Effective theory approach to a gauge invariant definition of the jet quenching parameter

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Detroit
August 21, 2012

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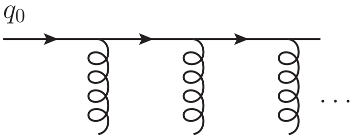
Outline

The jet quenching parameter $\hat{q}$

- **Jet broadening** refers to a process where the primary **jet** parton acquires a certain **transverse momentum** (to its original direction of motion) through interactions with a **medium**
- Define the **jet quenching parameter**

$$\hat{q} = \int d^2 k_\perp \, k_\perp^2 \, \frac{d\Gamma}{d^2 k_\perp}$$

- Γ is rate of elastic collisions of a parton with the medium particles



The jet quenching parameter $\hat{q}$

- Define $P(k_\perp)$ as the probability to acquire a transverse momentum $k_\perp$ after travelling through a medium with length L

$$\hat{q} = \frac{1}{L}\langle k_\perp^2 \rangle = \frac{1}{L} \int \frac{d^2 k_\perp}{(2\pi)^2} \, k_\perp^2 \, P(k_\perp)$$

- Does **not** include collinear radiation which changes the energy of the parton (fragmentation)
- Assume that the final virtuality is determined though **medium interactions** and not the initial hard process

Goal

Find field theoretic definition of $\hat{q}$

The effective field theory approach

- Several scales appear in the process, most notably

  The energy of the jet Q

  The scale of the medium (temperature) T
- Small dimensionless ratio $\lambda = T/Q \ll 1$

Conclusion

Use an effective field theory that provides a systematic expansion in λ

- When dealing with jets and their interactions with soft particles **Soft-Collinear Effective Theory** (SCET) is the appropriate EFT

  Bauer et al. '01; Beneke at al. '02

Soft-Collinear Effective Theory

- Classify modes by the scaling of their momentum components in the different light-cone directions $(n, \bar{n})$

  $(p^+, p^-, p_\perp) = (Q, Q, Q) \sim (1, 1, 1)$ is called hard

  $(p^+, p^-, p_\perp) = (T, T, T) \sim (\lambda, \lambda, \lambda)$ is called soft

  $(p^+, p^-, p_\perp) \sim (\lambda^2, 1, \lambda)$ is called collinear
- **Jets** have a collinear momentum, i.e., they have a large momentum component in one light cone direction, but only a small invariant mass
- Integrate out the hard modes and the off-cone components of the collinear modes to find the **SCET Lagrangian** for collinear fields

$$\mathcal{L} = \bar{\xi} i \bar{n} \cdot D \frac{\not{n}}{2} \xi + \bar{\xi} i \not{D}_\perp \frac{1}{i n \cdot D} i \not{D}_\perp \frac{\not{n}}{2} \xi + \mathcal{L}_{\text{Y.M.}}, \quad iD = i\partial + gA$$

In-medium Interactions

- Other possible modes interacting with a collinear quark
  $(p^+, p^-, p_\perp) \sim (\lambda^2, \lambda^2, \lambda^2)$ is called ultrasoft
  Decouple at leading power as proven in Bauer et al. '01
  $(p^+, p^-, p_\perp) \sim (\lambda^2, \lambda^2, \lambda)$ is called **Glauber**
  Introduction to SCET in Idilbi, Majumder '08
  $(p^+, p^-, p_\perp) \sim (\lambda^2, \lambda, \lambda)$ also Glauber ("longitudinal")
  Ovanesyan, Vitev '11; Qin, Majumder '12
- Introduce it into the SCET Lagrangian as an effective classical field of the medium particles

In-medium Interactions

- In order to determine the importance of these interactions the **scaling of the Glauber field itself** is relevant

$$A^{\mu}(x) = \int d^4y \, D^{\mu\nu}(x-y) J_{\nu}(y)$$

- Depends on the gauge used
- Covariant gauge: $A^{+,\mathrm{cov}} \sim \lambda^2$, $A_{\perp}^{\mathrm{cov}} \lesssim \lambda^2$ (for a soft source the longitudinal Glauber is more relevant)

  Idilbi, Majumder '08; Ovanesyan, Vitev '11

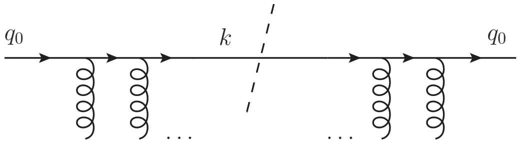
- At leading order in the power counting the extended SCET Lagrangian for the interaction of collinear particles with Glauber gluons is just

$$\mathcal{L} = \bar{\xi} i\bar{n} \cdot D \frac{\not{n}}{2} \xi, \quad iD = i\partial + gA$$

The Glauber mode

- Use SCET_G to calculate $\hat{q}$

  D'Eramo, Liu, Rajagopal '10
- Use optical theorem to determine scattering amplitude



- Initially on-shell quark scattering on an arbitrary number of medium particles via Glauber exchange

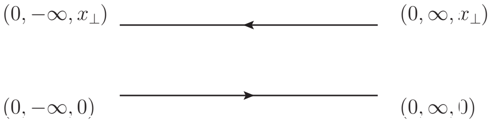
$\hat{q}$ in covariant gauge

- Result is the Fourier transform of the medium averaged expectation value of two Wilson lines

$$P(k_\perp) = \int d^2x_\perp e^{ik_\perp \cdot x_\perp} \frac{1}{d(\mathcal{R})} \left\langle \mathrm{Tr}\left[W_{\mathcal{R}}^\dagger[0, x_\perp] W_{\mathcal{R}}[0,0] \right] \right\rangle$$

$$W_{\mathcal{R}}[y^+, y_\perp] = \mathcal{P}\left\{ \exp\left[ig \int_{-\sqrt{2}L/2}^{\sqrt{2}L/2} dy^- A^+(y^+, y^-, y_\perp) \right] \right\}$$

- Agrees with Casalderrey-Solana, Salgado '07; Liang, Wang, Zhou '08



- Not gauge invariant ($W_{\mathcal{R}} = 1$ in light-cone gauge $A^+ = 0$)

Changes in arbitrary gauge

Goal

Want to show that SCET_G is complete and find a gauge invariant expression of $\hat{q}$ for applications

- In singular gauges, such as light-cone gauge the scaling of the Glauber field is different
  Idilbi, Majumder '08; Ovanesyan, Vitev '11
  $A_\perp^{\mathrm{cov}} \ll A_\perp^{\mathrm{lcg}}$
- This can be traced back to the factor $k_\perp/[k^+]$ appearing in the Fourier transform of the gluon propagator in light-cone gauge (the square brackets indicate an appropriate regularization for he light-cone singularity)
- Additional leading power interaction term in the Lagrangian becomes relevant

$$\bar{\xi} i \not{D}_\perp \frac{1}{Q} i \not{D}_\perp \frac{\not{n}}{2} \xi$$

Changes in arbitrary gauge

- Additional vertices for collinear-Glauber interaction

Will show

Sum over all possible interactions gives rise to a gauge invariant result

The gauge field at light-cone infinity

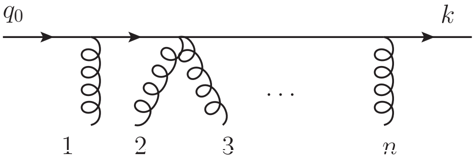
- The gluon field may be decomposed
  $A^i_\perp(x^+, x^-, x_\perp) =$
  $A^{\mathrm{cov},i}_\perp(x^+, x^-, x_\perp) + \theta(x^-) A^i_\perp(x^+, \infty, x_\perp) + \theta(-x^-) A^i_\perp(x^+, -\infty, x_\perp)$
  where $A^{\mathrm{cov},i}_\perp$ corresponds to the non-singular part of the propagator and vanishes at $\pm\infty^-$ and where the leading power comes from the terms at ∞^-

  Echevarria, Idilbi, Scimemi '11

- For $x^- \to \infty$ the field strength must vanish
  $\to A_\perp(x^+, \infty, x_\perp)$ is a pure gauge
  $A_\perp(x^+, \infty, x_\perp) = \nabla_\perp \phi(x^+, \infty, x_\perp)$
  $\phi(x^+, \infty, x_\perp) = -\int_{-\infty}^{0} ds\, l_\perp \cdot A_\perp(x^+, \infty, x_\perp + l_\perp s)$
  Belitsky, Ji, Yuan '02

Calculation

- Define the (amputated) diagram with n gluon interactions

$G_n(k) =$ 

- We can calculate this in a recursive fashion

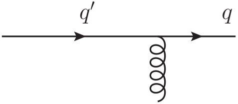
Calculation

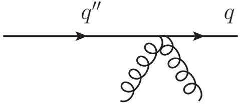
- Decompose into fields at $\pm\infty$

  $G_n(k^-, k_\perp) = \sum_{j=0}^{n} \int \frac{d^4q}{(2\pi)^4}\, G^+_{n-j}(k^-, k_\perp, q)\, \frac{iQ\,\bar{\not{n}}}{2Qq^+ - q_\perp^2 + i\epsilon}\, G^-_j(q)$

  where $G^\pm$ contains only the gluon at $\pm\infty$

- The recursive definition of G^- is then

$G^-_n(q) = \quad \int \frac{d^4q'}{(2\pi)^4}\, G^-_{n-1}(q')$ 

$+ \quad \int \frac{d^4q''}{(2\pi)^4}\, G^-_{n-2}(q'')$ 

- and G^+_n correspondingly

Results

- Squaring the amplitude and summing over any number of gluon interactions, we find

$$P(k_\perp) = \frac{1}{N_c}\int d^2x_\perp\, e^{ik_\perp\cdot x_\perp} \left\langle \mathrm{Tr}\left[T^\dagger(0,-\infty,x_\perp) T(0,\infty,x_\perp)\; T^\dagger(0,\infty,0) T(0,-\infty,0) \right] \right\rangle$$

- with
  $T(x_+,\pm\infty,x_\perp) = \mathcal{P}\, e^{-ig\int_{-\infty}^{0} ds\; l_\perp\cdot A_\perp(x_+,\pm\infty,x_\perp+l_\perp s)}$
  the transverse Wilson line

Results

- Wilson lines in the perpendicular plane at $\pm\infty^-$ for light-cone gauge

Results

- Combining the results with the ones in covariant gauge we find

$$\begin{aligned} P(k_\perp) &= \frac{1}{N_c}\int d^2x_\perp\, e^{ik_\perp\cdot x_\perp} \\ &\quad \Big\langle \mathrm{Tr}\Big[\, T^\dagger(0,-\infty,x_\perp)\, W_F^\dagger[0,x_\perp]\, T(0,\infty,x_\perp) \\ &\quad T^\dagger(0,\infty,0)\, W_F[0,0]\, T(0,-\infty,0)\Big]\Big\rangle \end{aligned}$$

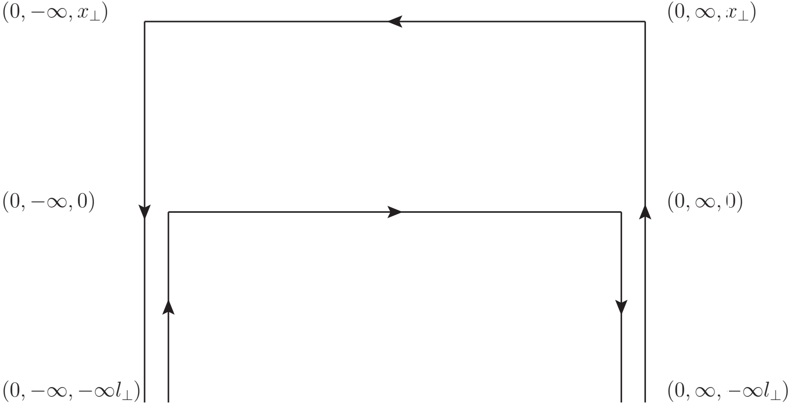
- The fields on the lower line are time ordered, the ones on the upper line anti-time ordered

  $\rightarrow$ Use Keldysh-Schwinger contour in path integral formalism

- Note that for certain regularizations of the light-cone singularity of the gluon propagator, the Glauber field might vanish at either $+\infty^-$ or $-\infty^-$ even in light-cone gauge

  Liang, Wang, Zhou '08

Results



$(0, -\infty, x_\perp)$
$(0, \infty, x_\perp)$
$(0, -\infty, 0)$
$(0, \infty, 0)$
$(0, -\infty, -\infty l_\perp)$
$(0, \infty, -\infty l_\perp)$

Conclusions

- SCET_G is a suitable framework for the calculation of gauge invariant results in jet quenching
- The jet quenching parameter $\hat{q}$ can be expressed as the medium average of two longitudinal and four transverse Wilson lines
- The operators are ordered along a Keldysh-Schwinger contour
- The results may be used to determine $\hat{q}$ in different frameworks also using lattice computations which require gauge invariant expressions

Thank you for your attention!

Bonus Slides

Soft-Collinear Effective Theory

- A jet originates from the fragmentation of a parton with high energy E_j and a much smaller invariant mass $m_j = \sqrt{p_j^2}$
  $\rightarrow$ Almost light-like $\rightarrow$ use light-cone coordinates

$$p^\mu = \frac{n^\mu}{2}\bar{n}\cdot p + \frac{\bar{n}^\mu}{2} n\cdot p + p_\perp^\mu \quad \text{with } n,\ \bar{n} \text{ light-cone vectors}$$
$$n\cdot p \sim E_j \gg p_\perp,\ \bar{n}\cdot p$$

- Introduce a **scaling parameter** $\lambda \ll 1$
  $n\cdot p \sim E_j \quad m_j \sim \lambda E_j$
  $p^2 = n\cdot p\,\bar{n}\cdot p + p_\perp^2$
  $\rightarrow p_\perp \sim \lambda E_j,\ \bar{n}\cdot p \sim \lambda^2 E_j^2$
- Jet momentum p_j scales like $(n\cdot p, \bar{n}\cdot p, p_\perp) \sim (\lambda^2, 1, \lambda)$
  "hard-collinear"

Scaling of the Glauber field

- Consider the form of the effective Glauber field

$$A^{\mu}(x) = \int d^4y \; D_G^{\mu\nu}(x-y)\, f_{\nu}(y)$$

$$D^{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} \, \frac{-i}{k^2+i\epsilon} \left(g^{\mu\nu} - \frac{k^{\mu}\bar{n}^{\nu} + k^{\nu}\bar{n}^{\mu}}{[k^+]} \right) e^{-ik(x-y)}$$

- Source f_{ν} only knows about the soft scale $\sim \lambda^3$