

New ways of searching for the primordial gravitational wave from Large Scale Structure

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Theoretical methods for non-linear cosmology TH institute
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References

Large-Scale clustering of galaxies in general relativity
DJ, Schmidt & Hirata [[arXiv:1107.5427](#)]

Clustering Fossils from the Early Universe
DJ & Kamionkowski [[arXiv:1203.0302](#)]

Cosmic Rulers
Schmidt & DJ [[arXiv:1204.3625](#)]

Large-Scale Structure with Gravitational Waves I: Galaxy Clustering
DJ & Schmidt [[arXiv:1205.1512](#)]

Large-Scale Structure with Gravitational Waves II: Shear
Schmidt & DJ [[arXiv:1205.1514](#)]

Introduction

Gravitational Wave 101

Gravitational wave (GW)

- is a **traceless transverse** (tensor) component of the metric perturbations:
(Einstein convention + Greek=0-4, Latin=1-3)

$$ds^2 = a^2(\eta) [-d\eta^2 + \{\delta_{ij} + h_{ij}(\eta, x)\} dx^i dx^j]$$

Traceless : $\text{Tr}[h_{ij}] = h_i^i = g^{ij} h_{ij} = 0$

Transverse : $\nabla_i h_{ij} = 0$

- There are
6 (symmetric 3x3 spatial matrix) - 3 (transverse) - 1 (traceless)
= 2 degrees of freedom = $\mathbf{h_x}$, $\mathbf{h_+}$

Primordial Gravitational Wave

- de-Sitter space generates stochastic gravitational waves with amplitude of ($m_{\text{pl}} = \sqrt{G_N}$)

$$\Delta_h^2(k) = \frac{k^3 P_T(k)}{2\pi^2} = \frac{64\pi}{m_{\text{pl}}^2} \left(\frac{H}{2\pi} \right)^2 \Big|_{k=aH}$$

+ Friedmann equation: $3H^2 \sim 8\pi G\rho$

where power spectrum is defined as ($P_T = 4P_h$)

$$\langle h_{ij}(k)h^{ij}(k') \rangle = (2\pi)^3 P_T(k) \delta^D(k - k')$$

- **Gravitational wave amplitude = energy scale of inflation!**

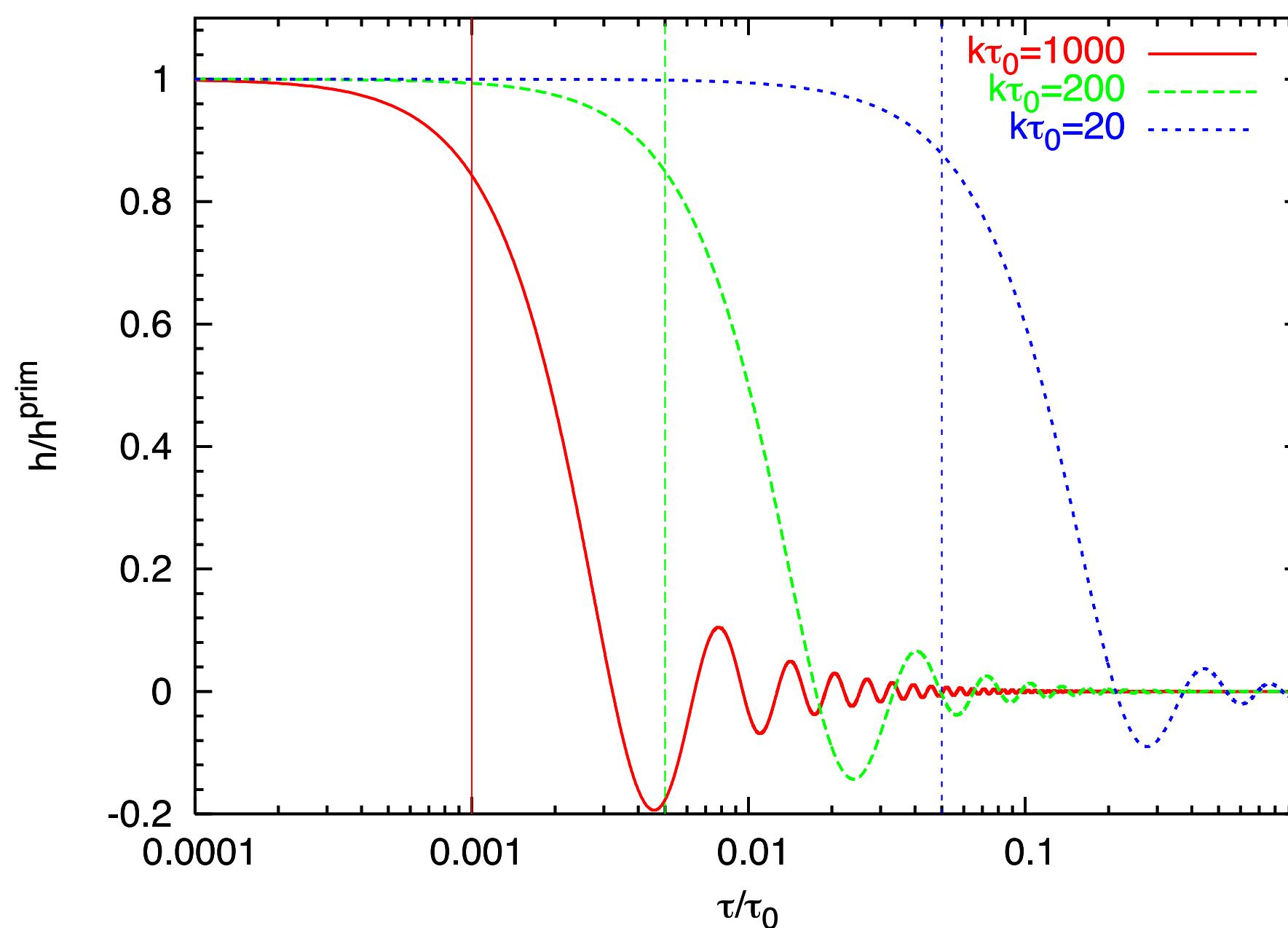
Evolution of GW

- Evolution of GW($p=+,x$) are described by K-G equation sourced by anisotropic stress ($\mathcal{H}=a'/a$ and $' = d/d\eta$):

$$-h_{ij;\nu}^{;\nu} = h_p''(\mathbf{k}) + \boxed{2\mathcal{H}h_p'(\mathbf{k})} + k^2h_p(\mathbf{k}) = 16\pi Ga^2\Pi_p(\mathbf{k})$$

Watanabe, Komatsu (2006)

Hubble damping term

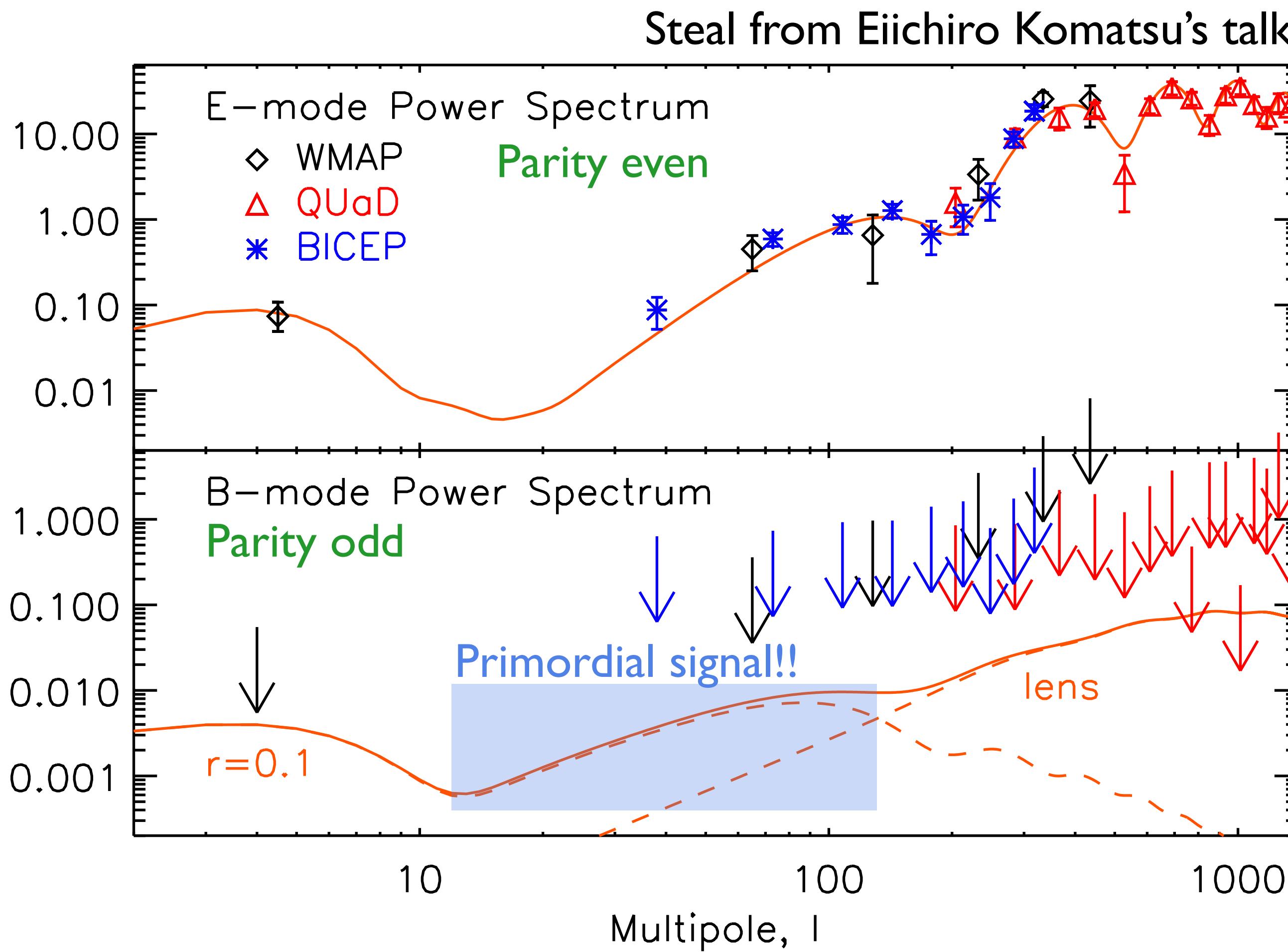


- GW decays once the mode enters the horizon. As effect from Π_p is small,

$$\text{RD : } h_p(\mathbf{k}, \eta) = j_0(k\eta)h_p^{\text{prim}}$$

$$\text{MD : } h_p(\mathbf{k}, \eta) = \frac{3j_1(k\eta)}{k\eta}h_p^{\text{prim}}$$

GW from CMB polarization



- **Parity-odd (B-mode) polarization is a window to the GW (or vector) in the primordial universe!**
- No B-mode yet...
- B-mode experiments:
Keck array, PIPER, CLASS,
LiteBIRD, PIXIE, ...
(e.g. 5σ for $r < 10^{-3}$)

GW from Large Scale Structure

- Two effects:
 - At the location of galaxies (**Source**)
 - Deflection of light from galaxies (**Line of sight**)
- Three possible ways of detecting GW from Large Scale Structure :
 - **Clustering** of galaxies in large scale structure (**S,L**)
 - Distortion on **shape** of galaxies, or cosmic shear (**S,L**)
 - Fossil memory at the **off-diagonal correlation** (**S**)

Galaxy clustering with GR

(as of March 2012)

- Gravitational waves are truly relativistic, so we need a fully relativistic description of galaxy clustering:
 - Scalar perturbations:
Yoo et al. 2009, Yoo 2010, Challinor&Lewis 2011, Bonvin& Durrer 2011, Jeong et al. 2011
 - Tensor perturbations:
 - Calculations were available (e.g. Yoo et al. 2009), but no quantitative analysis has been done.
 - Our gut knows that the effect should be small, anyway.
 - But, there was Masui & Pen (2010). Will come back soon...

Shape distortion (lensing) with GR

(as of March 2012)

- Do we have an equivalent formula for lensing?
- Yes, all the lensing literature are relativistic. But, with only scalar perturbations.
- To our best knowledge, other than scalars, **there was only one PRL article [Dodelson, Rozo and Stebbins (2003)]** with somewhat mysterious term of “metric shear”
- We need a covariant formula describing the shape distortion!
Cosmic Rulers (section II of this talk)

Galaxy clustering: $P_g(k)$ in the universe only with GW

Donghui Jeong, Fabian Schmidt & Christopher Hirata [arXiv:1107.5427]
Donghui Jeong & Fabian Schmidt [arXiv:1205.1512]

What did Masui & Pen said?

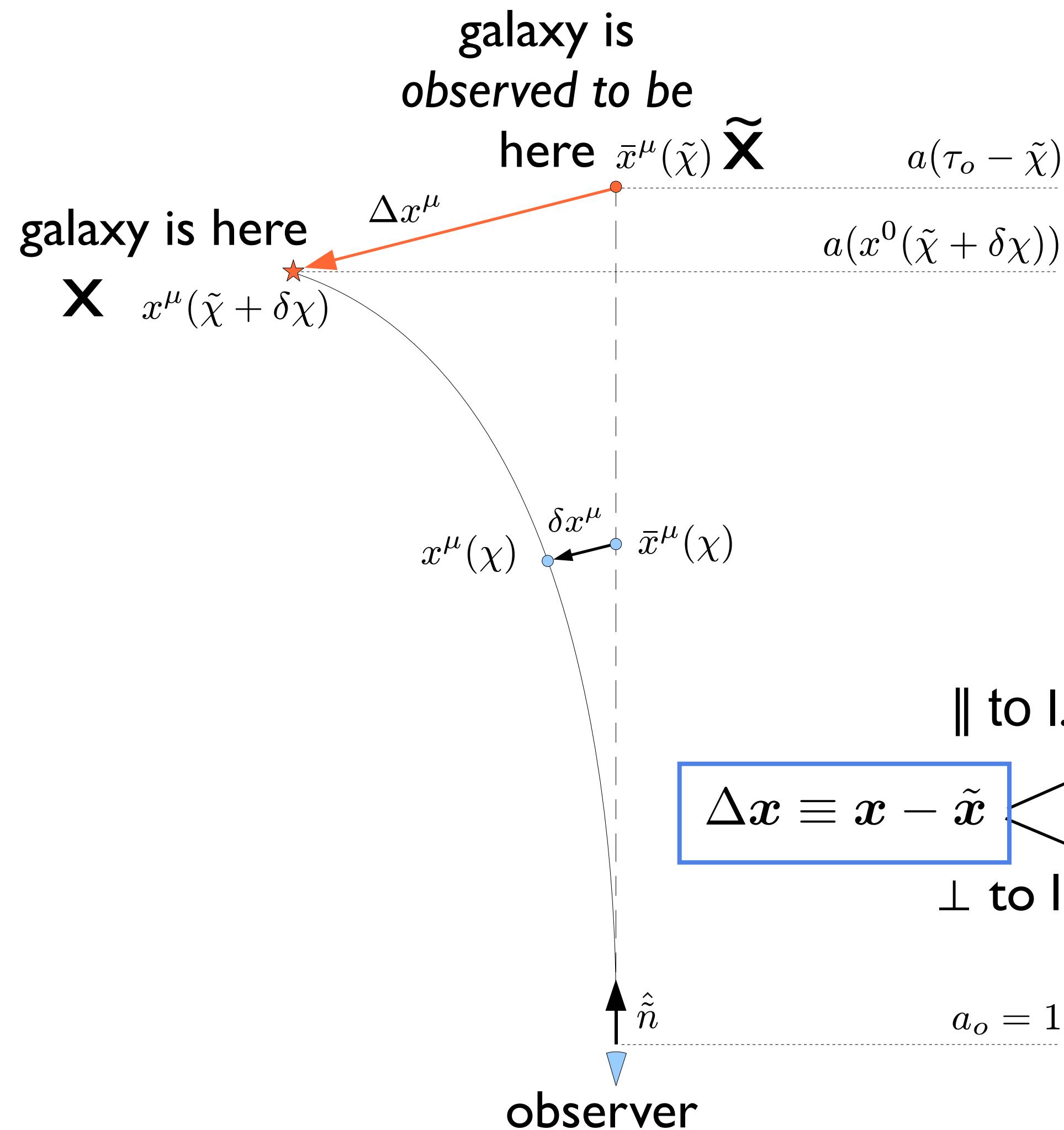
- The super-horizon tensor perturbations during inflation can generate anisotropy in the small scale matter distribution (e.g. power spectrum) Masui&Pen(2010)

$$P(k_a) = \tilde{P}(k) - \frac{k_i k_j h^{ij}}{2k} \frac{d\tilde{P}}{dk} + O\left(\frac{k_h}{k} h_{ij}\right) + O(h_{ij}^2)$$

- We can measure GW from correlation of 21cm fluctuation!
- This effect is order $\langle \delta \delta h \rangle$. If it is observable, why not $\langle hh \rangle$?
- Wait, can super-horizon modes do something ???
 - This is **without coupling**.
(will come back to the direct coupling later!)

$$ds^2 = a^2(\eta) [-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

Light deflection due to GW



- Deflection of photon changes the observed location of galaxies.

- From the geodesic equation, we calculate Δx : (Here, $h_{\parallel} = h_{ij} \hat{n}^i \hat{n}^j$)

time delay + l.o.s. displacement + redshift pert.

$$\Delta x_{\parallel} = -\frac{1}{2} \int_0^{\tilde{\chi}} d\chi h_{\parallel} - \frac{1 + \tilde{z}}{2H(\tilde{z})} \int_0^{\tilde{\chi}} d\chi h'_{\parallel}$$

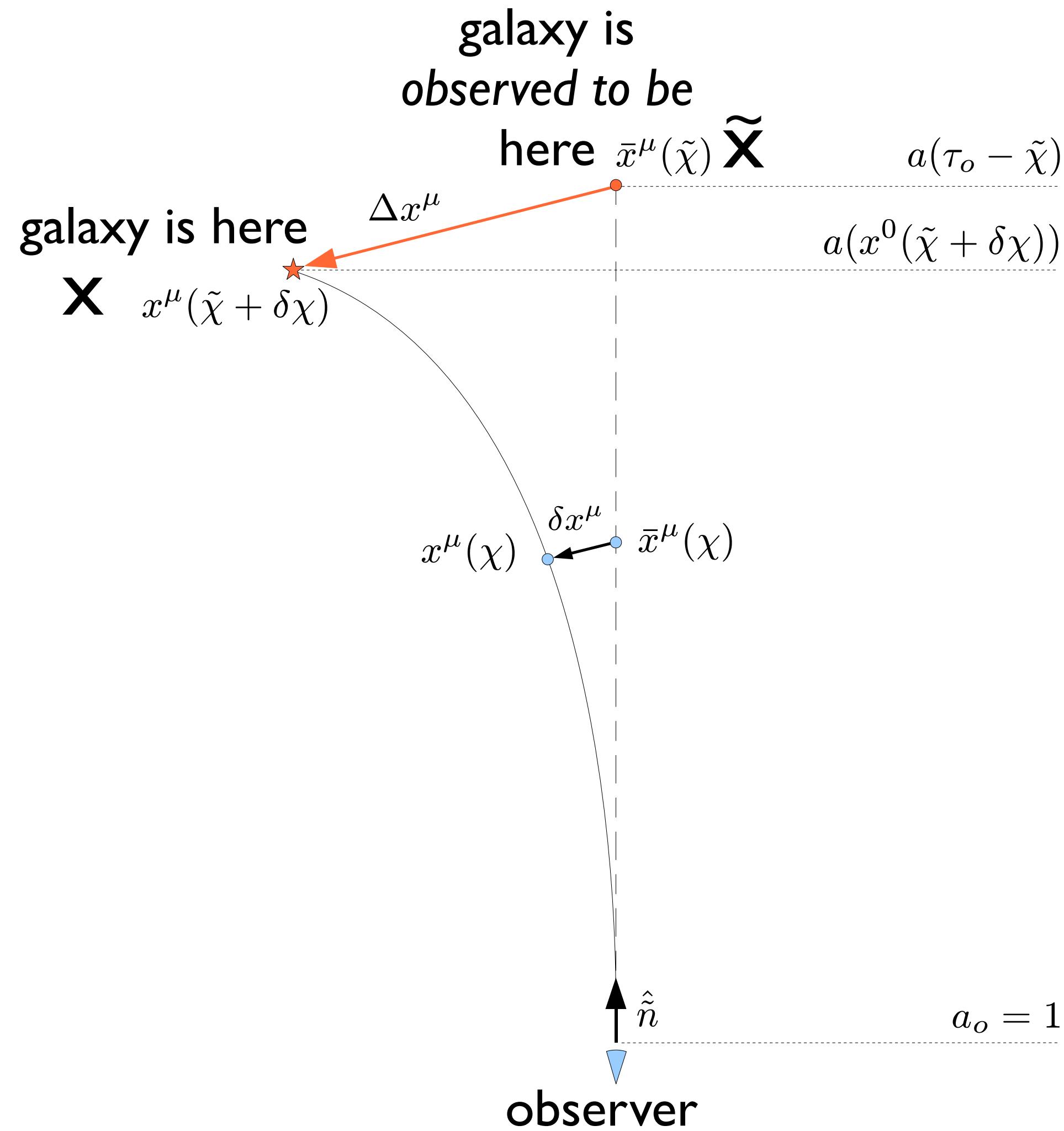
$$\Delta x_{\perp}^i = \frac{1}{2} \tilde{\chi} [(h_{ij})_0 \hat{n}^j - (h_{\parallel})_0 \hat{n}^i]$$

GW at the observer's position

$$+ \int_0^{\tilde{\chi}} d\chi \left\{ \frac{\tilde{\chi} - \chi}{2} \partial_{\perp}^i h_{\parallel} + \frac{c\tilde{h}i}{\chi} (h_{\parallel} \hat{n}^i - h_{ij} \hat{n}^j) \right\}$$

$$ds^2 = a^2(\eta) [-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

GW effect I. Volume distortion



- Then, the volume (number density) we inferred from the observed coordinate *is different from the true volume (number density)*:

$$N = \int_{\tilde{V}} \sqrt{-g} n_g(x^\alpha) \frac{1}{a(x^0)} \left| \frac{\partial x^i}{\partial \tilde{x}^j} \right| d^3 \tilde{x}$$

$\Delta x \equiv x - \tilde{x}$

$$\left| \frac{\partial x^i}{\partial \tilde{x}^j} \right| = 1 + \frac{\partial \Delta x^i}{\partial \tilde{x}^j} = \partial_{\parallel} \Delta x_{\parallel} + \frac{2\Delta x_{\parallel}}{\tilde{\chi}} + \partial_{\perp i} \Delta x_{\perp}^i$$

$$ds^2 = a^2(\eta) [-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

GW effect II. redshift perturbation

- Clustering measure: density contrast $\delta_g^{\text{obs}}(\tilde{z}, \hat{n}) = \frac{n(\tilde{z}, \hat{n}) - \bar{n}(\tilde{z})}{\bar{n}(\tilde{z})}$
- But, the measured redshift is different from the true redshift!

$$1 + \tilde{z} = (1 + z)(1 + \delta z) \quad \delta z = \frac{1}{2} \int_0^{\tilde{z}} d\chi h'_{||}$$

- That is, we **under-(over)** estimate the mean number density for **positive (negative)** δz [when there are more galaxies at lower redshifts].

$$\delta_g^{\text{obs}}(\tilde{z}, \hat{n}) = \delta_g^{\text{intrinsic}} + b_e \delta z$$

$$b_e \equiv \left. \frac{d \ln(a^3 \bar{n}_g)}{d \ln a} \right|_{\tilde{z}} = -(1 + \tilde{z}) \left. \frac{d \ln(a^3 \bar{n}_g)}{dz} \right|_{\tilde{z}}$$

$$ds^2 = a^2(\eta) [-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

GW effect III. Magnification

- If galaxies are selected by apparent magnitude, the magnification

$$\mathcal{M} \equiv \frac{D_L^{-2}}{\tilde{D}_L^{-2}(\tilde{z})} = \frac{D_A^{-2}}{\tilde{D}_A^{-2}(\tilde{z})}$$

also changes the density contrast ($Q = -d \ln \tilde{n}_g / d \ln F_{\text{cut}}$):

$$\tilde{\delta}_g = \tilde{\delta}_g(\text{no mag}) + \frac{\partial \ln \tilde{n}}{\partial \ln \mathcal{M}} (\mathcal{M} - 1) \equiv \tilde{\delta}_g(\text{no mag}) + Q\delta\mathcal{M}$$

$$ds^2 = a^2(\eta) \left[-d\eta^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right]$$

Galaxy density contrast with GW

- If gravitational waves are the ONLY source of the distortion, the “observed” galaxy density contrast becomes

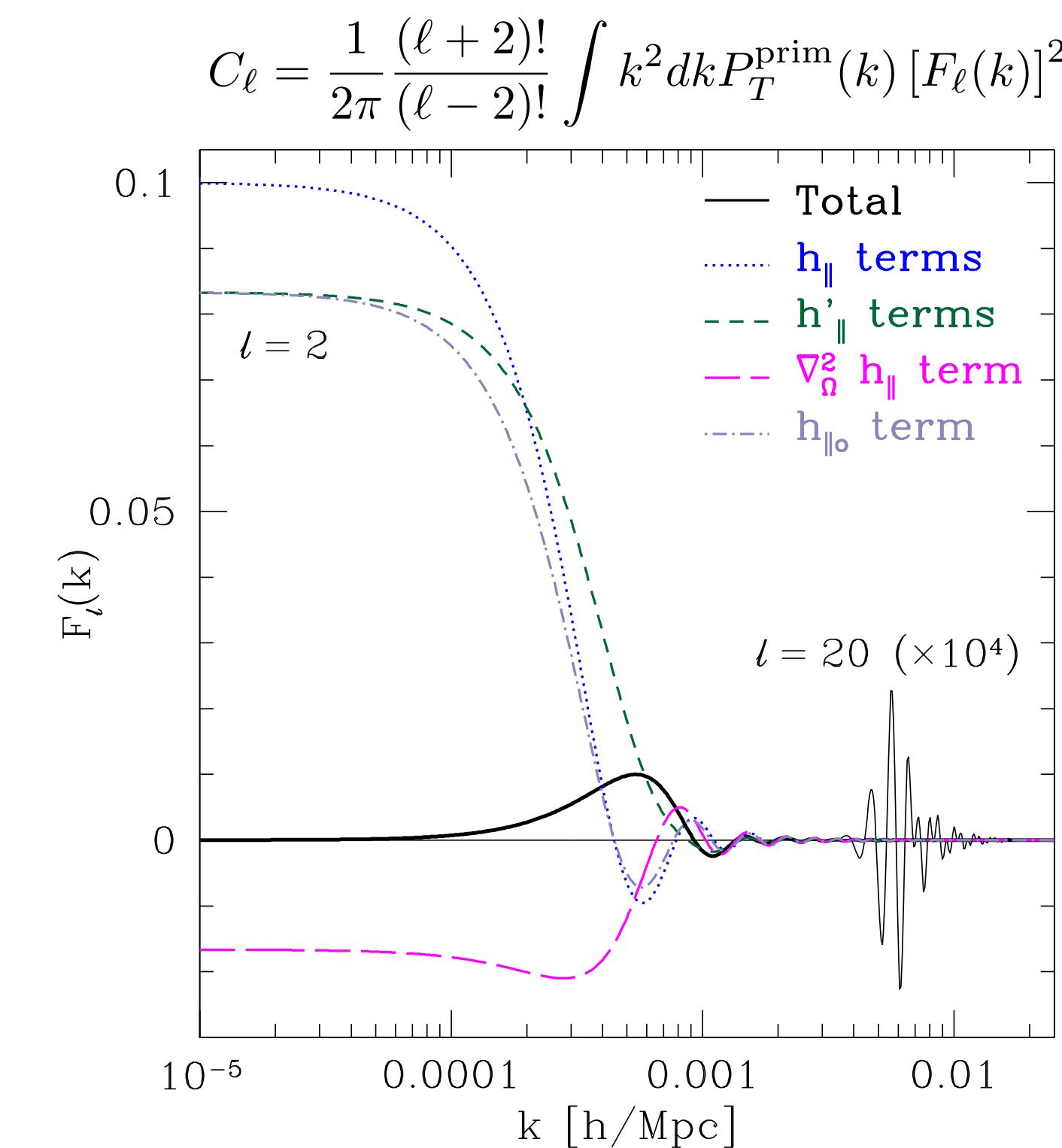
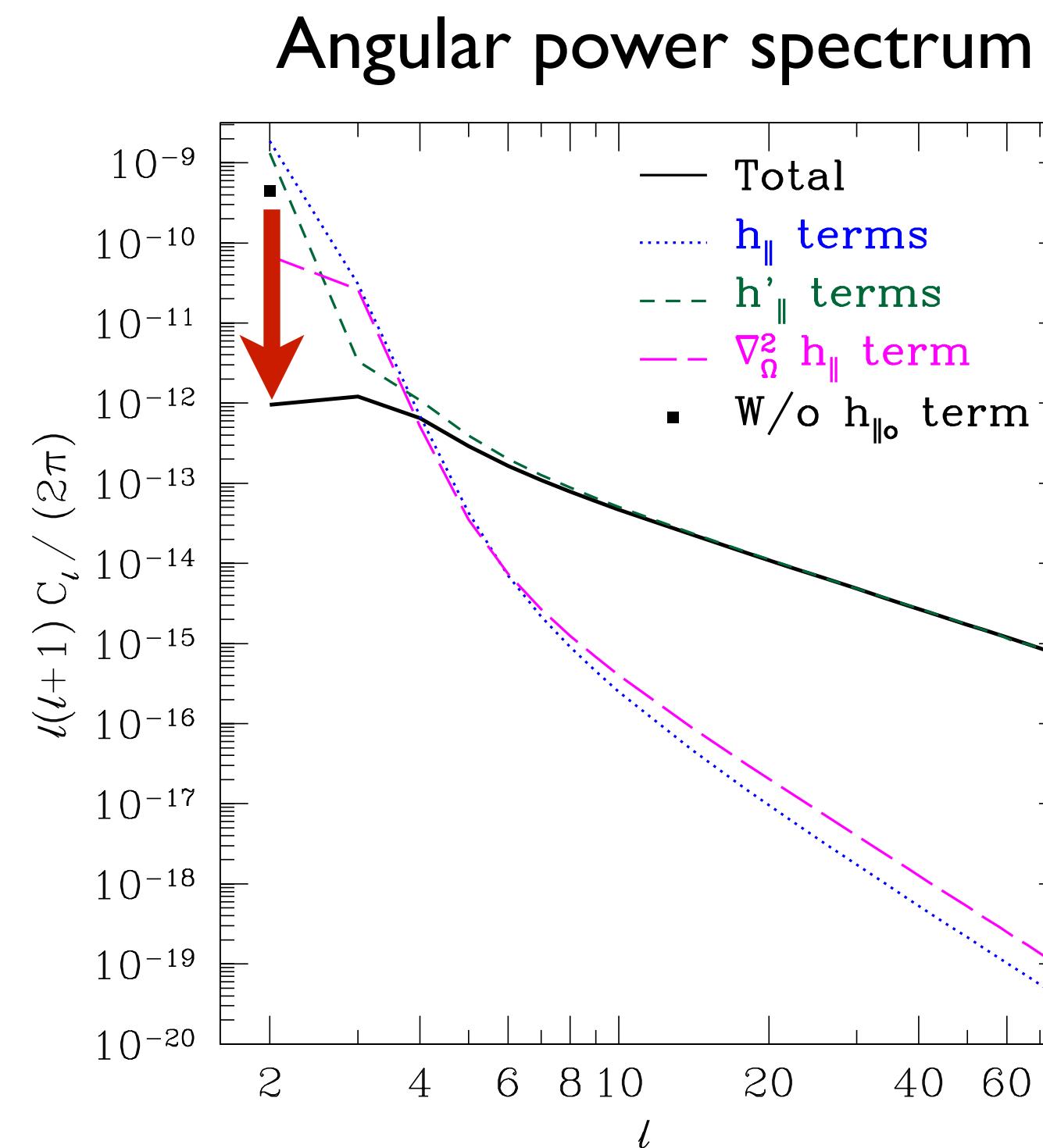
$$\begin{aligned}\tilde{\delta}_{gT} &= (b_e - 2Q)\delta z - 2(1 - Q)\hat{\kappa} - \frac{1 - Q}{2}h_{\parallel} - \frac{1 + \tilde{z}}{2H(\tilde{z})}h'_{\parallel} \\ &\quad - \frac{1 - Q}{\tilde{\chi}} \left[\int_0^{\tilde{\chi}} d\chi h_{\parallel} + \frac{1 + \tilde{z}}{H(\tilde{z})} \int_0^{\tilde{\chi}} d\chi h'_{\parallel} \right] \\ &\quad - \frac{H(\tilde{z})}{2} \frac{\partial}{\partial \tilde{z}} \left[\frac{1 + \tilde{z}}{H(\tilde{z})} \right] \int_0^{\tilde{\chi}} d\chi h'_{\parallel}.\end{aligned}$$

$$\hat{\kappa} = \boxed{\frac{5}{4}h_{\parallel o}} - \frac{1}{2}h_{\parallel} - \frac{1}{2} \int_0^{\tilde{\chi}} d\chi \left[h'_{\parallel} + \frac{3}{\chi}h_{\parallel} \right] - \frac{1}{4}\nabla_{\Omega}^2 \int_0^{\tilde{\chi}} d\chi \frac{\tilde{\chi} - \chi}{\chi\tilde{\chi}} h_{\parallel}$$

Observer term

Angular power spectrum with GW

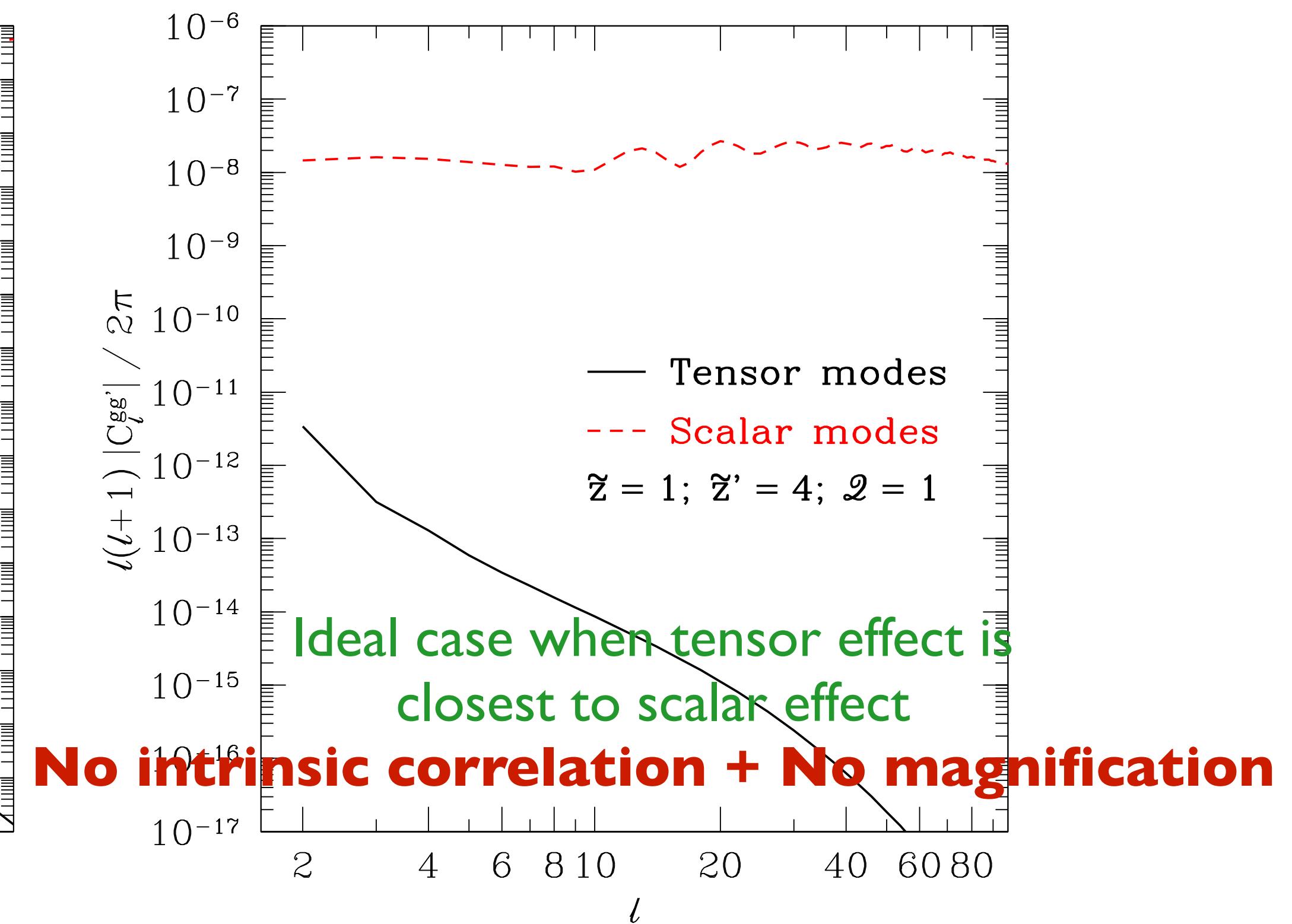
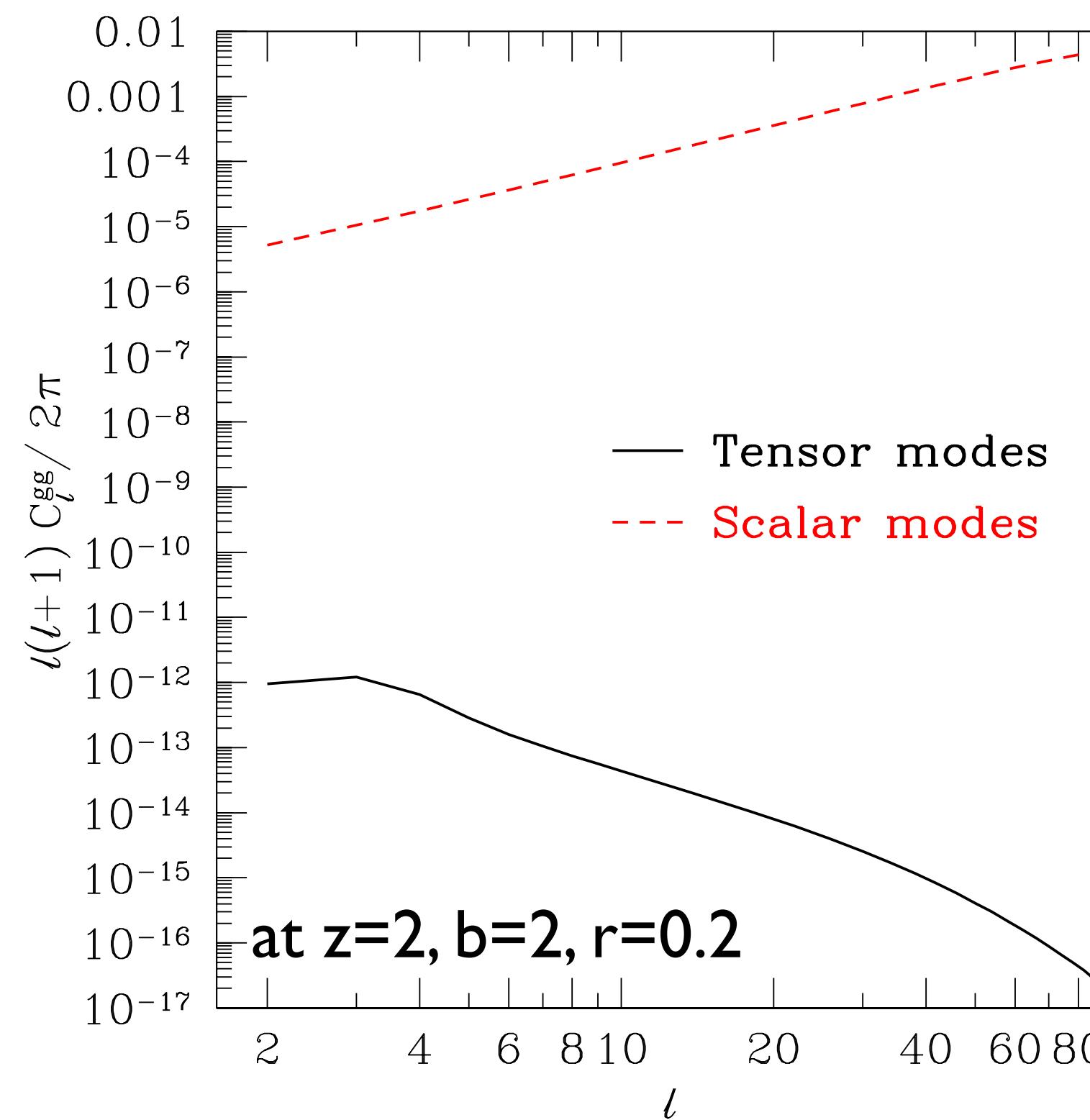
- For the sharp redshift slice at $z=2$ with $b_e=2.5, Q=1.5$



When including all effects, **NO** super horizon k-modes affect the sub-horizon clustering!!

We enjoyed physics, but too small!

- GW signal is way too small compared to the (1) intrinsic correlation and (2) the effect from scalar metric perturbations.



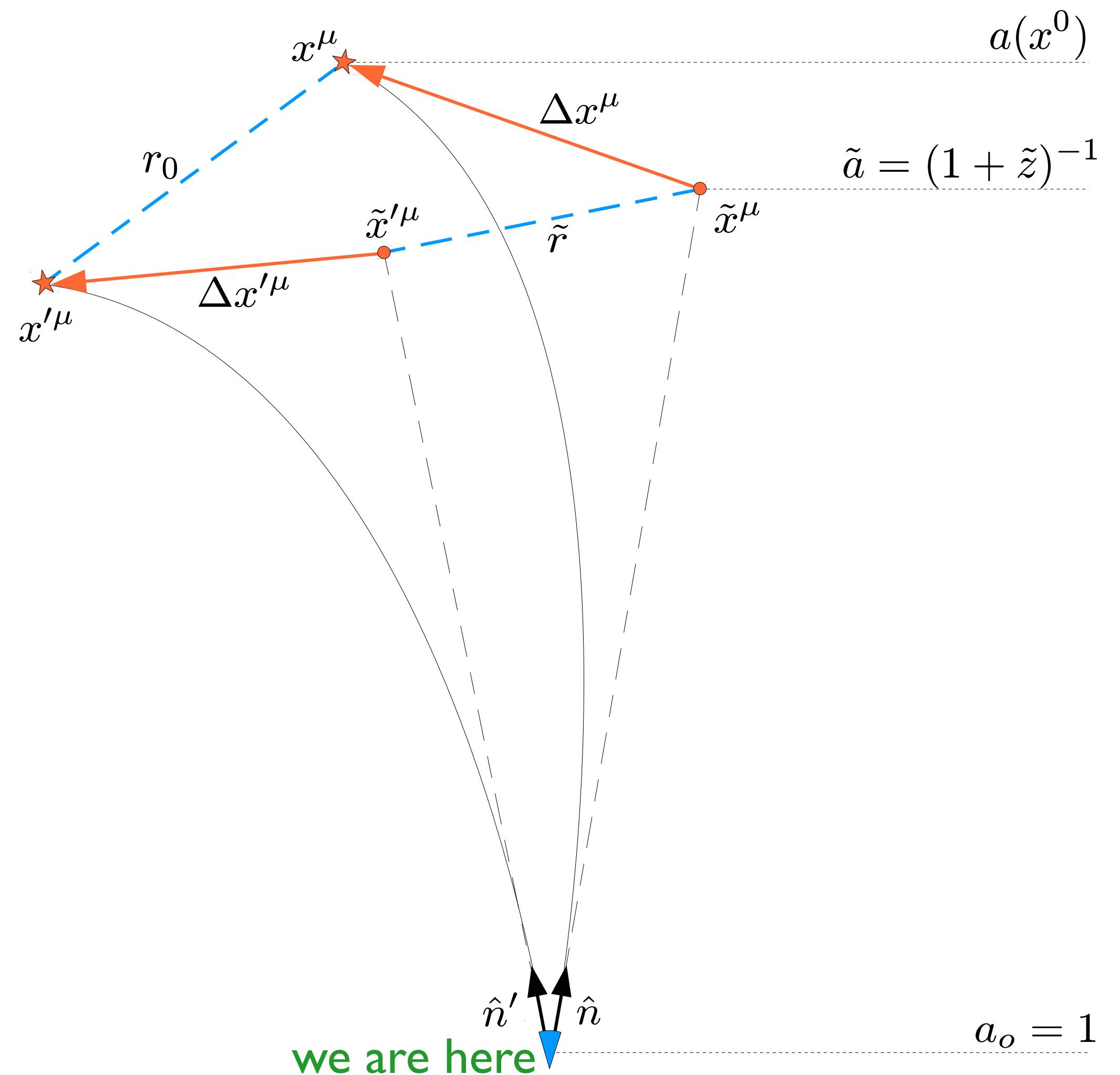
Cosmic Rulers

or, covariant formalism for the shape distortions

Fabian Schmidt & Donghui Jeong [arXiv:1204.3625]

$$ds^2 = a^2(\eta) \left[-(1 + 2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij} + h_{ij}) dx^i dx^j \right]$$

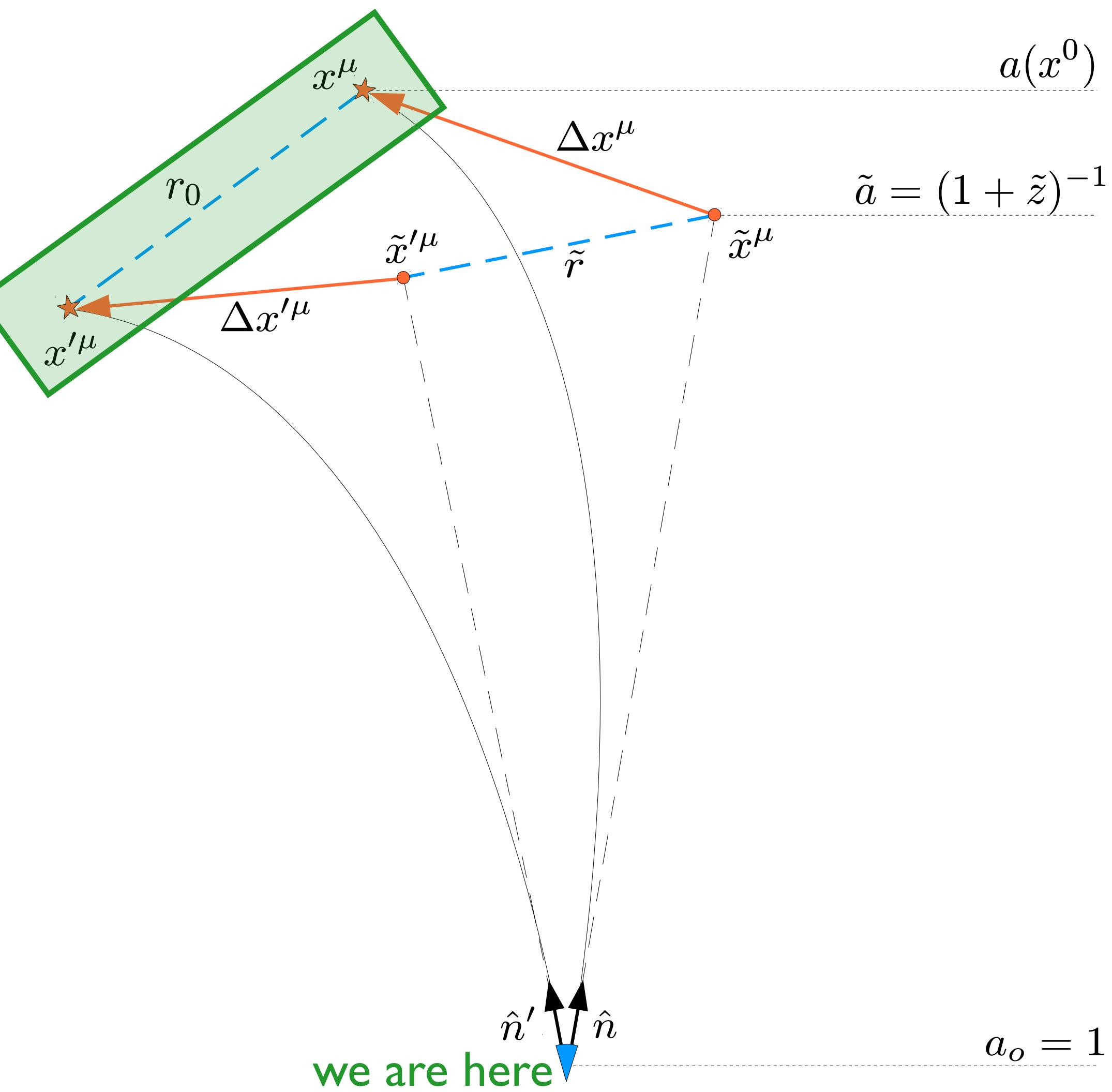
Cosmology with a high-z yardstick



- Consider a **shining yardstick** at high redshift, whose **proper length** is somehow known : r_0
- We observe (RA,DEC,z) for both ends of the stick, **infer** the length of the stick from them : \tilde{r}
- Due to perturbations, $\boxed{\tilde{r} \neq r_0}$ such a distortion to the size is an important tool to study perturbations!

$$ds^2 = a^2(\eta) [-(1 + 2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij} + h_{ij}) dx^i dx^j]$$

Who measures r_0 ?



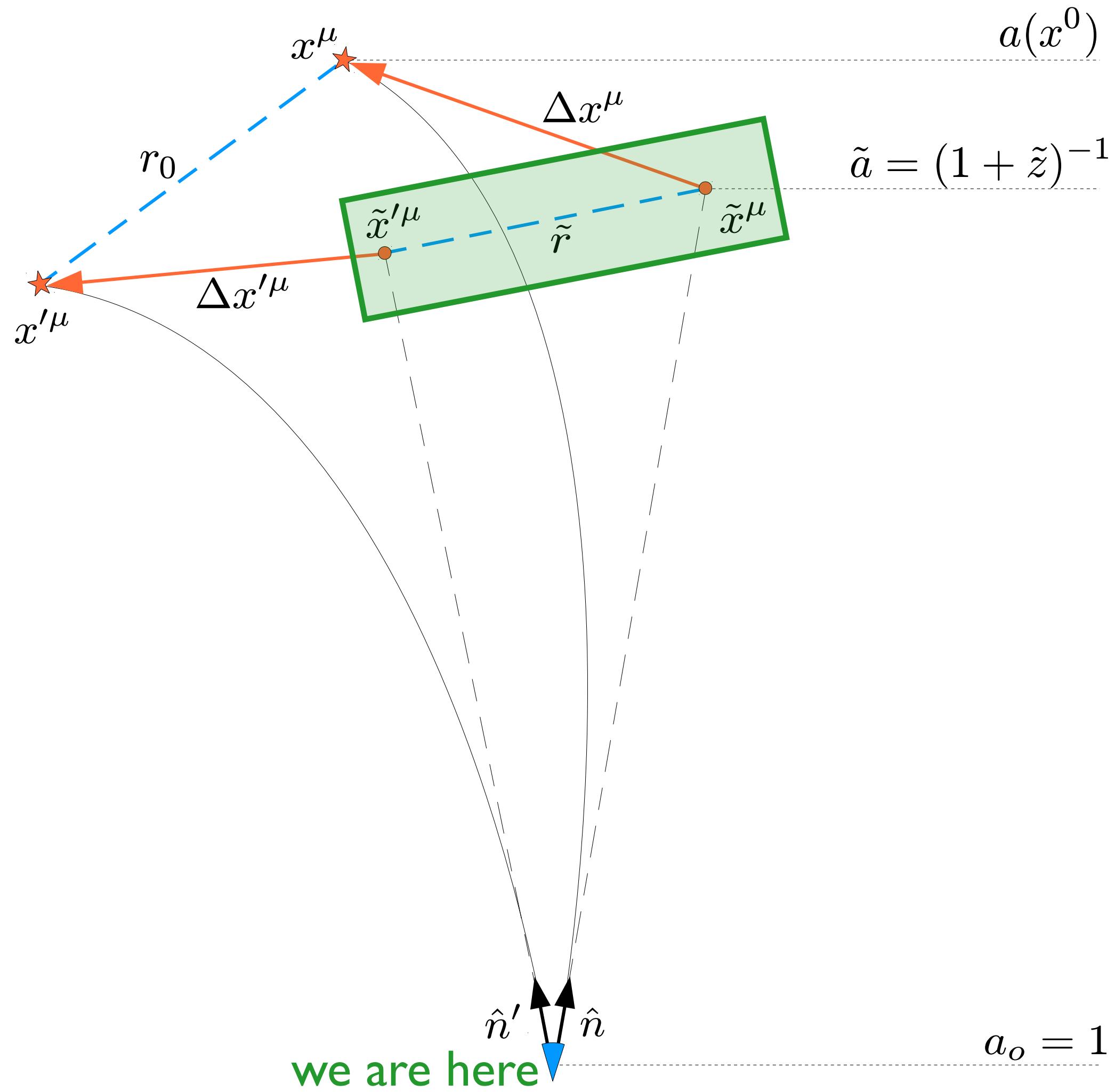
- An (imaginary) observer moving with the stick measures the length of the ruler:

$$r_0^2 = \frac{[g_{\mu\nu} + u_\mu u_\nu]}{\text{metric projected onto the constant-proper time hyper-surface of the comoving observer}} (x^\mu - x'^\mu)(x^\nu - x'^\nu)$$

$$g_{\mu\nu} + u_\mu u_\nu = a^2 \begin{pmatrix} 0 & -v_i \\ -v_i & \delta_{ij} + h_{ij} \end{pmatrix}$$

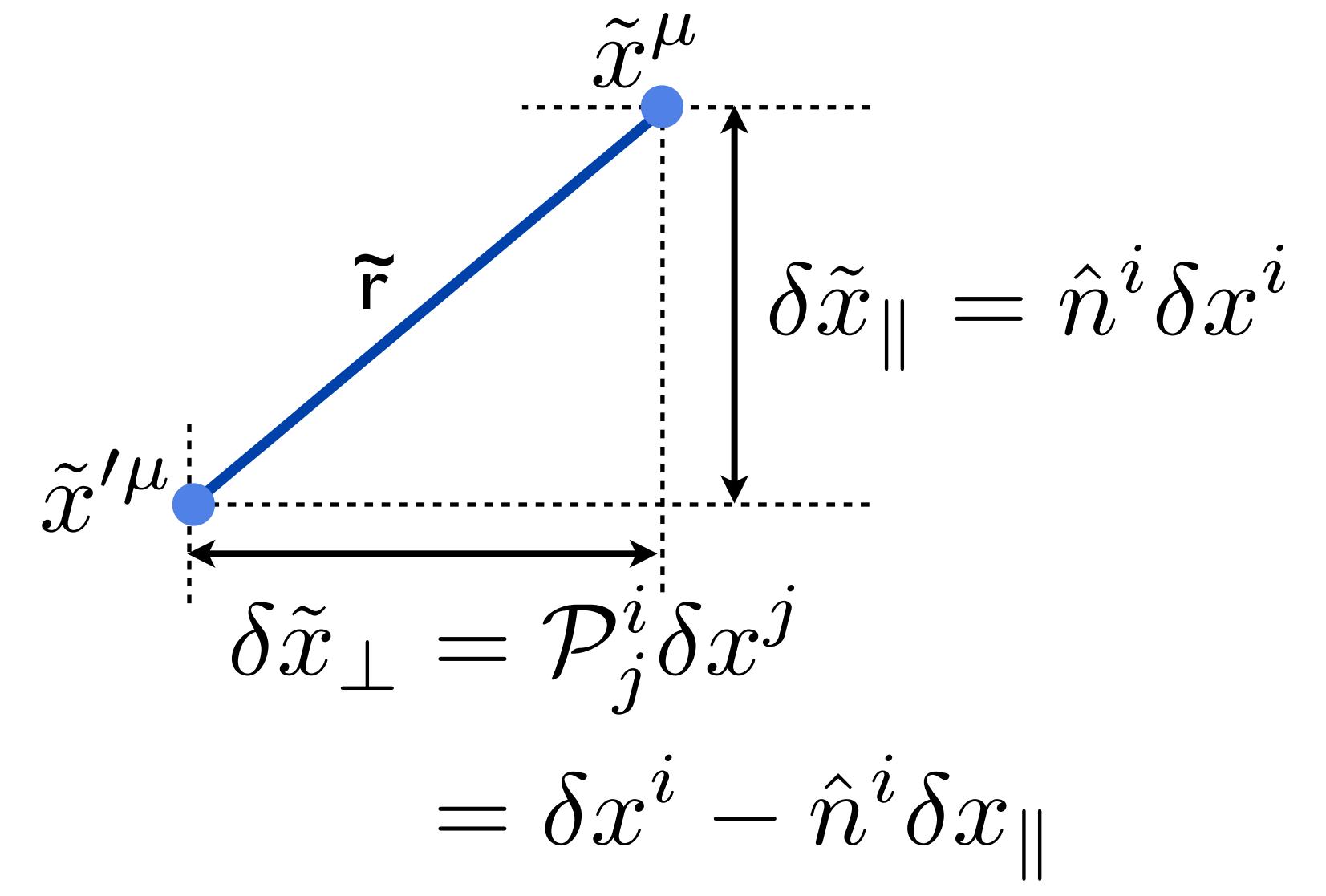
We assume a small ruler.

We measure \tilde{r} !



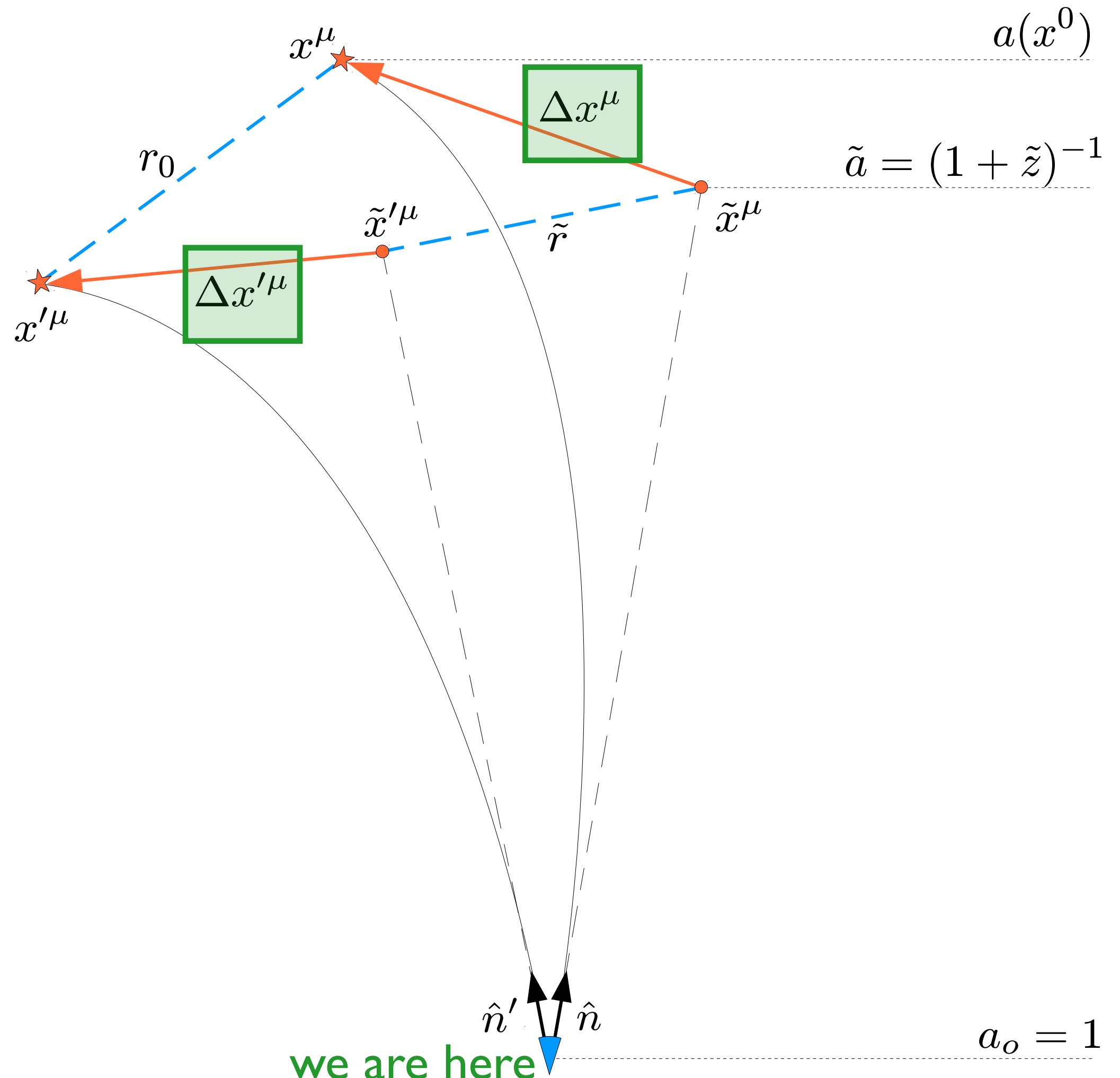
- We measure the angular and radial separations by using the **unperturbed metric**:

$$\tilde{r}^2 = \tilde{a}^2 \delta_{ij} (\tilde{x}^i - \tilde{x}'^i) (\tilde{x}^j - \tilde{x}'^j)$$



$$ds^2 = a^2(\eta) \left[-(1 + 2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij} + h_{ij}) dx^i dx^j \right]$$

Δx is from geodesic equations



Shift along the line of sight direction

$$\Delta x_{\parallel} = \int_0^{\tilde{x}} d\chi \left[A - B_{\parallel} - \frac{1}{2} h_{\parallel} \right] - \frac{1 + \tilde{z}}{H(\tilde{z})} \Delta \ln a$$

Shift along the perpendicular direction

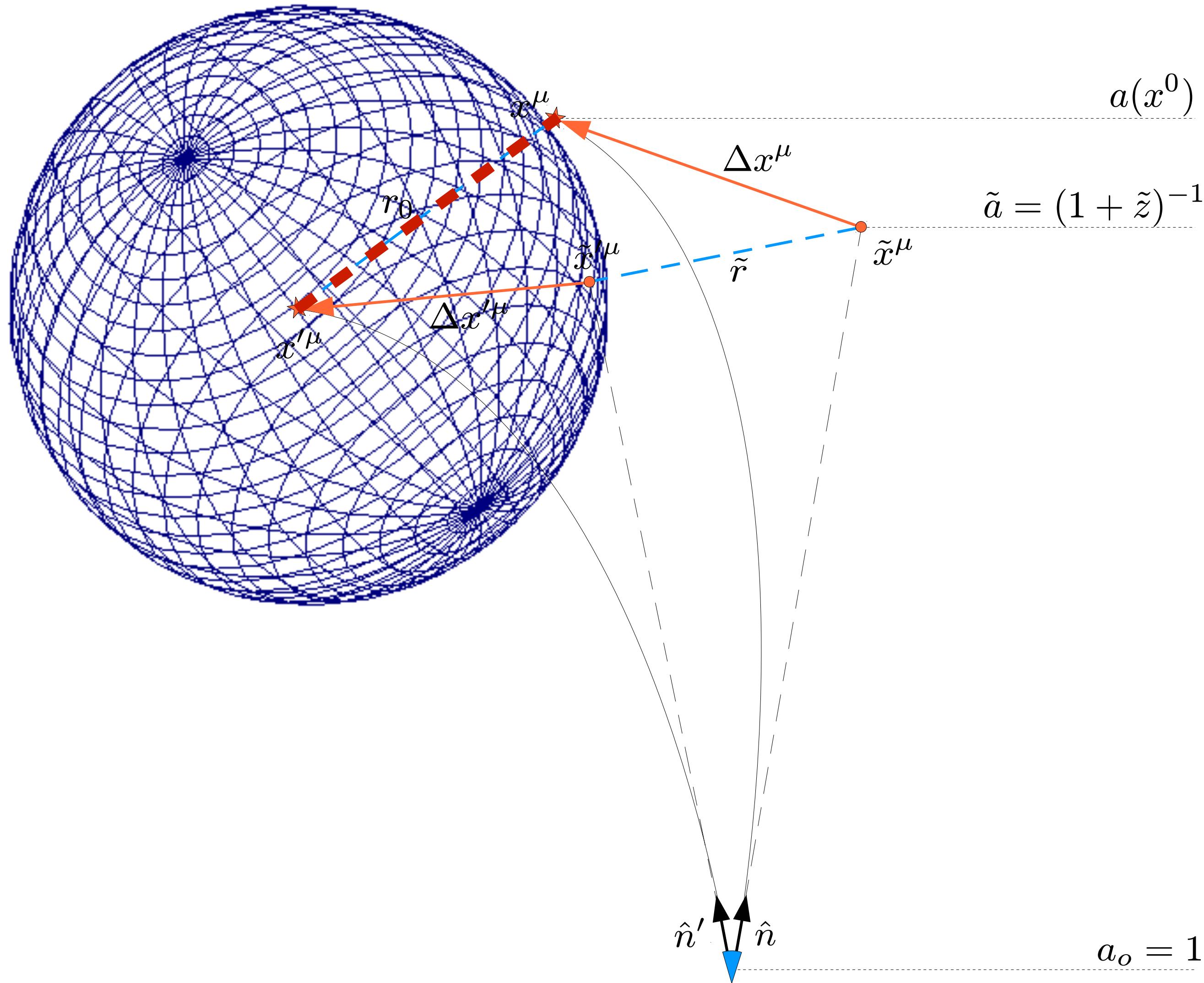
$$\begin{aligned} \Delta x_{\perp}^i &= \left[\frac{1}{2} \mathcal{P}^{ij} (h_{jk})_o \hat{n}^k + B_{\perp o}^i - v_{\perp o}^i \right] \tilde{\chi} \\ &\quad - \int_0^{\tilde{\chi}} d\chi \left[\frac{\tilde{\chi}}{\chi} (B_{\perp}^i + \mathcal{P}^{ij} h_{jk} \hat{n}^k) \right. \\ &\quad \left. + (\tilde{\chi} - \chi) \partial_{\perp}^i \left(A - B_{\parallel} - \frac{1}{2} h_{\parallel} \right) \right] \end{aligned}$$

perturbation to the scale factor at emission

$$\Delta \ln a = A_o - A + v_{\parallel} - v_{\parallel o} - \int_0^{\tilde{x}} d\chi \left[A - B_{\parallel} - \frac{1}{2} h_{\parallel} \right]'$$

Also, see Yoo et al. (2010)

Now, consider a spherical ruler

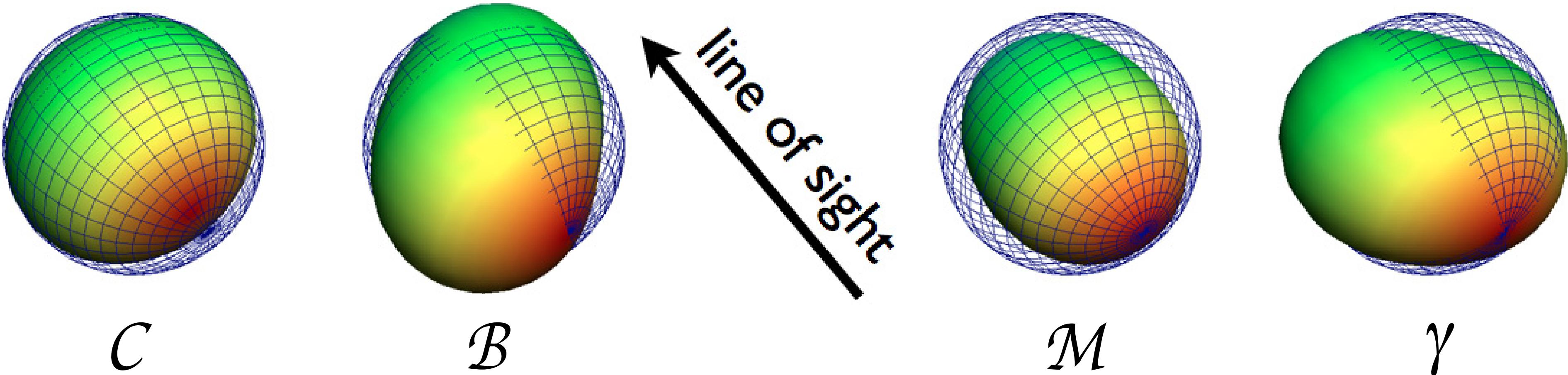


Classification of distortion

- We decompose the distortion as **Scalar**, **Vector** and **Tensor** according to their rotational property on sphere:

$$\frac{\tilde{r} - r_0}{\tilde{r}} = \mathcal{C} \frac{(\delta \tilde{x}_{\parallel})^2}{\tilde{r}_c^2} + \mathcal{B}_i \frac{\delta \tilde{x}_{\parallel} \delta \tilde{x}_{\perp}^i}{\tilde{r}_c^2} + \mathcal{A}_{ij} \frac{\delta \tilde{x}_{\perp}^i \delta \tilde{x}_{\perp}^j}{\tilde{r}_c^2}$$

longitudinal scalar Vector Magnification (trace) + shear (spin-2)



New!!

$$ds^2 = a^2(\eta) \left[-(1 + 2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij} + h_{ij}) dx^i dx^j \right]$$

Covariant formula for C, B, M

longitudinal scalar

$$\begin{aligned} \mathcal{C} = & -\Delta \ln a \left[1 - H(\tilde{z}) \frac{\partial}{\partial \tilde{z}} \left(\frac{1 + \tilde{z}}{H(\tilde{z})} \right) - \frac{\partial \ln r_0}{\partial \ln a} \right] \\ & - A - v_{\parallel} + B_{\parallel} \\ & + \frac{1 + \tilde{z}}{H(\tilde{z})} \left(-\partial_{\parallel} A + \partial_{\parallel} v_{\parallel} + B'_{\parallel} - v'_{\parallel} + \frac{1}{2} h'_{\parallel} \right) \end{aligned}$$

Vector

$$\begin{aligned} \mathcal{B}_i = & -\mathcal{P}_i{}^j h_{jk} \hat{n}^k - v_{\perp i} - \partial_{\perp i} \Delta x_{\parallel} - \partial_{\tilde{\chi}} \Delta x_{\perp i} + \frac{\Delta x_{\perp i}}{\tilde{\chi}} \\ = & -v_{\perp i} + B_{\perp i} + \frac{1 + \tilde{z}}{H(\tilde{z})} \partial_{\perp i} \Delta \ln a, \end{aligned}$$

Magnification (e.g. Yoo et al. 2009, Challinor&Lewis 2011, Bonvin& Durrer 2011, Jeong et al. 2011)

$$\mathcal{M} \equiv \mathcal{P}^{ij} \mathcal{A}_{ij} = -2\Delta \ln a \left[1 - \frac{\partial \ln r_0}{\partial \ln a} \right] - \frac{1}{2} (h^i{}_i - h_{\parallel}) + 2\hat{\kappa} - \frac{2}{\tilde{\chi}} \Delta x_{\parallel}.$$

convergence

$$\begin{aligned} \hat{\kappa} = & -\frac{1}{2} \left[\frac{1}{2} ((h^i{}_i)_o - 3(h_{\parallel})_o) - 2(B_{\parallel} - v_{\parallel})_o \right] \\ & + \frac{1}{2} \int_0^{\tilde{\chi}} d\chi \left[\partial_{\perp}^k B_k - \frac{2}{\chi} B_{\parallel} + (\partial_{\perp}^l h_{lk}) \hat{n}^k + \frac{1}{\chi} (h^i{}_i - 3h_{\parallel}) + (\tilde{\chi} - \chi) \frac{\chi}{\tilde{\chi}} \nabla_{\perp}^2 \left\{ A - B_{\parallel} - \frac{1}{2} h_{\parallel} \right\} \right] \end{aligned}$$

New!!

$$ds^2 = a^2(\eta) [-(1 + 2A)d\eta^2 - 2B_i d\eta dx^i + (\delta_{ij} + h_{ij}) dx^i dx^j]$$

... and γ!!

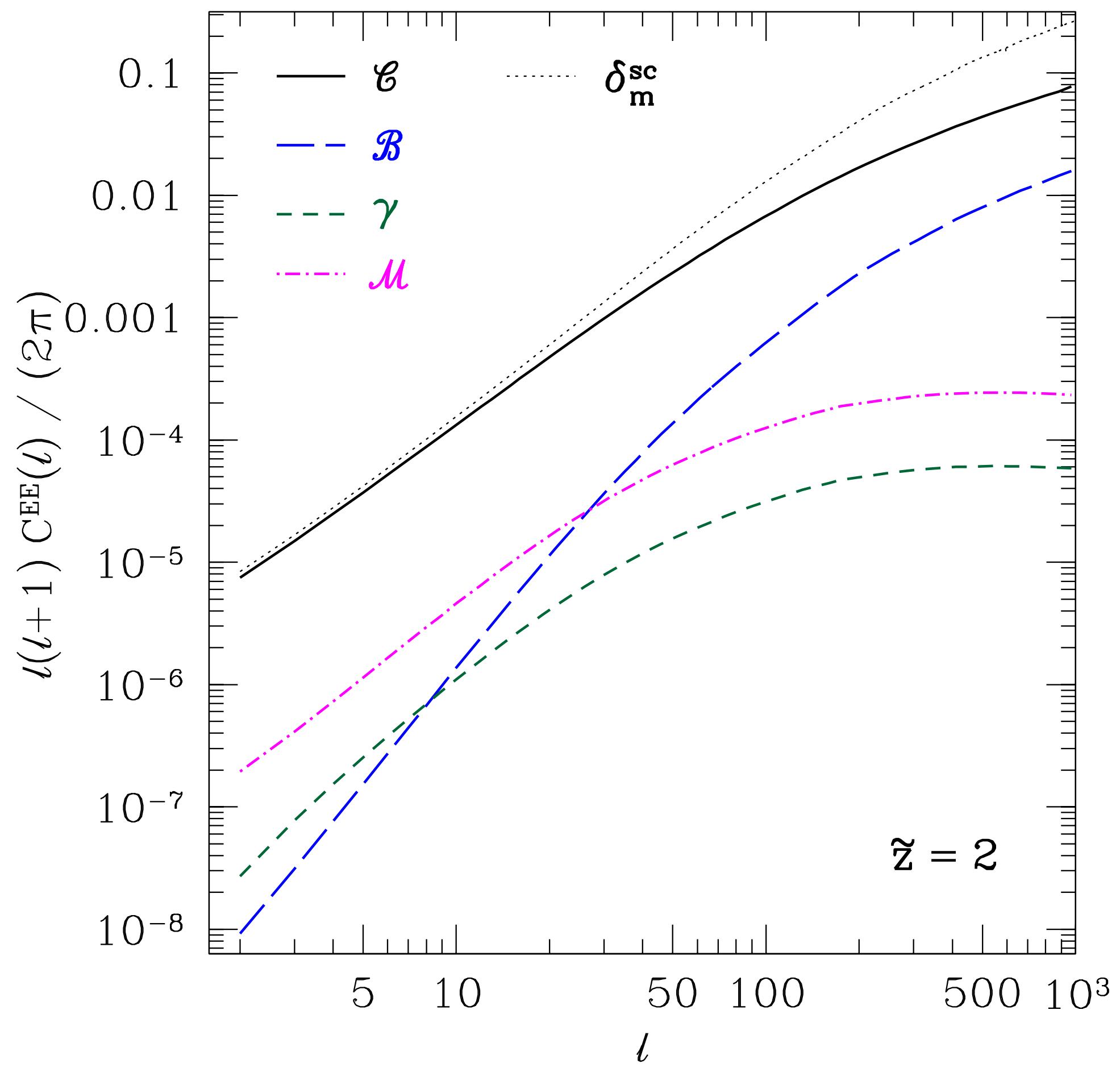
- First fully relativistic, covariant expression for the cosmic shear!!

$$\begin{aligned} \pm_2 \gamma &= -\frac{1}{2}h_\pm - \frac{1}{2}(h_\pm)_o - \int_0^{\tilde{\chi}} d\chi \left[\left(1 - 2\frac{\chi}{\tilde{\chi}}\right) [m_\mp^k \partial_\pm B_k + (\partial_\pm h_{lk}) m_\mp^l \hat{n}^k] - \frac{1}{\tilde{\chi}} h_\pm \right. \\ &\quad \left. + (\tilde{\chi} - \chi) \frac{\chi}{\tilde{\chi}} \left\{ -m_\mp^i m_\mp^j \partial_i \partial_j A + \hat{n}^k m_\mp^i m_\mp^j \partial_i \partial_j B_k + \frac{1}{2} m_\mp^i m_\mp^j (\partial_i \partial_j h_{kl}) \hat{n}^k \hat{n}^l \right\} \right] \end{aligned}$$

Here, $\pm_2 \gamma(\hat{n}) \equiv m_\mp^i m_\mp^j \mathcal{A}_{ij}$ is a spin ±2 component of the shear, where $m_\pm = \frac{1}{\sqrt{2}}(e_1 \mp ie_2)$ are spin ±1 vector field on sphere in the sense that it transforms $m_\pm \rightarrow m'_\pm = e^{\pm i\psi} m_\pm$ under the rotation $e_i \rightarrow e'_i$ with angle ψ .

- Conformal Newtonian gauge: $\pm_2 \gamma(\hat{n}) = \int_0^{\tilde{\chi}} d\chi (\tilde{\chi} - \chi) \frac{\chi}{\tilde{\chi}} m_\mp^i m_\mp^j \partial_i \partial_j (\Psi - \Phi)$

C, B, M, γ from scalar perturbations



- On small scales, C is dominated by line-of-sight velocity. When projecting onto sphere, velocity (solid line) and density (dotted line) have different slope
- Small scale B is dominated by perpendicular derivative of l.o.s. velocity.
- On small scales, $|M| = 2|\kappa| = 2|\gamma|$:
 $C_l^M = 4 C_l^\gamma$

Large-Scale Structure with GW II

: Shear

Fabian Schmidt & Donghui Jeong [arXiv:1205.1514]

$$ds^2 = a^2(\eta) [-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

Cosmic shear with GW

- With only tensor perturbation, shear expression becomes

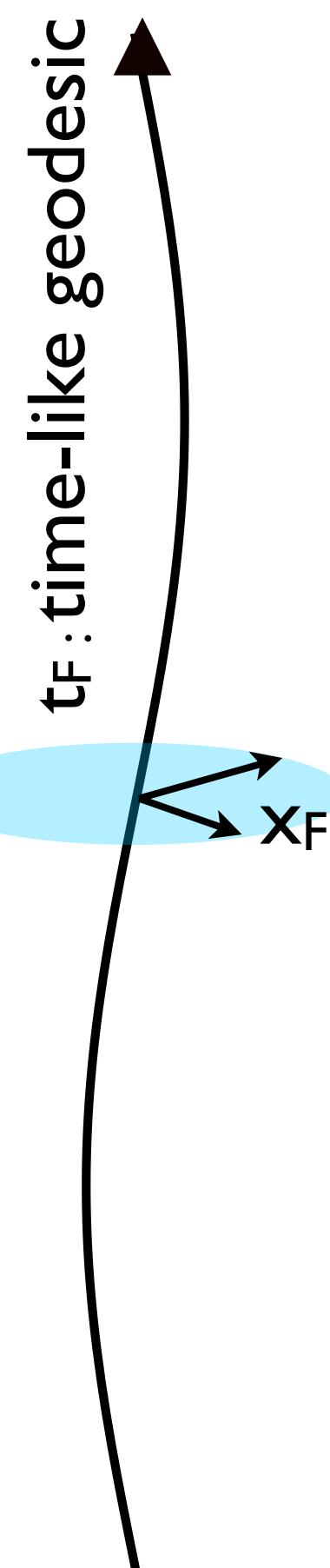
$$\pm_2 \gamma(\hat{n}) = -\frac{1}{2}h_{\pm o} - \frac{1}{2}h_{\pm} - \int_0^{\tilde{\chi}} d\chi \left\{ \frac{\tilde{\chi} - \chi}{2} \frac{\chi}{\tilde{\chi}} (m_{\mp}^i m_{\mp}^j \partial_i \partial_j h_{kl}) \hat{n}^k \hat{n}^l + \left(1 - 2 \frac{\chi}{\tilde{\chi}}\right) \hat{n}^l m_{\mp}^k m_{\mp}^i \partial_i h_{kl} - \frac{1}{\tilde{\chi}} h_{\pm} \right\}$$

Metric Shear

- Dodelson, Rozo & Stebbins (2003)
 “Assuming physical isotropy, we must add a ‘metric shear’ caused by the shearing of the coordinates with respect to physical space, i.e. $\Delta\gamma_{ij}$, which is just the traceless transverse projection of $-h_{ij}/2$ ”

$$ds^2 = a^2(\eta) \left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j \right]$$

What “metric shear” really is



- The cosmic shear measurement are referenced to **the frame within which galaxies are statistically round**.
- The most natural choice of such coordinate is the **local inertial frame defined along the time-like geodesic of the galactic center**, or so called **Fermi Normal Coordinate (FNC)**!
- Coordinate transformation from FRW to FNC coordinate:

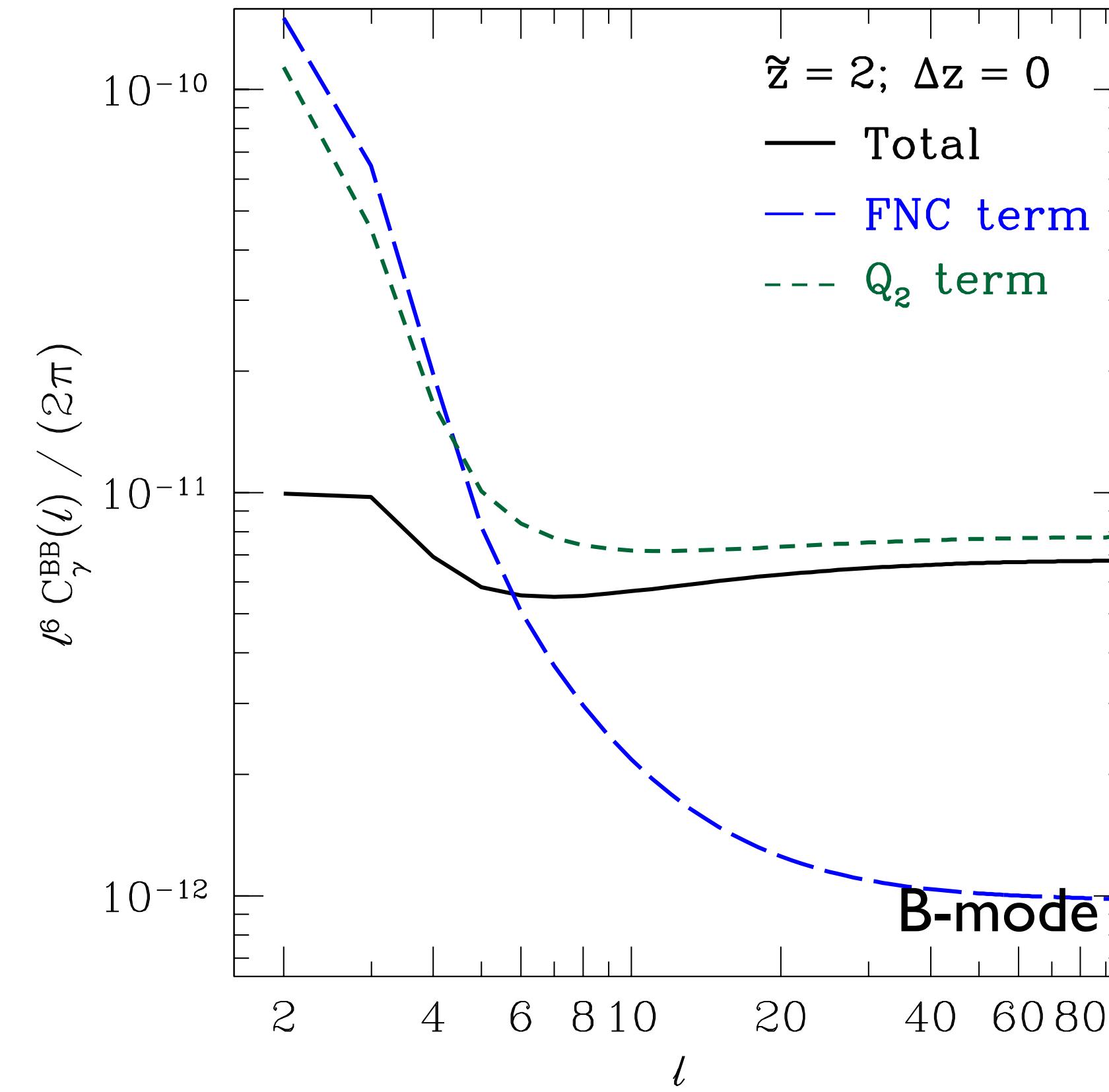
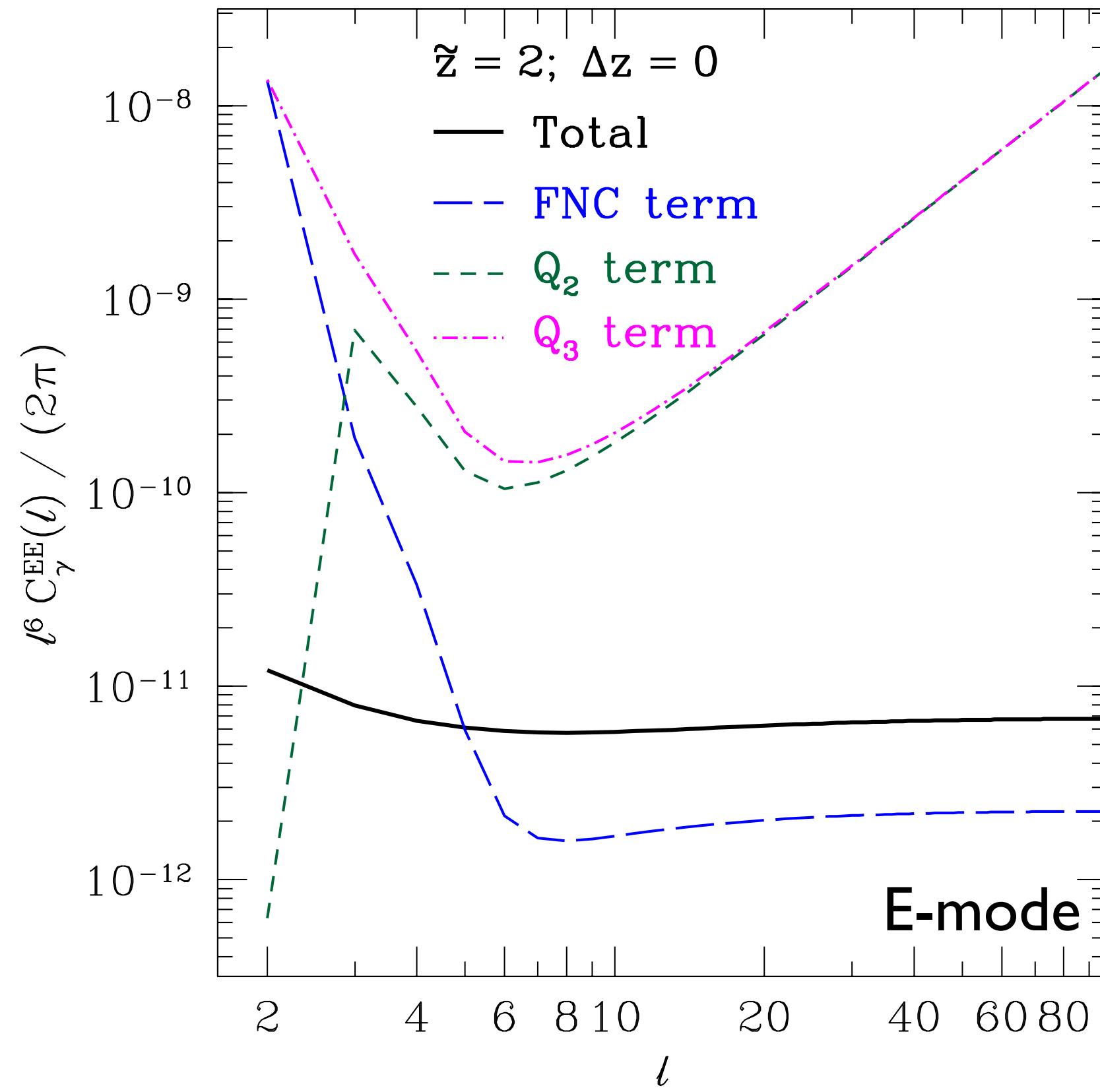
$$x_F^i = x^i - \frac{1}{2}h_{ij}x^j - \frac{1}{2}\Gamma_{jk}^i x^j x^k + \mathcal{O}(x^3)$$

FNC term

leads to an additional shear of

$$\partial_{\perp(i} \Delta x_{\perp j)} \rightarrow \partial_{\perp(i} \Delta x_{\perp j)} + \frac{1}{2} \mathcal{P}_i^k \mathcal{P}_j^l h_{kl} + \dots$$

Metric shear vs. I.o.s. integral



- They are about the same order of magnitude, but with opposite sign...

$$ds^2 = a^2(\eta) \left[-d\eta^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right]$$

FNC metric and tide

- The metric in the Fermi Normal Coordinate is given by

$$g_{00}^F = -1 + \left(\dot{H} + H^2 \right) r_F^2 + \left[\frac{1}{2} \ddot{h}_{lm} + H \dot{h}_{lm} \right] x_F^l x_F^m.$$

$$g_{0i}^F = \frac{1}{3} \left(\nabla_i \dot{h}_{lm} - \nabla_m \dot{h}_{li} \right) x_F^l x_F^m$$

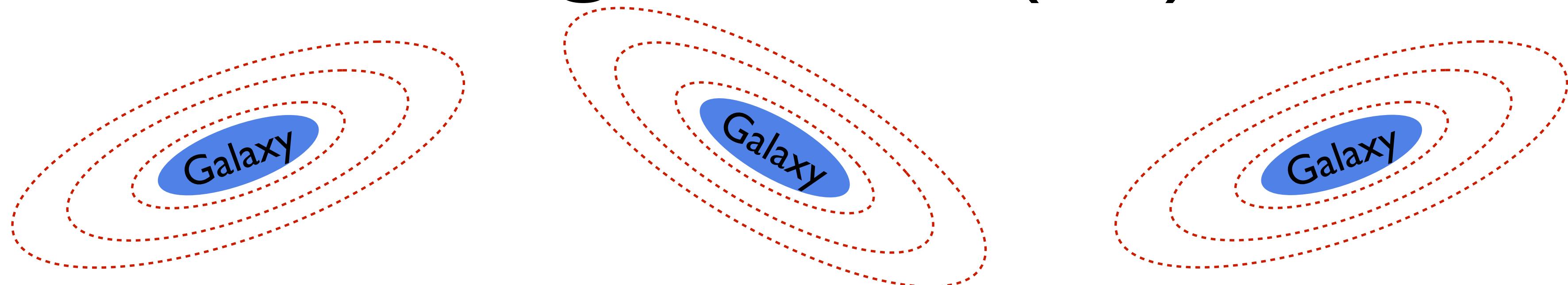
$$\begin{aligned} g_{ij}^F = & \delta_{ij} + \frac{H^2}{3} \left[x_F^i x_F^j - r_F^2 \delta_{ij} \right] + \frac{1}{6} \left(\nabla_i \nabla_j h_{ml} + \nabla_l \nabla_m h_{ij} - \nabla_l \nabla_j h_{im} - \nabla_i \nabla_m h_{jl} \right) x_F^l x_F^m \\ & + \frac{H}{6} \left(\dot{h}_{lj} x_F^l x_F^i + \dot{h}_{im} x_F^m x_F^j - \dot{h}_{ij} r_F^2 - \dot{h}_{lm} x_F^l x_F^m \delta_{ij} \right). \end{aligned}$$

- Equation of motion for non-relativistic body in FNC is determined by the effective gravitational potential $\Psi_{\text{eff}} = -\delta g_{00}/2$.

- Ψ generates tidal force: $t_{ij} = \left(\partial_i \partial_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) \Psi^F = - \left(\frac{1}{2} \ddot{h}_{lm} + H \dot{h}_{lm} \right)$

$$ds^2 = a^2(\eta) [-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

Intrinsic alignment (IA) model



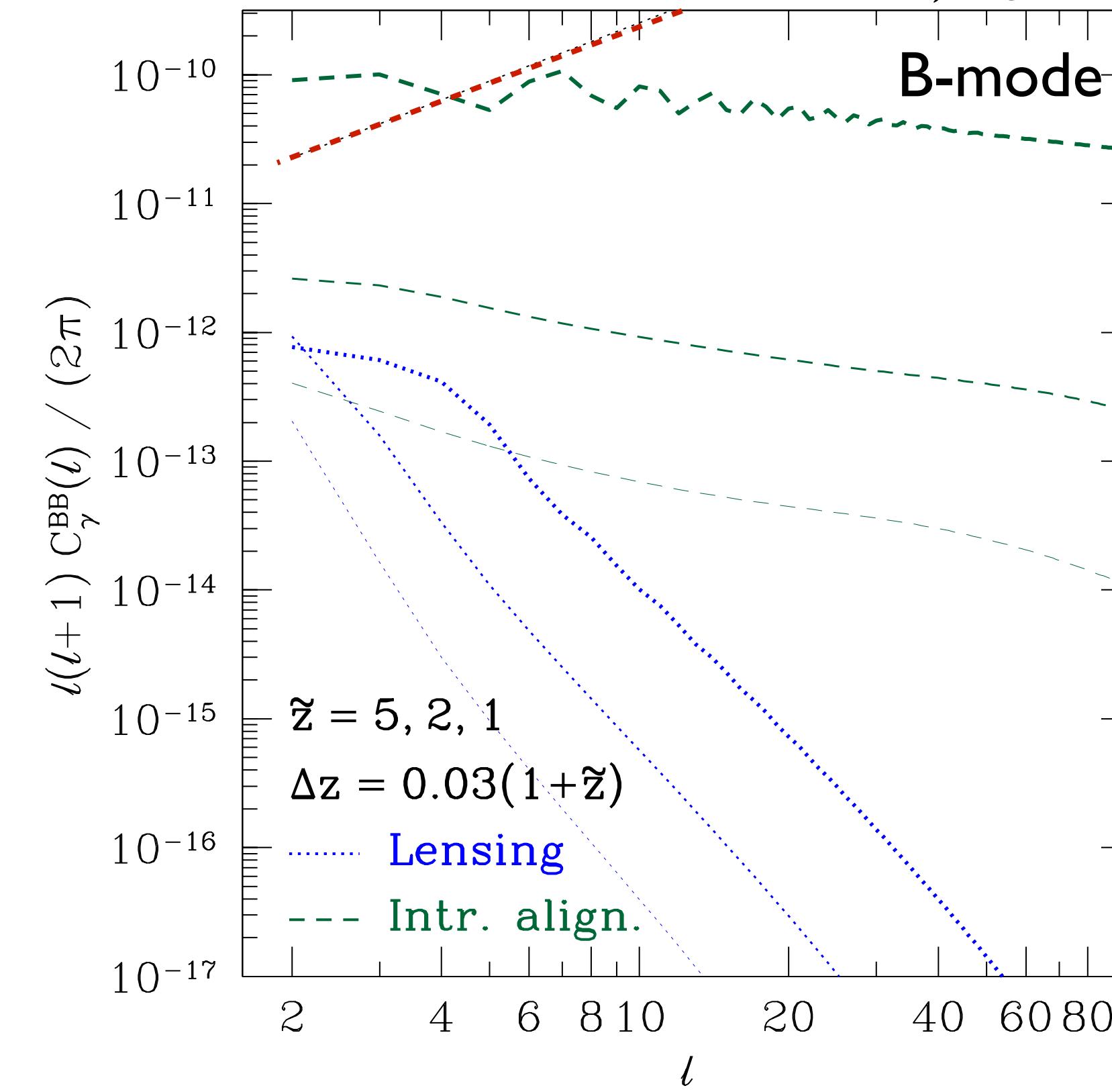
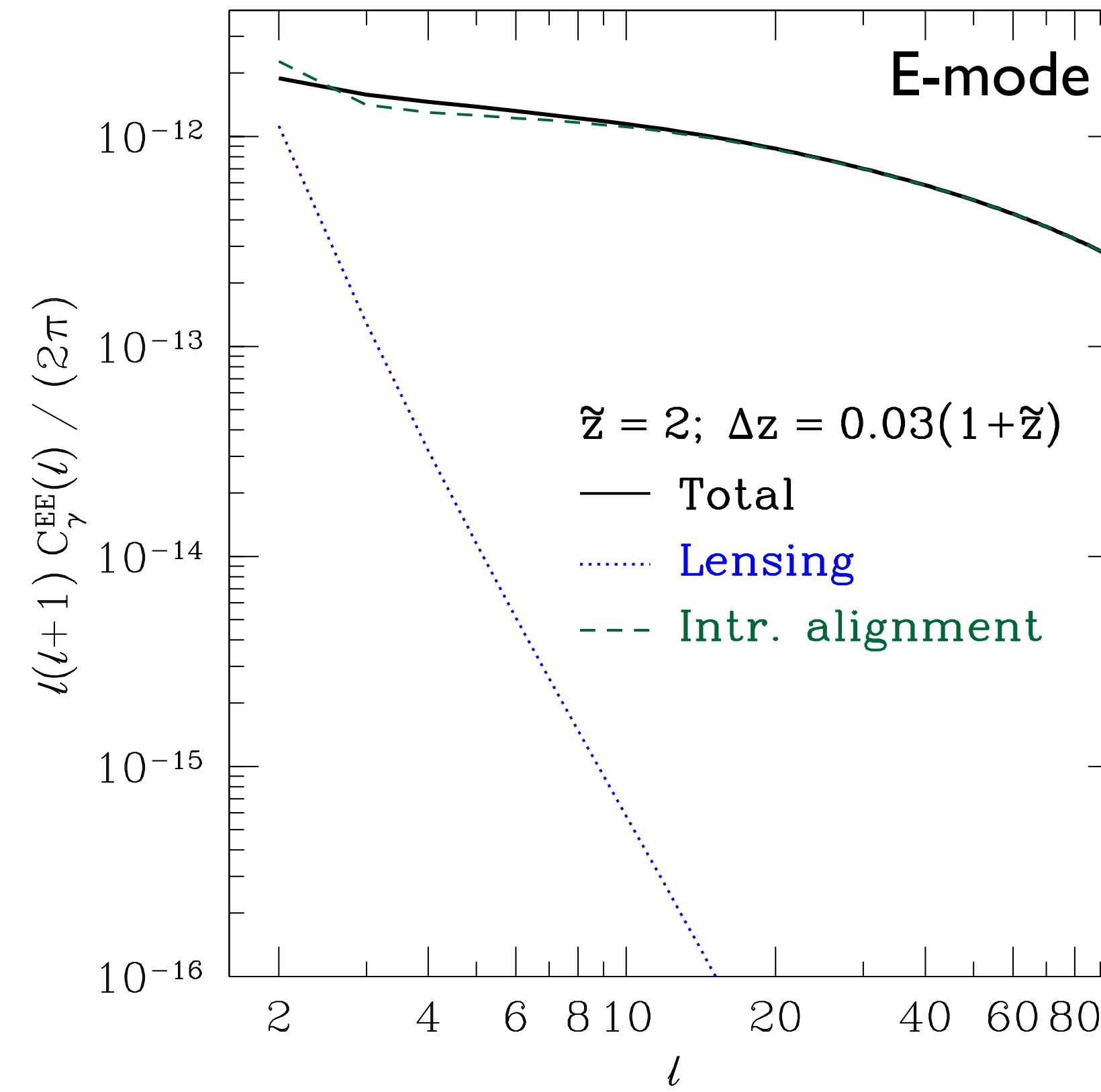
- Intrinsic alignment: **tidal fields (anisotropic gravitational potential) tends to align galaxies**
- Linear alignment model $\gamma_{ij}^{IA}(n) = -\frac{C_1}{4\pi G} \mathcal{P}_{ik} \mathcal{P}_{jl} t^{kl} = -\frac{2}{3} \frac{C_1 \rho_{\text{cr0}}}{H_0^2} \mathcal{P}_{ik} \mathcal{P}_{jl} t^{kl}$
- **consistent with observations on large (> 10 [Mpc/h]) scales**

Blazek+(2011), Joachimi+(2011)

$$\pm 2\gamma^{\text{IA}}(\hat{n}) = \frac{1}{3} \frac{C_1 \rho_{\text{cr0}}}{a^2 H_0^2} (h''_{\pm} + aH h'_{\pm})$$

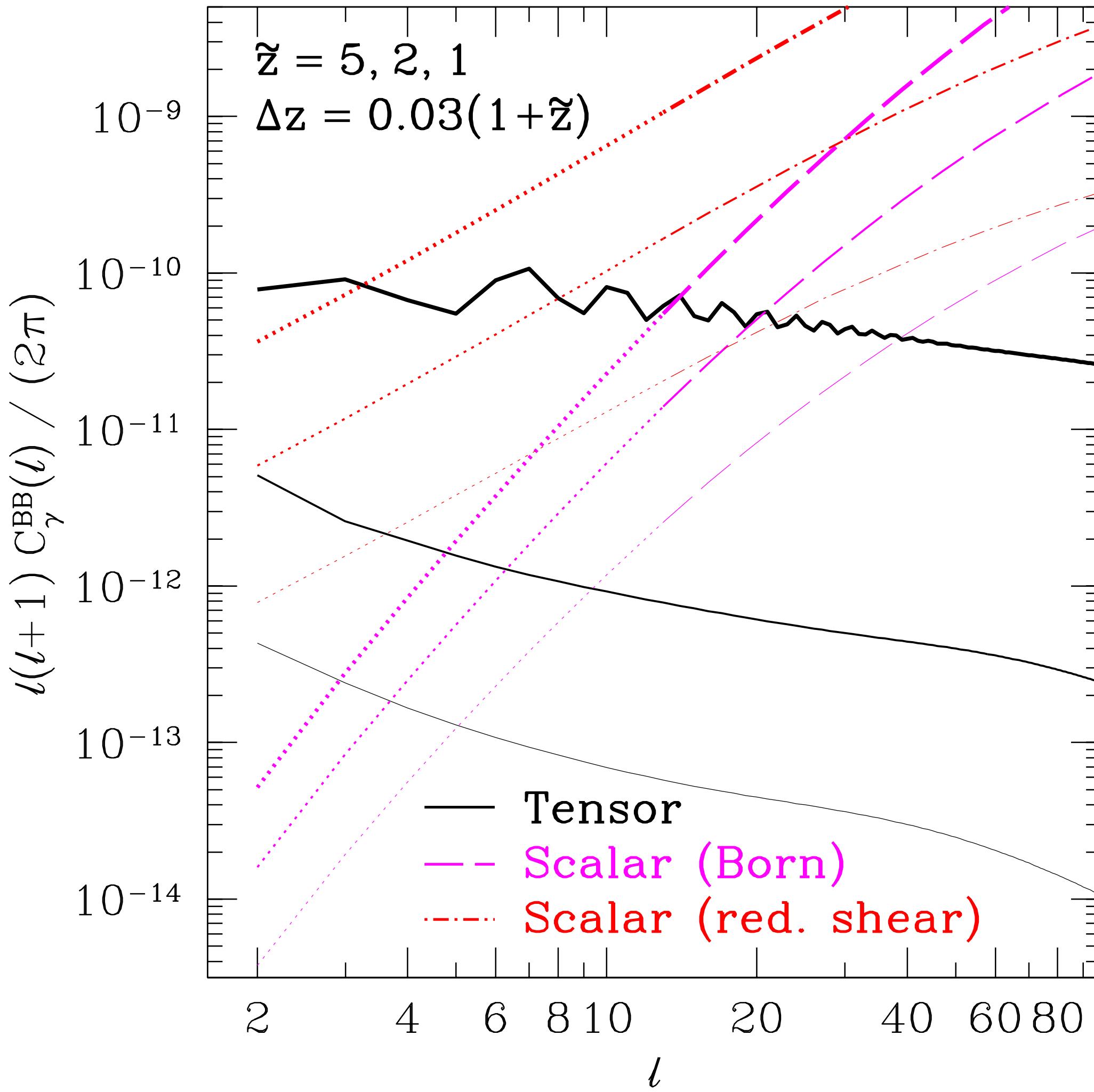
Shear vs. intrinsic alignment

noise for a half sky survey with
 $n=100/\text{arcmin}^2, \sigma_e=0.3$



- Intrinsic alignment dominates over the lensing signal, and IA signal increases at higher redshifts!

What about 2nd'ary B-modes?

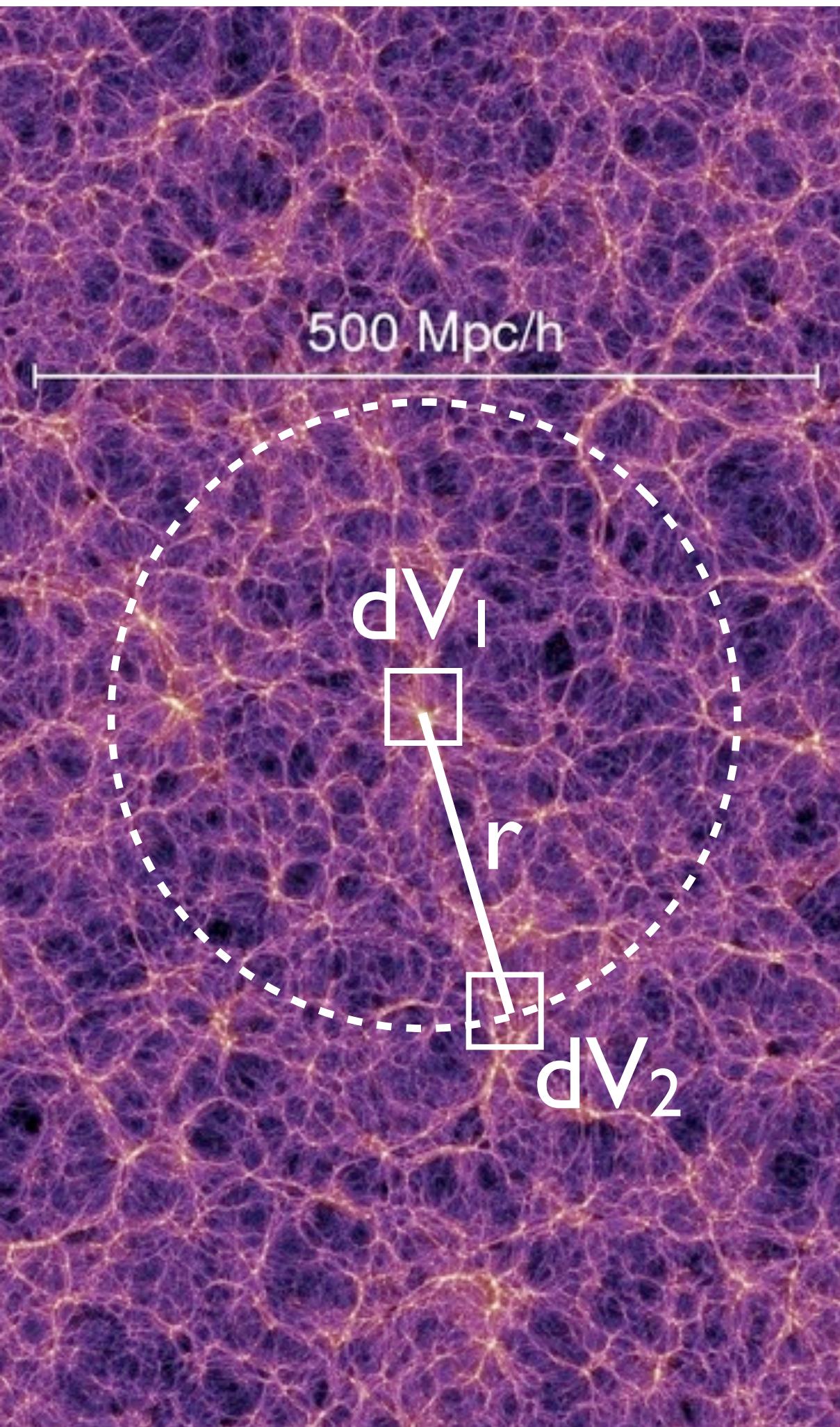


- The second order scalar perturbations can also generate parity odd (B-mode) lensing signal
- Induced GW $\sim 10^{-14}$
(Mollerach+2004; Bauman+2007)
- 2nd-order geodesic eqn.
(Hirata&Seljak 2003)
- reduced shear + lensing bias
(Schneider+1997, Dodelson+2006,
Schmidt+2009)

Clustering Fossils from the Early Universe

Donghui Jeong & Marc Kamionkowski [arXiv:1203.0302]

Two-point correlation functions



- Probability of finding two galaxies at separation r is given by the two-point correlation function:

$$P_2(r) = \bar{n}^2[1 + \xi(r)]dV_1dV_2$$

$$\xi(r) \equiv \overrightarrow{\langle \delta(x)\delta(x+r) \rangle}$$

statistical homogeneity (translational invariance)

- Power spectrum is the Fourier transform of it:

$$P(k) = \int d^3r \xi(r) e^{ik \cdot r}$$

or in terms of density contrast,

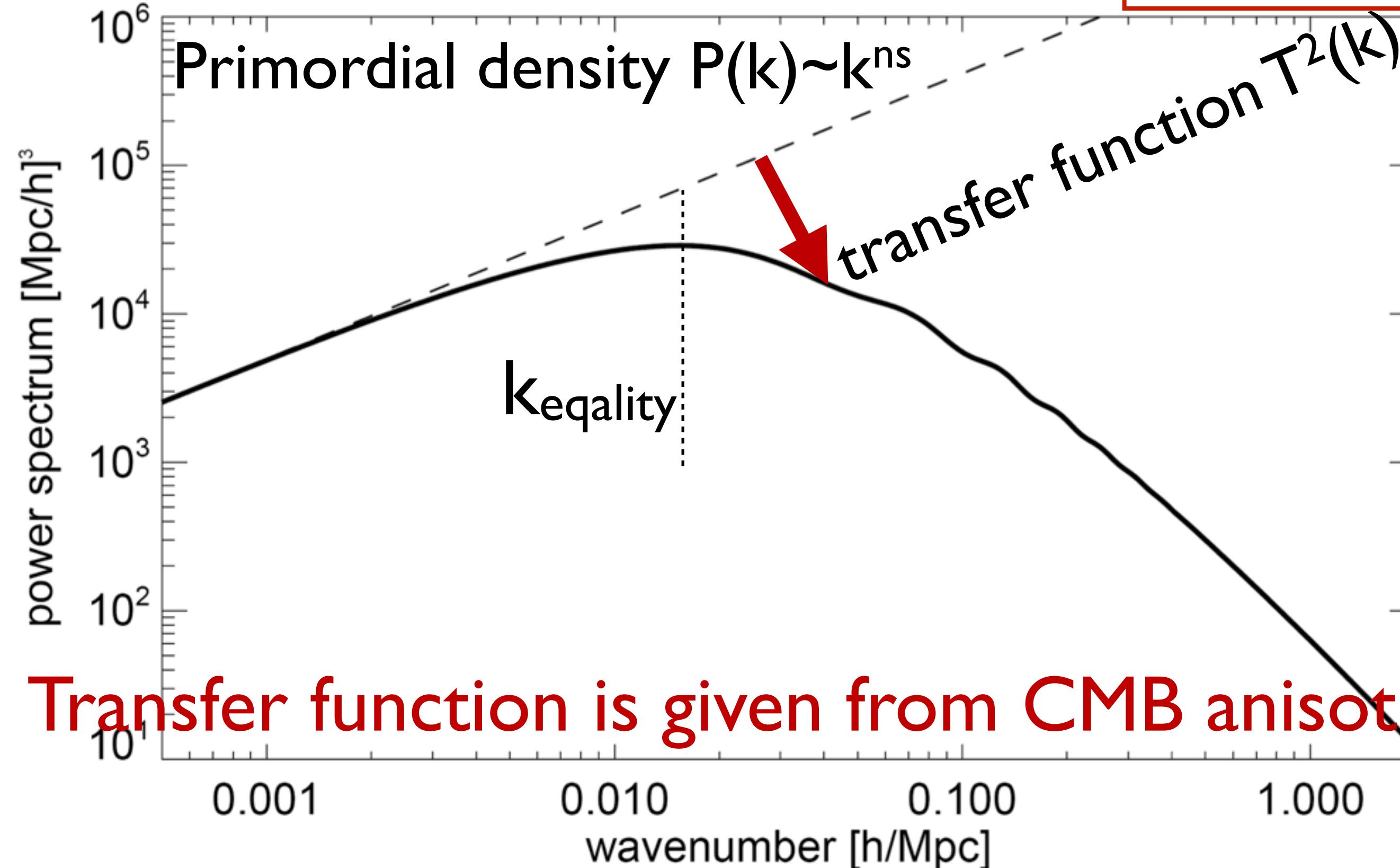
$$\langle \delta(k)\delta(k') \rangle = (2\pi)^3 P(k) \boxed{\delta^D(k+k')}$$

Linear evolution of power spectrum

$$\langle \delta_i(\mathbf{k})\delta_i(\mathbf{k}') \rangle = (2\pi)^3 P_{\text{initial}}(\mathbf{k}) \delta^D(\mathbf{k} + \mathbf{k}')$$

Initial statistical homogeneity is sustained!

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}') \rangle = (2\pi)^3 [T(\mathbf{k})]^2 P_{\text{initial}}(\mathbf{k}) \delta^D(\mathbf{k} + \mathbf{k}')$$



Non-Gaussianity and homogeneity

(local)

- **IF** we have a following non-linear coupling between primordial density fluctuations and **new field h_p** (JK coupling):

$$\langle \delta_i(\mathbf{k}_1) \delta_i(\mathbf{k}_2) h_p(\mathbf{K}) \rangle = (2\pi)^3 P_p(K) f_p(\mathbf{k}_1, \mathbf{k}_2) \epsilon_{ij}^p k_1^i k_2^j \delta^D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{K})$$

↓
 power spectrum of new field
 ↗
 coupling amplitude ↑
 polarization basis (scalar, vector, tensor)

- **THEN**, density power spectrum we observe now has ***non-zero off-diagonal*** components: **Fossil equation**

$$\langle \delta_i(\mathbf{k}_1) \delta_i(\mathbf{k}_2) \rangle|_{h_p(\mathbf{K})} = h_p(\mathbf{k}_1 + \mathbf{k}_2) f_p(\mathbf{k}_1, \mathbf{k}_2) \epsilon_{ij}^p k_1^i k_2^j \delta_{\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{K}}^D$$

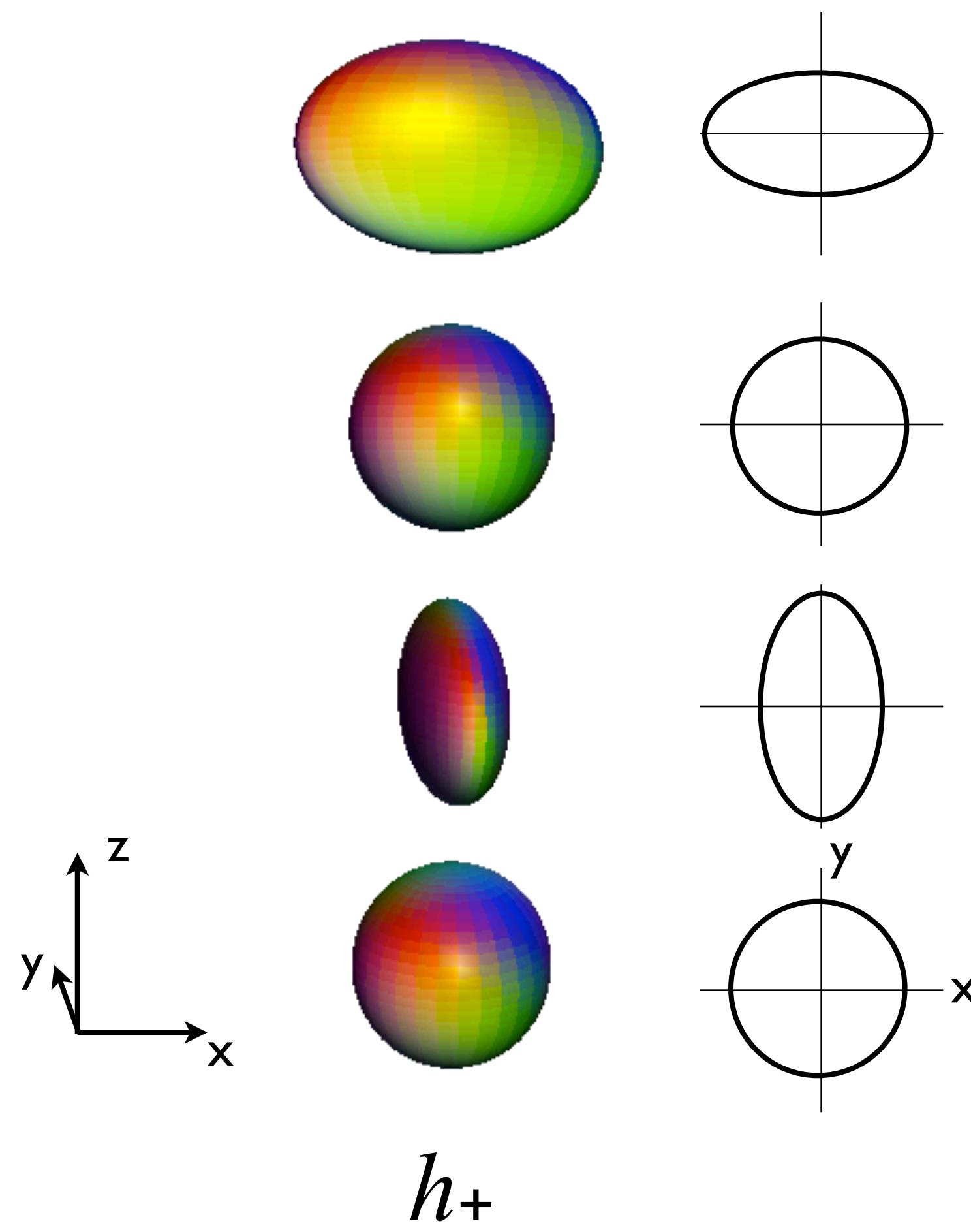
Why worrying about new fields?

- Inflaton(s) : a scalar field(s) responsible for inflation
- But, **inflaton might not be alone**. Many inflationary models need/introduce additional fields. But, direct detection of such fields turns out to be very hard:
 - Additional Scalar: not contributing to seed fluctuations
 - Vector: decays as $1/[\text{scale factor}]$
 - Tensor: decays after coming inside of comoving horizon
- **Off-diagonal correlation (Fossil equation) opens new way of detecting them!**

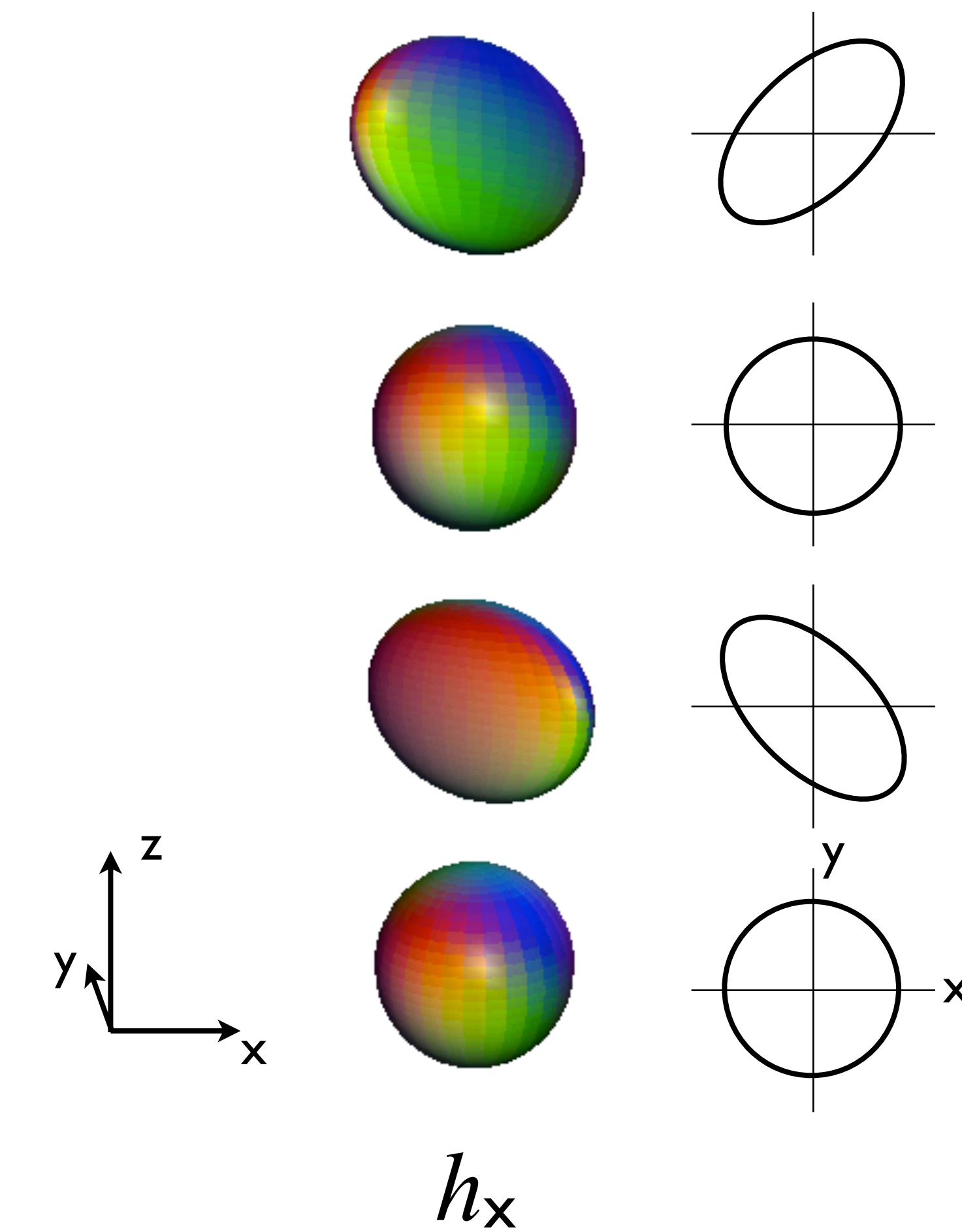
ϵ_{ij}^p : six independent modes

- In a symmetric 3×3 tensor, we have 6 degrees of freedom, which are further decomposed by Scalar, Vector and Tensor polarization modes.
- They are orthogonal: $\epsilon_{ij}^p \epsilon^{p',ij} = 2\delta_{pp'}$
 - Scalar ($p=0,z$): $\epsilon_{ij}^0 \propto \delta_{ij}$ $\epsilon_{ij}^z(\mathbf{K}) \propto K_i K_j - K^2/3$
 - Vector ($p=x,y$): $\epsilon_{ij}^{x,y}(K) \propto \frac{1}{2} (K_i e_j + K_j e_i)$ where $K_i e_i = 0$
 - Tensor ($p=x,+/-$): transverse and traceless
$$K_i \epsilon_{ij}^{+,\times}(\mathbf{K}) = 0 \quad \delta_{ij} \epsilon_{ij}^{+,\times}(\mathbf{K}) = 0$$

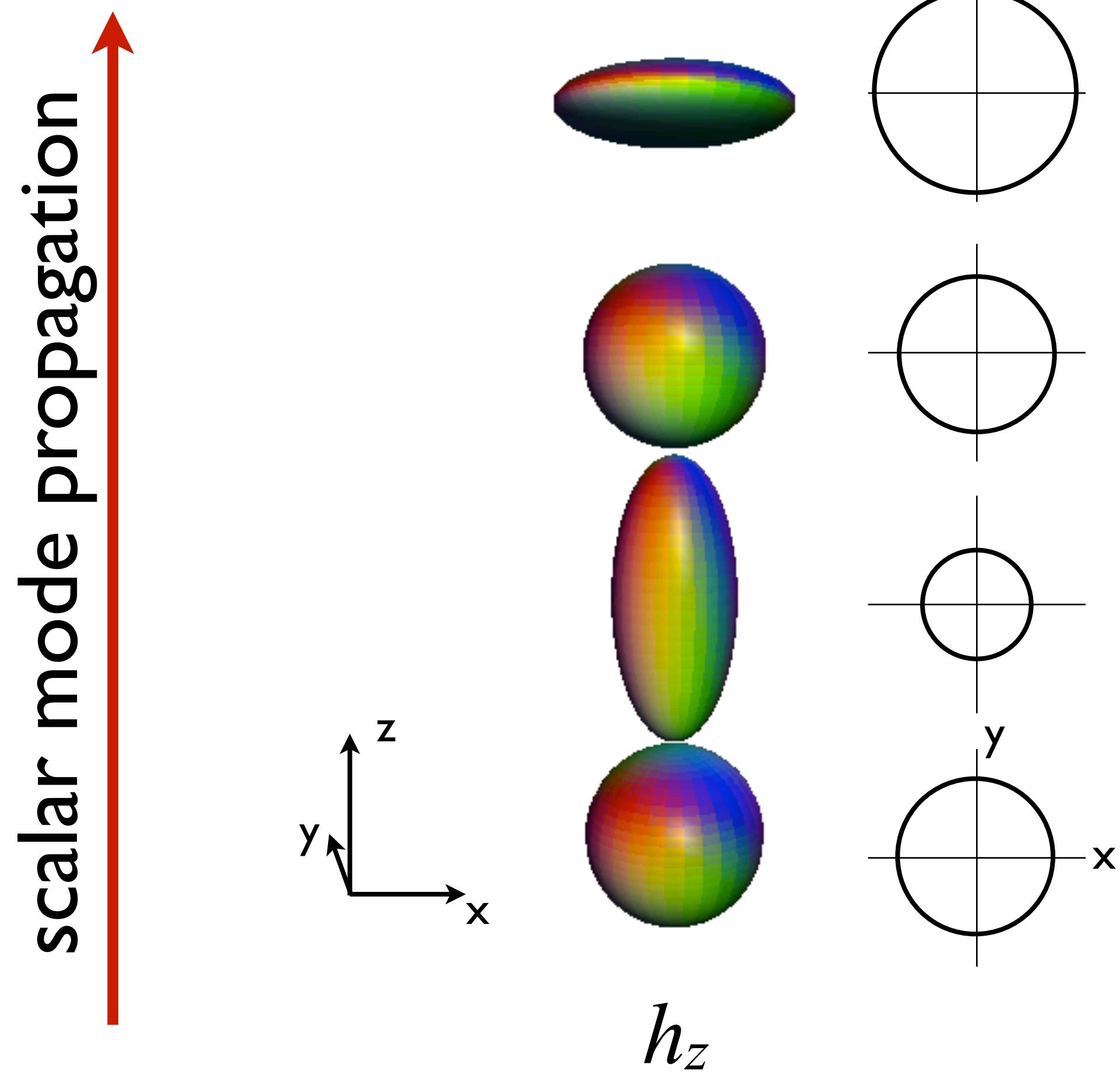
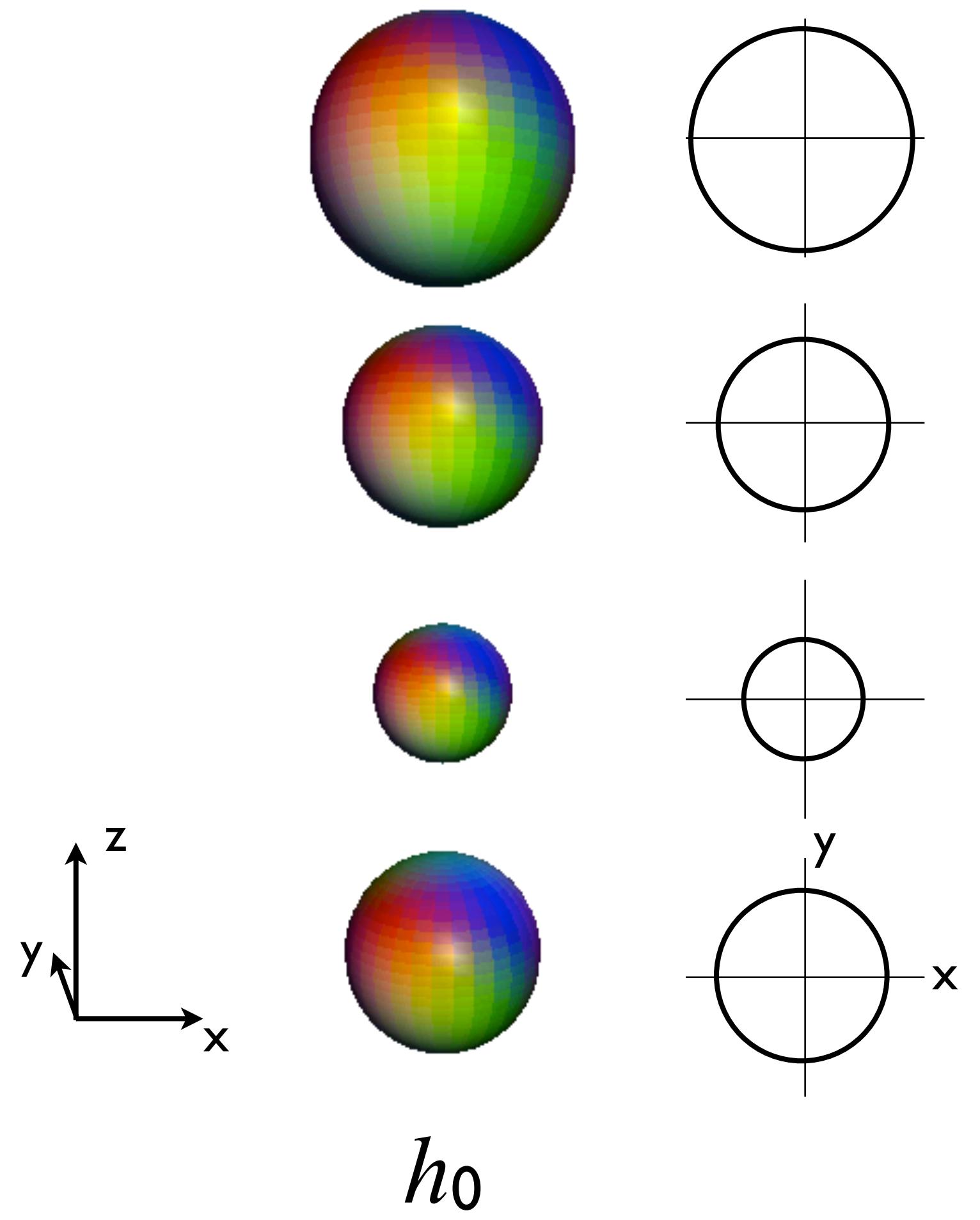
$\xi(\mathbf{r})$ with single tensor mode ($p=+,x$)



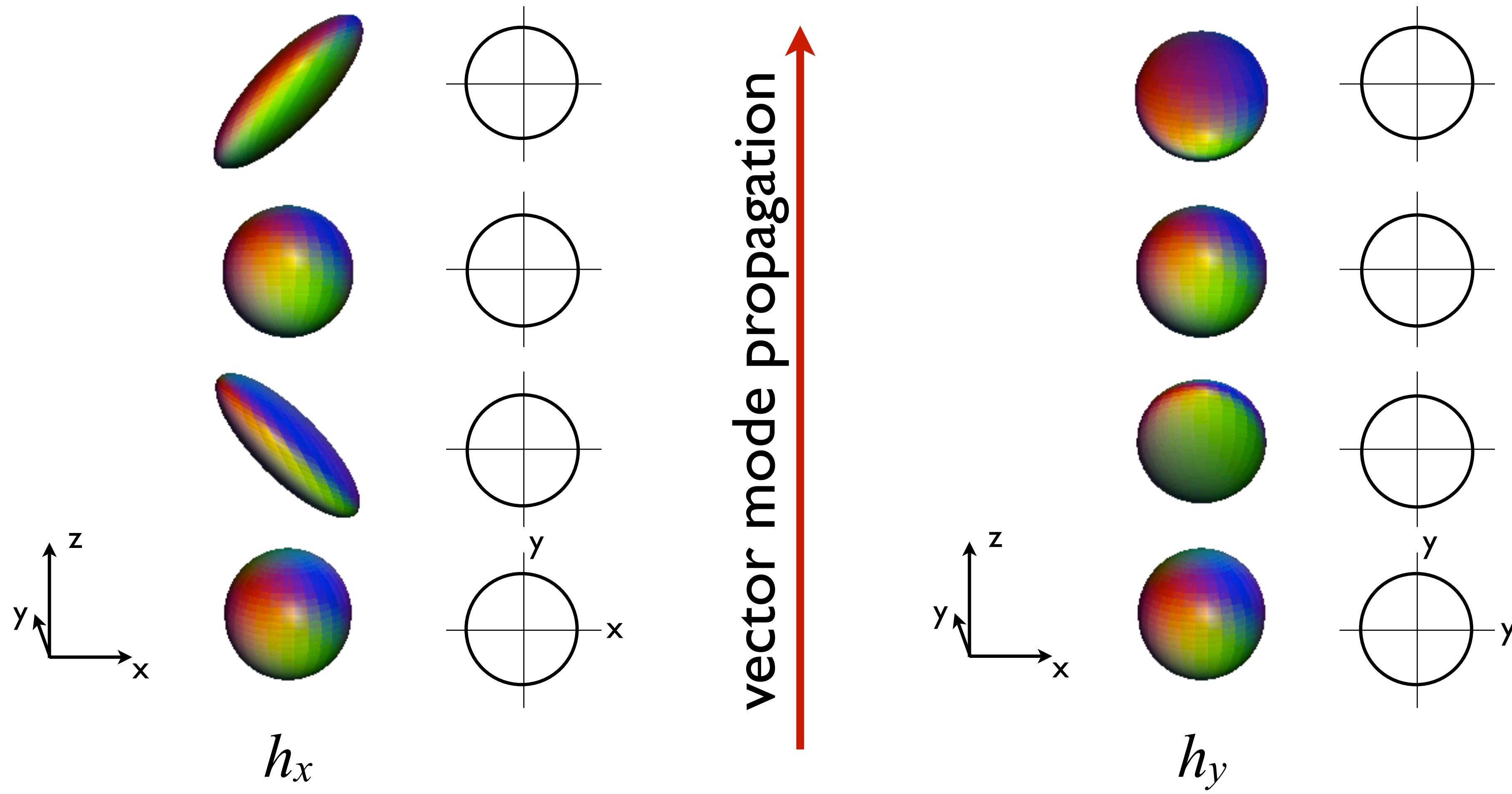
tensor mode propagation



$\xi(\mathbf{r})$ with single scalar mode ($p=0,z$)



$\xi(\mathbf{r})$ with single vector mode ($p=x,y$)

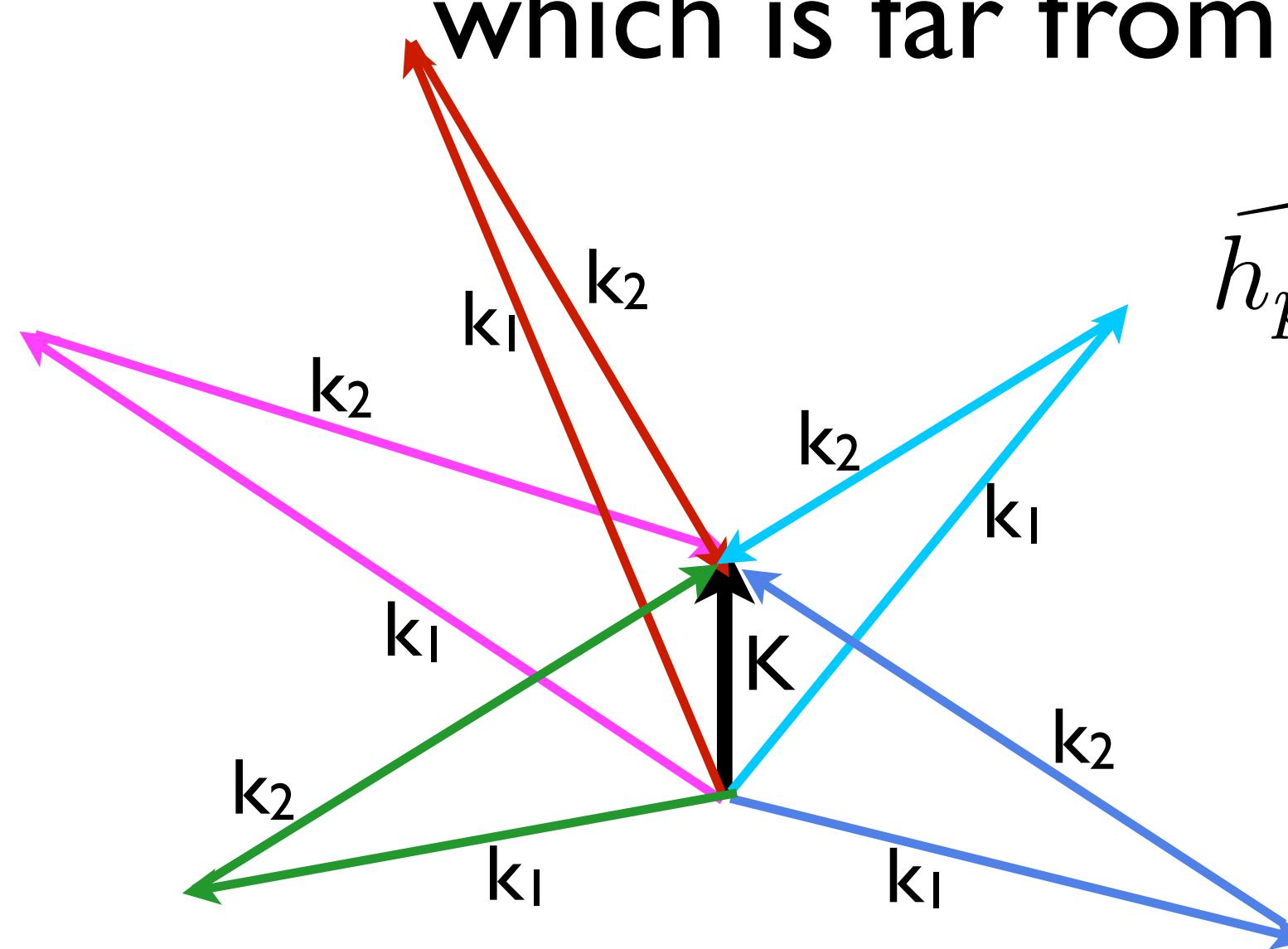


Naive estimator

- Let's start from Fossil equation

$$\langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \rangle|_{h_p(\mathbf{K})} = h_p(\mathbf{k}_1 + \mathbf{k}_2) f_p(\mathbf{k}_1, \mathbf{k}_2) \epsilon_{ij}^p k_1^i k_2^j \delta_{\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{K}}^D$$

- Rearranging it a bit, we get a naive estimator for the new field, which is far from optimal:



$$\widehat{h_p(\mathbf{K})} = \sum_{\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{K}} \frac{\delta(\mathbf{k}_1) \delta(\mathbf{k}_2)}{f_p(\mathbf{k}_1, \mathbf{k}_2) \epsilon_{ij}^p k_1^i k_2^j}$$

Optimal estimator for a single mode

- Inverse-variance weighting gives an optimal estimator for a single mode

$$\widehat{h_p(\mathbf{K})} = P_p^n(\mathbf{K}) \sum_{\mathbf{k}} \frac{f_p^*(\mathbf{k}, \mathbf{K} - \mathbf{k}) \epsilon_{ij}^p k^i (K - k)^j}{2V P^{\text{tot}}(k) P^{\text{tot}}(|\mathbf{K} - \mathbf{k}|)} \delta(\mathbf{k}) \delta(\mathbf{K} - \mathbf{k})$$

- With a noise power spectrum ($P_{\text{tot}} = P_{\text{galaxy}} + P_{\text{noise}}$)

$$P_p^n(K) = \left[\sum_{\mathbf{k}} \frac{|f_p(\mathbf{k}, \mathbf{K} - \mathbf{k}) \epsilon_{ij}^p k^i (K - k)^j|^2}{2V P^{\text{tot}}(k) P^{\text{tot}}(|\mathbf{K} - \mathbf{k}|)} \right]^{-1}$$

Optimal estimator for the power amplitude A_h

- For a stochastic background of new fields with power spectrum $P_p(K) = A_h P_h^f(K)$, we **optimally summed over different K-modes** to estimate the amplitude by (w/ NULL hypothesis):

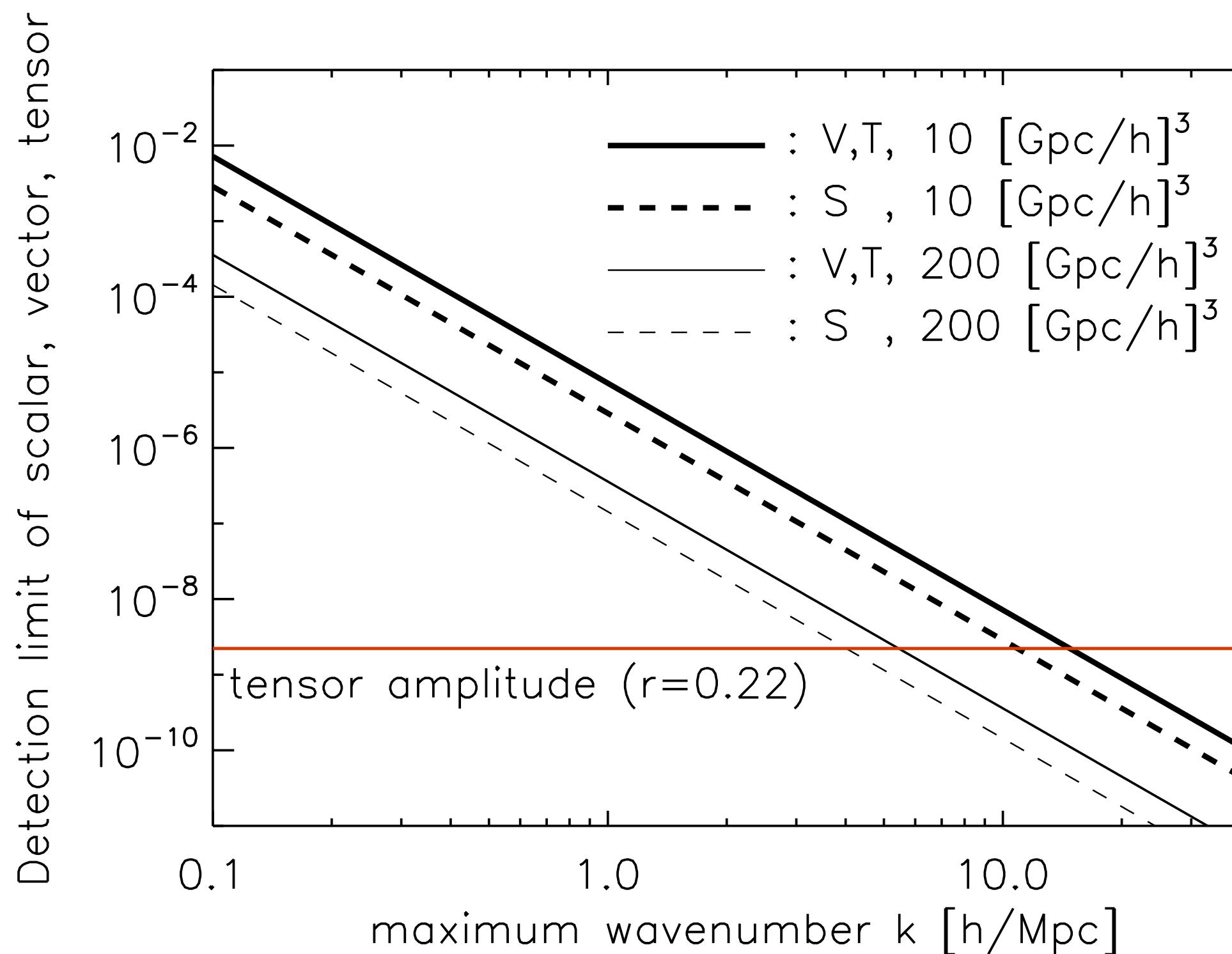
$$\widehat{A}_h = \sigma_h^2 \sum_{K,p} \frac{\left[P_h^f(K) \right]^2}{2 \left[P_p^n(K) \right]^2} \left(\frac{\left| \widehat{h_p}(K) \right|^2}{V} - P_p^n(K) \right)$$

- Here, the minimum uncertainty of measuring amplitude is

$$\sigma_h^{-2} = \sum_{K,p} \left[P_h^f(K) \right]^2 / 2 \left[P_p^n(K) \right]^2$$

When new “fields” are usual metric fluctuations

- Then, new field only rescales the wave-vector $k^2 \rightarrow k^2 - h_{ij}k_i k_j$, which reads $f_P = -3/2 P(k)/k^2$ (Maldacena, 2003)



- projected 3-sigma (99% C.L.) detection limit with galaxy survey parameters
- To detect the gravitational wave, we need a large dynamical range
- Current survey (e.g. SDSS) should set a limit on primordial V and T!

Conclusion

- We present three different ways of detecting primordial GW. For all three methods, effect at the source location is important as GW itself decays in time.
- **Galaxy clustering**: impossible to probe as the signal is too weak compared to that of scalar perturbations
- **Cosmic shear**: a bit challenge, but possible to detect GW on large scales thanks to the intrinsic alignment effect!
- **Fossil equation**: requires large dynamical range to beat the small signal (21cm map?). Interesting potential to detect primordial vector fields as well.