Monte-Carlos of jet quenching: an overview (II)

Konrad Tywoniuk

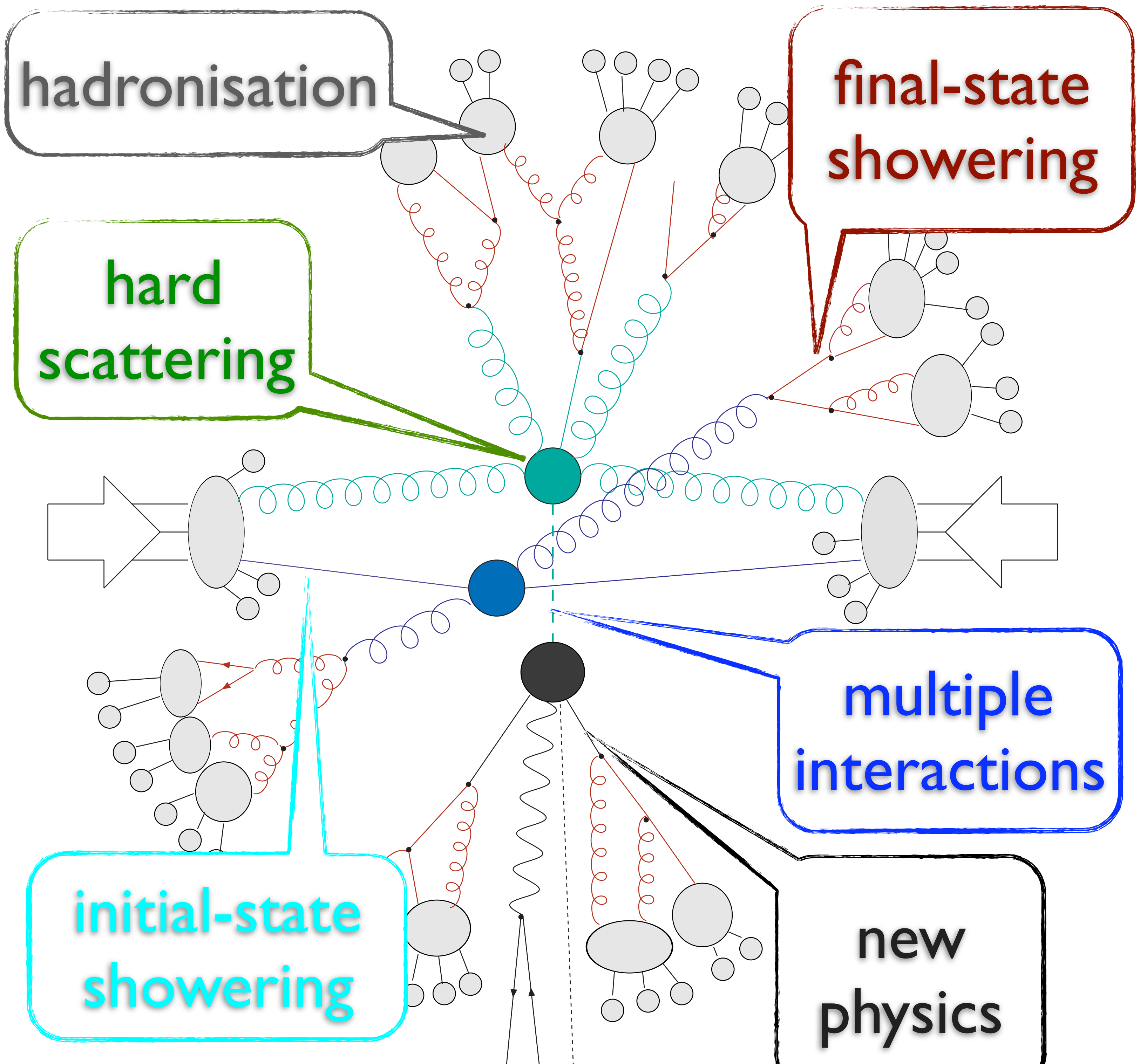
Jet Modification in the RHIC and LHC era
(QM12 Satellite Workshop)
20-24 August 2012, Wayne State University

Outline

- classical vs. quantum
  - when is a probabilistic interpretation viable?
- key effects & concepts
- perspectives & challenges

[disclaimer: devil is often in the details...]

typical pp event [à la PYTHIA, HERWIG, SHERPA]



Courtesy: S. Prestel

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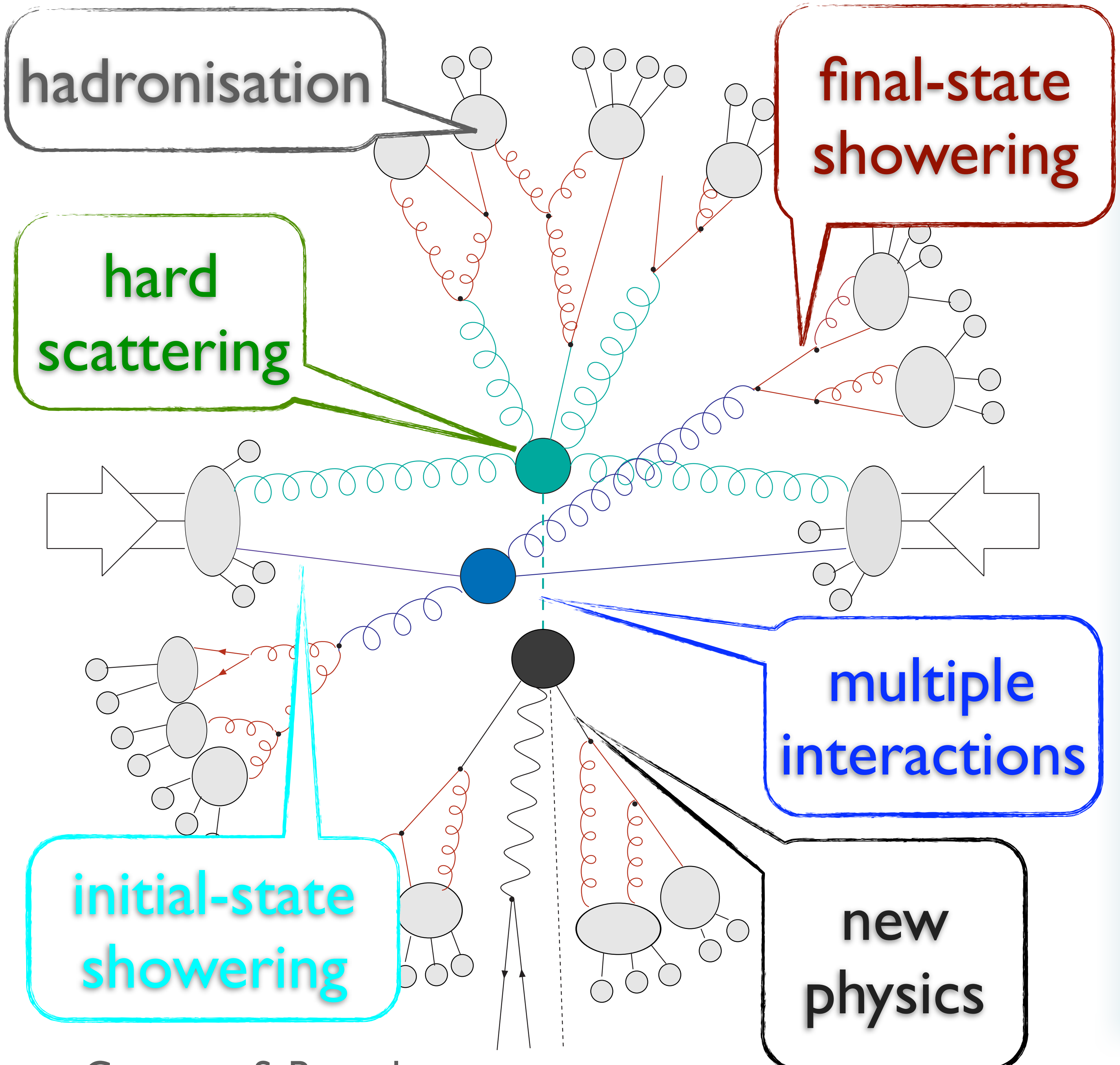

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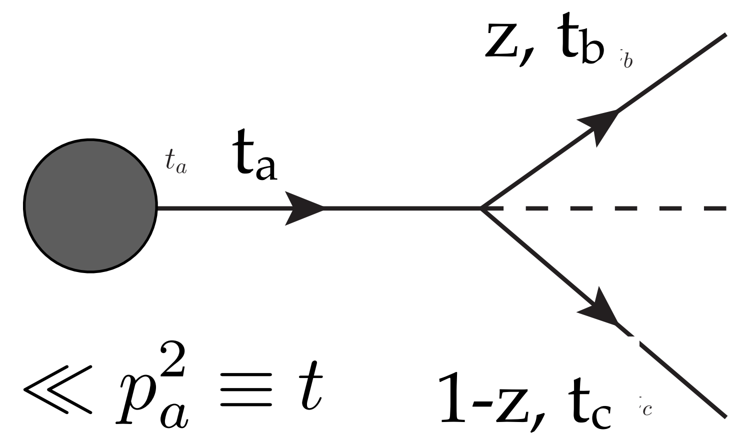
Issues in HIC:

- factorization?
  - soft physics
  - back-reaction
- showering
  - final-state
  - inital-state
- hadronisation
- other...

MC shower: primer



- factorization of phase-space & splitting $p_b^2, p_c^2 \ll p_a^2 \equiv t$

$$d\sigma_{n+1} = d\sigma_n \frac{dt}{t}\frac{d\phi}{2\pi}dz\,\frac{\alpha_s}{2\pi}P(z)$$

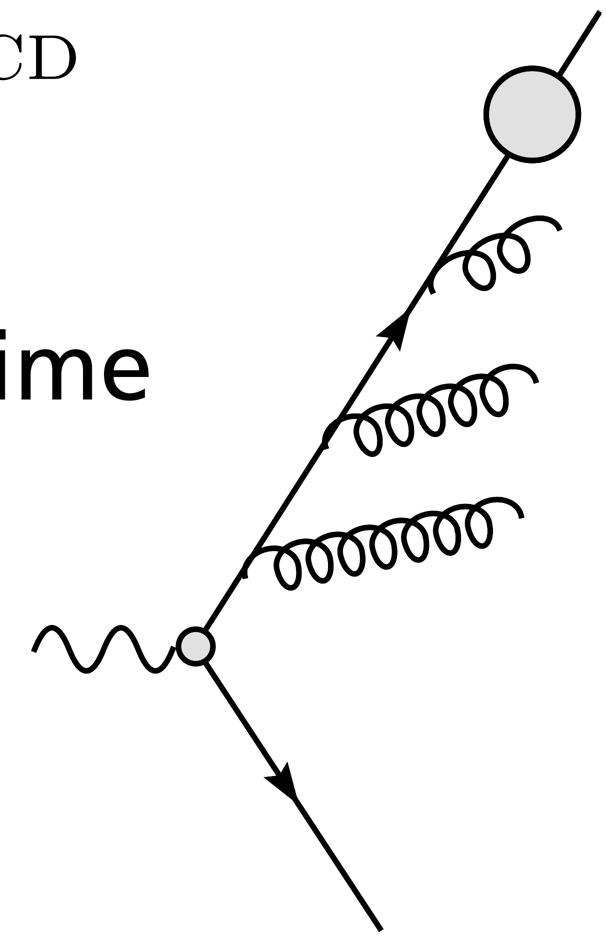
$$t = 2E_bE_c(1-\cos\theta) = z(1-z)E_a^2\theta^2$$

- collinear & soft singularities
- coherence effects (soft)

not discernible at this level yet...

DGLAP evolution

$$t_0 \gg t_1 \gg t_2 \gg \ldots \gg \Lambda^2_{\rm QCD}$$



- strong ordering in virtuality
- strong ordering in formation time

$$t_f = \frac{2\omega}{k_\perp^2}$$

- allows for a probabilistic interpretation!

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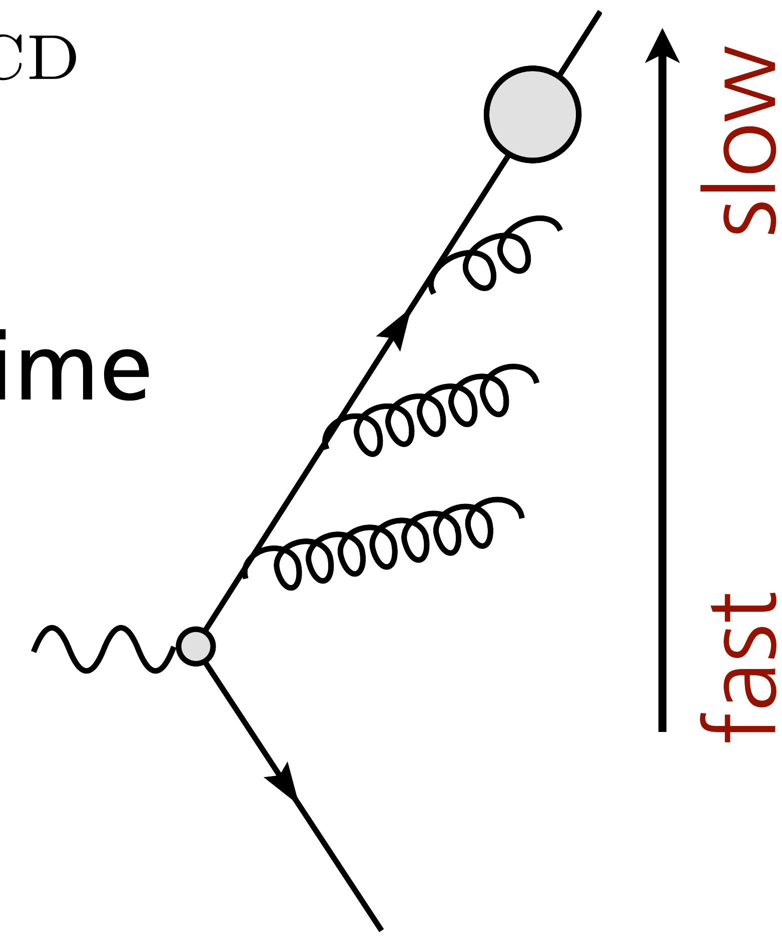

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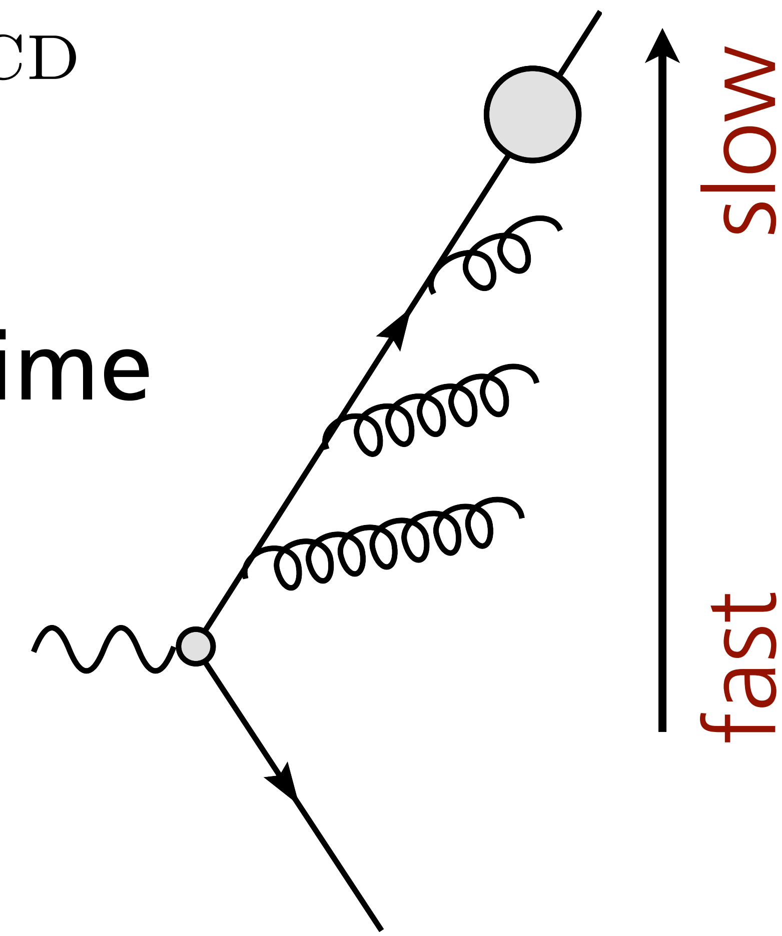
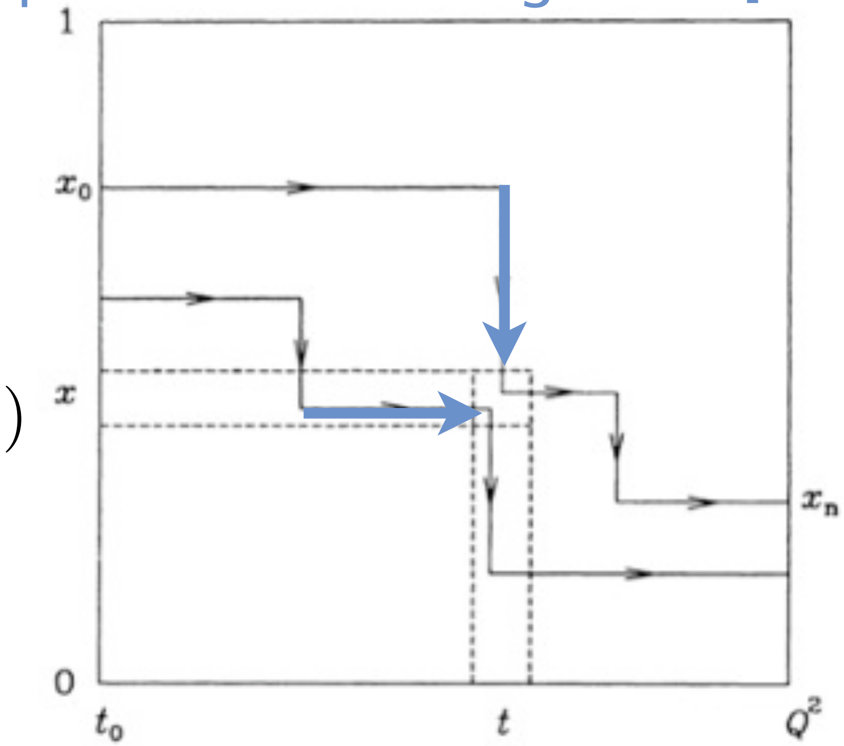


Distribution of gluons with mom fraction ***x*** and virtuality *Q*²

$$t\frac{\partial f_i(x,t)}{\partial t} = \sum_j \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi}P_{ij}(z)f_j\left(\frac{x}{z},t\right)$$

Stochastic process

[space-like branching here…]



$$\mathcal{P}\left(a \to b(z) + c(1-z) \text{ when } t \to t + \delta t\right) = \frac{\delta t}{t}\frac{\alpha_s}{2\pi}P_{ba}(z)$$

tap: $$\delta f_{in}(x,t) = \frac{\delta t}{t}\int_x^1 dx' dz\, \frac{\alpha_s}{2\pi}P(z) f(x',t)\, \delta(x - zx')$$

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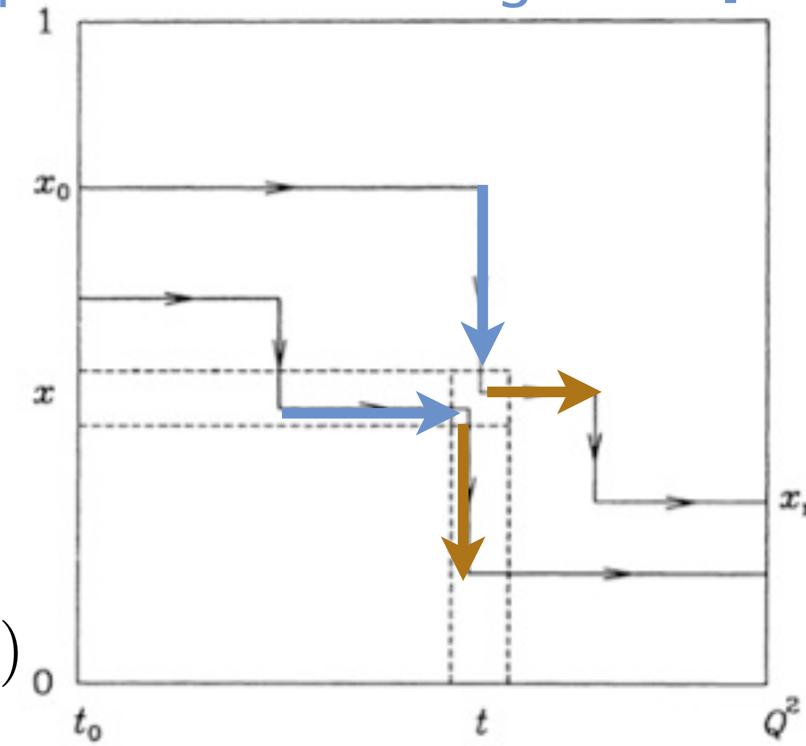
[space-like branching here...]



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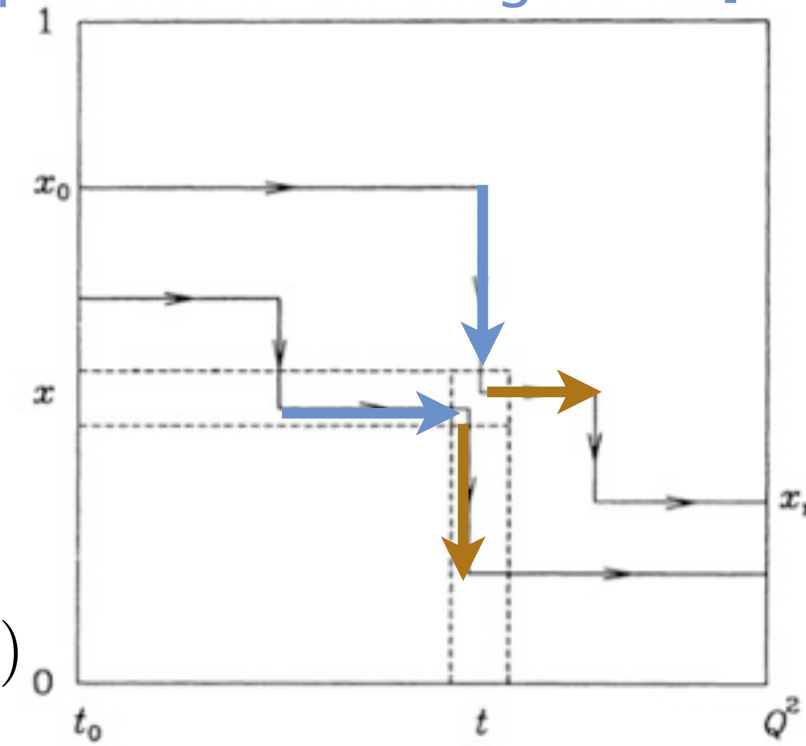

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$$\delta f(x,t) = \delta f_{in}(x,t) - \delta f_{out}(x,t) = \frac{\delta t}{t}\int_0^1 dz \, \frac{\alpha_s}{2\pi} P(z) \left(\frac{1}{z} f\left(\frac{x}{z},t\right) - f(x,t)\right)$$

virtual terms

- evolution "time" = virtuality

Sudakov form factor

$$\Delta(t_1, t_2) = \exp\left[-\int_{t_1}^{t_2} \frac{dt}{t} \int_{z_{\min}(t)}^{1-z_{\min}(t)} dz \, \frac{\alpha_s}{2\pi} P(z)\right]$$

probability of no emission in $[t_1, t_2]$ interval

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Integral equation:

$$f(x,t) = \Delta(t_0,t) f(x,t_0) + \int_{t_0}^{t} \frac{dt'}{t'} \Delta(t',t) \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, t'\right)$$

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Veto algorithm:

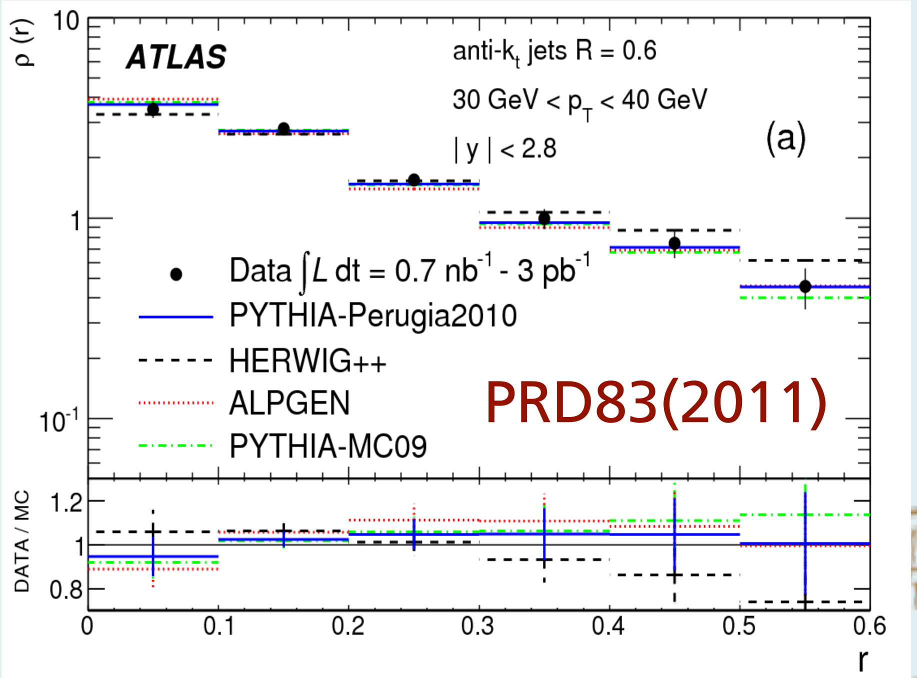
dicing down a step in the ladder using two random numbers

$$\Delta(t,t') = \mathcal{R}_1$$

$$\int_{z_{\min}(t')}^{z'} dz\, \frac{\alpha_s}{2\pi} P(z) = \mathcal{R}_2 \int_{z_{\min}(t')}^{1-z_{\min}(t')} dz\, \frac{\alpha_s}{2\pi} P(z)$$

Coherence effects

- soft gluon radiation implicate large angles
- another type of evolution equation!
- in MC: accounted for in an "average" sense
  - angular ordering
  - good enough for inclusive & collinear observables
  - inter-jet activity



[see Yacine's talk, Mon]

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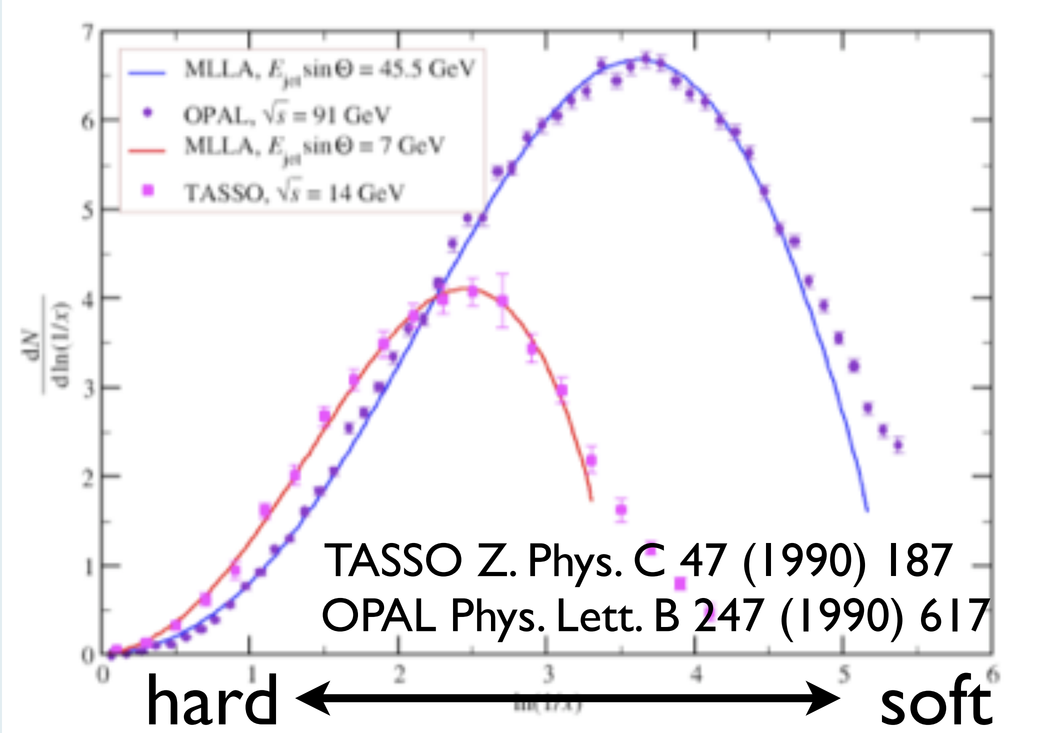
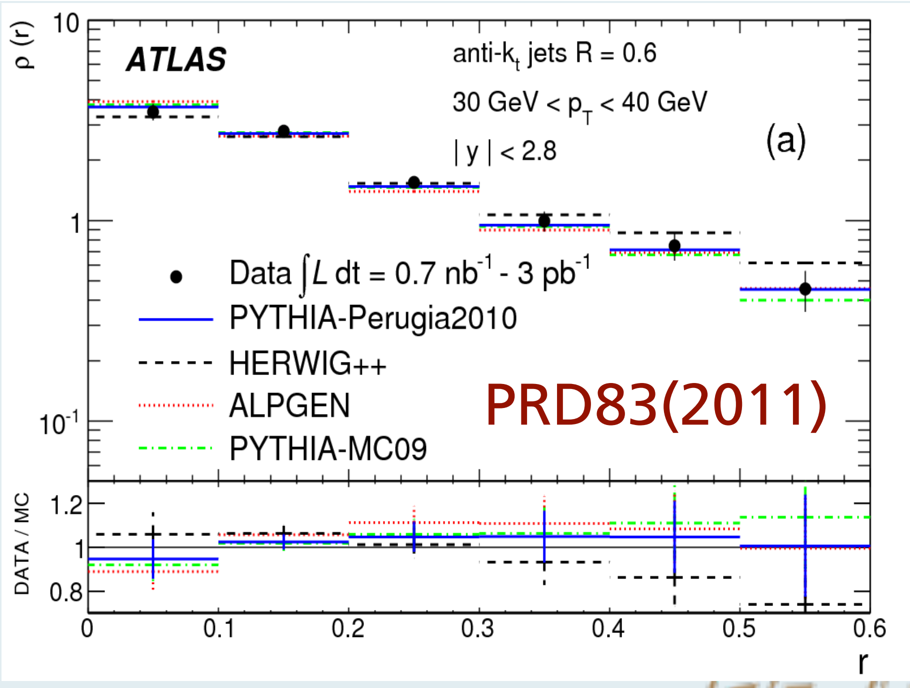
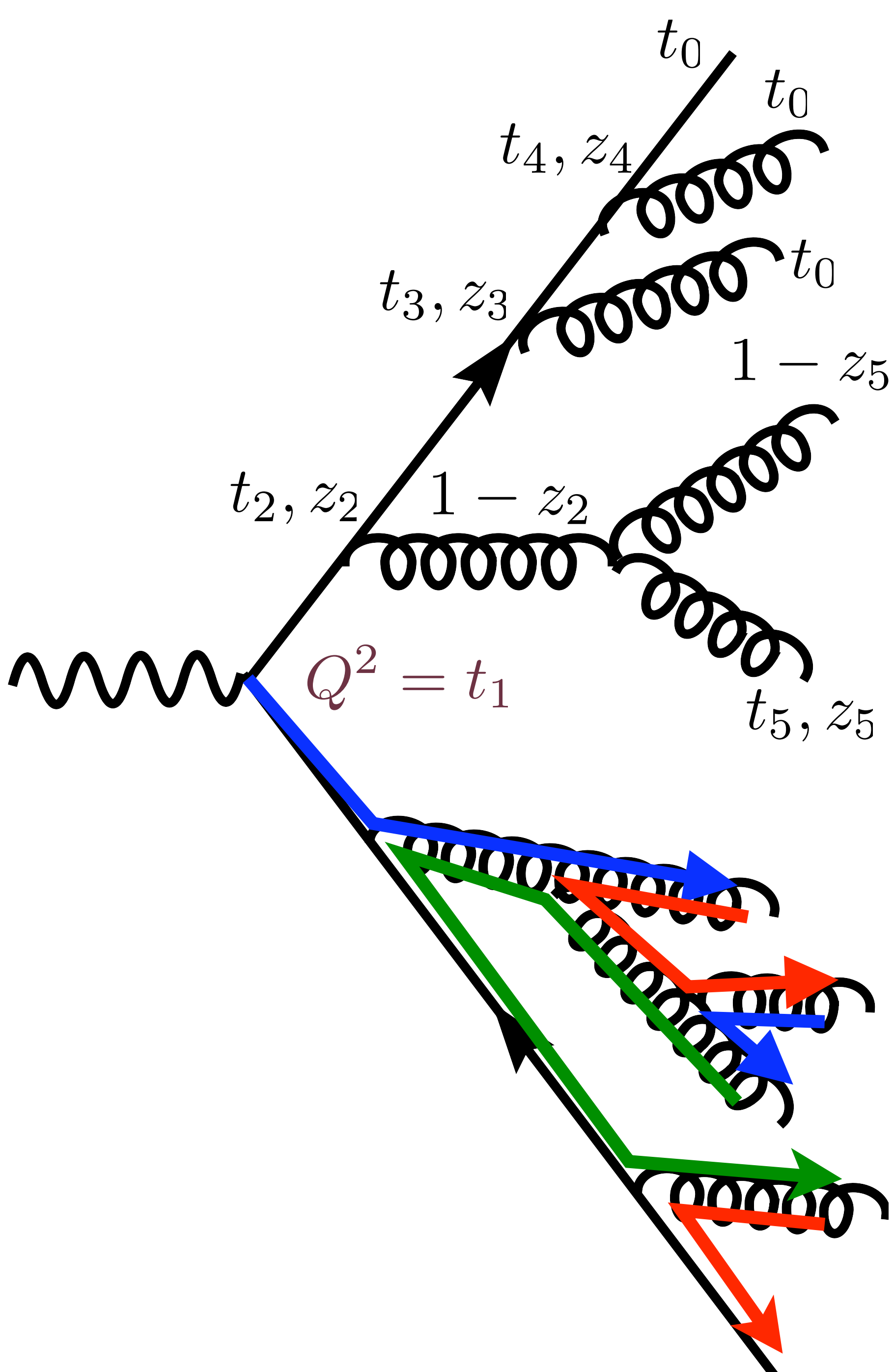




[see Yacine’s talk, Mon]

Parton-showers in vacuum



- no reference to space-time
- large N_c-limit (planar)
  - only closest dipoles can radiate
- multi-jet and matching
- non-perturbative effects
  - color reconnections(!)
  - hadronization at Q_0
    - Lund string model
    - cluster hadronization

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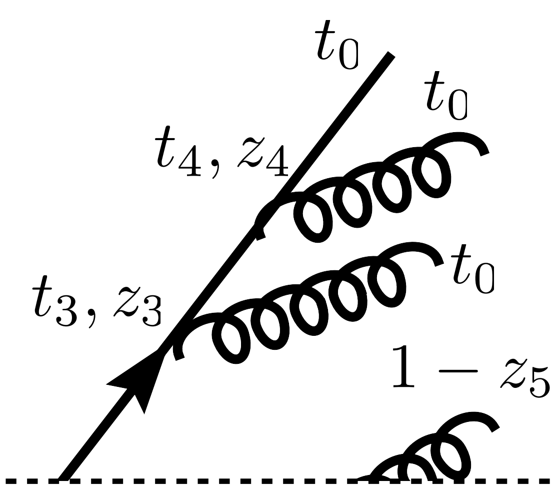

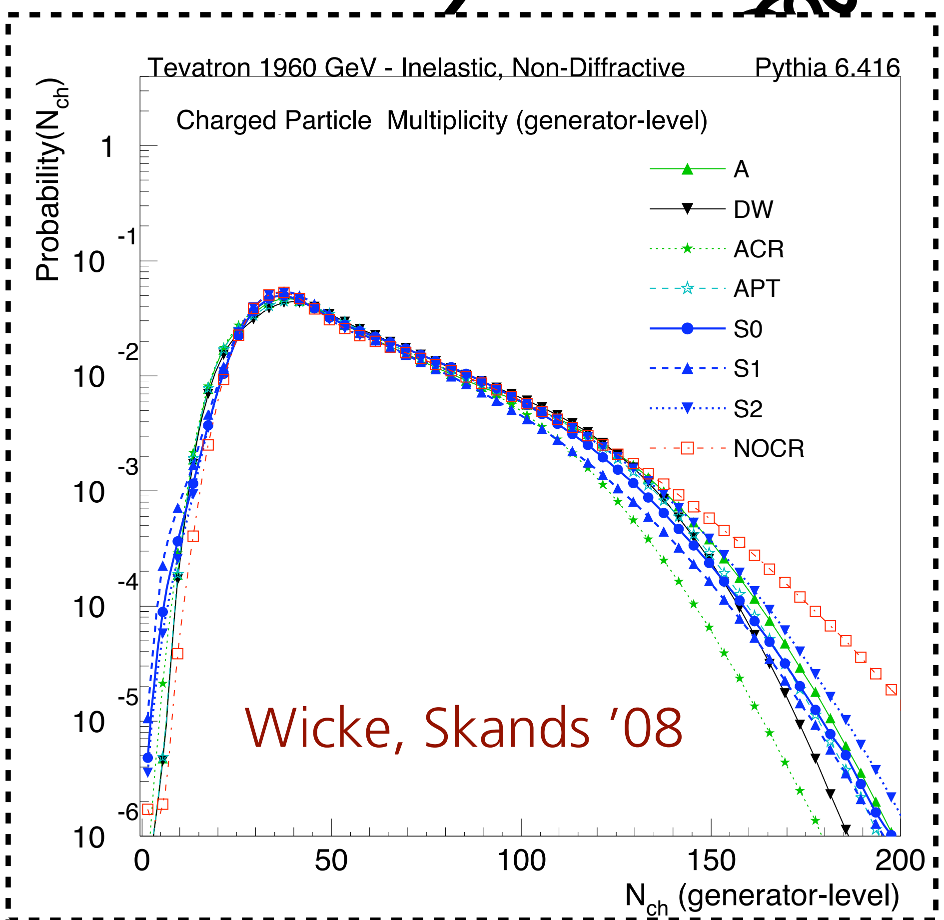

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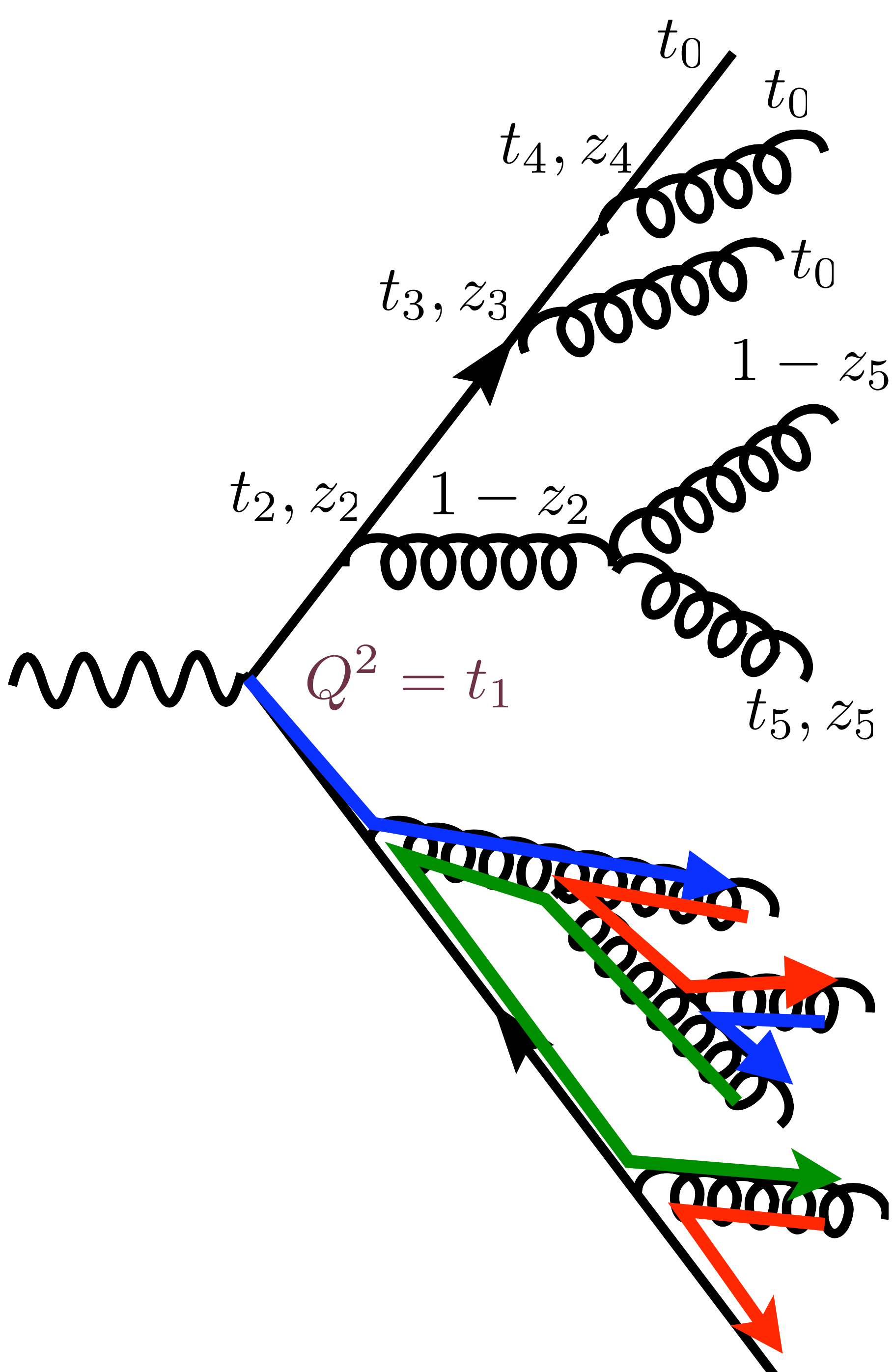

- probabilistic interpretation ensured!
- PYTHIA
  - virtuality ($k_\perp$) ordered (veto on angular ordering)
- HERWIG
  - angular ordered

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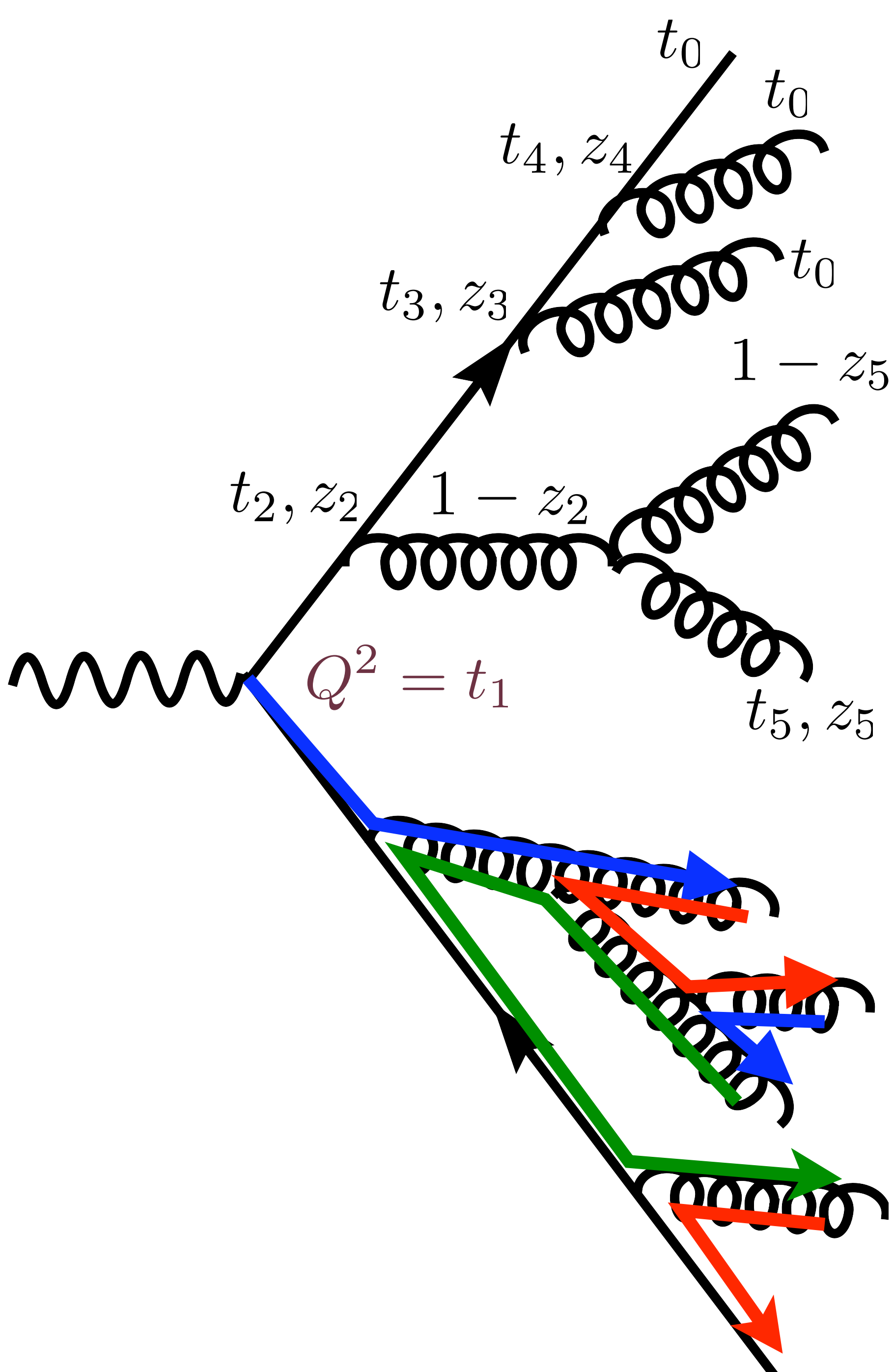

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Issues in HIC:

- two types of radiation
- dispersion of momentum
- reference to space-time
- ...“(un)kown unkowns”

Models: an overview

[not comprehensive..! well of transport formulations!]

	vacuum rad.	med-induced rad.	elastic e.-loss	remarks
HIJING	✔	✔	(?)	full generator
PYQUEN	✔	✔		rough BDMPS
YaJEM	✔	✔	✔	mod kinematics
JEWEL1.0	✔	✔	✔	mod splitting functions + kinematics
JEWEL-LPM		✔		‘exact’ induced radiation
MARTINI	✔	✔	✔	rate equations
Q-PYTHIA	✔	✔		vacuum baseline
Q-HERWIG	✔	✔		vacuum baseline

Wang, Gyulassy PRD 44 (1991) 3501, Comput.Phys.Commun. 83 (1994) 307
Lokhtin, Snigirev EPJC 45 (2006) 211, Renk, PRC 78 (2008) 034908,
Ingelman, Rathsman, Stachel, Wiedemann, Zapp EPJC 60 (2009) 617,
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Q-PYTHIA	✔	✔		vacuum baseline
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only one Comput. Phys. Commun!

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Standard features

Radiative processes

- $2 \rightarrow 3$ induced radiation (Gunion-Bertsch)
- medium-modified splitting functions
- absorptive reactions

Elastic processes

- transverse momentum broadening
- energy transfer, drag effects
- randomization of color
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quantum ⇔ **classical**

[pQCD] [Boltzman eq., …]

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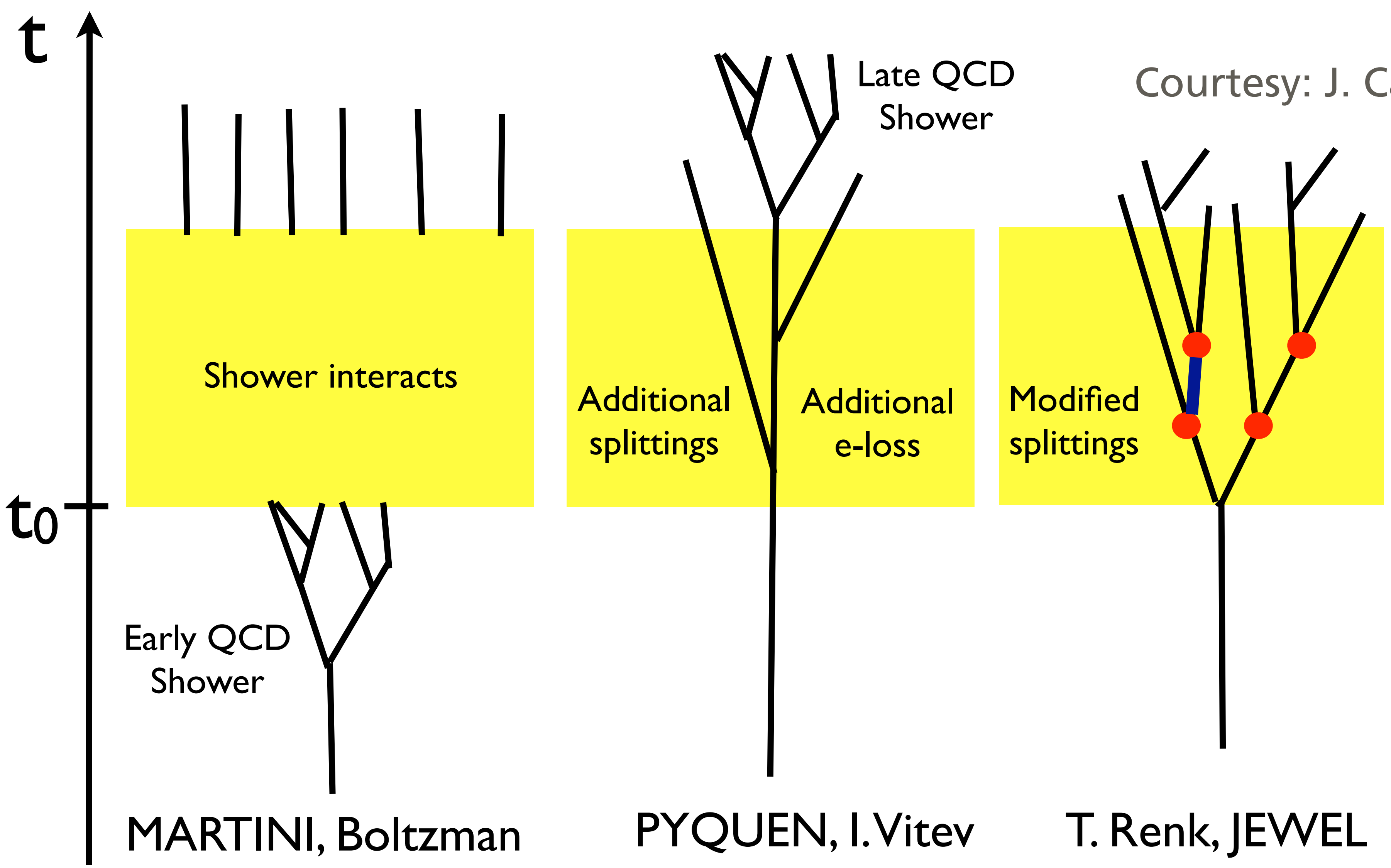
[pQCD] [Boltzman eq., ...]

Differences in evolution



Courtesy: J. Casalderrey-Solana

Is it reasonable to assume a separation of these processes?

We need guidance from theory!

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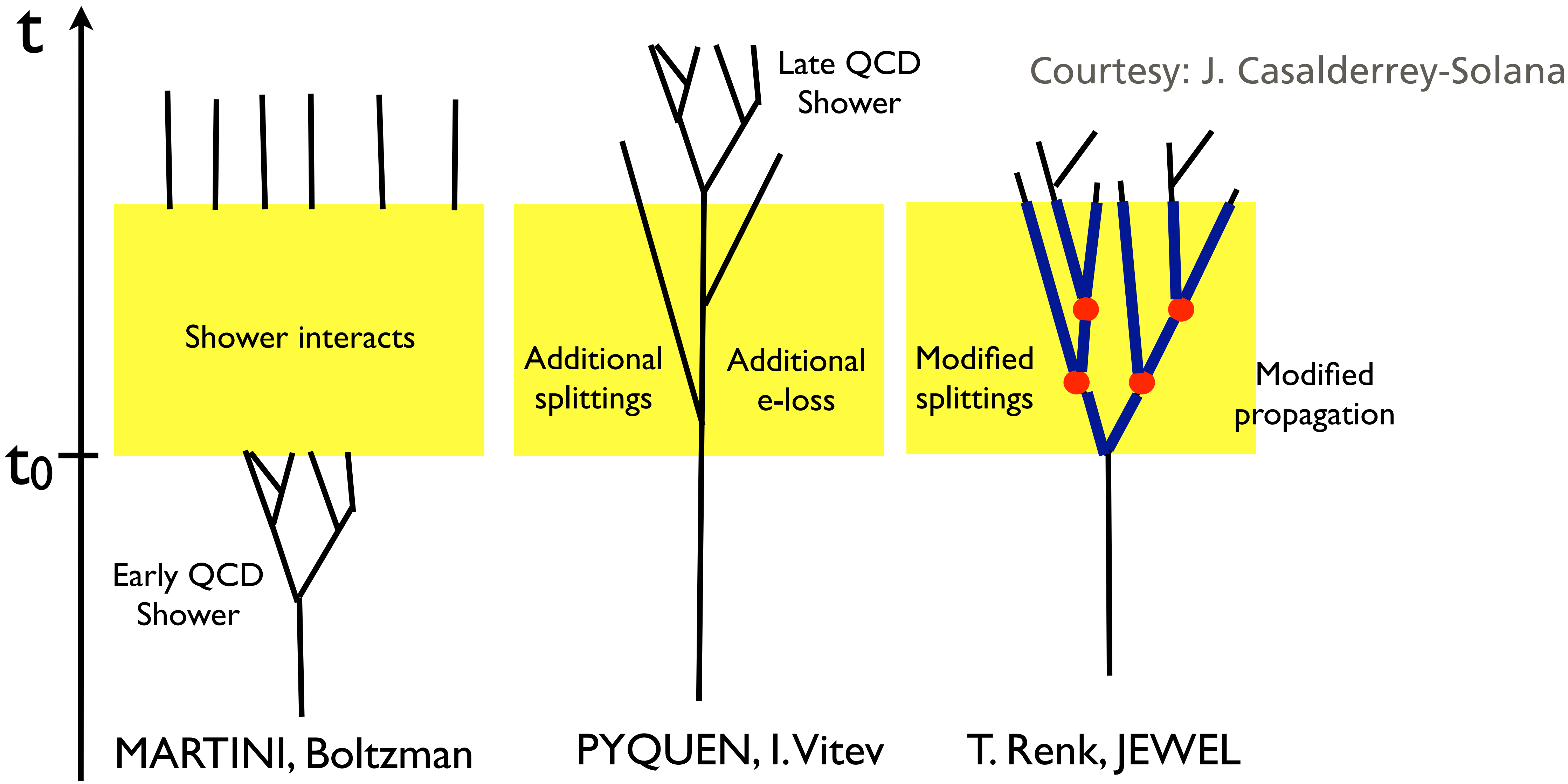

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YaJEM

[lifetime of parton i+1]

$$\langle \tau_{i+1} \rangle = \frac{E_{i+1}}{Q^2_{i+1}} - \frac{E_{i+1}}{Q^2_i}$$

[emergence of parton i+1]

$$\tau^0_{i+1} = \sum_{j=1}^{i} \tau^0_j$$

[smearing of lifetime]

$$P(\tau_i) = \exp\left(- \tau_i / \langle \tau_i \rangle \right)$$

Renk, PRC 78 (2008) 034908

- modifies PYSHOW
  - implements space-time via formation time estimate
- in between splittings
  - “drag”
  - “broadening”
- if $Q^2 \ll \Delta Q^2$: t_f is found iteratively

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[smearing of lifetime]

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$$\Delta Q^2_i = \int_{\tau^0_i}^{\tau^0_i + \tau_i} d\xi \ \hat{q}(\xi)$$

$$\Delta E_i = \int_{\tau^0_i}^{\tau^0_i + \tau_i} d\xi \ D\rho(\xi)$$

Renk, PRC 78 (2008) 034908

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  - "broadening"
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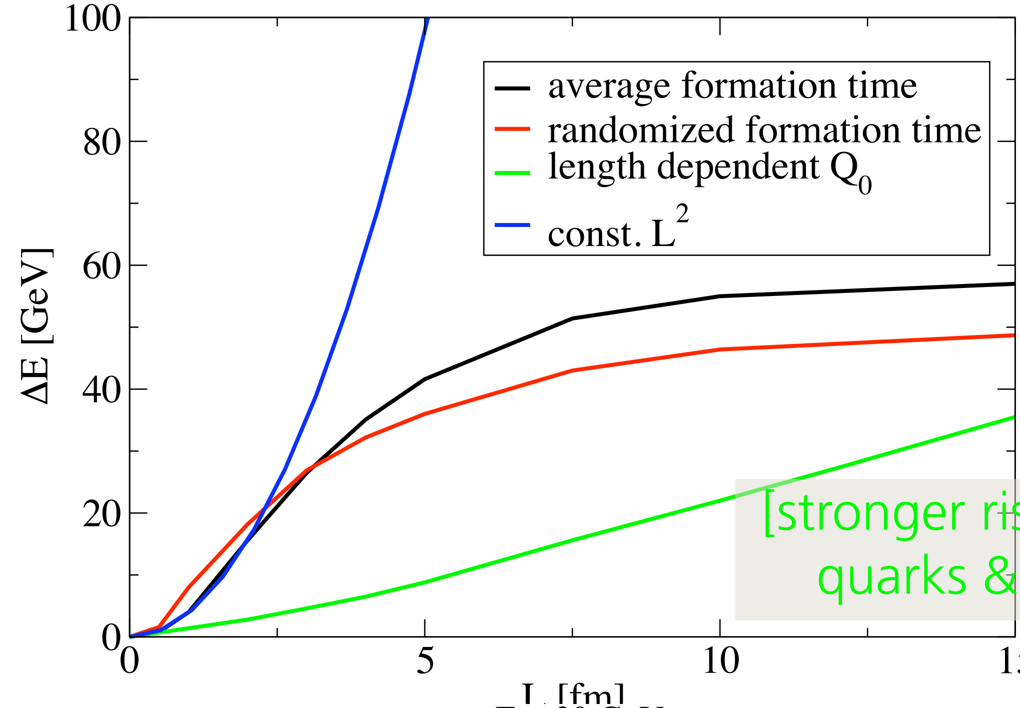
$$\hat{q}(\xi) = 2K \, \varepsilon(\xi)^{3/4} \big[\cosh \rho(\xi) - \sinh \rho(\xi) \cos \psi \big]$$

Path-dependence

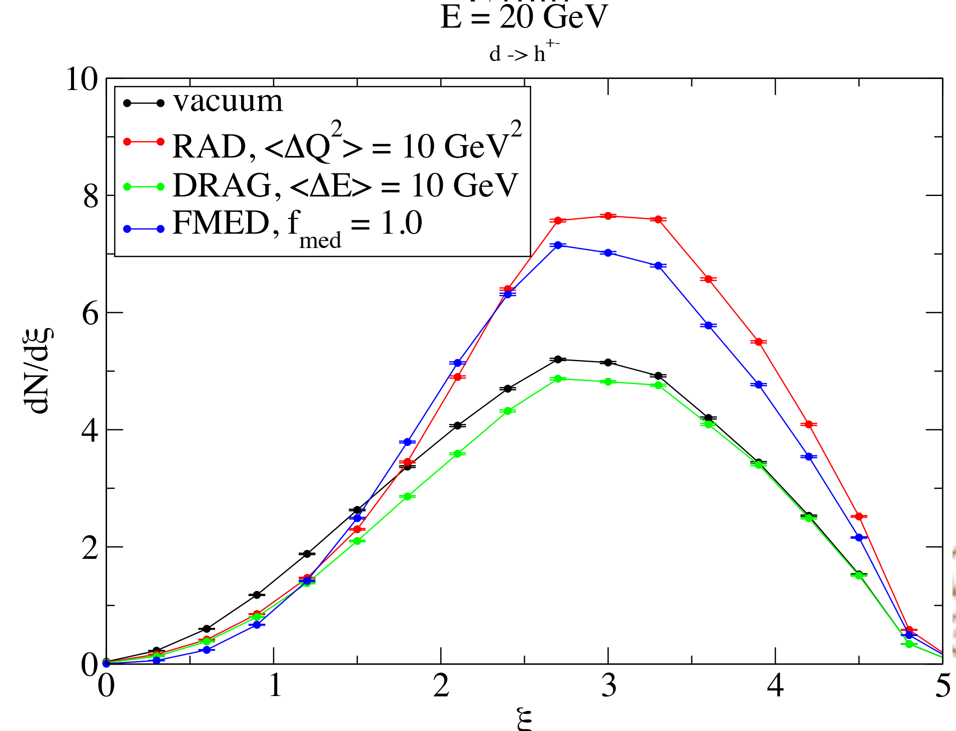
- many modes
  - RAD: only radiative
  - DRAG: only drag
  - FMED: enhanced singularity in vacuum splitting function
  - ASW: ?
- YaJEM-D: enhanced path-length dependence

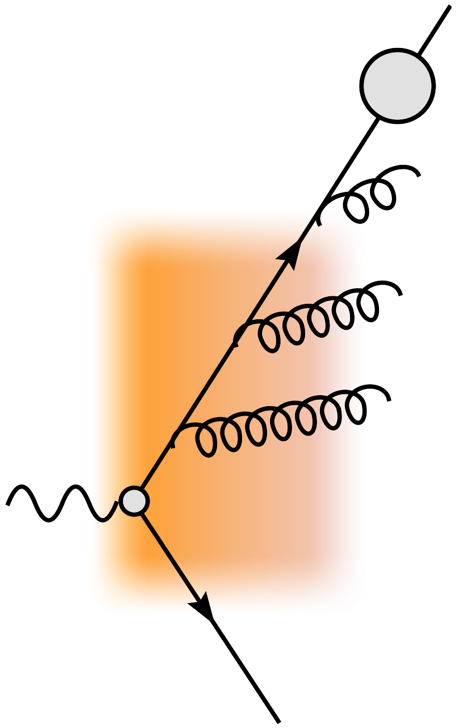
[dynamical cut-off in medium]

$$Q_0 = \sqrt{\frac{E}{L}}$$



[stronger rise for light quarks & gluons]

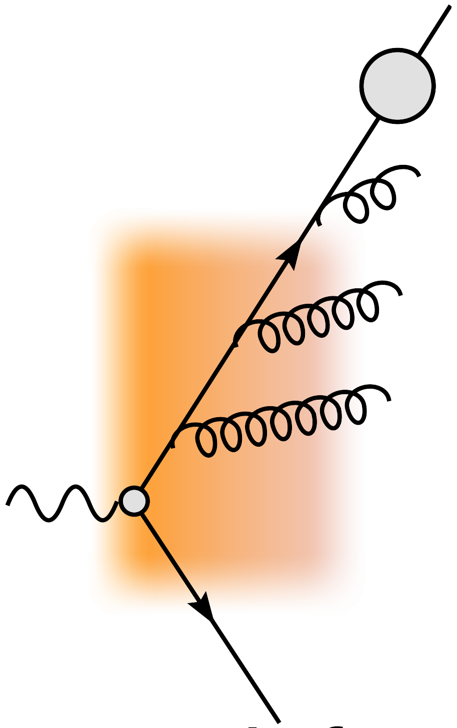




Q-PYTHIA

$$\frac{dN}{dz dk_\perp^2} = \frac{dN^{\mathrm{med}}}{dz dk_\perp^2} + \frac{\alpha_s}{2\pi}\frac{P(z)}{k_\perp^2}$$

Armesto, Cunqueiro, Salgado EPJC 61 (2009) 775



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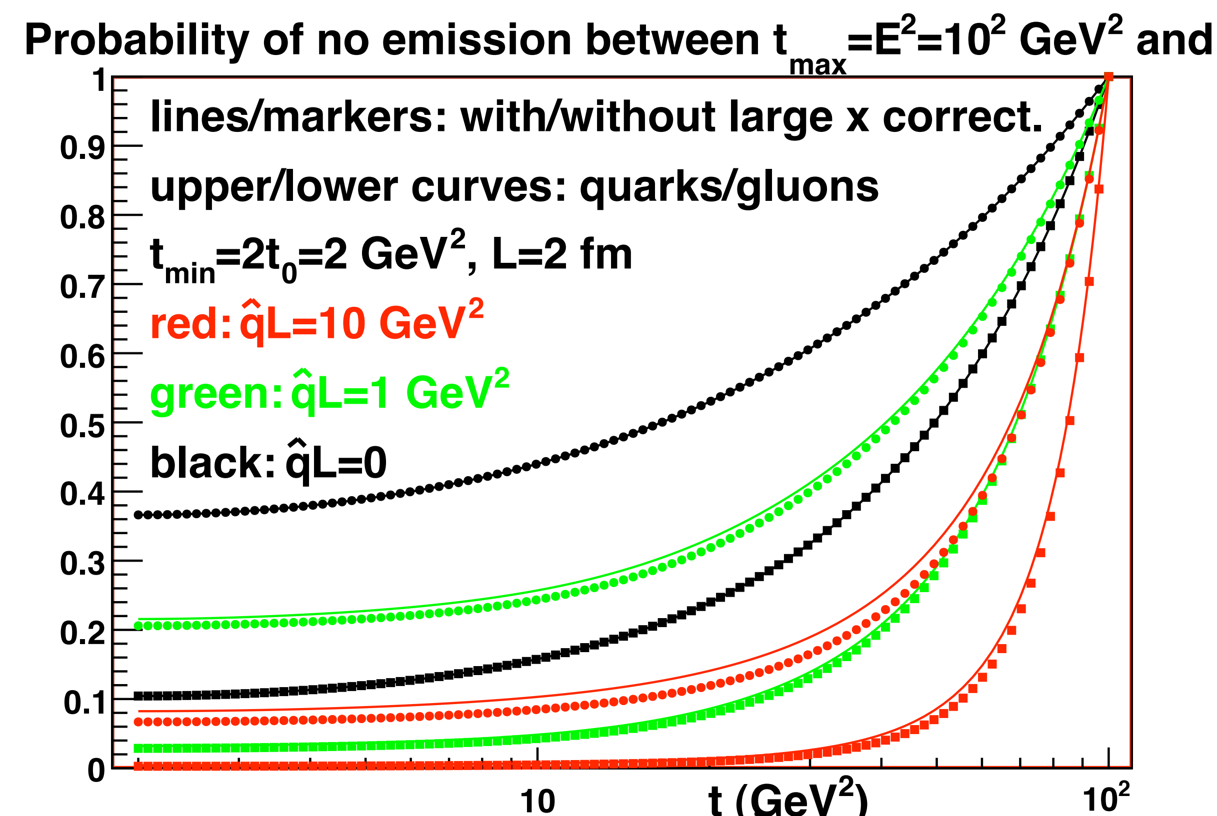
E=10 GeV



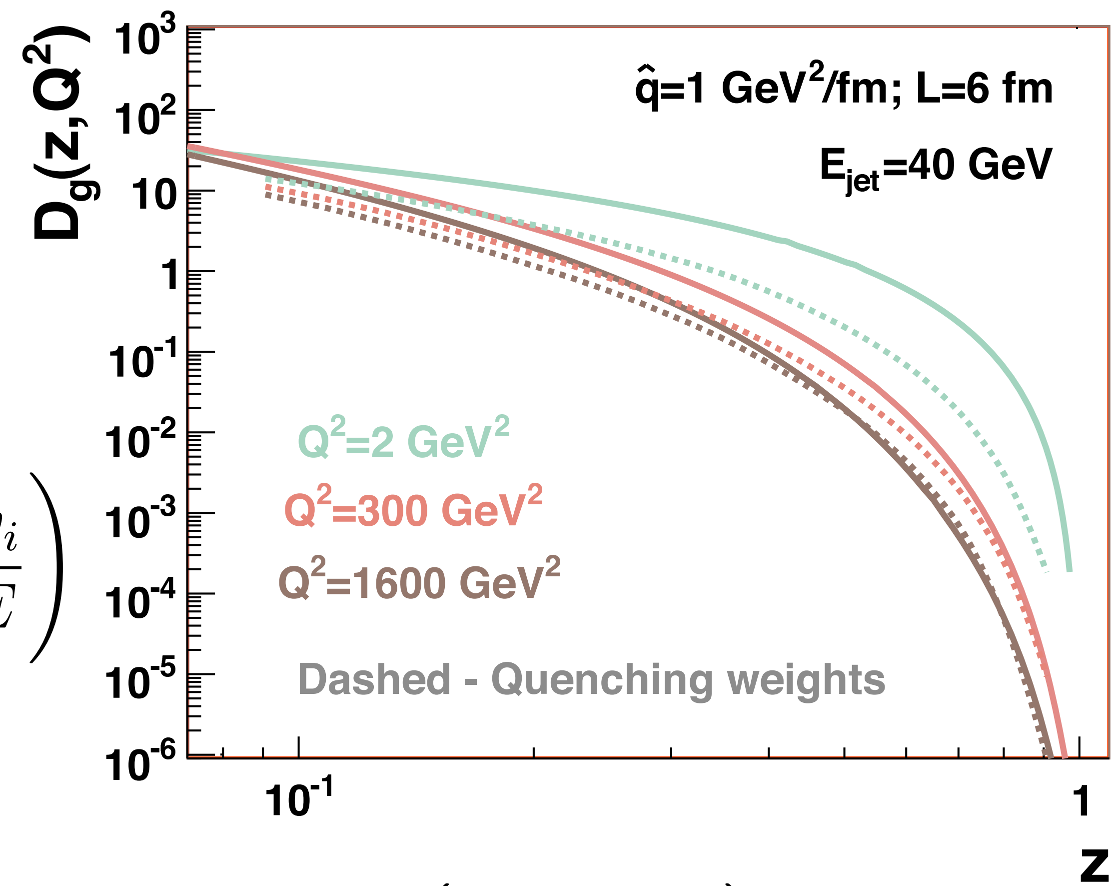
- define a medium-modified splitting function
- showering á la DGLAP as implemented in PYTHIA
  - also as Q-HERWIG
- similar in spirit to HT

$$\Delta(t_1,t_2) = \exp\left\{-\int_{t_1}^{t_2}\frac{dt}{t}\int_{z_{\rm min}(t)}^{1-z_{\rm min}(t)} dz\,\frac{\alpha_s}{2\pi}\left[P(z)+\Delta P(z)\right]\right\}$$

Some motivation

Armesto, Cunqueiro, Salgado, Xiang JHEP 0802 (2008) 048

- neglecting ordering in virtuality: back to quenching weights!



$$p(\epsilon) = p_0 \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int d\omega_i \int d\mathbf{k}_{\perp i} \frac{dI^{\rm med}}{d\omega_i d\mathbf{k}_{\perp i}} \delta\left(\epsilon - \sum_{j=1}^{n} \frac{\omega_i}{E}\right)$$

$$p_0 = \exp\left[-\int d\omega \int d\mathbf{k}_\perp \frac{dI^{\rm med}}{d\omega d\mathbf{k}_\perp}\right]$$

$$D(x,t) \simeq p_0\, D^{\rm vac}(x,t) + \int \frac{d\epsilon}{1-\epsilon}\, p(\epsilon)\, D^{\rm vac}\left(\frac{x}{1-\epsilon}, t\right)$$

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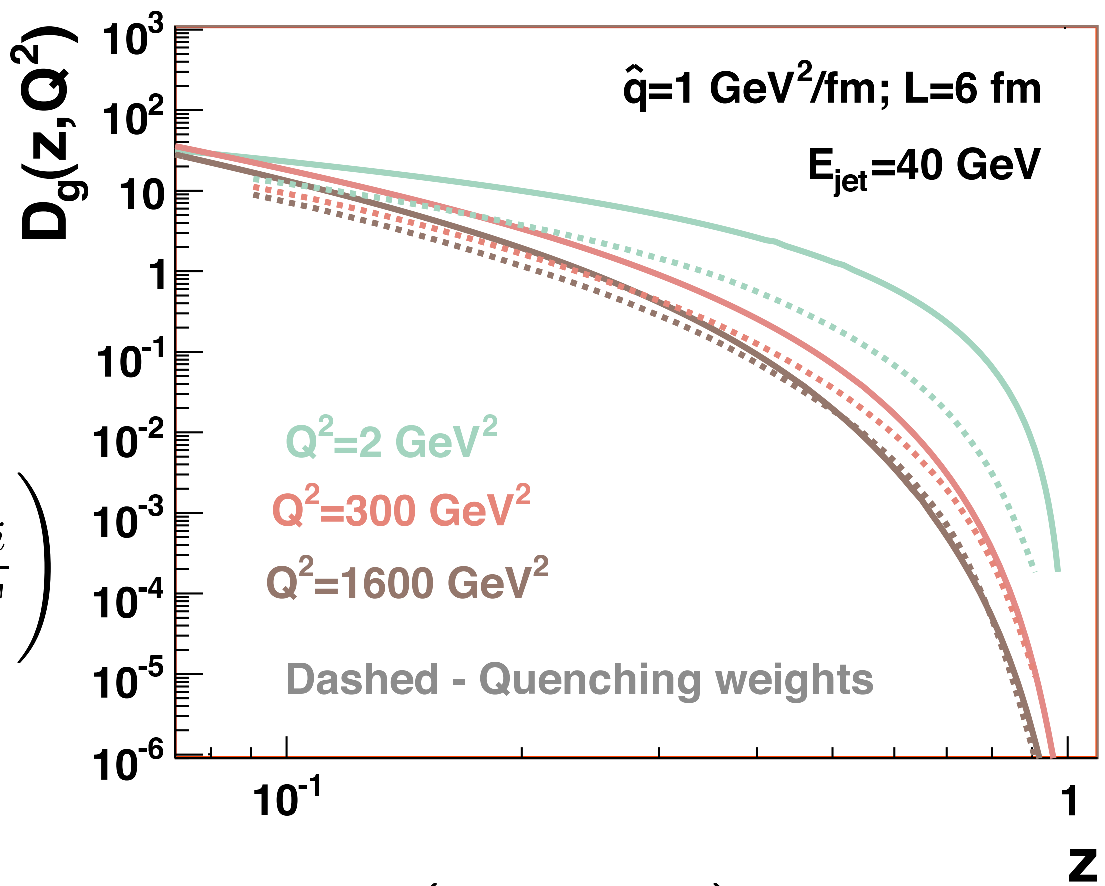

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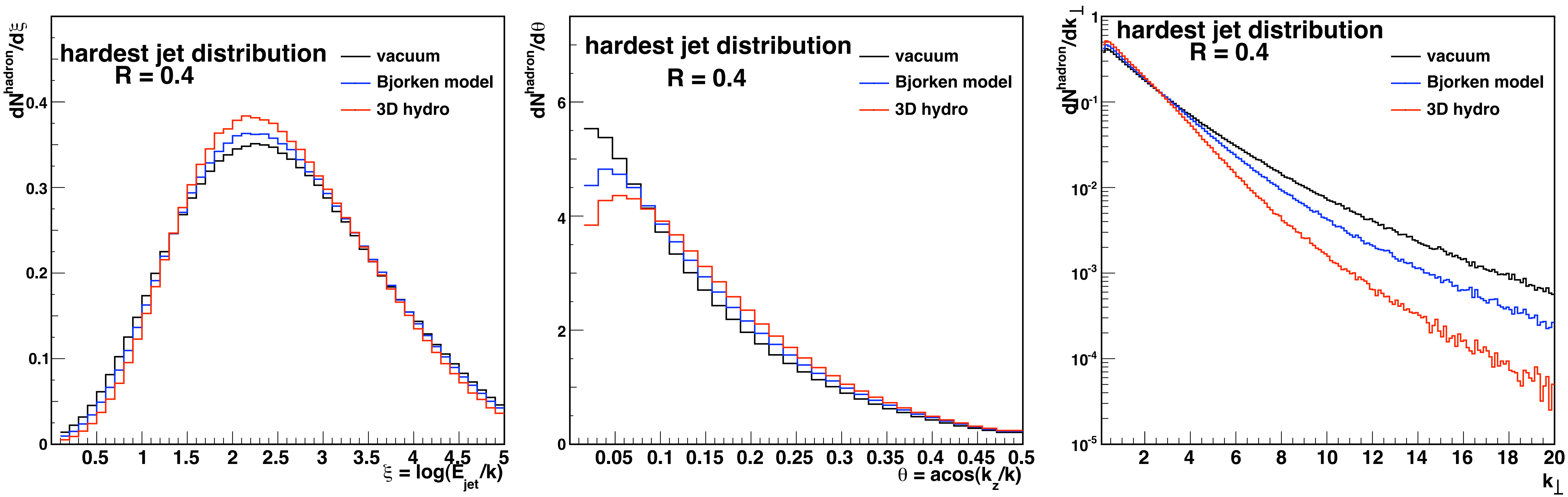
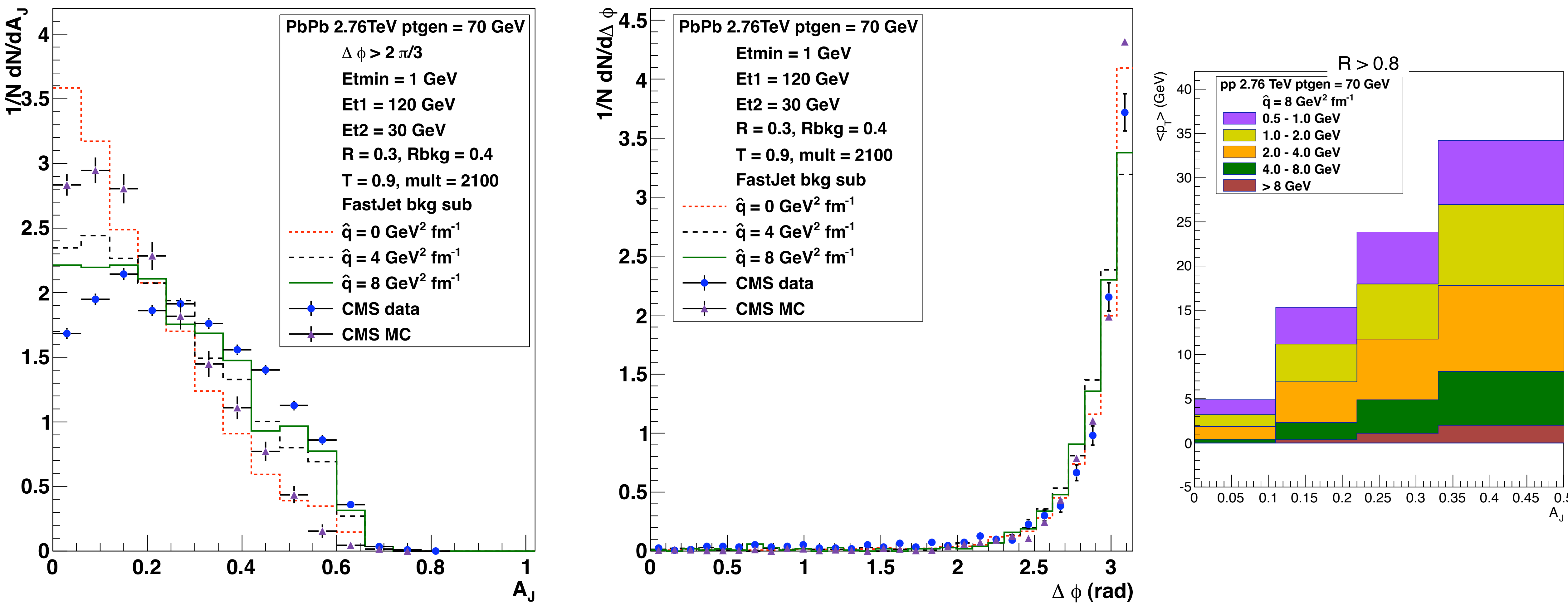
Underlying: medium-rad small correction to virtuality-driven evolution

Selected results



- inter-jet distributions
- effects of expanding medium on jet quenching

Dijet asymmetry



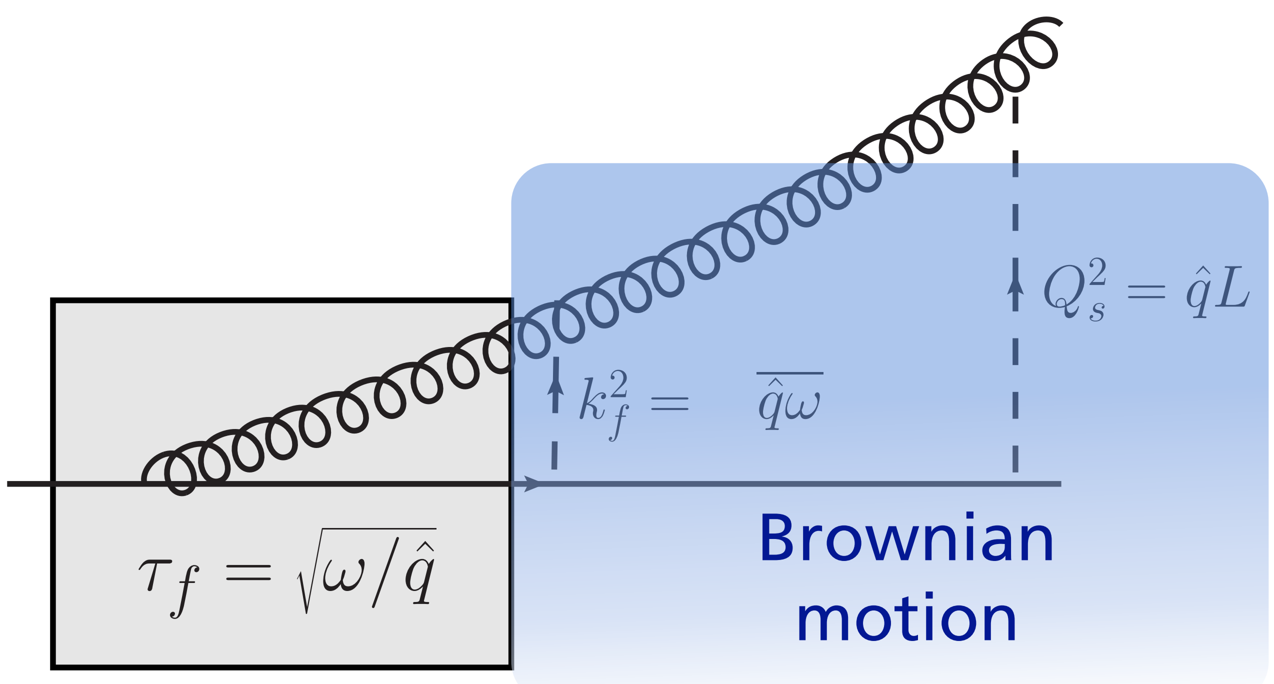
- embedded in background (PSM)
- versatile tool

Apollinario, Armesto, Cunqueiro
arXiv:1207.6587

A local implementation of LPM

Stachel, Wiedemann, Zapp PRL 103 (2009) 152302, JHEP 1107 (2011) 118

- medium-induced radiation: longitudinal coherence
- probabilistic picture interpolating between known limits



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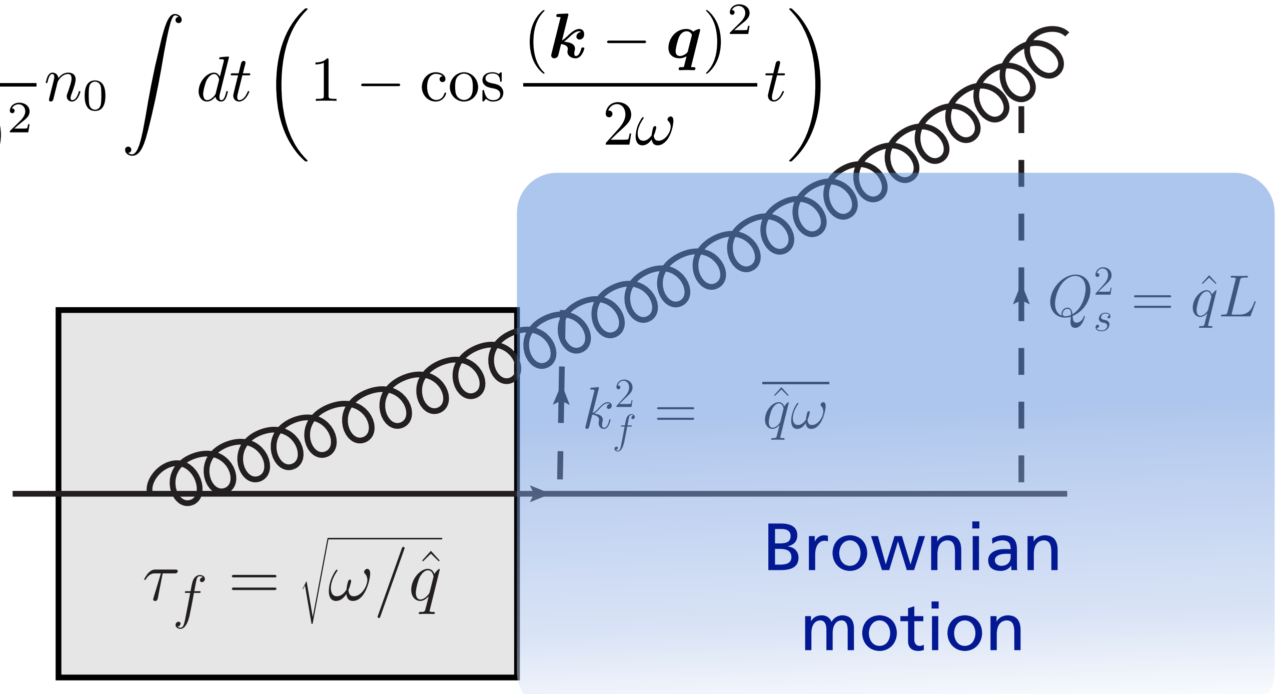
Stachel, Wiedemann, Zapp PRL 103 (2009) 152302, JHEP 1107 (2011) 118

- medium-induced radiation: longitudinal coherence
- probabilistic picture interpolating between known limits

$$\omega \frac{dN^{(1)}}{d\omega d^2\boldsymbol{k} d^2\boldsymbol{q}} = \frac{\alpha_s C_R}{\pi^2} \frac{|A(\boldsymbol{q})|^2}{(2\pi)^2} \frac{2\,\boldsymbol{k}\cdot\boldsymbol{q}}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} n_0 \int dt \left(1 - \cos\frac{(\boldsymbol{k}-\boldsymbol{q})^2}{2\omega} t\right)$$

[at first order in medium opacity]



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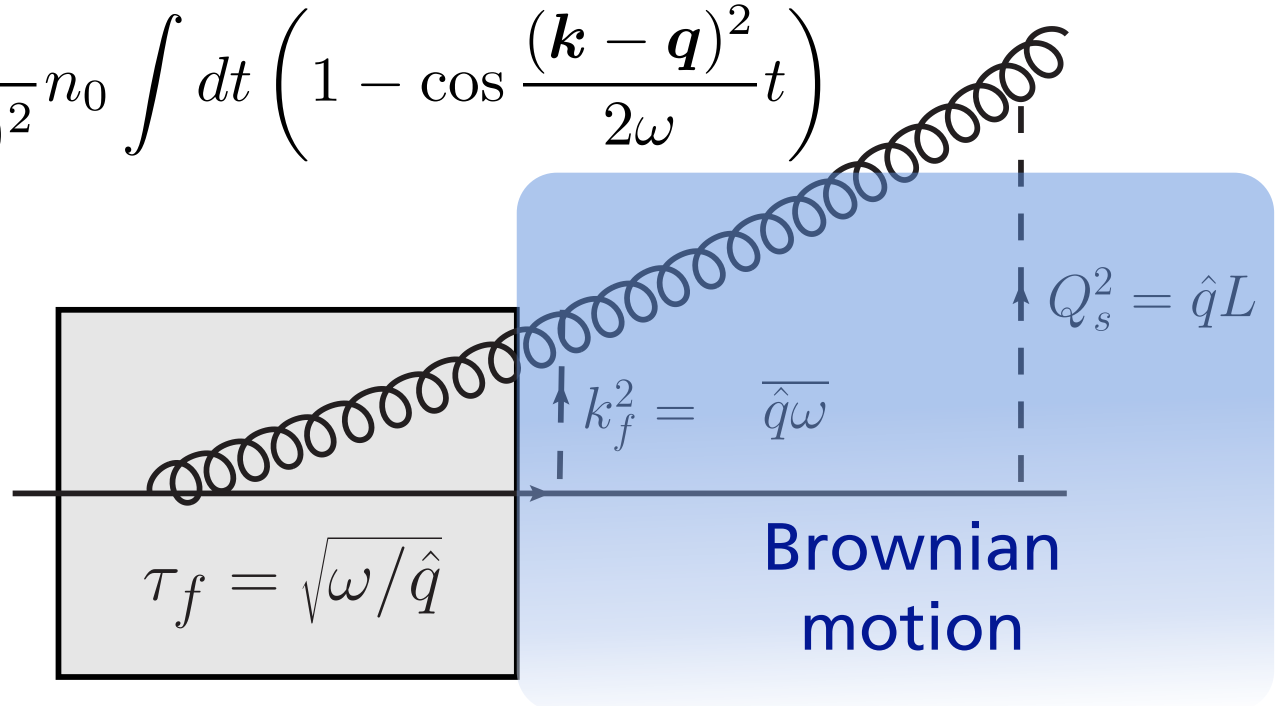
$$V_{\rm tot} = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} |A(\boldsymbol{q})|^2 = \frac{\alpha_s C_A}{\pi} \int d^2\boldsymbol{q} \frac{m_D^2}{(\boldsymbol{q}^2+m_D^2)^2}$$

[prob of scattering with the medium]



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[formation time prior to rescattering]

$$t_f^{(1)} = \frac{2\omega}{(\boldsymbol{k}-\boldsymbol{q})^2}$$

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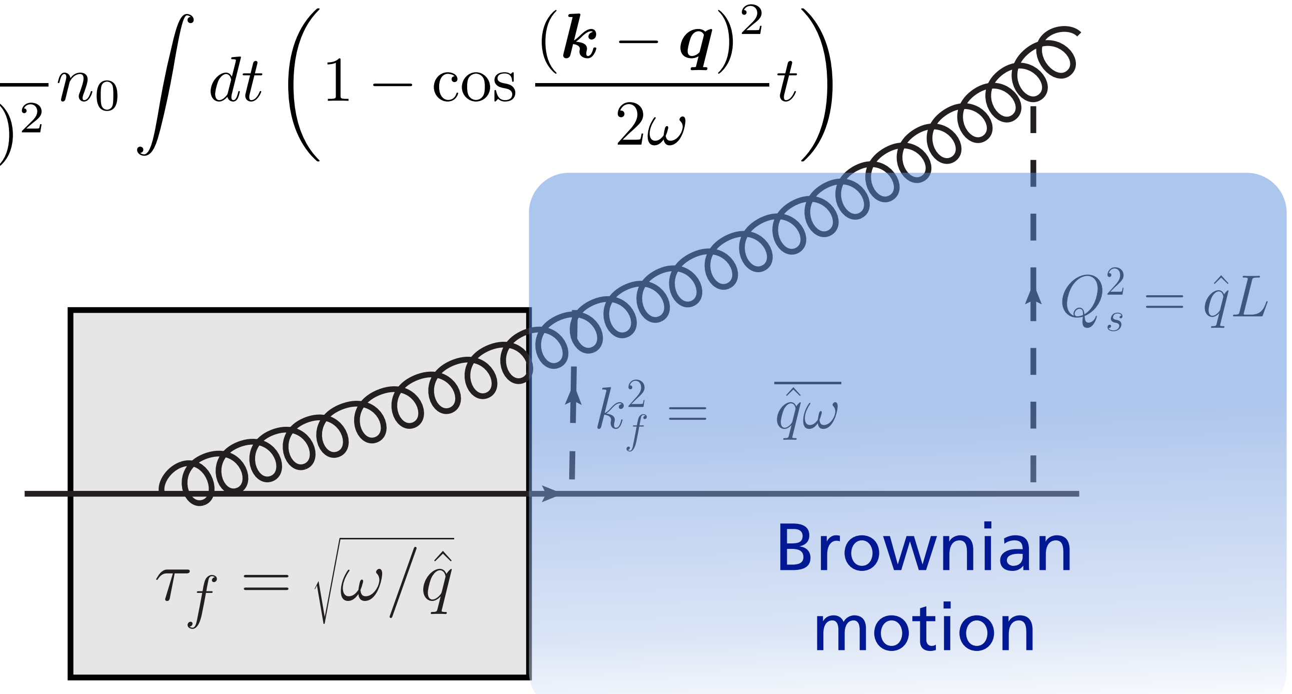

Two illustrative limits

Coherent limit: $L \ll t_f^{(1)}$ $\qquad \omega \frac{dN^{(1)}}{d\omega d^2\boldsymbol{k} d^2\boldsymbol{q}} = 0$

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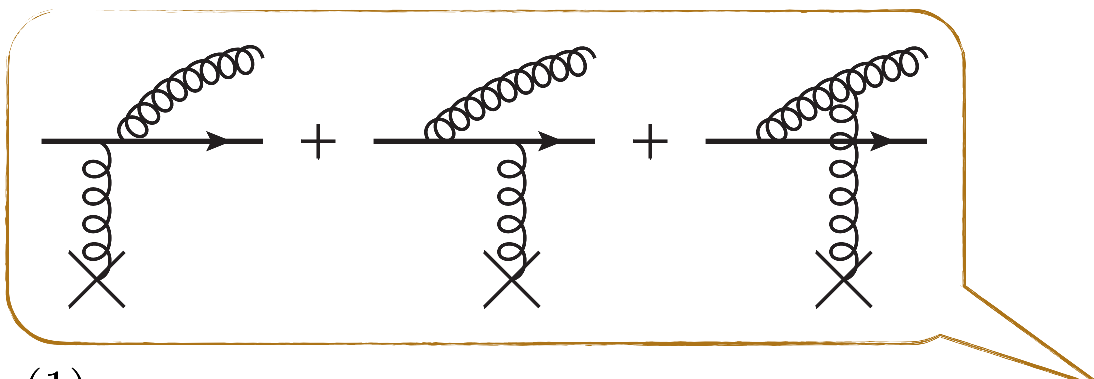

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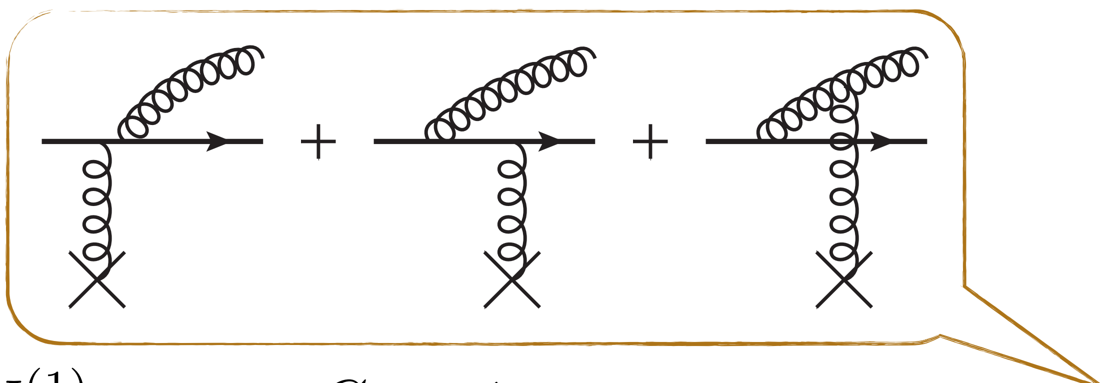

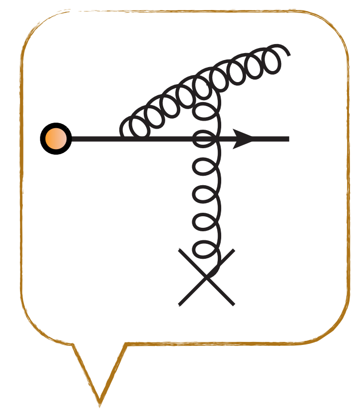

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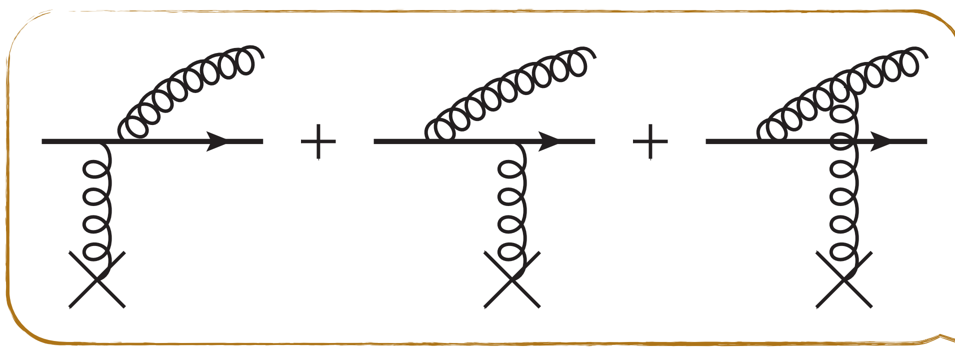

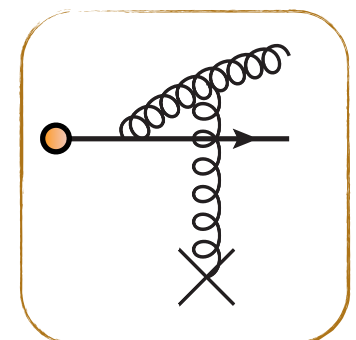

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Conservation of probability!

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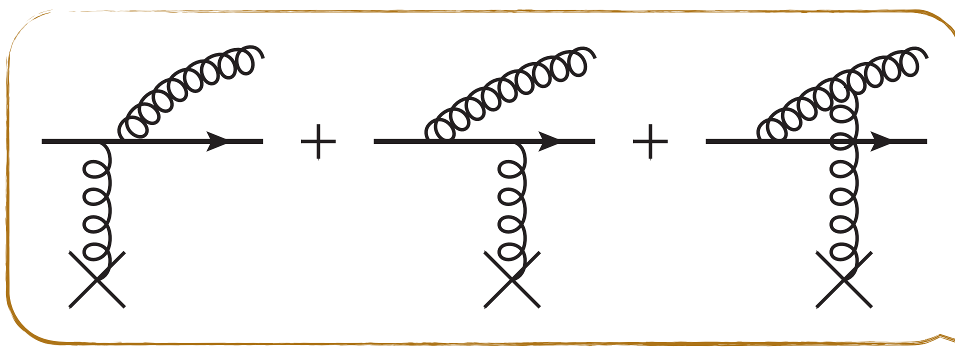

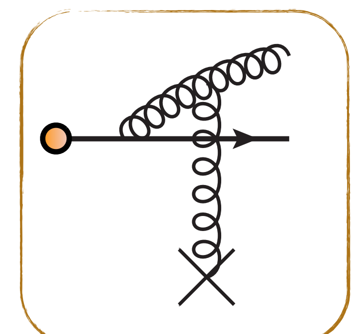

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Conservation of probability!

Resummed form factor: $S_{\rm el} = \exp\left[- n_0 L V_{\rm tot} \right]$

MC implementation

- neglecting final-state rescattering of vacuum radiation
  - equivalent to radiation off asymptotic charge
  - perfectly ok for $\omega\, dN/d\omega$
  - rescattering of induced radiation included in $R(k+\sum q_{rescat}, \sum q_{induced})$
- veto radiation that hasn't been formed inside the medium (qualitative guidance)
- dynamical determination of t_f
- numerical simplification: $R(\boldsymbol{k},\boldsymbol{q}) \sim \delta\big(\boldsymbol{k}-\boldsymbol{q}\big)$

Space-time propagation

- define mean path lengths for rescattering and radiation
- emitter scatters only inelastically, "emitee" scatters only elastically

$$\lambda_{\rm el} \equiv \frac{1}{n_0 V_{\rm tot}} \qquad \lambda_{\rm inel} \equiv \frac{1}{n_0 \sigma_{\rm inel}}$$

BDMPS-Z limit: $\lambda_{\rm el} \ll \lambda_{\rm inel}$

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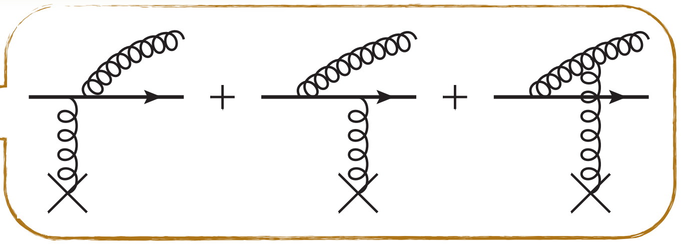

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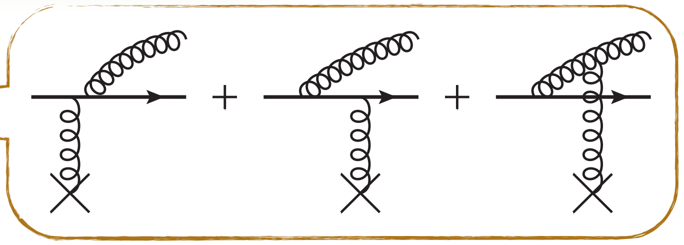
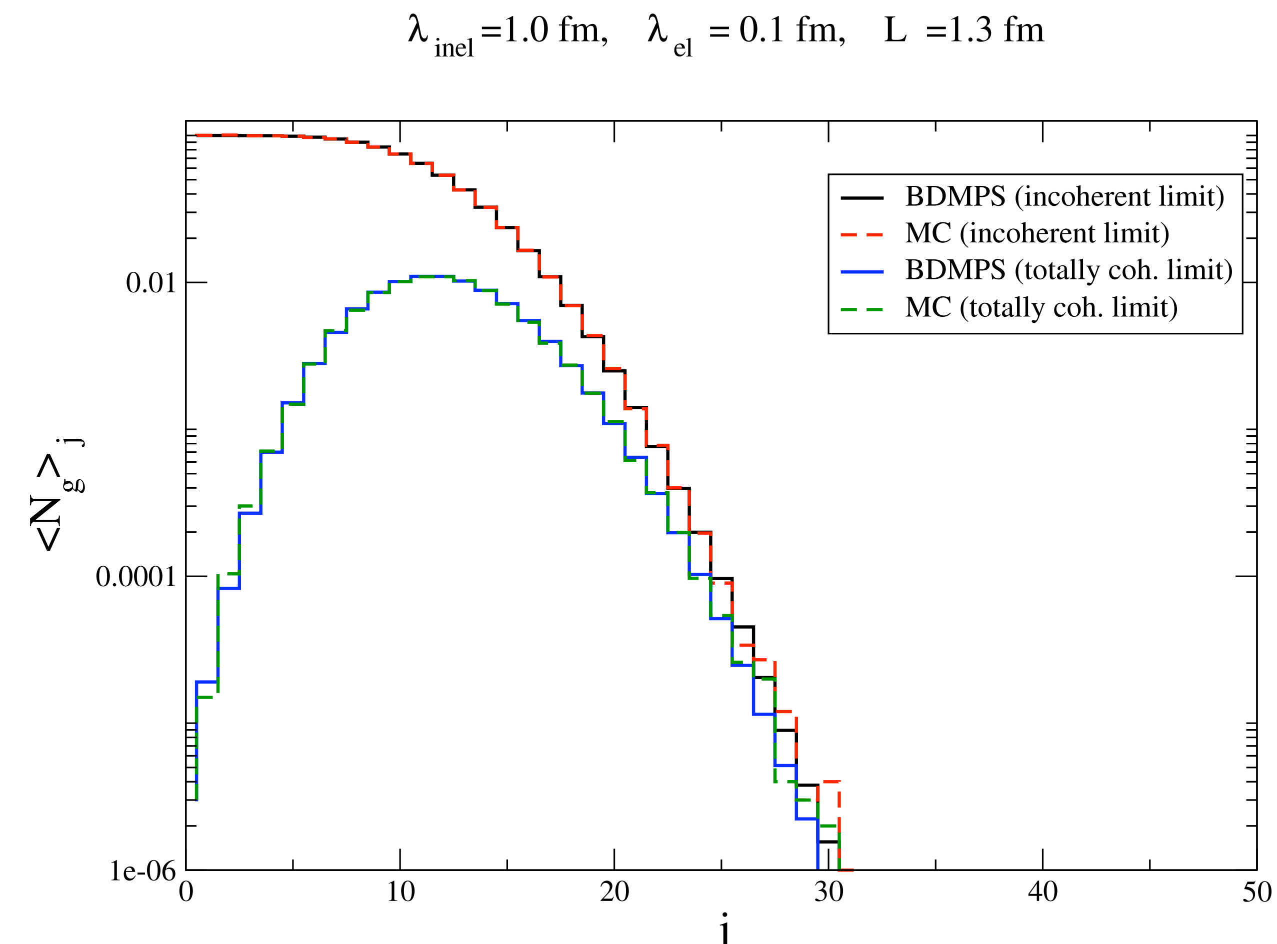


$$S_{\rm no}^{\rm (in)el}(L) = \exp\left(-L/\lambda_{\rm (in)el}\right)$$

Subtleties due to coherence

- t_f establishes a "zone" for emission
  - $t_f \gg L$: elastic re-scatterings can take place along the **whole** (not remaining) path
- re-weight by $(N_s)^{-1}$



[Ns = number of scatterings during formation]

$$\langle N_g^{\rm incoh} \rangle (N_s) = N_s \, \langle N_g^{\rm coh} \rangle (N_s)$$

MC algorithm

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- determine position of (next) inelastic scattering: $1\text{-}S^{inel}_{no}(L\text{-}\xi)$

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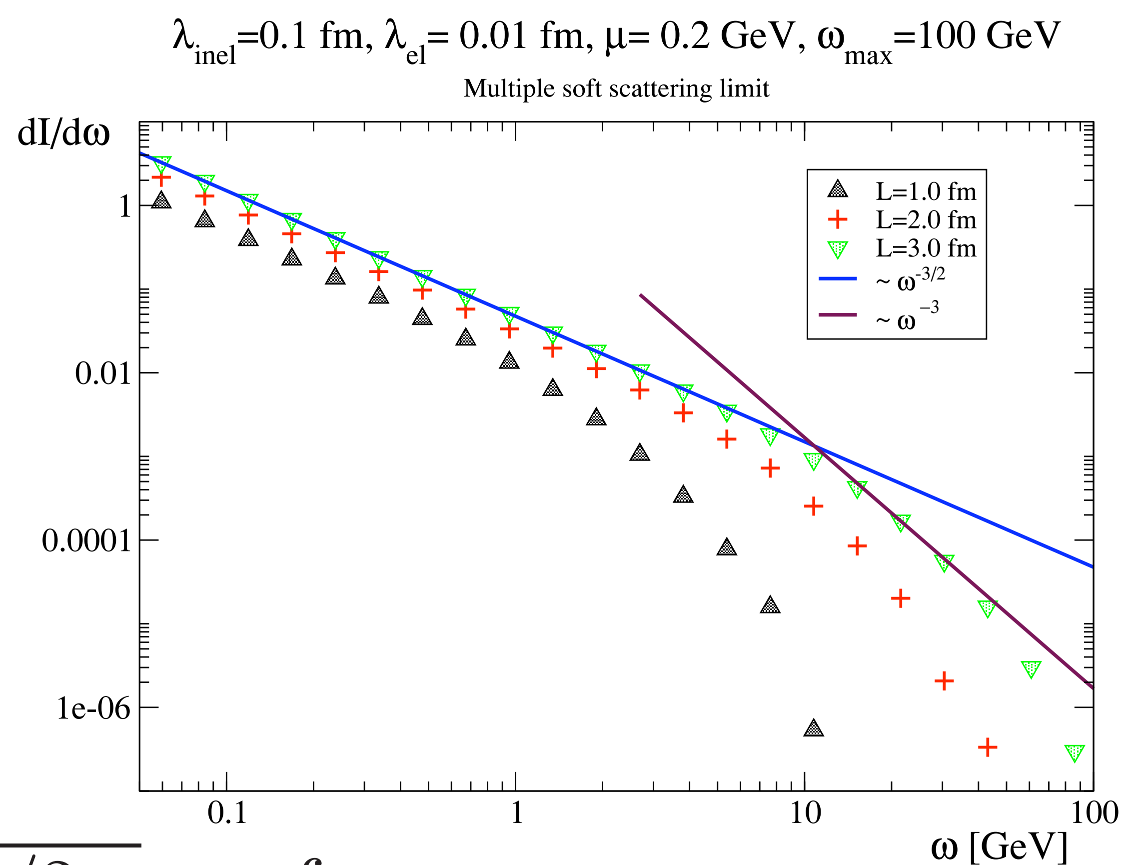
  - accept gluon with prob N_s^{-1} and determine a random production point in a box with size t_f around ξ
  - propagate further to find remaining elastic scatterings

Limit: BDMPS-Z spectrum

- "BDMPS-Z scenario"
  - only soft scattering
  - only projectile is allowed to radiate
- spectrum is nicely reproduced with applied approximations
- allows to go beyond!

[see Fabio's talk, Wed]

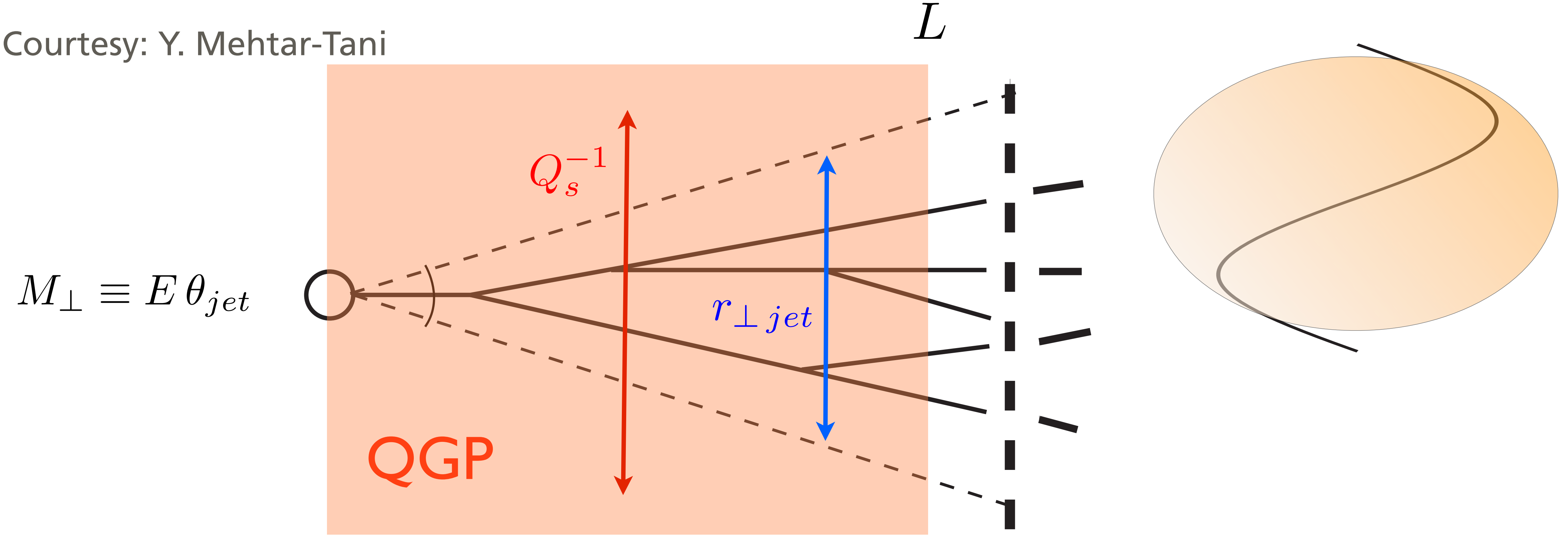
$$|A(\boldsymbol{q})|^2 \rightarrow |A(\boldsymbol{q})|^2 \; \Theta(2m_D - \boldsymbol{q})$$



$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \simeq \frac{2\alpha_s C_R}{\pi} \begin{cases} \sqrt{\omega_c/2\,\omega} & \text{for} \quad \omega \ll \omega_c \\ \frac{1}{12}\left(\frac{\omega_c}{\omega}\right)^2 & \text{for} \quad \omega \gg \omega_c \end{cases}$$

Vacuum-medium interface

Courtesy: Y. Mehtar-Tani



- bremsstrahlung is treated quite differently in the various codes
- separation of scales at LHC!

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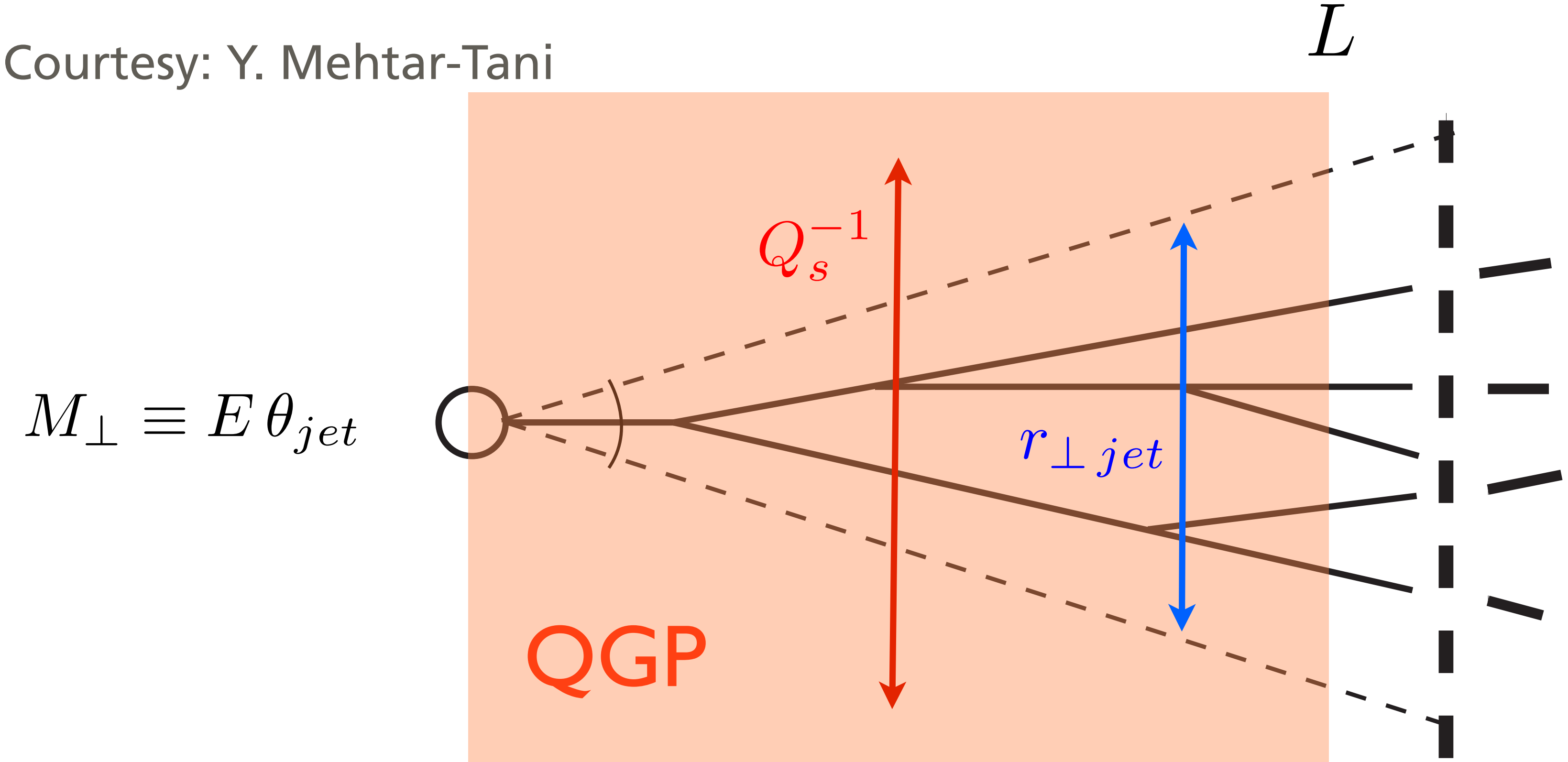

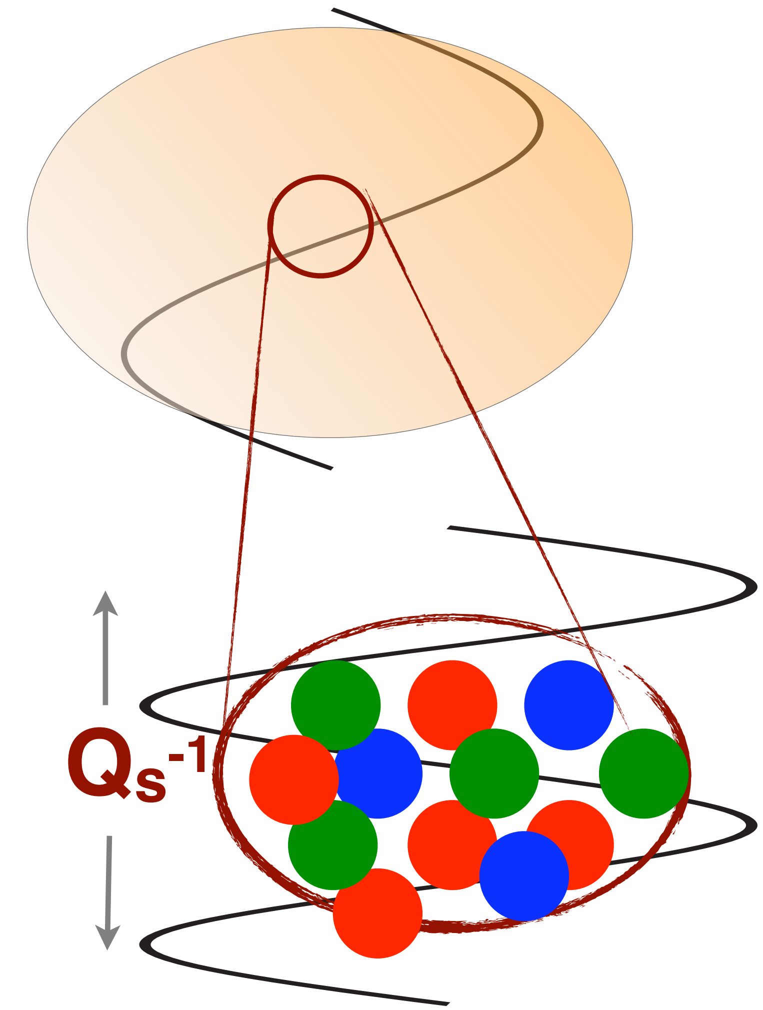

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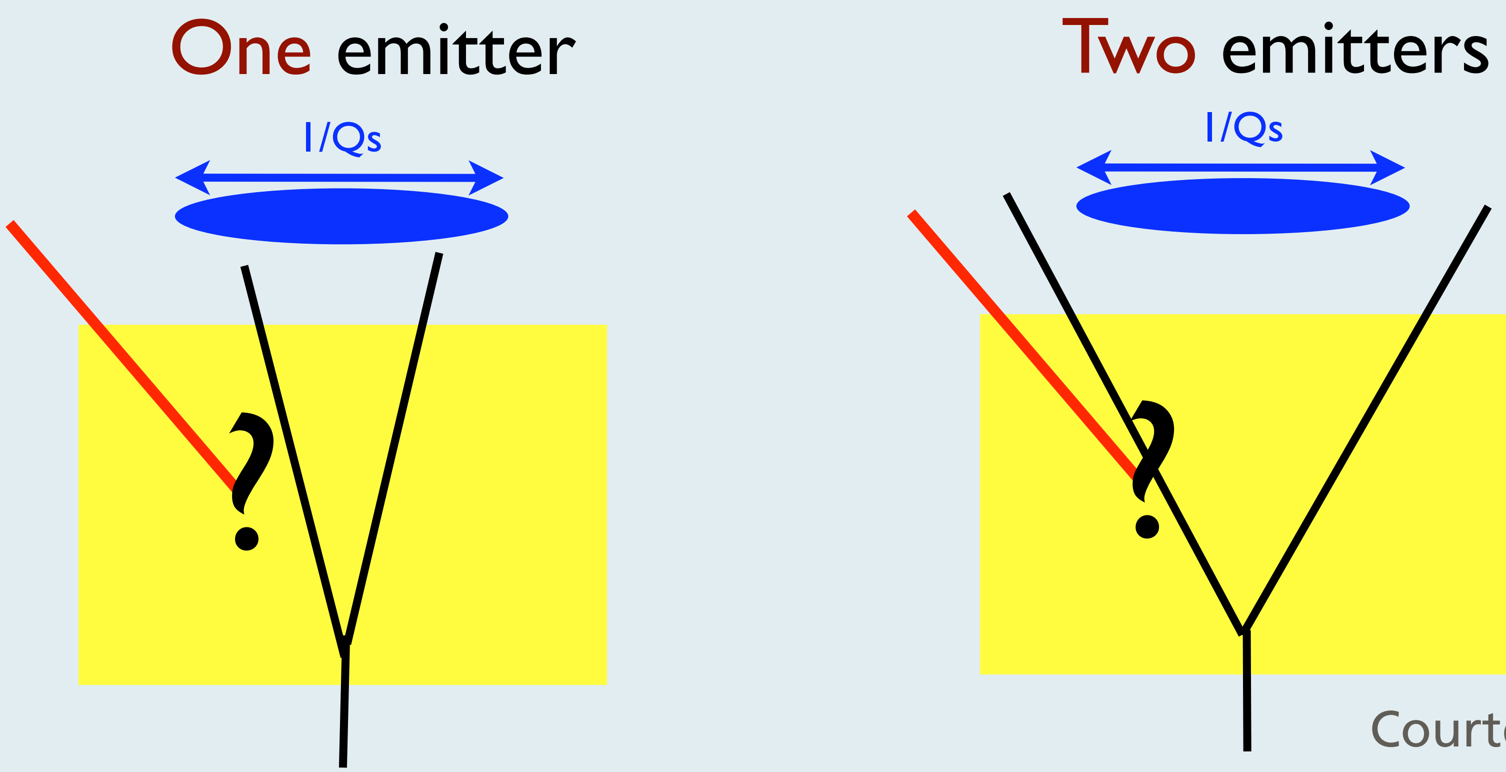
Guidance from the antenna



Courtesy: J. Casalderrey-Solana

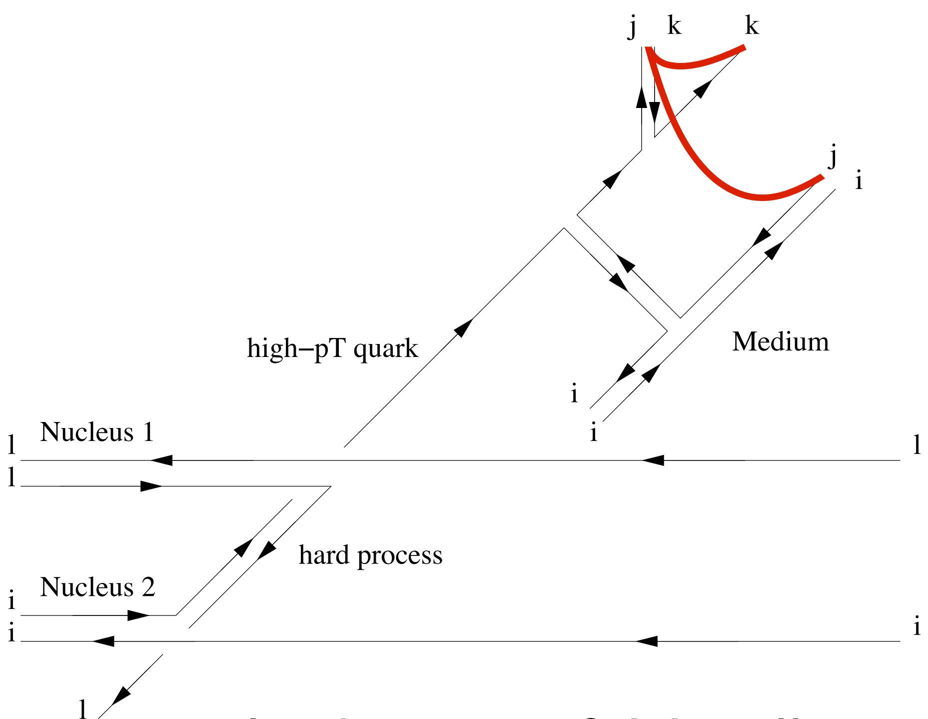
- Shower transverse size < 1/Qs ⇒ radiation as a single parton
- Shower transverse size > 1/Qs ⇒ radiation as a independent partons

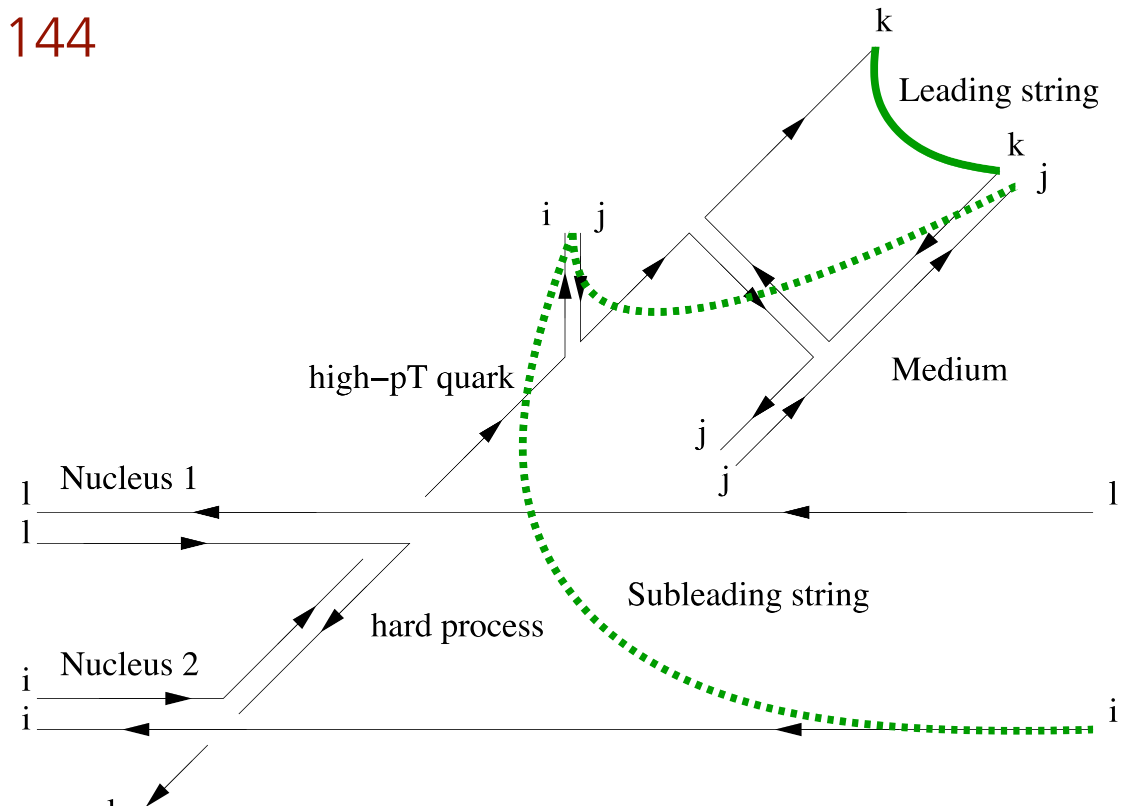
Genuine pQCD effect: color transparency

[tunneling]

An important point...

Beraudo, Milhano, Wiedemann JHEP 1207 (2012) 144





In the case of AA collisions a naive convolution

$$\text{Parton Energy loss} \otimes \text{Vacuum Fragmentation}$$

without accounting for the modified color-flow would result into a too hard hadron spectrum: fitting the experimental amount of quenching would require an overestimate of the energy loss at the partonic level;

Andrea Beraudo, Hard Probes 2012

MC prospects

- versatile tools vs. detailed descriptions
- exact kinematics
  - conservation of energy-momentum
- track all particles
- implement 'advanced' space-time picture
  - evolution of energy-density, flow fields
- must be checked against well-controlled limits!
- extensions beyond theory
  - recoil/back-reaction: source terms

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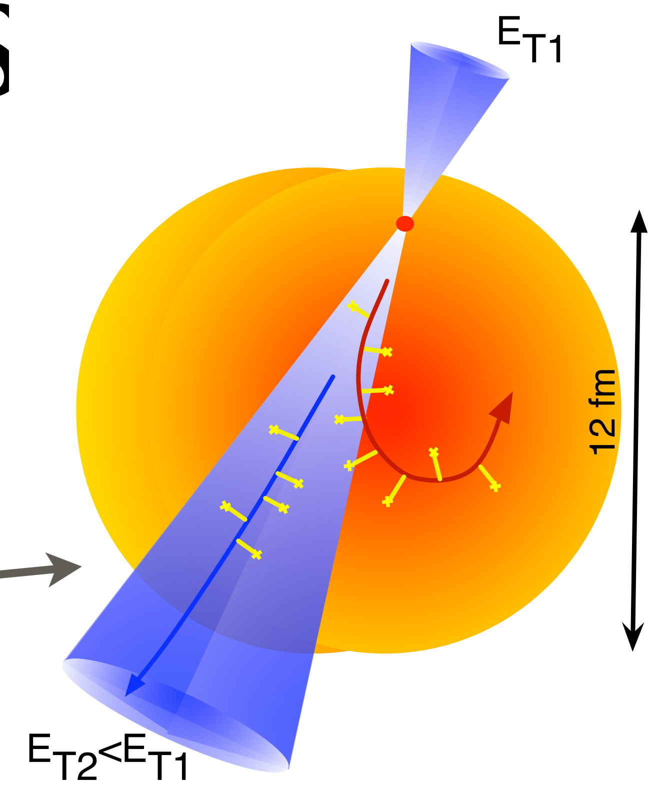

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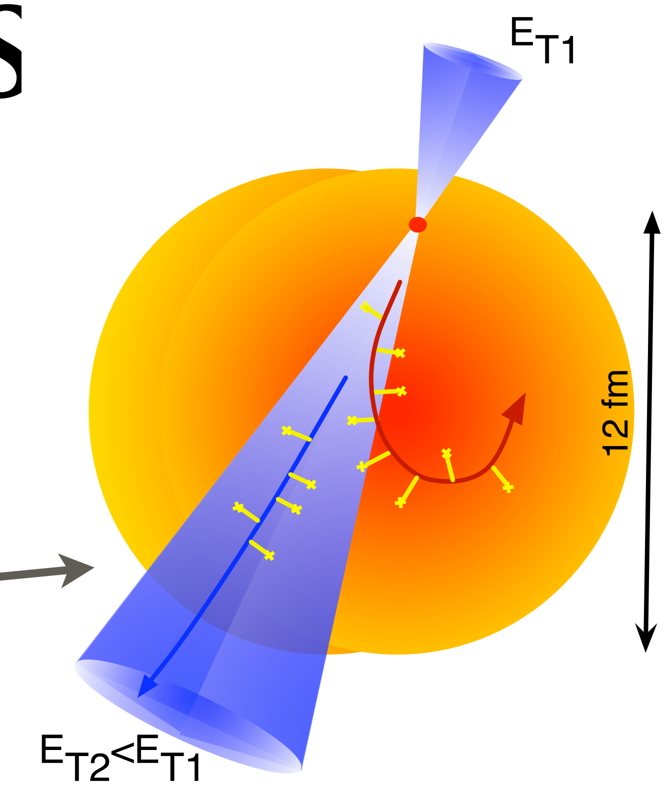

the "truth" is out there...

Outlook

- generic features: energy loss & softening
- how robust are the experimental signals to:
  - collisional (drag), radiative, collimation (broadening), NLO, non-perturbative (hadronization)........
  - how to get a handle?
- do we need to rethink approach to the problem?