

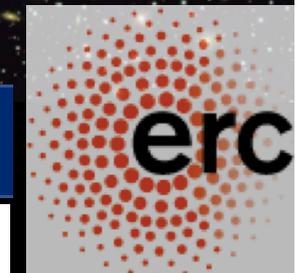
Licia Verde

**Modeling large-scale structures statistics  
towards the non-linear regime**

<http://icc.ub.edu/~liciaverde>



Institut de Ciències  
del Cosmos



# Outline

Perturbation theory approach for the power spectrum:  
from dark matter in real space to halos in reshift space

Gil-Marín, Héctor ; Wagner, Christian; Verde, Licia; Porciani Cristiano, Jimenez, Raul  
2012 (to appear in arXiv soon)

A new fitting formula for the dark matter bispectrum

Gil-Marín, Héctor; Wagner, Christian; Fragkoudi, Frantzeska; Jimenez, Raul; Verde, Licia  
2012 (JCAP)

Part of the PhD thesis of Hector Gil-Marin

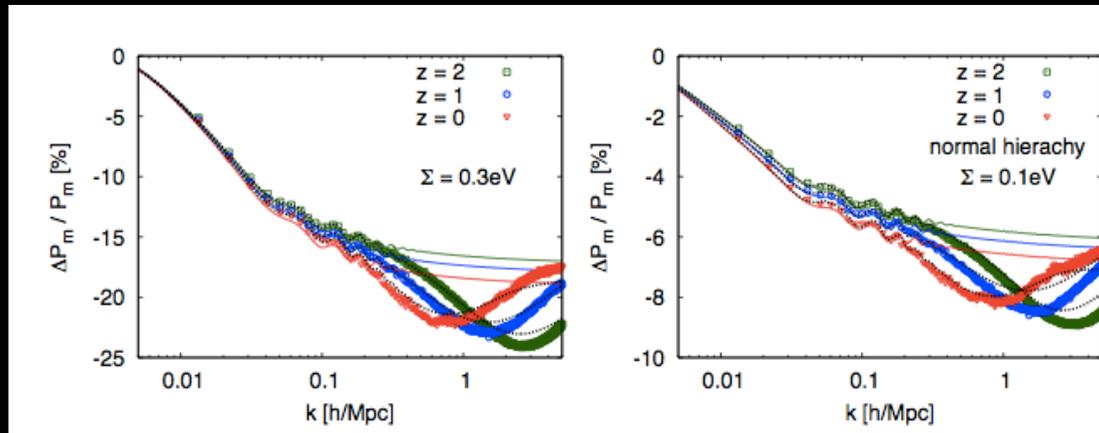
# Preaching to the converted...

- A lot of signal to noise at small scales
- Non-linearities are important (compared to error-bars) on relatively large scales for forthcoming surveys
- “Precision cosmology” should also be “accurate cosmology”
- N-body simulations are a great resource but are still “slow” to run and impossible to simulate the full survey.
- Value in calibrate analytics on simulations (both for modeling signal and errors)

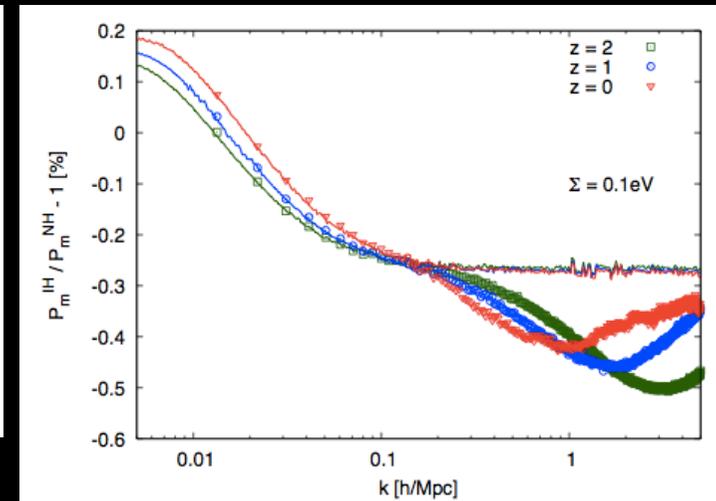
# Aside for discussion

- Future surveys (e.g. EUCLID) yield in principle tiny error-bars
- Are *linear* predictions up to the job?

Example for dark matter -think weak lensing- for neutrino properties:



From: Wagner, LV, Jimenez, 2012, ApJLett



Non-linearities enhance the effect!!!! But the effect is small, below % level.

One needs theoretical predictions of the absolute non-linear power spectrum at least accurate to the 0.1% level ; different linear Einstein-Boltzmann codes (e.g. CAMB (Lewis et al. 2000) and CLASS (Blas et al. 2011)) still do not agree to 0.1% precision on the relevant scales.

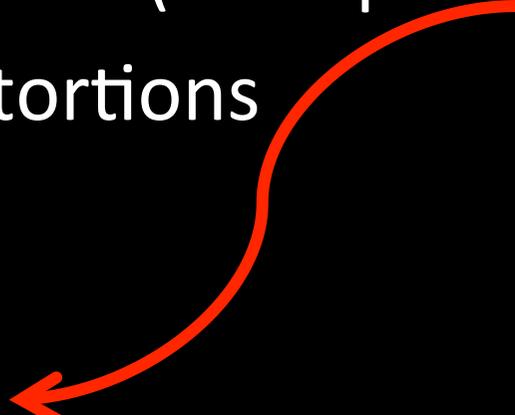
# Back to business... The importance of a (semi)analytic description

- Even for present-day surveys it is challenging to N-body simulate the full survey volume once. How do you estimate errors? (not by Monte Carlo!!!)
- What if you want to explore cosmology dependence?
- Effects to account for: NON-LINEARITIES, real world effects and, if not weak lensing, “bias” of tracers.

# Perturbation theory approach to power spectrum

- Dark matter evolution (real space)
- Redshift-space distortions
- (biasing)

Resummed perturbation theory quite successful



$$P^{\text{SPT}}(\mathbf{k}) = P^{(0)}(\mathbf{k}) + P^{(1)}(\mathbf{k}) + P^{(2)}(\mathbf{k}) + \dots$$

Standard perturbation theory

Where:

$$P^{(0)}(\mathbf{k}) = P^{\text{lin}}(\mathbf{k}),$$

$$P^{(1)}(\mathbf{k}) = 2P_{13}(\mathbf{k}) + P_{22}(\mathbf{k})$$

$$P^{(2)}(\mathbf{k}) = 2P_{15}(\mathbf{k}) + 2P_{24}(\mathbf{k}) + P_{33}(\mathbf{k}).$$

Each term is a multi-dimensional integral of convolutions with “kernels”

$$P^{\text{RPT}-\mathcal{N}_i}(\mathbf{k}, z) = \left[ P^{\text{lin}}(\mathbf{k}, z) + P_{22}(\mathbf{k}, z) + P_{33}^{2L}(\mathbf{k}, z) + \dots + P_{nn}^{(n-1)L}(\mathbf{k}, z) + \dots \right] \mathcal{N}_i(\mathbf{k})^2$$

Re-organized expansion factorizing common terms.

Must still be truncated but works better for the “same” computational cost.

The form of the function  $\mathcal{N}$  depends on the approximation of the kernels in the resummation.

In this way each term that is added is positive and does not make the approximation “oscillate” around the true solution as more terms are added...

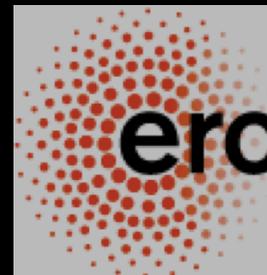
# Simulations

- 160 independent runs,  $768^3$  particles, 2.4Gpc/h box side

Need a big computer and somebody that can use it!

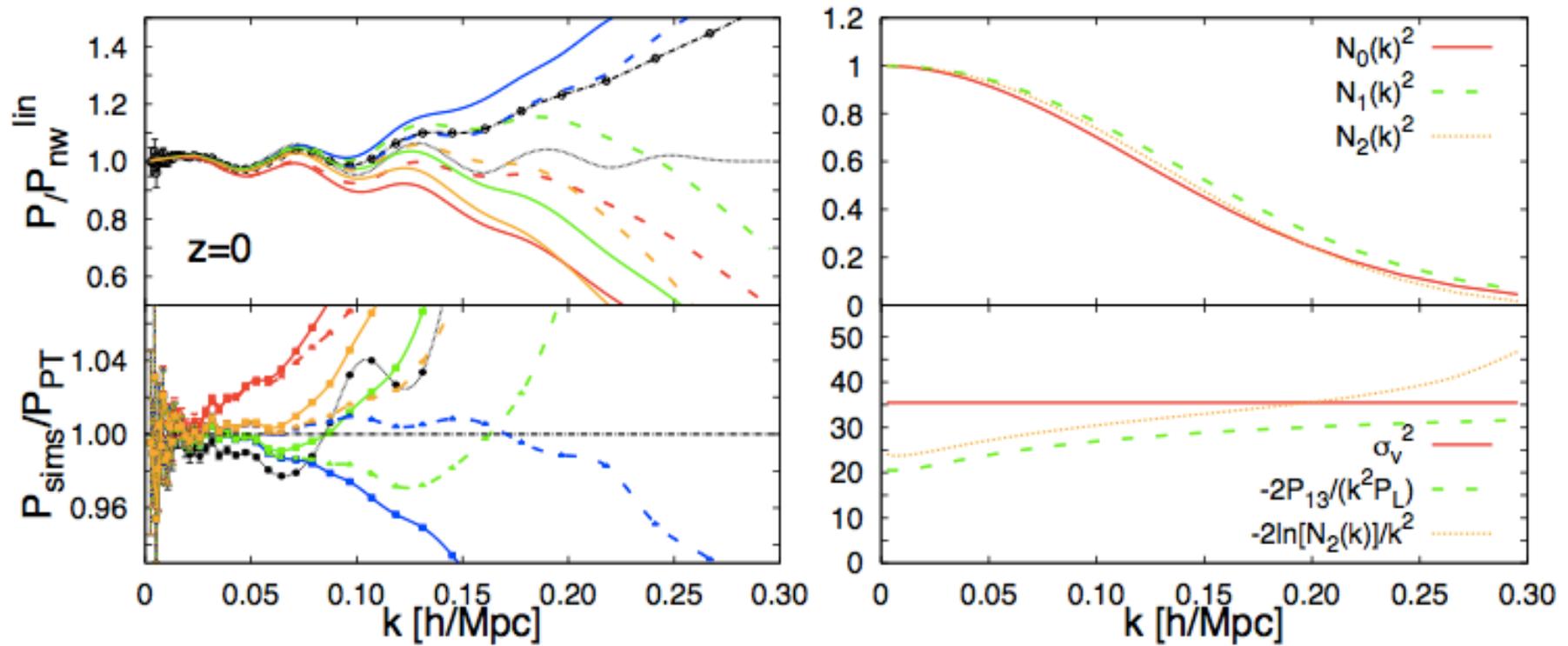


Christian Wagner



Hipatia

# Dark matter real space



..... Linear theory

SPT

RPT  $N_0$

RPT  $N_1$

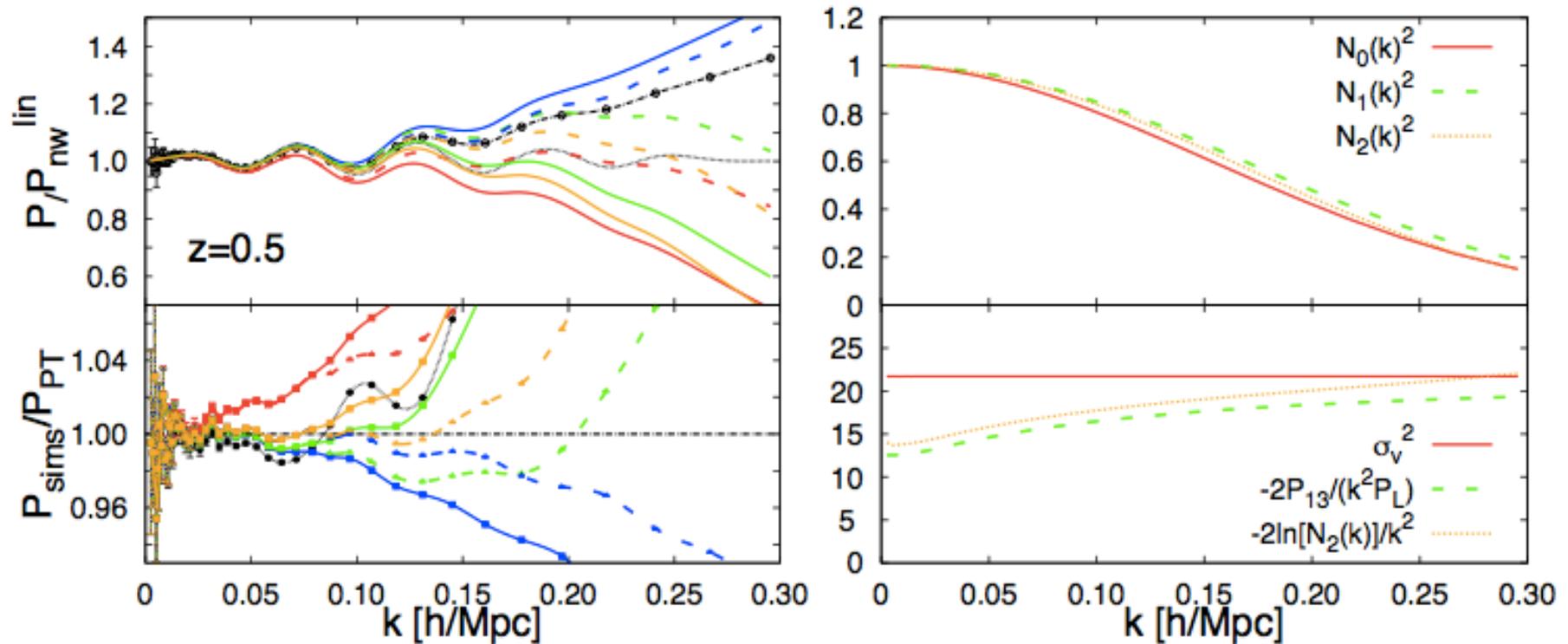
RPT  $N_2$

----- 2 loops

----- Simulations

\_\_\_\_\_ 1 loop

# Dark matter real space



..... Linear theory

SPT

RPT  $N_0$

RPT  $N_1$

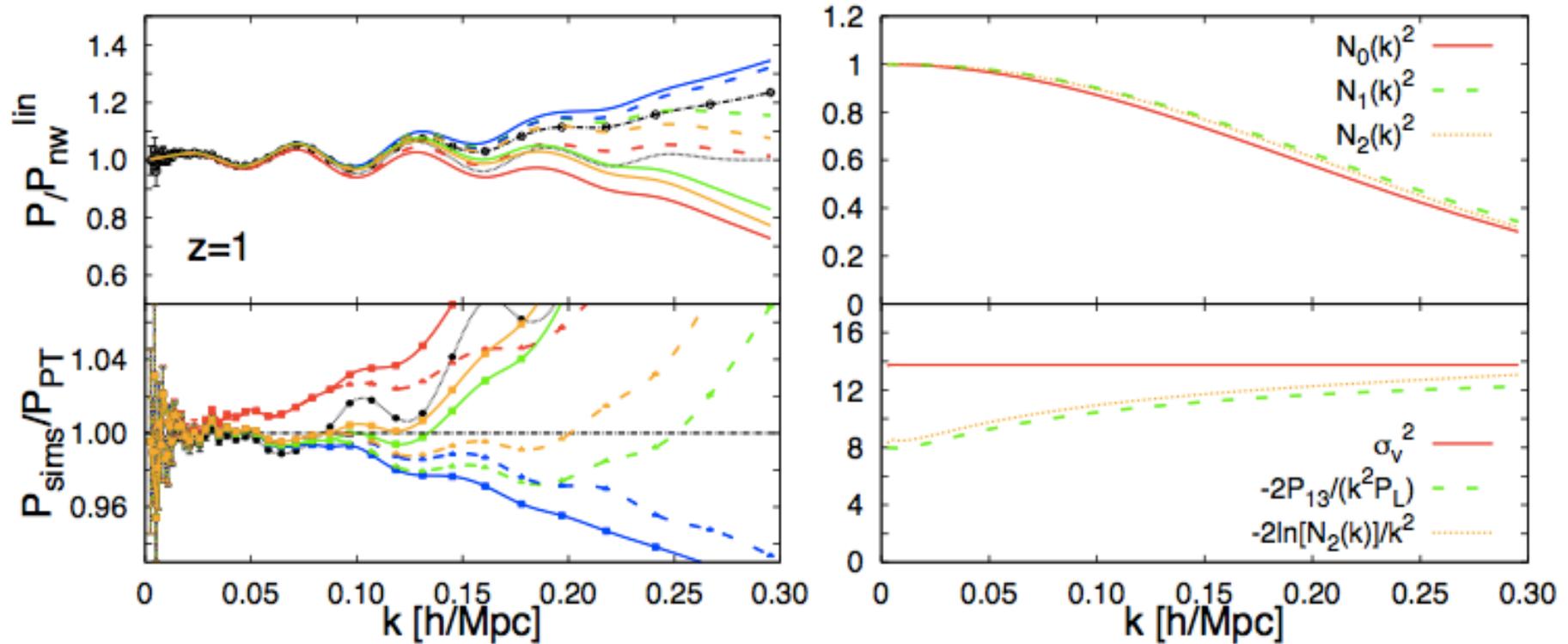
RPT  $N_2$

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# Dark matter real space



..... Linear theory

SPT

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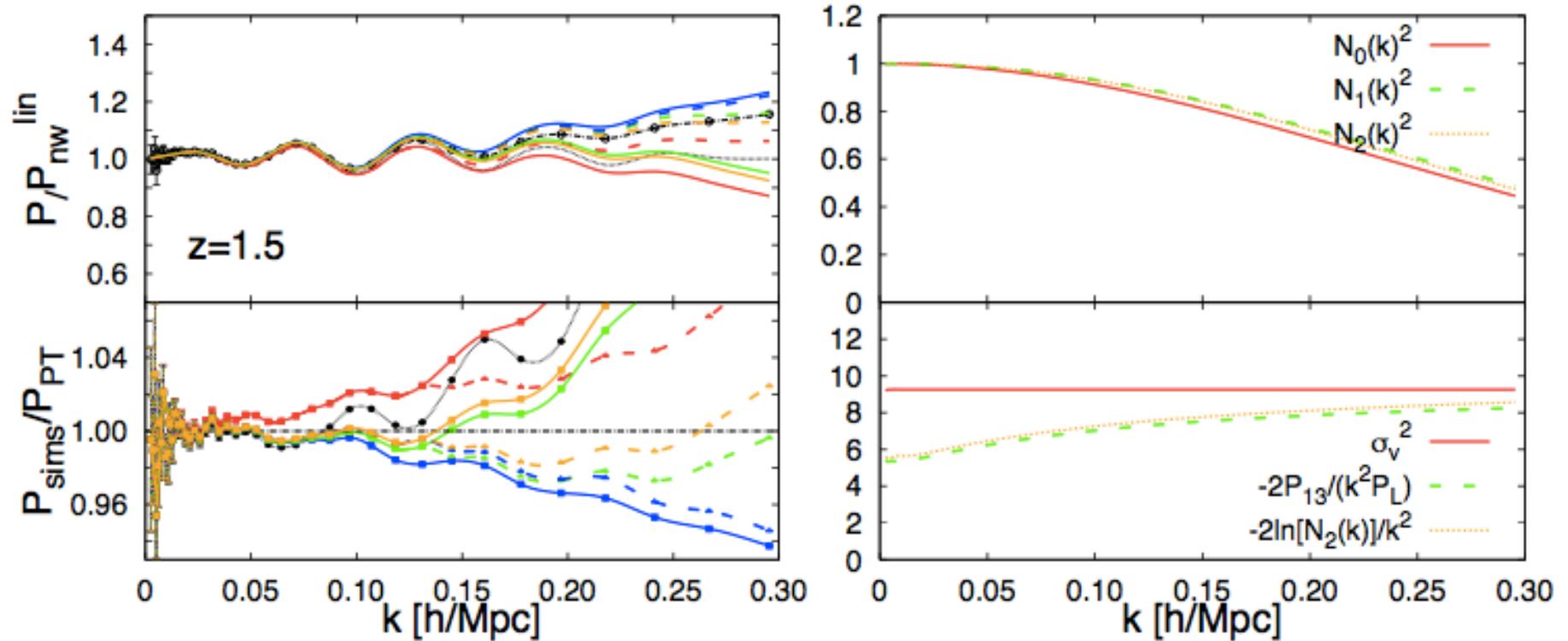
RPT  $N_2$

----- 2 loops

----- Simulations

----- 1 loop

# Dark matter real space



..... Linear theory

SPT

RPT  $N_0$

RPT  $N_1$

RPT  $N_2$

----- 2 loops

----- Simulations

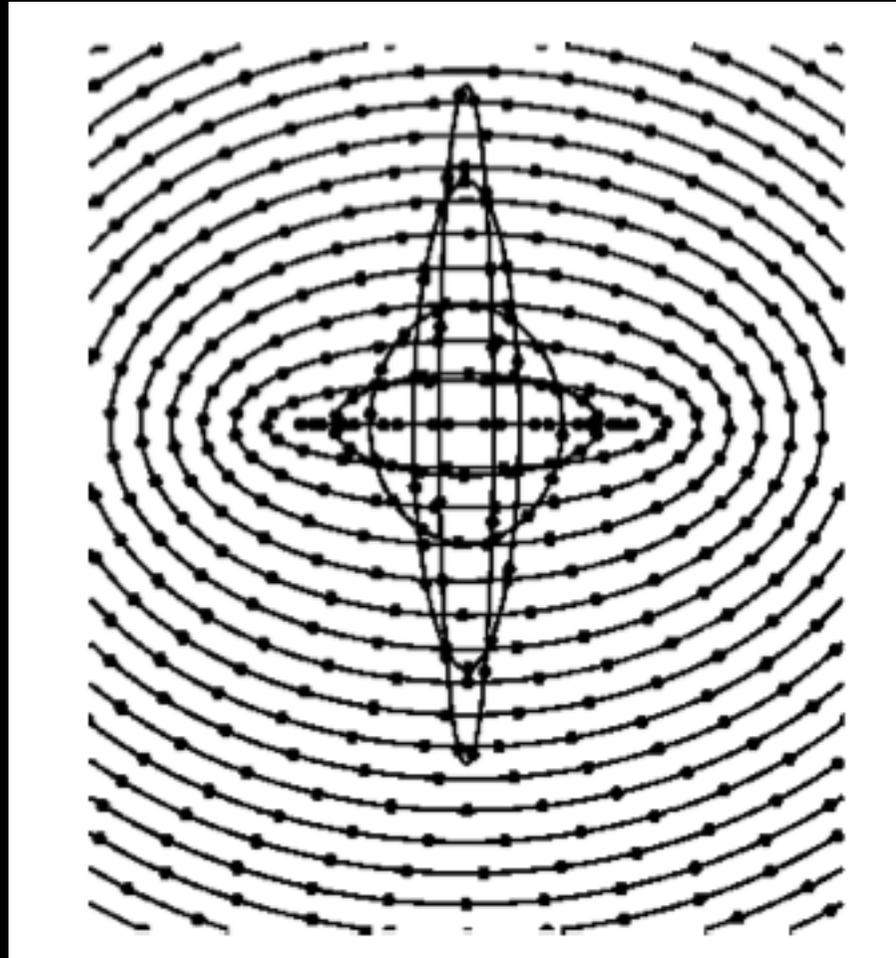
----- 1 loop

RPT N2 at 2 loops works very well.

Unfortunately in many applications we can't probe the dark matter in real space directly

# Redshift space distortions

They are well known for not being well described perturbatively



Hamilton 1997, Kaiser, 1987

# Redshift space distortions

They are well known for not being well described perturbatively

Consider:

Kaiser model (Kaiser, 1987)  $P^s(k, \mu) = (b(k) + f^2 \mu)^2 P_{\delta\delta}(k),$

Scoccimarro 2004 model  $P^s(k, \mu) = [b(k)^2 P_{\delta\delta}(k) + 2b(k)\mu^2 f P_{\delta\theta}(k) + f^2 \mu^4 P_{\theta\theta}(k)]$

where  $P_{\delta\delta}$ ,  $P_{\delta\theta}$  and  $P_{\theta\theta}$  are the *matter-matter*, *velocity-matter* and *velocity-velocity* power spectra

Taruya Nishimichi, Saito 2010, model

$$P^s(k, \mu) = [b(k)^2 P_{\delta\delta}(k) + 2b(k)\mu^2 f P_{\delta\theta}(k) + f^2 \mu^4 P_{\theta\theta}(k) + A(k, \mu, b) + B(k, \mu, b)]$$

Plus Fingers of God modeling as a multiplicative Damping function (Davis & Peebles 1983)

# (angular) Multipoles

$$P_\ell(k) = (2\ell + 1) \int_0^1 d\mu P^s(k, \mu) L_\ell(\mu),$$

Legendre polynomials

Recall: linear theory the only non-zero terms are:

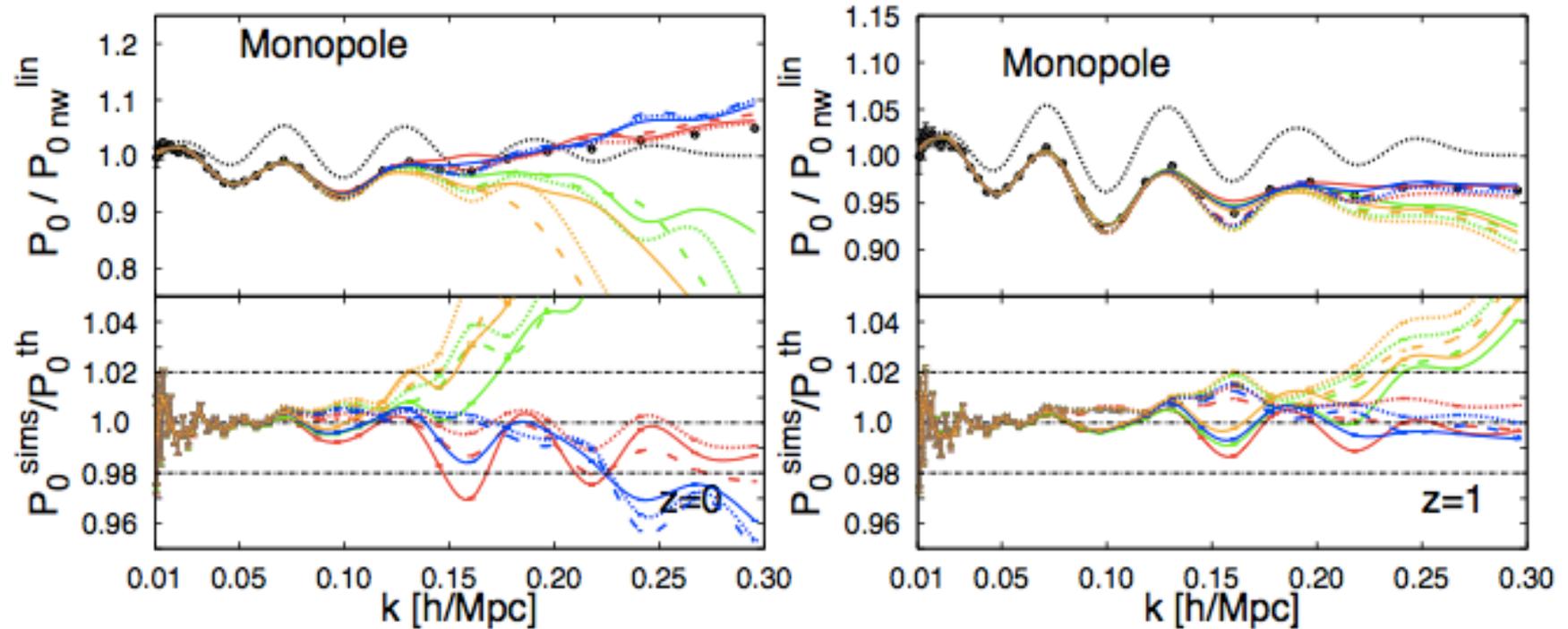
$$P_0(k) = P^{\text{lin}}(k) \left( b(k)^2 + \frac{2}{3}b(k)f + \frac{1}{5}f^2 \right),$$

$$P_2(k) = P^{\text{lin}}(k) \left( \frac{4}{3}b(k)f + \frac{4}{7}f^2 \right),$$

$$P_4(k) = P^{\text{lin}}(k) \left( \frac{8}{35}f^2 \right).$$

Hint: extract the growth function and study dark energy with it (or modified gravity)

# Dark matter Monopole



.....

Kaiser

Linear theory

$\sigma$  free parameter fitted to Nbody

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Scoccimarro

SPT 1loop

SPT 2 loops

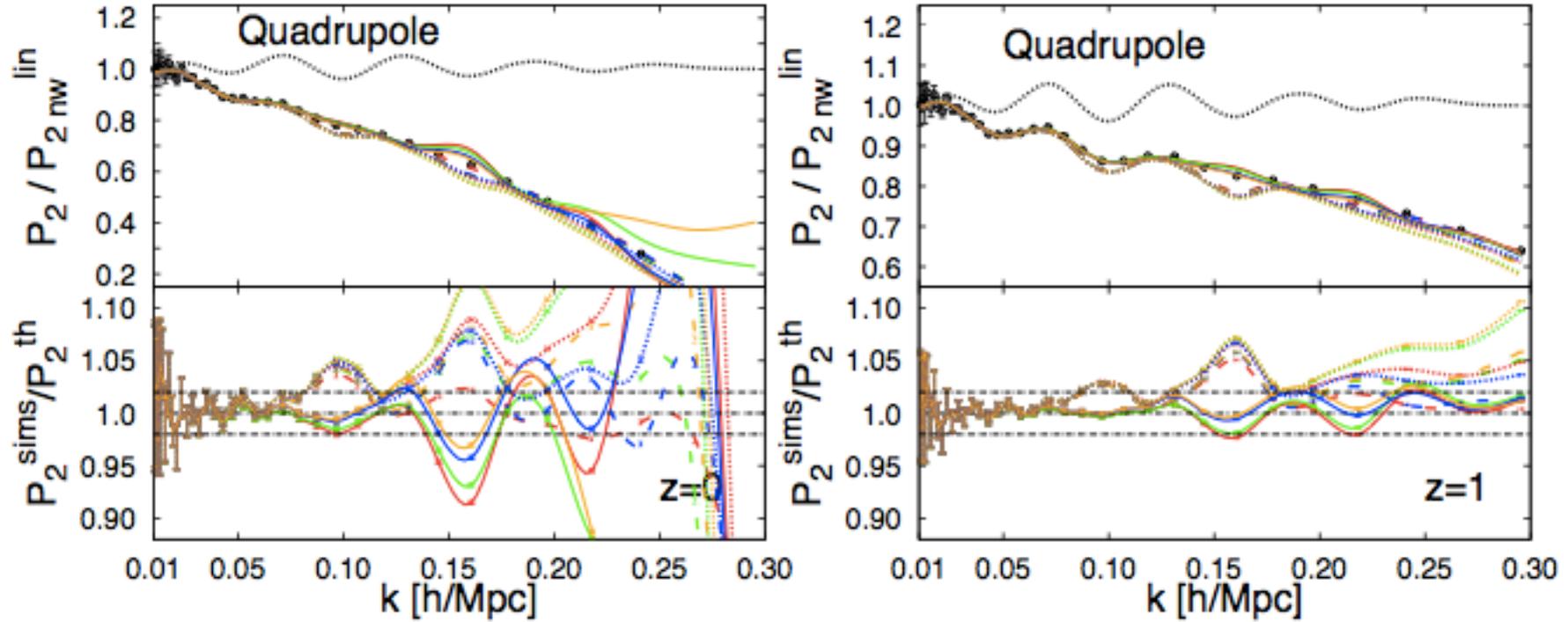
2 loops RPT  $N_1$

2 loops RPT  $N_2$

—

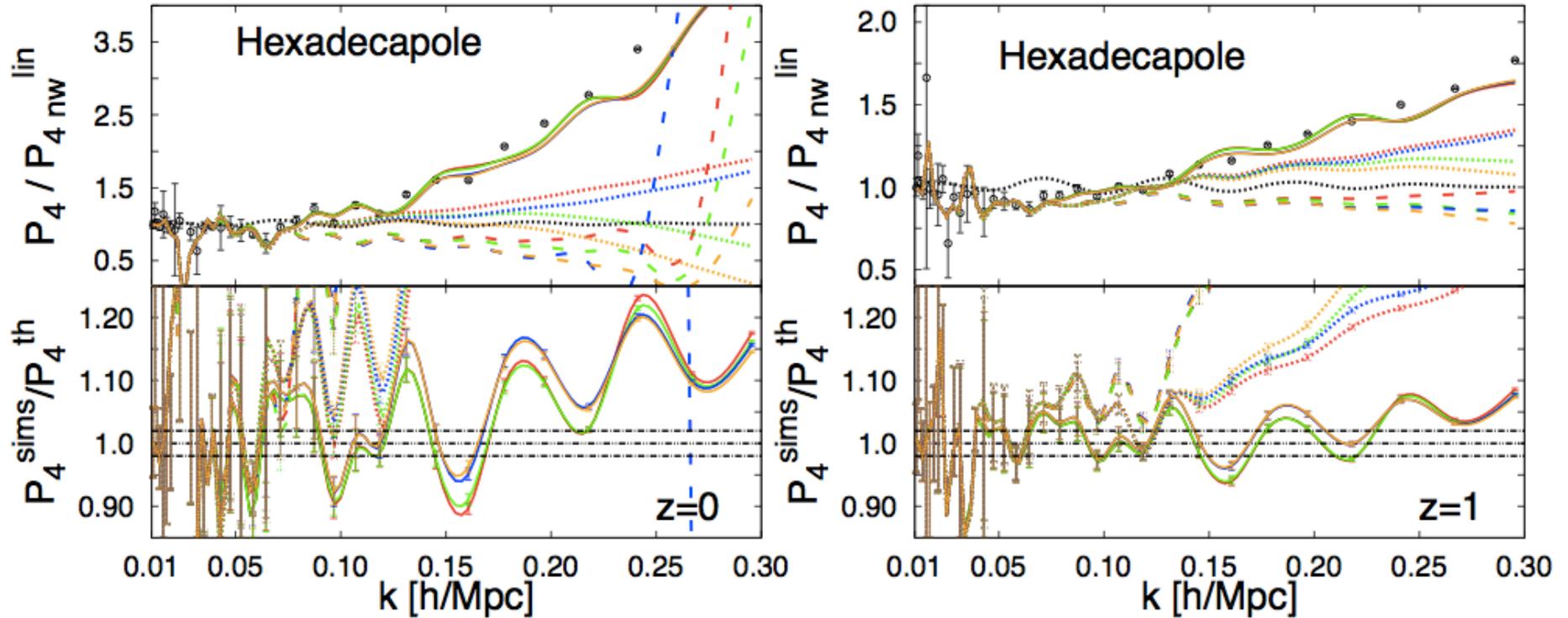
Taruya

# Dark matter Quadrupole

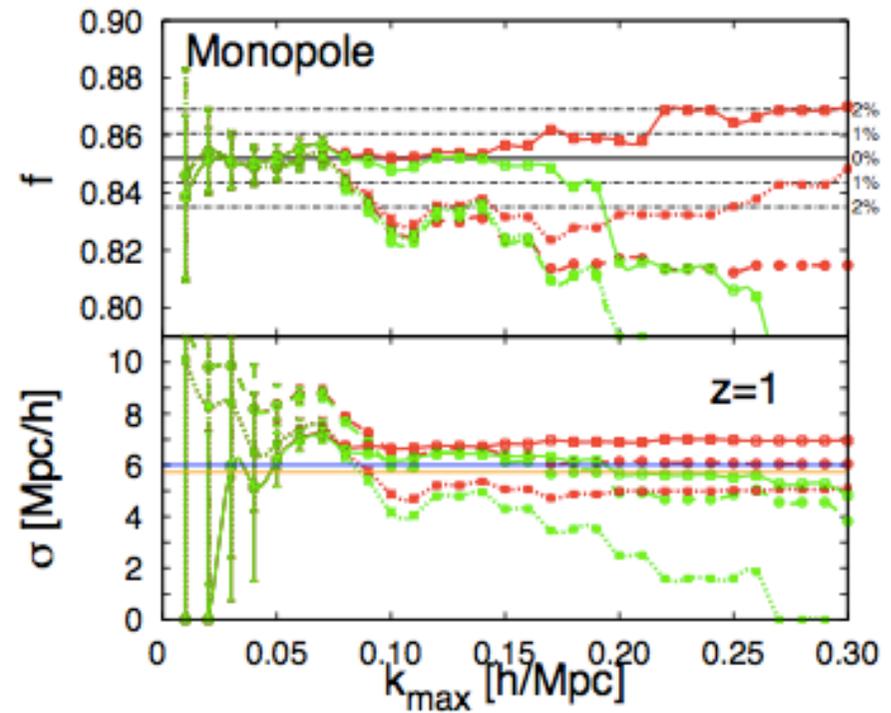
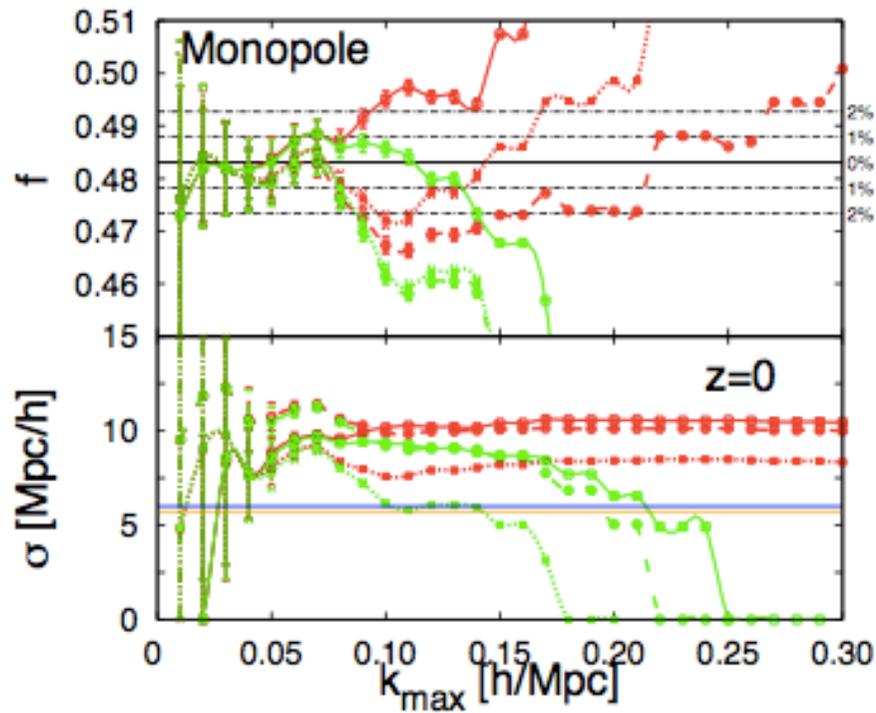


- ..... Kaiser
  - Scoccimarro
  - Taruya
  - Linear theory
  - SPT 1loop
  - SPT 2 loops
  - 2 loops RPT  $N_1$
  - 2 loops RPT  $N_2$
- $\sigma$  free parameter fitted to Nbody

# Dark matter Hexadecapole



# Recovering $f$

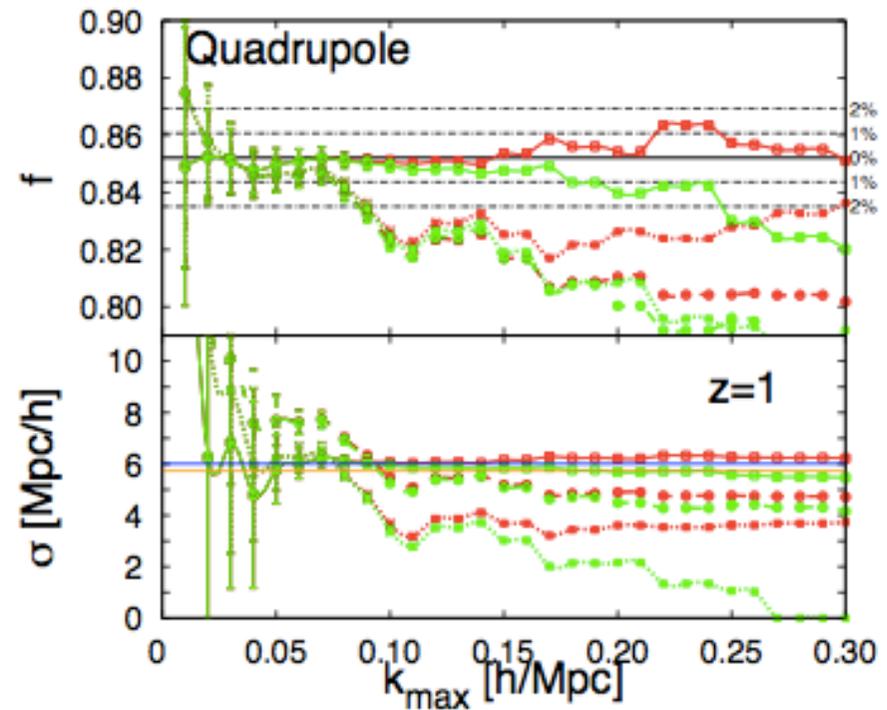
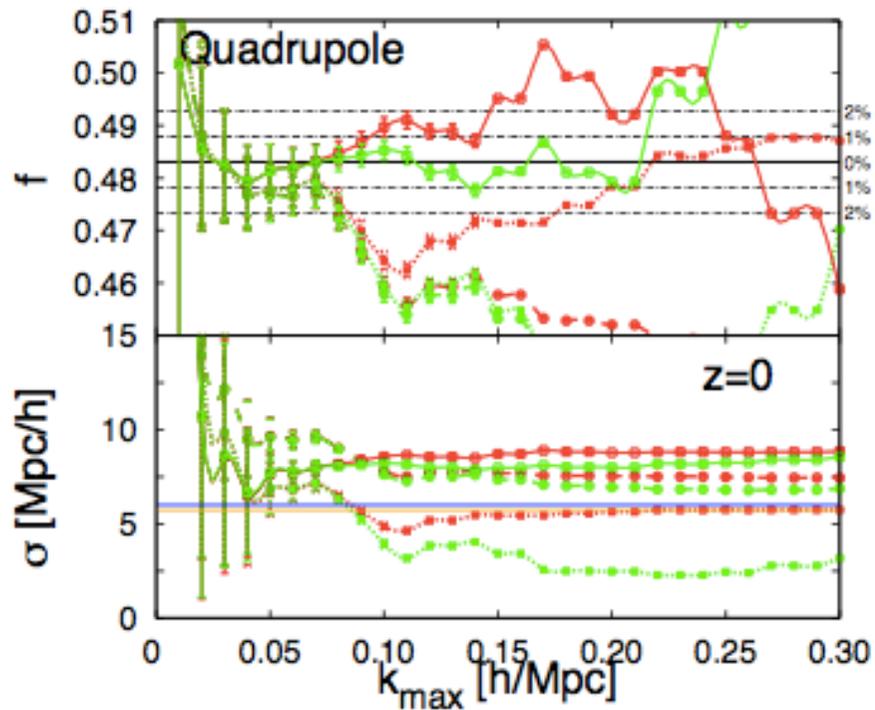


- ..... Kaiser
- Scoccimarro
- Taruya

2 loops RPT  $N_1$

SPT 1loop

# Recovering f

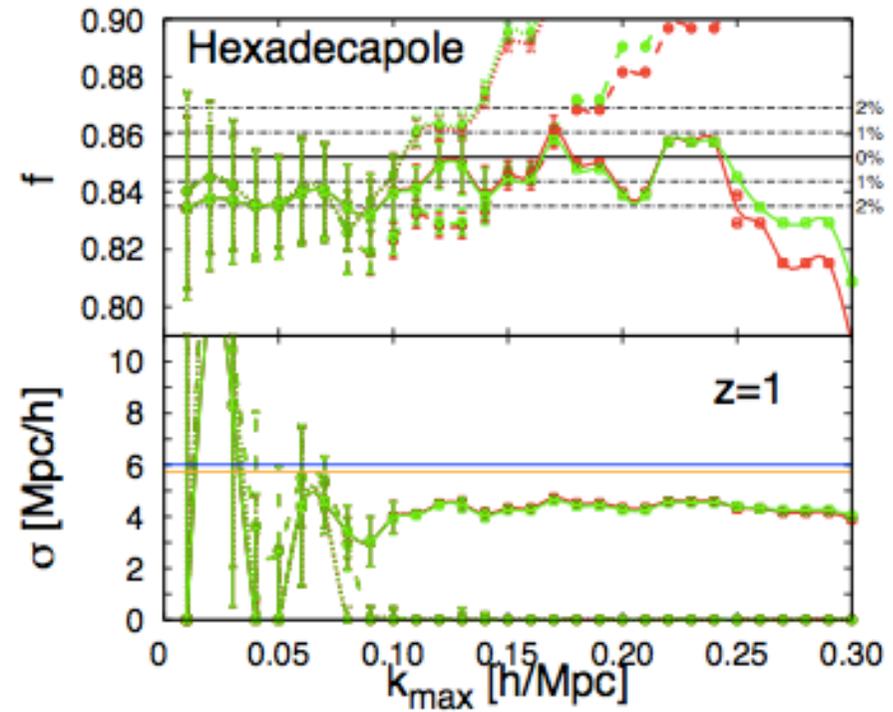
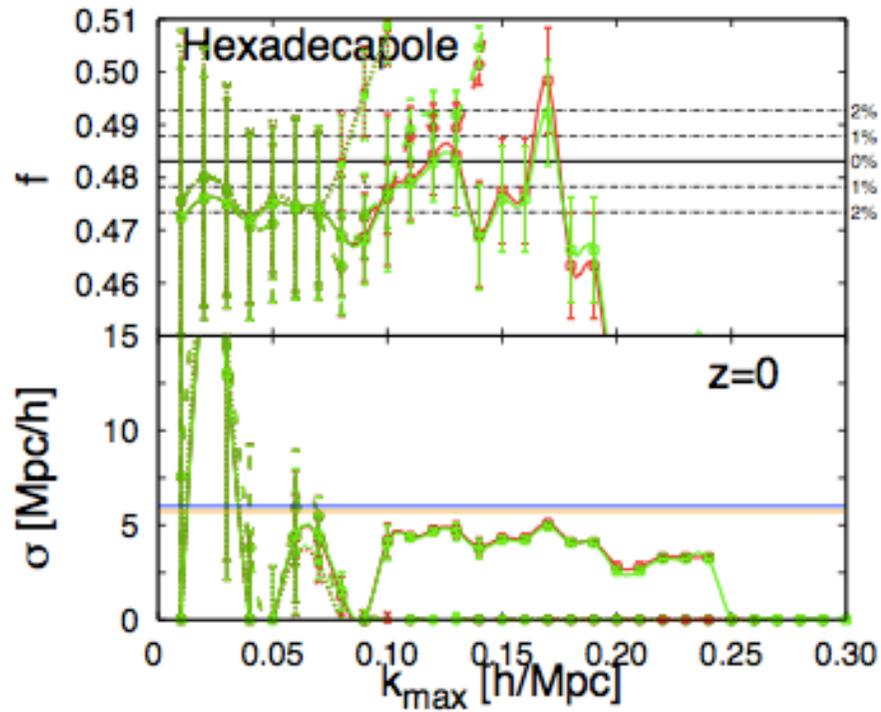


- ..... Kaiser
- - - - Scoccimarro
- Taruya

2 loops RPT  $N_1$

SPT 1loop

# Recovering f



Kaiser



Scoccimarro



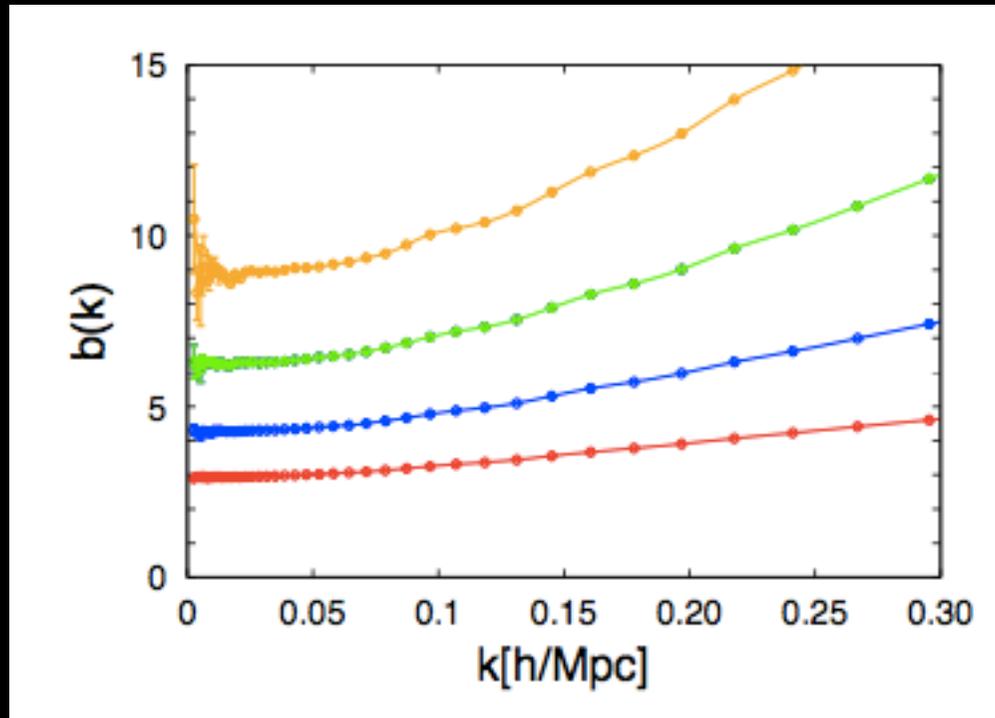
Taruya

2 loops RPT  $N_1$

SPT 1loop

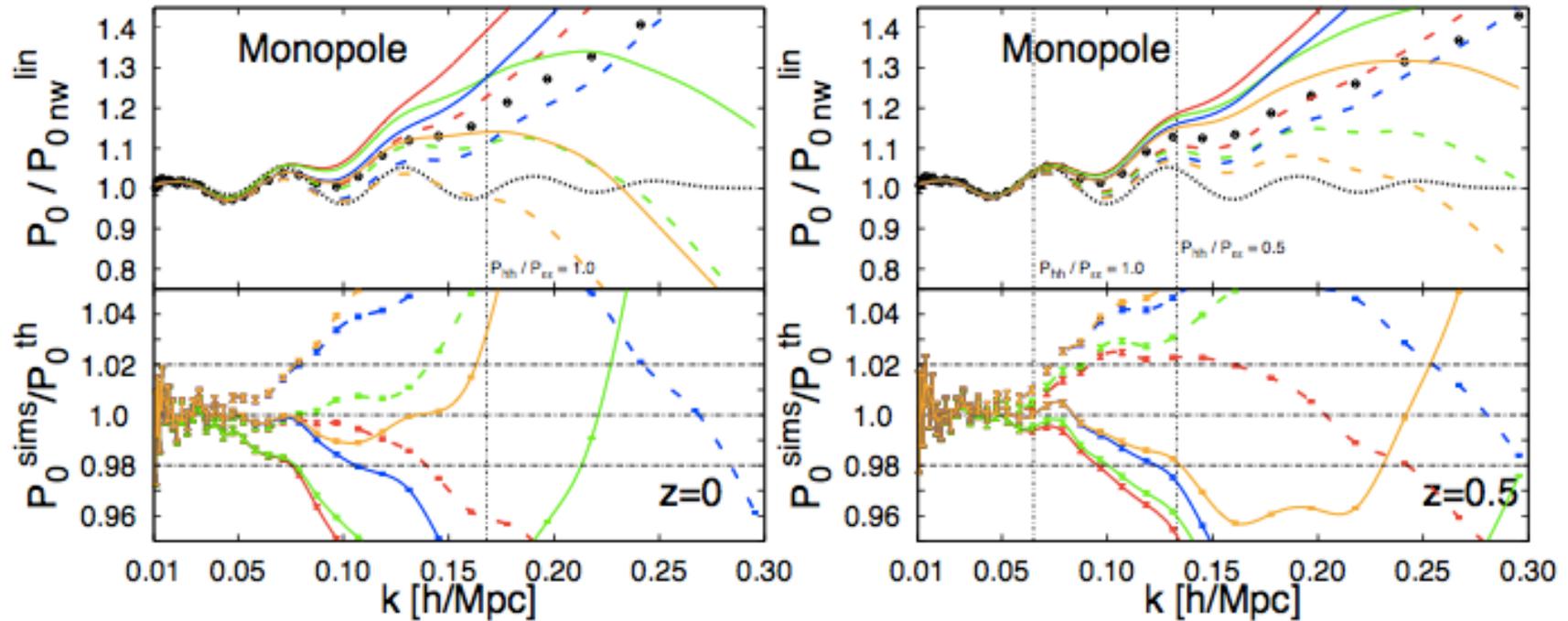
# HALOS (still not galaxies...)

Say that we know the bias (from simulations)



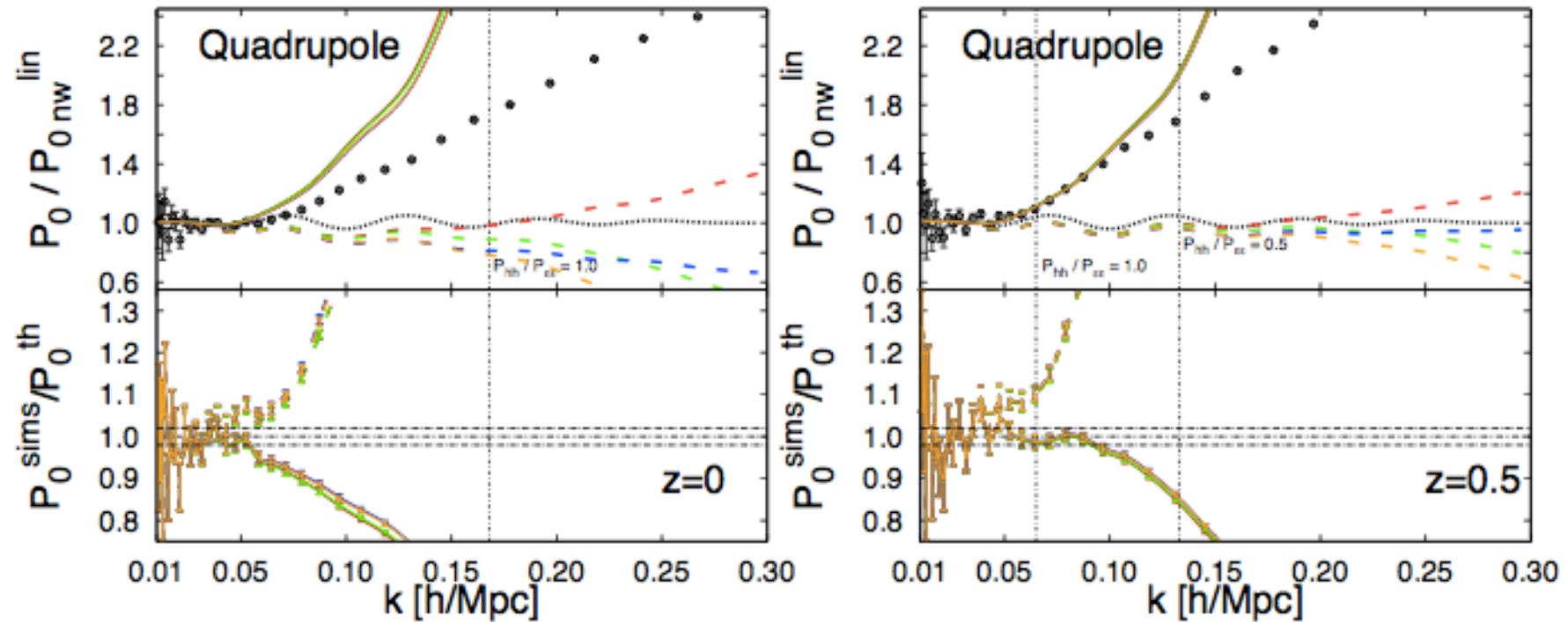
$M > 10^{14} M_{\text{sun}}$

# HALOS (still not galaxies...) multipoles



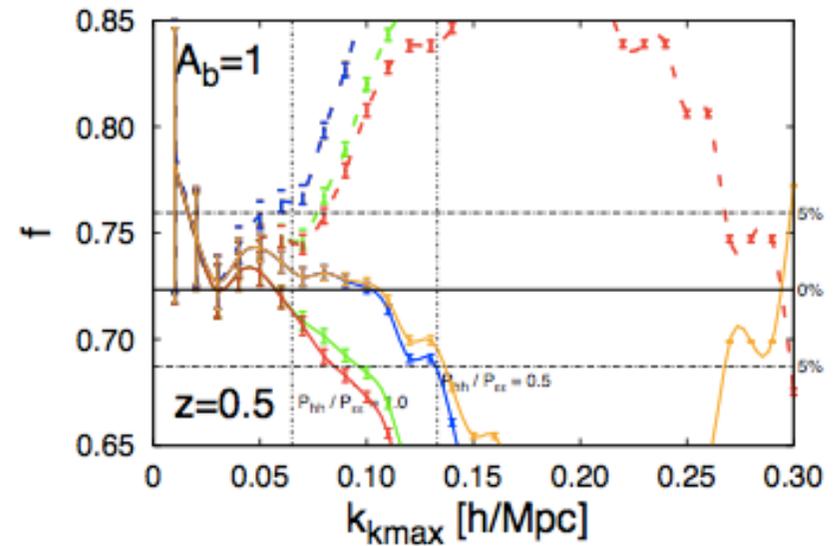
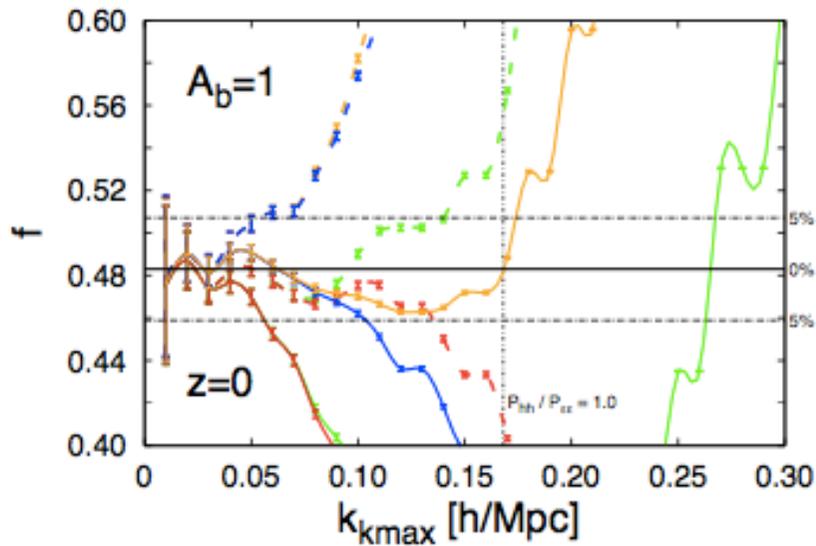
- ..... Kaiser
- - - - - Scoccimarro
  - SPT 1loop
  - SPT 2 loops
  - 2 loops RPT  $N_1$
  - 2 loops RPT  $N_2$
- Taruya

# HALOS (still not galaxies...) multipoles



- ..... Kaiser
- - - - - Scoccimarro
  - SPT 1loop
  - SPT 2 loops
  - 2 loops RPT  $N_1$
  - 2 loops RPT  $N_2$
- Taruya

# Recovering f...



If you know bias

.....

Kaiser

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Scoccimarro

SPT 1loop

SPT 2 loops

2 loops RPT  $N_1$

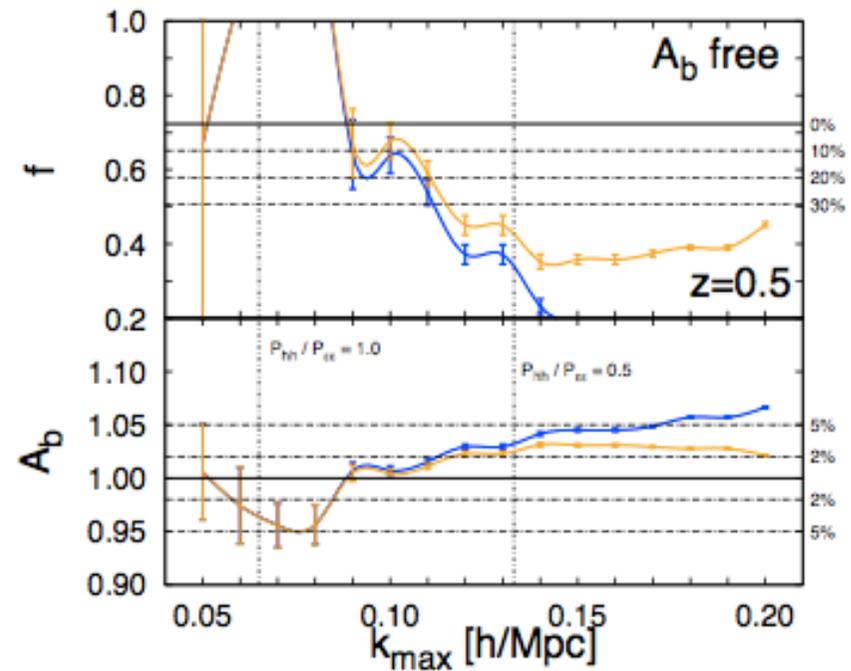
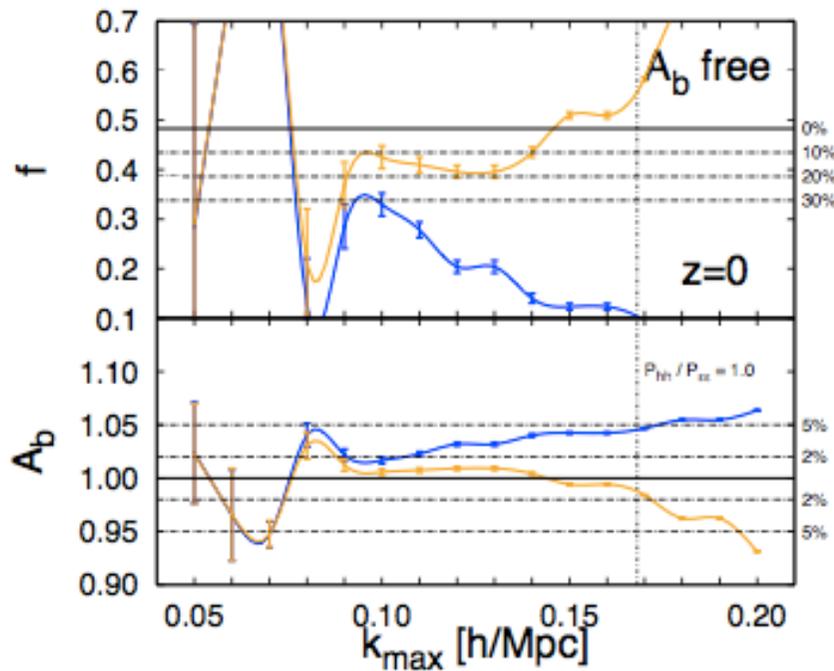
2 loops RPT  $N_2$

—

Taruya

# Recovering $f$ ...

But if you don't.....



Taruya+2loops SPT

Taruya+2 loops RPT  $N_2$

## To summarize...

- Dark matter real space under control (2 loops RPT N2 works quite well... % accuracy up to  $k=0.2h/\text{Mpc}$ )
- Redshift space distortions complicate things but for dark matter  $f$  is unbiased ( at % level)
- Halos are a problem even if you know bias
- A small error on  $b$  propagates into a large error on  $f$

Disclaimer: We used massive halos, for less massive ones things might be different.

# Let's talk about Bispectrum

- Motivation:

Bias, gravitational instability etc.

- analytic modeling based on PT is accurate only to  $\sim 30\%$  and fail to reproduce the BAO features (Emiliano knows)
- So, fit to simulations...
- Re-do Scoccimarro and Couchman 2001 with bigger simulations; study redshift dependence

# Bispectrum

$$B_{123} = 2F_2^s(\mathbf{k}_1, \mathbf{k}_2)P_1^L P_2^L + \text{cyc. perm.},$$

2 order PT tree level

$$F_2^s(\mathbf{k}_i, \mathbf{k}_j) = \frac{5}{7} + \frac{1}{2} \cos(\theta_{ij}) \left( \frac{k_i}{k_j} + \frac{k_j}{k_i} \right) + \frac{2}{7} \cos^2(\theta_{ij}),$$

# SC 2001

$$F_2^{\text{eff}}(\mathbf{k}_i, \mathbf{k}_j) = \frac{5}{7}a(n_i, k_i)a(n_j, k_j) + \frac{1}{2}\cos(\theta_{ij})\left(\frac{k_i}{k_j} + \frac{k_j}{k_i}\right)b(n_i, k_i)b(n_j, k_j) + \frac{2}{7}\cos^2(\theta_{ij})c(n_i, k_i)c(n_j, k_j),$$

$$a(n, k) = \frac{1 + \sigma_8^{a_6}(z)[0.7Q_3(n)]^{1/2}(qa_1)^{n+a_2}}{1 + (qa_1)^{n+a_2}},$$

$$b(n, k) = \frac{1 + 0.2a_3(n+3)q^{n+3}}{1 + q^{n+3.5}},$$

$$c(n, k) = \frac{1 + 4.5a_4/[1.5 + (n+3)^4](qa_5)^{n+3}}{1 + (qa_5)^{n+3.5}}.$$

$$n \equiv \frac{d \log P^L(k)}{d \log k}$$

$$q \equiv k/k_{\text{nl}}$$

$$\frac{k_{\text{nl}}^3 P^L(k_{\text{nl}})}{2\pi^2} \equiv 1;$$

$$a_1 = 0.25, a_2 = 3.5, a_3 = 2, a_4 = 1, a_5 = 2, a_6 = -0.2.$$

$$Q_3(n) = \frac{4 - 2^n}{1 + 2^{n+1}}.$$

# Simulations

	A	B
$L_b$ [Mpc/h]	2400	1875
$N_p$	$768^3$	$1024^3$
$N_r$	40	3
$k_N/4$ [h/Mpc]	0.25	0.43
softening $\epsilon$ [kpc/h]	90	40
PM grid	$2048^3$	$2048^3$
ErrTolForceAcc $\alpha$	0.005	0.005
initial scale factor $a_i$	0.05	0.02
maximum $\Delta \log a$	0.025	0.025
ErrTolIntAccuracy $\eta$	0.025	0.025
# time steps	$\sim 1300$	$\sim 2500$

# modifications

$$\tilde{a}(n, k) = \frac{1 + \sigma_8^{a_6}(z)[0.7Q_3(n)]^{1/2}(qa_1)^{n+a_2}}{1 + (qa_1)^{n+a_2}},$$

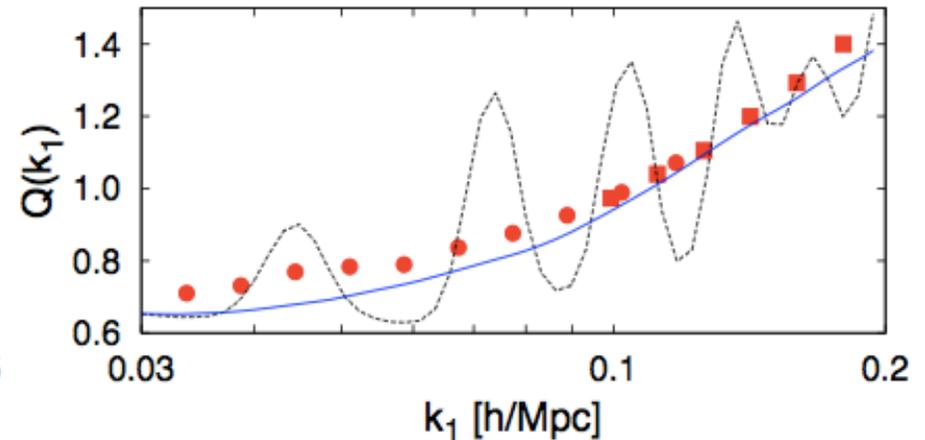
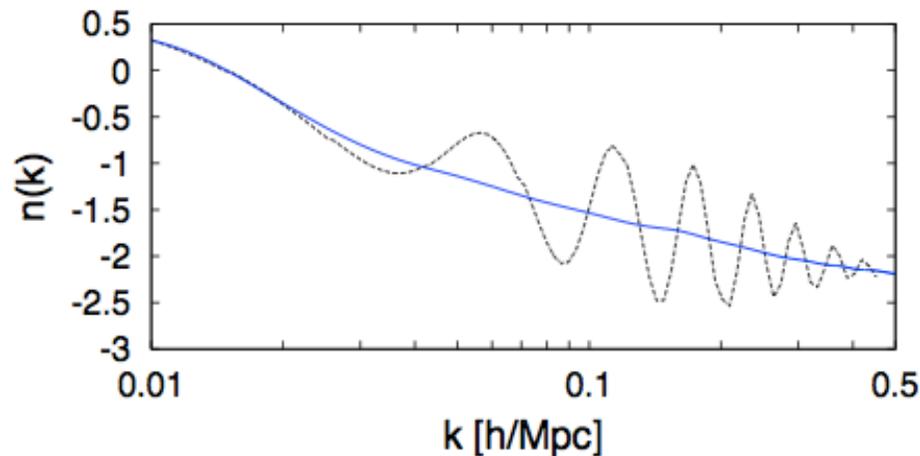
$$\tilde{b}(n, k) = \frac{1 + 0.2a_3(n+3)(qa_7)^{n+3+a_8}}{1 + (qa_7)^{n+3.5+a_8}},$$

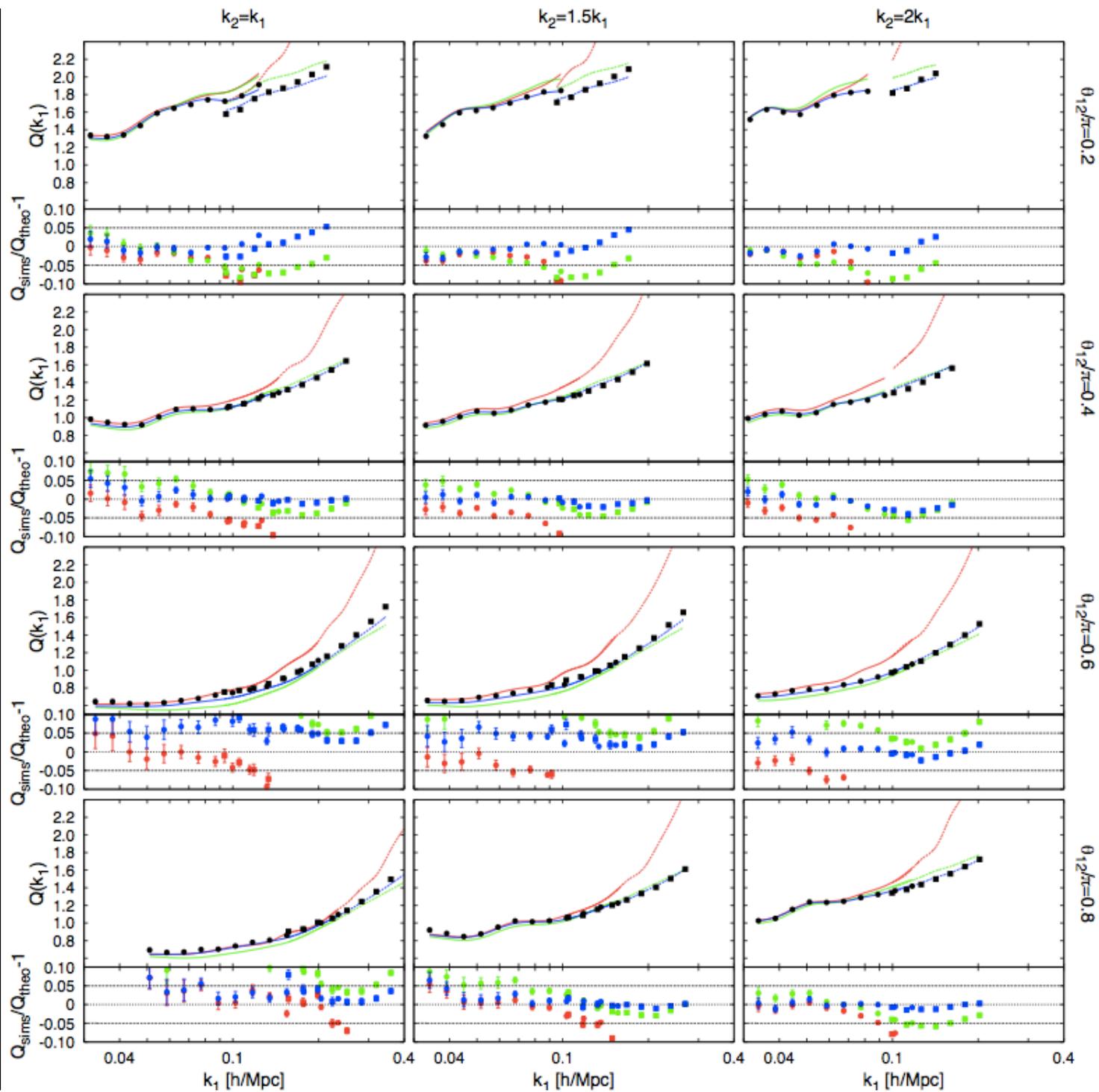
$$\tilde{c}(n, k) = \frac{1 + 4.5a_4/[1.5 + (n+3)^4](qa_5)^{n+3+a_9}}{1 + (qa_5)^{n+3.5+a_9}}.$$

3 new parameters

Recovers SC2001 for  $a_7 \rightarrow 1$  and  $a_8, a_9 \rightarrow 0$

Dewiggle the linear  $P(k)$



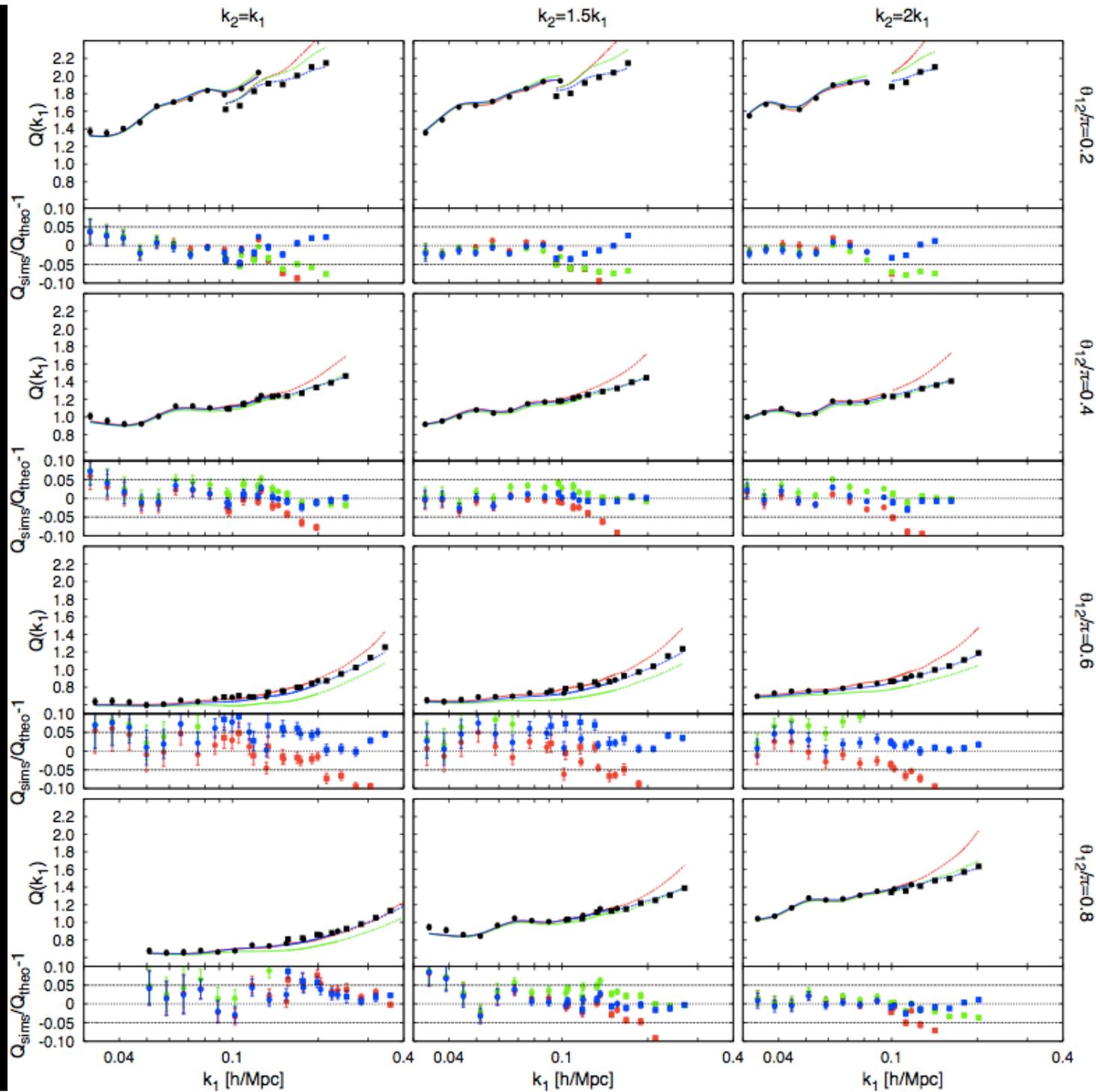


1loop

SC

This work

Z=0



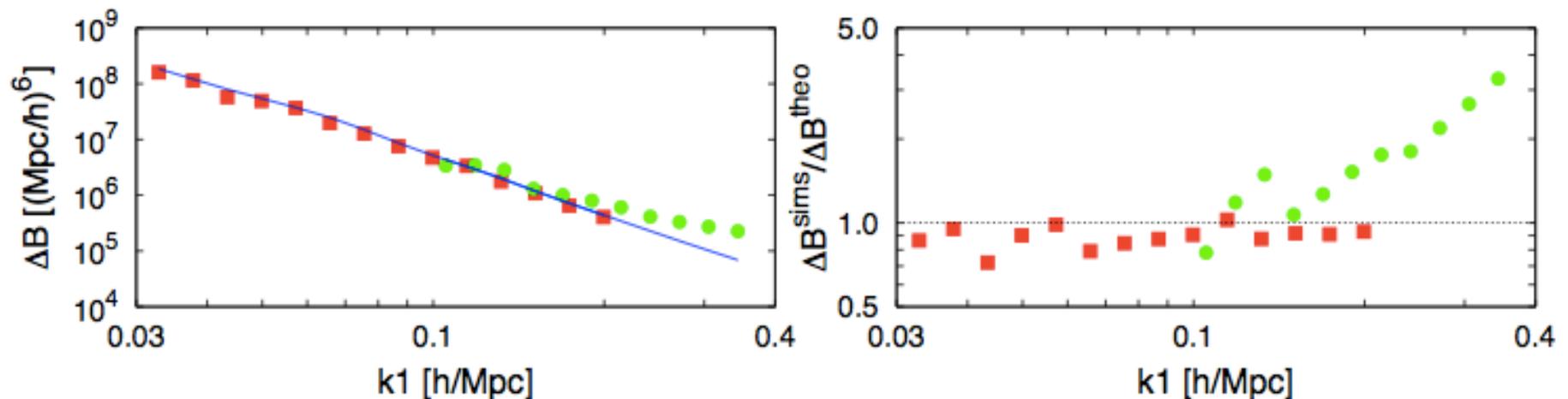
1loop

SC

This work

$Z=1$

# Aside: beware when estimating errors



$k_2/k_1 = 1$  and  $\theta_{12} = 0.6\pi$

$$\Delta \hat{B}^2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = s_B \frac{V_f}{V_B} (2\pi)^3 P(k_1) P(k_2) P(k_3)$$

$$V_f = (2\pi)^3 / L_b^3$$

$$\simeq 8\pi^2 k_1 k_2 k_3 \Delta k^3$$

# Performance & summary

For  $z < 1.5$  and  $k < 0.4 \text{ h/Mpc}$

Typically better than 5% at  $z=0$   
and never worse than 10% (equilateral configurations)

For non-standard LCDM model i.e. an  $f(R)$  model the formula works to 3% typically (10% in the worst cases  $\sim$  equilateral configurations)

Disclaimer: this is for the dark matter and in real space!

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END