$B \to V$ form factors

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Outline

- Small recoil (large q^2): lattice QCD
- Large recoil (low q^2): light-cone sum rules
- Comparison
- Combination

Form factor definitions

$$\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^{\mu}b|B(p)\rangle = \frac{2iV(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_{\nu}p'_{\rho}p_{\sigma}$$

$$\begin{aligned}\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^{\mu}\hat{\gamma}^5 b|B(p)\rangle &= 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^{\mu} \\ &+(m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^{\mu}\right) \\ &-A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+p')^{\mu}-\frac{m_B^2-m_V^2}{q^2}q^{\mu}\right)\end{aligned}$$

$$q^{\nu}\langle V(p',\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle = 2T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^{\tau}p'^{\sigma}$$

$$\begin{aligned}q^{\nu}\langle V(p',\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle &= iT_2(q^2)[\varepsilon^*_{\mu}(m_B^2-m_V^2)-(\varepsilon^*\cdot q)(p+p')_{\mu}] \\ &+iT_3(q^2)(\varepsilon^*\cdot q)\left[q_{\mu}-\frac{q^2}{m_B^2-m_V^2}(p+p')_{\mu}\right]\end{aligned}$$

LQCD

Correlation functions

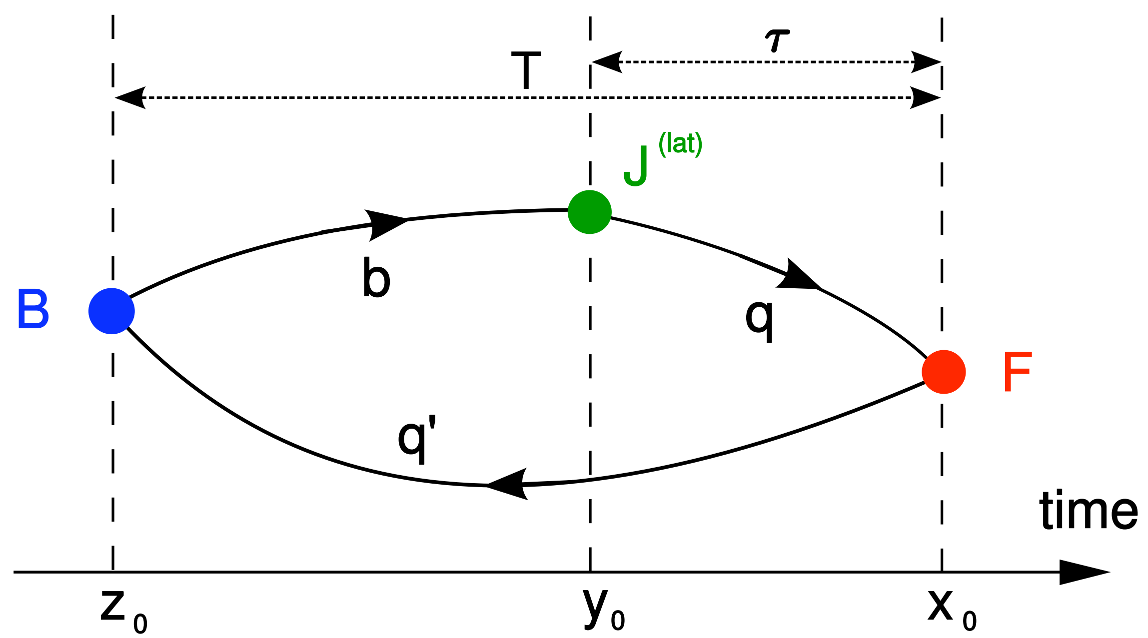
3-point function

$$C_{FJB}(\mathbf{p}', \mathbf{p}, x_0, y_0, z_0) = \sum_{\mathbf{y}} \sum_{\mathbf{z}} \left\langle \Phi_F(x)\, J(y)\, \Phi_B^\dagger(z) \right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})} e^{-i\mathbf{p}\cdot(\mathbf{y}-\mathbf{z})}$$

2-point functions

$$C_{BB}(\mathbf{p}, x_0, y_0) = \sum_{\mathbf{x}} \left\langle \Phi_B(x)\, \Phi_B^\dagger(y) \right\rangle e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})},$$

$$C_{FF}(\mathbf{p}', x_0, y_0) = \sum_{\mathbf{x}} \left\langle \Phi_F(x)\, \Phi_F^\dagger(y) \right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})}.$$



Interpolating operators

$$\Phi_V = \bar{u}\gamma_j s$$

$$\Phi_B = \bar{u}\gamma_5 b$$

Correlation functions

Large Euclidean-time behavior

$$
\begin{aligned}
C_{FJB}(\mathbf{p}',\,\mathbf{p},\,\tau,\,T) &\rightarrow A^{(FJB)} e^{-E_F\tau}\, e^{-E_B(T-\tau)}, \\
C_{FF}(\mathbf{p},\,\tau) &\rightarrow A^{(FF)}\, e^{-E_F\tau}, \\
C_{BB}(\mathbf{p},\,\tau) &\rightarrow A^{(BB)}\, e^{-E_B\tau},
\end{aligned}
$$

$$
A^{(FJB)} = \frac{\sqrt{Z_V}}{2E_V}\frac{\sqrt{Z_B}}{2E_B} \boxed{\sum_s \varepsilon_j(p',s)\,\langle V\,\big(p',\varepsilon(p',s)\big)\,|\,J\,|B(p)\rangle,}
$$

$$
A^{(FF)} = \sum_s \frac{Z_V}{2E_V}\,\varepsilon_j^*(p',s)\varepsilon_j(p',s)
$$

$$
A^{(BB)} = \frac{Z_B}{2E_B},
$$

LQCD Results

Unquenched LQCD calculation

Horgan, Liu, Meinel, Wingate, in preparation

MILC lattices (2+1 asqtad staggered)

label	#	$N_x^3 \times N_t$	$am_\ell^{\rm sea}/am_s^{\rm sea}$	r_1/a	$1/a$ (GeV)
c007	2109	$20^3 \times 64$	0.007/0.05	2.625(3)	1.660(12)
c02	2052	$20^3 \times 64$	0.02/0.05	2.644(3)	1.665(12)
f0062	1910	$28^3 \times 96$	0.0062/0.031	3.699(3)	2.330(17)

ensemble	m_B (GeV)	m_{B_s} (GeV)	m_π (MeV)	m_K (MeV)	m_{η_s} (MeV)	m_ρ (MeV)	m_{K^*} (MeV)	m_ϕ (MeV)
c007	5.5439(32)	5.6233(7)	313.4(1)	563.1(1)	731.9(1)	892(28)	1045(6)	1142(3)
c02	5.5903(44)	5.6344(15)	519.2(1)	633.4(1)	730.6(1)	1050(7)	1106(4)	1162(3)
f0062	5.5785(22)	5.6629(13)	344.3(1)	589.3(2)	762.0(1)	971(7)	1035(4)	1134(2)
“physical”	5.279	5.366	140	495	686	775	892	1020

NRQCD b quarks

- Effective field theory, cutoff by lattice
- HQET power counting: requires working with low recoil
- Current matching

$$(\bar{q}\Gamma_\mu^{V,A} b)|_{\text{cont}} \doteq (1+\alpha_s \rho^{(\mu)})(\bar{c}\Gamma_\mu^{V,A} b)|_{\text{latt}}$$

$$(\bar{q}\hat{\sigma}_{\mu\nu} b)|_{\text{cont}} \doteq (1+\alpha_s c^{(T\nu)})(\bar{q}\hat{\sigma}_{\mu\nu} b)|_{\text{latt}}$$

ensemble	C_v	$\rho^{(0)}$	$\rho^{(k)}$	$c^{(T0)}$	$c^{(Tj)}$
c	2.825	0.043	0.270	0.076	0.076
f	1.996	−0.058	0.332	0.320	0.320

Gulez et al., PRD69 (2003), PRD73 (2006); Mueller et al., PRD83 (2011)

Effective pole models

$$F(t) = \frac{r_1}{1 - t/m_R^2} + \frac{r_2}{1 - t/m_{\rm fit}^2} + \frac{r_3}{(1 - t/m_{\rm fit}^2)^2}$$

$$r_3 = 0 \text{ for } V, A_0, T_1$$

$$r_1 = r_3 = 0 \text{ for } A_1, T_2$$

$$r_1 = 0 \text{ for } A_2, T_3$$

Becirevic & Kaidalov;
Ball & Zwicky

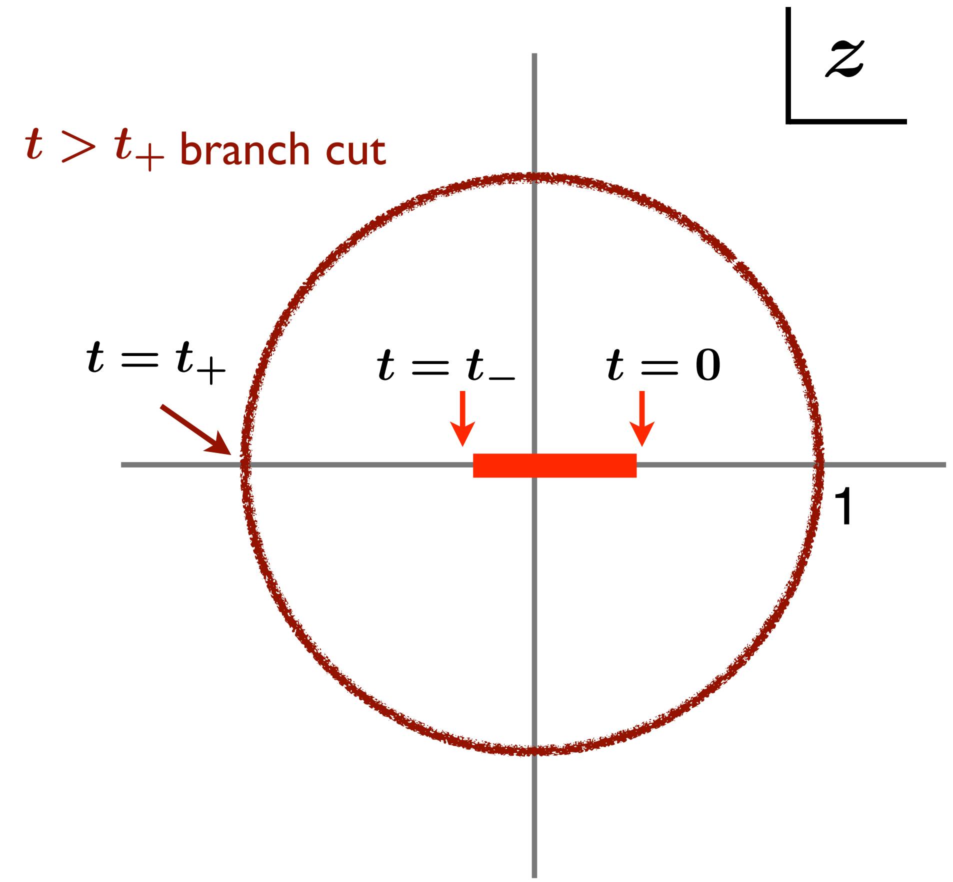
Form factor shape

Series (z) expansion

$$t = q^2 \qquad t_\pm = (m_B \pm m_F)^2$$

Choose, *e.g.* $t_0 = 12\ \text{GeV}^2$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$



Simplified series expansion

$$F(t) = \frac{1}{1 - t/m_{\text{res}}^2} \sum_n a_n z^n$$

Bourrely, Caprini, Lellouch PRD **79** (2009) following Okubo; Bourrely, Machet, de Rafael; Boyd, Grinstein, Lebed; Boyd & Savage; Arneson *et al.*; FNAL/MILC lattice collab; ...

Kinematic-continuum-mass fits

HPQCD

$$\equiv 1/P(t)$$

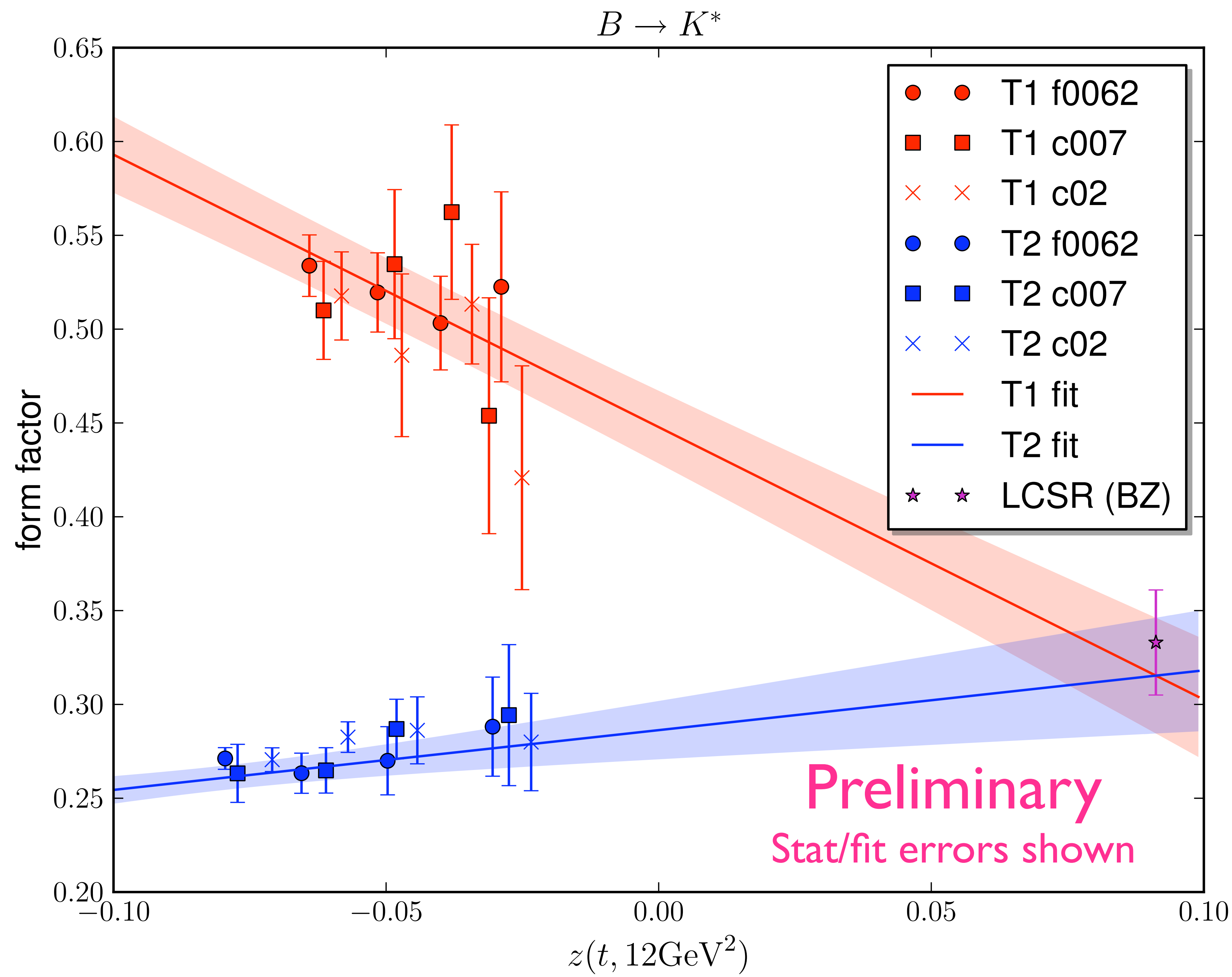
$$F(t) = \boxed{\frac{1}{1 - t/m_{\rm res}^2}}[1 + b_1(aE_F)^2 + \ldots]\sum_n a_n d_n z^n$$

discretization errors

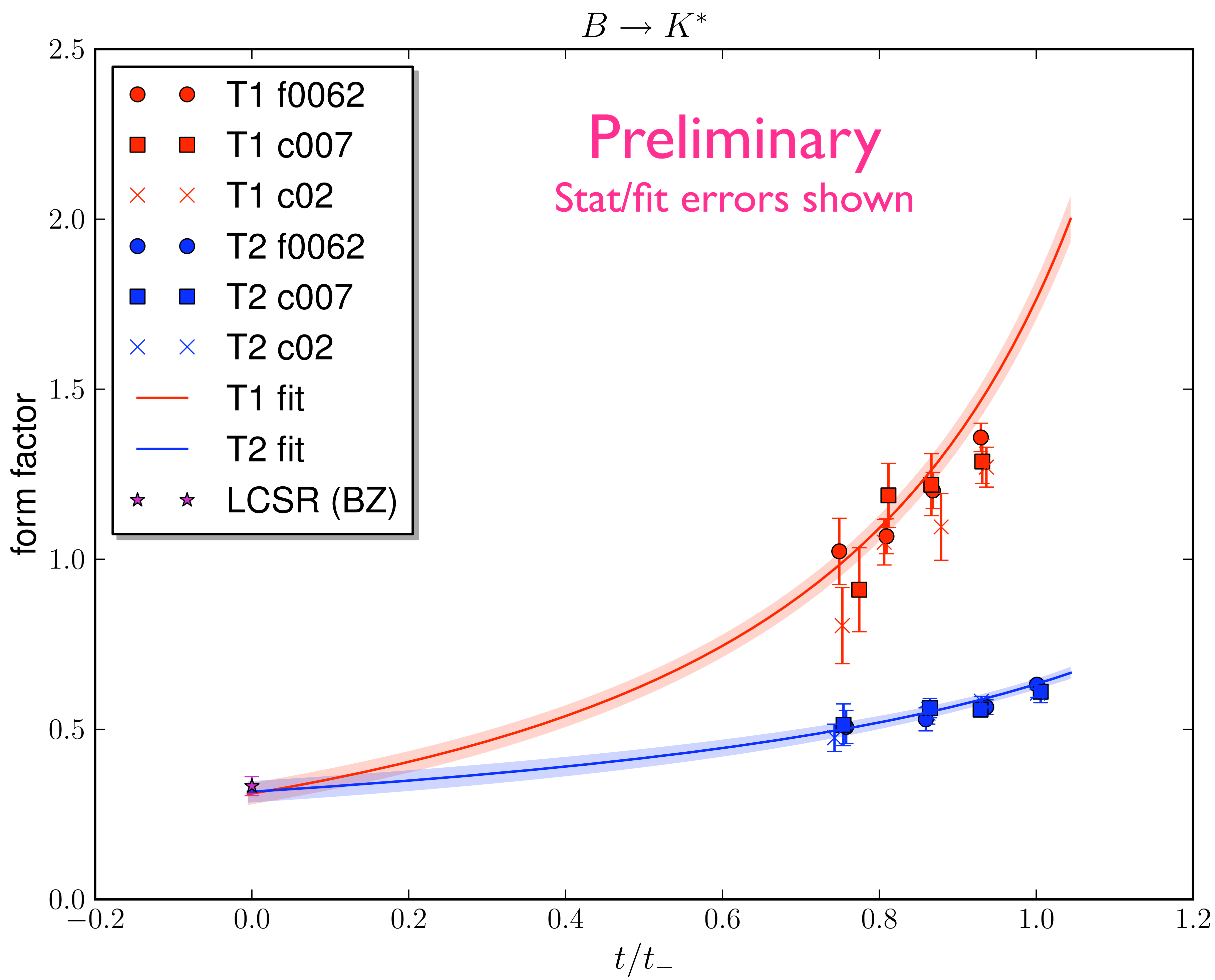
$$d_n = 1 + c_{n1}\frac{m_P^2}{(4\pi f)^2} + \ldots$$

quark mass dependence

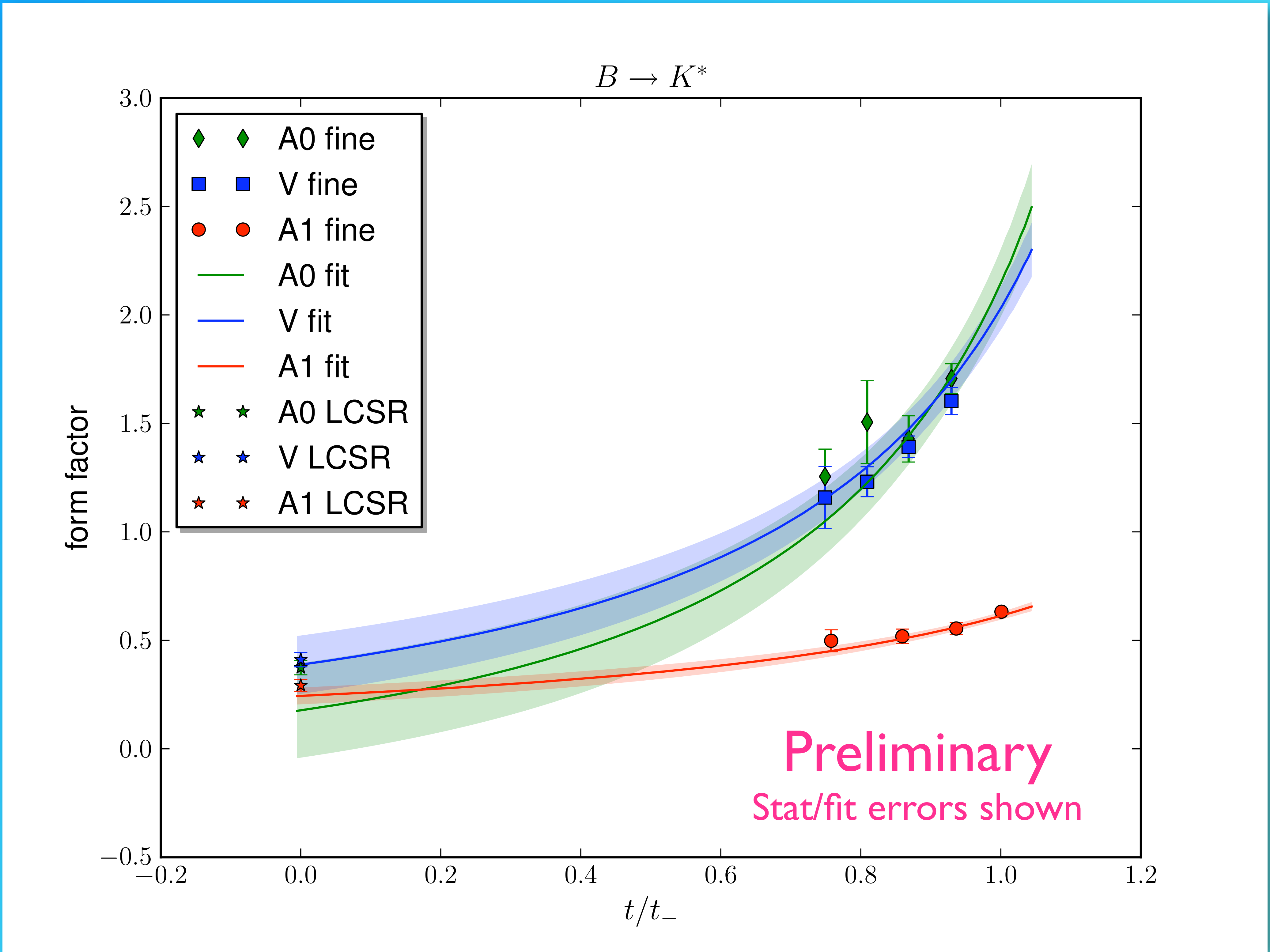
$B \to K^*$, $P(t)T_1$ & $P(t)T_2$, vs. z



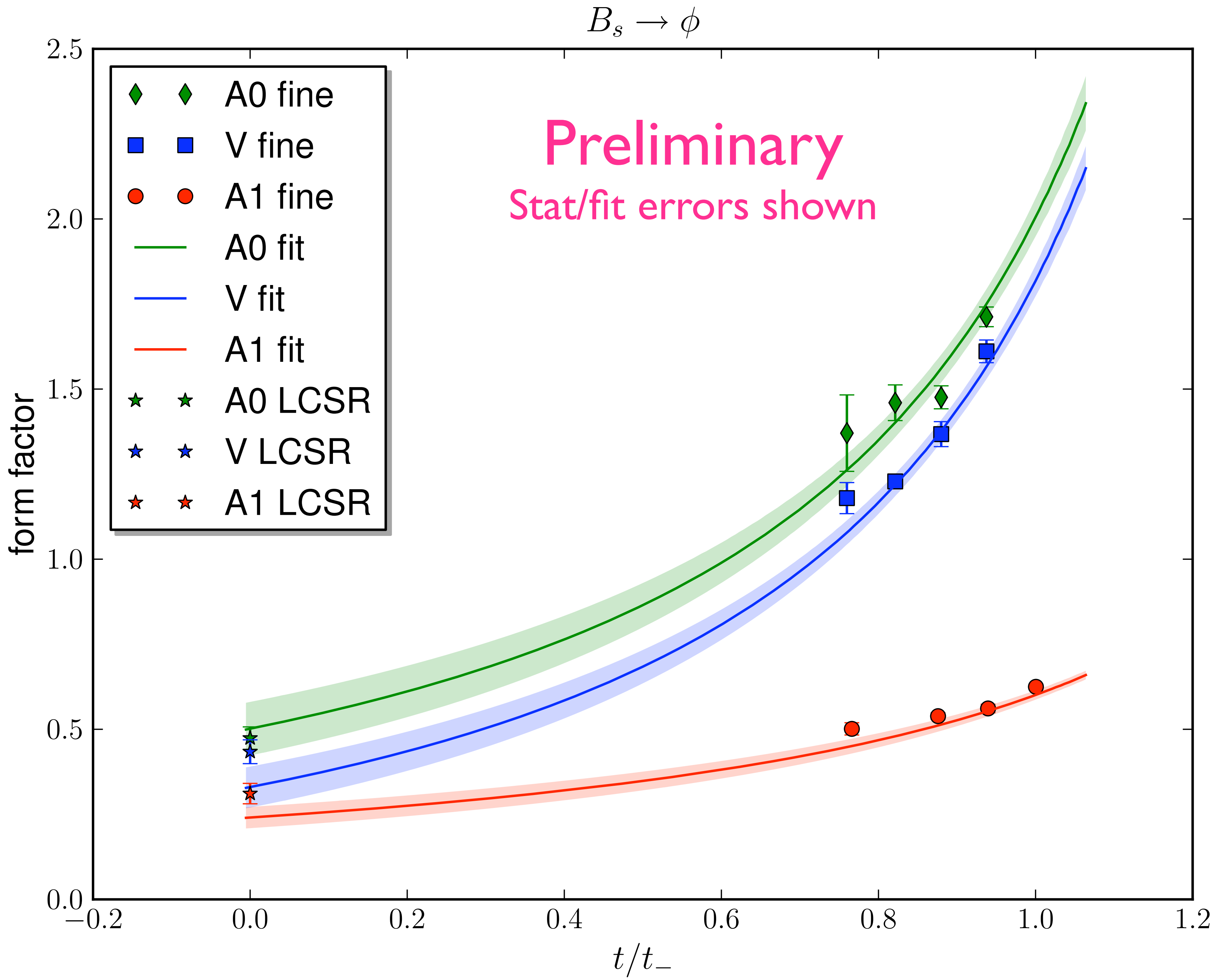
$B \to K^*$, T_1 & T_2, vs. q^2/q^2_{max}



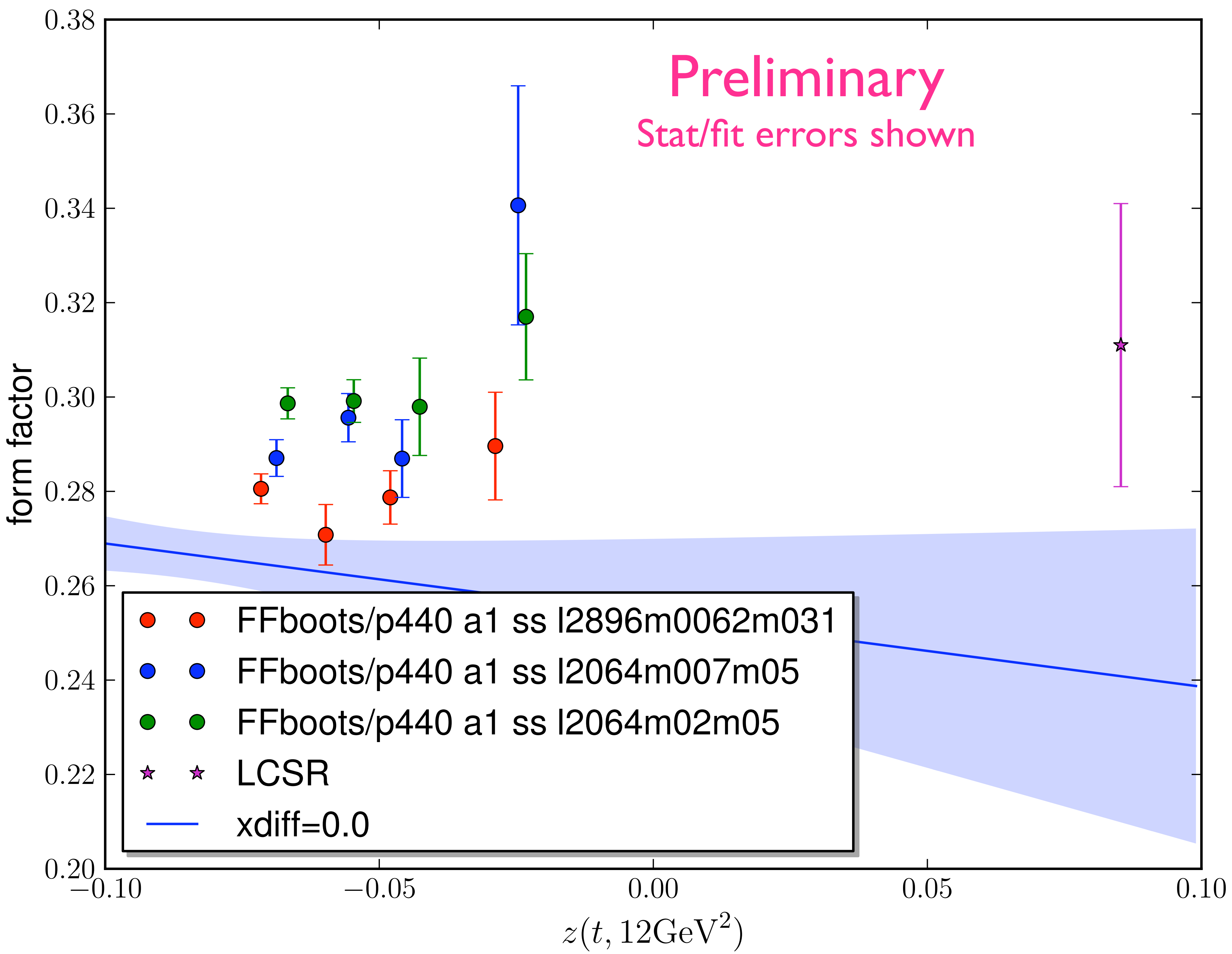
$B \to K^*$, V, A_0, A_1, vs. q^2/q^2_{max}



$B_s \to \varphi$, V, A_0, A_1, vs. q^2/q^2_{max}



Discretization errors



LQCD form factors

- Now removed quenched uncertainty
- Calculation done with low recoil kinematics, compl. LCSR
- Reduced statistical error below systematic (except for $B \to \rho$)
- Dominant systematic is due to perturbative operator matching
- Caveat: effect of narrow width approximation?

LCSR

Light cone sum rules (LCSR)

- Light cone expansion: correlation functions factorized as nonperturbative distribution amplitudes convolved with perturbative amplitudes

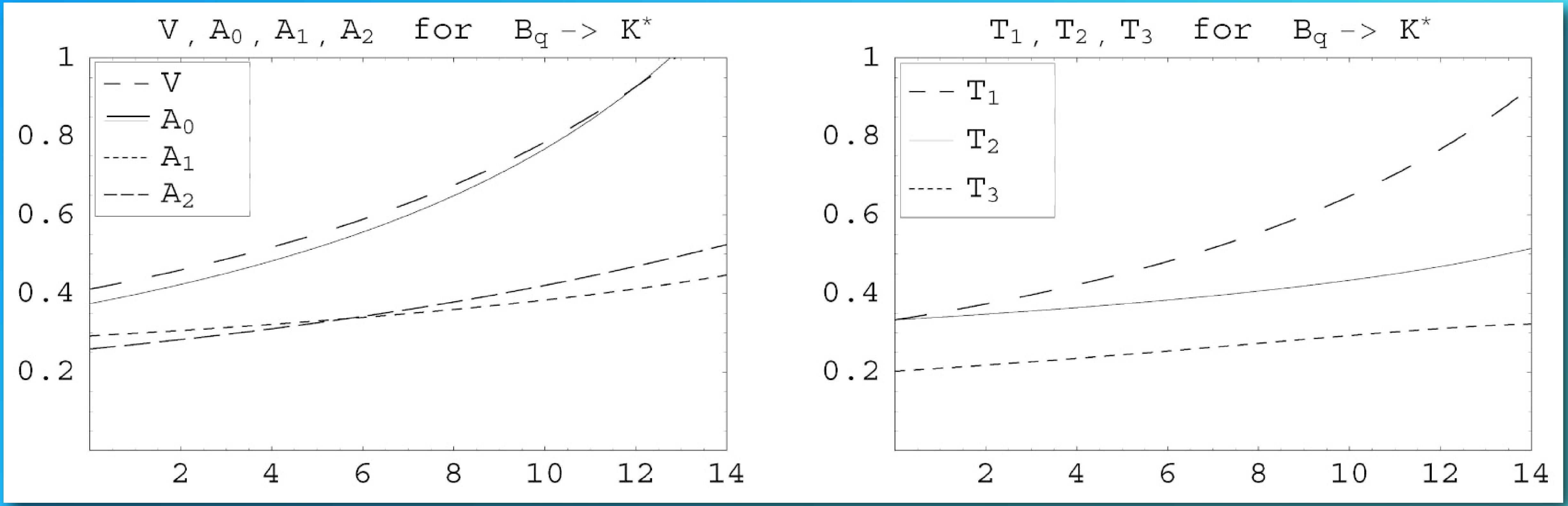
- Valid for low q^2, where $E_V \gg \Lambda_{\mathrm{QCD}}$

- Dispersion relations for correlation functions

- Quark-hadron duality to isolate B contribution

Light cone sum rule results

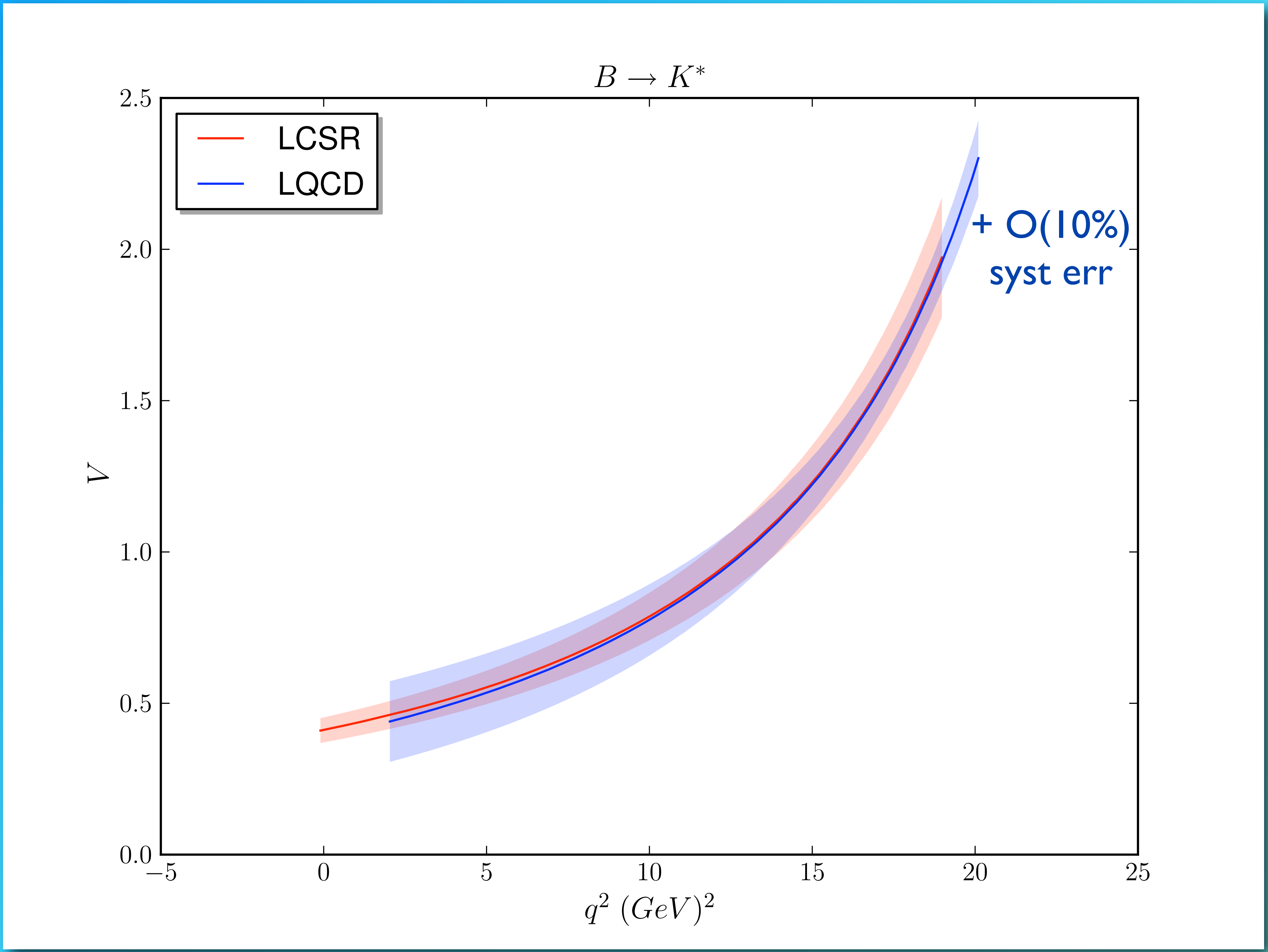
Ball & Zwicky, Phys. Rev. D71, 014029 (2005)



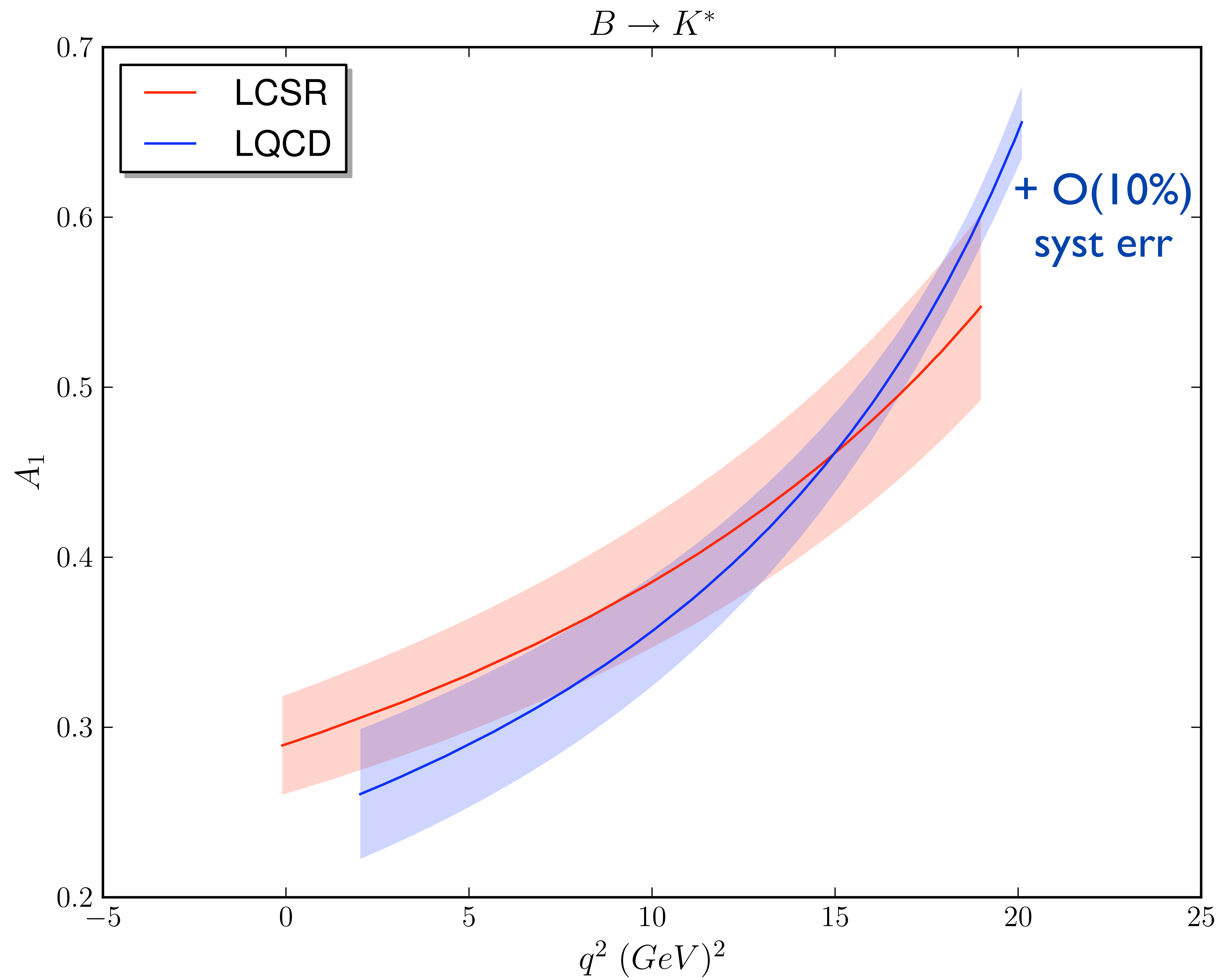
- 10% uncertainty, cannot reduce below 7%
- Uncertainty grows as one extrapolates to large q^2
- Narrow width approximation

LCSR & LQCD

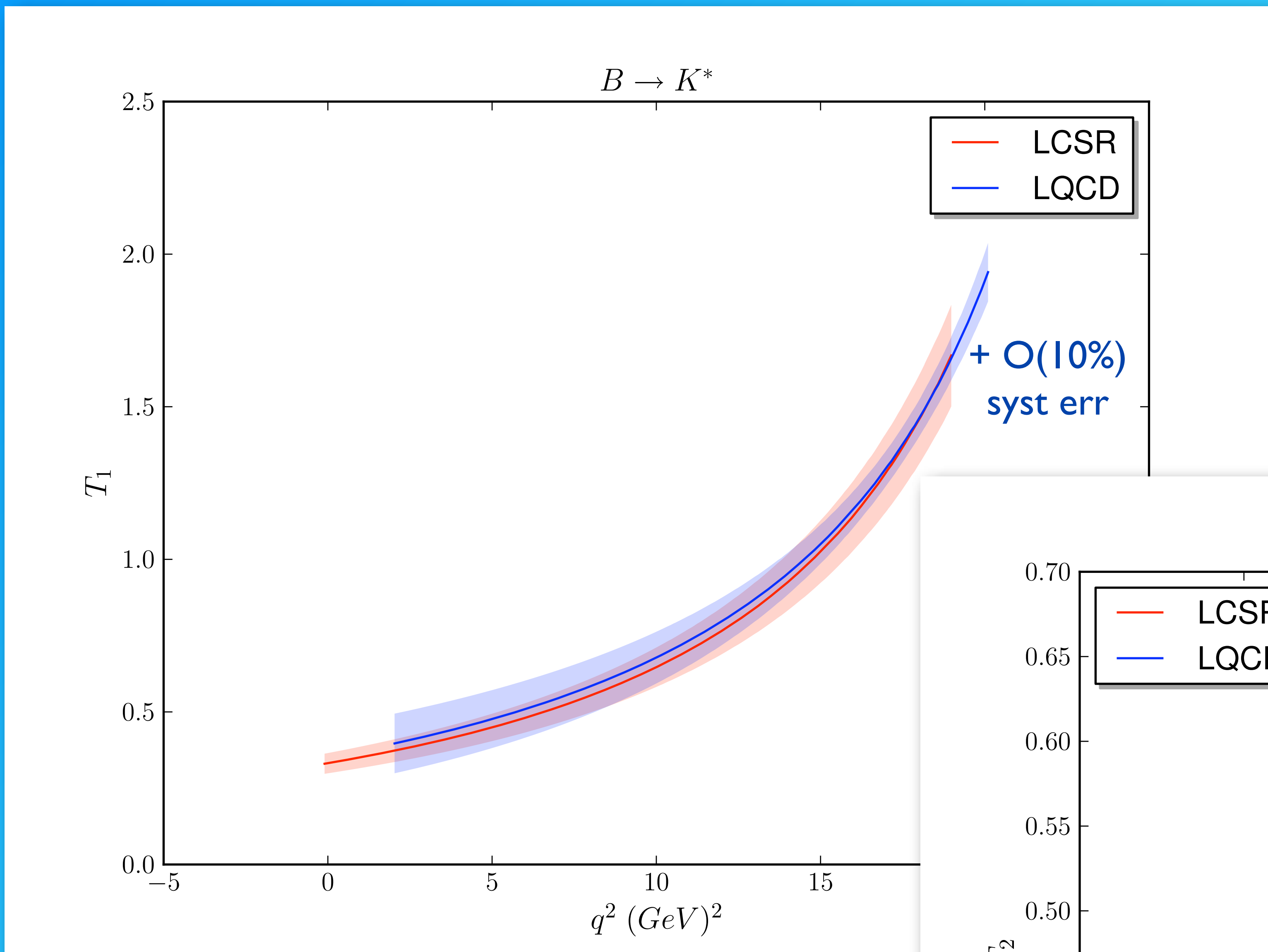
V comparison

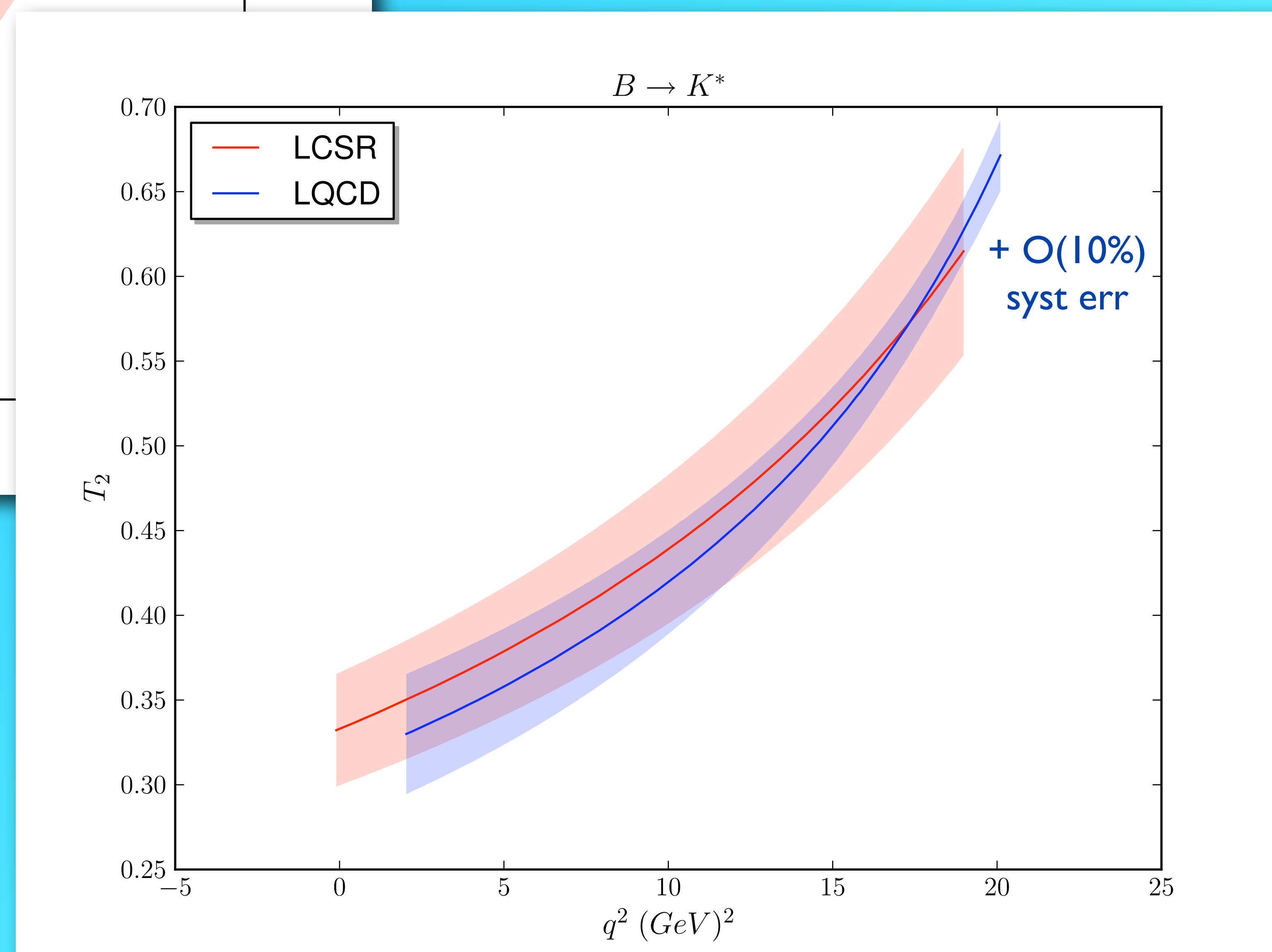


A_1 comparison

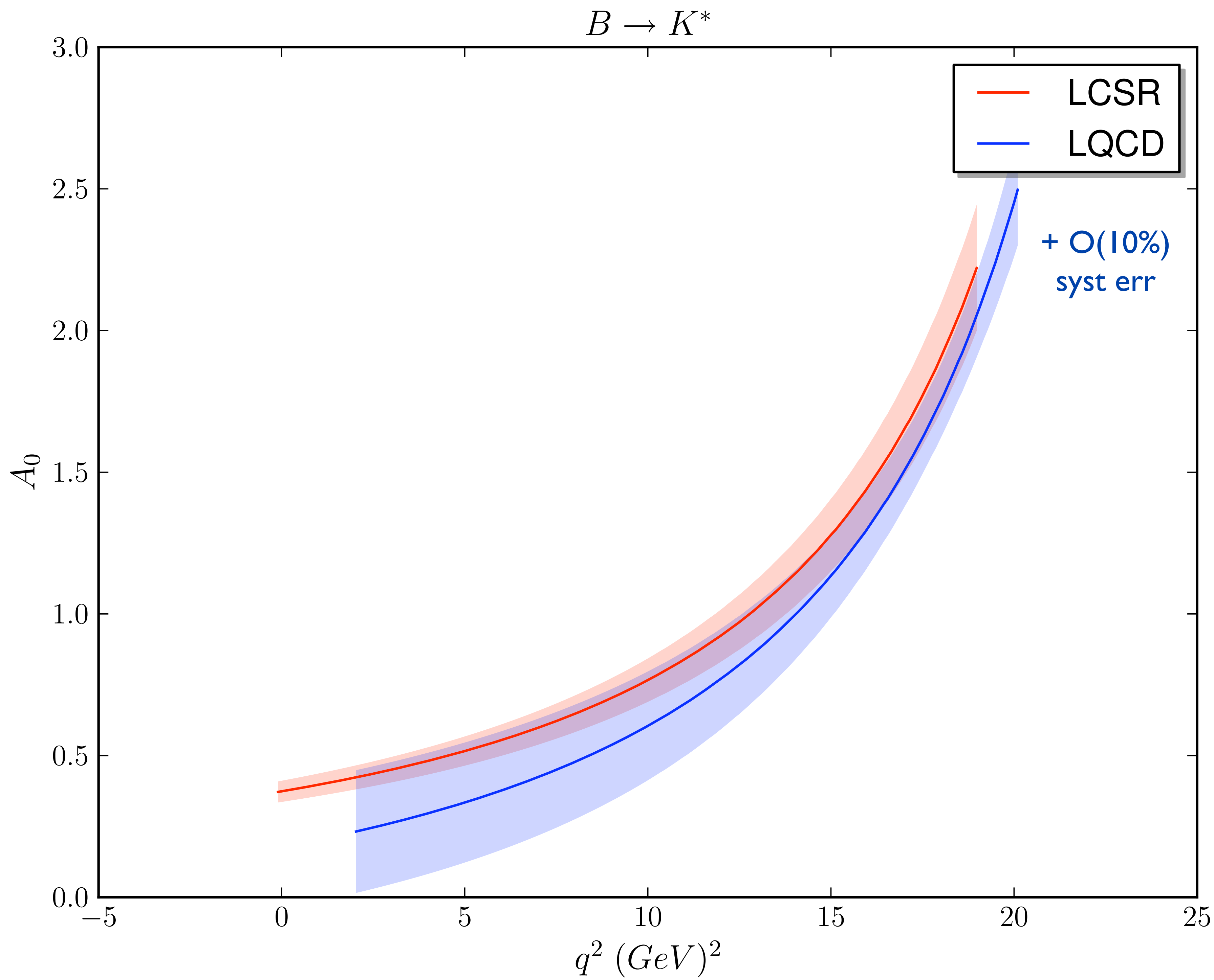


T_1 & T_2 comparisons

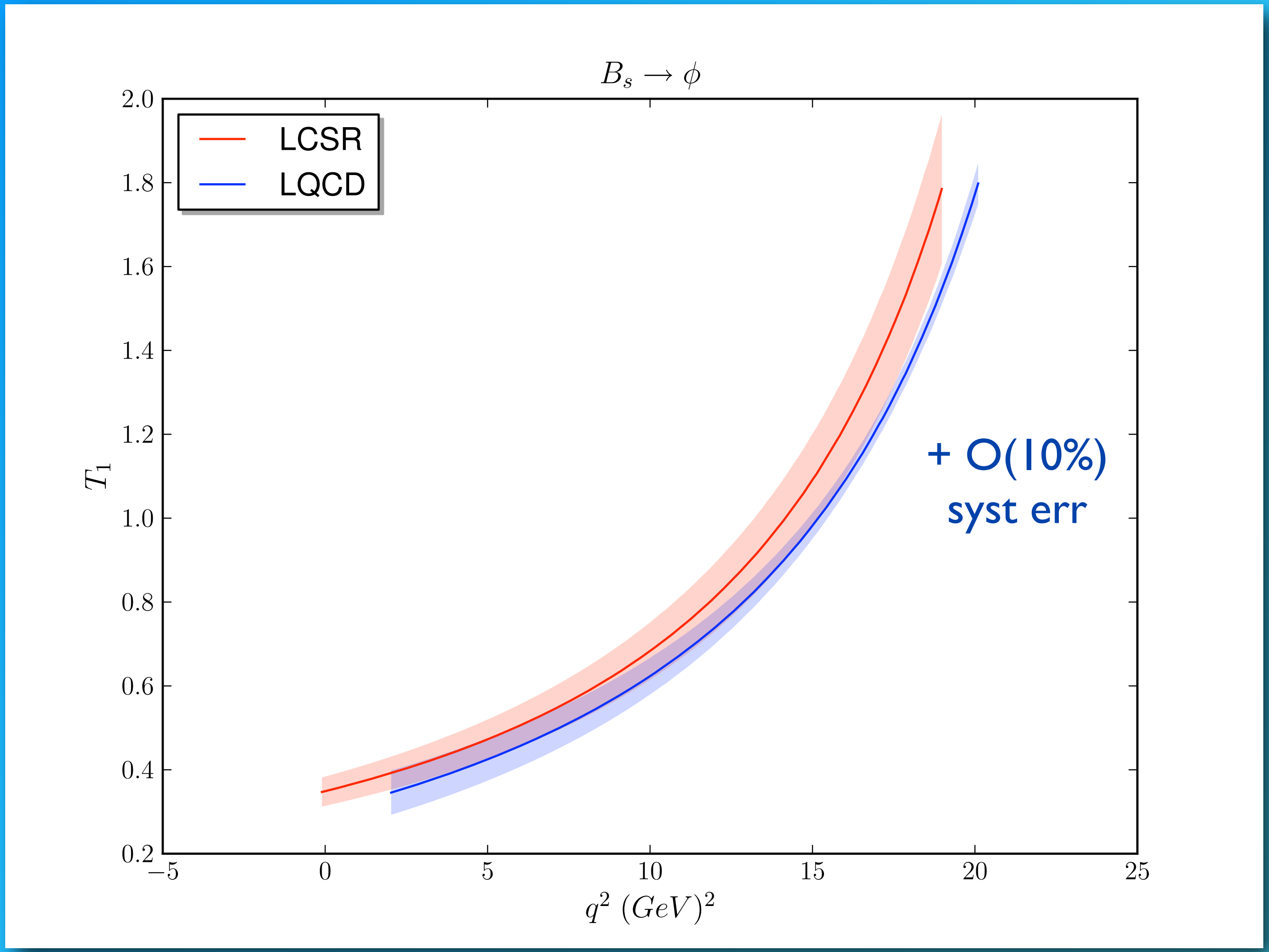


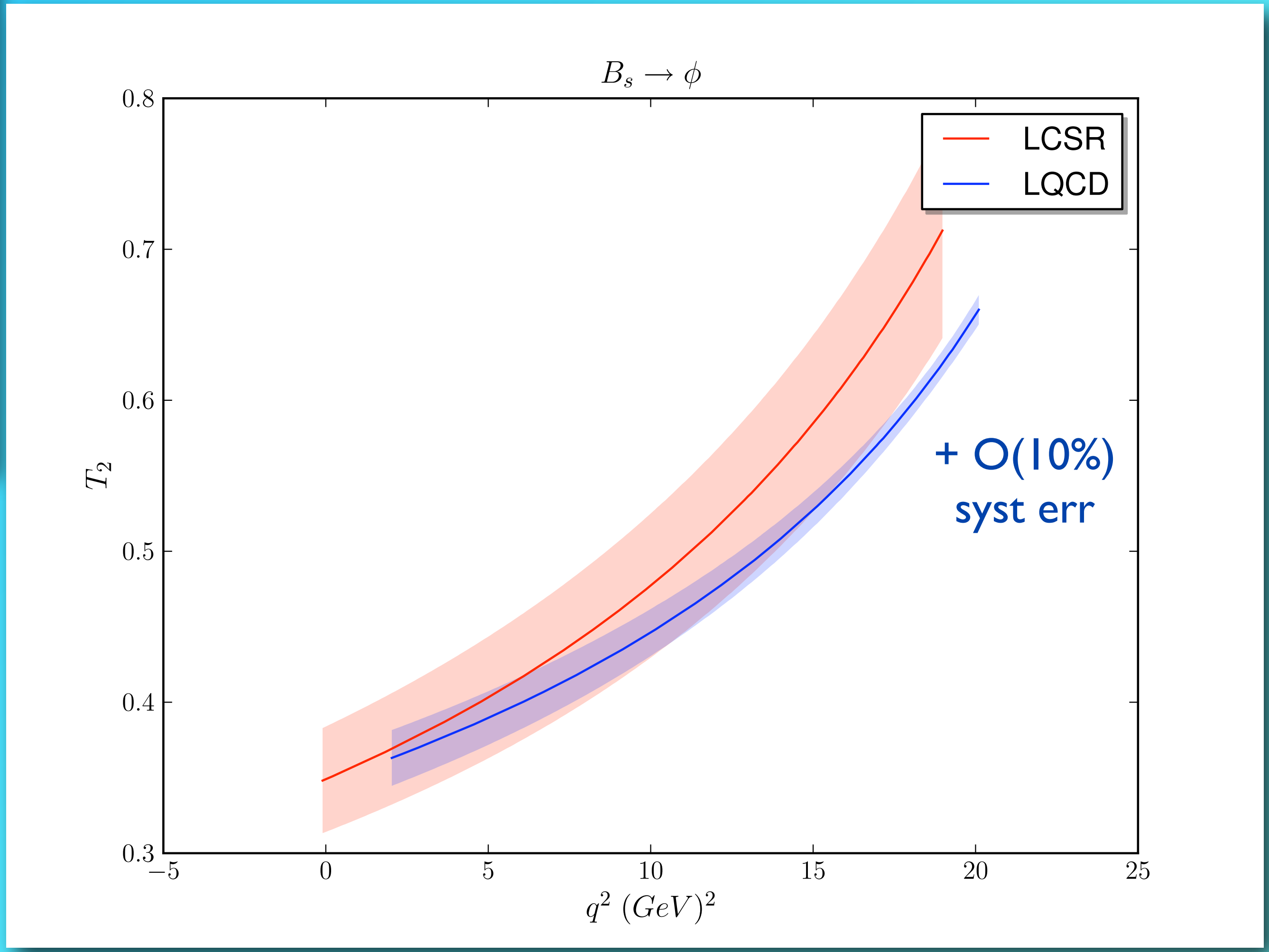


A_0 comparison

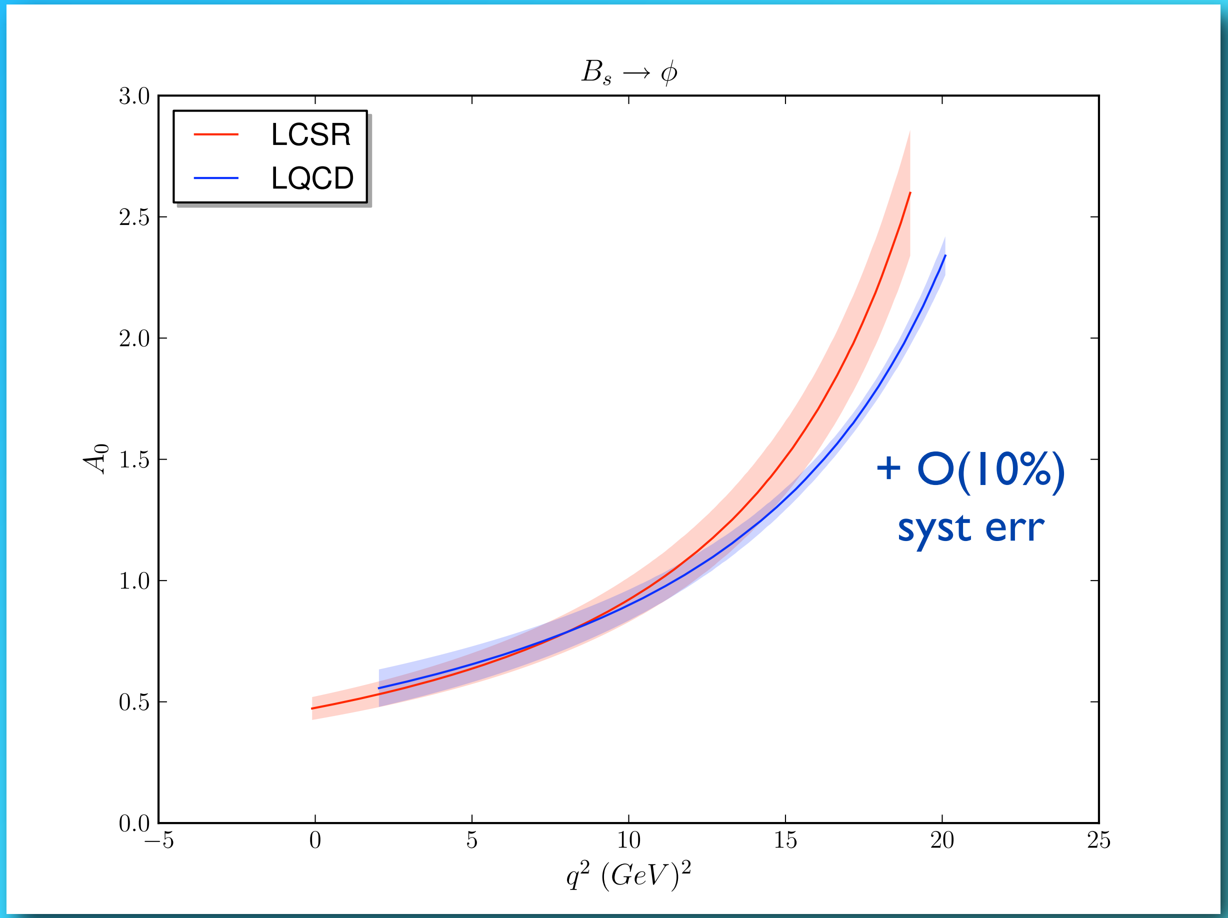


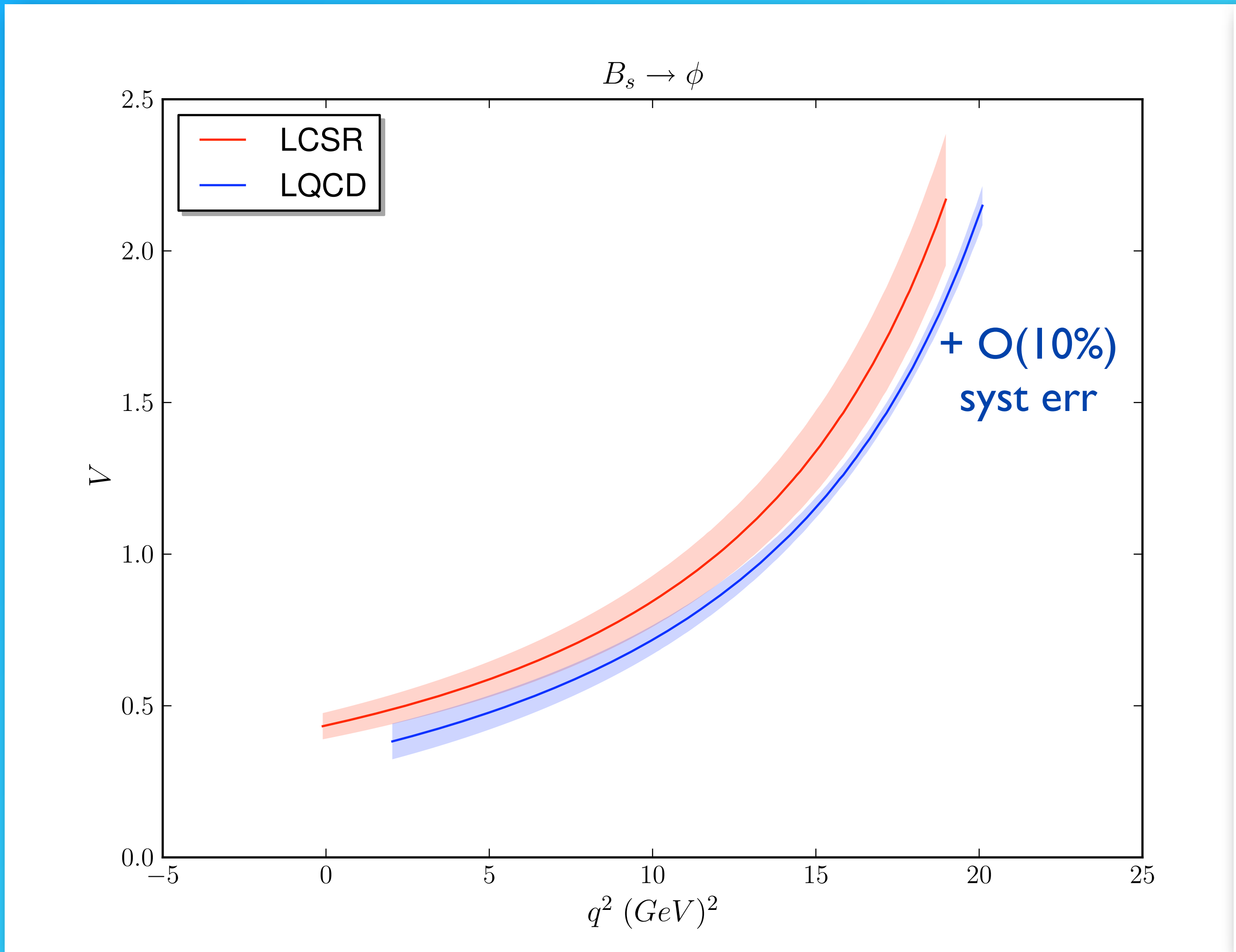
$B_s \to \varphi$

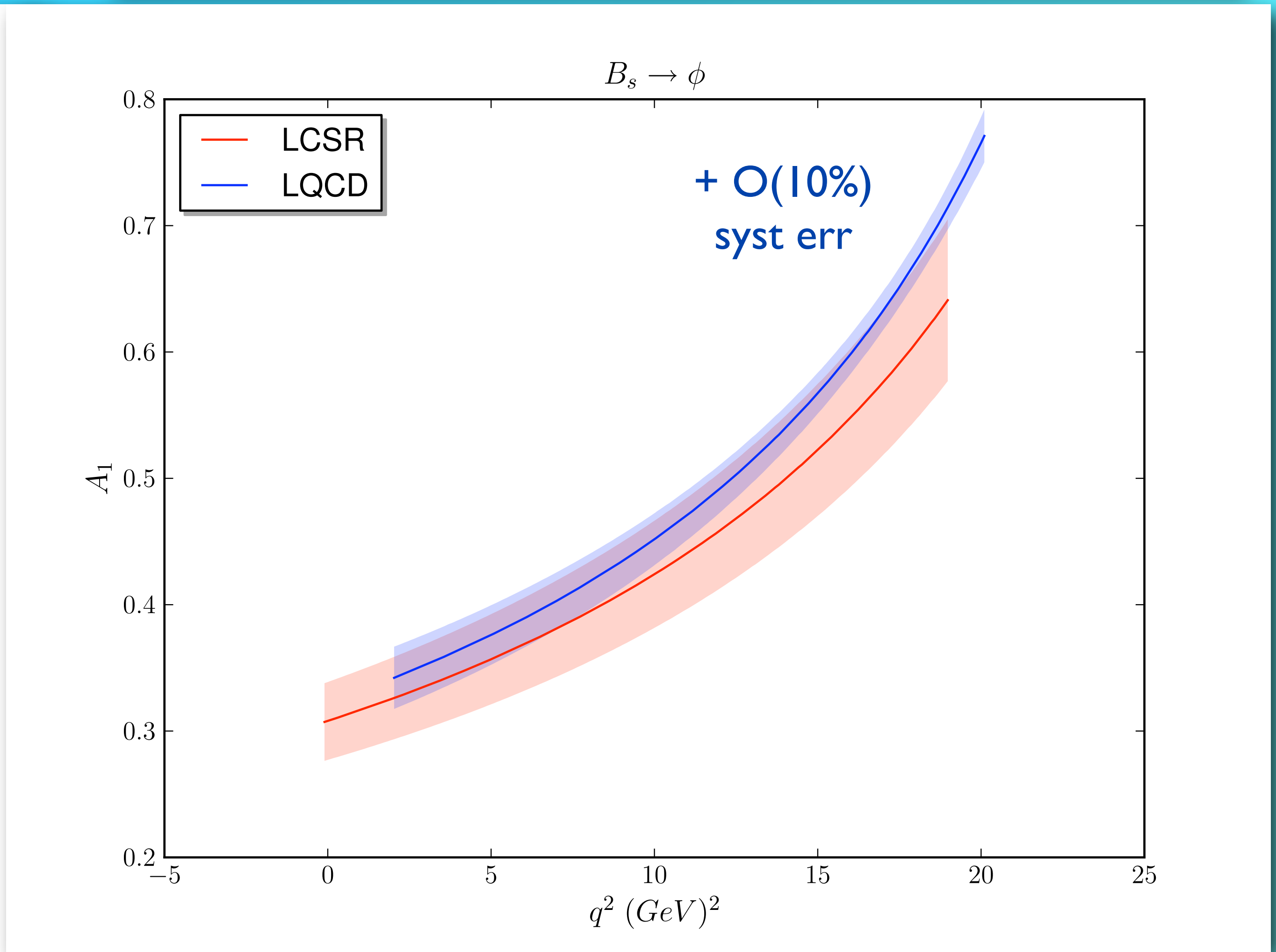

$B_s \to \phi$
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
T_1
LCSR
LQCD
+ O(10%)
syst err
−5
0
5
10
15
20
25
$q^2\ (GeV)^2$


$B_s \to \phi$
0.8
0.7
0.6
0.5
0.4
0.3
T_2
LCSR
LQCD
+ O(10%)
syst err
−5
0
5
10
15
20
25
$q^2\ (GeV)^2$

$B_s \to \varphi$


$B_s \to \phi$
LCSR
LQCD
A_0
+ O(10%)
syst err
$q^2\ (GeV)^2$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−5
0
5
10
15
20
25


$B_s \to \phi$
LCSR
LQCD
V
+ O(10%)
syst err
$q^2\ (GeV)^2$
0.0
0.5
1.0
1.5
2.0
2.5
−5
0
5
10
15
20
25


$B_s \to \phi$
LCSR
LQCD
A_1
+ O(10%)
syst err
$q^2\ (GeV)^2$
0.2
0.3
0.4
0.5
0.6
0.7
0.8
−5
0
5
10
15
20
25

Combinations

Combined LCSR-LQCD fit

Bharucha, Feldmann, Wick, JHEP 09 (2010) 090

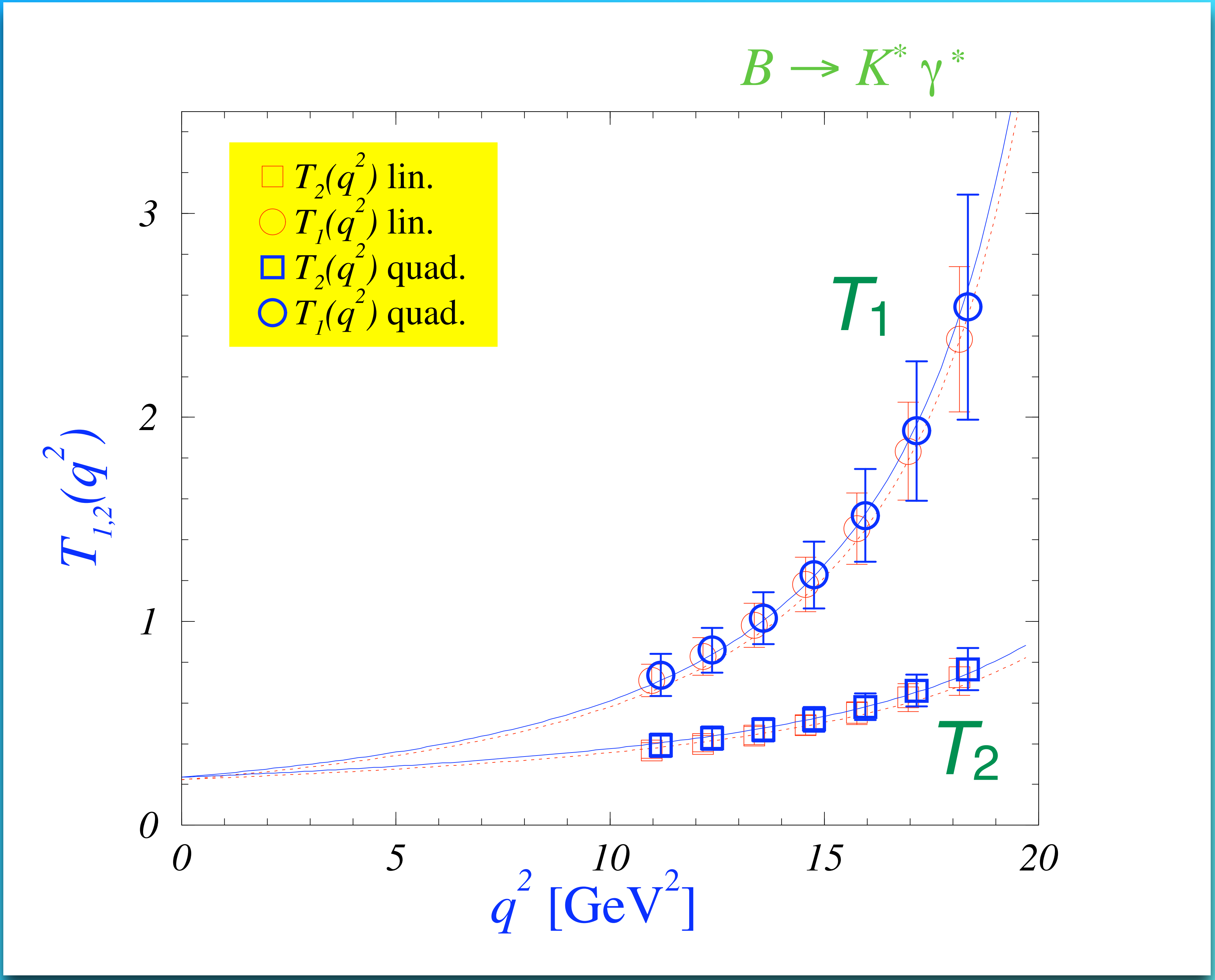
- LCSR and LQCD data, including correlations
- Series expansion or simplified series expansion
- Implement dispersive bounds (if possible: probably need A_2 and T_3)

Summary

- Unquenched LQCD at high q^2, with statistics and other systematics also improved (RR Horgan, Z Liu, S Meinel, MW)
- LCSR at low q^2 (P Ball & R Zwicky)
- To do: combined fit to LQCD & LCSR results, include dispersive bounds (as in Bharucha, Feldmann, Wick) if possible (may need A_2, T_3)
- Open question: errors due to narrow width approximation? Can we learn something by studying simpler matrix elements through threshold?

Quenched T_1 & T_2

Bećirević-Lubicz-Mescia, Nucl. Phys. B769, 31 (2007)



Quenched V, A_0, A_1, A_2

Bowler, Gill, Maynard, Flynn, JHEP 05 (2004) 035

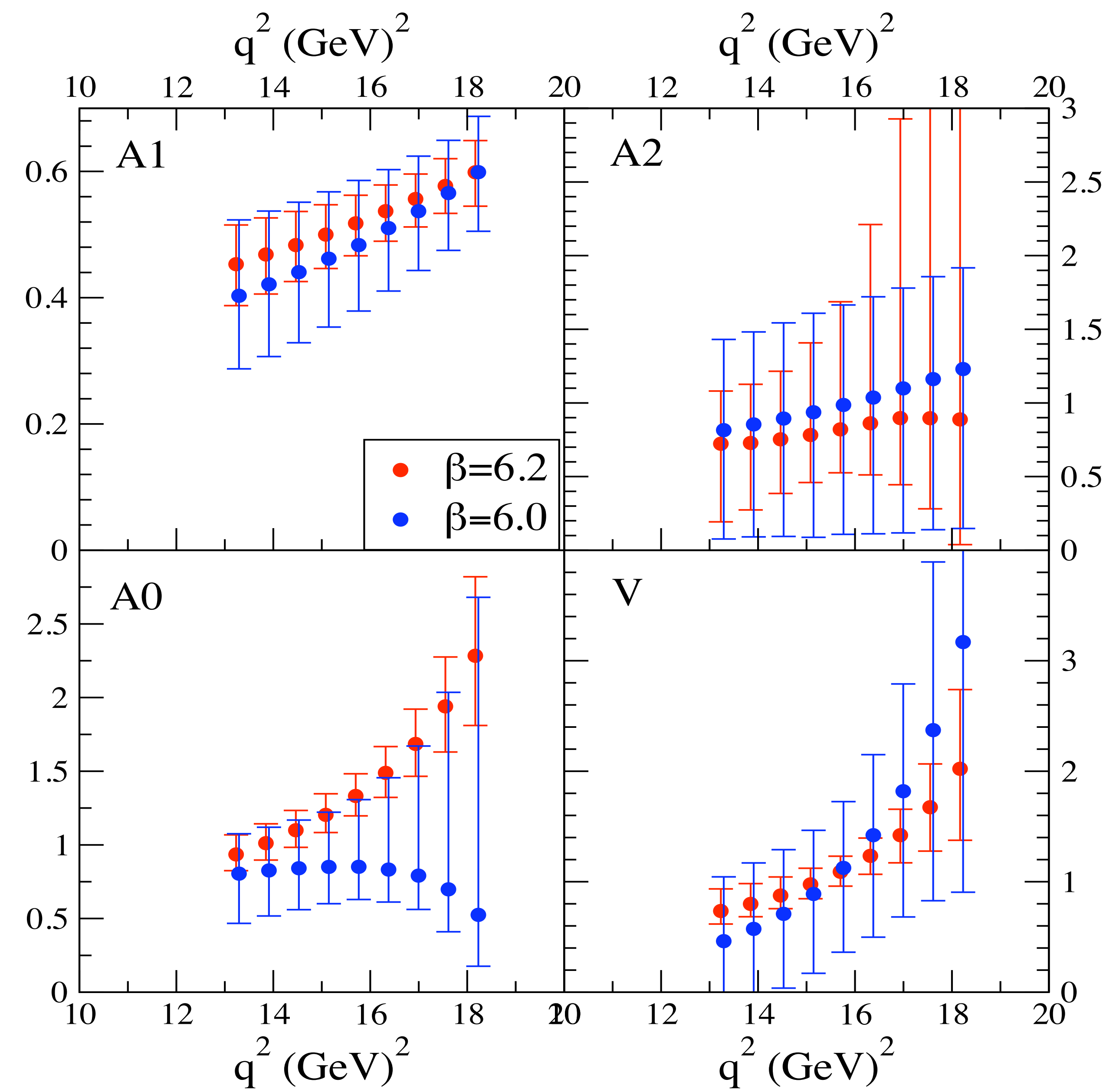


Figure 4: The form factors on both lattices. The vertical scale is different for each form factor.