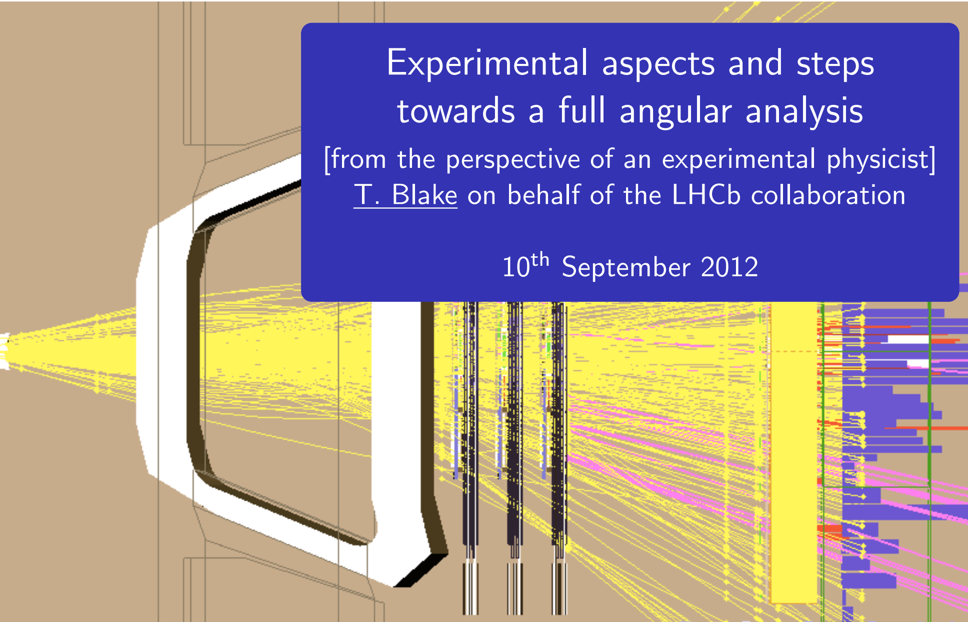
Experimental aspects and steps towards a full angular analysis

[from the perspective of an experimental physicist]

T. Blake on behalf of the LHCb collaboration

10th September 2012



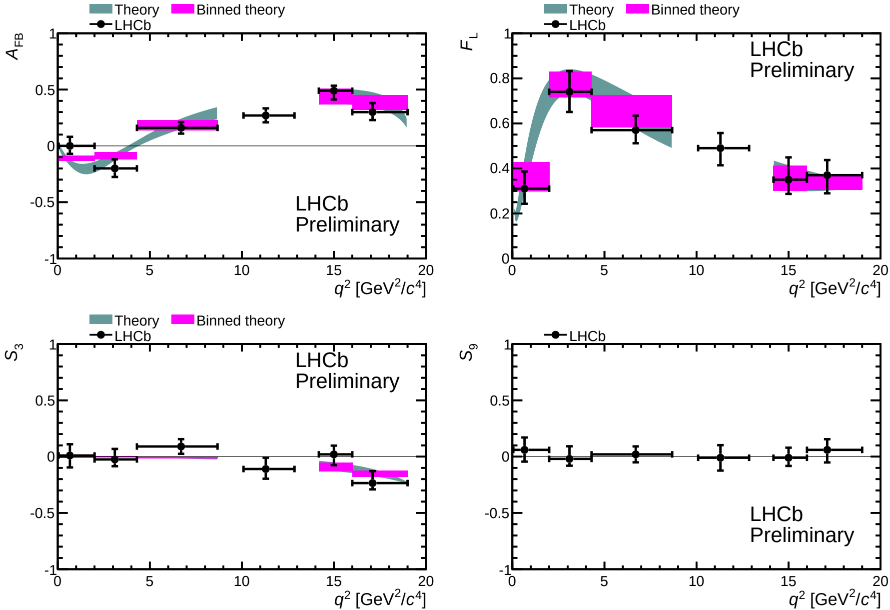
~~Outline~~ Things that aren't in this talk

I won't talk at all about:

- $B^0 \to K^{*0}\ell\ell$ at low-q^2 (and $B^0 \to K^{*0}e^-e^-$) $\to$ Marie-Helene's talk.
- $\Lambda_b \to pK\mu^+\mu^-$ and $\Lambda_b \to \Lambda^0\mu^+\mu^-$ $\to$ Michal's talk.
- Isospin analysis $\to$ Ulrik's talk.
- $B^+ \to K^+\mu^+\mu^-$, $B^+ \to \pi^+\mu^+\mu^-$, $B^0 \to \rho\mu^+\mu^-$ etc.

I won't even talk much about LHCb's $B^0 \to K^{*0}\mu^+\mu^-$ results.
Instead focus on:

- Experimental issues that appear in the $B^0 \to K^{*0}\mu^+\mu^-$ angular analysis.
- Steps towards a full angular analysis.



LHCb $1\,\mathrm{fb}^{-1}$ [LHCb-CONF-2012-008]

Angular definition for $B^0 \to K^{*0}\mu^+\mu^-$

- The decay $B^0 \to K^{*0}\mu^+\mu^-$ ($K^{*0} \to K^+\pi^-$) is self-tagging. The angular definition becomes important.
- The angular convention used by LHCb is **not** the same as the theory convention:
  - It uses the same definition for $\cos\theta_\ell$ and $\cos\theta_K$ as BaBar, Belle and CDF.
  - It has a different definition of the ϕ angle from CDF (S_9 vs A_9).
- The angular basis is defined from the B^0 and the basis for the $\overline{B}^0$ obtained using CP.
  - It is given explicitly in the following slides.

Angular definition: θ_ℓ

- For the B^0 ($\overline{B}^0$) decay, θ_ℓ is defined by the angle between the μ^+ (μ^-) in the dimuon rest frame and the dimuon in the B^0 ($\overline{B}^0$) rest frame.
- Equivalently, the angle between the μ^+ (μ^-) and the direction opposite that of the B^0 ($\overline{B}^0$) in the dimuon rest frame:

$$
\begin{aligned}
\cos\theta_\ell &= \frac{\vec{p}^{\,\mu\mu}_{\mu^-} \cdot \vec{p}^{\,B}_{\mu\mu}}{|\vec{p}^{\,\mu\mu}_{\mu^-}||\vec{p}^{\,B}_{\mu\mu}|} \\
&= -\frac{\vec{p}^{\,\mu\mu}_{\mu^-} \cdot \vec{p}^{\,\mu\mu}_{B}}{|\vec{p}^{\,\mu\mu}_{\mu^-}||\vec{p}^{\,\mu\mu}_{B}|} = -\frac{\vec{p}^{\,\mu\mu}_{\mu^-} \cdot \vec{p}^{\,\mu\mu}_{K\pi}}{|\vec{p}^{\,\mu\mu}_{\mu^-}||\vec{p}^{\,\mu\mu}_{K\pi}|}.
\end{aligned}
$$

NB This has the opposite sign convention to the "theory basis" $\rightarrow$ a different sign convention for A_{FB}.

Angular definition: θ_K

- The angle θ_K is defined as the angle between the K^+ (K^-) in the K^{*0} ($\overline{K}^{*0}$) rest frame and the K^{*0} ($\overline{K}^{*0}$) in the B^0 ($\overline{B}^0$) rest frame.

$$\cos\theta_K = \frac{\vec{p}_{\mathrm{K}}^{K\pi}\cdot\vec{p}_{K\pi}^{B}}{|\vec{p}_{\mathrm{K}}^{K\pi}||\vec{p}_{K\pi}^{B}|} = -\frac{\vec{p}_{\mathrm{K}}^{K\pi}\cdot\vec{p}_{B}^{K\pi}}{|\vec{p}_{\mathrm{K}}^{K\pi}||\vec{p}_{B}^{K\pi}|} = -\frac{\vec{p}_{\mathrm{K}}^{K\pi}\cdot\vec{p}_{\mu\mu}^{K\pi}}{|\vec{p}_{\mathrm{K}}^{K\pi}||\vec{p}_{\mu\mu}^{K\pi}|}.$$

Angular definition: ϕ

- The angle ϕ is defined by the angle between the plane containing the $\mu^+\mu^-$ and the plane containing the $K^\pm\pi^\mp$ in the B^0 ($\bar{B}^0$) rest frame.
- For the B^0:

$$\begin{aligned} \cos\phi &= \vec{n}^B_{\mu^+\mu^-} \cdot \vec{n}^B_{\mathrm{K}^+\pi^-} \text{ and} \\ \sin\phi &= (\vec{n}^B_{\mu^+\mu^-} \times \vec{n}^B_{\mathrm{K}^+\pi^-}) \cdot \frac{\vec{p}^B_{K^+\pi^-}}{|\vec{p}^B_{K^+\pi^-}|}. \end{aligned}$$

- For the $\bar{B}^0$:

$$\begin{aligned} \cos\phi &= -\vec{n}^B_{\mu^+\mu^-} \cdot \vec{n}^B_{\mathrm{K}^-\pi^+} = \vec{n}^B_{\mu^-\mu^+} \cdot \vec{n}^B_{\mathrm{K}^-\pi^+} \text{ and} \\ \sin\phi &= (\vec{n}^B_{\mu^+\mu^-} \times \vec{n}^B_{\mathrm{K}^-\pi^+}) \cdot \frac{\vec{p}^B_{K^-\pi^+}}{|\vec{p}^B_{K^-\pi^+}|} = -(\vec{n}^B_{\mu^-\mu^+} \times \vec{n}^B_{\mathrm{K}^-\pi^+}) \cdot \frac{\vec{p}^B_{K^-\pi^+}}{|\vec{p}^B_{K^-\pi^+}|} \end{aligned}$$

NB the sign convention between the B^0 and $\bar{B}^0$ is important and arises from the CP transformation. The behaviour is different for $B^0_s \to \phi\mu^+\mu^-$ decays, where $\mathrm{C}(K^+) \to K^-$.

Angular definition: ϕ

NB We can measure A_9 in LHCb by swapping the sign of ϕ for $\overline{B}^0$ decays only (a la CDF).

where:

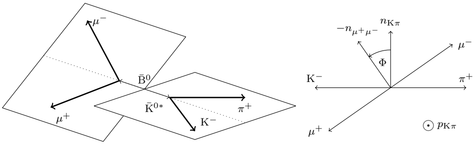
$$S_9 \propto \sin\delta_S \cos\delta_W$$

and

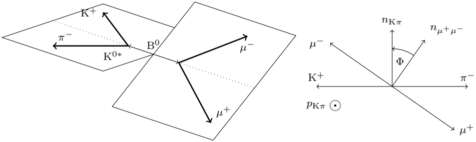
$$A_9 \propto \cos\delta_S \sin\delta_W$$

and δ_S is a strong phase difference and δ_W a weak phase difference.

ϕ definition:



(a) Φ for the $\overline{B}^0$ decay



(b) Φ for the B^0 decay

- For $B^0_s \to \phi\mu^+\mu^-$, $B^0_s \to \overline{B}^0_s$ corresponds to $\theta_\ell \to \pi - \theta_\ell$, $\theta_K \to \pi - \theta_K$ and $\phi \to -\phi$ in the angular distribution.

Angular convention and observables

- Following the convention of [Altmannshofer et. al. JHEP 01 (2009)], LHCb measures:

$$\frac{\mathrm{d}^4\Gamma[B^0 \to K^{*0}\mu^+\mu^-]}{\mathrm{d}q^2\mathrm{d}\cos\theta_\ell\mathrm{d}\cos\theta_K\mathrm{d}\phi} + \frac{\mathrm{d}^4\Gamma[\overline{B}^0 \to \overline{K}^{*0}\mu^+\mu^-]}{\mathrm{d}q^2\mathrm{d}\cos\theta_\ell\mathrm{d}\cos\theta_K\mathrm{d}\phi} = \frac{9}{32\pi}\sum_{i=0}^{9} S_i f_i(\cos\theta_\ell, \theta_K, \phi)$$

- Where:

$$S_i = \left(I[B^0 \to K^{*0}\mu^+\mu^-] + I[\overline{B}^0 \to \overline{K}^{*0}\mu^+\mu^-]\right)$$

- This only works in the absence of any production, detection or direct CP ($\Gamma[B^0 \to K^{*0}\mu^+\mu^-] \neq \Gamma[\overline{B}^0 \to \overline{K}^{*0}\mu^+\mu^-]$) asymmetry.
- In reality there is a small production asymmetry, $\mathcal{O}(1\%)$, between B^0 and $\overline{B}^0$ that mixes S_i with A_i and we are only sensitive to direct CP asymmetries at the level of $5-10\%$ (sets an upper limit on the mixing between A and S at 10%).

Simplifying the angular distribution

- With limited statistics (in individual q^2 bins) it is not yet possible to perform a full angular analysis of the $B^0 \to K^{*0}\mu^+\mu^-$ decay. Instead we try to reduce the number of parameters.
- Can either integrate over two of the angles to give:

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}q^2} \Big/ \frac{d\Gamma}{dq^2} = \frac{3}{4}F_L(1-\cos^2\theta_\ell) + \frac{3}{8}(1-F_L)(1+\cos^2\theta_\ell) + A_{FB}\cos\theta_\ell \quad ,$$

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}\cos\theta_K\,\mathrm{d}q^2} \Big/ \frac{d\Gamma}{dq^2} = \frac{3}{2}F_L\cos^2\theta_K + \frac{3}{4}(1-F_L)(1-\cos^2\theta_K)$$

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}\phi\,\mathrm{d}q^2} \Big/ \frac{d\Gamma}{dq^2} = \frac{1}{2\pi}\left[1 + S_3\cos 2\phi + A_{Im}\sin 2\phi\right]$$

NB Can replace $S_3 = \frac{1}{2}(1-F_L)A_T^2$ throughout these slides.

Simplifying the angular distribution

Or try to fold the angular distribution to remove the terms. For example "folding" ϕ such that:

$$\hat{\phi} = \begin{cases} \phi + \pi & \text{if } \phi < 0 \\ \phi & \text{otherwise} \end{cases}$$

- Giving:

$$\frac{\mathrm{d}^4\Gamma}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\hat{\phi}\,\mathrm{d}q^2} \Big/ \frac{d\Gamma}{dq^2} = \frac{9}{16\pi}\Big[F_L\cos^2\theta_K + \frac{3}{4}(1-F_L)(1-\cos^2\theta_K) - \\ F_L\cos^2\theta_K(2\cos^2\theta_\ell - 1) + \\ \frac{1}{4}(1-F_L)(1-\cos^2\theta_K)(2\cos^2\theta_\ell - 1) + \\ S_3(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\cos 2\hat{\phi} + \\ \frac{4}{3}A_{FB}(1-\cos^2\theta_K)\cos\theta_\ell + \\ S_9(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\sin 2\hat{\phi} \Big]$$

Simplifying the angular distribution

- Can also try other foldings to have access to the other terms in the angular expression.

e.g. for S_5:

$$\hat{\phi} = \begin{cases} -\phi & \text{if } \phi < 0 \\ \phi & \text{otherwise} \end{cases} \quad \text{and} \quad \hat{\theta}_\ell = \begin{cases} \pi - \theta_\ell & \text{if } \theta_\ell > \frac{\pi}{2} \\ \theta_\ell & \text{otherwise} \end{cases}$$

$$\frac{\mathrm{d}^4\Gamma}{\mathrm{d}\cos\hat{\theta}_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\hat{\phi}\,\mathrm{d}q^2} \Big/ \frac{d\Gamma}{dq^2} \propto \Big[\frac{3}{4}(1-F_L)(1-\cos^2\theta_K) + F_L\cos^2\theta_K$$

$$\frac{1}{4}(1-F_L)(1-\cos^2\theta_K)(2\cos^2\hat{\theta}_\ell - 1) \; -$$

$$F_L\cos^2\theta_K(2\cos^2\hat{\theta}_\ell - 1) \; +$$

$$S_3(1-\cos^2\theta_K)(1-\cos^2\hat{\theta}_\ell)\cos 2\hat{\phi} \; +$$

$$S_5(2\cos^2\theta_K - 1)\sin\hat{\theta}_\ell\cos\hat{\phi}\Big]$$

What do we mean by observables?

- Due to the limited statistics we perform measurements in wide q^2 regions, the observables that we measure are rate averages of the physics quantities over the q^2 region:

e.g.

$$< F_L >= \frac{\left[\int_{q^2_{\min}}^{q^2_{\max}} F_L(q^2)\frac{d\Gamma}{dq^2}dq^2\right]}{\left[\int_{q^2_{\min}}^{q^2_{\max}} \frac{d\Gamma}{dq^2}dq^2\right]}$$

- The situation gets more complicated if there are products of two (or more) q^2 dependent terms in the angular expression.
- In general:

$$< A_{\perp L}(q^2)A^*_{\parallel L}(q^2) > \neq < A_{\perp L}(q^2) >< A^*_{\parallel L}(q^2) >$$

unless one of the amplitudes is constant.

What do we mean by observables?

- This has implications for performing the full angular fit.
  - Do we need to take into account the q^2 dependence to get unbiased results?
  - What form should we use for the q^2 dependence?

NB For the transverse observables we can write:

$$< (1 - F_L(q^2))A_T^2(q^2) > = < (1 - F_L(q^2) > < \tilde{A}_T^2 >$$

where:

$$< \tilde{A}_T^2 > = \frac{\left[\int_{q^2_{\rm min}}^{q^2_{\rm max}} A_T^2 \frac{d\Gamma}{dq^2}(1 - F_L(q^2))dq^2\right]}{\left[\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\Gamma}{dq^2}(1 - F_L(q^2))dq^2\right]}$$

i.e. a transverse rate-average (which should again be a theoretically “clean” observable [Descotes-Genon et al arXiv:1207.2753]).

Can we neglect terms in $4m_\mu^2/q^2$?

- When writing down the angular expressions we have assumed that:

$$4m_\mu^2 \ll q^2$$

  This assumption clearly breaks down for $q^2 < 1\,\mathrm{GeV}^2/c^4$.

- Marie-Helene will go into more detail, but this has several consequences for the angular analysis and the interpretation of experimental results in the $4m_\mu^2 < q^2 < 2\,\mathrm{GeV}^2/c^4$ mass window.
- Importantly:

$$I_1^c \neq -I_2^c \quad \text{and} \quad I_1^s \neq 3I_2^s$$

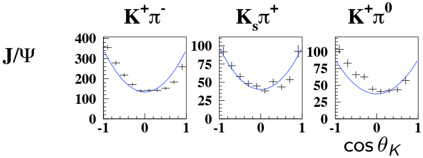
- In a SM simulation, the bias on the observables from neglecting these terms is $\mathcal{O}(20\%)$.

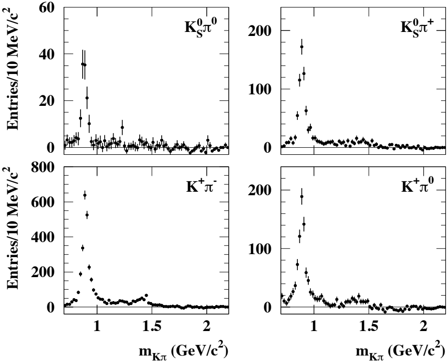
NB As $q^2 \to 4m_\mu^2$, the angular distribution becomes isotropic and sensitivity to the "physics" parameters is completely lost.

S-wave interference

- The angular distribution is based on a narrow width approximation of the K^{*0}. In reality we consider a $200\,\mathrm{MeV}/c^2$ $m_{K^+\pi^-}$ window.
- Can also have contributions from the tails of higher $K^+\pi^-$ resonances, e.g. $K_0^{*0}(1430)$, or non-resosant S-wave $K^+\pi^-$ (supported by data from LASS).
- Interference between a P- and S-wave $K^+\pi^-$ system is seen in $B^0 \to K^+\pi^- J/\psi$ data at the B-factories.

[BaBar PRD 76 (2007)]





[BaBar PRD 71 (2005)]

S-wave intereference

- Based on [Lü & Wang PRD 85 (2012)], introduce a S-wave $K^+\pi^-$ by replacing:

$$A^{J=1}_{0\,L,R}\cos\theta_K \rightarrow \frac{1}{\sqrt{3}}A^0_{0\,L,R} + A^1_{0\,L,R}\cos\theta_K$$

  - Should we expect $A^0_{0\,L,R}$ and $A^1_{0\,L,R}$ to have similar (the same) q^2 dependence?
  - Do we expect a large S-wave contribution where F_L is largest?

- To simplify the discussion, define:

$$F_S = \frac{|A^0_0|^2}{\Gamma_{\text{tot.}}} \quad \text{and} \quad A_{S\,L,R} \propto |A^0_{0\,L,R}||A^1_{0\,L,R}|\cos\delta_{SL,R}$$

- In $B^0 \rightarrow K^+\pi^- J/\psi$, $F_S \sim 7-8\%$ and the interference term is $\sim 20\%$.

S-wave interference

- The impact of the S-wave interference is felt in two places:
  1. It dilutes F_L, A_{FB}, A_{Im} and S_3.
  2. It introduces new terms into the angular expression (F_S and A_S).

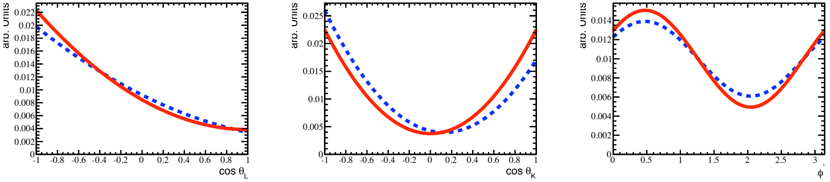
  Giving:

$$
\begin{aligned}
\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_\ell\, d\hat{\phi}} \Big/ \frac{d\Gamma}{dq^2} = \frac{9}{16\pi} \Big[& \frac{2}{3} F_S (1-\cos^2\theta_\ell) + \frac{4}{3} A_S \cos\theta_K (1-\cos^2\theta_\ell) \;+ \\
& 2(1-F_S) F_L \cos^2\theta_K (1-\cos^2\theta_\ell) \;+ \\
& \frac{1}{2}(1-F_S)(1-F_L)(1-\cos^2\theta_K)(1+\cos^2\theta_\ell) \;+ \\
& (1-F_S) S_3 (1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\cos 2\hat{\phi} \;+ \\
& \frac{4}{3}(1-F_S) A_{FB} (1-\cos^2\theta_K)\cos\theta_\ell \;+ \\
& (1-F_S) A_{Im} (1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\sin 2\hat{\phi} \Big] .
\end{aligned}
$$

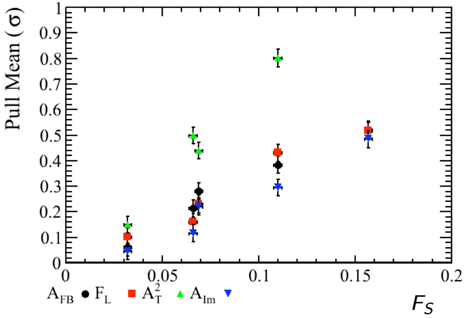
  it will also modify the angular distribution associated to I_4, I_5, I_7 and I_8 in a full angular analysis.

S-wave interference

$A_{FB} = -0.18$, $F_L = 0.66$, $A_T^2 = 0.3$, $A_{Im} = 0.07$ and $F_S = 0.07$ with 20% interference term.



- With current statistical precision an 8% S-wave contribution gives a significant experimental bias.
- We do not yet have enough statistics to directly measure F_S in the data in a 200 MeV/c^2 mass window.



Fit stability and clean observables from a practical perspective

- When we perform a likelihood fit to the distribution of events we see in the data we are sensitive to the relative contribution from the different $f_i(\cos\theta_\ell, \cos\theta_K, \phi)$ terms in:

$$\frac{\mathrm{d}^4\Gamma[B^0 \to K^{*0}\mu^+\mu^-]}{\mathrm{d}q^2\mathrm{d}\cos\theta_\ell\mathrm{d}\cos\theta_K\mathrm{d}\phi} + \frac{\mathrm{d}^4\Gamma[\overline{B}^0 \to \overline{K}^{*0}\mu^+\mu^-]}{\mathrm{d}q^2\mathrm{d}\cos\theta_\ell\mathrm{d}\cos\theta_K\mathrm{d}\phi} = \frac{9}{32\pi}\sum_{i=0}^{9} S_i f_i(\cos\theta_\ell, \theta_K, \phi)$$

  $\rightarrow$ From an experimental perspective, S_i are a natural choice of observable.

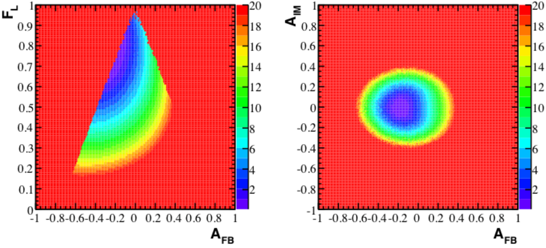
- The transverse variables:
  - A_T^2 [Krüger & Mattias PRD 71 (2005)]
  - A_T^{Re} and A_T^{Im} [Becirevic & Schneider arXiv:1106.3283]

  appear in combination with $1 - F_L$ and can be correlated to F_L in the fit. Moreover they become ill defined with $F_L \rightarrow 1$.

- The transverse variables do however have some nice experimental benefits . . .

Boundaries in parameter space

- We talk about F_L, S_3 (A_T^2), A_{FB} (A_T^{Re}) and A_{Im} (A_T^{Im}) being observables, but in reality they are not independent. They depend on the same transversity amplitudes.



e.g. For A_{FB} to be large, $A_{\parallel}$ and $A_{\perp}$ must be large compared to $A_0 \rightarrow F_L$ must be small.

- For the angular distribution to remain +ve definite:

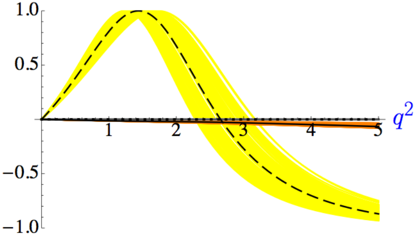
$$A_{FB} \leq \frac{3}{4}(1 - F_L) \quad , \quad A_{Im} \leq \frac{1}{2}(1 - F_L) \;\; \text{and} \;\; S_3 \leq \frac{1}{2}(1 - F_L)$$

NB these are the same transformations needed to go to the transverse variables (A_T^{Re}, A_T^{Im} and A_T^2).

A_T^{Re} as an illustration of boundary effects

- In the SM A_T^2, S_9 and $A_9 \sim 0$ and boundary issues in the fit are limited to $A_{FB} : F_L$.

- To illustrate possible problems look at A_T^{Re} [Becirevic & Schneider arXiv:1106.3283].
- In the SM A_T^{Re} becomes maximal around $1 - 2\,\mathrm{GeV}^2/c^4$ and again at higher q^2.



$\therefore$ SM favours values of A_{FB} that sit on the edge of the allowed parameter space where the numerical approaches adopted by `MINUIT` are least stable.

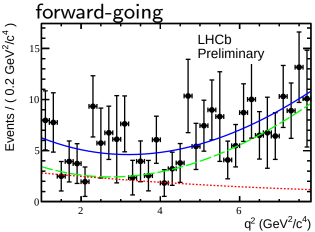
The meaning of experimental errors

- The confidence intervals quoted on the angular observables are constructed treating the other angular observables as nuisance parameters.
- What does this mean in practice?
  - It tells you, for example, that A_{FB} is in the range $XX < A_{FB} < YY$ at 68% confidence level without making any inference on the value of F_L.
- The intervals may not and in practice are not simultaneously valid at this confidence level.
  - To put it more simply, due to correlations and boundary effects, you can not recreate the full likelihood from the confidence intervals on the individual parameters.
- The correlations are important for global fits, so how should we make this information available?
  - It's complicated now with 4 parameters, but will get much worse when we move to a full angular analysis.

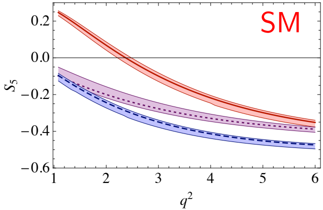
Zero crossing point of A_{FB}

- The LHCb zero-crossing measurement does not come from an angular analysis. Instead we fit the q^2 distribution of forward and backward going events.

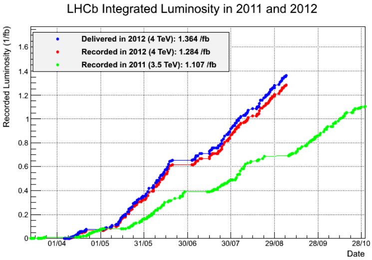
$$A_{FB} \quad q_0^2 = 4.9^{+1.1}_{-1.3}\,\mathrm{GeV}^2/c^4$$



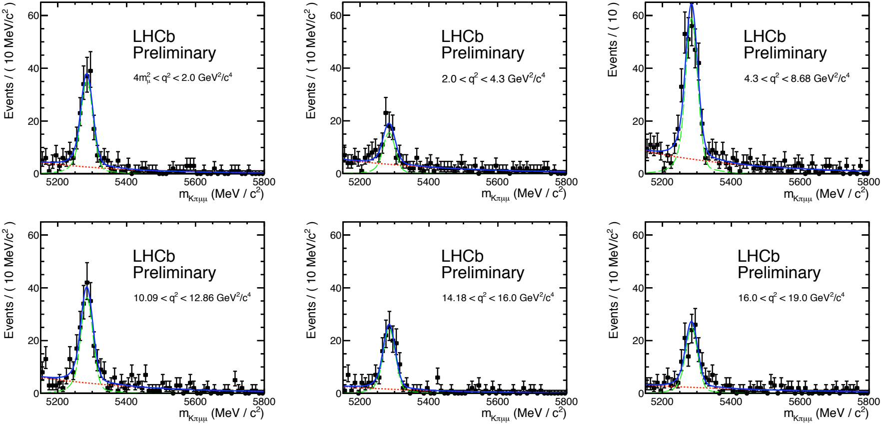
- Can apply the same technique to other observables that can be measured using counting experiments, e.g. S_5 [Bharucha & Reece, Eur. Phys. J. C69 (2010)]
- From SM MC simulation, expect a similar precision on the zero crossing point of S_5.



- Preliminary result from LHCb based on $\sim$ 900 candidates in $1\,\text{fb}^{-1}$.
  - $\rightarrow$ Updated to include A_9, A_T^{Re} and A_T^{Im}.
- Expect a combined 2011+2012 data set of $3\,\text{fb}^{-1}$ with $\sim$ 3000 signal candidates (assuming no analysis improvements).
- At the end of the year we go into LS1 of the LHC.



Yields in $1\,\text{fb}^{-1}$:



→ expect ~ 540 candidates in the $1 < q^2 < 6\,\mathrm{GeV}^2/c^4$ range in the 2011+2012 data sample.

Towards a full angular analysis and open questions

- With the combined data set we start to have enough data to contemplate a full angular fit but:
  - What variables do we fit for?
    - Transversity amplitudes? S_i's and A_i's? P_i's? [Descotes-Genon et al arXiv:1207.2753]

    We already know that we expect to see problems fitting for the S_i's and A_i's due to correlations between the parameters.
  - Do we need to take into account the q^2 dependence or can we bin in q^2?
  - How do we deal with the S-wave interference in a full angular analysis?
  - Do we separate B^0 and $\overline{B}^0$ in the fit?
  - Could we (and should we) consider fitting for the Wilson coefficients directly at low q^2?

Towards a full angular analysis and open questions

- If we do not parameterise the q^2 dependence, should we consider changing the q^2 binning that we currently use?
  - Are we better off binning in hadronic re-coil at high q^2?

- We now have a mess of naming conventions for different observables:
  - S_i and A_i, $A_T^{2,3,4}$, A_T^{Re} and A_T^{Im}, P_i, $H_T^{(1,2,3)}$ [Bobeth et al arXiv:1006.5013]].

  It could get even worse. For example does it make sense to talk about CP averages or asymmetries of the P_i's? If so what should we call them . . . ?

Discussion