Non-linear structure formation in modified gravity models

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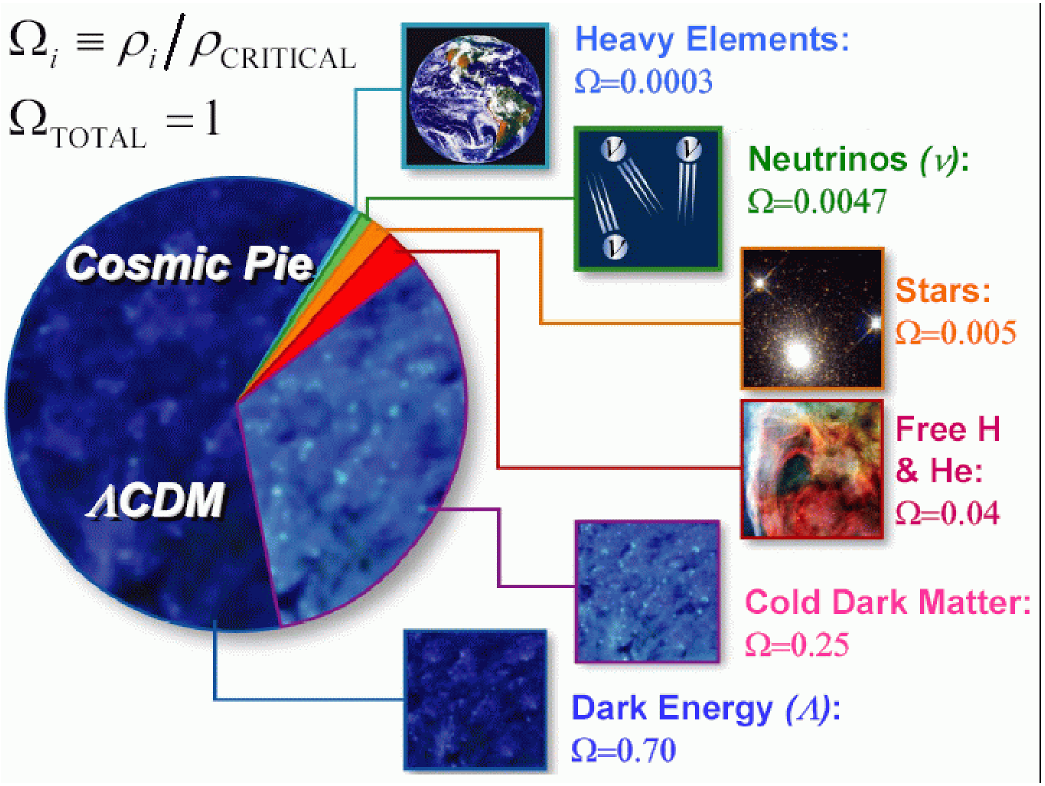
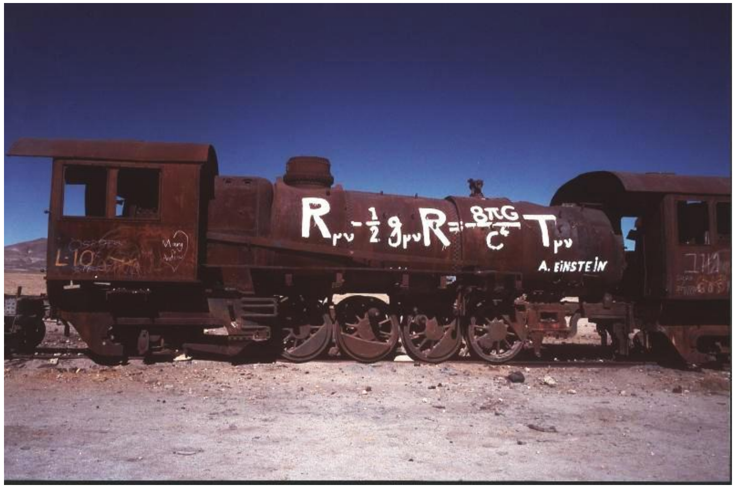
Dark energy v modified gravity





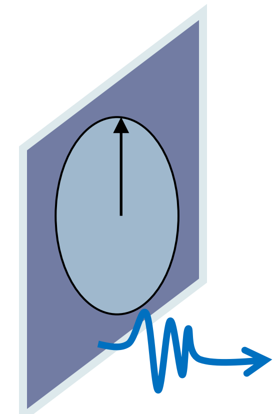
Is cosmology probing the breakdown of general relativity at large distance?

Examples

- Dvali-Gabada d model

  gravity leaks les and the Universe

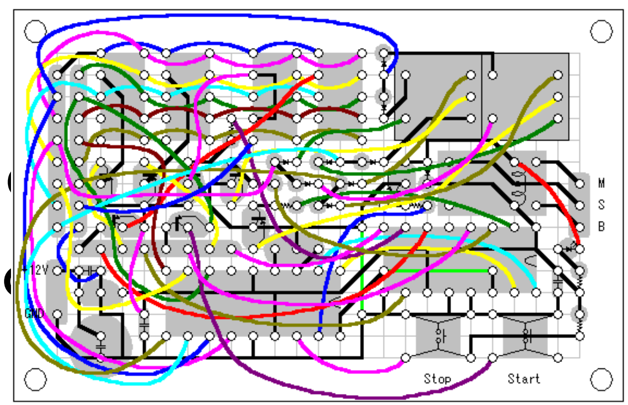
  self-accelera cal constant



- f(R) gravity

  there is no at low energies yet

  the expansi accelerate



It is extremely difficult to construct a consistent theory

General picture

- Largest scales

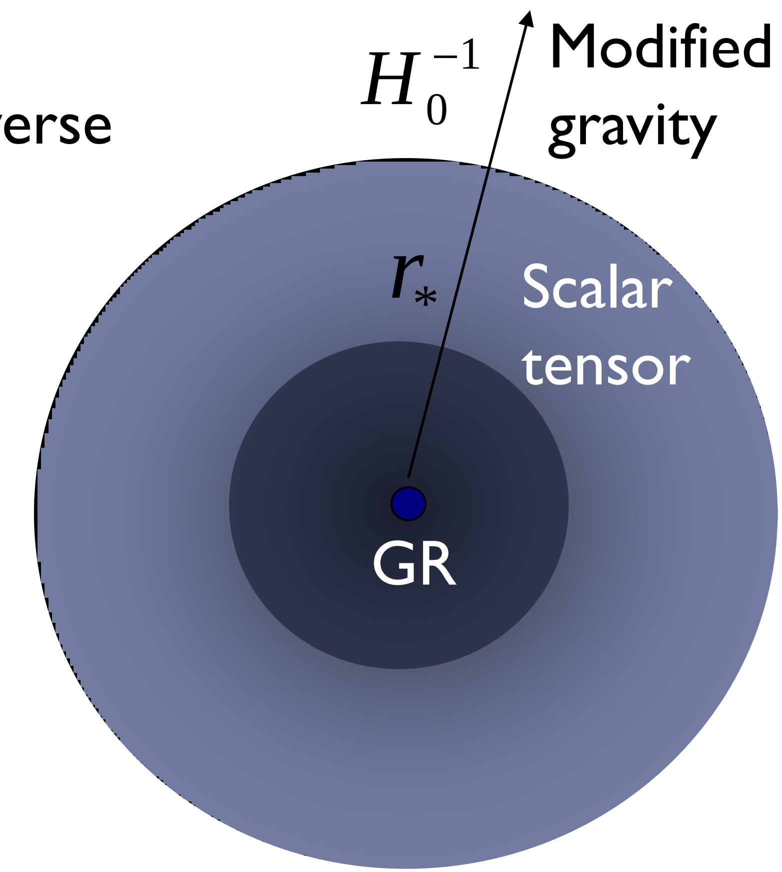
  gravity is modified so that the universe accelerates without dark energy

- Large scale structure scales

  gravity is still modified by a fifth force from scalar graviton

- Small scales (solar system)

  GR is recovered



From linear to non-linear scales

- Linear scales

  Model independent parametrisation of modified Einstein equations is possible (two functions of time and space)

  many ways to parametrise these functions directly or indirectly (i.e. parametrisation of the growth rate)

  Pogosian, Silverstri, KK, Zhao 1002.2383

  Principal component analysis provides model independent tests

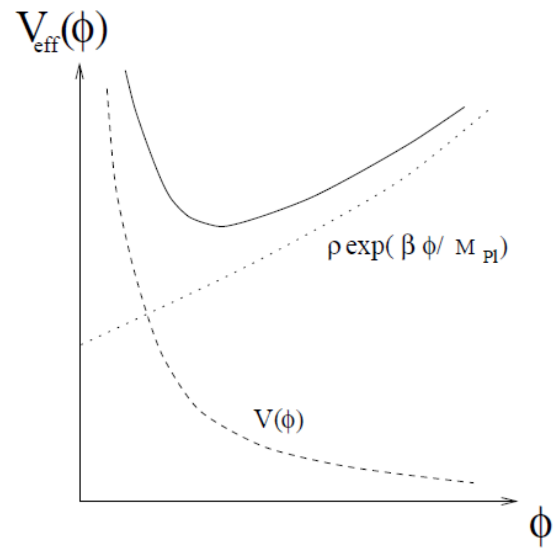
  Zhao et.al. 0908.1568, 1003.001, Hojjati et.al. 1111.3960

- Non-linear scales

  Mechanisms to recover GR on small scales are model dependent

How to recover GR on small scales?

On non-liner scales, the fifth force must be screened by some mechanisms



- Chameleon mechanism

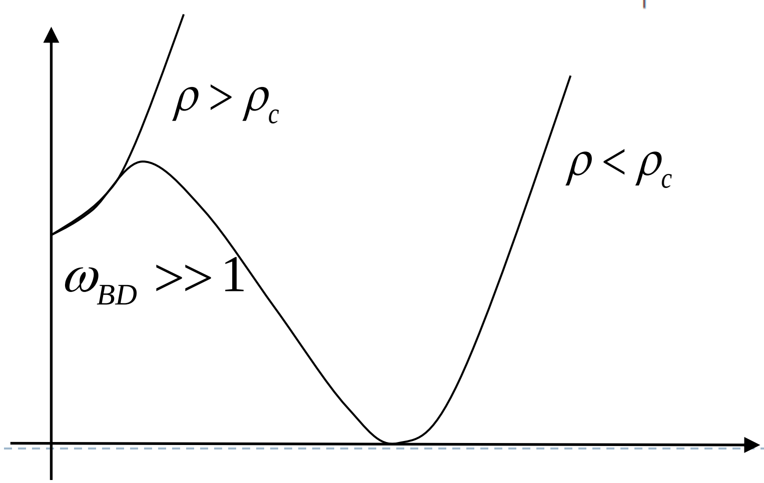
  Mass of the scalar mode becomes large in dense regions

- Symmetron mechanism

  The kinetic term becomes large in dense region

- Vainshtein mechanism

  Non-liner derivative self-interactions becomes large in a dense region

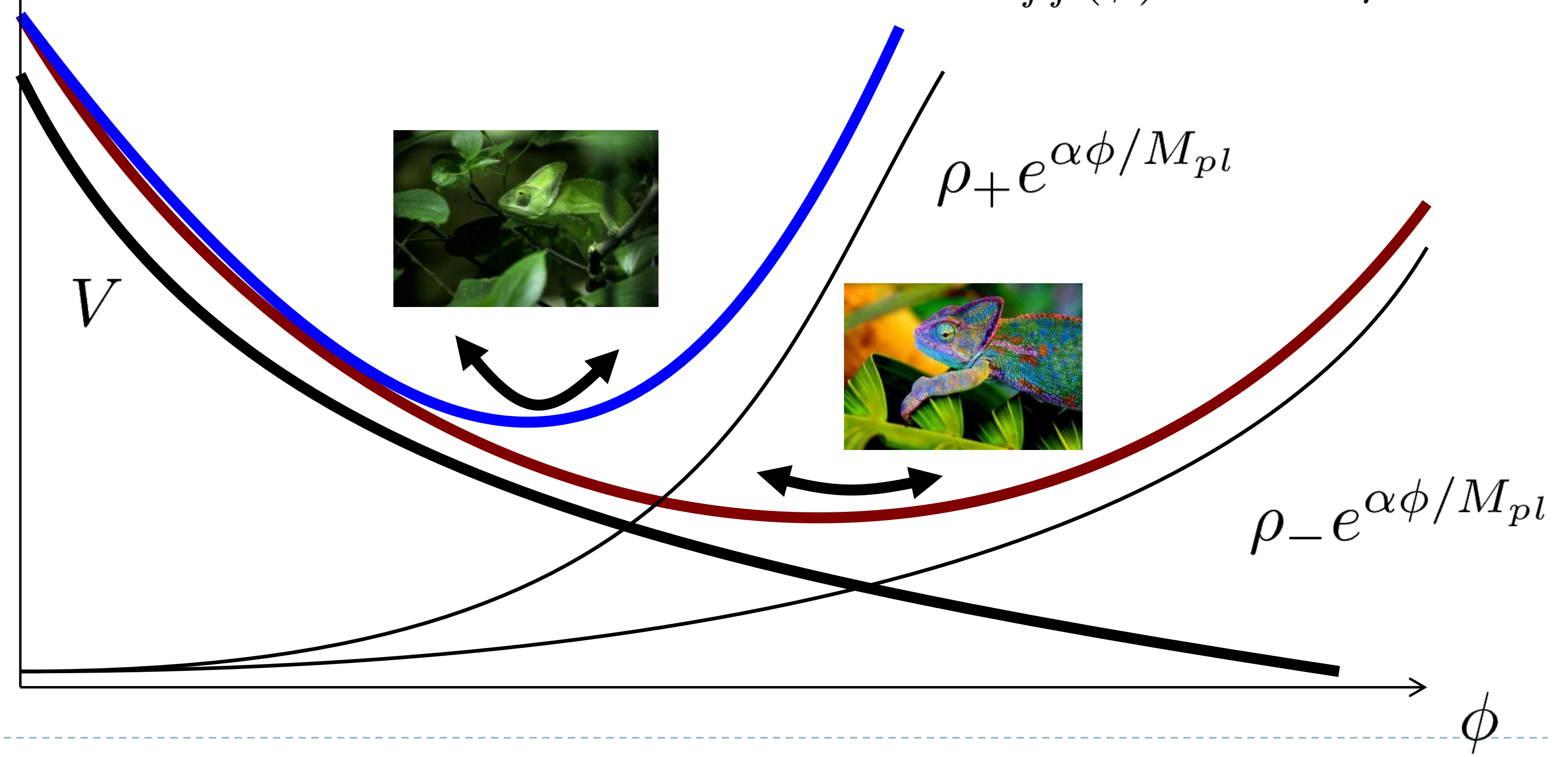


How we recover GR on small scales

- **Chameleon mechanism** (Khoury & Weltman)

$$\nabla^2\phi = \partial_\phi V + \frac{\alpha}{M_{pl}}\rho e^{\alpha\phi/M_{pl}}$$

$$V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$



Example – f(R) gravity

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R+f(R)}{16\pi G} + \mathcal{L}_{\mathrm{M}}\right]$$

$$\nabla^2\Phi = \frac{16\pi G}{3}a^2\delta\rho_{\mathrm{M}} + \frac{a^2}{6}\delta R(f_R) \qquad f_R \equiv \frac{\mathrm{d}f(R)}{\mathrm{d}R}$$

$$\nabla^2\delta f_R = -\frac{a^2}{3}[\delta R(f_R) + 8\pi G\delta\rho_{\mathrm{M}}],$$

Two limits

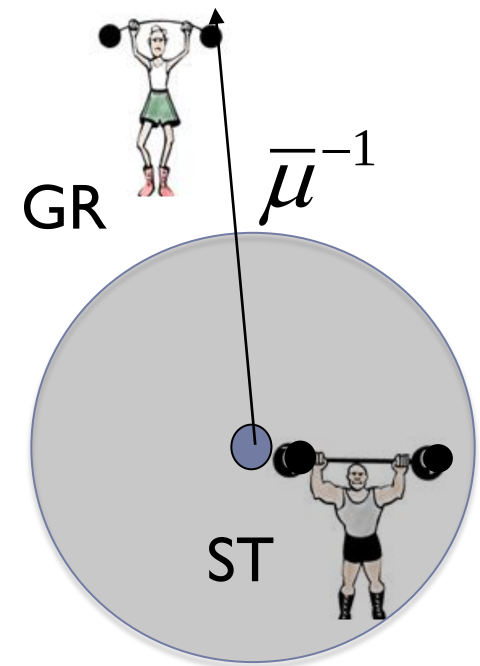
- GR $\delta f_R = 0$ $\delta R = -8\pi G\delta\rho_{\mathrm{M}} \Rightarrow \nabla^2\Phi = 4\pi Ga^2\delta\rho_{\mathrm{M}}.$
- Scalar-Tensor(ST) $\delta R = 0 \Rightarrow \nabla^2\Phi = \dfrac{16\pi G}{3}a^2\delta\rho_{\mathrm{M}}$

  The fifth force has a similar strength as gravity

Linear regime

- Linearise the equation

$$\nabla^2 \delta f_R = a^2 \bar{\mu}^2 \delta f_R - \frac{8\pi G}{3} a^2 \delta \rho_{\rm M}$$



The fifth force does not propagate beyond the Compton wavelength $\bar{\mu}^{-1}$ (GR limit)

$$\nabla^2 \Phi = 4\pi G a^2 \delta \rho_{\rm M}$$

Below the Compton wavelength, gravity is enhanced (ST limit)

$$\nabla^2 \Phi = \frac{16\pi G}{3} a^2 \delta \rho_{\rm M}$$

Chameleon mechanism

- Fifth force is strongly constrained at solar system

  the post-Newtonian parameter is $\gamma = 1/2$ not $\gamma = 1$

- Chameleon mechanism

  the mass of the scalar mode becomes heavy

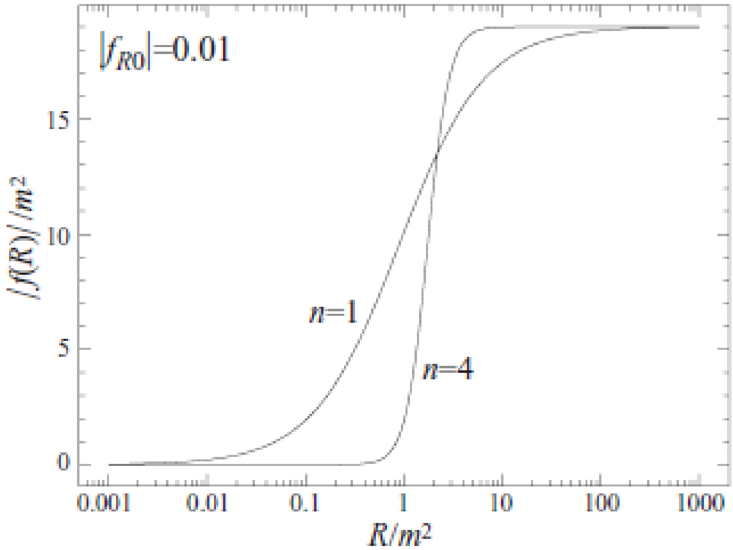
  in a dense environment

$$\nabla^2 \delta f_R = -\frac{a^2}{3}\left[\delta R(f_R) + 8\pi G \delta\rho_{\rm M}\right]$$

Engineering f(R) gravity model

$$f(R) = -m^2 \frac{c_1(-R/m^2)^n}{c_2(-R/m^2)^n + 1}$$

Hu & Sawicki



Non-linear regime

- Chameleon mechanism

$$f_R = -\frac{c_1}{c_2^2}\frac{n(-R/m^2)^{n-1}}{[(-R/m^2)^n+1]^2} \approx -\frac{nc_1}{c_2^2}\left(\frac{m^2}{-R}\right)^{n+1} \qquad -\bar{R} \approx 41m^2$$

Present day Ricci curvature of the Universe today

$$\bar{\rho} \approx 10^{-30} g/cm^3, \quad \rho_{solar} \approx 10 g/cm^3 \Rightarrow R_{solar} >> \bar{R}, \quad f_R \to 0$$

In a dense region, linearisation fails and GR is recovered

It is required to solve a non-linear Klein-Gordon equation of the scalar field self-consistently

$$\nabla^2 \delta f_R = -\frac{a^2}{3}[\delta R(f_R) + 8\pi G \delta\rho_{\rm M}]$$

$$\delta f_R = f_R(R) - f_R(\bar{R}), \delta R = R - \bar{R}$$

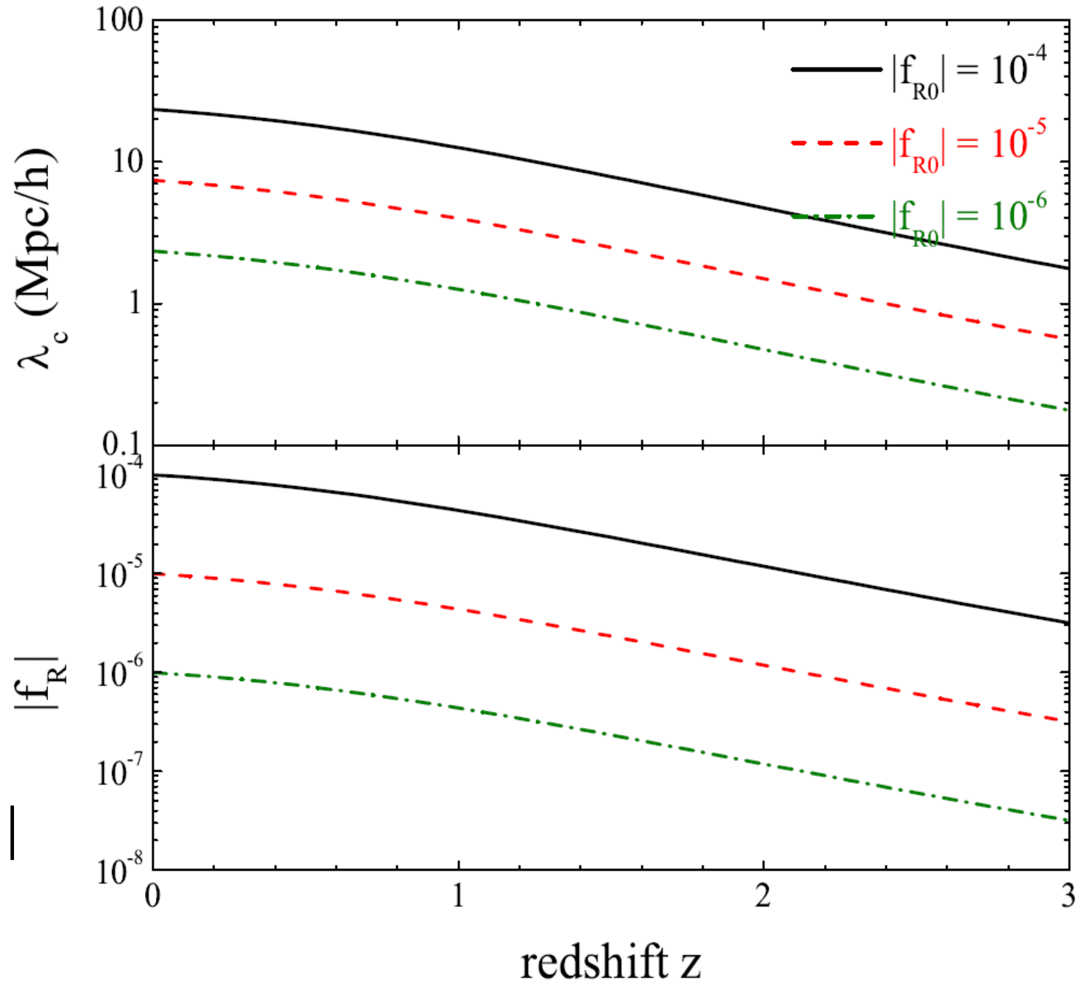
Parameter $|f_{R0}|$ $\quad f_R \equiv \frac{\mathrm{d}f(R)}{\mathrm{d}R}$

- ## Compton wavelength

For a larger $|f_{R0}|$,

the Compton wavelength

is longer



- ## Chameleon mechanism

The Chameleon works

when $|\bar{f}_R| \ll \delta f_R$ and the

linearisation fails

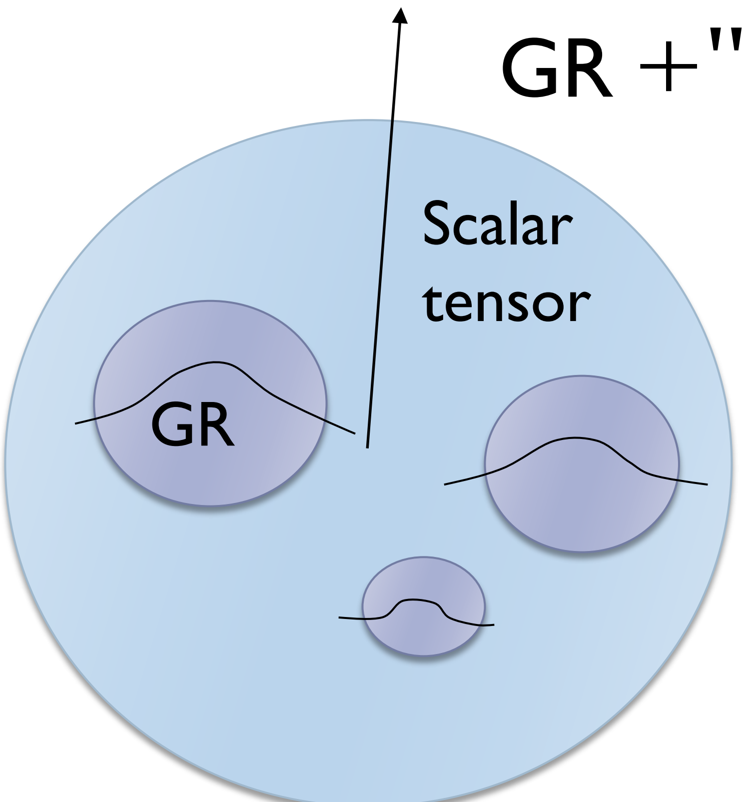
It works better for smaller $|f_{R0}|$

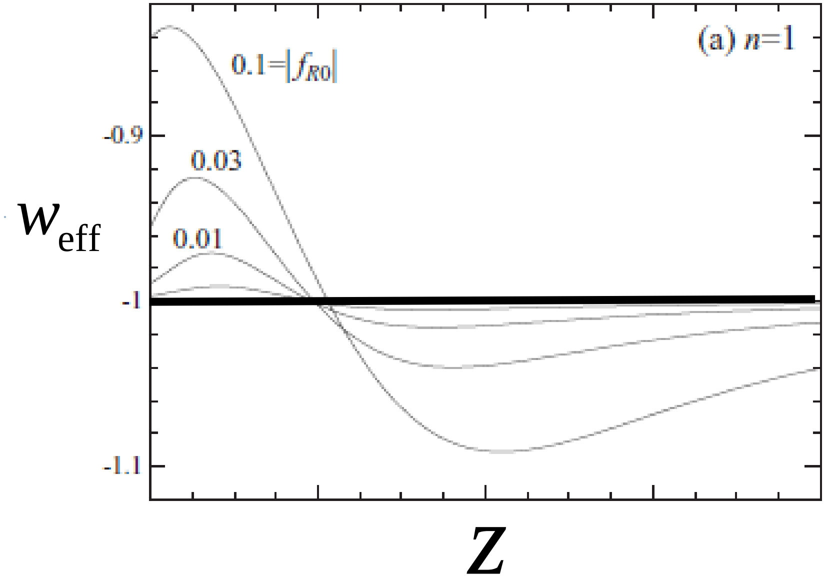
and earlier times

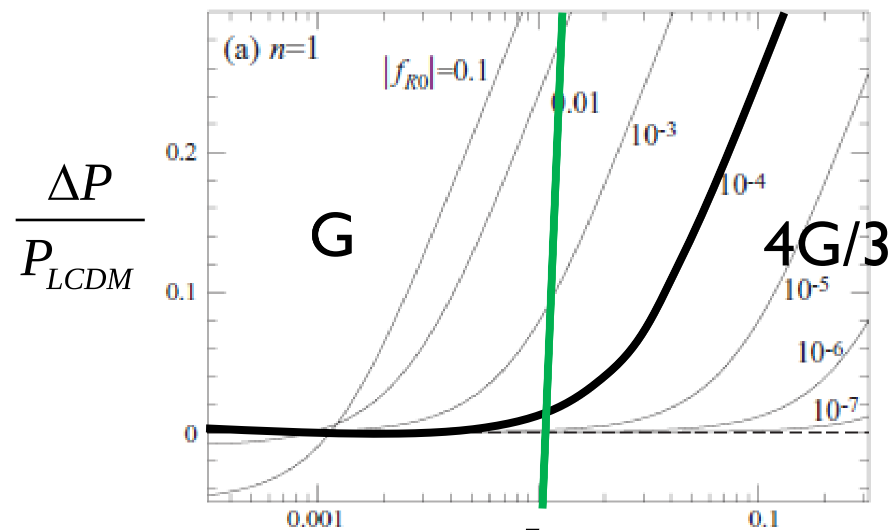
Behaviour of gravity

There regimes of gravity

GR + "Λ"







$$\nabla^2 \delta f_R = -\frac{a^2}{3}[\delta R(f_R) + 8\pi G \delta \rho_{\mathrm{M}}]$$

Understandings of non-linear clustering require N-body simulations

Models $n = 1, |f_{R0}| = 10^{-4}, 10^{-5}, 10^{-6}$

- Full f(R) simulations

  solve the non-linear scalar equation

$$\nabla^2 \delta f_R = -\frac{a^2}{3}[\delta R(f_R) + 8\pi G \delta \rho_{\mathrm{M}}]$$

- Non-Chameleon simulations

  artificially suppress the Chameleon by linearising the scalar equation to remove the Chameleon effect

$$\nabla^2 \delta f_R = a^2 \bar{\mu}^2 \delta f_R - \frac{8\pi G}{3} a^2 \delta \rho_{\mathrm{M}}$$

- LCDM

Oyaizu et.al. PRD78 123524 2008, Schmidt et.al. PRD79 083518 2009

N-body Simulations

- **MLAPM code** Li, Zhao 0906.3880, Li, Barrow 1005.4231
  Zhao, Li, Koyama 1011.1257

	Box size (Mpc/h) 64	128	256
N_{sim}	10	10	10
N_{p}	256^3	256^3	256^3
N_{grid}	128	128	128
$k_{N/2}$(h/Mpc)	0.79	1.57	3.14
k_*(h/Mpc)	5.5	11.0	22.0
Refinement levels	10	9	8
Force resolution (Kpc/h)	12	23	94
Mass resolution ($10^{11} M_\odot$/h)	13.3	1.75	0.21

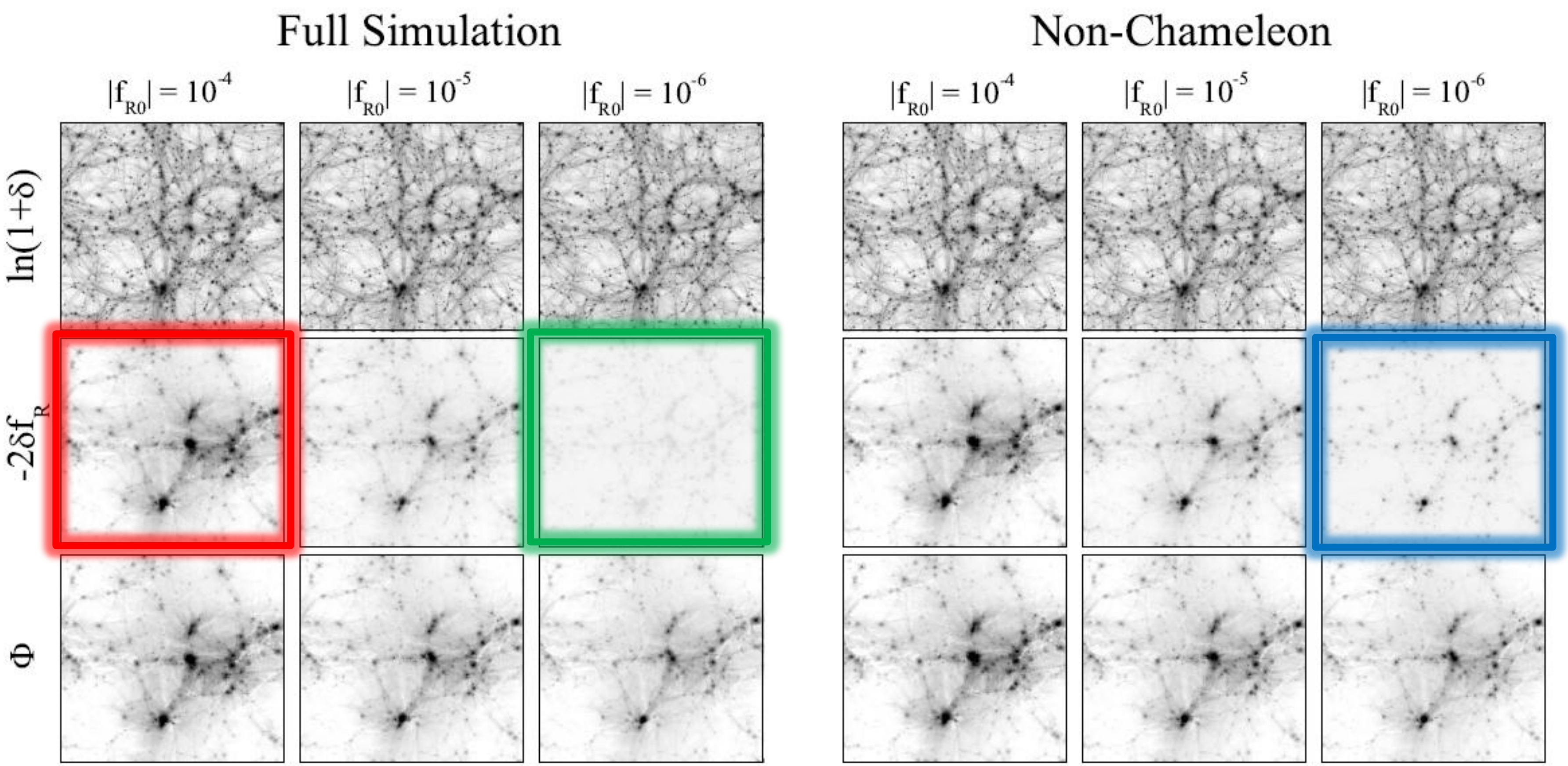
- **ECOSMOG code** (based on RAMSES) Li, Zhao, Teyssier, Koyama 1110.1379
  Braxet.al. 1206.3568

models	L_{box}	no. of particles	k_{Nyq} [h/Mpc]	force resolution [h^{-1} kpc]	convergence criterion	realisations
ΛCDM, F6, F5, F4	$1.5h^{-1}$Gpc	1024^3	2.14	22.9	$\lvert\epsilon\rvert < 10^{-12}/10^{-8}$	6
ΛCDM, F6, F5, F4	$1.0h^{-1}$Gpc	1024^3	3.21	15.26	$\lvert\epsilon\rvert < 10^{-12}/10^{-8}$	1
ΛCDM, F6, F5, F4	$500h^{-1}$Mpc	512^3	3.21	30.52	$\lvert\epsilon\rvert < 10^{-12}/10^{-8}$	1
ΛCDM, F6, F5, F4	$250h^{-1}$Mpc	512^3	6.43	7.63	$\lvert\epsilon\rvert < 10^{-12}/10^{-8}$	1

Snapshots at z=0

Zhao, Li, Koyama 1011.1257

- If the fifth force is not suppressed, we have $-2\delta f_R = \Phi.$



Fifth force is not suppressed

Chameleon is working

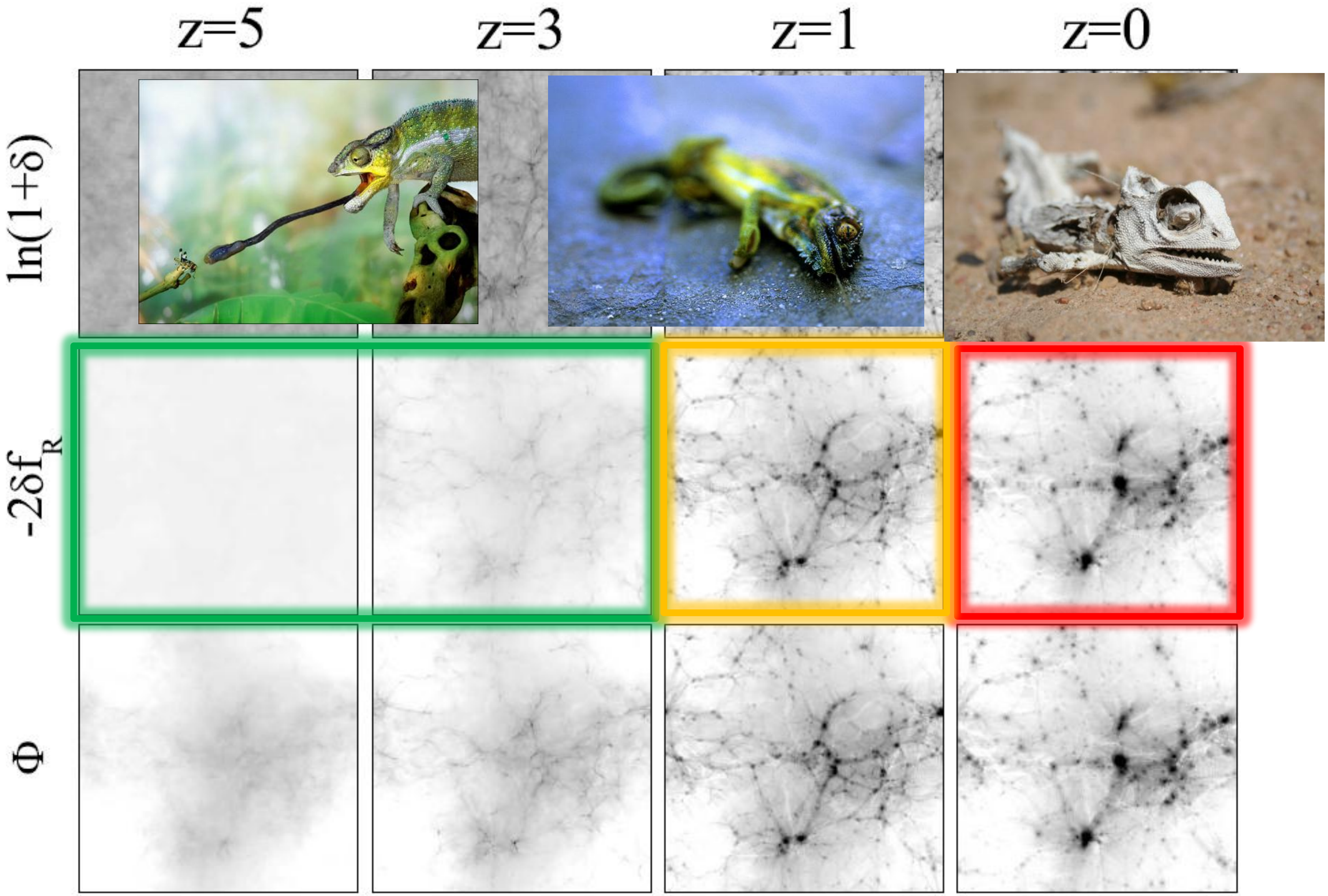
Compton wavelength is short

Snapshots

Chameleon is working

Chameleon starts to hibernate

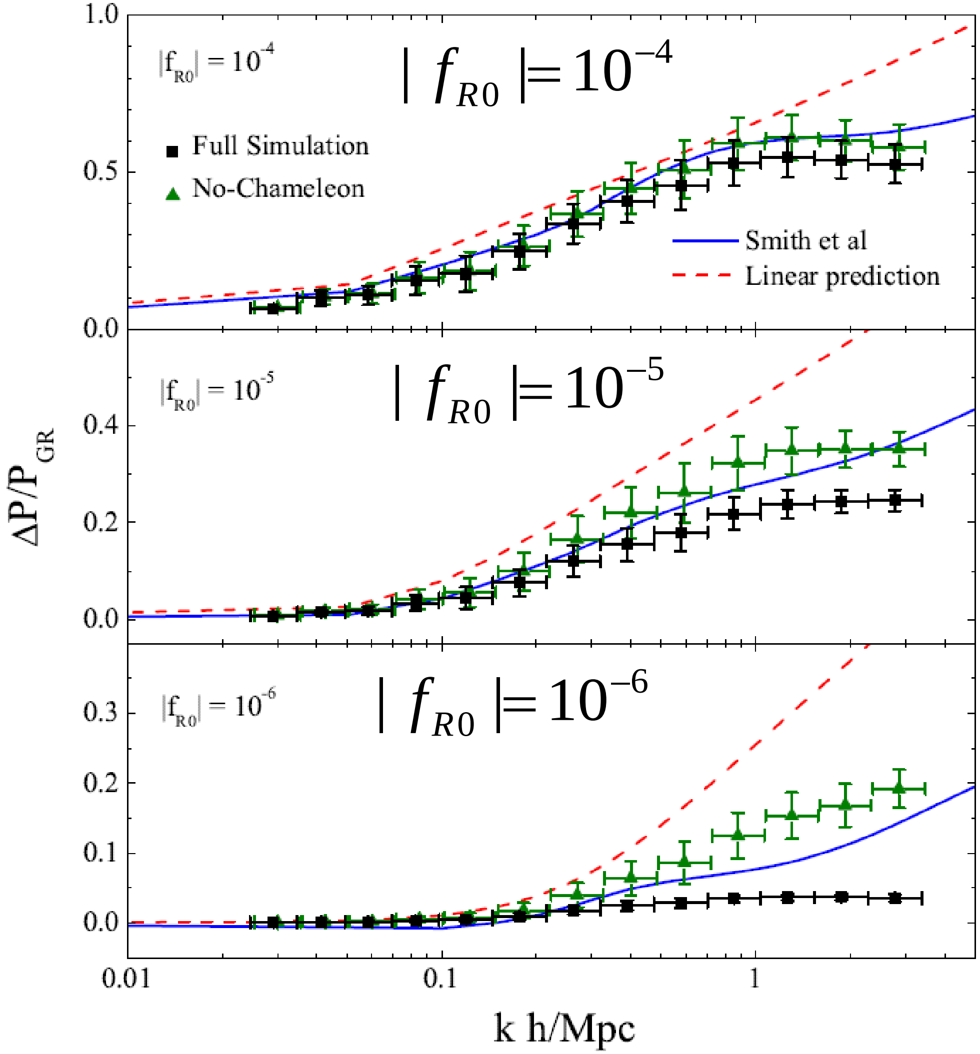
Chameleon stops working



$|f_{R0}| = 10^{-4}$

Power spectrum (z=0)

Zhao, Li, Koyama 1011.1257



On large scales, simulations agree with linear predictions

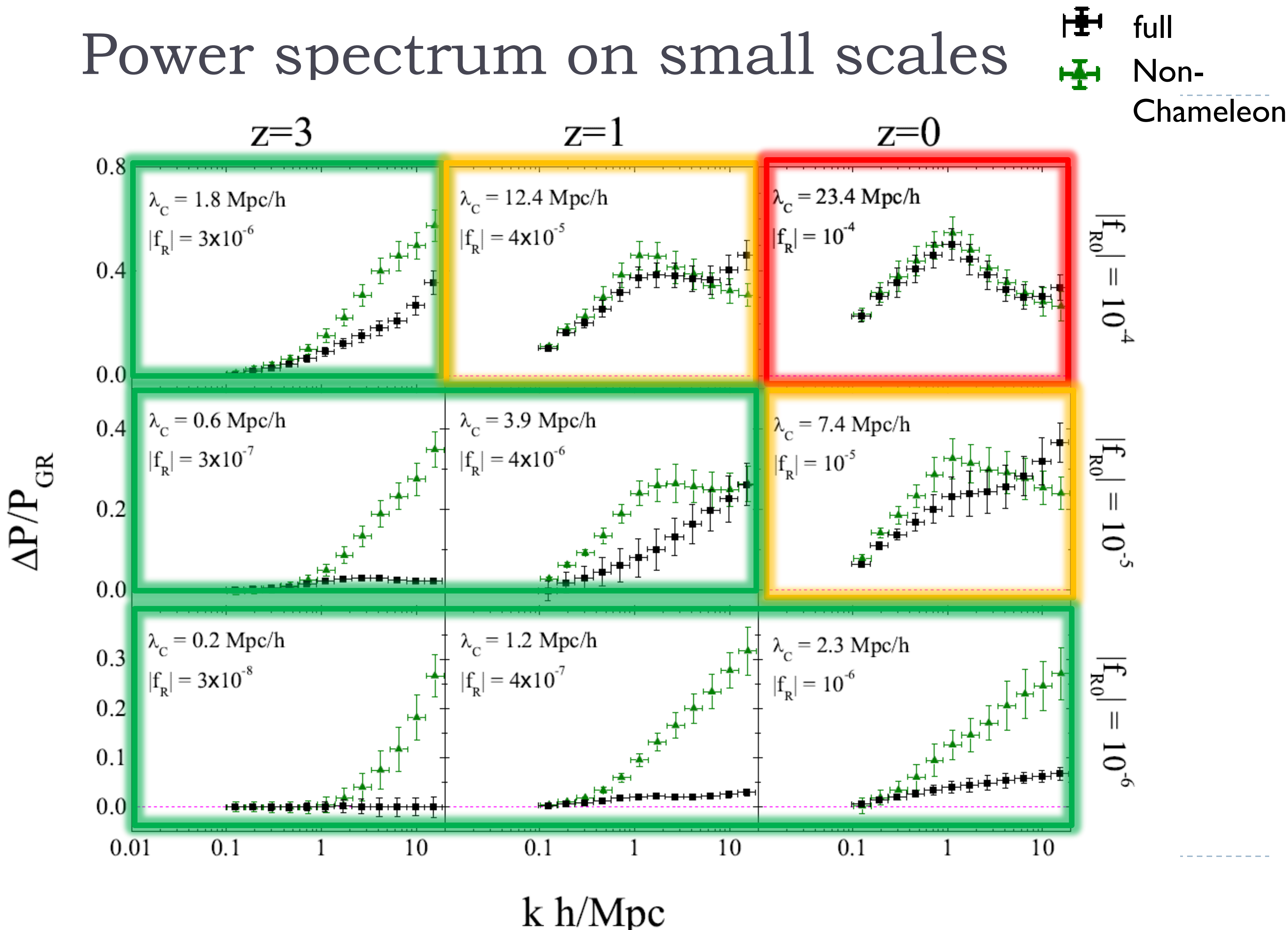
A naïve use of Halofit overestimets the power on smaller scales (fully consistent with previous simulations)

full

Non-Chameleon

Oyaizu et.al. PRD78 123524 2008, Schmidt et.al. PRD79 083518 2009

Power spectrum on small scales



Power spectrum

- Chameleon starts to fail when $|\bar{f}_R| \square 10^{-5}$

  - At early times, the background field is small and the Chameleon is working Deviations from the GR power spectrum are strongly suppressed

  - Once the background field becomes large , the Chameleon starts to fail

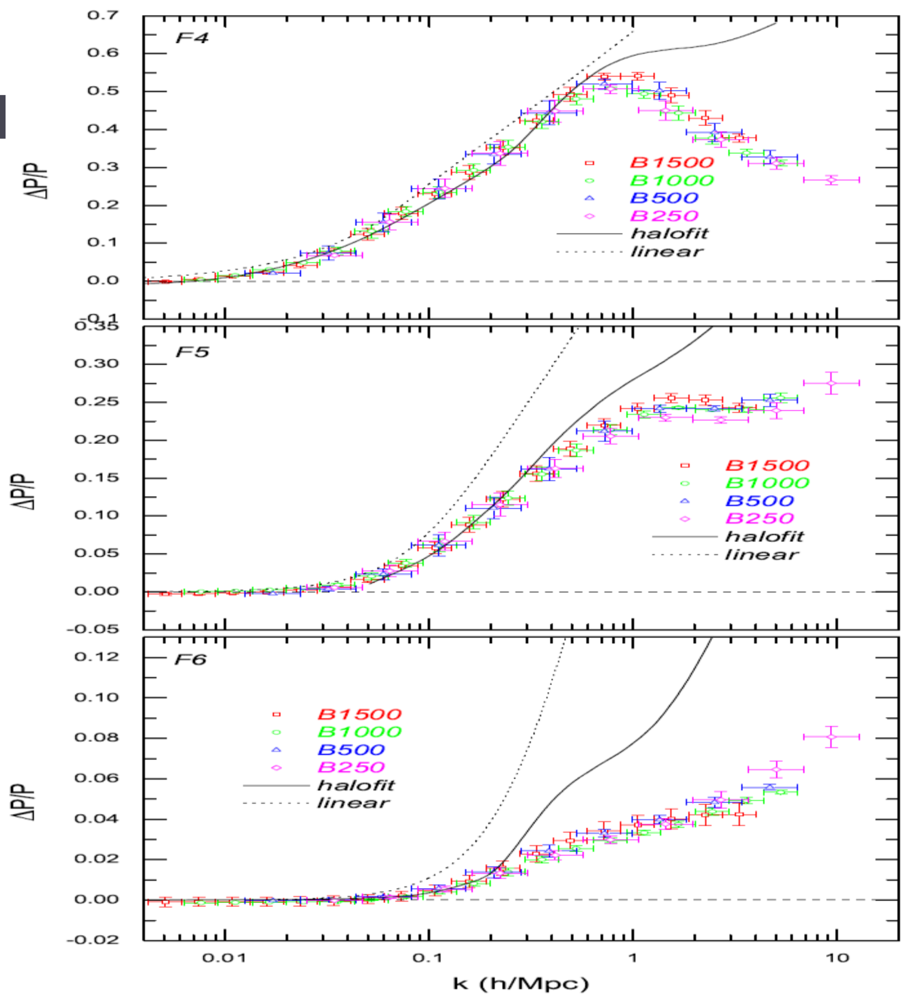
$$|\bar{f}_R| > \delta f_R \leq \Phi \square 10^{-5}$$

  - After some time, the power spectrum approaches that in non-Chameleon simulations

A naïve use of halofit gives wrong results for large k

New simulations

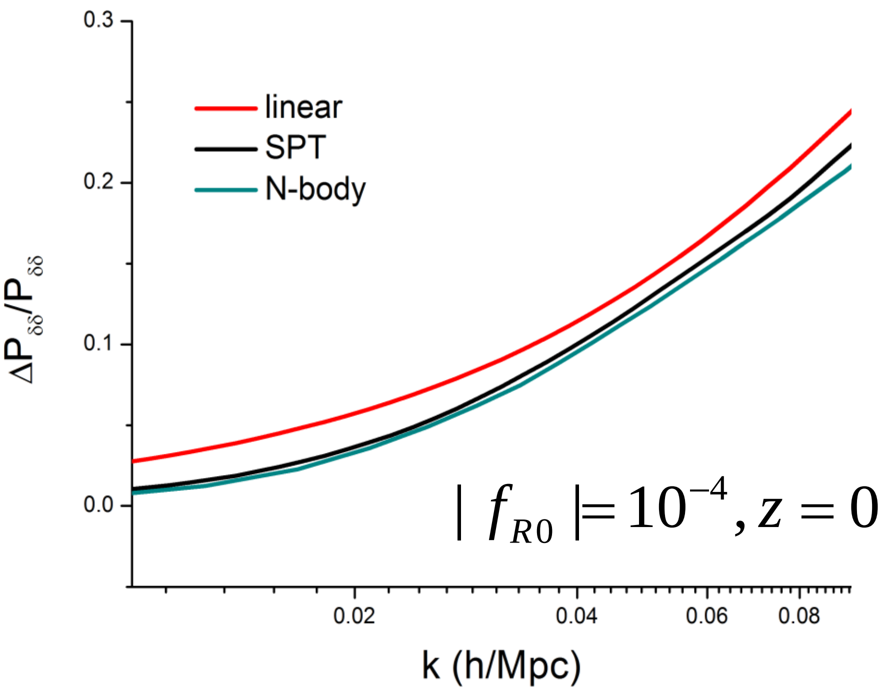
- ECOSMOG code
  - Based on a fully parallelised code RAMSES
  - This enabled us to run large box size simulations

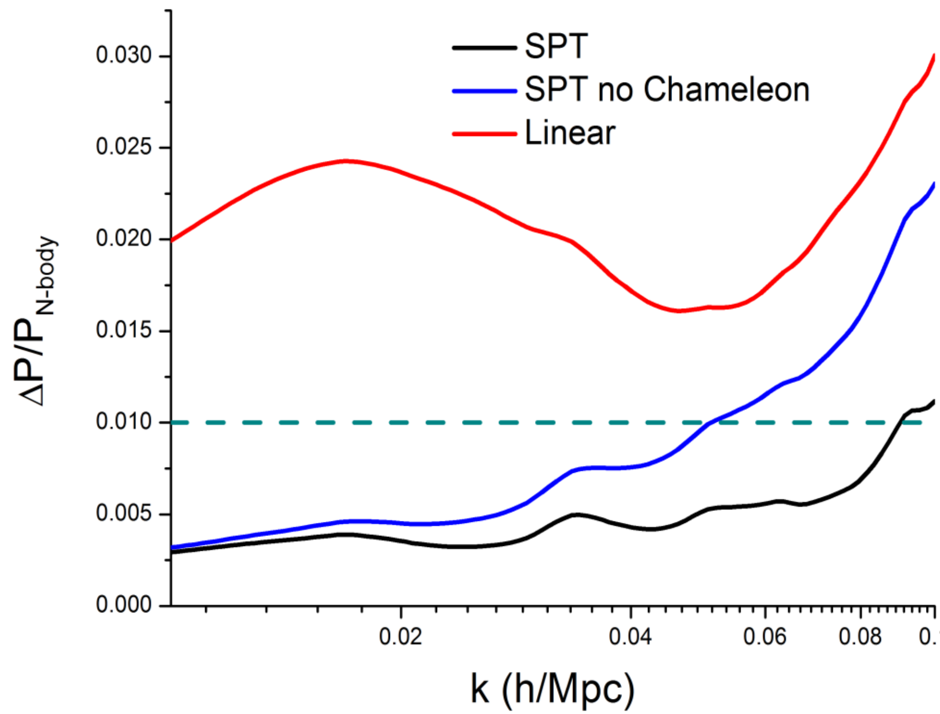


Li, Hellwing, KK, Zhao, Jennings, Baugh 1206.4317

Quasi non-linear scales

- Standard perturbation theory predictions (KK, Taruya, Hiramatsu 0902.0618)





Even in F4, inclusion of Chameleon effects is important below k<0.1 h/Mpc

SPT agrees with N-body results at 1% level at k<0.09 h/Mpc (z=0)

- Bernardeau, Brax 1102.1907, Brax, Valageas 1205.6583

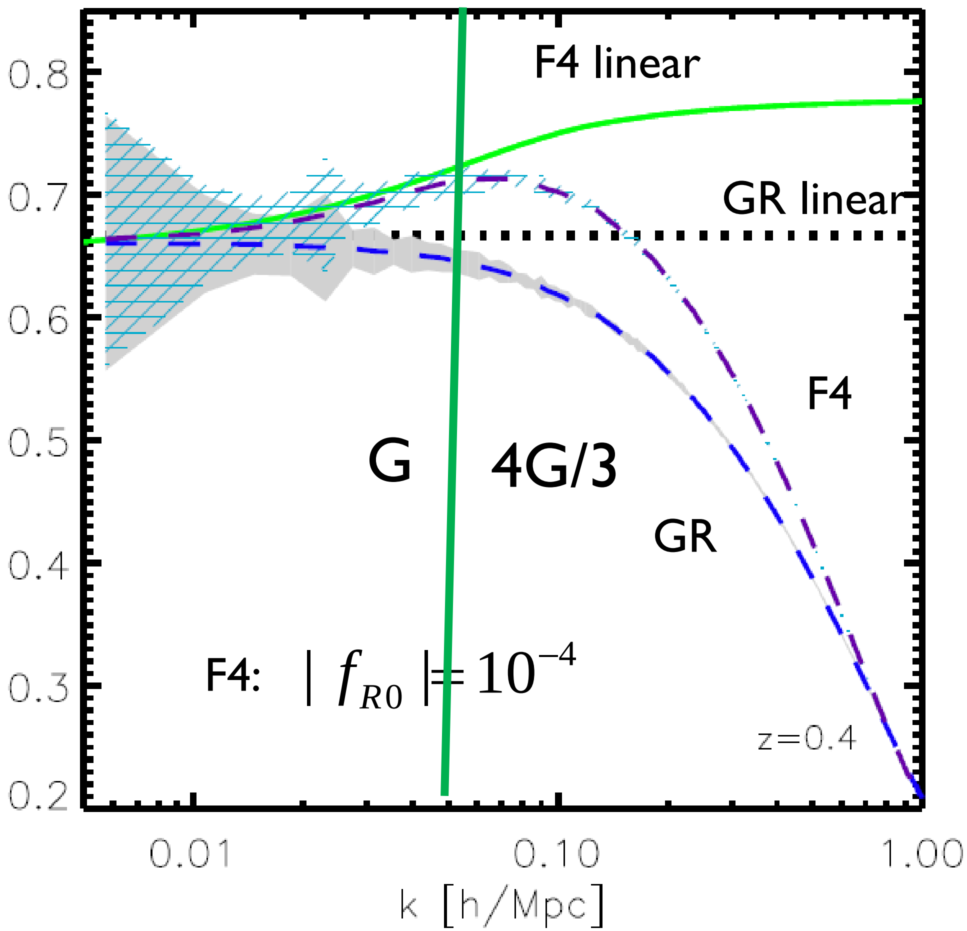
Growth rate

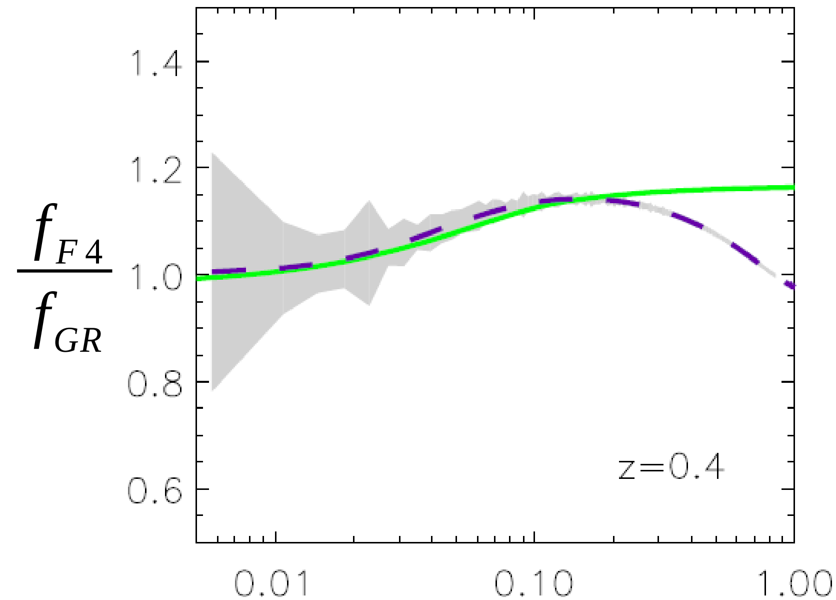
Jennings, Baugh, Li, Zhao, Koyama, 1205.2698

- Growth rate

  on linear scales it is defined as

$$f = \sqrt{\frac{P_{\theta\theta}}{P_{\delta\delta}}}, \quad \theta = \nabla \cdot v$$





Stronger gravity enhances linear growth rate as well as non-linear damping

Jennings, Baugh, Li, Zhao, Koyama, 1205.2698

Redshift space distortions

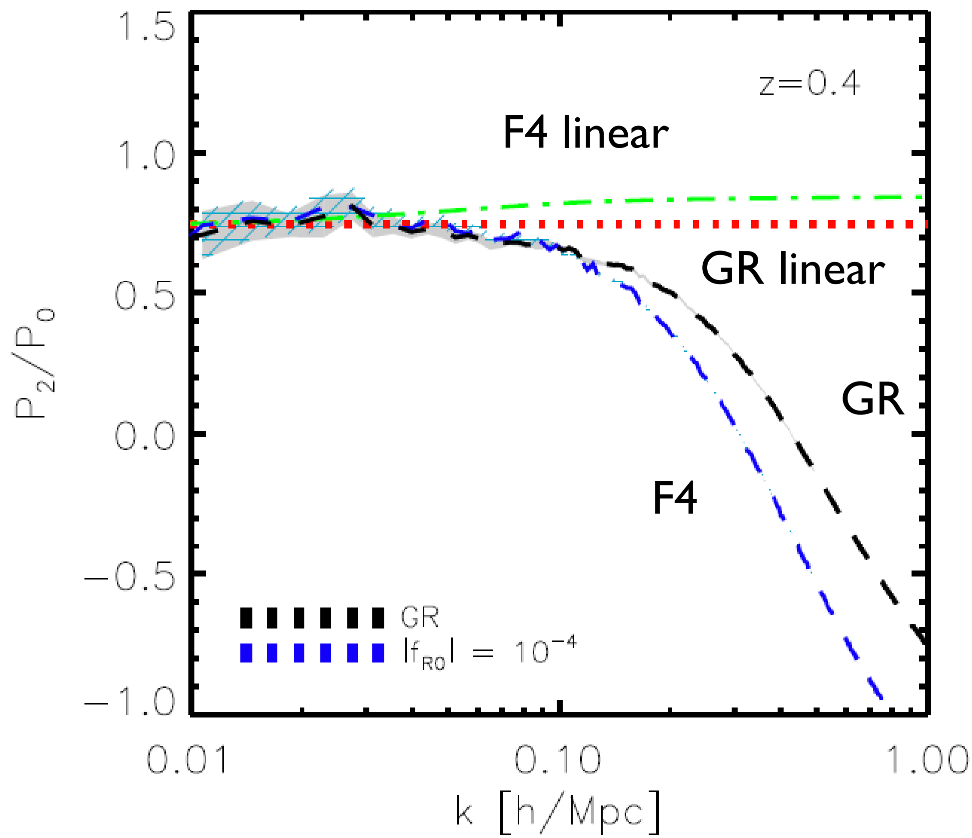
- Power spectrum in redshift space become anisotropic

$$P(k,\mu), \quad \mu = k_{\parallel} / k$$

- Multipole decomposition

$$P(k,\mu) = \sum_{\ell} P_{\ell}(k) L_{\ell}(\mu)$$

$$\left.\frac{P_2}{P_0}\right|_{linear} = \frac{\frac{4}{3} f + \frac{4}{7} f^2}{1 + \frac{2}{3} f + \frac{1}{5} f^2}$$



Modelling of non-linear effects is crucial to extract the differences in the linear growth rate between GR and f(R) gravity models

- Taruya, Nishimichi, Saito 1006.0699, Nishimichi, Taruya 1106.4562

Halos

Zhao, Li, Koyama 1011.1257

- MHF (default halo identifier of MLAPM)

  Use TSC interpolation to assign particles to grids and identify halos using the spherical over density method

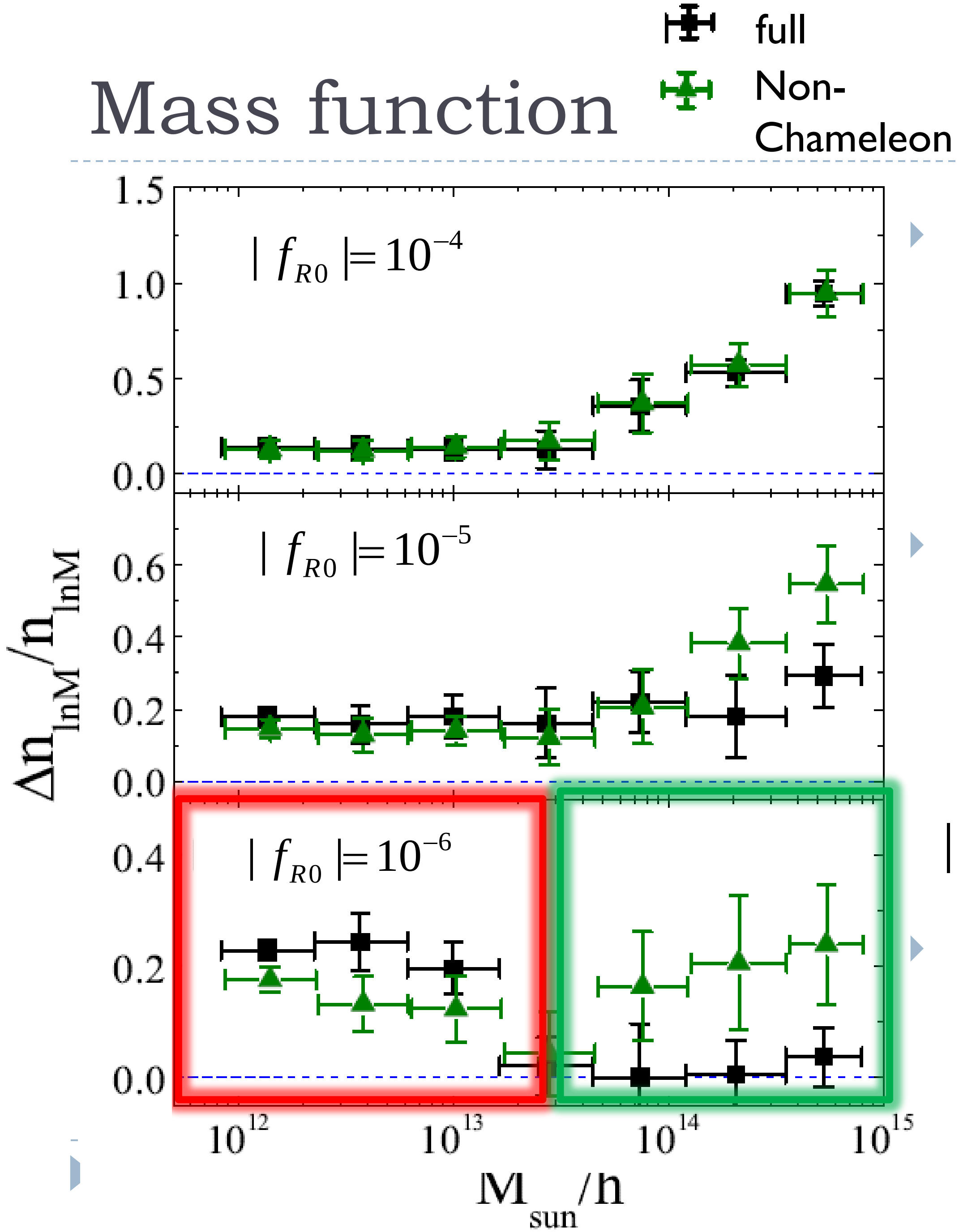
- Spherical over-density

  We use the virial over-density in LCDM

  $\Delta_{vir} = 373.76$ at z=0 and $\Delta_{vir} = 242.71$ at z=1

- Minimum number of particles in halos is 800

Mass function



- If Chameleon is not working, strong gravity creates more and more heavy halos and the abundance of massive halos is enhanced
- Cluster abundance gives the tightest constraint so far

$$|f_{R0}| < 1.65 \times 10^{-4}$$

$$|f_{R0}| < 10^{-5}$$

- Chameleon works better for heavier halos and it suppresses the abundance of large halos

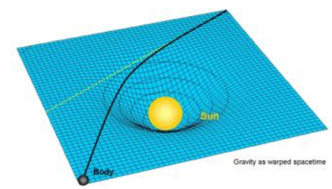
Environmental dependence

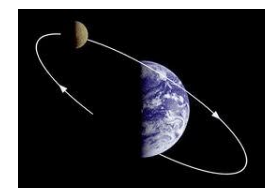
Zhao, Li, Koyama 1105.0922

- In modified gravity models, dynamical mass inferred from velocity dispersions and lensing mass can be different

$$k^2(\Phi+\Psi)/2 = 4\pi G a^2 \Sigma(k,a)\rho_m \Delta_m$$

$$k^2\Phi = 4\pi G a^2 \mu(k,a)\rho_m \Delta_m$$





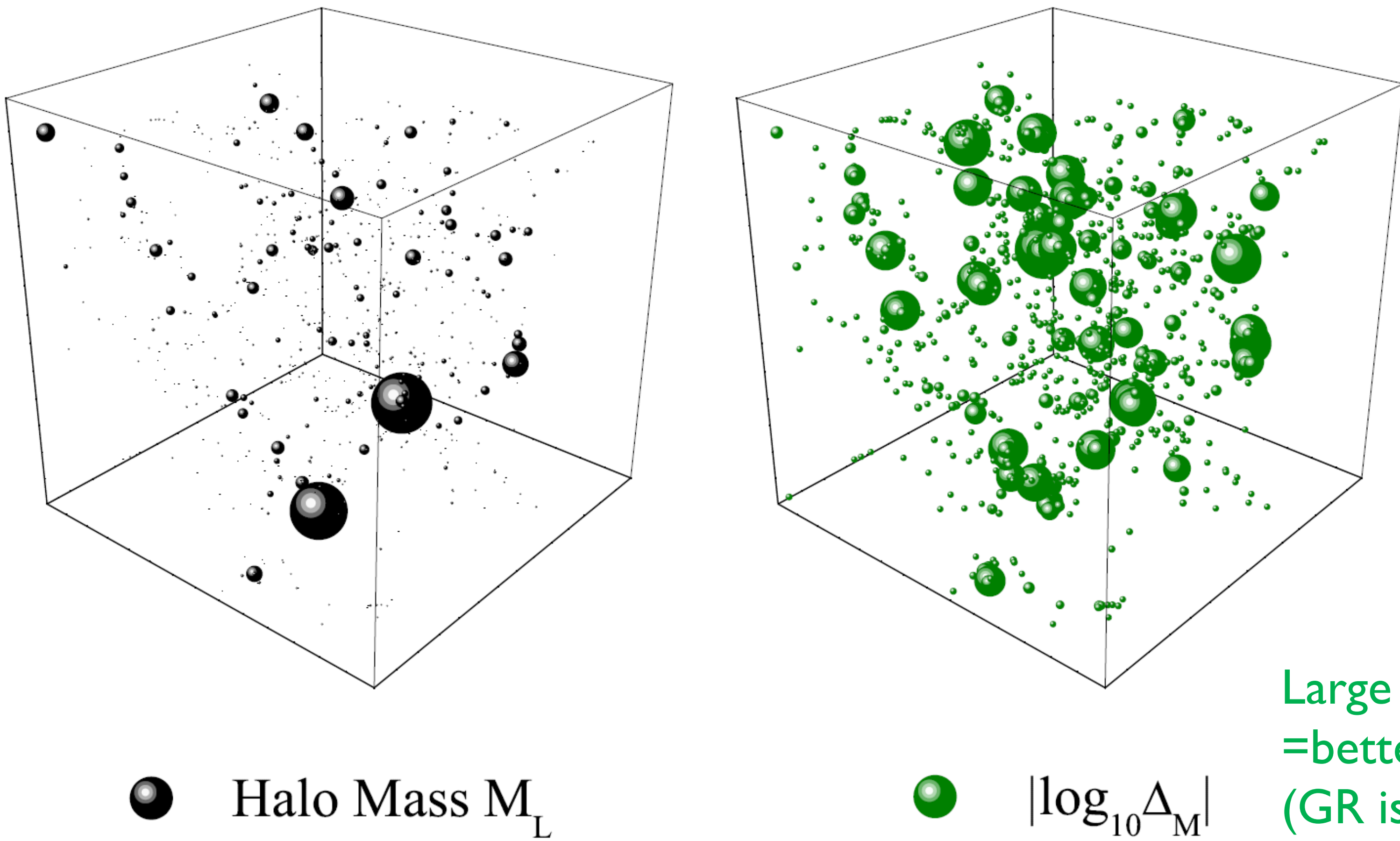
- f(R) $\Sigma \simeq 1$ The fifth force does not change geodesics of photon

  $\mu = [1:4/3]$ The fifth force enhances Newtonian gravity below the Compton wavelength

- Difference between dynamical and lensing masses

$$\Delta_M(r) = \frac{d\Phi(r)/dr}{d\Phi_+(r)/dr} - 1, \quad \Phi_+ \equiv (\Phi+\Psi)/2 \qquad \Delta_M = [0:1/3]$$

- Difference in lensing and dynamical masses small for massive halos that are better screened

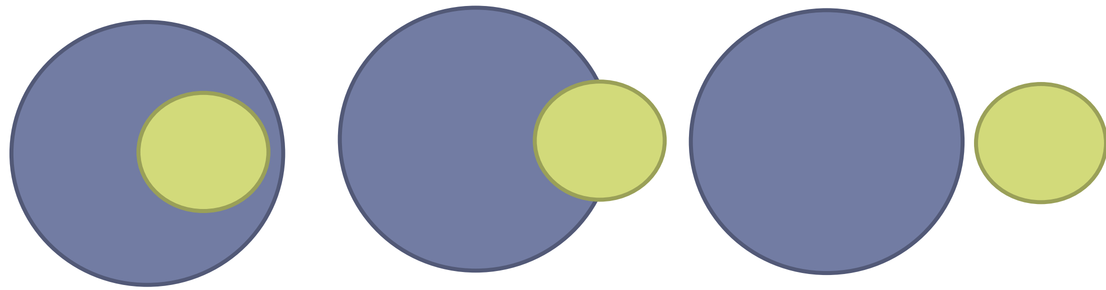


Large bubbles
=better screened
(GR is recovered)

There is another variable that determines the screeening of halos

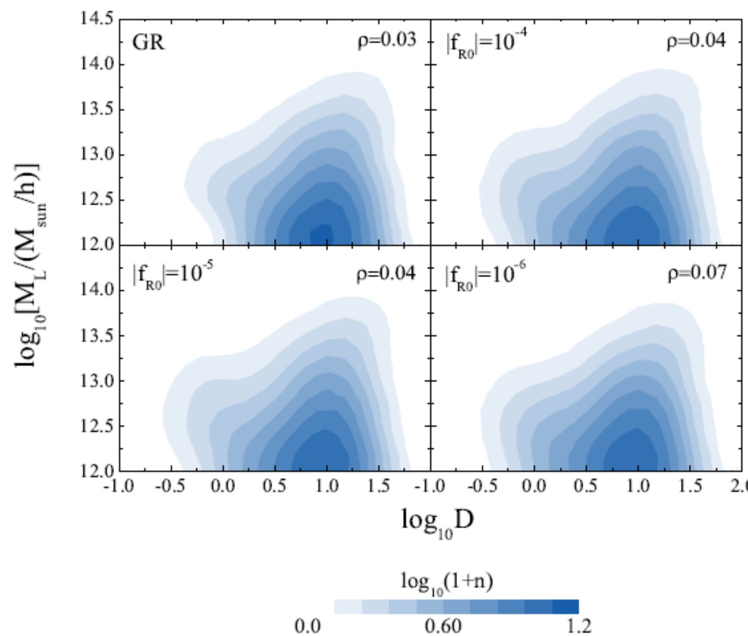
- Small halos nearby big halos are well screened

$$D = \frac{d}{r_{NB}}, \quad M_{NB} > M$$

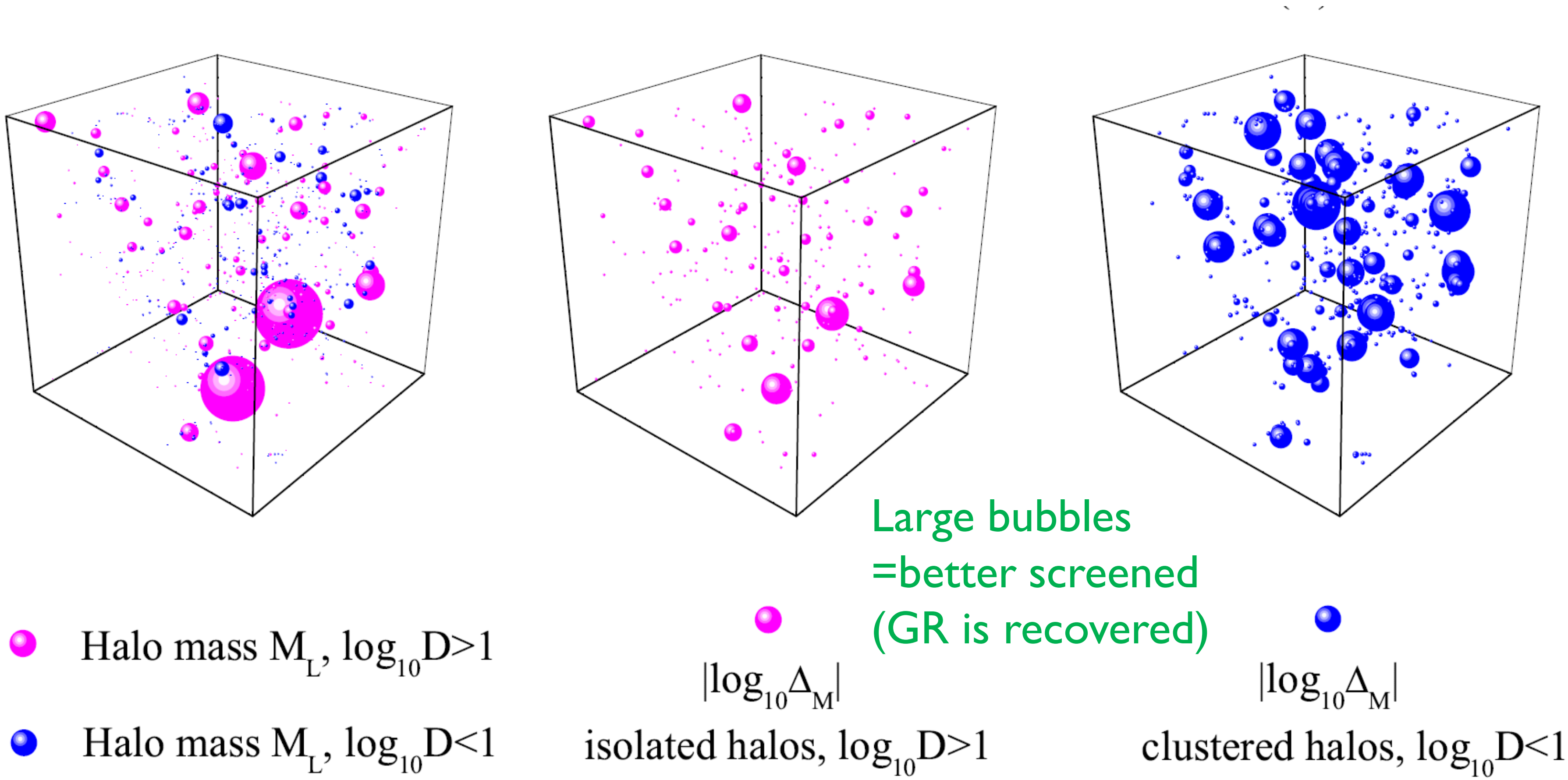


$D<1$ $\quad D=1$ $\quad D>1$

- D is almost uncorrelated with the halo mass

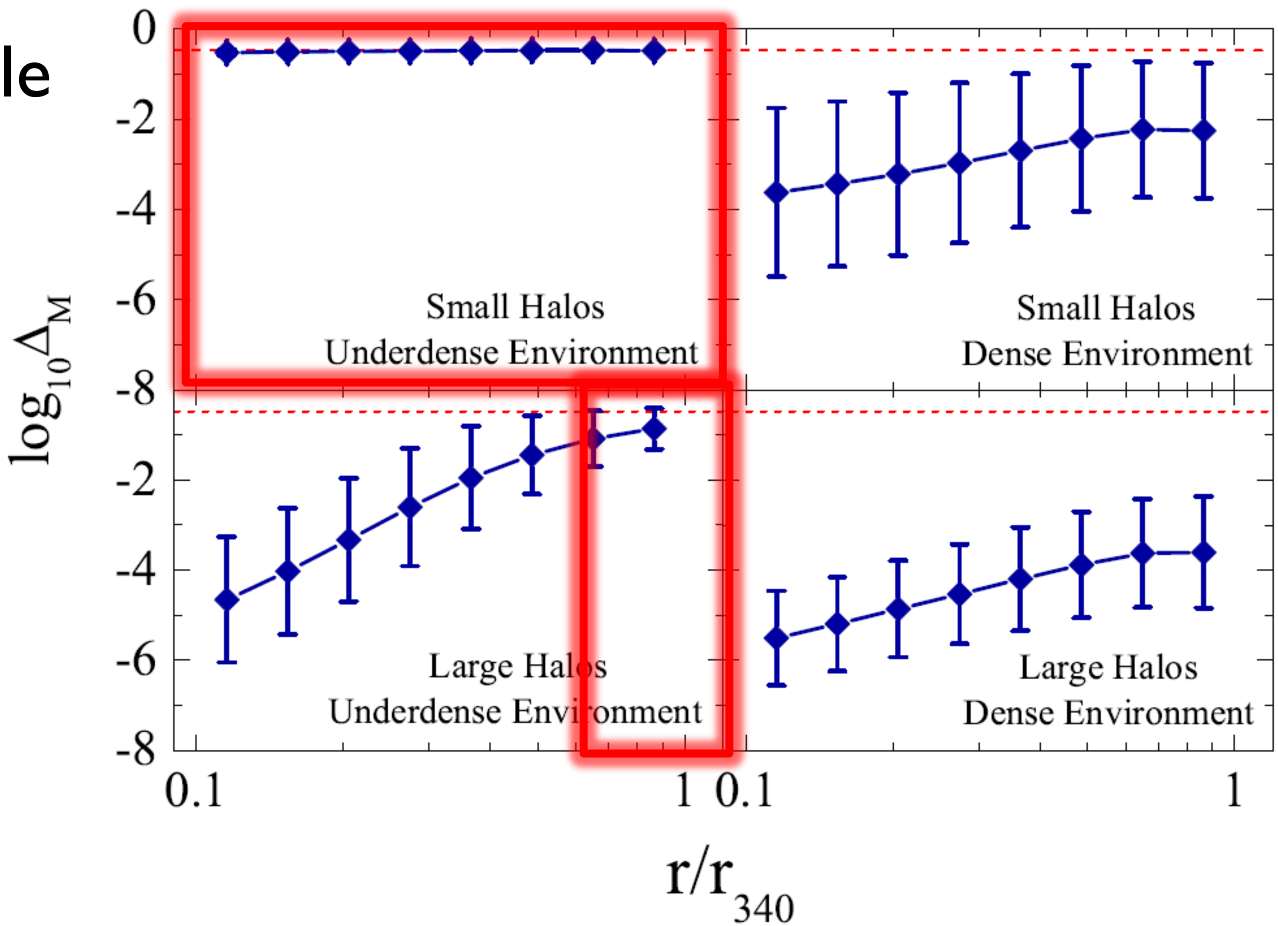


Hass et.al.
arXiv:1103.0547



Recovery of GR depends on both mass of dark matter halos and environment

- Profile



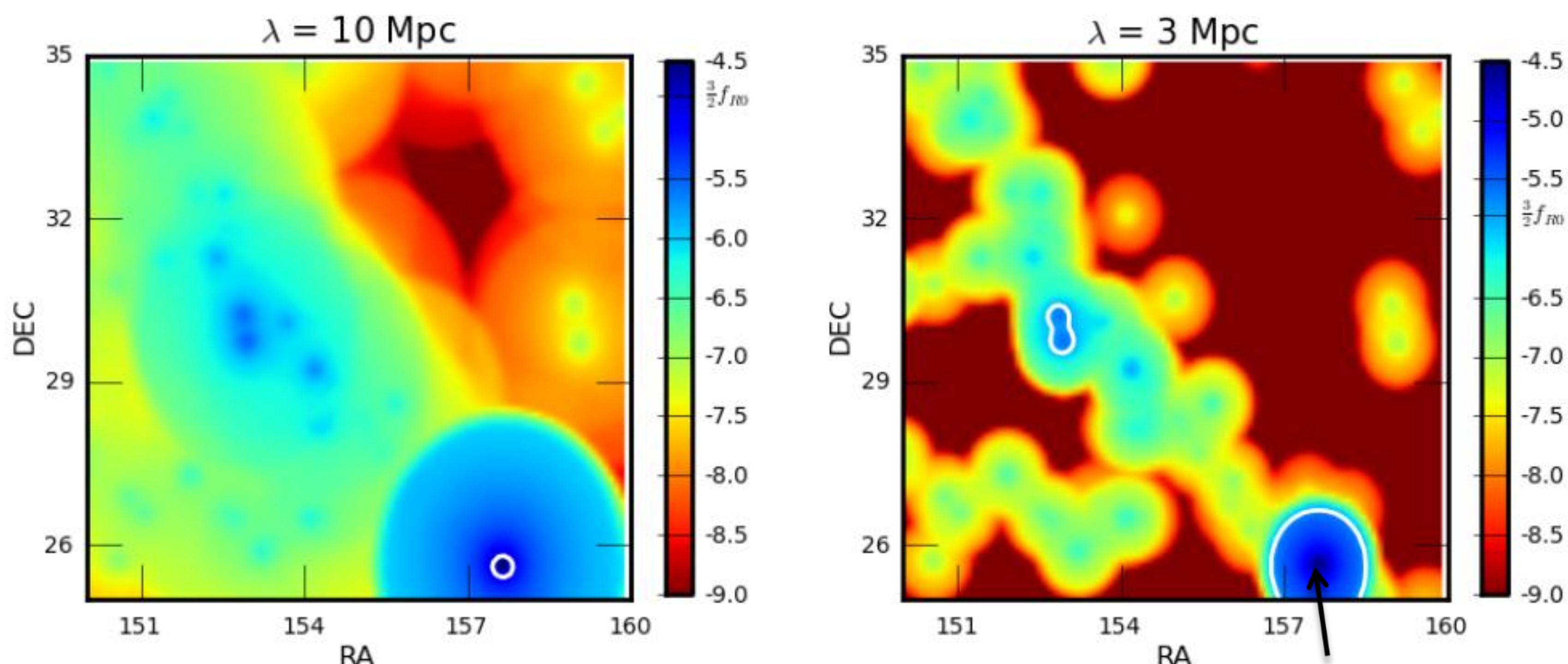
- Environmental dependence will help us disentangle other observational systematic errors
- It is possible to distiguish between different screening mechanisms

(i.e. in the case of Vainshtein, the recovery of GR is almost independent of halos mass and environment, Schmidt'10)

Creating a screening map

- It is essential to find places where GR is not recovered
  - Small galaxies in underdense regions
  - SDSS galaxies within 200 Mpc

Cabre, Vikram, Zhao, Jain, KK 1204.6046



$|f_{R0}| = 10^{-5}$ $\quad$ $|f_{R0}| = 10^{-6}$

GR is recovered

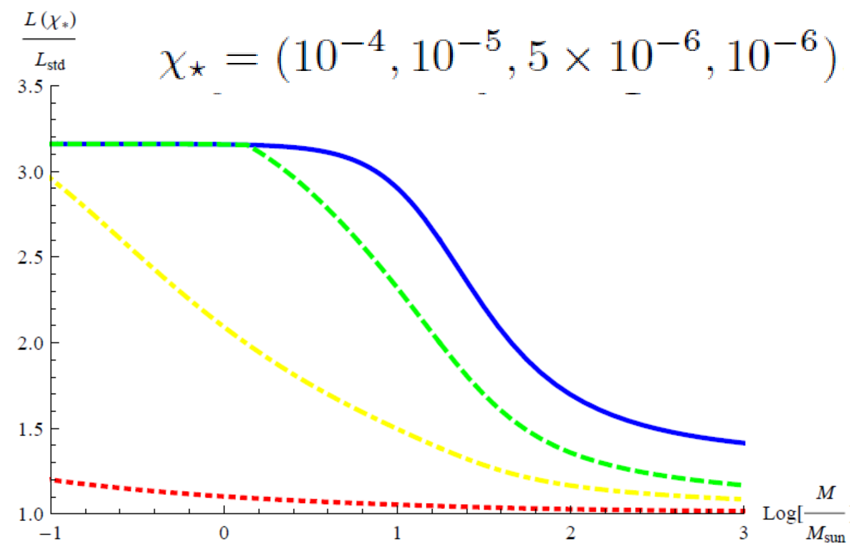
Tests of gravity on small scales

- **dwarf galaxies in voids**

  shallow potentials $\Psi \leq 10^{-7}$

  unscreened

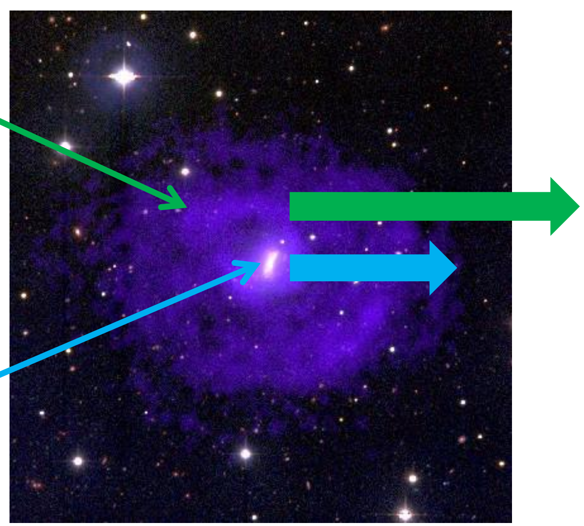
  - Galaxies are brighter
  - A displacement of the stellar disks from HI gases



Davis et.al. 1102.5278

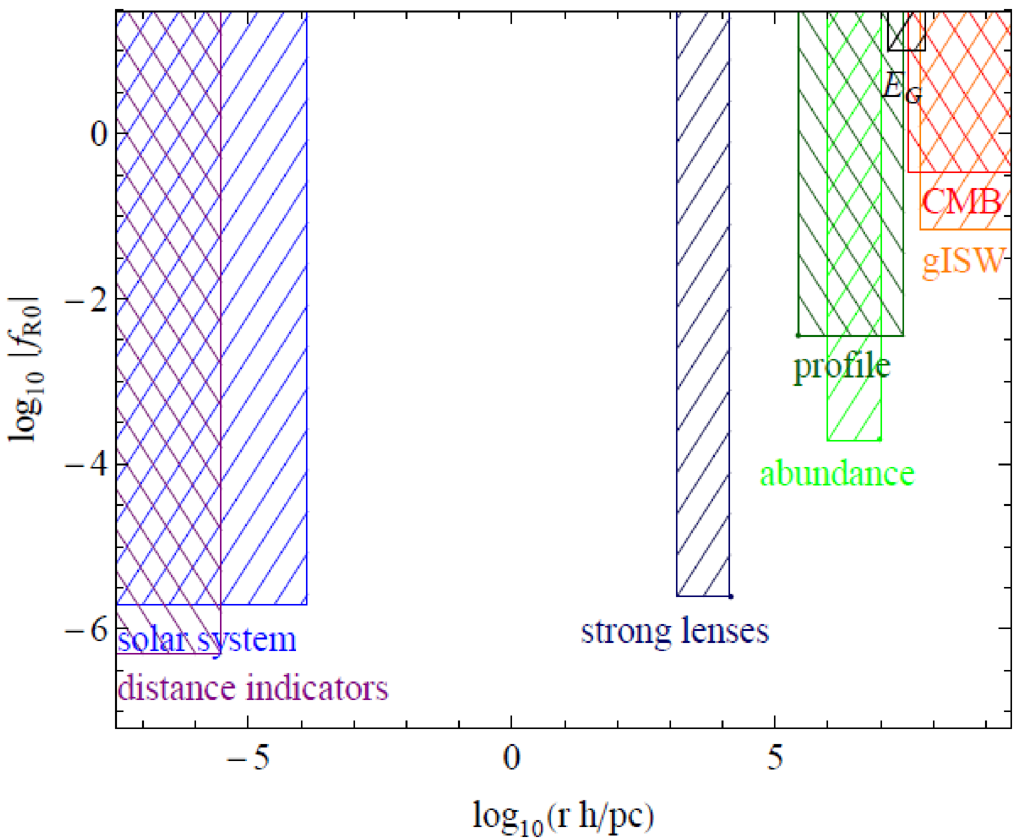
HI gas: unscreened

Stellar disk: screened



Jain & VanderPlas 1106.0065

Constraints on f_{R0} on various scales



CMB temperature [15, 31]

Cross correlation of integrated Sachs-Wolfe effect with foreground galaxies [15, 32]

Relation of galaxy clustering to lensing and velocities [33]

Density profiles of SDSS maxBCG clusters [14]

Abundance of massive clusters from Chandra X-ray data [34] and SDSS MaxBCG data [15]

Einstein rings and stellar velocity dispersion from SLACS strong lenses [35]

Cassini mission [36]

Cepheids and tip of the red giant branch stars [37]

By Lucas Lombriser

Summary

- Non-linear clustering

  mechanisms to recover GR play a crucial role

  - The power spectrum tends to go back to the one in GR with the same expansion history
  - GR is better recovered in massive halos
  - Details of the recovery of GR depend on screening mechanisms

- A challenge for theoretical predictions

  need to solve non-linear Poisson equation for the scalar

  - Perturbation theory approach (KK, Taruya, Hiramatsu 0902.0618)
  - N-body simulations

- Need to find the best places to detect deviations from GR
  - Fifth force can significantly changes stellar evolution in unscreened galaxies (Chang & Hui, Davis et.al.)
  - Stellar discs can be self-screened in unscreened dwarf galaxies (Jain & VanderPlas)