

# THEORY OF $B \rightarrow K^{(*)}l^+l^-$ DECAYS AT HIGH $q^2$

---

Gerhard Buchalla

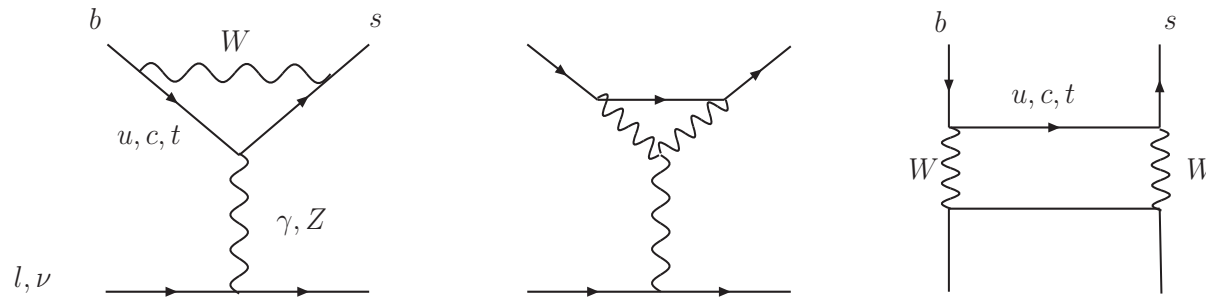
LMU München

Workshop on the physics reach of rare and exclusive  $B$  decays

University of Sussex, Brighton, 10 – 11 September 2012

- OPE for hadronic contribution to  $B \rightarrow K^{(*)}l^+l^-$
- Quark-hadron duality
- Conclusions

*Beylich, G.B., Feldmann*

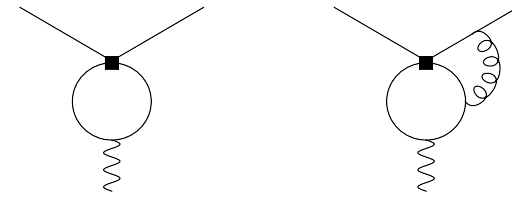


$$\frac{dB(\bar{B} \rightarrow \bar{K}l^+l^-)}{ds} = \tau_B \frac{G_F^2 \alpha^2 m_B^5}{1536\pi^5} |V_{ts}V_{tb}|^2 \lambda_K^{3/2}(s) f_+^2(s) (|a_9|^2 + |a_{10}|^2)$$

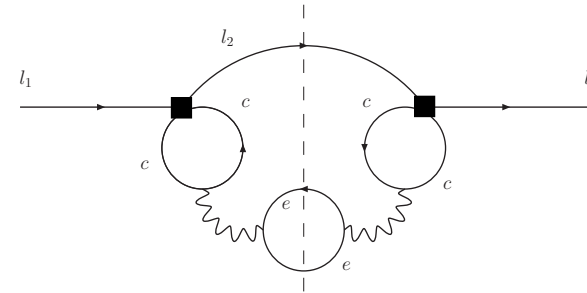
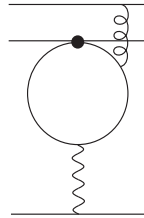
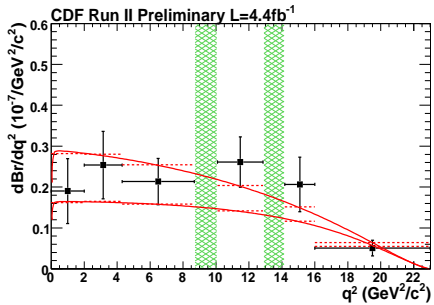
$$A(\bar{B} \rightarrow \bar{M} l^+ l^-) = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi} \lambda_t \left[ A_9^\mu \bar{l} \gamma_\mu l + A_{10}^\mu \bar{l} \gamma_\mu \gamma_5 l \right]$$

$$A_9^\mu = C_9 \langle \bar{M} | \bar{s} \gamma^\mu (1 - \gamma_5) b | \bar{B} \rangle + \langle \bar{M} | \mathcal{K}_H^\mu(q) | \bar{B} \rangle + \dots$$

$$\mathcal{K}_H^\mu(q) = -\frac{8\pi^2}{q^2} i \int d^4x e^{iq \cdot x} T j^\mu(x) H^c(0)$$



- semileptonic contribution
- hadronic contribution (“charm loops”)



$$|a_9|^2 = |C_9 + \Delta|^2 = C_9^2 + 2C_9\text{Re}\Delta + |\Delta|^2$$

$$\Delta_{\text{res}} = \frac{-f^2}{q^2 - M^2 + iM\Gamma}$$

$$|\Delta_{\text{res}}|^2 = \frac{f^2}{M\Gamma} \text{Im}\Delta_{\text{res}}$$

*Beneke, G.B., Neubert, Sachrajda*

•  $B \rightarrow K\psi^{(\prime)} \rightarrow Kl^+l^-$  dominates by factor  $10^2 \rightarrow$  remove by cuts

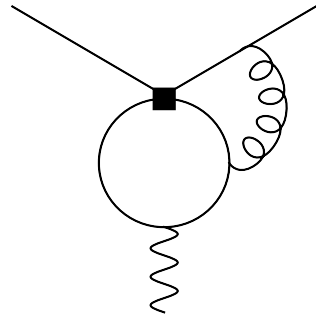
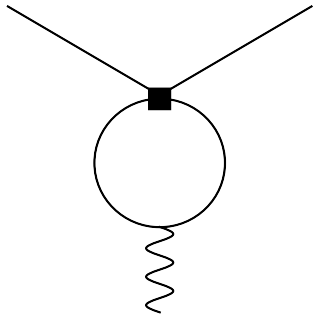
• low- $q^2$ : QCD factorization

*Beneke, Feldmann, Seidel*

• high- $q^2$ : OPE

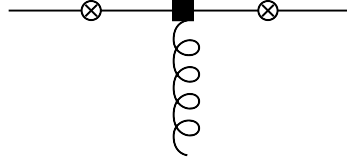
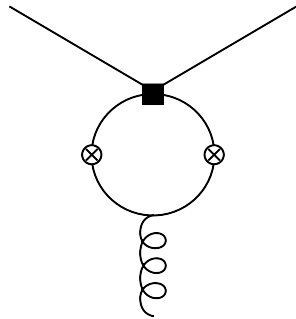
*G.B., Isidori; Grinstein, Pirjol*

$$\mathcal{K}_H^\mu(q) = -\frac{8\pi^2}{q^2} i \int d^4x e^{iq \cdot x} T j^\mu(x) H^c(0) = \sum_{d,n} C_{d,n}(q) \mathcal{O}_{d,n}^\mu \quad \Lambda, E_K \ll \sqrt{q^2}$$

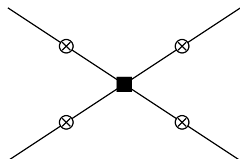


$$\mathcal{O}_{3,1}^\mu = \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{s} \gamma_\nu (1 - \gamma_5) b$$

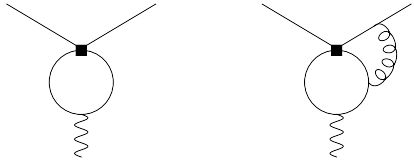
$$\mathcal{O}_{3,2}^\mu = \frac{im_b}{q^2} q_\lambda \bar{s} \sigma^{\lambda\mu} (1 + \gamma_5) b$$



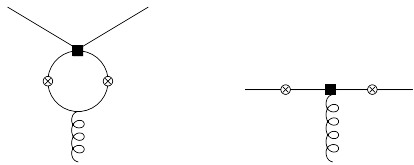
$$\mathcal{O}_{5,n}^\mu = \bar{s} (g G \Gamma_n)^\mu b$$



$$\mathcal{O}_{6ann,n}^\mu = (\bar{r} \Gamma_1 b \bar{s} \Gamma_2 r)_n^\mu$$



$$\mathcal{K}_{H3}^\mu = \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \bar{s} \gamma_\nu (1 - \gamma_5) b \cdot h(x) (C_1 + 3C_2) + \dots, \quad x = \frac{4m_c^2}{q^2}$$



$$\mathcal{K}_{H5}^\mu = \left[ \varepsilon^{\alpha\beta\lambda\rho} \frac{q_\beta q^\mu}{q^2} + \varepsilon^{\beta\mu\lambda\rho} \frac{q_\beta q^\alpha}{q^2} - \varepsilon^{\alpha\mu\lambda\rho} \right] \bar{s} \gamma_\lambda (1 - \gamma_5) g G_{\alpha\rho} b \frac{C_1 Q_c}{q^2} f(x) - \frac{q_\lambda}{m_B} \bar{s} g G_{\alpha\beta} (g^{\alpha\lambda} \sigma^{\beta\mu} - g^{\alpha\mu} \sigma^{\beta\lambda}) (1 + \gamma_5) b \frac{4C_8 Q_b}{q^2}$$

$$f(x) = \frac{x}{\sqrt{1-x}} \left( \ln \frac{1-\sqrt{1-x}}{1+\sqrt{1-x}} + i\pi \right) - 2$$

*G.B., Isidori, Rey*

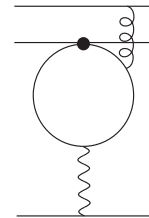
$$a_9 \approx 4$$

$$\Delta a_{9,H5}(K) = -\frac{\pi\alpha_s(E_K)C_F}{2N} C_1 Q_c f(x) \frac{m_B f_B f_K}{\lambda_B f_+(q^2) q^2} \approx 0.019 - 0.012i$$

$$\Delta a_{9,H5}(K_{\perp}^*) = -\frac{\pi\alpha_s(E_K)C_F}{4N} \frac{m_B f_B f_{\perp}}{\lambda_B V(q^2) q^2} (C_1 Q_c f(x) + 8C_8 Q_b) \approx 0.008 - 0.006i$$

(for  $q^2 = 15 \text{ GeV}^2$ )

- explicit calculation of 2nd order power corrections for  $\Lambda \ll E_K \ll \sqrt{q^2}$  ( $q^2 \gtrsim 15 \text{ GeV}^2$ )
- consistent with QCDF ( $\Lambda \ll E_K$ , any  $q^2$ )
- impact below 1%



*Beneke, G.B., Neubert, Sachrajda*

$$\mathcal{H}_{\text{eff}} = \frac{G}{\sqrt{2}} \left[ (\bar{l}_2 l_1)_{V-A} (\bar{c} c)_{V-A} - (\bar{l}_2 l_1)_{V-A} (\bar{t} t)_{V-A} \right]$$

$$A(l_1 \rightarrow l_2 e^+ e^-) = -\frac{G}{\sqrt{2}} e_c e^2 \Pi(q^2) \bar{l}_2 \gamma^\mu (1 - \gamma_5) l_1 \bar{e} \gamma_\mu e$$

$$\Pi \equiv \Pi_c - \Pi_t, \quad \Pi(0) = \frac{N}{12\pi^2} \ln \frac{m_t^2}{m_c^2}$$

$$\frac{d\Gamma(l_1 \rightarrow l_2 e^+ e^-)}{ds} = \frac{G^2 \alpha^2 m_1^5}{108\pi^5} (1-s)^2 (1+2s) |C + \Delta(q^2)|^2, \quad s = \frac{q^2}{m_1^2}$$

$$C \equiv 2\pi^2 \Pi(0) \quad \Delta(q^2) \equiv 2\pi^2 (\Pi(q^2) - \Pi(0))$$

$$|C + \Delta|^2 = C^2 + 2C \text{Re}\Delta + |\Delta|^2$$

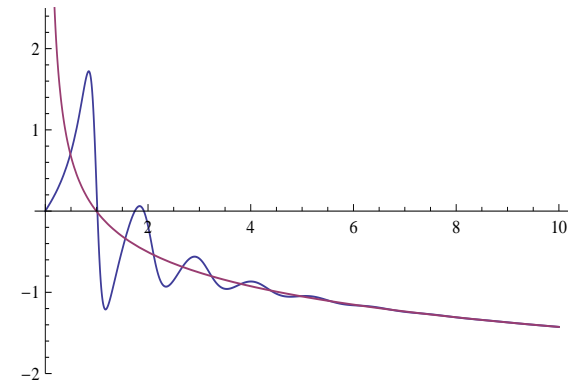


$$\Pi(q^2) = c \sum_{n=1}^{\infty} \frac{1}{z+n}, \quad z = \left( \frac{-q^2 - i\epsilon}{\lambda^2} \right)^{1-b/\pi}, \quad b = \frac{\Gamma_n}{M_n}$$

$$\Rightarrow \Pi(q^2) - \Pi(0) = -\frac{N}{12\pi^2} \frac{1}{1-b/\pi} [\psi(z+1) + \gamma] \rightarrow -\frac{N}{12\pi^2} \ln \frac{-q^2 - i\epsilon}{\lambda^2}$$

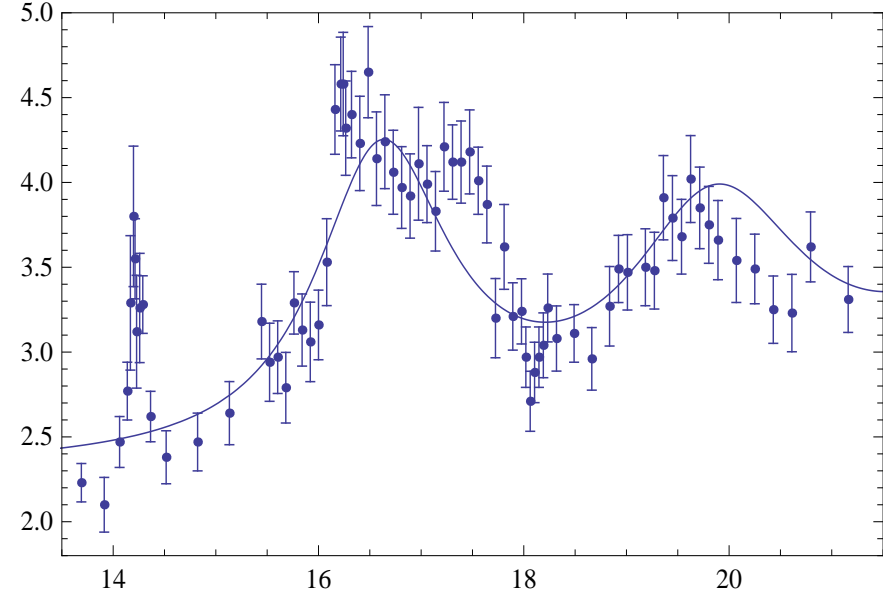
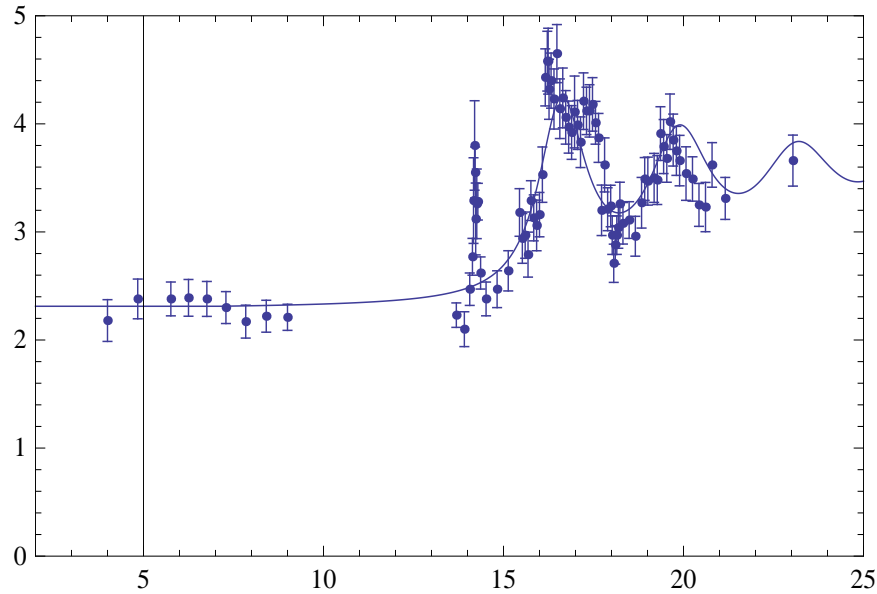
asymptotic series:  $\psi(z+1) = \ln z - \sum_{k=1}^{\infty} \frac{B_k}{k} \frac{1}{z^k}$

$$\text{Re}\Delta(q^2/\lambda^2)$$



$$\psi(z+1) + \gamma \equiv [\psi(-z) + \gamma - i\pi]_{OPE} + [-\pi \cot \pi z + i\pi]_{DV}$$

$$[-\pi \cot \pi z + i\pi] \approx 2\pi \exp\left(\frac{-2\pi b q^2}{\lambda^2}\right) \left[ \sin\left(\frac{2\pi q^2}{\lambda^2}\right) - i \cos\left(\frac{2\pi q^2}{\lambda^2}\right) \right]$$



$$R = R_{\text{light}} - \frac{4}{3} \frac{1}{(1-b/\pi)\pi} \text{Im} \psi(3+z), \quad z = \left( -\frac{q^2 - 4m_c^2 + i\epsilon}{\lambda^2} \right)^{1-b/\pi}$$

$$\lambda^2 = 3.08 \text{ GeV}^2, \quad m_c = 1.33 \text{ GeV}, \quad b = 0.082$$

$$\langle \mathcal{K}_H^\mu \rangle = \frac{16\pi^2}{3} a_2 \langle (\bar{s}b)_{V-A} \rangle^\mu \Pi_c(q^2), \quad a_2 \approx 0.3$$

$$\Delta a_9 = a_2 d, \quad d \equiv \frac{16\pi^2}{3} (\Pi_c(q^2) - \Pi_c(0))$$

$$d = -\frac{4}{3} \frac{1}{1-b/\pi} [\psi(z+3) - \psi(z_0+3)]$$

$$\rightarrow d_{DV} \approx -\frac{8\pi}{3} \exp(-2\pi br) (\sin 2\pi r - i \cos 2\pi r)$$

$$r = (q^2 - 4m_c^2)/\lambda^2, \quad s = q^2/m_B^2$$

$$|R_{DV,1}| = \left| \frac{2a_2}{a_9} \frac{\int_{s_0}^{sm} ds \varphi(s) \text{Red}_2}{\int_{s_0}^{sm} ds \varphi(s)} \right| \lesssim \frac{8}{3} \frac{a_2}{a_9} \frac{\varphi(s_0)}{\int_{s_0}^{sm} ds \varphi(s)} \frac{\lambda^2}{m_B^2} \exp(-2\pi b(q_0^2 - 4m_c^2)/\lambda^2)$$

impact on high- $q^2$  rate  $\sim 1.5\%$

# Conclusions

---

- $B \rightarrow K\psi \rightarrow Kl^+l^-$  background:  
huge duality violation for  $\psi(1S)$ ,  $\psi(2S)$  region ( $\leftrightarrow |\Pi(q^2)|^2$ )
- high  $q^2$ : OPE (in  $\Lambda/m_b$ ) for hadronic amplitude in  $B \rightarrow K^{(*)}l^+l^-$ :  
no 1st order power corrections, 2nd order small
- high  $q^2$ : quantitative model estimate of duality violation (small)
- excellent control of  $B \rightarrow K^{(*)}l^+l^-$  at high (and low)  $q^2$
- items for discussion: intermediate  $q^2$   
 $B \rightarrow K^{(*)}\psi_n, \psi_n \rightarrow l^+l^-$   
duality violation at low  $q^2$

---

# Backup

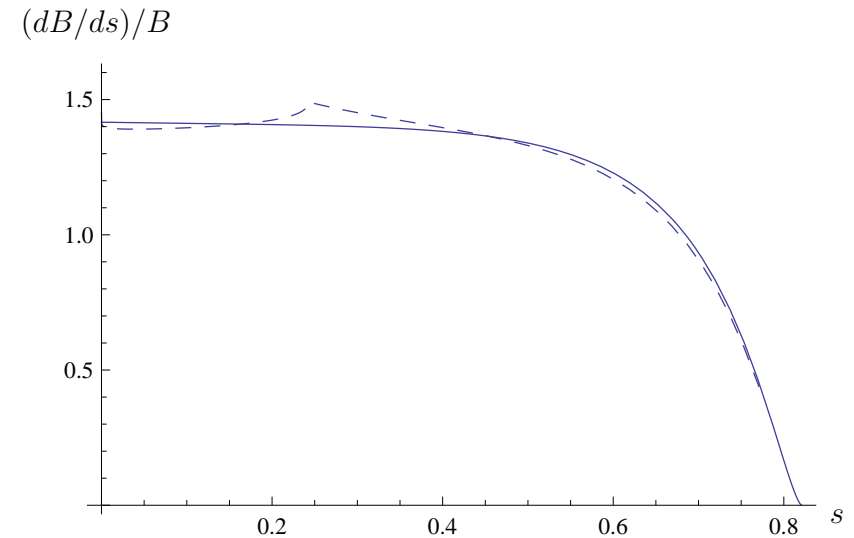
*Hurth, Wyler; Bartsch, Beylich, G.B., Gao*

normalized spectrum:

$B \rightarrow K \nu \bar{\nu}$  (solid)

$B \rightarrow K l^+ l^-$  (dashed)

$$s = q^2 / m_B^2$$



$$0 \leq s \leq 0.25 \text{ (low } s) \quad 0.6 \leq s \leq s_m = 0.82 \text{ (high } s)$$

$$R_{256} \equiv \frac{\int_0^{s_m} ds dB(B^- \rightarrow K^- \nu \bar{\nu})/ds}{\int_0^{0.25} ds dB(B^- \rightarrow K^- l^+ l^-)/ds + \int_{0.6}^{s_m} ds dB(B^- \rightarrow K^- l^+ l^-)/ds} \equiv$$

$$14.60^{+0.28}_{-0.38} (a_0) \quad ^{+0.10}_{-0.02} (b_1) \quad ^{-0.80}_{+0.62} (\mu)$$

- modified  $Z$ -penguin (SUSY) could suppress  $R_{256}$

*Altmannshofer et al.*

modification of  $B \rightarrow K \nu \bar{\nu}$

- $B \rightarrow K S S$  light invisible scalars

*Bird et al.*

- $B \rightarrow K \tilde{\chi}_1^0 \tilde{\chi}_1^0$  light neutralinos

*Dreiner et al.*

- $Z'$  models

*Altmannshofer et al.*

- $B \rightarrow K \nu_\tau \bar{\nu}_\tau$

topcolor assisted technicolor

$Z'$  with FCNC at tree level

*G.B., Burdman, Hill, Kominis*