}(q^2) - \left[3\mathcal{I}_1^{s}(q^2) - \mathcal{I}_2^{s}(q^2)\right]\Big\}\int |BW_{K^*}|^2 dm_{K\pi}^2$$

Studying the **separate** distributions one cannot extract $3\mathcal{I}_1^s(q^2) - \mathcal{I}_2^s(q^2)$ and avoid the contribution from $B \to K_0^*\ell^+\ell^-$.

$$a_{\theta_K}(q^2) = \frac{1}{8}\Big\{ \big[3\mathcal{I}_1^{c\,\prime}(q^2) - \mathcal{I}_2^{c\,\prime}(q^2)\big] \int |BW_{K_0^*}|^2 dm^2_{K\pi}$$

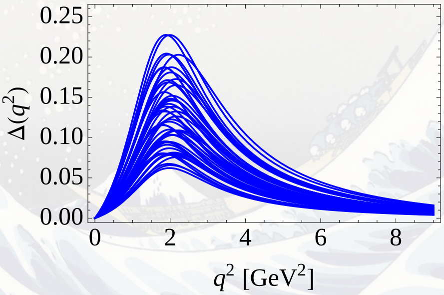
$$+ 3\big[3\mathcal{I}_1^{s}(q^2) - \mathcal{I}_2^{s}(q^2)\big] \int |BW_{K^*}|^2 dm^2_{K\pi} \Big\}$$

$$= \frac{3}{8}\big[3\mathcal{I}_1^{s}(q^2) - \mathcal{I}_2^{s}(q^2)\big](1 + \Delta(q^2)) \int |BW_{K^*}|^2 dm^2_{K\pi}$$

Note that $\Delta(q^2)$ is not suppressed by factor m_ℓ^2/q^2.

Around $q^2 \approx 2$ GeV^2 as many as 25% events, recognized as $a_{\theta_K}(q^2)$ of $B \to K^*\ell^+\ell^-$, might be coming from $B \to K_0^*\ell^+\ell^-$ decay.

[Becirevic&AT('12), arXiv:1207.4004]



Similar situation occurs in $B_s \to \phi(\to K^+K^-)\ell^+\ell^-$ except that the effect of $B_s \to f_0(\to K^+K^-)\ell^+\ell^-$ is smaller: it remains under 15% around $q^2 \approx 2.5$ GeV^2.

$\mathcal{I}_2^s$ extraction from the uniangular distributions in the $m_\ell = 0$ limit

However, as was pointed out in ref. [Matias('12), arXiv:1209.1525], in the massless limit, $\mathcal{I}_2^s(q^2)$ can be determined from the combination of

$$a_\phi(q^2) = \frac{1}{3\pi}\Big\{\left[3\mathcal{I}_1^c{}'(q^2) - \mathcal{I}_2^c{}'(q^2)\right]\int |BW_{K_0^*}|^2 dm_{K\pi}^2$$

$$+ \left[3\mathcal{I}_1^c(q^2) - \mathcal{I}_2^c(q^2) + 6\mathcal{I}_1^s(q^2) - 2\mathcal{I}_2^s(q^2)\right]\int |BW_{K^*}|^2 dm_{K\pi}^2\Big\}$$

$$\simeq \frac{4}{3\pi}\left\{\left[4\mathcal{I}_2^s(q^2) - \mathcal{I}_2^c(q^2)\right]\int |BW_{K^*}|^2 dm_{K\pi}^2 - \mathcal{I}_2^c{}'(q^2)\int |BW_{K_0^*}|^2 dm_{K\pi}^2\right\}$$

$$a_{\theta_\ell}(q^2) = \left[\mathcal{I}_1^c{}'(q^2) - \mathcal{I}_2^c{}'(q^2)\right]\int |BW_{K_0^*}|^2 dm_{K\pi}^2$$

$$+ \left[\mathcal{I}_1^c(q^2) - \mathcal{I}_2^c(q^2) + 2\mathcal{I}_1^s(q^2) - 2\mathcal{I}_2^s(q^2)\right]\int |BW_{K^*}|^2 dm_{K\pi}^2$$

$$\simeq 2\left\{\left[2\mathcal{I}_2^s(q^2) - \mathcal{I}_2^c(q^2)\right]\int |BW_{K^*}|^2 dm_{K\pi}^2 - \mathcal{I}_2^c{}'(q^2)\int |BW_{K_0^*}|^2 dm_{K\pi}^2\right\}$$

$$\Longrightarrow \boxed{\mathcal{I}_2^s(q^2) \simeq \frac{1}{\int |BW_{K^*}|^2 dm_{K\pi}^2}\left[\pi a_\phi(q^2) - \frac{2}{3}a_{\theta_\ell}(q^2)\right] + \mathcal{O}(m_\ell^2/q^2)}$$

NB: The above assumption $m_\ell \to 0$ requires to abandon the principle of the transversity which then requires the knowledge of the $A_{0,2}(q^2)$ and $T_3(q^2)$. Corrections $\propto m_\ell^2/q^2$ are NOT negligible and *might be* problematic for $1 < q^2 < 3$ GeV^2 where $\mathcal{I}_2^s(q^2)$ has minimum.

Scalar contribution to the integrated decay rate

Integrated over 3 angles and $m^2_{K\pi}$, the distribution can be written as

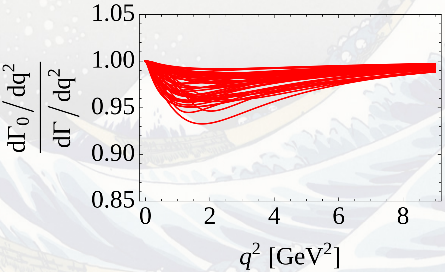
$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_0}{dq^2} + \frac{d\Gamma_S}{dq^2}$$

$$\frac{d\Gamma_0}{dq^2} = \frac{1}{4}\left[3\mathcal{I}_1^c(q^2) - \mathcal{I}_2^c(q^2) + 2(3\mathcal{I}_1^s(q^2) - \mathcal{I}_2^s(q^2))\right] \int_{(m_{K^*}-\delta)^2}^{(m_{K^*}+\delta)^2} |BW_{K^*}|^2 dm^2_{K\pi}$$

$$\frac{d\Gamma_S}{dq^2} = \frac{1}{4}\left[3\mathcal{I}_1^{c\prime}(q^2) - \mathcal{I}_2^{c\prime}(q^2)\right] \int_{(m_{K^*}-\delta)^2}^{(m_{K^*}+\delta)^2} |BW_{K_0^*}|^2 dm^2_{K\pi}$$

Using $\delta^{\exp} \simeq 100$ MeV, the inclusion of $K\pi$ from K_0^* amounts to at most 10% excess with respect to the desired $d\Gamma_0/dq^2$.

[Becirevic&AT('12), arXiv:1207.4004]



Conclusions and perspectives

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5. At large $q^2 \gtrsim 14$ GeV^2, instead, the effect of $B \to K_0^*\ell^+\ell^-$ and $B_s \to f_0(980)\ell^+\ell^-$ on $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ respectively, is completely negligible.

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6. This uncertainty, together with the one related to the charm loop, and the controllable uncertainties on the ratios of the $B \to K^*$ form factors, suggests that the overall error on $\mathcal{A}_T^{(2,\mathrm{im},\mathrm{re})}(q^2)$ is under about 30%, and therefore at that level of accuracy the measurement of $\mathcal{A}_T^{(2,\mathrm{im},\mathrm{re})}(q^2)$ remains a good tool for detecting the NP signal.

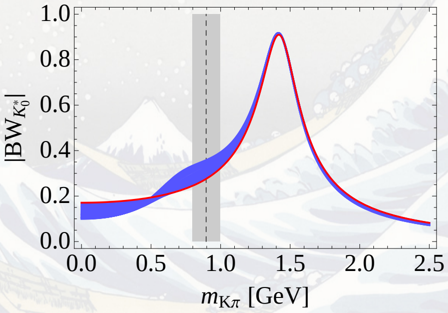
BACKUP SLIDES

$$BW_{K_0^*}(m_{K\pi}^2) = \mathcal{N}\left[\frac{g_\kappa}{m_{K\pi}^2 - (m_\kappa - i\Gamma_\kappa/2)^2} - \frac{1}{m_{K\pi}^2 - (m_{K_0^*} - i\Gamma_{K_0^*}/2)^2}\right]$$

where $m_\kappa = 658(13)$ MeV, $\Gamma_\kappa = 557(24)$ MeV, that was identified as a pole in the amplitude of the $K\pi \to K\pi$ scattering [Descotes-Genon&Moussallam, hep-ph/0607133]. We vary $g_\kappa \in [0,\ 0.2]$. The constant $\mathcal{N}$ is obtained from the normalization to unity,

$$\int_{-\infty}^{\infty} |BW_{K_0^*}(m_{K\pi}^2)|^2 dm_{K\pi}^2 = 1$$

We checked that in the region of $m_{K\pi} \in [m_{K^*} - \delta, m_{K^*} + \delta]$, with $\delta \approx 100$ MeV, our $BW_{K_0^*}(m_{K\pi}^2)$ reproduces the shapes of the corresponding $K\pi$ form factors.



$\mathcal{I}_i(q^2)$ functions

$$\mathcal{I}_1^s(q^2) = \frac{2+\beta_\ell^2}{4}\left[|\mathcal{M}_\perp^{\ell_L}|^2 + |\mathcal{M}_\perp^{\ell_R}|^2 + |\mathcal{M}_\parallel^{\ell_L}|^2 + |\mathcal{M}_\parallel^{\ell_R}|^2\right] + \frac{4m_\ell^2}{q^2}\mathcal{R}e\left[\mathcal{M}_\perp^{\ell_L}\mathcal{M}_\perp^{\ell_R *} + \mathcal{M}_\parallel^{\ell_L}\mathcal{M}_\parallel^{\ell_R *}\right]$$

$$\mathcal{I}_2^s(q^2) = \frac{\beta_\ell^2}{4}\left[|\mathcal{M}_\perp^{\ell_L}|^2 + |\mathcal{M}_\perp^{\ell_R}|^2 + |\mathcal{M}_\parallel^{\ell_L}|^2 + |\mathcal{M}_\parallel^{\ell_R}|^2\right]$$

$$\mathcal{I}_3(q^2) = \frac{\beta_\ell^2}{2}\left[|\mathcal{M}_\perp^{\ell_L}|^2 + |\mathcal{M}_\perp^{\ell_R}|^2 - |\mathcal{M}_\parallel^{\ell_L}|^2 - |\mathcal{M}_\parallel^{\ell_R}|^2\right]$$

$$\mathcal{I}_6^s(q^2) = 2\beta_\ell\mathcal{R}e\left[\mathcal{M}_\parallel^{\ell_L}\mathcal{M}_\perp^{\ell_L *} - \mathcal{M}_\parallel^{\ell_R}\mathcal{M}_\perp^{\ell_R *}\right]$$

$$\mathcal{I}_9(q^2) = \beta_\ell^2\mathcal{I}m\left[\mathcal{M}_\perp^{\ell_L}\mathcal{M}_\parallel^{\ell_L *} + \mathcal{M}_\perp^{\ell_R}\mathcal{M}_\parallel^{\ell_R *}\right]$$

$$\mathcal{I}_1^{c(\prime)}(q^2) = |\mathcal{M}_0^{\ell_L(\prime)}|^2 + |\mathcal{M}_0^{\ell_R(\prime)}|^2 + \frac{4m_\ell^2}{q^2}\left(|\mathcal{M}_t^{(\prime)}|^2 + 2\mathcal{R}e\left[\mathcal{M}_0^{\ell_L(\prime)}\mathcal{M}_0^{\ell_R(\prime)*}\right]\right)$$

$$\mathcal{I}_2^{c(\prime)}(q^2) = -\beta_\ell^2\left[|\mathcal{M}_0^{\ell_L(\prime)}|^2 + |\mathcal{M}_0^{\ell_R(\prime)}|^2\right]$$

($\mathcal{M}_i$ and $\mathcal{M}_i'$ denote respectively the spin amplitudes of $B \to K^*\ell^+\ell^-$ and $B \to K_0^*\ell^+\ell^-$)

Spin amplitudes of $B \to K^* \ell^+ \ell^-$

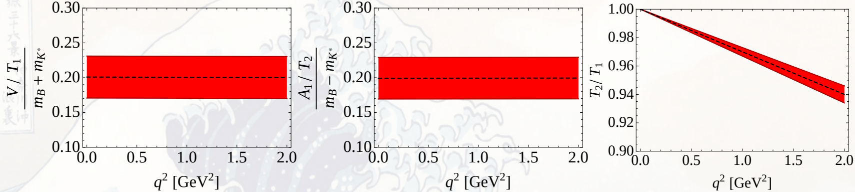
$$\mathcal{M}_{\perp}^{\ell_{L,R}}(q^2) = N(q^2)\sqrt{2\lambda(q^2)}\left\{\frac{2m_b}{q^2}(C_{7\gamma} + C'_{7\gamma})T_1(q^2)\right.$$

$$\left. + \left[(C_9 + C_9) \mp (C_{10} + C'_{10})\right]\frac{V(q^2)}{m_B + m_{K^*}}\right\}$$

$$\mathcal{M}_{\parallel}^{\ell_{L,R}}(q^2) = -N(q^2)\sqrt{2}(m_B^2 - m_{K^*}^2)\left\{\frac{2m_b}{q^2}(C_{7\gamma} - C'_{7\gamma})T_2(q^2)\right.$$

$$\left. + \left[(C_9 - C'_9) \mp (C_{10} - C'_{10})\right]\frac{A_1(q^2)}{m_B - m_{K^*}}\right\}$$

- The advantage of using the quantities that include only $\mathcal{M}_{\parallel,\perp}$ is that they do not require a detailed knowledge of hadronic form factors $T_3(q^2)$ and $A_{2,0}(q^2)$ which are quite hard to compute using the lattice QCD simulations.
- Moreover, the ratios $A_1(q^2)/T_2(q^2)$ and $V(q^2)/T_1(q^2)$ are flat in the low q^2-region which makes the relevant hadronic uncertainties to be better controlled.



The ratios of the form factors that have similar q^2-behavior in the heavy quark limit and in the limit of large energy of K^*, are kept as constants, namely

$$\frac{V(q^2)/T_1(q^2)}{m_B + m_{K^*}} \approx \frac{A_1(q^2)/T_2(q^2)}{m_B - m_{K^*}} \approx 0.2\ \mathrm{GeV}^{-1}$$

which in practice we vary between $(0.17 \div 0.23)\ \mathrm{GeV}^{-1}$. For the ratio of the tensor form factors we use the approximation

$$\frac{T_2(q^2)}{T_1(q^2)} \approx 1 + zq^2$$

with $z = -0.030(3)$.

[Ball&Zwicky, Phys.Rev.D71('05), Becirevic et al., Nucl.Phys.B769('07), Colangelo et al.('96), arXiv:9510403]