Excursion sets, peaks and scale-dependent bias

Aseem Paranjape

The Abdus Salam ICTP, Trieste

AP & Sheth: arXiv:1206.3506
Desjacques, AP & Sheth: in preparation

and in collaboration with Sefusatti, Monaco, Borgani

CERN, Geneva, September 2012

Plan

1. Excursion sets
    - Original framework...
    - ... and why it's wrong
    - Interlude: correlated steps

Plan

1. Excursion sets
   - Original framework...
   - ... and why it's wrong
   - Interlude: correlated steps

2. Modifying the framework
   - Peaks theory: counting the correct walks
   - Comparison with simulations: Sqrt barrier with LN scatter

Plan

1. Excursion sets
   - Original framework...
   - ... and why it's wrong
   - Interlude: correlated steps
2. Modifying the framework
   - Peaks theory: counting the correct walks
   - Comparison with simulations: Sqrt barrier with LN scatter
3. Lagrangian halo bias
   - Real space Vs. Fourier space
   - Comparison with simulations

Plan

Nonlinear gravity
Separating dynamics from statistics

Dark matter halos are fully nonlinear collapsed gravitational systems.

But at first approximation, they form when a “dense enough” clump falls into itself due to gravity. Calling the collapse a spherical cow allows dynamics to be separated from statistics of initial conditions.

Nonlinear gravity
Separating dynamics from statistics

Dark matter halos are fully nonlinear collapsed gravitational systems.

But at first approximation, they form when a "dense enough" clump falls into itself due to gravity. Calling the collapse a spherical cow allows dynamics to be separated from statistics of initial conditions.

Also, language simplifies in the right variables: use smoothed, linearly extrapolated density contrast

$$\delta_R = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} W(kR)\delta(\vec{k}) \;\; ; \;\; s \equiv \sigma_R^2 = \langle \, \delta_R^2 \, \rangle = \int \mathrm{d}\ln k \; \Delta^2(k) W(kR)^2$$

where $\Delta^2(k) = k^3 P(k)/2\pi^2$. E.g.: $W(y) = \mathrm{e}^{-y^2/2}$ or $W(y) = (3/y) j_1(y)$.
Also, frequently use $\nu \equiv \delta_c/\sqrt{s}$; $\delta_c \simeq 1.686$.

Nonlinear gravity
Separating dynamics from statistics

Dark matter halos are fully nonlinear collapsed gravitational systems.

But at first approximation, they form when a “dense enough” clump falls into itself due to gravity. Calling the collapse a spherical cow allows dynamics to be separated from statistics of initial conditions.

Also, language simplifies in the right variables: use smoothed, linearly extrapolated density contrast

$$\delta_R = \int \frac{\mathrm{d}^3k}{(2\pi)^3} W(kR)\delta(\vec{k}) \; ; \; s \equiv \sigma_R^2 = \langle\, \delta_R^2 \,\rangle = \int \mathrm{d}\ln k\; \Delta^2(k) W(kR)^2$$

where $\Delta^2(k) = k^3 P(k)/2\pi^2$. E.g.: $W(y) = \mathrm{e}^{-y^2/2}$ or $W(y) = (3/y)j_1(y)$.
Also, frequently use $\nu \equiv \delta_c/\sqrt{s}$; $\delta_c \simeq 1.686$.

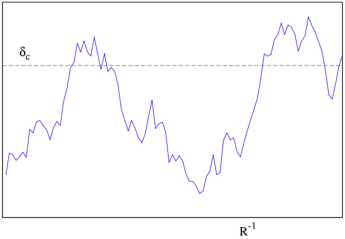
Approximation of dynamics (Press & Schechter 74) :

If δ_R exceeds a critical threshold δ_c, then a collapsed object of mass $m \propto R^3$ forms today. [So σ_R^2 is a label for mass m.]

Excursion set ansatz
"Pick a card, any card".

Original ansatz (Bond et al. 91):

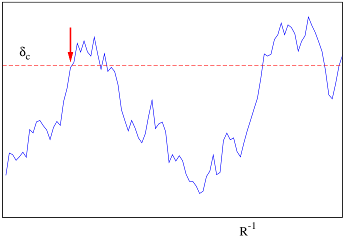
- Pick a location in space at random.
- Construct sequence of values $\delta_R \sim \sum W(kR)\delta(\vec{k})$ as R is decreased from a large value. This is a random walk.
- Some of these values will lie above δ_c. Corresponding smoothing scales are candidate halo masses.



Excursion set ansatz
"Pick a card, any card".

Original ansatz (Bond et al. 91):

- Pick a location in space at random.
- Construct sequence of values $\delta_R \sim \sum W(kR)\delta(\vec{k})$ as R is decreased from a large value. This is a random walk.
- Some of these values will lie above δ_c. Corresponding smoothing scales are candidate halo masses.
- To avoid overcounting, look for *first* crossing for a given walk. [All other crossings correspond to smaller scales, which will be eaten up by the largest – "cloud-in-cloud".]



Excursion set ansatz
"Pick a card, any card".

Original ansatz (Bond et al. 91):

- Pick a location in space at random.
- Construct sequence of values $\delta_R \sim \sum W(kR)\delta(\vec{k})$ as R is decreased from a large value. This is a random walk.
- Some of these values will lie above δ_c. Corresponding smoothing scales are candidate halo masses.
- To avoid overcounting, look for *first* crossing for a given walk. [All other crossings correspond to smaller scales, which will be eaten up by the largest – "cloud-in-cloud".]

Excursion set ansatz :

Distribution of first-crossing scales $f(s)\mathrm{d}s$ in initial conditions, gives mass fraction $f(m)\mathrm{d}m$ in collapsed objects of mass m.
$f(m)$ then gives number density of m-halos or "halo mass function" $n(m)$ through simple transformation.

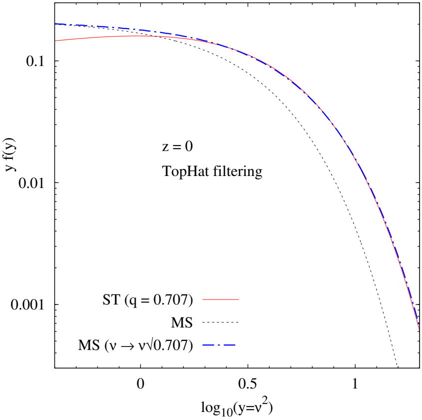
Excursion set approach also makes predictions for halo bias, merger rates, etc.

Plan

1. Excursion sets
   - Original framework...
   - ... and why it's wrong
   - Interlude: correlated steps
2. Modifying the framework
   - Peaks theory: counting the correct walks
   - Comparison with simulations: Sqrt barrier with LN scatter
3. Lagrangian halo bias
   - Real space Vs. Fourier space
   - Comparison with simulations

Empirical problem
Random walks confront halos

Well-known order-of-magnitude discrepancy



AP & Sheth 12

Theoretical problem
"Any" card?!

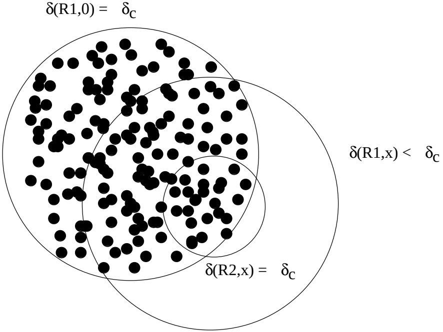
Picking a location at random cannot be correct, because halos form at special locations (e.g., near peaks in the initial density).

The effect of this inaccurate choice can be easily understood (Sheth, Mo & Tormen 01; Sheth 11):

Theoretical problem
"Any" card?!

Picking a location at random cannot be correct, because halos form at special locations (e.g., near peaks in the initial density).

The effect of this inaccurate choice can be easily understood (Sheth, Mo & Tormen 01; Sheth 11):



I.e., masses will be underestimated, and hence also the tail of the mass function.

Plan

First crossing distributions
Analytical approximations

Analytical solutions in closed form are known for the sharp-k filter, for barriers that are constant (Bond et al 91) or linear in $s = \sigma_R^2$ (e.g., Sheth 98). [*This case is tractable since steps in* $\delta_R \sim \sum W(kR)\delta(\vec{k})$ *are uncorrelated.*]

For other filters and barrier shapes, approximations are needed, mainly because the steps in the walks are correlated (Peacock & Heavens 90; Maggiore & Riotto 10; AP, Lam & Sheth 12; Musso & Sheth 12).

First crossing distributions
Analytical approximations

Analytical solutions in closed form are known for the sharp-k filter, for barriers that are constant (Bond et al 91) or linear in $s = \sigma_R^2$ (e.g., Sheth 98). [*This case is tractable since steps in $\delta_R \sim \sum W(kR)\delta(\vec{k})$ are uncorrelated.*]

For other filters and barrier shapes, approximations are needed, mainly because the steps in the walks are correlated (Peacock & Heavens 90; Maggiore & Riotto 10; AP, Lam & Sheth 12; Musso & Sheth 12).

The most accurate is due to Musso & Sheth 12. For a constant barrier:

$$f_{\rm MS}(s) = \int_0^\infty \mathrm{d}v\, v\, p(\delta_c, v)\,,$$

where $v \equiv \mathrm{d}\delta/\mathrm{d}s$ and $p(\delta, v)$ is a bivariate Gaussian with zero mean and covariance matrix that depends on choice of filter and power spectrum $P(k)$.

Intuitively, because walks with correlated steps are smooth, "first crossing of δ_c" $\approx$ "crossing with positive slope".

First crossing distributions
Analytical approximations

Analytical solutions in closed form are known for the sharp-k filter, for barriers that are constant (Bond et al 91) or linear in $s = \sigma_R^2$ (e.g., Sheth 98). [*This case is tractable since steps in* $\delta_R \sim \sum W(kR)\delta(\vec{k})$ *are uncorrelated.*]

For other filters and barrier shapes, approximations are needed, mainly because the steps in the walks are correlated (Peacock & Heavens 90; Maggiore & Riotto 10; AP, Lam & Sheth 12; Musso & Sheth 12).

The most accurate is due to Musso & Sheth 12. For a constant barrier:

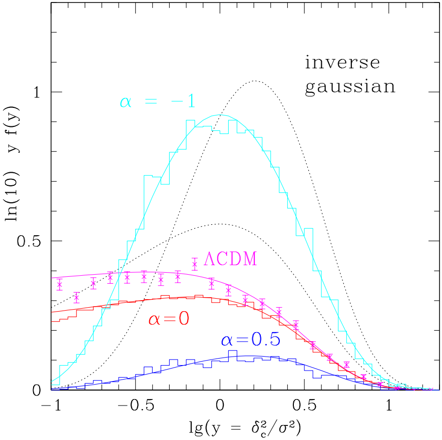
$$f_{\rm MS}(s) = \int_0^\infty \mathrm{d}v\, v\, p(\delta_c, v)\,,$$

where $v \equiv \mathrm{d}\delta/\mathrm{d}s$ and $p(\delta, v)$ is a bivariate Gaussian with zero mean and covariance matrix that depends on choice of filter and power spectrum $P(k)$.

Intuitively, because walks with correlated steps are smooth, "first crossing of δ_c" $\approx$ "crossing with positive slope".

Integral is analytical. Generalisation to moving barriers is straightforward: $\delta_c \to B(s)$ and $\int_0^\infty \mathrm{d}v\, v \ldots \to \int_{B'}^\infty \mathrm{d}v (v - B') \ldots$

Random walks
First crossing distributions: Analytical approximations



Musso & Sheth 12

Plan

Halos and peaks

To good approximation, massive halo center-of-mass $\approx$ density peak in initial conditions (Ludlow & Porciani 11). So we'd like to only count peaks.

BBKS 86 showed how to do this: basically, constrain allowed values of first derivative $\vec{\nabla}\delta$ $(= 0)$ and curvature $\nabla^2\delta$ (< 0).

Cropping the ensemble of walks
The Appel & Jones distribution

Rewriting MS12 (i.e., all-walks distribution) to standardise the notation:

- define $x \equiv 2\gamma\sqrt{s}v$, $\gamma^2 \equiv \langle\, \delta v \,\rangle^2 / (\langle\, \delta^2 \,\rangle \langle\, v^2 \,\rangle) = \langle\, x\, \nu \,\rangle^2$
- denote Normal distribution $p_{\rm G}(x - \mu; \Sigma^2)$

$$f_{\rm MS}(\nu) = \frac{{\rm e}^{-\nu^2/2}}{\sqrt{2\pi}} \frac{1}{\gamma\nu} \int_0^\infty {\rm d}x\, x\, p_{\rm G}(x - \gamma\nu; 1 - \gamma^2)$$

Cropping the ensemble of walks
The Appel & Jones distribution

Rewriting MS12 (i.e., all-walks distribution) to standardise the notation:

- define $x \equiv 2\gamma\sqrt{s}v$, $\gamma^2 \equiv \langle \delta v \rangle^2/(\langle \delta^2 \rangle \langle v^2 \rangle) = \langle x\, \nu \rangle^2$
- denote Normal distribution $p_{\rm G}(x-\mu;\Sigma^2)$

$$f_{\rm MS}(\nu) = \frac{e^{-\nu^2/2}}{\sqrt{2\pi}}\frac{1}{\gamma\nu}\int_0^\infty {\rm d}x\, x\, p_{\rm G}(x-\gamma\nu;1-\gamma^2)$$

A technical simplification when Gaussian filtering: $x = -\nabla^2\delta/\sqrt{\langle (\nabla^2\delta)^2 \rangle} =$ peak curvature.

Cropping the ensemble of walks
The Appel & Jones distribution

Rewriting MS12 (i.e., all-walks distribution) to standardise the notation:

- define $x \equiv 2\gamma\sqrt{s}v$, $\gamma^2 \equiv \langle \delta v \rangle^2/(\langle \delta^2 \rangle \langle v^2 \rangle) = \langle x\, \nu \rangle^2$
- denote Normal distribution $p_{\rm G}(x-\mu;\Sigma^2)$

$$f_{\rm MS}(\nu) = \frac{e^{-\nu^2/2}}{\sqrt{2\pi}}\frac{1}{\gamma\nu}\int_0^\infty {\rm d}x\, x\, p_{\rm G}(x-\gamma\nu;1-\gamma^2)$$

A technical simplification when Gaussian filtering: $x = -\nabla^2\delta/\sqrt{\langle (\nabla^2\delta)^2 \rangle}$ = peak curvature.

In this case, counting peaks amounts to introducing $F(x)(V/V_*)$ under the integral.

- V/V_* = ratio of peak Lagrangian volume $V(= m/\bar{\rho})$ to characteristic volume V_* (*which depends on filter and* $P(k)$).
- $F(x)$ = peak curvature function (▸ *BBKS eqn A15* *... bunch of error functions and Gaussians*).

Cropping the ensemble of walks
The Appel & Jones distribution

Rewriting MS12 (i.e., all-walks distribution) to standardise the notation:

- define $x \equiv 2\gamma\sqrt{s}v$, $\gamma^2 \equiv \langle \delta v \rangle^2/(\langle \delta^2 \rangle \langle v^2 \rangle) = \langle x\, \nu \rangle^2$
- denote Normal distribution $p_{\rm G}(x-\mu;\Sigma^2)$

$$f_{\rm MS}(\nu) = \frac{e^{-\nu^2/2}}{\sqrt{2\pi}} \frac{1}{\gamma\nu} \int_0^\infty {\rm d}x\, x\, p_{\rm G}(x-\gamma\nu; 1-\gamma^2)$$

A technical simplification when Gaussian filtering: $x = -\nabla^2\delta/\sqrt{\langle (\nabla^2\delta)^2 \rangle} =$ peak curvature.

In this case, counting peaks amounts to introducing $F(x)(V/V_*)$ under the integral.

- $V/V_* =$ ratio of peak Lagrangian volume $V (= m/\bar{\rho})$ to characteristic volume V_* (*which depends on filter and $P(k)$*).
- $F(x) =$ peak curvature function (BBKS eqn A15 ... *bunch of error functions and Gaussians*).

Result: rederivation of Appel & Jones 90 distribution

$$f_{\rm AJ}(\nu) = \frac{e^{-\nu^2/2}}{\sqrt{2\pi}} \left(\frac{V}{V_*}\right) \frac{1}{\gamma\nu} \int_0^\infty {\rm d}x\, x\, F(x) p_{\rm G}(x-\gamma\nu; 1-\gamma^2)$$

Excursion set peaks
Re-interpreting Appel & Jones 90

Alternate point of view (with same result):

Excursion set peaks
Re-interpreting Appel & Jones 90

Alternate point of view (with same result):

- BBKS peaks number density defined on a single smoothing scale.

Excursion set peaks
Re-interpreting Appel & Jones 90

Alternate point of view (with same result):

- BBKS peaks number density defined on a single smoothing scale.
- So "peak-in-peak" problem not solved: critical peak on one scale can also be critical peak on larger scale.

Excursion set peaks
Re-interpreting Appel & Jones 90

Alternate point of view (with same result):

- BBKS peaks number density defined on a single smoothing scale.
- So "peak-in-peak" problem not solved: critical peak on one scale can also be critical peak on larger scale.
- Musso-Sheth prescription shows how to solve this:
  $\longrightarrow$ Halo of given mass $=$ region that is a peak of critical height on scale of interest, but not on any larger scale.
  $\longrightarrow$ I.e., solve first crossing distribution for peaks.
  $\longrightarrow$ Correlated steps $\Longrightarrow$ "any larger scale" $\approx$ "next larger scale".

Excursion set peaks
Re-interpreting Appel & Jones 90

Alternate point of view (with same result):

- BBKS peaks number density defined on a single smoothing scale.
- So “peak-in-peak” problem not solved: critical peak on one scale can also be critical peak on larger scale.
- Musso-Sheth prescription shows how to solve this:
  $\longrightarrow$ Halo of given mass $=$ region that is a peak of critical height on scale of interest, but not on any larger scale.
  $\longrightarrow$ I.e., solve first crossing distribution for peaks.
  $\longrightarrow$ Correlated steps $\Longrightarrow$ “any larger scale” $\approx$ “next larger scale”.

Implementing this: With $\mathcal{N}_{\rm BBKS}(\nu,x) = (1/V_*)\left(\mathrm{e}^{-\nu^2/2}/\sqrt{2\pi}\right) F(x)p_{\rm G}(x-\gamma\nu;1-\gamma^2)$,

$$\mathcal{N}_{\rm BBKS}(\nu)\Delta\nu = \int_0^\infty \mathrm{d}x\,\Delta\nu\,\mathcal{N}_{\rm BBKS}(\nu,x) = \int_0^\infty \mathrm{d}x \int_{\delta_c}^{\delta_c+\Delta\nu\sqrt{s}} \frac{\mathrm{d}\delta}{\sqrt{s}}\,\mathcal{N}_{\rm BBKS}(\delta/\sqrt{s},x)\,,$$

Excursion set peaks
Re-interpreting Appel & Jones 90

Alternate point of view (with same result):

- BBKS peaks number density defined on a single smoothing scale.
- So “peak-in-peak” problem not solved: critical peak on one scale can also be critical peak on larger scale.
- Musso-Sheth prescription shows how to solve this:
  $\longrightarrow$ Halo of given mass = region that is a peak of critical height on scale of interest, but not on any larger scale.
  $\longrightarrow$ I.e., solve first crossing distribution for peaks.
  $\longrightarrow$ Correlated steps $\Longrightarrow$ “any larger scale” $\approx$ “next larger scale”.

Implementing this: With $\mathcal{N}_{\rm BBKS}(\nu,x) = (1/V_*)\left(e^{-\nu^2/2}/\sqrt{2\pi}\right) F(x)p_{\rm G}(x-\gamma\nu;1-\gamma^2)$,

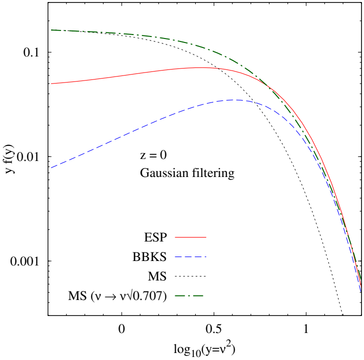
$$\mathcal{N}_{\rm BBKS}(\nu)\Delta\nu = \int_0^\infty {\rm d}x\, \Delta\nu\, \mathcal{N}_{\rm BBKS}(\nu,x) = \int_0^\infty {\rm d}x \int_{\delta_c}^{\delta_c+\Delta\nu\sqrt{s}} \frac{{\rm d}\delta}{\sqrt{s}} \mathcal{N}_{\rm BBKS}(\delta/\sqrt{s},x)\,,$$

Using $\Delta\nu\sqrt{s} \to v\,\Delta s = \left(x/2\gamma\sqrt{s}\right)\,\Delta s$ (*Musso & Sheth 12 prescription*), and $f(\nu) = V\,\mathcal{N}(\nu)$ gives

$$f_{\rm ESP}(\nu) = \frac{e^{-\nu^2/2}}{\sqrt{2\pi}}\left(\frac{V}{V_*}\right)\frac{1}{\gamma\nu}\int_0^\infty {\rm d}x\, x\, F(x)p_{\rm G}(x-\gamma\nu;1-\gamma^2) = f_{\rm AJ}(\nu)$$

Result
Enhancement at high masses (*as expected*)

$$\text{At large } \nu\text{: } f_{\rm ESP}(\nu) \to \frac{e^{-\nu^2/2}}{\sqrt{2\pi}}\,(\nu^3 - 3\nu)\,\frac{V\gamma^3}{V_*} \approx f_{\rm MS}(\nu)\,\frac{V\gamma^3\nu^3}{V_*}.$$

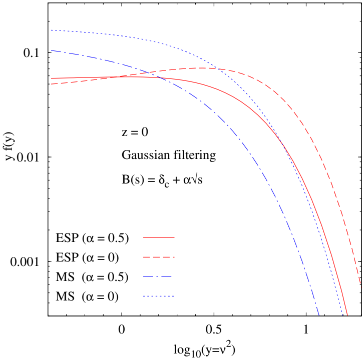


AP & Sheth 12

Changing barrier shapes
... again, is trivial

If $\delta_c \to B(s)$,

$$\mathcal{N}_{\rm ESP}(\nu) = \frac{1}{\gamma\nu}\int_{x_{\rm min}}^{\infty} \mathrm{d}x\,(x - 2\gamma\sqrt{s}B')\,\mathcal{N}_{\rm BBKS}(B/\sqrt{s}, x)\,; \quad x_{\rm min} = \max\{0, 2\gamma\sqrt{s}B'\}$$



AP & Sheth 12

To mixed filtering

Plan

Realistic filtering
... and technical complications

Halo mass is typically defined using sharp cuts in real space, e.g. using spherical TopHat. [*This is also the reason for not comparing ESP and N-body fits in previous plot, since $s \leftrightarrow m$ relation depends on filter.*]
So we'd like to switch from Gaussian to TopHat.

Realistic filtering
... and technical complications

Halo mass is typically defined using sharp cuts in real space, e.g. using spherical TopHat. [*This is also the reason for not comparing ESP and N-body fits in previous plot, since* $s \leftrightarrow m$ *relation depends on filter.*]
So we'd like to switch from Gaussian to TopHat.

Formally, not a problem.
One can even keep track of the difference between $v = \mathrm{d}\delta/\mathrm{d}s$ and $\nabla^2\delta$.

Unfortunately, $\langle\, (\nabla^2\delta)^2 \,\rangle$ diverges for most $P(k)$ of interest, including ΛCDM.

Realistic filtering
... and technical complications

Halo mass is typically defined using sharp cuts in real space, e.g. using spherical TopHat. [*This is also the reason for not comparing ESP and N-body fits in previous plot, since $s \leftrightarrow m$ relation depends on filter.*]
So we'd like to switch from Gaussian to TopHat.

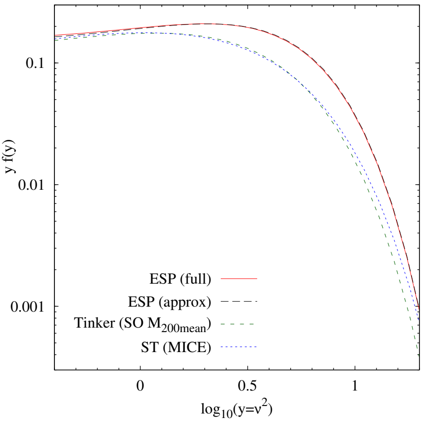
Formally, not a problem.
One can even keep track of the difference between $v = \mathrm{d}\delta/\mathrm{d}s$ and $\nabla^2\delta$.

Unfortunately, $\langle\, (\nabla^2\delta)^2 \,\rangle$ diverges for most $P(k)$ of interest, including ΛCDM.

There is a way out:

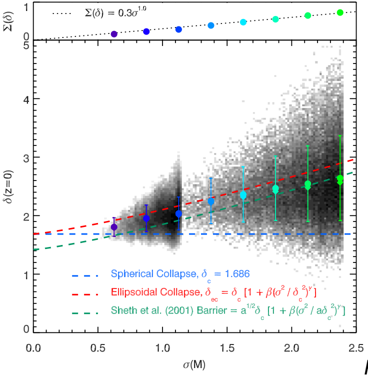
- Use TopHat when computing $\nu = \delta_c/\sqrt{s}$, so masses come out right.
- Use Gaussian when computing spatial derivatives.
- Match scales by requiring $\langle\, \delta_{\rm G} | \delta_{\rm TH} \,\rangle = \delta_{\rm TH}$, i.e., $\langle\, \delta_{\rm G}\delta_{\rm TH} \,\rangle = \langle\, \delta_{\rm TH}^2 \,\rangle = s$.

Mixed ESP
Constant barrier $B(s) = \delta_c = 1.686$



More realistic barriers
Results from simulations

We know that barrier cannot be one constant number (Sheth, Mo & Tormen 01). Recent simulations show mean trend $\sim \sqrt{s}$ and $\sim$ LogNormal scatter $\propto \sqrt{s}$ (Robertson et al. 09).



Robertson et al. 09

Consistent with $B(s) = \delta_c + \beta\sqrt{s}$ where $\beta \sim$ LogNormal with $\langle\, \beta \,\rangle = 0.43$; $\sqrt{\mathrm{Var}(\beta)} = 0.3$.

Square-root barrier with scatter
What should we expect?

Square-root barrier without scatter will dramatically decrease large-mass counts (we saw this here).

Adding scatter will lead to an Eddington bias-like effect... preferential upscattering from low-mass to high-mass. This will increase large-mass counts.

Square-root barrier with scatter
What should we expect?

Square-root barrier without scatter will dramatically decrease large-mass counts
(we saw this here).

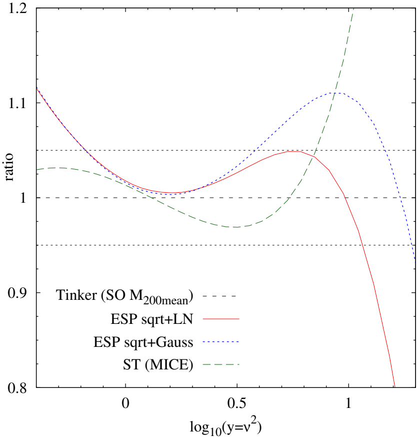
Adding scatter will lead to an Eddington bias-like effect... preferential upscattering from low-mass to high-mass. This will increase large-mass counts.

To see the full effect, we must calculate $f_{\rm ESP}(\nu) = \int {\rm d}\beta\, p(\beta)\, f_{\rm ESP}(\nu|\beta)$ where

$$
\begin{aligned}
f_{\rm ESP}(\nu|\beta) &= V\,\mathcal{N}_{\rm ESP}(\nu|\beta) \\
&= \left(\frac{V}{V_*}\right)\frac{{\rm e}^{-(\nu+\beta)^2/2}}{\sqrt{2\pi}}\,\frac{1}{\gamma\nu}\int_{\beta\gamma}^{\infty}{\rm d}x\,(x-\beta\gamma)F(x)p_{\rm G}(x-\beta\gamma-\gamma\nu;1-\gamma^2)\,,
\end{aligned}
$$

and $p(\beta)$ is LogNormal with mean 0.43 and std. deviation 0.3.

Square-root barrier with scatter
Result



Compare all-walks result

Plan

General result
Linear Lagrangian bias

Suppose that $\delta_0(\mathbf{k}) = \delta(\mathbf{k})W(kR_0)$ and $\delta_{\rm h}(\mathbf{k}) = b_1(\mathbf{k})\delta(\mathbf{k})W(kR)$ and $W(y) = \mathrm{e}^{-y^2/2}$.
We'd like to calculate cross-correlation bias in real space: $b_1 = \langle\, \delta_{\rm h}\delta_0 \,\rangle / \langle\, \delta_0^2 \,\rangle$.

General result
Linear Lagrangian bias

Suppose that $\delta_0(\mathbf{k}) = \delta(\mathbf{k})W(kR_0)$ and $\delta_{\rm h}(\mathbf{k}) = b_1(\mathbf{k})\delta(\mathbf{k})W(kR)$ and $W(y) = {\rm e}^{-y^2/2}$.
We'd like to calculate cross-correlation bias in real space: $b_1 = \langle\,\delta_{\rm h}\delta_0\,\rangle/\langle\,\delta_0^2\,\rangle$.

Suppose in k-space: $b_1(\mathbf{k}) = b_{10} = $ constant

Then in real space : $b_1 = (S_\times/S_0)b_{10}$; $[S_\times = \int {\rm d}\ln k\,\Delta^2(k)W(kR)W(kR_0)]$

(AP & Sheth 12, arXiv:1105.2261)

General result
Linear Lagrangian bias

Suppose that $\delta_0(\mathbf{k}) = \delta(\mathbf{k})W(kR_0)$ and $\delta_\mathrm{h}(\mathbf{k}) = b_1(\mathbf{k})\delta(\mathbf{k})W(kR)$ and $W(y) = \mathrm{e}^{-y^2/2}$.
We'd like to calculate cross-correlation bias in real space: $b_1 = \langle\, \delta_\mathrm{h}\delta_0 \,\rangle / \langle\, \delta_0^2 \,\rangle$.

Suppose in k-space: $b_1(\mathbf{k}) = b_{10} = \text{constant}$
Then in real space : $b_1 = (S_\times/S_0)b_{10};$ $[S_\times = \int \mathrm{d}\ln k\, \Delta^2(k)W(kR)W(kR_0)]$
(AP & Sheth 12, arXiv:1105.2261)

Suppose in k-space: $b_1(\mathbf{k}) = b_{10} + (k^2 s/\sigma_1^2)b_{11};$ $[\sigma_1^2 = \int \mathrm{d}\ln k\, \Delta^2(k)k^2 W(kR)^2]$
Then in real space : $b_1 = (S_\times/S_0)[b_{10} + \epsilon_\times b_{11}]\,;$ $\epsilon_\times = 2\mathrm{d}\ln S_\times/\mathrm{d}\ln s$

General result
Linear Lagrangian bias

Suppose that $\delta_0(\mathbf{k}) = \delta(\mathbf{k})W(kR_0)$ and $\delta_{\rm h}(\mathbf{k}) = b_1(\mathbf{k})\delta(\mathbf{k})W(kR)$ and $W(y) = \mathrm{e}^{-y^2/2}$.
We'd like to calculate cross-correlation bias in real space: $b_1 = \langle\,\delta_{\rm h}\delta_0\,\rangle/\langle\,\delta_0^2\,\rangle$.

Suppose in k-space: $b_1(\mathbf{k}) = b_{10} =$ constant
Then in real space : $b_1 = (S_\times/S_0)b_{10}$; $[S_\times = \int \mathrm{d}\ln k\,\Delta^2(k)W(kR)W(kR_0)]$
(AP & Sheth 12, arXiv:1105.2261)

Suppose in k-space: $b_1(\mathbf{k}) = b_{10} + (k^2 s/\sigma_1^2)b_{11}$; $[\sigma_1^2 = \int \mathrm{d}\ln k\,\Delta^2(k)k^2W(kR)^2]$
Then in real space : $b_1 = (S_\times/S_0)[b_{10} + \epsilon_\times b_{11}]$; $\epsilon_\times = 2\mathrm{d}\ln S_\times/\mathrm{d}\ln s$

Excursion sets analysis naturally gives this form for the linear bias. (Musso, AP & Sheth 12)
As a corollary, so does ESP. (AP & Sheth 12)

[These arguments generalise in a very simple way to include scale dependence in all higher order b_n, but that's a separate talk!]

Linear Lagrangian bias
For ESP + square-root barrier with scatter...

... in full glory

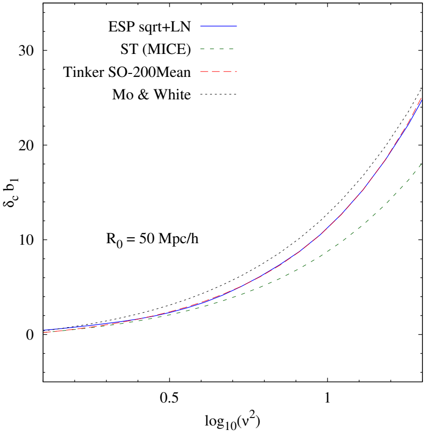
$$\delta_c b_1 = \left(\frac{S_\times}{S_0}\right) \frac{\int \mathrm{d}\beta\, p(\beta) \mathcal{B}_{1,\mathrm{ESP}}(\nu, \epsilon_\times|\beta)}{\int \mathrm{d}\beta\, p(\beta) f_{\mathrm{ESP}}(\nu|\beta)} \,,$$

where

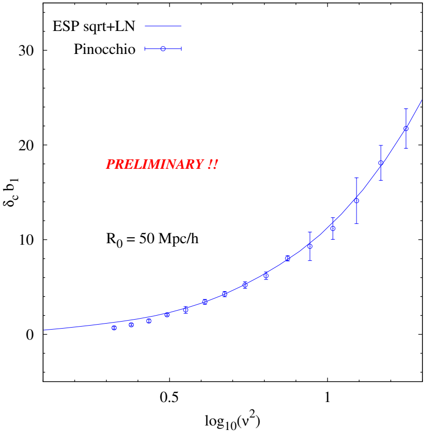
$$\begin{aligned}\mathcal{B}_{1,\mathrm{ESP}}(\nu, \epsilon_\times|\beta) \equiv \left(\frac{V}{V_*}\right) & \frac{\mathrm{e}^{-(\nu+\beta)^2/2}}{\sqrt{2\pi}} \\ & \times \frac{1}{\gamma\nu} \int_{x_{\min}}^{\infty} \mathrm{d}x\, (x - \beta\gamma) F(x) p_{\mathrm{G}}(x - \beta\gamma - \gamma\nu; 1 - \gamma^2) \\ & \qquad \times \left[\nu(\nu + \beta) - (1 - \epsilon_\times) \frac{\gamma\nu}{1 - \gamma^2} (x - \beta\gamma - \gamma\nu)\right] \,,\end{aligned}$$

Plan

Comparing to *N*-body fits



Comparing to Pinocchio



▸ Bias at $R_0 = 25h^{-1}$Mpc ▸ Mass function

Summary

- Excursion set ansatz can't be right because halos form in special places.
- Excursion set ansatz can be modified to only count walks centered on density peaks.
- Equivalently, peaks theory can be modified to account for "peak-in-peak" problem.
- Resulting mass function and bias (at large masses) agree well with results of N-body simulations.
- Similar ideas should apply to voids too.
- Would be nice to have a physical model for $p(\beta)$.

Summary

- Excursion set ansatz can't be right because halos form in special places.
- Excursion set ansatz can be modified to only count walks centered on density peaks.
- Equivalently, peaks theory can be modified to account for "peak-in-peak" problem.
- Resulting mass function and bias (at large masses) agree well with results of N-body simulations.
- Similar ideas should apply to voids too.
- Would be nice to have a physical model for $p(\beta)$.

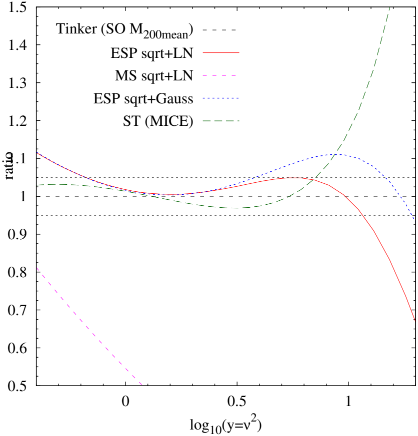
Thank you.

Peaks curvature function
BBKS eqn A15

$$F(x) = \frac{1}{2}\left(x^3 - 3x\right)\left\{\mathrm{erf}\left(x\sqrt{\frac{5}{2}}\right) + \mathrm{erf}\left(x\sqrt{\frac{5}{8}}\right)\right\}$$
$$+ \sqrt{\frac{2}{5\pi}}\left[\left(\frac{31x^2}{4} + \frac{8}{5}\right)\mathrm{e}^{-5x^2/8}\right.$$
$$\left. + \left(\frac{x^2}{2} - \frac{8}{5}\right)\mathrm{e}^{-5x^2/2}\right]$$

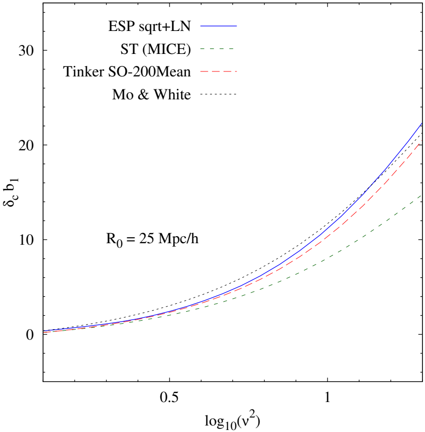
Back

Square root barrier with scatter
Comparing all-walks result



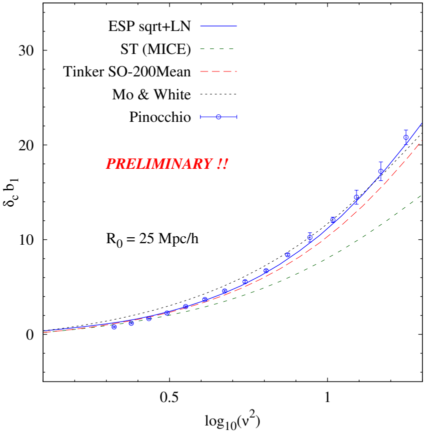
Back

Comparing to N-body fits and Pinocchio
Seeing k-dependence in real space



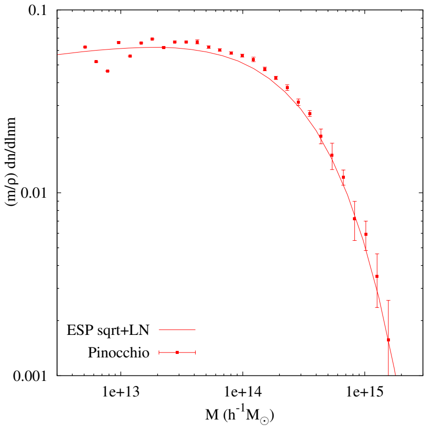
Back

Comparing to N-body fits and Pinocchio
Seeing k-dependence in real space



Back

Pinocchio mass function



Back