Discussion on clean observables for the binned analysis of $B \to K^* l^+ l^-$

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PLAN of the TALK

- I. Angular distribution of $B \to K^*(\to K\pi)l^+l^-$.
  Rationale of a complete description in terms of the basis of clean observables P_i.
- II. Methodology to obtain Rare B decay constraints in the space of correlations between Wilson Coefficients and Implications.
- III. Control on two systematics: lepton masses and S-wave pollution.

After the last results from LHCb:

Main Conclusion:

"To see New Physics in Flavour will be extremely subtle"

Main Implications:

- Model Building: The Era of "Order of Magnitude" NP in Flavour (checking only $B \to X_s\gamma$) is gone
- Flavour Physicists: Redefine strategies to Focus when possible on Precise and Clean Observables. (Example $B \to K^*(\to K\pi)l^+l^-$)

Main impact on Rare B decays:

The fruitful theory+experimental effort:

UT $\to$ Wilson Coefficient correlation planes.

$$B \to K^*(\to K\pi)l^+l^-$$

Differential decay distributions

The decay $\bar{\mathbf{B}}_{\mathbf{d}} \to \bar{\mathbf{K}}^{*\mathbf{0}}(\to \mathbf{K}^-\pi^+)\mathbf{l}^+\mathbf{l}^-$ with the K^{*0} on the mass shell is described by $s = q^2$ and three angles $\theta_{\mathbf{l}}$, $\theta_{\mathbf{K}}$ and ϕ

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$

The differential distribution splits in J_i coefficients:

$$J(q^2, \theta_l, \theta_K, \phi) =$$
$$J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l + J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$
$$+J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi + (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l$$
$$+J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi + J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\,.$$

The information on

- the helicity/transversity amplitudes of the K^* ($H_{\pm 1,0}$ or $A_{\perp,\|,0}$) is inside the coefficients J_i.
- short distance physics C_i is encoded in ($H_{\pm 1,0}$ or $A_{\perp,\|,0}$)

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R\,*} + A_\parallel^L A_\parallel^{R\,*}\right),$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R\,*})\right] + \beta_\ell^2|A_S|^2,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + (L\to R)\right], \quad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + (L\to R)\right],$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + (L\to R)\right], \quad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L\,*}) + (L\to R)\right],$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L\,*}) - (L\to R) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^R A_S^*)\right],$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L\,*}) - (L\to R)\right], \quad J_{6c} = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}\left[A_0^L A_S^* + (L\to R)\right],$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L\,*}) - (L\to R) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* + A_\perp^R A_S^*)\right],$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L\,*}) + (L\to R)\right], \quad J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L\,*} A_\perp^L) + (L\to R)\right]$$

In red lepton mass terms.

Differential q^2-dependent observables

How to provide a complete and clean description of this 4-body decay?

J.M, F. Mescia, M. Ramon, J. Virto,'12

- Construct a basis of observables:
  - Number of observables=number of independent J=$2n_A - n_S$, where n_A number of Amplitudes, n_S number of Symmetries Egede et al'10.
  - Maximize (**flexible**) the number of clean (cancel FF exactly at LO) observables P_i, M_i,... We call them **FFI** (Form-Factor-Independent).

Different cases (FFD, FFI):

Massless lepton: $n_A = 6$ ($A^{L,R}_{\perp,\|,0}$), $n_S = 4$ symmetries.

$B^{massless}$: $\left\{F_T, \frac{d\Gamma}{dq^2}, P_1, P_2, P_3, P_4, P_5, P_6\right\}$ or $F_T \leftrightarrow A_{\rm FB}$

Massive leptons:$n_A = 7$ ($A^{L,R}_{\perp,\|,0}, A_t$), $n_S = 4$ symmetries.

$B^{massive}$: $\left\{F_T, \frac{d\Gamma}{dq^2}, P_1, P_2, P_3, P_4, P_5, P_6, M_1, M_2\right\}$

Massive leptons+scalars: $n_A = 8$ ($A^{L,R}_{\perp,\|,0}, A_{t,S}$), $n_S = 4$ symmetries.

B^{full}: $\left\{F_T, \frac{d\Gamma}{dq^2}, P_1, P_2, P_3, P_4, P_5, P_6, M_1, M_2, \mathcal{S}_1, \mathcal{S}_2\right\}$

A different basis (complete but it does not fulfill FFI requirements) associate to each $J_i \to S_i, A_i$ (all are **FFD**).

Egede et al.'10

What is the rationale of this basis?

$\Rightarrow$ Geometrical interpretation of the distribution ($m_\ell = 0$)

$$n_\| = (A_\|^L, A_\|^{R^*}) \quad n_\perp = (A_\perp^L, -A_\perp^{R^*}) \quad n_0 = (A_0^L, A_0^{R^*})$$

All physical information of the massless distribution encoded in 3 moduli + 3 complex scalar products - 1 constraint (**relation among n_i**): $3 + 3 \times 2 - 1 = 8$

$$|n_\||^2 = \frac{2}{3}J_{1s} - J_3\,, \qquad |n_\perp|^2 = \frac{2}{3}J_{1s} + J_3\,, \qquad |n_0|^2 = J_{1c}$$

$$n_\perp^\dagger n_\| = \frac{J_{6s}}{2} - iJ_9\,, \qquad n_0^\dagger n_\| = \sqrt{2}J_4 - i\frac{J_7}{\sqrt{2}}\,, \qquad n_0^\dagger n_\perp = \frac{J_5}{\sqrt{2}} - i\sqrt{2}J_8$$

Those are the building blocks of any observable (only 8 independent) conveniently normalized, this is the rationale behind the P_i basis

$$|n_i|^2, \quad \mathrm{Re}(n_i^\dagger n_j), \quad \mathrm{Im}(n_i^\dagger n_j)$$

For the six (clean) FFI observables we chose originally the set:

J.M, F. Mescia, M. Ramon, J. Virto,'12

$$P_1 = \frac{|n_\perp|^2 - |n_\||^2}{|n_\perp|^2 + |n_\||^2} = \frac{J_3}{2J_{2s}} \to A_T^{(2)}, \text{Kruger, JM , 05}$$

$$P_2 = \frac{\mathrm{Re}(n_\perp^\dagger n_\|)}{|n_\||^2 + |n_\perp|^2} = \beta_\ell \frac{J_{6s}}{8J_{2s}} \to A_T^{(re)}, \text{Becirevic et al., 12}$$

$$P_3 = \frac{\mathrm{Im}(n_\perp^\dagger n_\|)}{|n_\||^2 + |n_\perp|^2} = -\frac{J_9}{4J_{2s}} \to A_T^{(im)}, \text{Becirevic et al., 12}$$

$$P_4 = \frac{\mathrm{Re}(n_0^\dagger n_\|)}{\sqrt{|n_\||^2|n_0|^2}} = \frac{\sqrt{2}J_4}{\sqrt{-J_{2c}(2J_{2s}-J_3)}} \to H_T^{(1)}(low-recoil), \text{Bobeth et al., 08}$$

$$P_5 = \frac{\mathrm{Re}(n_0^\dagger n_\perp)}{\sqrt{|n_\perp|^2|n_0|^2}} = \frac{\beta_\ell J_5}{\sqrt{-2J_{2c}(2J_{2s}+J_3)}} \to H_T^{(2)}(low-recoil), \text{Bobeth et al., 08}$$

$$P_6 = \frac{\mathrm{Im}(n_0^\dagger n_\|)}{\sqrt{|n_\||^2|n_0|^2}} = -\frac{\beta_\ell J_7}{\sqrt{-2J_{2c}(2J_{2s}-J_3)}} , \text{J.M, F.Mescia, M.Ramon, J.Virto, 12}$$

Their form in terms of amplitudes is the same for massive and massless leptons. And a redundant

$$Q = \frac{\mathrm{Im}(n_0^\dagger n_\perp)}{\sqrt{|n_0|^2|n_\perp|^2}} = -\frac{\sqrt{2}J_8}{\sqrt{-J_{2c}(2J_{2s}+J_3)}} .$$

But now became more natural a change in the basis:

S. Descotes, J.M, M. Ramon, J. Virto,'12

$$
\begin{aligned}
P_1 &= \frac{|n_\perp|^2 - |n_\||^2}{|n_\perp|^2 + |n_\||^2} = \frac{J_3}{2J_{2s}} \to A_T^{(2)}, \\
P_2 &= \frac{\mathrm{Re}(n_\perp^\dagger\, n_\|)}{|n_\||^2 + |n_\perp|^2} = \beta_\ell \frac{J_{6s}}{8J_{2s}} \to A_T^{(re)}, \\
P_3 &= \frac{\mathrm{Im}(n_\perp^\dagger\, n_\|)}{|n_\||^2 + |n_\perp|^2} = -\frac{J_9}{4J_{2s}} \to A_T^{(im)}, \\
P_4' &= P_4\sqrt{1-P_1} = \frac{J_4}{\sqrt{-J_{2c}J_{2s}}}, \\
P_5' &= P_5\sqrt{1+P_1} = \frac{\beta_\ell J_5}{2\sqrt{-J_{2c}J_{2s}}}, \\
P_6' &= P_6\sqrt{1-P_1} = -\frac{\beta_\ell J_7}{2\sqrt{-J_{2c}J_{2s}}}\ ,
\end{aligned}
$$

and a redundant

$$Q' = Q\sqrt{1+P_1}$$

Notice that $P'_{4,5,6}$ are also clean and emerged naturally in the massless and massive distribution. Why?

This is seen when expressing the coefficients $\rightarrow$ in terms of observables($m_\ell = 0$)

$$J_{1s} = \frac{3}{4} F_T \frac{d\Gamma}{dq^2} = \frac{3}{4} c_4 \, , \qquad J_{2s} = \frac{1}{4} F_T \frac{d\Gamma}{dq^2} = \frac{1}{4} c_4$$

$$J_{1c} = F_L \frac{d\Gamma}{dq^2} = c_0 - c_4 \, , \qquad J_{2c} = -F_L \frac{d\Gamma}{dq^2} = c_4 - c_0 \, ,$$

$$J_3 = \frac{1}{2} P_1 F_T \frac{d\Gamma}{dq^2} = \frac{1}{2} c_1 \, , \qquad J_{6s} = 2 P_2 F_T \frac{d\Gamma}{dq^2} = 2 c_2 \, ,$$

$$J_4 = \frac{1}{2} P_4' \sqrt{F_T F_L} \frac{d\Gamma}{dq^2} \, , \qquad J_9 = -P_3 F_T \frac{d\Gamma}{dq^2} = -c_3 \, ,$$

$$J_5 = P_5' \sqrt{F_T F_L} \frac{d\Gamma}{dq^2} \, , \qquad J_7 = -P_6' \sqrt{F_T F_L} \frac{d\Gamma}{dq^2} \, ,$$

A simple translation table between **FFD** and **FFI** observables is:

$$P_1 = 2 \frac{S_3}{F_T} \, , \quad P_2 = \frac{S_{6s}}{2 F_T} \, , \quad P_3 = -\frac{S_9}{F_T} \, ,$$

and

$$P_4' = 2 \frac{S_4}{\sqrt{F_T F_L}} \, , \quad P_5' = \frac{S_5}{\sqrt{F_T F_L}} \, , \quad P_6' = -\frac{S_7}{\sqrt{F_T F_L}} .$$

Same cleaning mechanism.

In the massive case one alternative exact parametrization is the basis J.M.'12 $\{\hat{F}_T, \hat{F}_L, \tilde{F}_T, \tilde{F}_L, P_1, P_2, P_3, P_4', P_5', P_6'\}$ instead of $B_{massive}$:

$$J_{1s} = \frac{3}{4}\hat{F}_T\frac{d\Gamma_{K^*}}{dq^2}, \qquad J_{2s} = \frac{1}{4}\beta_\ell^2\tilde{F}_T\frac{d\Gamma_{K^*}}{dq^2},$$

$$J_{1c} = \hat{F}_L\frac{d\Gamma_{K^*}}{dq^2}, \qquad J_{2c} = -\beta_\ell^2\tilde{F}_L\frac{d\Gamma_{K^*}}{dq^2},$$

$$J_3 = \frac{1}{2}\beta_\ell^2 P_1\tilde{F}_T\frac{d\Gamma_{K^*}}{dq^2}, \qquad J_{6s} = 2\beta_\ell P_2\tilde{F}_T\frac{d\Gamma_{K^*}}{dq^2},$$

$$J_4 = \frac{1}{2}\beta_\ell^2 P_4'\sqrt{\tilde{F}_T\tilde{F}_L}\frac{d\Gamma_{K^*}}{dq^2}, \qquad J_9 = -\beta_\ell^2 P_3\tilde{F}_T\frac{d\Gamma_{K^*}}{dq^2},$$

$$J_5 = \beta_\ell P_5'\sqrt{\tilde{F}_T\tilde{F}_L}\frac{d\Gamma_{K^*}}{dq^2}, \qquad J_7 = -\beta_\ell P_6'\sqrt{\tilde{F}_T\tilde{F}_L}\frac{d\Gamma_{K^*}}{dq^2},$$

where a factor $1/X$ in each J_i due to the BW resonance has to be included

$$\frac{\hat{F}_T}{\tilde{F}_T} = \frac{1}{3}(2+(1+4M_1)\beta_\ell^2), \quad \frac{\hat{F}_L}{\tilde{F}_L} = 1+M_2, \quad \hat{F}_L+\hat{F}_T = 1+\hat{\delta}, \quad \tilde{F}_T+\tilde{F}_L = 1+\delta$$

Experimentally is instrumental to define an **improved massless limit** (I & II) that reduces to $< 1.3\%$ the **extraction** error in neglecting **lepton mass terms**:

- I. Keep the β_ℓ dependence
- II. Reexpress $\hat{F}_T$, $\hat{F}_L$ in terms of $\tilde{F}_T$: ($\hat{F}_L = 1-\tilde{F}_T$ and $\hat{F}_T = \tilde{F}_T$). Effective reduced basis $\{\frac{d\Gamma}{dq^2}, \tilde{F}_T, \tilde{F}_L, P_1, P_2, P_3, P_4', P_5', P_6'\}$ (equivalent to $M_1 \to 0$)
- III If $\tilde{F}_L = 1-\tilde{F}_T$ (keep β_ℓ and neglect δ) larger error (not recommended)

The reason of the suppressed error size is the cancellation of errors in $\hat{F}_L$:

$$\begin{aligned}\hat{F}_T &= \tilde{F}_T + (R_T - 1)\tilde{F}_T \equiv \tilde{F}_T + L_1 \\ \hat{F}_L &= 1 - \tilde{F}_T + \delta + (R_L - 1)(1 - \tilde{F}_T) + \mathcal{O}((R_L - 1)\delta) \equiv 1 - \tilde{F}_T + L_2\end{aligned}$$

error in % are $\mathrm{Er}(L_1/\tilde{F}_T) < 1.2\%$ and $\mathrm{Er}(L_2/(1 - \tilde{F}_T)) < $ **1.3%**

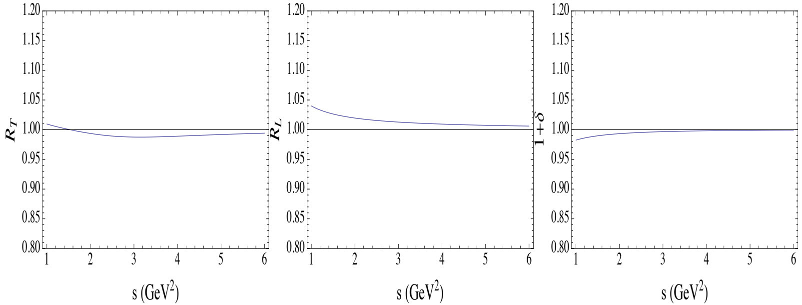


Figure: (left) ratio $R_T = \hat{F}_T/\tilde{F}_T$ shows that the error is always below 1.2%, (middle) ratio $R_L = \hat{F}_L/\tilde{F}_L$ the error range here is between 1% to less than 4% in the SM (right) δ represents the deviation from one of $\tilde{F}_L + \tilde{F}_T$.

The conclusion can be extended to NP given the small deviations of $M_{1,2}$

What are the symmetry transformations? They correspond to invariances of the angular distribution under a rotation in the space of transversity amplitudes.

- **Massless**: Unitary transformations that leave the modulus and scalar products among the n_i invariant. *Symmetries*: two global phase transformations (ϕ_L and ϕ_R), and two rotations θ, $\tilde{\theta}$ among the real and imaginary components of the transversity amplitudes.
- **Massive**: The symmetries of the massive distribution are also symmetries of the massless case $\Rightarrow$ same form of rotation matrices. *Symmetries*:
  - Common global phase transformation for both L and R components

    $$n_i^{'} = U_0(\phi) n_i = \begin{bmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{bmatrix} n_i \,,$$

    for $i = \|, \perp, 0$.
  - Independent global phase transformation for the A_t amplitude:

    $$A_t' = e^{i\phi_t} A_t .$$
  - Two rotations $U_1(\theta)$ and $U_2(\tilde{\theta})$ between the real and imaginary components of the transversity amplitudes $A_i^{L,R}$:

    $$n_i' = U_1(\theta) n_i \,, \qquad n_i' = U_2(\tilde{\theta}) n_i \,,$$

    ($i = \perp, \|, 0$) with similar structure to the massless case, except for a phase $e^{-i\delta(\theta)}$ in $U_1(\theta)$ and $e^{-i\tilde{\delta}(\tilde{\theta})}$ in $U_2(\tilde{\theta})$. $\delta(\theta)$ and $\tilde{\delta}(\tilde{\theta})$ are non-linear functions of the transversity amplitudes.

- **Massive+scalars**:
  - $A_\perp$ and $A_\|$ transform exactly as in the massive case.
  - A_0 and A_S get mixed in the transformation and, contrary to the massive case, an explicit dependence on the lepton mass appears in the symmetry transformation. This has an important consequence for the construction of observables: while the lepton mass terms in J_{1s} is invariant by itself, the mass terms in J_5 and J_7 are not.
  - Another fundamental difference with the previous cases, is that the use of the compact two-component complex vector n_i is no longer possible for A_0 and A_S. A new formalism in terms of four-component vectors is required.

Physical interpretation of massless symmetries:

- First and second symmetry: since L and R amplitudes do not interfere here, there is the freedom to chose their phases.
- The interpretation of the third and fourth symmetry is that they transform a helicity $+1$ final state with a LH current into a helicity -1 state with a RH current. The simultaneous change of helicity and handedness of the current cannot be measured experimentally.

What are the advantages of the P_i in front of other parametrizations?

- S_i, A_i:
  - **Pros:** Simple, each J_i is associated to a S_i and A_i, normalized to the $d\Gamma/dq^2$.
  - **Cons:** I. Strong dependence on form factor error's size:
    - Observables whose value is near zero are less sensitive.
    - In presence of NP this problem becomes very important.
  - II. If scalars are neglected the set of J_i become redundant.
  - III. Definitions depend on lepton mass terms.
  - IV. Intrinsic S-wave pollution directly in their definition.
- P_i and P_j':
  - **Pros:** I. Robust observables, free from Soft Form Factor uncertainties at LO, dependence at NLO mild in SM and beyond.
  - II. Definitions independent of lepton mass terms
  - III. P_i free from S-wave pollution
  - IV. Basis of observables flexible and non redundant.
  - **Cons:** They appear in products with other observables. Specifically: F_T, $\sqrt{F_T F_L}$.

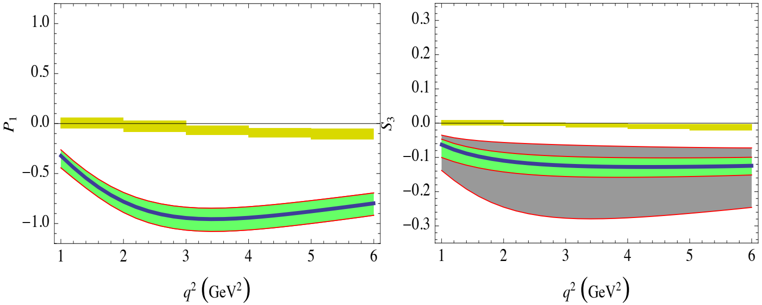
The wide spread of different errors in literature associated to form factors:

$V(0) = 0.31 \pm 0.04$ and $A(0) = 0.33 \pm 0.03$ W. Altmannshofer et al. '09

$V(0) = 0.36 \pm 0.17$ and $A(0) = 0.29 \pm 0.10$ A. Khodjamirian et al. '10

has a relevant impact on observables. Even central values have shifted significantly, for instance $V(0) = 0.41 \pm 0.05$ P. Ball and R. Zwicky,'05

- The SM prediction for P_1 is insensitive to the choice of form factors. Also S_3 is insensitive due to the fact that $S_3 \sim 0$.
- The NP predictions for P_1 is insensitive to the choice of form factors. S_3 is very sensitive and the hadronic form factors x3, reducing the ability of S_3 to disentangle among different NP curves.



Integrated observables

Contact theory and experiment:

Indeed the observables are measured in bins.

This requires a redefinition of observables in bins: (**massless**)

$$\left\langle A_T^{(2)} \right\rangle_{\rm bin} \equiv \langle P_1 \rangle_{\rm bin} = \frac{\int_{\rm bin} dq^2 J_3}{2 \int_{\rm bin} dq^2 J_{2s}} = \frac{\int_{\rm bin} dq^2 F_T P_1 \frac{d\Gamma}{dq^2}}{\int_{\rm bin} dq^2 F_T \frac{d\Gamma}{dq^2}} = \frac{\int_{\rm bin} dq^2 c_1(q^2)}{\int_{\rm bin} dq^2 c_4(q^2)},$$

$$\langle P_2 \rangle_{\rm bin} = \frac{\int_{\rm bin} dq^2 J_{6s}}{8 \int_{\rm bin} dq^2 J_{2s}} = \frac{\int_{\rm bin} dq^2 F_T P_2 \frac{d\Gamma}{dq^2}}{\int_{\rm bin} dq^2 F_T \frac{d\Gamma}{dq^2}} = \frac{\int_{\rm bin} dq^2 c_2(q^2)}{\int_{\rm bin} dq^2 c_4(q^2)},$$

$$\langle P_3 \rangle_{\rm bin} = -\frac{\int_{\rm bin} dq^2 J_9}{4 \int_{\rm bin} dq^2 J_{2s}} = \frac{\int_{\rm bin} dq^2 F_T P_3 \frac{d\Gamma}{dq^2}}{\int_{\rm bin} dq^2 F_T \frac{d\Gamma}{dq^2}} = \frac{\int_{\rm bin} dq^2 c_3(q^2)}{\int_{\rm bin} dq^2 c_4(q^2)}.$$

where $c_i(q^2) = P_i F_T \frac{d\Gamma}{dq^2}$ (i=1,2,3) and $c_4(q^2) = F_T \frac{d\Gamma}{dq^2}$

But they are already indirectly measured via S_3, A_{im}, A_{FB}, F_L (**and already provide constraints**)

Contact theory and experiment:

Indeed the observables are measured in bins.

This requires a redefinition of observables in bins: (**massive**) J.M.'12

$$\left\langle A_T^{(2)} \right\rangle_{\rm bin} \equiv \left\langle P_1^{m_\ell \neq 0} \right\rangle_{\rm bin} = \frac{\int_{\rm bin} dq^2 J_3}{2 \int_{\rm bin} dq^2 J_{2s}} = \frac{\int_{\rm bin} dq^2 \beta_\ell^2 \tilde{F}_T P_1 \frac{d\Gamma}{dq^2}}{\int_{\rm bin} dq^2 \beta_\ell^2 \tilde{F}_T \frac{d\Gamma}{dq^2}} = \frac{\int_{\rm bin} dq^2 c_1(q^2)}{\int_{\rm bin} dq^2 c_4(q^2)},$$

$$\left\langle P_2^{m_\ell \neq 0} \right\rangle_{\rm bin} = \frac{\int_{\rm bin} dq^2 J_{6s}}{8 \int_{\rm bin} dq^2 J_{2s}} = \frac{\int_{\rm bin} dq^2 \beta_\ell \tilde{F}_T P_2 \frac{d\Gamma}{dq^2}}{\int_{\rm bin} dq^2 \beta_\ell^2 \tilde{F}_T \frac{d\Gamma}{dq^2}} = \frac{\int_{\rm bin} dq^2 c_2(q^2)}{\int_{\rm bin} dq^2 c_4(q^2)},$$

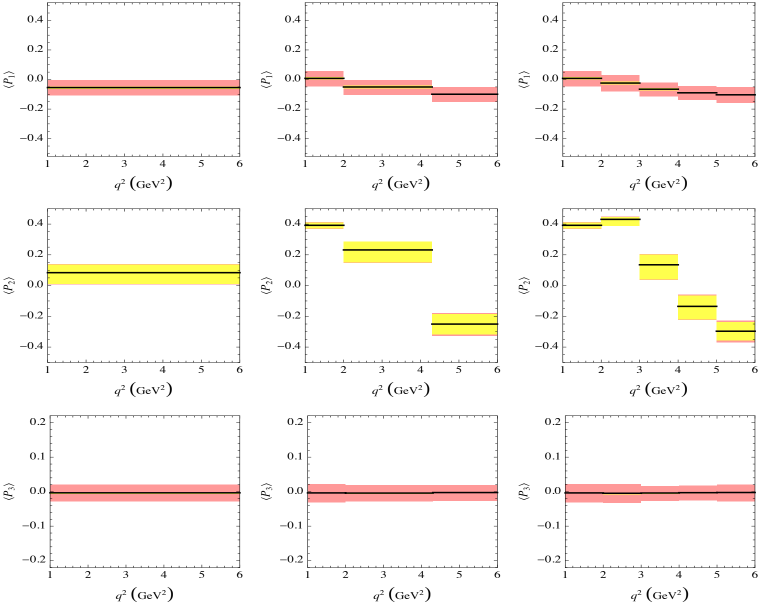
$$\left\langle P_3^{m_\ell \neq 0} \right\rangle_{\rm bin} = -\frac{\int_{\rm bin} dq^2 J_9}{4 \int_{\rm bin} dq^2 J_{2s}} = \frac{\int_{\rm bin} dq^2 \beta_\ell^2 \tilde{F}_T P_3 \frac{d\Gamma}{dq^2}}{\int_{\rm bin} dq^2 \beta_\ell^2 \tilde{F}_T \frac{d\Gamma}{dq^2}} = \frac{\int_{\rm bin} dq^2 c_3(q^2)}{\int_{\rm bin} dq^2 c_4(q^2)}.$$

where $c_i(q^2) = \beta_\ell^2 P_i \tilde{F}_T \frac{d\Gamma}{dq^2}$ (i=1,3), $c_2(q^2) = \beta_\ell P_2 \tilde{F}_T \frac{d\Gamma}{dq^2}$, $c_4(q^2) = \beta_\ell^2 \tilde{F}_T \frac{d\Gamma}{dq^2}$

Notice that $P_1^{m_\ell \neq 0} = P_1$, $P_2^{m_\ell \neq 0} = P_2/\beta_\ell$ and $P_3^{m_\ell \neq 0} = P_3$

The difference between the SM prediction for the integrated bin in 1 to 6 GeV^2 in the massless case and massive case is below 0.8%. This is the maximal **theory** error if the massless theoretical predictions are used for the integrated $P_{1,2,3}$.

Binned SM prediction for $P_{1,2,3}$ observables (massless):



The integrated version of observables $P'_{4,5,6}$ are defined by

$$\langle P'_4\rangle_{\rm bin} = \frac{2\int_{\rm bin} dq^2 J_4(q^2)}{\sqrt{\int_{\rm bin} dq^2 c_4(q^2) \int_{\rm bin} dq^2 (c_0(q^2) - c_4(q^2))}} = \frac{\langle J_4\rangle_{\rm bin}}{\sqrt{-\langle J_{2s}\rangle_{\rm bin}\langle J_{2c}\rangle_{\rm bin}}},$$

$$\langle P'_5\rangle_{\rm bin} = \frac{\int_{\rm bin} dq^2 J_5(q^2)}{\sqrt{\int_{\rm bin} dq^2 c_4(q^2) \int_{\rm bin} dq^2 (c_0(q^2) - c_4(q^2))}} = \frac{\langle J_5\rangle_{\rm bin}}{2\sqrt{-\langle J_{2s}\rangle_{\rm bin}\langle J_{2c}\rangle_{\rm bin}}},$$

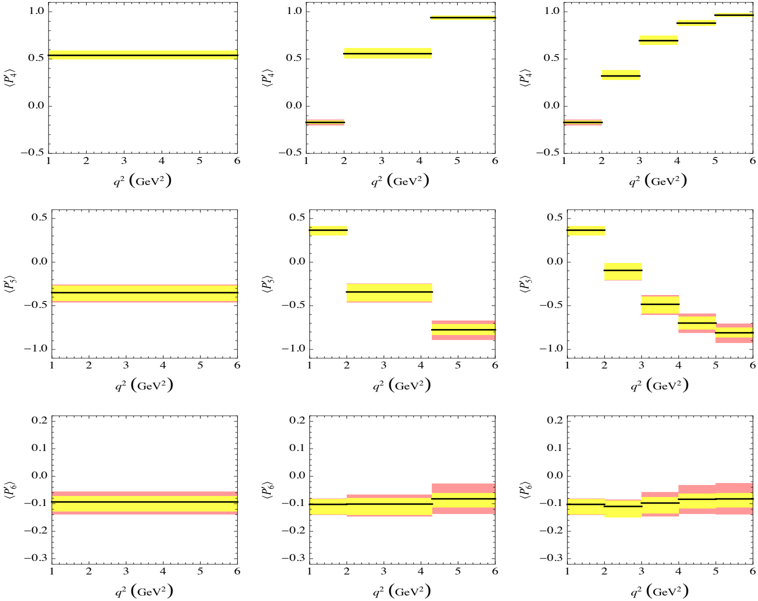
$$\langle P'_6\rangle_{\rm bin} = \frac{-\int_{\rm bin} dq^2 J_7(q^2)}{\sqrt{\int_{\rm bin} dq^2 c_4(q^2) \int_{\rm bin} dq^2 (c_0(q^2) - c_4(q^2))}} = \frac{-\langle J_7\rangle_{\rm bin}}{2\sqrt{-\langle J_{2s}\rangle_{\rm bin}\langle J_{2c}\rangle_{\rm bin}}}.$$

- They are not yet measured but the double-folded distributions give access to these observables. There exist massive generalizations for $P'_{4,5,6}$ similar to the ones of $P_{1,2,3}$.
- There is also a redundant clean observable Q' (if there are no scalars) associated to J_8 that can be introduced for practical reasons:

$$\langle Q'\rangle_{\rm bin} = \frac{-2\int_{\rm bin} dq^2 J_8(q^2)}{\sqrt{\int_{\rm bin} dq^2 c_4(q^2) \int_{\rm bin} dq^2 (c_0(q^2) - c_4(q^2))}} = \frac{-\langle J_8\rangle_{\rm bin}}{\sqrt{-\langle J_{2s}\rangle_{\rm bin}\langle J_{2c}\rangle_{\rm bin}}}$$

Notice that $Q' = f(P_i)$ **but** $\langle Q'\rangle_{\rm bin} \neq f(\langle P_i\rangle_{\rm bin})$.

Binned SM prediction for $P'_{4,5,6}$ observables:



Discussion on constraints on WC from radiative and leptonic B decays should be addressed in a given framework, specific scenarios & observables

S. Descotes, D. Ghosh, JM., M. Ramon, '11

- Framework: NP in C_7, C_9, C_{10} and $C_{7'}, C_{9'}, C_{10'}$ [chirally-flipped operators $\gamma_5 \to -\gamma_5$] as a real shift in the Wilson coefficients
- Scenarios (depending on the specific model)
  - A : NP in 7,7' only
  - B : NP in 7,7', 9,10 only
  - B': NP in 7,7', 9',10' only
  - C : NP in 7,7',9,10,9',10' only
- Classes within a Framework
  - I: observables sensitive only to 7,7'
  - II: observables sensitive only to 7,7',9,9',10,10'
  - III: observables sensitive to 7,7',9,9',10,10' and more (scalars...)

Other model-independent analysis:

Bobeth, Hiller, van Dyk 1105.0376
Altmannshofer, Paradisi, Straub 1111.1257
Bobeth, Hiller, van Dyk,Wacker 1111.2558
Beaujean, Bobeth, van Dyk,Wacker 1205.1838
Altmannshofer, Straub 1206.0273
Becirevic, Kou, Le Yaouanc,Tayduganov 12061502
....

Also specific model analysis:

M. Blanke, B. Shakya, P. Tanedo, Y. Tsai, 1203.6650
F. Mahmoudi, S. Neshatpour and J. Orloff, 1205.1845
Nejc Kosnik, 1206.2970
T. Hurth and F. Mahmoudi, 1207.0688
....

Observables

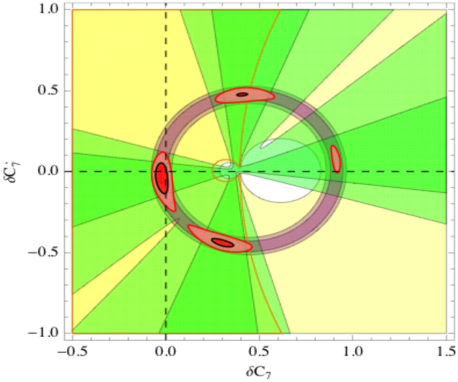
Limited sensitivity to hadronic inputs, or strong impact on analysis

- Class-I
  - $\mathcal{B}(B \to X_s\gamma)$ with $E_\gamma > 1.6\,\mathrm{GeV}$ [Misiak, Steinhauser, Haisch]
  - exclusive time-dependent CP asymmetry $S_{K^*\gamma}$
  - isospin asymmetry $A_I(B \to K^*\gamma)$ [Beneke, Feldman, Seidel] [Kagan, Neubert, Feldman, J.M.]
- Class-II
  - Integrated transverse asymmetries $\tilde{A}_{\mathrm{T}}^2 = P_1$, P_2 and P_3 in $B \to K^* l^+ l^-$ over low-q^2 region in bins. [Kruger and J.M.]
- Class-III
  - $\mathcal{B}(B \to X_s l^+ l^-)$ [Bobeth et al., Huber, Lunghi et al.]
  - Integrated $\tilde{F}_{\mathrm{L}}$ and $\tilde{A}_{\mathrm{FB}}$ in $B \to K^* l^+ l^-$ [1-6 GeV^2]

Simple numerical parametrisation as $\delta C_i = C_i(\mu_b) - C_i^{SM}(\mu_b)$

We provide the numerical expressions for the integrated observables $\langle A_{FB}\rangle, \langle F_L\rangle, \langle P_{1,2,3}\rangle$ and $\langle P_{4',5',6'}\rangle$ as a function of the NP Wilson coefficients, for different choices of the q^2-binning. S. Descotes, JM., J. Virto, M. Ramon '12

$\delta C_7 - \delta C_{7'}$ plane : constraints at 68.3% and 95.5% C.L.



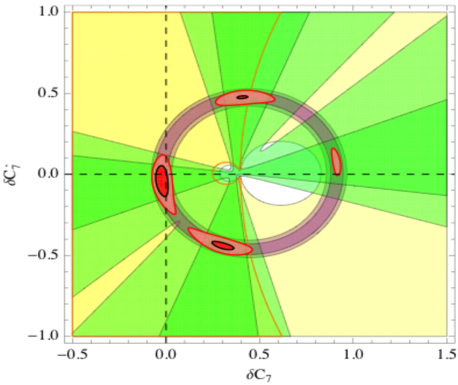
Class I observables (only $O_{7,7'}$)
dark 68.3%, light 95.5% CL

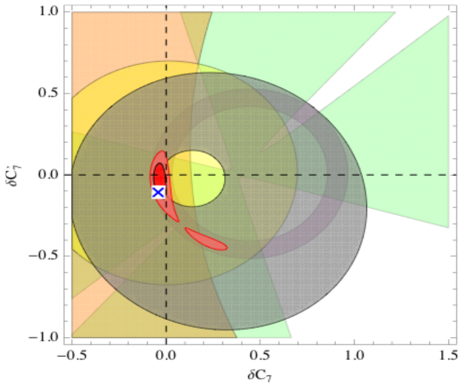
- A_I (yellow)
- B($B \to X_s\gamma$) (purple)
- $S_{K^*\gamma}$ (green)

Overlap regions (red dark and light)

- Region around SM favoured: solid black countour red dark $(\delta C_7, \delta C_{7'}) \sim (0,0)$.
- three non-SM solutions also allowed $(\delta C_7, \delta C_{7'}) \simeq (-C_7^{SM}, \pm 0.4), (0.9, 0)$

- A_I disfavours at 68.3% CL changed-sign solution $(C_7, C_{7'}) = (C_7^{SM} + 0.9, 0)$
  $\Longrightarrow$Same conclusion as [Gambino, Haisch, Misiak], without using Class-III $B \to X_s \ell^+ \ell^-$. Constraints independent of other WCs.

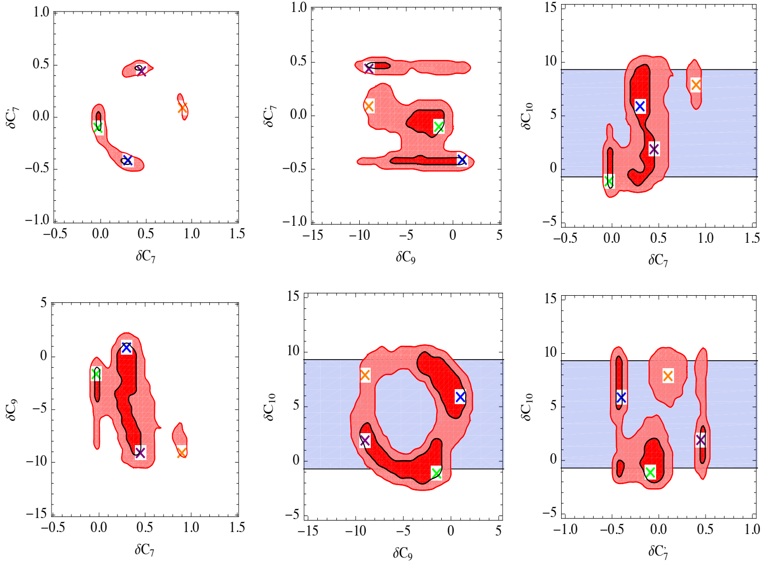




$\Longrightarrow$class-III observables ($< A_{\rm FB} >_{[1,6]}$, $< F_{\rm L} >_{[1,6]}$, **BR($B \to X_s l^+ l^-$)**) constrain further the shifts $\delta C_7, \delta C_{7'}$ (if all other NP WC to zero))

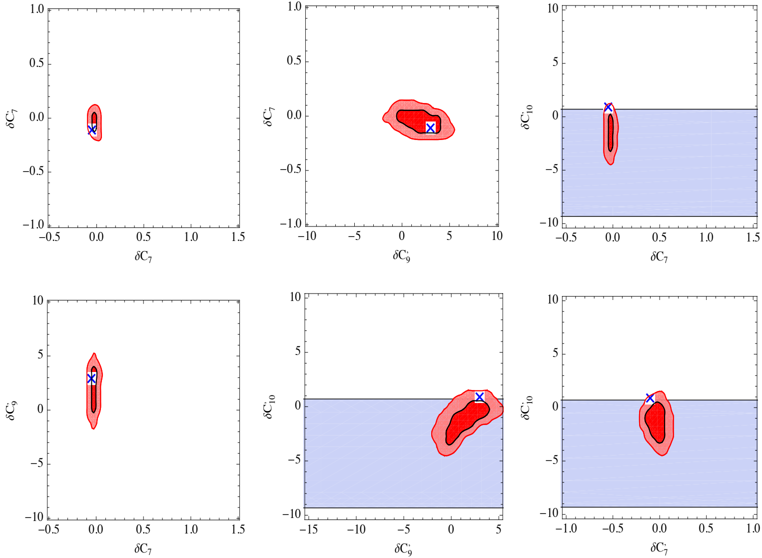
- **BR**($B \to X_s \mu^+ \mu^-$) favours SM-like region and two non-SM regions.
- $< A_{\rm FB} >_{[1,6]}$ selects SM region and one non-SM region. $< F_{\rm L} >_{[1,6]}$ does not discriminate any region.
- All combined observables disfavour changed sign solution at more than 95.5 % CL

Scenario B ($C_{7,7',9,10}$) with all constraints

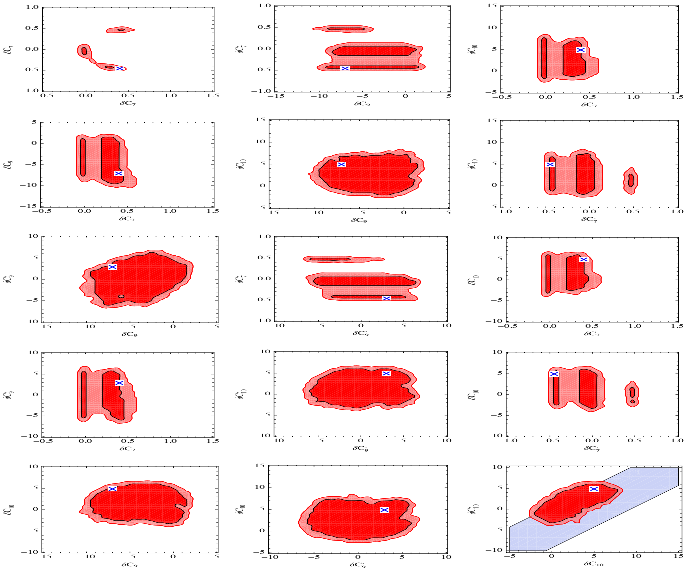


Four islands in the space of Wilson Coefficients ("four benchmark points" projected in all planes). Blue band is $BR(B_s \to \mu^+\mu^-)$ constraint.

Scenario B' ($C_{7,7',9',10'}$) with all constraints



One island in the space of WCs. Blue band is $BR(B_s \to \mu^+\mu^-)$ constraint. Changed-sign solution for C_7 reduce its statistical significance.



Constraints from measured $P_{1,2,3}$

Using

$$\langle P_1\rangle_{\rm bin} = \frac{2\,\langle S_3\rangle_{\rm bin}}{1-\langle F_L\rangle_{\rm bin}}\ ,\ \langle P_2\rangle_{\rm bin} = -\frac{2}{3}\frac{\langle A_{\rm FB}\rangle_{\rm bin}}{(1-\langle F_L\rangle_{\rm bin})}\ ,\ \langle P_3\rangle_{\rm bin} = -\frac{\langle A_{im}\rangle_{\rm bin}}{(1-\langle F_L\rangle_{\rm bin})}\ .$$

Observable	Experiment	SM prediction
$\langle P_1\rangle_{[2,4.3]}$	-0.19 ± 0.58	-0.051 ± 0.050
$\langle P_1\rangle_{[4.3,8.68]}$	0.42 ± 0.31	-0.117 ± 0.059
$\langle P_1\rangle_{[1,6]}$	0.29 ± 0.47	-0.055 ± 0.051
$\langle P_2\rangle_{[2,4.3]}$	0.51 ± 0.27	0.232 ± 0.069
$\langle P_2\rangle_{[4.3,8.68]}$	-0.25 ± 0.08	-0.405 ± 0.064
$\langle P_2\rangle_{[1,6]}$	0.35 ± 0.14	0.084 ± 0.066
$\langle P_3\rangle_{[2,4.3]}$	0.08 ± 0.35	-0.004 ± 0.024
$\langle P_3\rangle_{[4.3,8.68]}$	-0.05 ± 0.16	-0.001 ± 0.027
$\langle P_3\rangle_{[1,6]}$	-0.21 ± 0.21	-0.003 ± 0.024

Table: Experimental values for the clean observables P_1, P_2 and P_3 within different q^2-bins, extracted from the measurements of S_3, A_{im}, A_{fb} and F_L, and their SM predictions.

The exp. error bars in red are large since they do not contain correlations $\Rightarrow$ one expects to reduced them substantially once correlations are included.

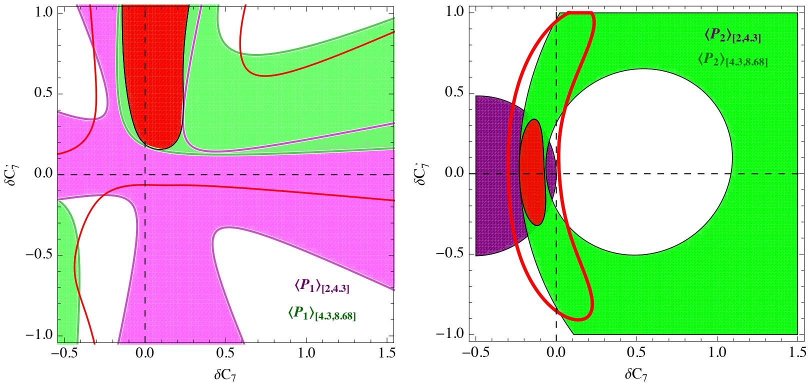


Figure: Left: Individual 68.3% CL constraints in the $\delta C_7(\mu_b)$-$\delta C_7'(\mu_b)$ plane from the integrated clean observables $\langle P_1\rangle_{[2,4.30]}$ and $\langle P_1\rangle_{[4.30,8.68]}$, together with the combined result. The red region and contour correspond to the combined 68.3% and 95.5% CL.

- P_1 in the bin [4.30,8.68] deviates a bit from the SM, but the other bins are OK.
- All bins in P_2 point towards a negative NP contribution to C7. The combined fit for P_2 is away from the SM point at 95.5 % CL.
- P_3 gives no constraints for the moment.

Constraints from binned A_{FB} and F_L.

What is the impact of including binning?

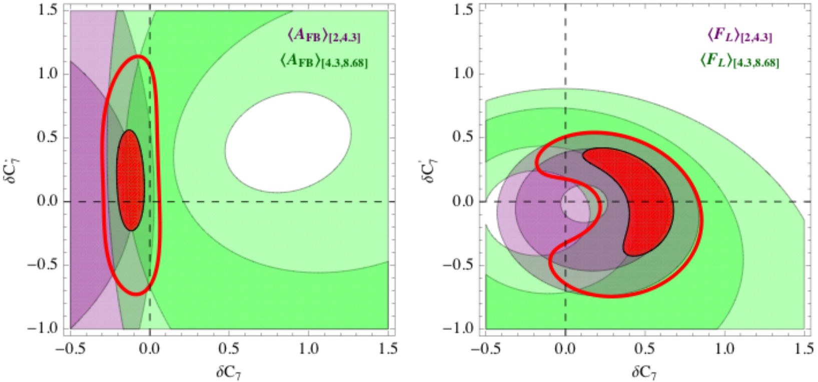


Figure: Left: Individual 68.3% and 95.5% CL constraints in the $\delta C_7(\mu_b)$-$\delta C_7'(\mu_b)$ plane from $\langle A_{FB}\rangle_{[2,4.30]}$ and $\langle A_{FB}\rangle_{[4.30,8.68]}$, together with the combined constraint. Right: Same analysis for $\langle F_L\rangle_{[2,4.30]}$ and $\langle F_L\rangle_{[4.30,8.68]}$.

- A_{FB} in the bins consistent with the SM at 95.5% CL.
- F_L in the bins is a bit off the SM at 95.5% CL.
- Could be that the errors have been underestimated?

Future Prospects

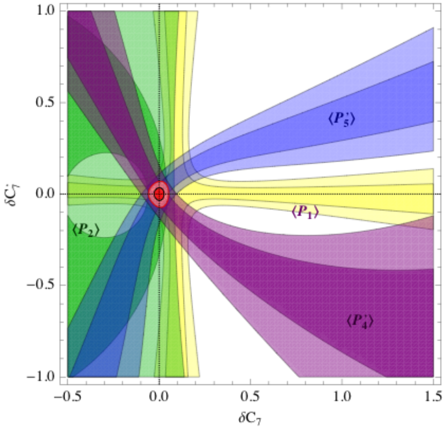


Figure: Individual constraints in the $\delta C_7 - \delta C_7'$ plane from hypothetical measurements of the observables $\langle P_1 \rangle_{[2,4.3]}$ (yellow), $\langle P_2 \rangle_{[2,4.3]}$ (green), $\langle P_4' \rangle_{[2,4.3]}$ (purple) and $\langle P_5' \rangle_{[2,4.3]}$ (blue), corresponding to central values equal to the SM predictions and an experimental uncertainty $\sigma_{exp} = 0.10$. The combined 68.3% (dark red) and 95.5% (light red) C.L. regions are also shown.

S-wave pollution

Becirevic, Tayduganov,'12

It has been argued recently that transverse asymmetries **normalized** by $3J_1^s - J_2^s$ extracted from **uniangular distributions** can suffer a pollution from S-wave $(K\pi)$ pairs coming from the companion decay into the scalar K_0^* of up to 23% around $q^2 = 2\ \mathrm{GeV}^2$ and less than 10% in the rest.

J.M '12

Here I will show a procedure, using the P_i instead (**normalized** to J_2^s) to extract these observables from **folded distributions** with **ZERO pollution**. And even if one uses uniangular distributions one can design an strategy to extract the normalization with an error given by $\mathcal{O}(m_\ell^2/q^2)$ suppressed terms.

The full differential decay distribution is now

$$\frac{d\Gamma_{full}}{dq^2} = \frac{d\Gamma_{K^*}}{dq^2} + \frac{d\Gamma_{K_0^*}}{dq^2}$$

with

$$\frac{d\Gamma_{K^*}}{dq^2} = \frac{1}{4}\left(3J_1^c + 6J_1^s - J_2^c - 2J_2^s\right)X, \quad \frac{d\Gamma_{K_0^*}}{dq^2} = +2\tilde{J}_{1a}^c - \frac{2}{3}\tilde{J}_{2a}^c$$

$X = \int dm_{K\pi}^2 |BW_K^*(m_{K\pi}^2)|^2$ takes into account the width of the resonance and $\tilde{J}_i$ are interference terms of companion decay. J.M'12 from Becirevic'12

Exact solution including lepton masses

Substitute uniangular distributions $\to$ folded distributions

The identification of $\phi \leftrightarrow \phi + \pi$ ($\phi < 0$) produces a "folded" angle $\hat{\phi} \in [0, \pi]$ in terms of which a (folded) differential rate $d\hat{\Gamma}(\hat{\phi}) = d\Gamma(\phi) + d\Gamma(\phi - \pi)$ is:

$$\frac{d^4\hat{\Gamma}}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\hat{\phi}} = \frac{9}{16\pi}\Big[J_{1c}\cos^2\theta_K + J_{1s}(1-\cos^2\theta_K) +$$
$$+J_{2c}\cos^2\theta_K\cos 2\theta_\ell + J_{2s}(1-\cos^2\theta_K)(2\cos^2\theta_\ell - 1) + J_3(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\cos 2\hat{\phi}$$
$$+J_{6s}(1-\cos^2\theta_K)\cos\theta_\ell + J_9(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\sin 2\hat{\phi}\Big]X + W_1$$

and

$$W_1 = \frac{1}{2\pi}\left[\tilde{J}^c_{1a} + \tilde{J}^c_{1b}\cos\theta_K + \left(\tilde{J}^c_{2a} + \tilde{J}^c_{2b}\cos\theta_K\right)(2\cos^2\theta_\ell - 1)\right]$$

Advantages:

- Folding reduces the # of coefficients to a manageable experimentally subset. In this case: 11 (10) obs + 8 $\tilde{J}$ $\to$ 7 obs + 4 $\tilde{J}$
- Unwanted S-wave pollution has a distinct angular dependence: W_1. It can be parametrized by

$$W_1 = a_W + b_W(2\cos^2\theta_\ell - 1) + c_W\cos\theta_K + d_W\cos\theta_K(2\cos^2\theta_\ell - 1)$$

Fitting for these four parameters (a_W, b_W, c_W, d_W) one can disentangle the S-wave pollution but also extract the coefficients $J_{3,6s,9,2s}$ involved in the definition of the $P_{1,2,3}$ **completely free from any S-wave pollution**.

Or in terms of observables: The generalization of Eq.(1) of LHCb note to include lepton mass corrections and the S-wave pollution in a minimal form

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\hat{\phi}} = \frac{9}{16\pi}\Big[\hat{F}_L\cos^2\theta_K + \frac{3}{4}\hat{F}_T(1-\cos^2\theta_K) - \beta_\ell^2\tilde{F}_L\cos^2\theta_K\cos 2\theta_\ell$$
$$+\frac{1}{4}\beta_\ell^2\tilde{F}_T(1-\cos^2\theta_K)(2\cos^2\theta_\ell-1) + \frac{1}{2}\beta_\ell^2 P_1\tilde{F}_T(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\cos 2\hat{\phi}$$
$$+2\beta_\ell P_2\tilde{F}_T(1-\cos^2\theta_K)\cos\theta_\ell - \beta_\ell^2 P_3\tilde{F}_T(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\sin 2\hat{\phi}\Big]\frac{d\Gamma_{K*}}{dq^2} + W_1$$

or in the massless-improved limit:

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\hat{\phi}} = \frac{9}{16\pi}\Big[(1-\tilde{F}_T)\cos^2\theta_K + \frac{3}{4}\tilde{F}_T(1-\cos^2\theta_K) - \beta_\ell^2\tilde{F}_L\cos^2\theta_K\cos 2\theta_\ell$$
$$+\frac{1}{4}\beta_\ell^2\tilde{F}_T(1-\cos^2\theta_K)(2\cos^2\theta_\ell-1) + \frac{1}{2}\beta_\ell^2 P_1\tilde{F}_T(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\cos 2\hat{\phi}$$
$$+2\beta_\ell P_2\tilde{F}_T(1-\cos^2\theta_K)\cos\theta_\ell - \beta_\ell^2 P_3\tilde{F}_T(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\sin 2\hat{\phi}\Big]\frac{d\Gamma_{K*}}{dq^2} + W_1\ ,$$

where the β_ℓ dependence is kept, only $\hat{F}_{L,T}$ is substituted and $L_{1,2}$ terms have been dropped off. The error of this approximation is below 1.3%.
In this improved limit ONLY 6 observables: $\{\ \tilde{F}_{L,T},\ P_{1,2,3},\ \frac{d\Gamma_{K*}}{dq^2}\ \} + 4\ \tilde{J}$

Other double folded distributions can be **more selective** and allow to extract P_1, P_2, P_3 and P_4', P_5', P_6'. Some examples:

I. Identify $\phi \leftrightarrow -\phi$ $(\phi < 0)$ and $\theta_K \leftrightarrow \pi - \theta_K$ $(\theta_K > \frac{\pi}{2})$ with $\hat{\phi} \in [0, \pi]$, $\hat{\theta}_K \in [0, \pi/2]$

$$d\hat{\Gamma} = d\Gamma(\hat{\phi}, \theta_\ell, \hat{\theta}_K) + d\Gamma(\hat{\phi}, \theta_\ell, \pi - \hat{\theta}_K) + d\Gamma(-\hat{\phi}, \theta_\ell, \hat{\theta}_K) + d\Gamma(-\hat{\phi}, \theta_\ell, \pi - \hat{\theta}_K)$$

$$\frac{d^4\hat{\Gamma}}{dq^2 d\cos\hat{\theta}_K d\cos\theta_\ell d\hat{\phi}} = \frac{9}{32\pi}\Big[4\cos^2\hat{\theta}_K(\hat{F}_L - \beta_\ell^2\tilde{F}_L\cos 2\theta_\ell) + (3\hat{F}_T + \beta_\ell^2\tilde{F}_T\cos 2\theta_\ell)\sin^2\hat{\theta}_K$$
$$+2\tilde{F}_T(\beta_\ell^2 P_1\cos 2\hat{\phi}\sin^2\hat{\theta}_K\sin^2\theta_\ell + 4\beta_\ell P_2\cos\theta_\ell\sin^2\hat{\theta}_K)\Big]\frac{d\Gamma_{K*}}{dq^2} + W_8$$

where $W_8 = \frac{1}{\pi}\left[\tilde{J}_{1a}^c + \tilde{J}_{2a}^c\cos 2\theta_\ell + \cos\hat{\phi}(\tilde{J}_5 + 2\tilde{J}_4\cos\theta_\ell)\sin\hat{\theta}_K\sin\theta_\ell\right]$

II. Identify $\phi \leftrightarrow -\phi$ $(\phi < 0)$ and $\theta_\ell \leftrightarrow \theta_\ell - \frac{\pi}{2}$ $(\theta_\ell > \frac{\pi}{2})$ with $\hat{\phi} \in [0, \pi]$, $\hat{\theta}_\ell \in [0, \pi/2]$

$$d\hat{\Gamma} = d\Gamma(\hat{\phi}, \hat{\theta}_\ell, \theta_K) + d\Gamma(\hat{\phi}, \hat{\theta}_\ell + \frac{\pi}{2}, \theta_K) + d\Gamma(-\hat{\phi}, \hat{\theta}_\ell, \theta_K) + d\Gamma(-\hat{\phi}, \hat{\theta}_\ell + \frac{\pi}{2}, \theta_K)$$

$$\frac{d^4\hat{\Gamma}}{dq^2\, d\cos\theta_K\, d\cos\hat{\theta}_l\, d\hat{\phi}} = \frac{9}{32\pi}\Big[\frac{1}{2}(4\hat{F}_L + 3\hat{F}_T + (4\hat{F}_L - 3\hat{F}_T)\cos 2\theta_K) +$$
$$+2\beta_\ell\sqrt{\tilde{F}_L\tilde{F}_T}P_5'\cos\hat{\phi}\sin 2\theta_K(\sin\hat{\theta}_\ell + \cos\hat{\theta}_\ell) +$$
$$+\tilde{F}_T\sin^2\theta_K(\beta_\ell^2 P_1\cos 2\hat{\phi} + 4\beta_\ell P_2(\cos\hat{\theta}_\ell - \sin\hat{\theta}_\ell))\Big]\frac{d\Gamma_{K*}}{dq^2} + W_{13}$$

where $W_{13} = \frac{1}{2\pi}\left[2\tilde{J}_{1a}^c + 2\tilde{J}_{1b}^c\cos\theta_K + \tilde{J}_5\cos\hat{\phi}\sin\theta_k(\cos\hat{\theta}_\ell + \sin\hat{\theta}_\ell)\right]$

The last important point is to discern which observable is inherently clean and which suffers from S-wave pollution. The products

$$\tilde{F}_T \frac{d\Gamma_{K^*}}{dq^2}, \quad \tilde{F}_L \frac{d\Gamma_{K^*}}{dq^2}, \quad P_i \tilde{F}_T \frac{d\Gamma_{K^*}}{dq^2} \quad P'_j \sqrt{\tilde{F}_L \tilde{F}_T} \frac{d\Gamma_{K^*}}{dq^2}$$

with $i = 1, 2, 3$ and $j = 4, 5, 6$ are all free from this contamination. **Notice** that they enter in numerator and denominator of **integrated observables**.

In conclusion both $P_i^{(\prime)}$ i=1..6 and the corresponding integrated observables extracted from folded distributions are free from S-wave pollution.

On the contrary, the S_i observables normalized by the full differential decay distribution will be directly affected by the S-wave pollution since the DDD is largely affected by this disease. **Consequently**

$$S_i = \frac{J_i + \bar{J}_i}{\frac{d\Gamma_{full}}{dq^2}}$$

and any other observable (for example a $\bar{F}_L$) normalized by $\frac{d\Gamma_{full}}{dq^2}$ is S-wave polluted, but $\bar{F}_L \frac{d\Gamma_{full}}{dq^2}$ is clean.

Work in progress

There are two types of extensions of this basis that we are exploring at present:

- P_i^{CP} is a corresponding basis of CP violating observables.
  In collaboration with J. Virto
- A complete basis of clean observables in the low-recoil region is now constructed: P_i^{low}.
  In collaboration with S. Descotes and J. Virto

Translation Table

J.M., F. Mescia, M. Ramon, J. Virto'12 $\rightarrow$ hep-ph 1202.4266
Egede et al'10 $\rightarrow$ hep-ph 1005.0571
F. Kruger, J.M'05 $\rightarrow$ hep-ph 0502060
Becirevic et al'12 $\rightarrow$ hep-ph 1106.3283
Bobeth et al'08 $\rightarrow$ hep-ph/ 0805.2525, 1006.5013
S. Descotes, D. Ghosh, JM, M. Ramon'11 $\rightarrow$ hep-ph 1104.3342, 1202.2172
S. Descotes, J.M., M. Ramon, J. Virto'12 $\rightarrow$ hep-ph 1207.2753
J.M'12 $\rightarrow$ **today's arxive** hep-ph 1209.1525
Altmannshofer et al.'09 $\rightarrow$ hep-ph 0811.1214
Khodjamirian et al.'10 $\rightarrow$ hep-ph 1006.4945
P. Ball and R. Zwicky,'05 $\rightarrow$ hep-ph 0412079
Becirevic, Tayduganov'12 $\rightarrow$ hep-ph 1207.4004