Theory status of $B^0 \to K^{(*)0}\mu^+\mu^-$ [and $B \to K^*\gamma$]

(— large hadronic recoil — low q^2 —)

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Outline

Naively Factorizing Contributions ($\rightarrow$ Form Factors)

- advantage of helicity-based form factors

QCD Factorization of Hadronic Operators

- charm-loop effects
- light-quark loops

QCDF at NLO

- vertex corrections
- specator scattering and annihilation
- power corrections

Conclusions

Guide-line: *Search for potential sources of theoretical uncertainties …*

Naively Factorizing Contributions

($\rightarrow$ Form Factors)

Semi-leptonic and electromagnetic Operators

$$\mathcal{O}_7 = -\frac{g_{\rm em} m_b}{8\pi^2}(\bar s \sigma^{\mu\nu}(1+\gamma_5)b)F_{\mu\nu}\,, \qquad \mathcal{O}_{9,10} = \frac{\alpha_{\rm em}}{2\pi}(\bar s \gamma_\mu(1-\gamma_5)b)(\bar\ell\ell)_{V,A}$$

("trivial") Factorization:

$$\begin{aligned} &\langle \bar K^{(*)}\,\ell\ell\,|\,(\bar s\, b)_{V-A}\,(\bar\ell\,\ell)_{V,A}\,|\bar B\rangle \\ =&\langle \bar K^{(*)}|\bar s\gamma_\mu(1-\gamma_5)b|\bar B\rangle \times \langle \ell\ell|(\bar\ell\,\ell)_{V,A}|0\rangle + \mathcal{O}(\alpha_{\rm em}) \end{aligned}$$

and similar for $\mathcal{O}_7$.

- Non-perturbative Input: Hadronic Form Factors

- At which level of precision do (non-factorizable) electromagnetic effects become important ?

Form-Factor Parametrizations $(B \to K)$

Convenient Definition ("helicity-based"): [e.g. Boyd/Savage 97, Bharucha/TF/Wick 10]

$$\mathcal{A}_{V,\sigma}(q^2) = \sqrt{\frac{q^2}{\lambda}}\, \varepsilon_\sigma^{*\mu}(q)\, \langle \bar K | \bar s \, \gamma_\mu \, b | \bar B \rangle$$

$$\mathcal{A}_{T,\sigma}(q^2) = (-i)\sqrt{\frac{1}{\lambda}}\, \varepsilon_\sigma^{*\mu}(q)\, \langle \bar K | \bar s \, \sigma_{\mu\nu} q^\nu \, b | \bar B \rangle$$

- $\varepsilon_\sigma(q)$: transverse, longitudinal, or time-like polarization vectors, $\sigma = \{\pm; 0; t\}$ or $\{1,2; 0; t\}$
- normalization: $\lambda = \left((M-m)^2 - q^2\right)\left((M-m)^2 - q^2\right)$

$$A_{V,0} \equiv f_+ \,, \qquad A_{V,t} \equiv \frac{M^2 - m^2}{\sqrt{\lambda}}\, f_0 \,, \qquad A_{T,0} \equiv \frac{\sqrt{q^2}}{M+m}\, f_T$$

Form-Factor Parametrizations $(B \to K^*)$

Similar for $B \to K^*$:

$$B_{V,0} \propto \; A_2 - \frac{(M+m)^2(M^2-m^2-q^2)}{\lambda} \, A_1$$

$$B_{T,0} \propto \; T_3 - \frac{(M^2-m^2)(M^2+3m^2-q^2)}{\lambda} \, T_2$$

$$B_{V,t} \propto A_0 \,, \quad B_{V,1} \propto V \,, \quad B_{V,2} \propto A_1 \,, \quad B_{T,1} \propto T_1 \,, \quad B_{T,2} \propto T_2$$

Non-perturbative calculations (lattice, sum rules)
should directly determine the linear combinations in

$B_{V,0}$ and $B_{T,0}$ instead of A_2 and T_3 !

Advantage of Helicity Form Factors

- definite spin-parity / diagonalization of unitarity relations ($\rightarrow$ simpler expressions for unitarity bounds)
- simple form of HQET/SCET symmetry relations for small/large recoil
- relatively simple expressions for observables in factorization approximation, e.g.

$$\frac{d\Gamma[B \to K\ell^+\ell^-]}{dq^2} \propto |C_{10}|^2 \, (A_{V,0})^2 + \left| C_9 \, A_{V,0} + \frac{2m_b \, C_7^{\rm eff}}{\sqrt{q^2}} \, A_{T,0} \right|^2$$

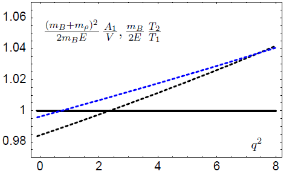
$$\stackrel{\rm SCET}{\longrightarrow} \left(|C_{10}|^2 + \left| C_9 + \frac{2m_b \, C_7^{\rm eff}}{M} \right|^2 \right) (A_{V,0})^2$$

similar for transverse/longitudinal rate and FB asymmetry

- in $B \to K^* \ell^+\ell^-$ [Bharucha/TF/Wick 10]
- in $\Lambda_b \to \Lambda \, \ell^+\ell^-$ [TF/Yip 2011]

- QCDF to $\mathcal{O}(\alpha_s)$ [Beneke/TF 00]
- Radiative corrections to $\mathcal{O}(\alpha_s^2)$ and SCET resummation

  [Beneke/Kiyo/Yang 04, Becher/Hill et al. 04, Beneke/Yang 05]



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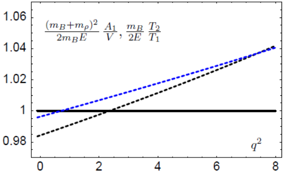

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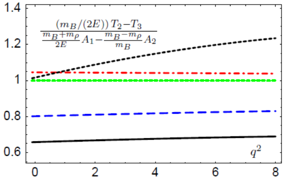

— Relation between $B_{T,0}$ and $B_{V,0}$ receives large spectator-scattering corrections, up to 40% (depending on B-meson LCDA)

(– – : without NLO+LL corrections to spectator term)

(– – : without any spectator corrections)

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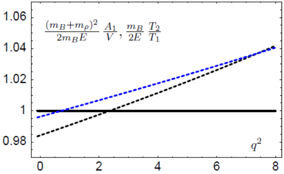

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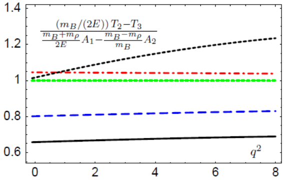

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- - QCD-SR estimate rather uncertain (power corrections, cancellations?)

- Relevant FF ratio for FB-Asymmetry receives $\mathcal{O}(10\%)$ corrections (!) (dependent on LCDAs for heavy and light meson) [Beneke/Yang 05]

Lessons from Leading Approximation

Basis for Global SM Fit or NP Constraints:

- Normalization and shape of differential decay rates proportional to

$$|V_{ts} V_{tb}^*|^2 \qquad \text{and} \qquad (\text{SCET form factors})^2$$

- Certain decay asymmetries are sensitive to

$$\frac{2m_b\, C_7^{\text{eff}}}{M}\,, \quad C_9\,, \quad C_{10} \qquad \text{and/or} \qquad \text{form-factor RATIOS}$$

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- How well do we know the hadronic form factors,
  in particular for decays into unstable particles like K^* or ρ **?**
  → Also use experimental data to find best-fitting values for FFs **!**
- Precision of SCET symmetry relations and estimates for its corrections **?**

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  $\rightarrow$ Also use experimental data to find best-fitting values for FFs **!**
- Precision of SCET symmetry relations and estimates for its corrections **?**

- How do (non-factorizable) effects from hadronic operators affect
  - Theoretical predictions for decay rates and asymmetries **?**
  - Modelling of $B \to (K\pi)\,\ell\ell$ (non-resonant) background **?**

QCDF of Hadronic Operators

- Perturbative Analysis, $\alpha_s \ll 1$
  - hard momentum modes $\mu \sim m_b$ in vertex corrections
  - hard-collinear modes $\mu \sim \sqrt{m_b \Lambda}$ in spectator scattering
  - soft and collinear modes in B-meson and kaon LCDAs.
- Dominated (?) by leading power in $1/m_b$ and $1/E_K$
  - non-factorizable contributions (at sub-leading power)

Hadronic Operators (contributions from long-distance photons)

[à la Beneke/TF/Seidel 01]

Additional Contributions from $\mathcal{O}_{1-6}$, $\mathcal{O}_8^g$ + Photon Radiation (in the SM)

$$C_9 + \frac{2m_b\, C_7^{\rm eff}}{\sqrt{q^2}}\,\frac{A_{T,0}}{A_{V,0}} + \ldots \longrightarrow \mathcal{C}_9^{(K)}(q^2) \equiv C_9 + \frac{2m_b}{M}\,\frac{\mathcal{T}^{(K)}(q^2)}{A_{V,0}(q^2)}$$

- All information contained in q^2-dependent functions $\mathcal{T}(q^2)$, encoding the hadronic matrix element $\langle \bar{K}^{(*)}\gamma^*|H_{\rm eff}|\bar{B}\rangle$.

[à la Beneke/TF/Seidel 01]

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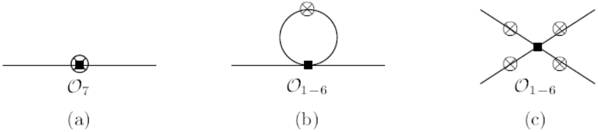
$$C_9 + \frac{2m_b\, C_7^{\rm eff}}{\sqrt{q^2}}\,\frac{B_{T,0}}{B_{V,0}} + \ldots \longrightarrow \mathcal{C}_{9\parallel}^{(K^*)}(q^2) \equiv C_9 - \frac{2m_b}{M}\,\frac{\mathcal{T}_\parallel^{(K^*)}(q^2)}{B_{V,0}(q^2)}$$

$$C_9 + \frac{2m_b\, C_7^{\rm eff}}{\sqrt{q^2}}\,\frac{B_{T,i}}{B_{V,i}} + \ldots \longrightarrow \mathcal{C}_{9\perp}^{(K^*)}(q^2) \equiv C_9 + \frac{2m_b M}{q^2}\,\frac{\mathcal{T}_\perp^{(K^*)}(q^2)}{B_{V,i}(q^2)}$$

$(i = 1, 2)$

- All information contained in q^2-dependent functions $\mathcal{T}(q^2)$, encoding the hadronic matrix element $\langle \bar K^{(*)}\gamma^*|H_{\rm eff}|\bar B\rangle$.
- In the large recoil limit, $\mathcal{T}_{\perp,1} = \mathcal{T}_{\perp,2} \equiv \mathcal{T}_\perp$.
- For $B \to K^*\gamma$ and FBA, only $\mathcal{T}_\perp$ contributes.

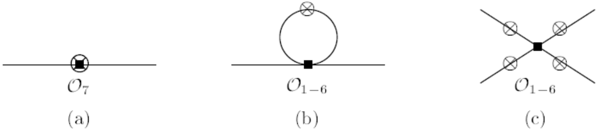
Contributions to $\mathcal{T}_X(q^2)$ at zero$^{\rm th}$ order in QCDF



(a) Photon from $\mathcal{O}_7$, (spectator quark not drawn)
included in Naive Factorization $\surd$

(see above)

Contributions to $\mathcal{T}_X(q^2)$ at zero$^{\text{th}}$ order in QCDF



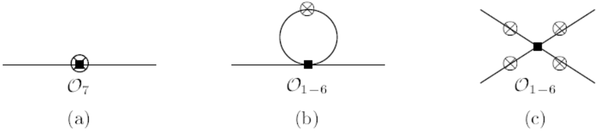
(b) Photon from Quark Loops with hadronic operators $\mathcal{O}_{1-6}$
(spectator quark not drawn)

$$C_7 \to C_7^{\text{eff}}\,, \qquad C_9 \to C_9 + Y(q^2) \qquad (\surd)$$

- Generates imaginary part for $q^2 \geq 4m_q^2$!
- Perturbative description valid near resonances ?

(see below)

Contributions to $\mathcal{T}_X(q^2)$ at zero$^{\text{th}}$ order in QCDF



(c) Photon from Annihilation Topologies with $\mathcal{O}_{1-6}$

- enters with small penguin coefficients $C_{3,4}$ (or with $C_{1,2}$ and small CKM factor)
- leading contribution if photon radiated from light quark in B-meson

proportional to $$\int_0^\infty \frac{d\omega}{\omega - q^2/M - i\epsilon}\,\phi_B^-(\omega) \quad \rightarrow \quad \text{imaginary part (!)}$$

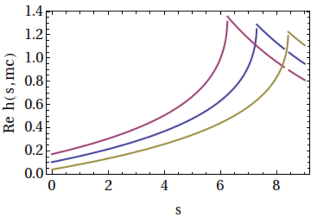
- sub-leading terms important for isospin asymmetries !

(see below)

Threshold-dependence in function $Y(q^2)$

$$Y(s) = h(s, m_c) \left(3\bar{C}_1 + \bar{C}_2 + 3\bar{C}_3 + \bar{C}_4 + 3\bar{C}_5 + \bar{C}_6\right) - \frac{1}{2}\, h(s,0) \left(\bar{C}_3 + 3\bar{C}_4\right)$$
$$- \frac{1}{2}\, h(s, m_b) \left(4\,(\bar{C}_3 + \bar{C}_4) + 3\bar{C}_5 + \bar{C}_6\right) + \frac{2}{9} \left(\frac{2}{3}\bar{C}_3 + 2\bar{C}_4 + \frac{16}{3}\bar{C}_5\right)$$

Charm Loop – Function $h(s = q^2, m_c)$



enters with large Wilson coefficients $C_{1,2}$

- Sensitivity to $m_c = \{1.25, 1.35, 1.45\}$ GeV
- Perturbatively stable, **IF** $q^2 \ll 4m_c^2$
- Irreducible error around $q^2 \gtrsim 6$ GeV^2

$\rightarrow$ Estimate of systematic error in QCDF due to charm resonances *below* threshold !

— need for model-dependent “improvement” ? — [Khodjamirian et al. 2010]

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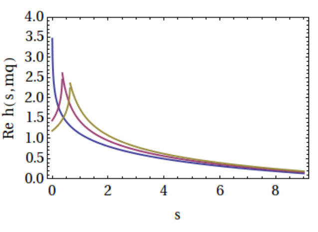
Light-Quark Loop – Function $h(s = q^2, m_q \to 0)$



enters with small penguin coefficients $C_{3,4}$
(or with $C_{1,2}$ and small CKM factor)

- Sensitivity to $m_q = \{0.003, 0.3, 0.4\}$ GeV
- Perturbatively stable, **IF** $q^2 \gg 1$ GeV^2
- Irreducible error around $q^2 \lesssim 2$ GeV^2

(?) Estimate of systematic error in QCDF
due to light resonances *above* threshold (?)

— in any case sub-leading for $B \to K^{(*)}$ — error not included in [Beneke/TF/Seidel]

Threshold-dependence in function $Y(q^2)$

$$Y(s) = h(s, m_c) \left(3\bar{C}_1 + \bar{C}_2 + 3\bar{C}_3 + \bar{C}_4 + 3\bar{C}_5 + \bar{C}_6\right) - \frac{1}{2}\, h(s,0) \left(\bar{C}_3 + 3\bar{C}_4\right)$$
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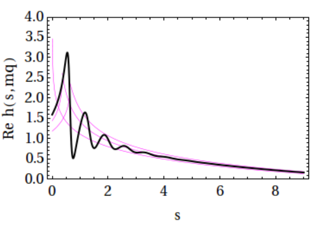

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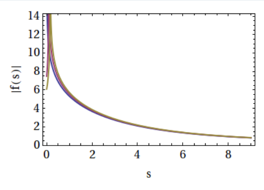
(?) Estimate of systematic error in QCDF
vs. Shifman-like model for light resonances

— for illustration only —

IR-Sensitivity from B-Meson LCDA

$$f(q^2) \equiv \int_0^\infty d\omega \, \frac{\phi_B^{(-)}(\omega)}{\omega - q^2/M + m_q^2/M - i\epsilon}\,, \qquad \phi_B^{(-)}(\omega) = \frac{e^{-\omega/\omega_0}}{\omega_0}$$

$f(s)$ enters annihilation and hard-scattering diagrams



- Sensitivity to $m_q = \{0.003, 0.3, 0.4\}$ GeV
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- Small uncertainty for $q^2 < 2$ GeV^2

$\rightarrow$ Systematic error in QCDF

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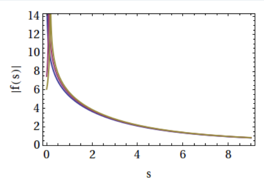

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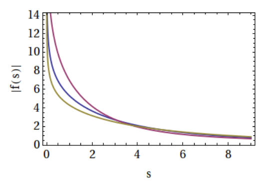
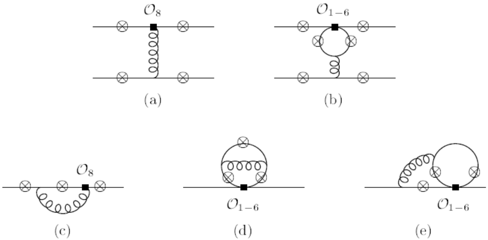


- Sensitivity to $\omega_0 = \{0.2, 0.35, 0.5\}$ GeV
- Affects region where $q^2 \sim \omega_0 M$.

$\rightarrow$ Parametric error in QCDF

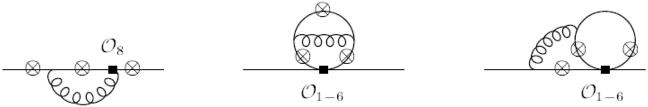
QCDF at NLO



$\mathcal{O}_8$
$\mathcal{O}_{1-6}$
(a)
(b)
$\mathcal{O}_8$
$\mathcal{O}_{1-6}$
$\mathcal{O}_{1-6}$
(c)
(d)
(e)

Non-factorizable Vertex Corrections

(Factorizable vertex corrections included in form-factor ratios.)

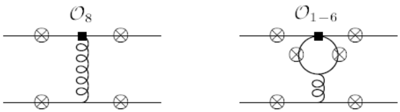


- Independent of spectator quark — Same as for inclusive spectra.
- Essential to reduce renormalization-scale ambiguities in LO result. $\surd$
- α_s and m_c-dependence contribute to error budget !

Uncertainties mainly a perturbative issue (away from resonances).

Non-factorizable Spectator Scattering

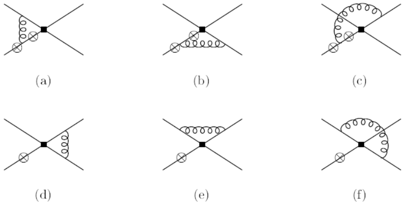
(Factorizable spectator scattering included in form-factor ratios.)



- Depends on spectator charge $\rightarrow$ isospin asymmetry.
- Depends on LCDAs for heavy and light mesons !
  $\rightarrow$ included in hadronic (parametric) uncertainties. ($\sim$ 50% uncertain)
- Introduces new source of scale ambiguity !
  $\rightarrow$ included in perturbative uncertainties.
- Non-factorizable quark-loop contributions !
  $\rightarrow$ more complicated intermediate partonic/hadronic states.

Deserves further studies ...

Annihilation at NLO



- Not included in [Beneke/TF/Seidel]
  $\to$ Scale ambiguity from penguin coefficients C_{3-6} not fully resolved. !

Potentially relevant for (accurate) isospin asymmetries ...

Power Corrections

- Power corrections (generally) not well-defined in QCDF ?
  sensitivity to endpoint configurations:

$$\int_0^1 \frac{du}{1-u}\,\frac{\phi_K(u)}{1-u} = ?\,, \qquad \int_0^\infty \frac{d\omega}{\omega}\,\frac{\phi_B^{(+)}(\omega)}{\omega} = ?$$

- Ad-hoc regularization
  $\to$ poor man's error estimate in QCDF !(?)

$$\frac{du}{1-u} \longrightarrow \left(1+\rho e^{i\phi}\right)\frac{du}{1-u}\,\theta\left[1-\frac{\Lambda}{M}-u\right]$$

- Alternative approaches (sum rules, $k_\perp$-factorization, ...)
  $\to$ hadronic modeling (spectral functions, wave functions, ...)

(Still) awaits resolution within SCET ...

Conclusions

Theoretically Sound:

- QFT Formalism: Heavy-quark/large-recoil expansion – QCDF — SCET.
- Transparent predictions for LO contributions:
  Wilson coefficients & helicity-based FFs.
- Reasonable accuracy for factorizable and non-factorizable NLO corrections.
- Reasonable control on (factorizable) quark-loop contributions.

Open Issues:

- Theoretical and experimental treatment of unstable particles (K^*, $\rho, \ldots$).
- Ultimate accuracy for form factors and form-factor ratios.
  [see also discussion by Matthew Wingate]
- Better control on spectator scattering, power corrections, annihilation.
- Significant duality violation through non-factorizable quark loops (?)
- Non-factorizable electromagnetic corrections.
- ...

Some issues may actually be resolved from experimental data !

Backup Slides

- Annihilation:

$$\Delta\mathcal{T}_\perp^{(K^*)}\Big|_{\rm ann} = -e_q\,\frac{4\pi^2}{3}\,\frac{f_B f_\perp^{(K^*)}}{m_b M}\left(C_3+\frac{4}{3}(C_4+3C_5+4C_6)\right)\int_0^1 du\,\frac{\phi_\perp^{(K^*)}(u)}{\bar u+u\hat s} + e_q\,\frac{2\pi^2}{3}\,\frac{f_B f_\parallel^{(K^*)}}{m_b M}\left(C_3+\frac{4}{3}(C_4+12C_5+16C_6)\right)\frac{m_{K^*}}{(1-\hat s)\,\lambda_{B,+}(q^2)}$$

- Hard-Scattering:

$$\Delta\mathcal{T}_\perp^{(K^*)}\Big|_{\rm hsa} = e_q\,\frac{\alpha_s C_F}{4\pi}\,\frac{\pi^2 f_B}{N_c m_b M}\Bigg\{12C_8^{\rm eff}\,\frac{m_b}{M}\,f_\perp^{(K^*)}X_\perp(\hat s) + 8f_\perp^{(K^*)}\int_0^1 du\,\frac{\phi_\perp^{(K^*)}(u)}{\bar u+u\hat s}\,F_V(\bar u+u\hat s) - \frac{4m_{K^*}f_\parallel^{(K^*)}}{(1-\hat s)\,\lambda_{B,+}(q^2)}\int_0^1 du\int_0^u dv\,\frac{\phi_\parallel^{(K^*)}(v)}{\bar v}\,F_V(\bar u+u\hat s)\Bigg\}$$

  - threshold dependence of quark-loop function $F_V(\bar u+u\hat s)$
  - endpoint divergence in function $X_\perp(\hat s)$
  - ...