Taming cosmological statistics: Logs, weightings, and Gaussianization

Mark Neyrinck
Johns Hopkins University

Main Collaborators:

- Nuala McCullagh, Bridget Falck, Xin Wang, Miguel Aragón-Calvo, Alex Szalay (JHU), István Szapudi, Julien Carron* (IfA, Hawaii)

*- see his talk, too

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Outline

- Problem: two-point statistics poorly describe highly non-Gaussian fields
  - Solution: Gaussianization
  - Can be considered as a weighting more by volume than mass
  - Information gain: measurements from simulations
  - A new analytic model of the transformed-density correlation function
    - (detour: spherical-collapse local Lagrangian approximation)

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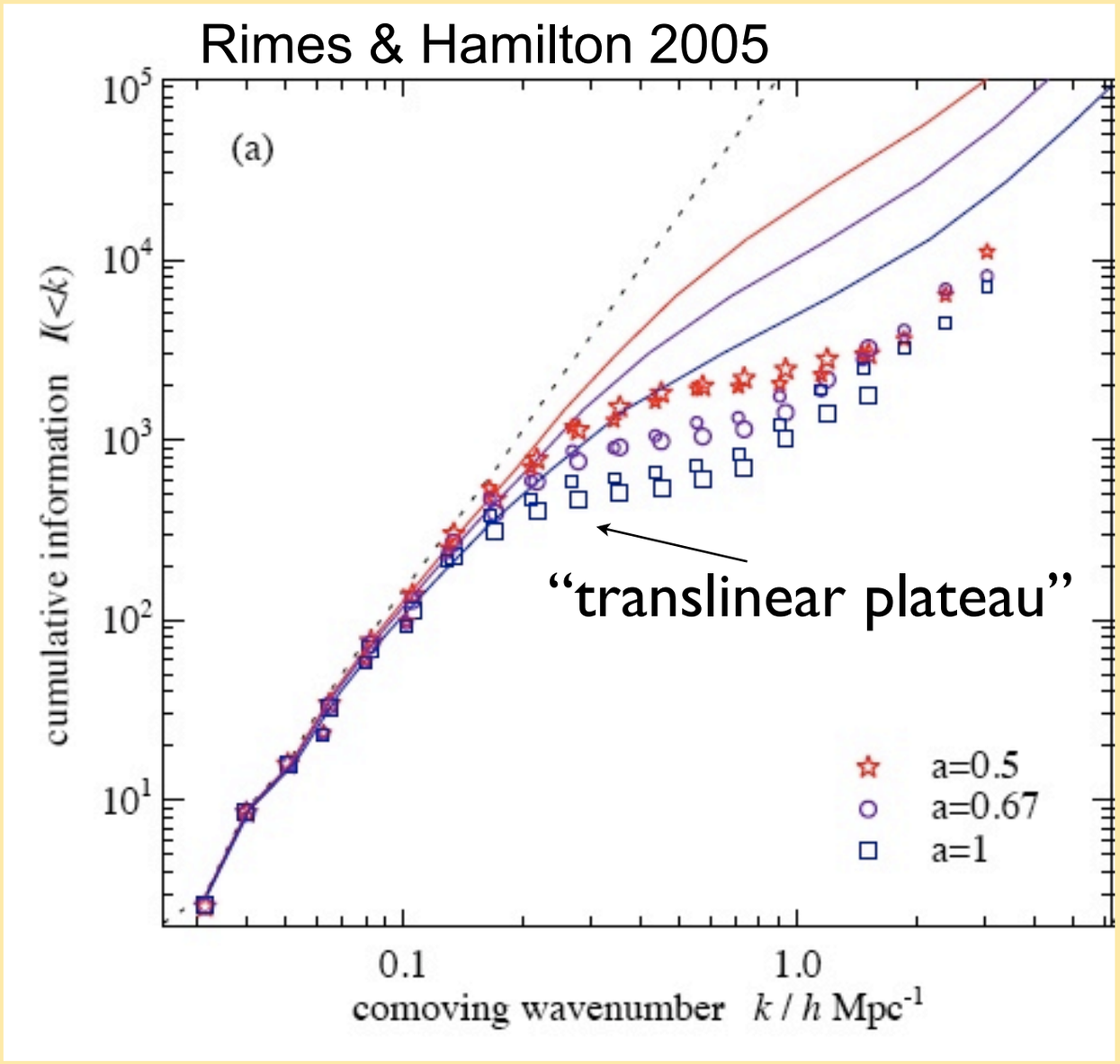
Scales		Stats
Hubble radius	Many Gpc	Cosmological parameters
Linear, Gaussian scales	x > ~100 Mpc/h k < ~0.1 h/Mpc	Power spectrum
Nonlinear scales	x < ~50 Mpc/h	Morphological statistics?

Why not use the power spectrum on small scales?

Fisher-information loss (small-scale covariance)
⇔ Fourier-mode coupling, loss in degrees of freedom

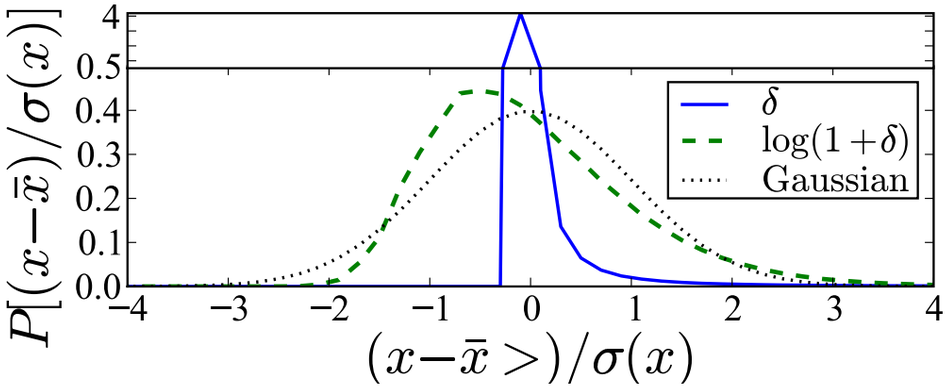
independent Fourier modes

large, linear scales



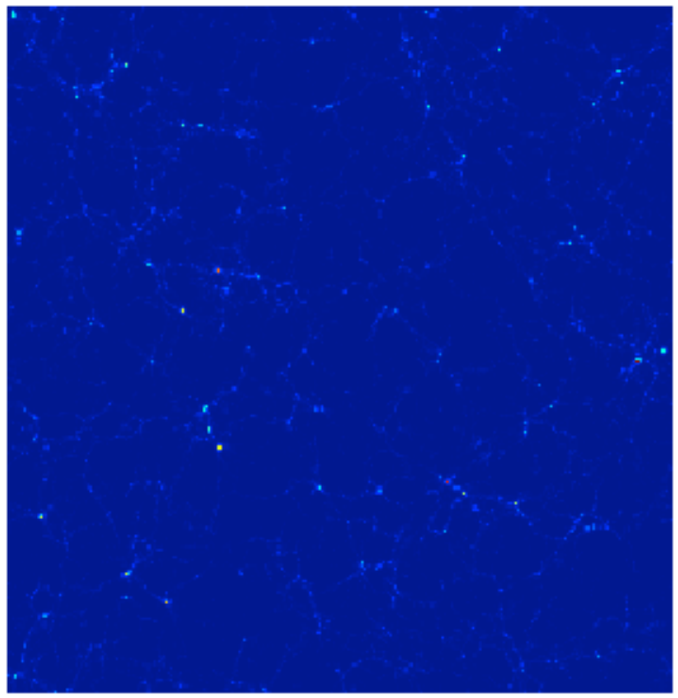
small, halo scales

Sharp peaks overweighted in mass-weighting -- try a ~volume-weighting.

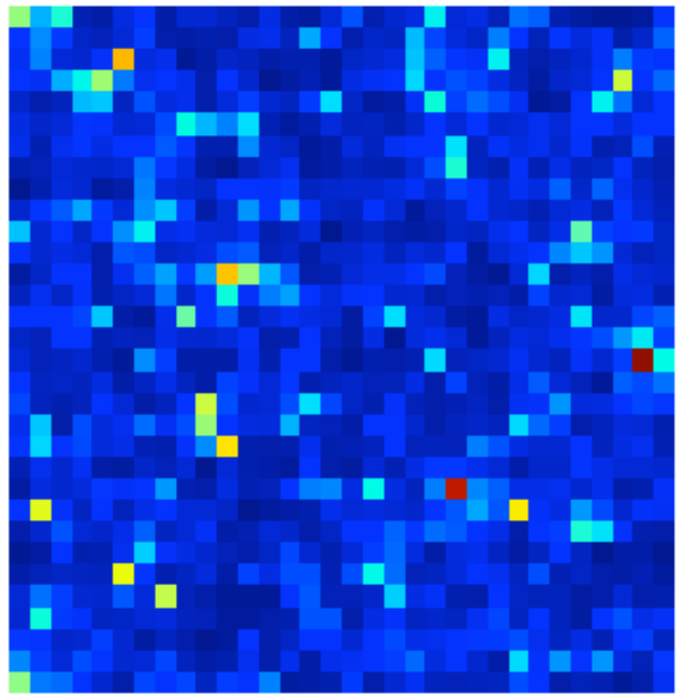


$$\delta(\mathbf{x}) \equiv \frac{\rho(\mathbf{x})}{\bar{\rho}} - 1$$

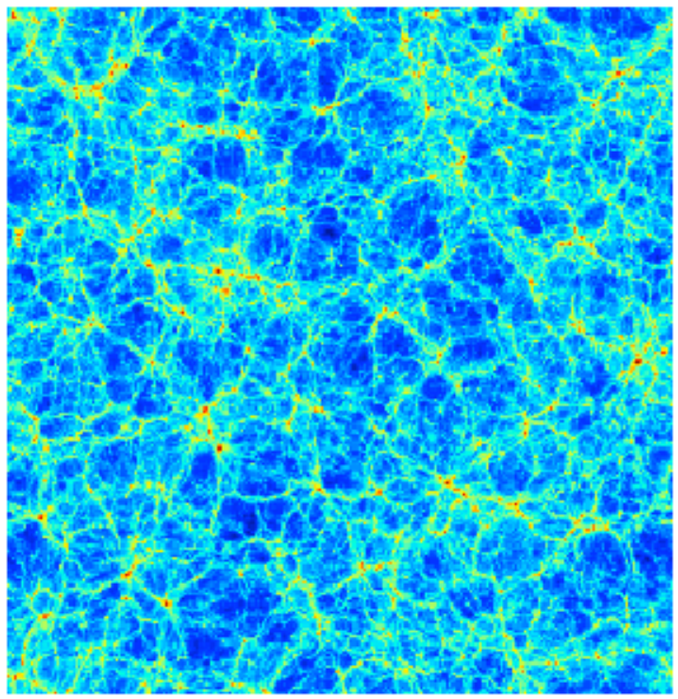
Overdensity δ



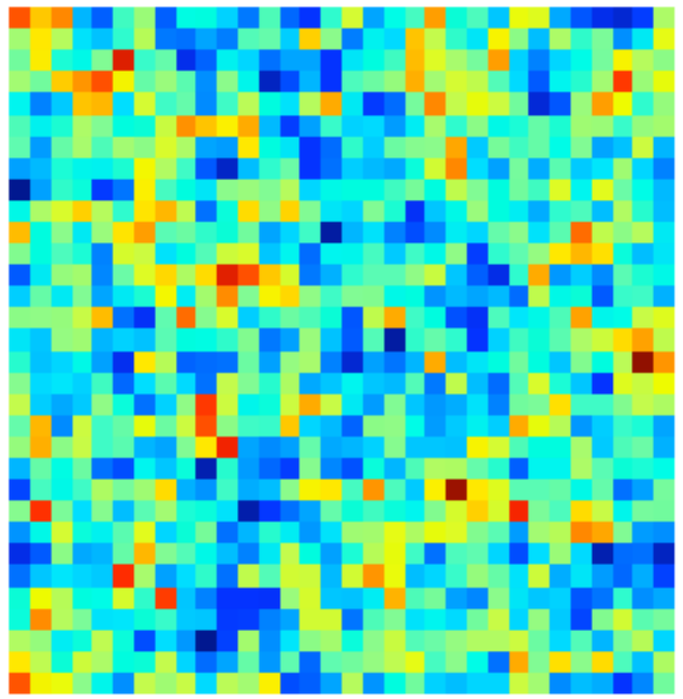
What we analyze



$\ln(1+\delta)$



$\ln(1+\delta)$ (low−res)



Millennium Sim. (Springel et al. 2005)

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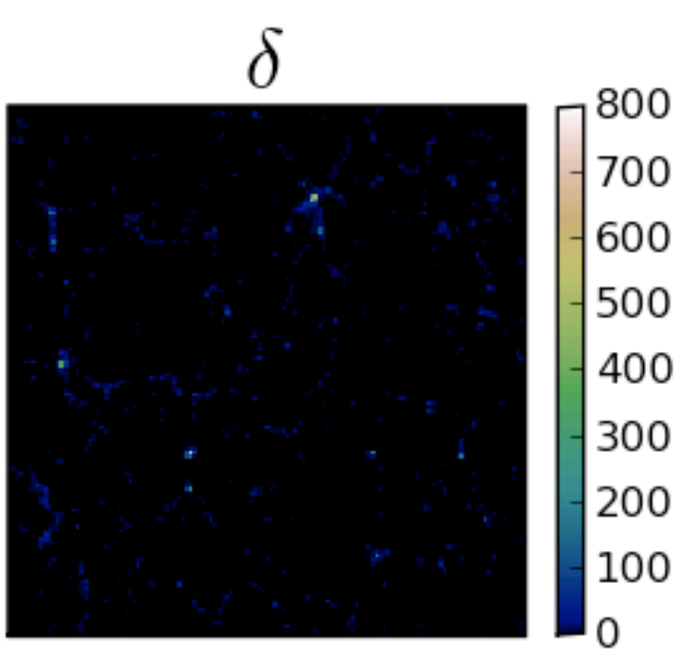
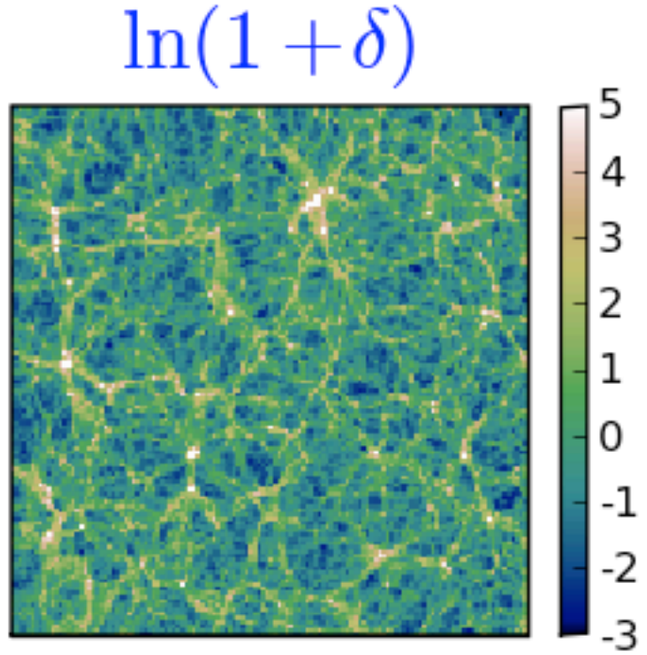
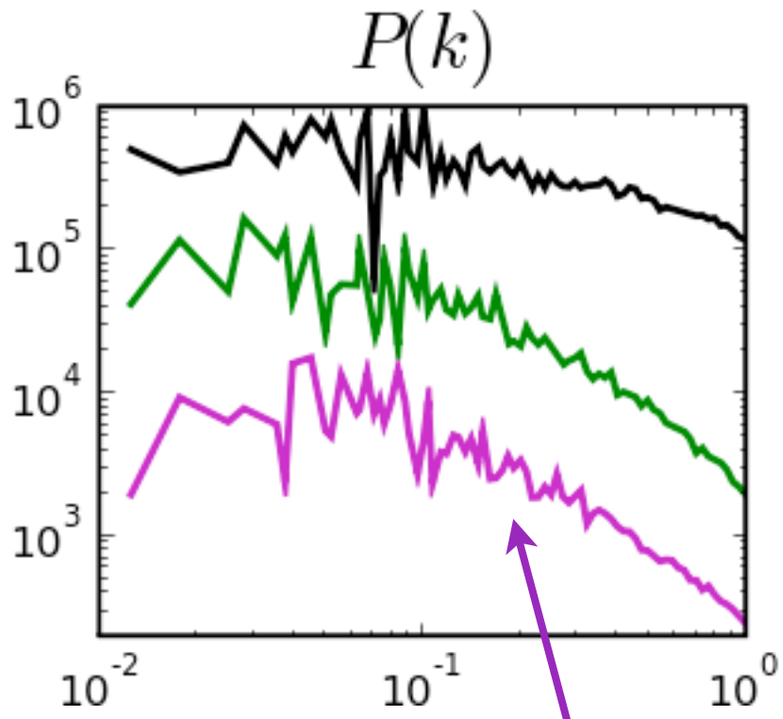
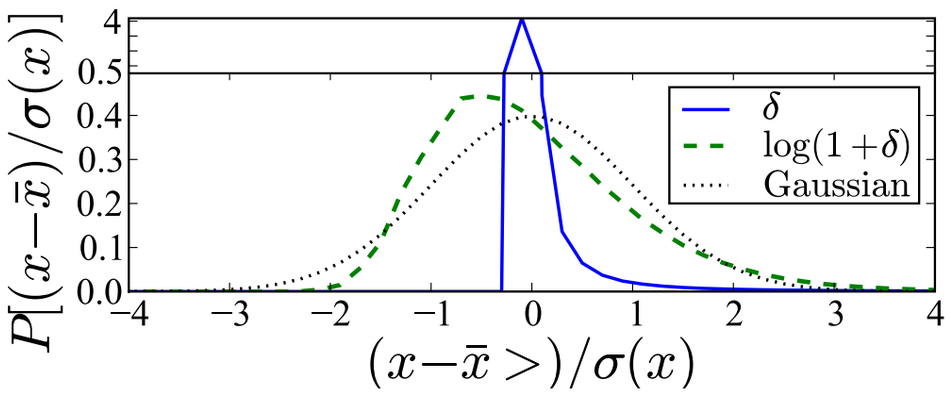






Initial power spectrum

Substantial dependence of the power spectrum on the 1-pt PDF. Suggests decoupling the 1-pt from 2-pt statistics.



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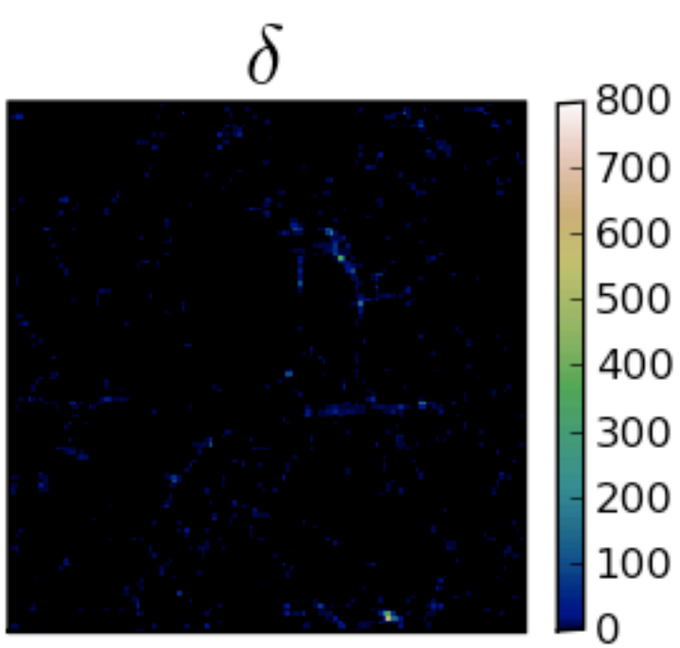

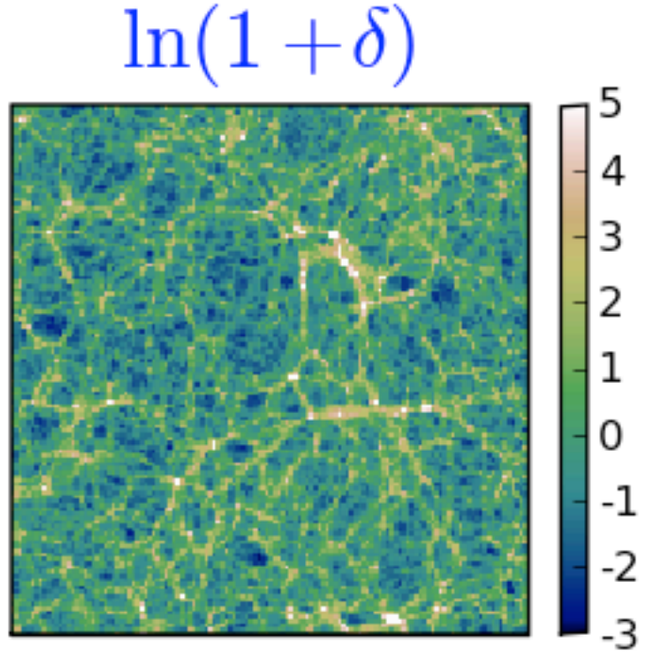

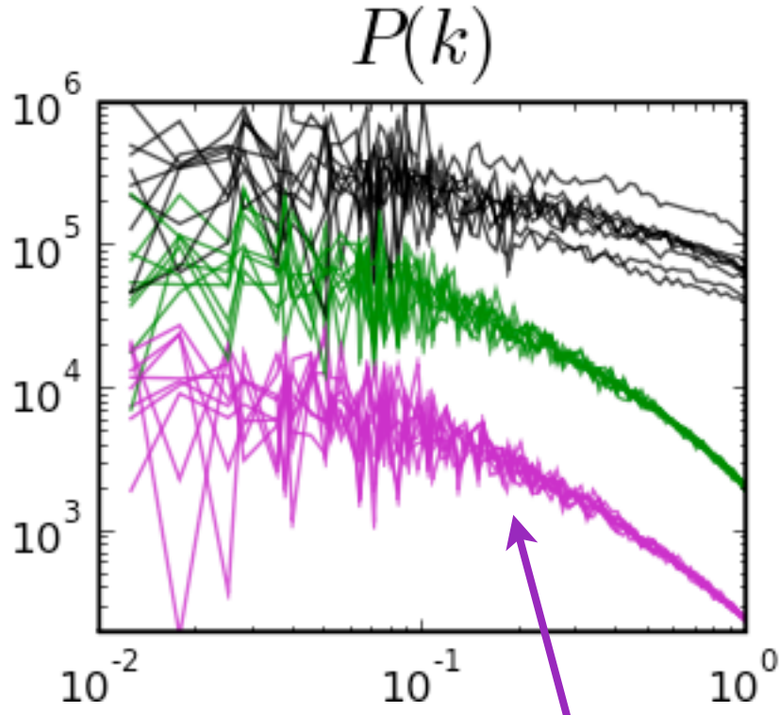

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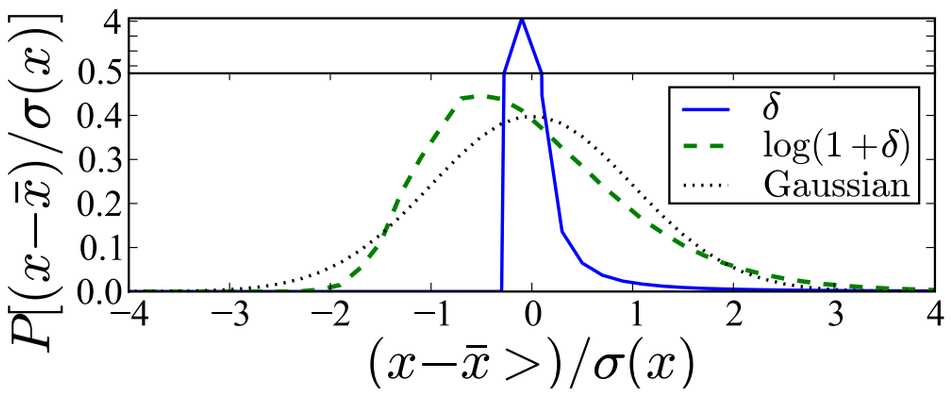

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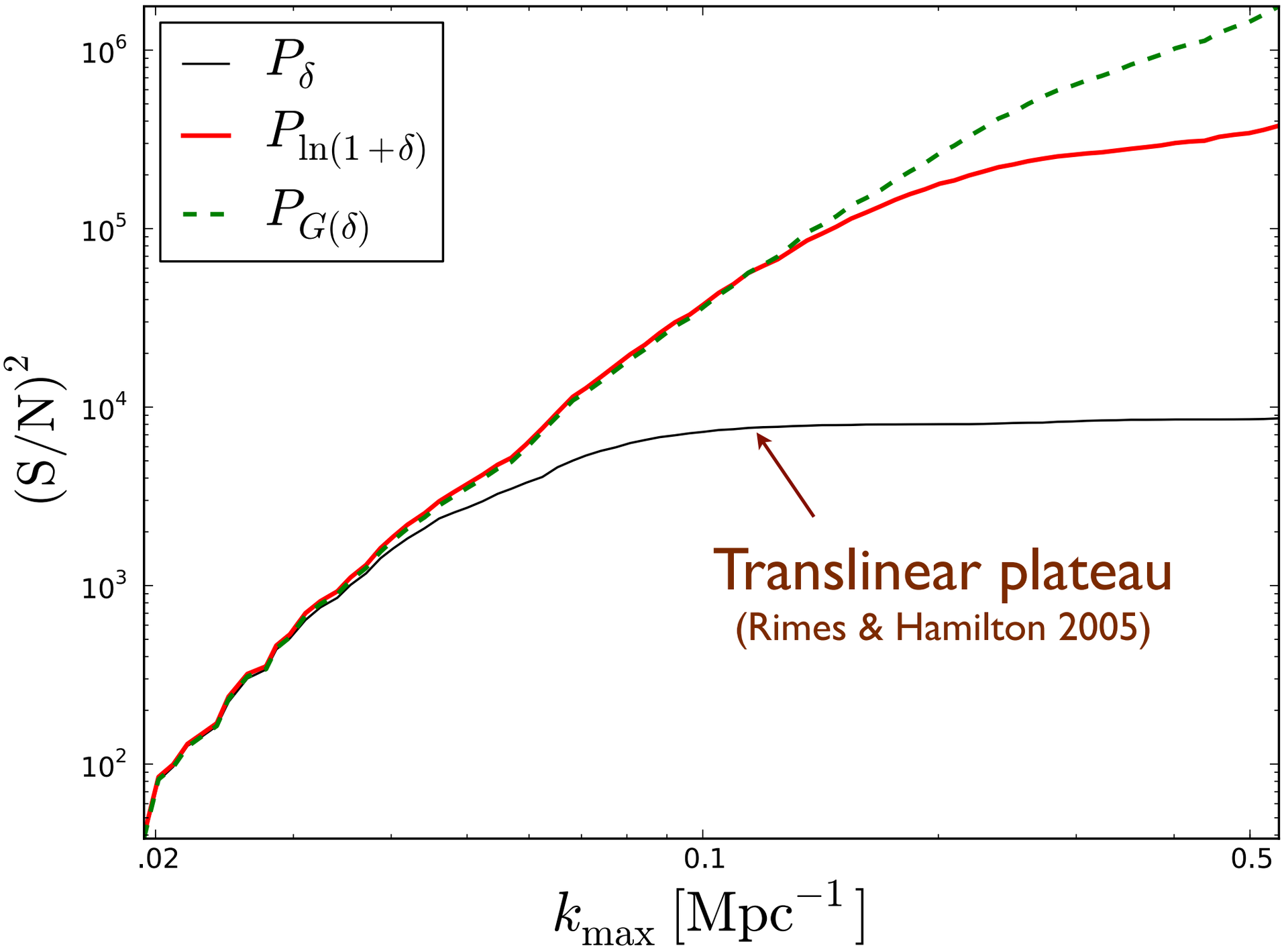
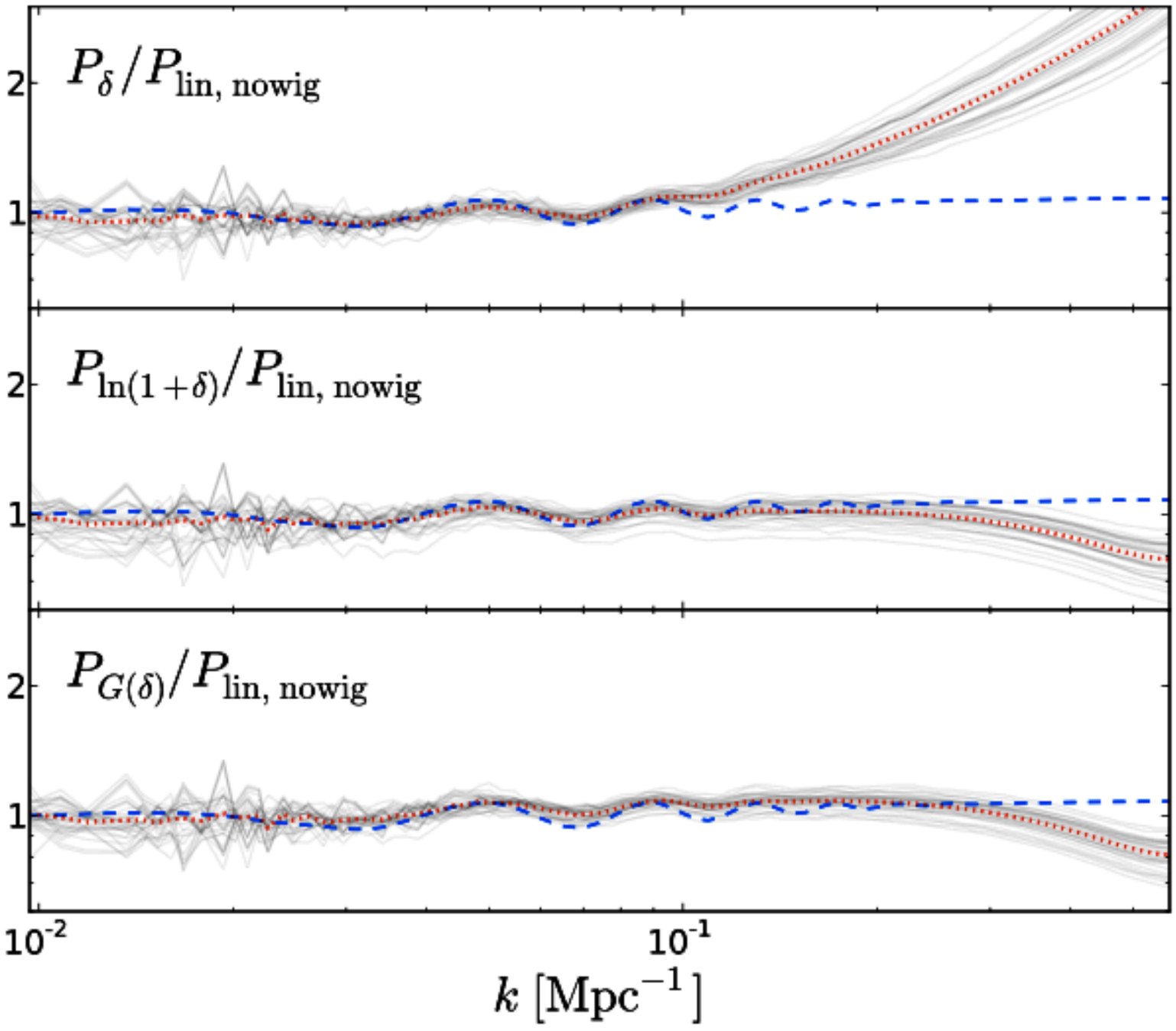
Intrinsic Fisher information greatly enhanced

statistically independent Fourier modes as a function of resolution (MN et al. 2009, 2011)

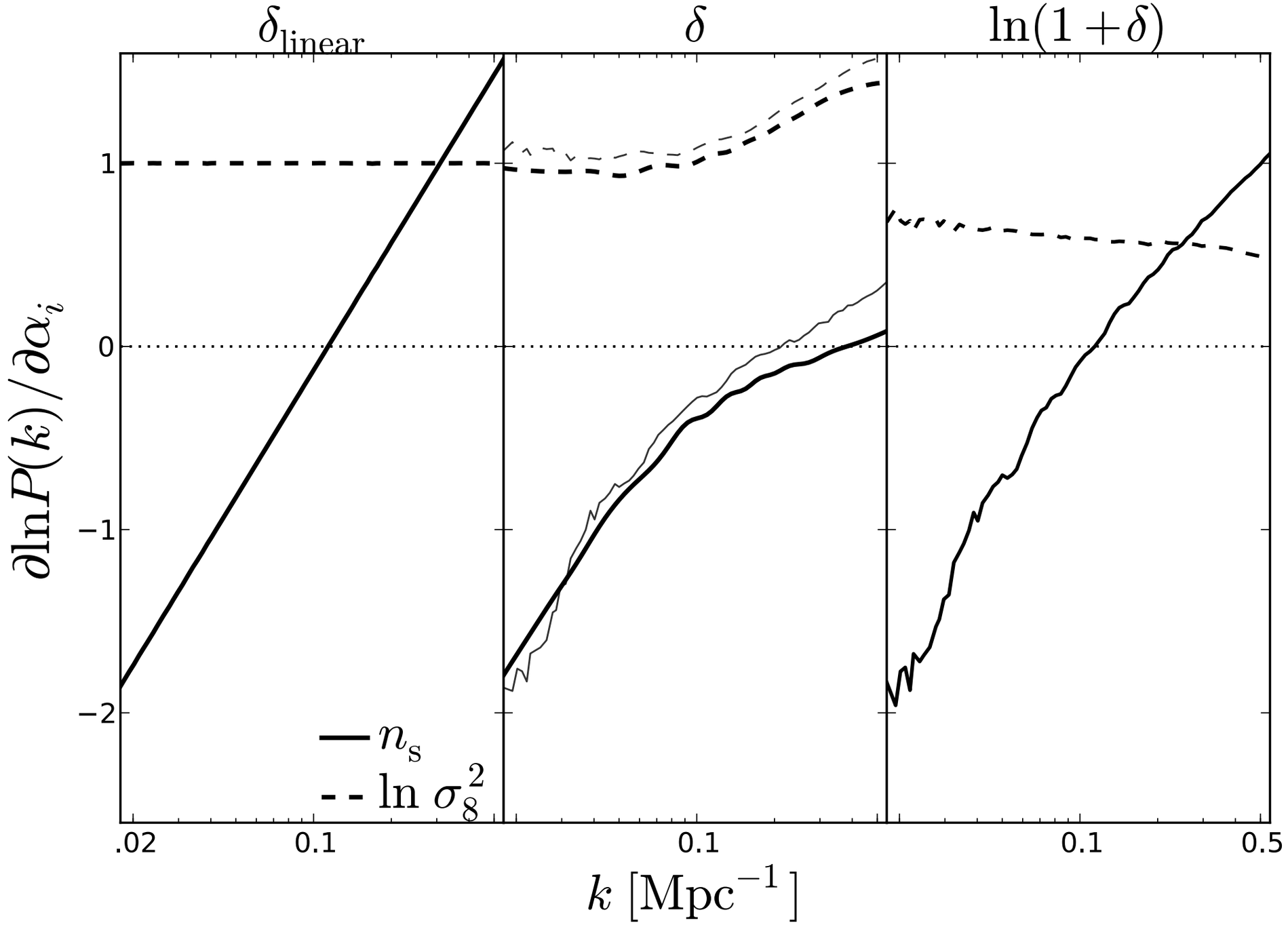


A fairer weighting also straightens the linear → nonlinear transfer function

37 Coyote Universe simulations shown, spanning a range of concordance cosmological params



Dependence on parameters



Measured from the Coyote Universe simulations (Heitmann et al, 2008, 2009, 2010).

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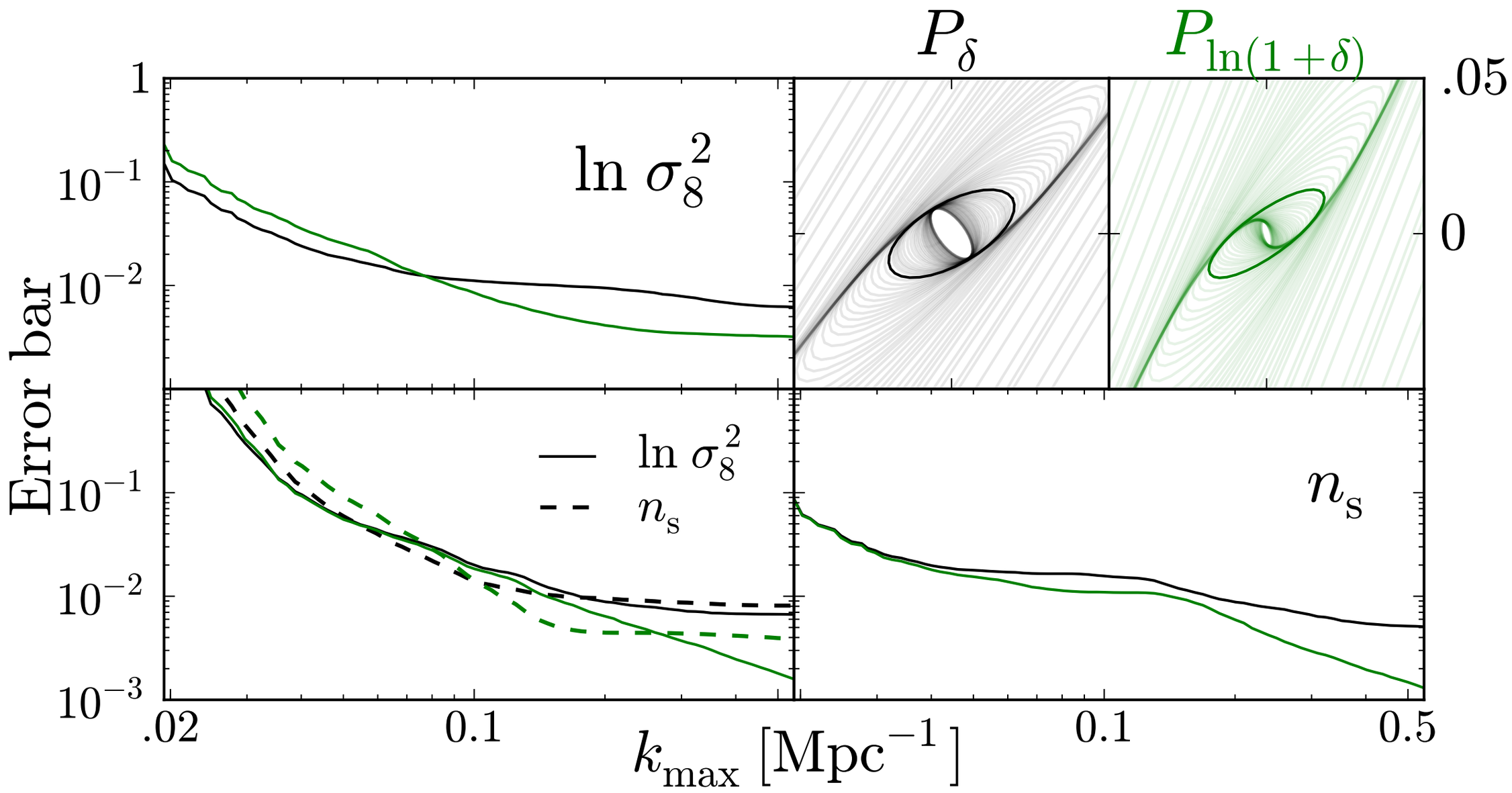
$P_{\ln(1+\delta)}$ significantly more cosmologically informative.



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What if we move densities, not mass, around? Violates mass conservation, but should give a "better" weighting.
(i.e. statistically more friendly, and with more fidelity to initial conditions)

* Decouples mass-element bulk motion from gravitational enhancement

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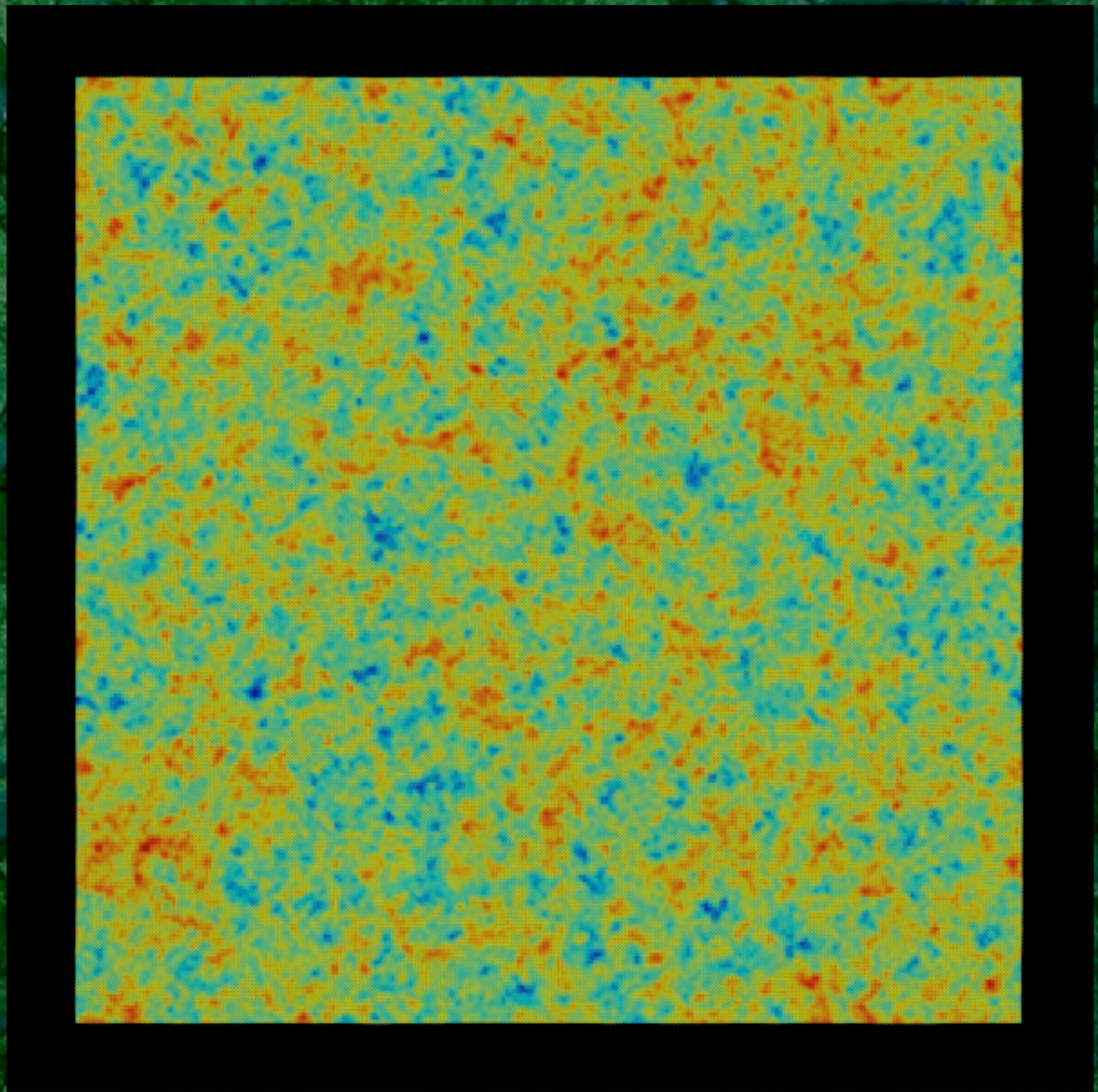

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How to model analytically?
First, interlude about Bernardeau's (1992, 1994) spherical-collapse approximation, true in the limit $\Omega_\Lambda=0$, $\Omega_m \to 0$,

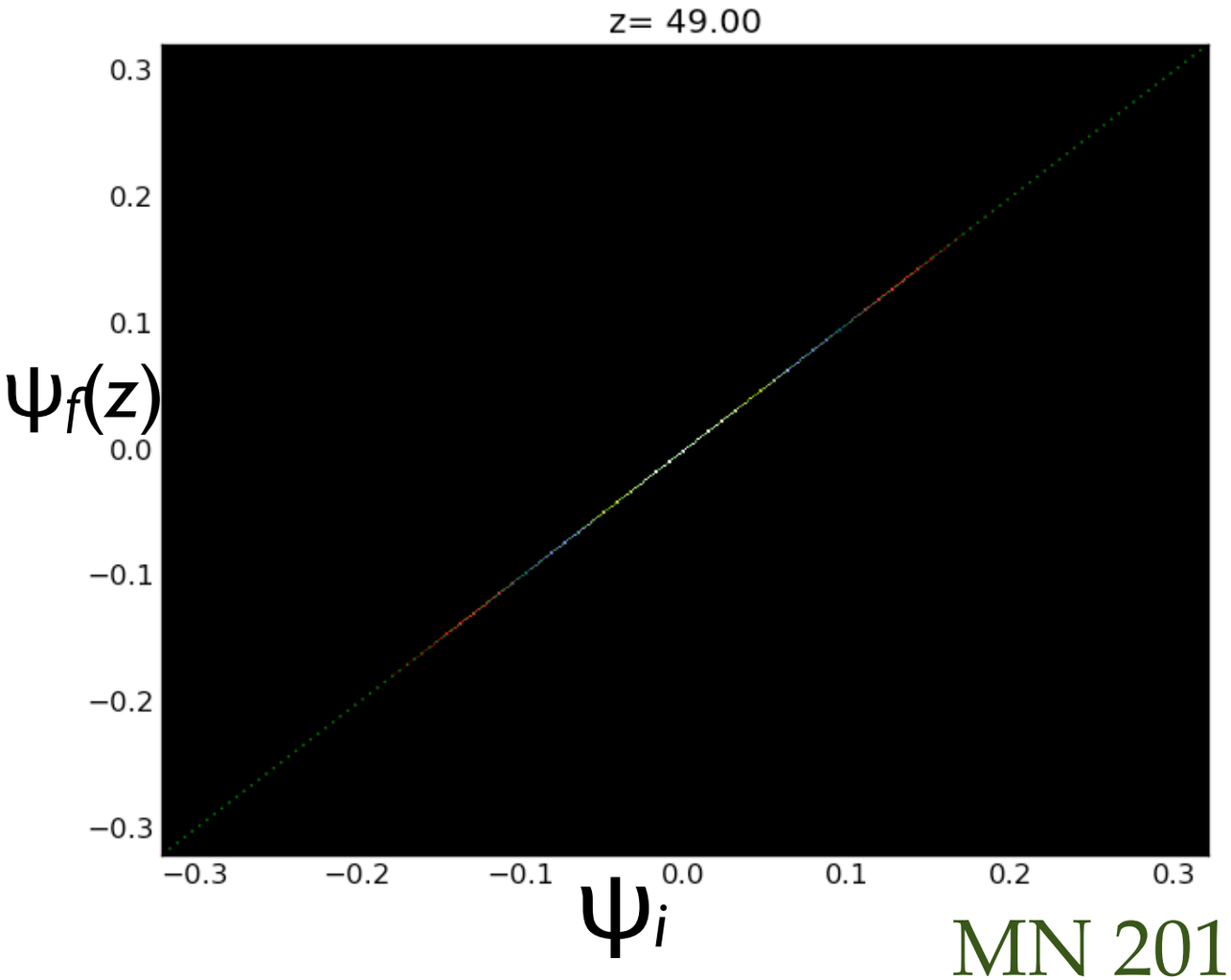
$$V(t) = V_0 \left(1 - \frac{2}{3}\delta_{\rm lin}\right)^{3/2}$$

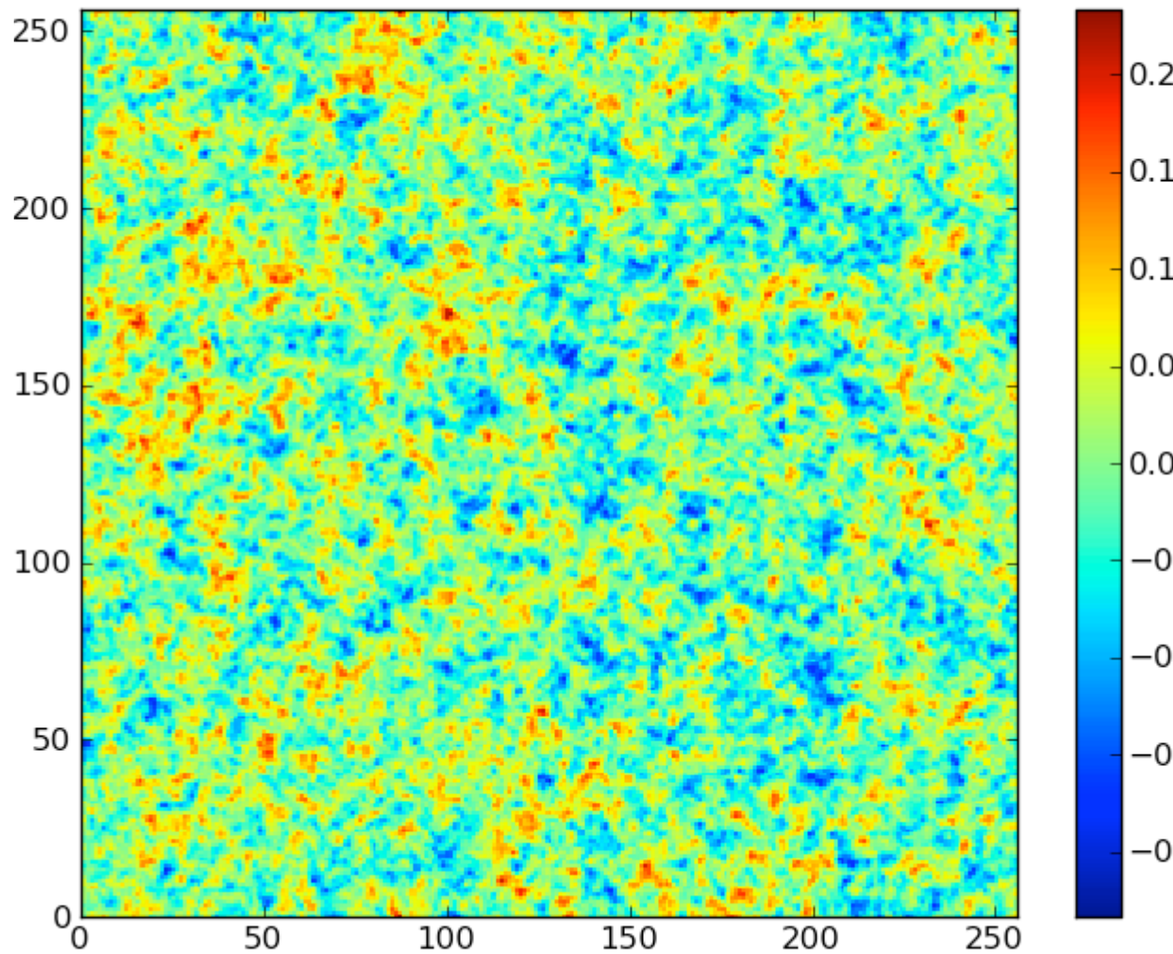
Allows a good model of the Lagrangian divergence of the displacement field, $\psi(\mathbf{q}) \equiv \boldsymbol{\nabla}_q \cdot \boldsymbol{\Psi}(\mathbf{q})$ (Mohayaee et al. 2006, Neyrinck 2012):

$$\psi_{\rm sc} = 3\left[\left(1 + \frac{2}{3}\psi_{\rm lin}\right)^{1/2} - 1\right]$$

$$\psi(\mathbf{q}) \equiv \nabla_q \cdot \mathbf{\Psi}(\mathbf{q})$$ behavior

- Zel'dovich (1970): $\psi = -\delta_{linear}$.
- $\psi = -3$: halo formation, where $\nabla_L \cdot x_f = 0$.





MN 2012, arXiv:1204.1326

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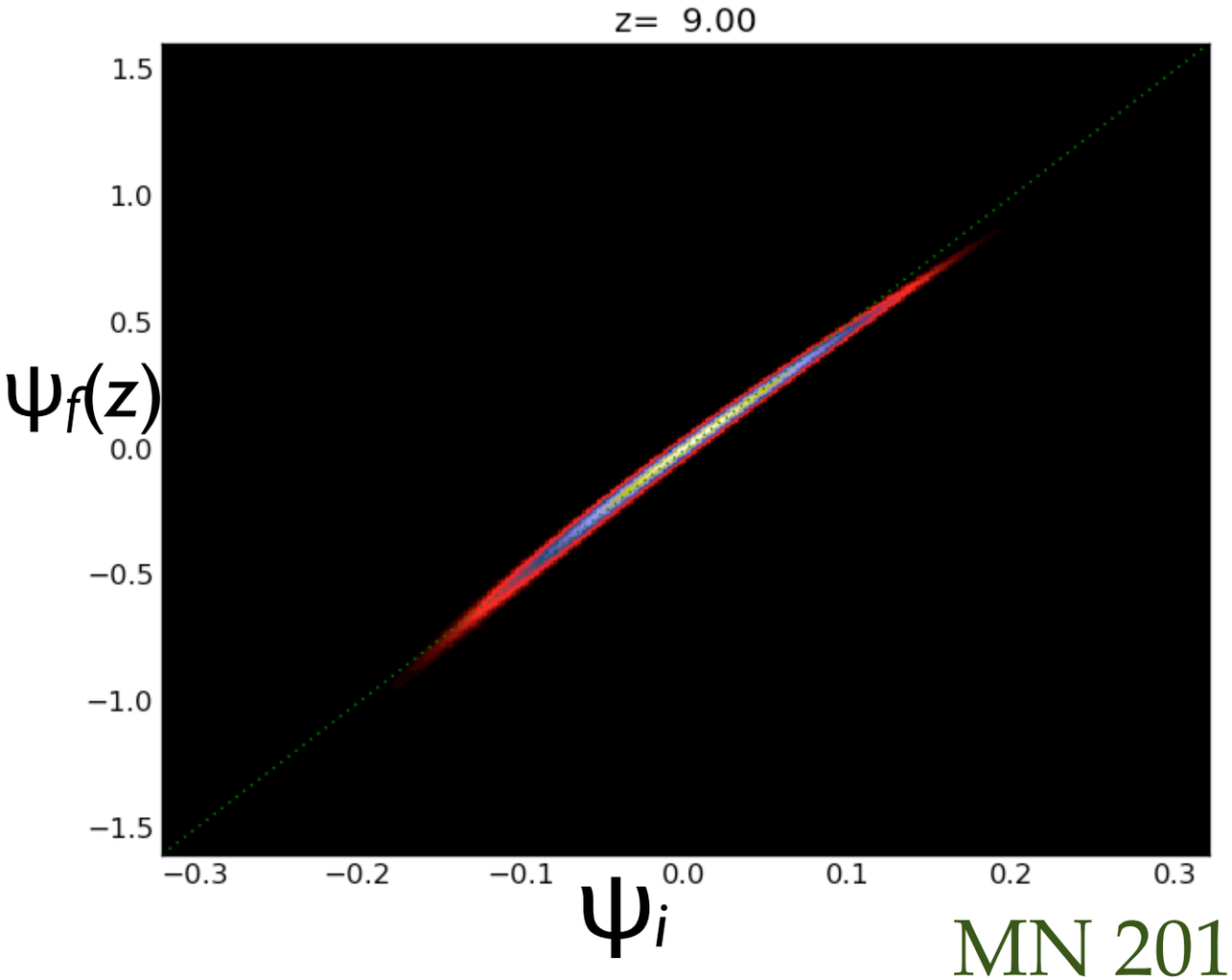

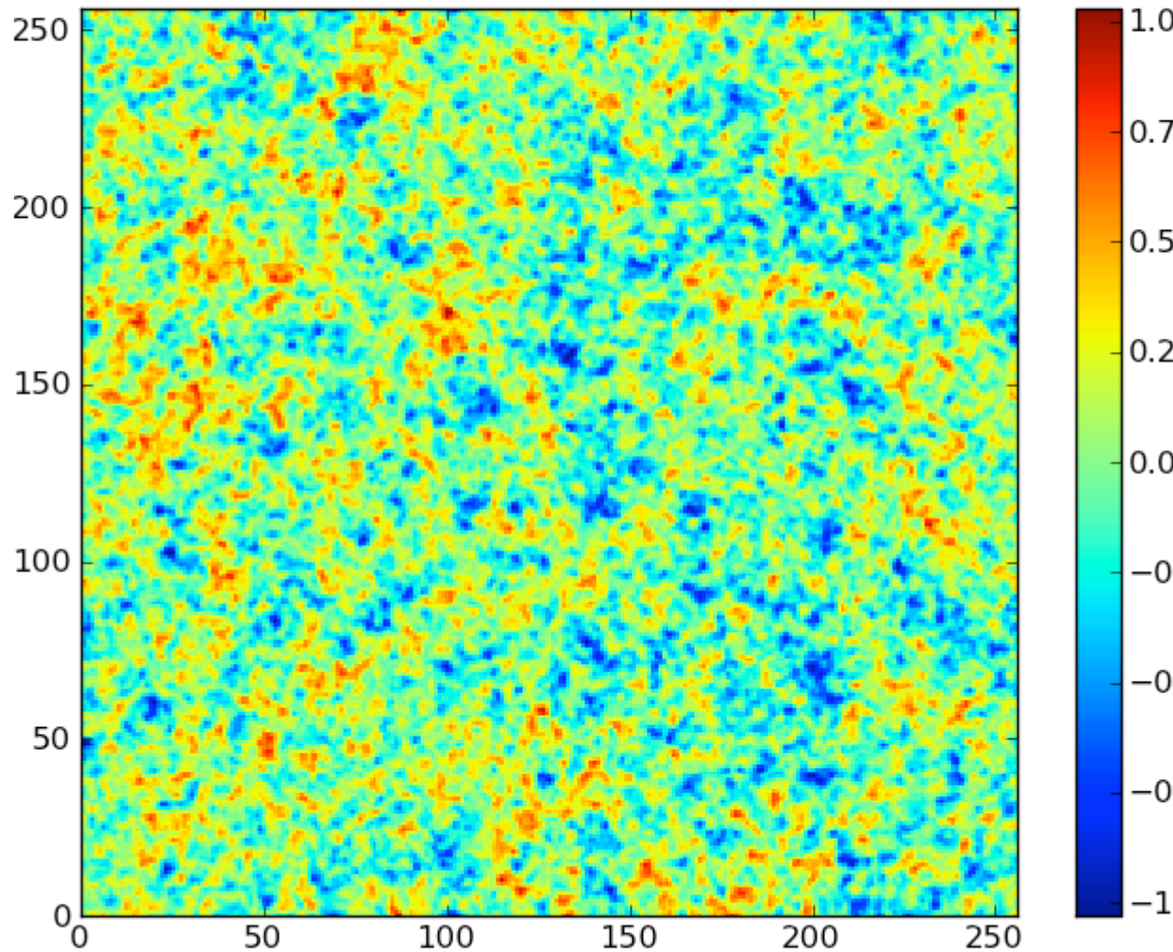

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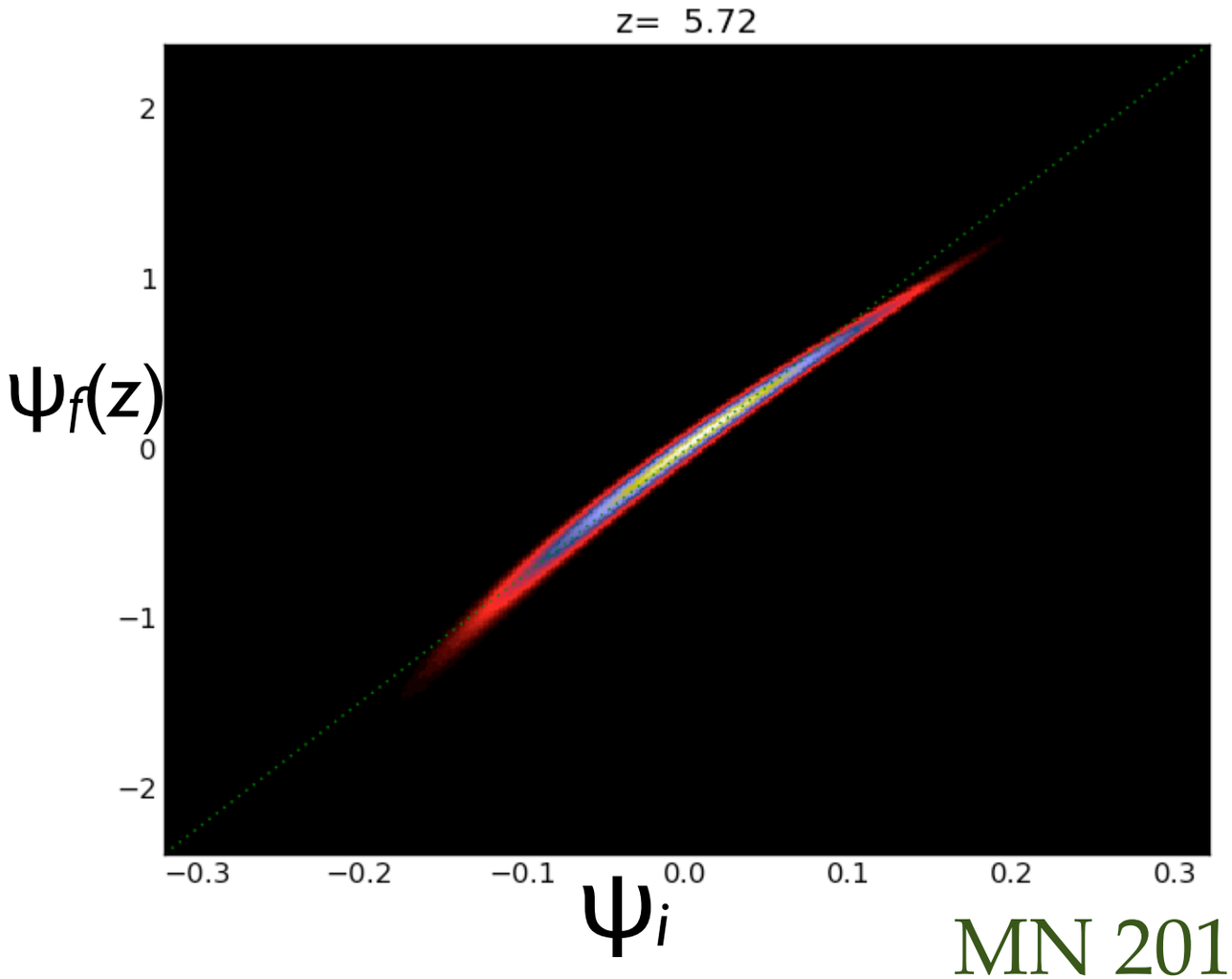

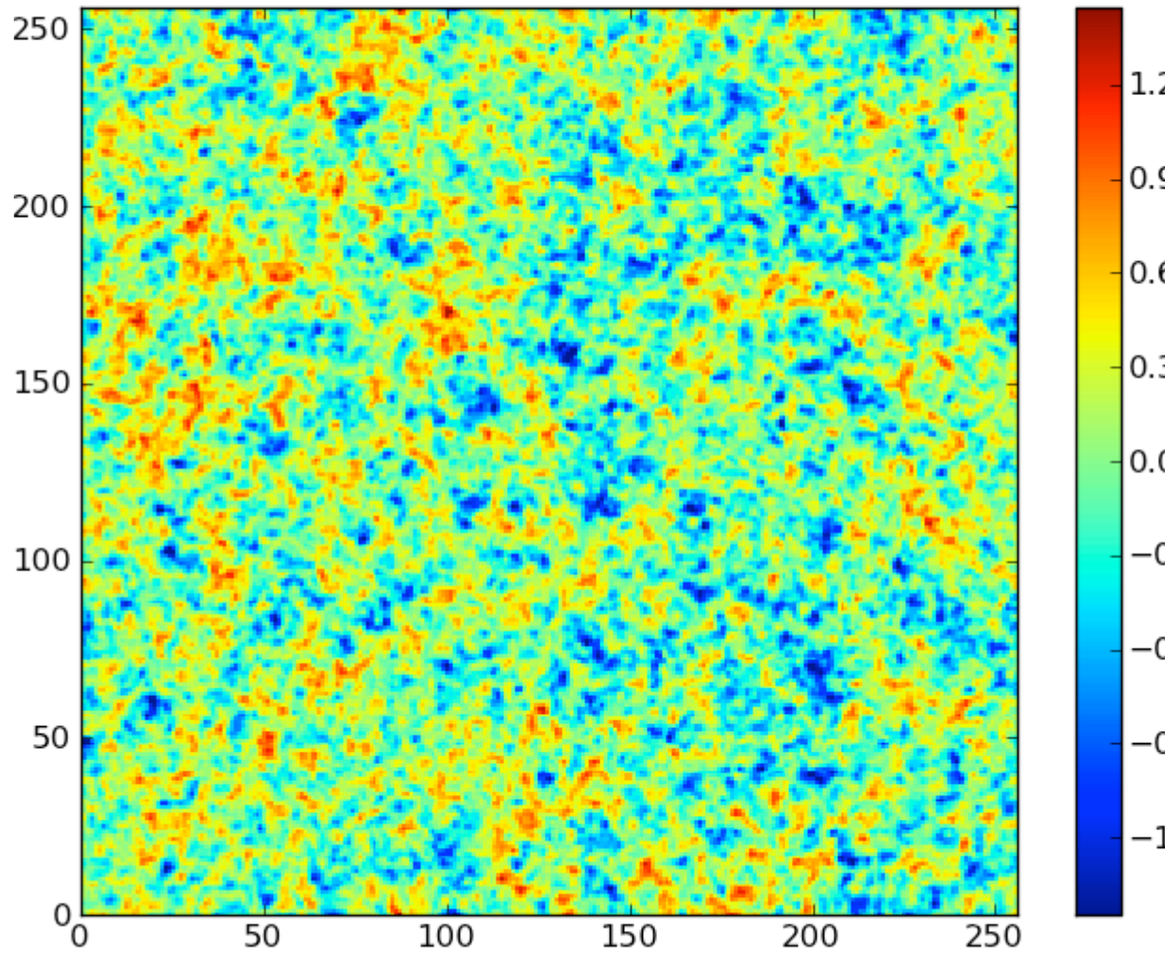

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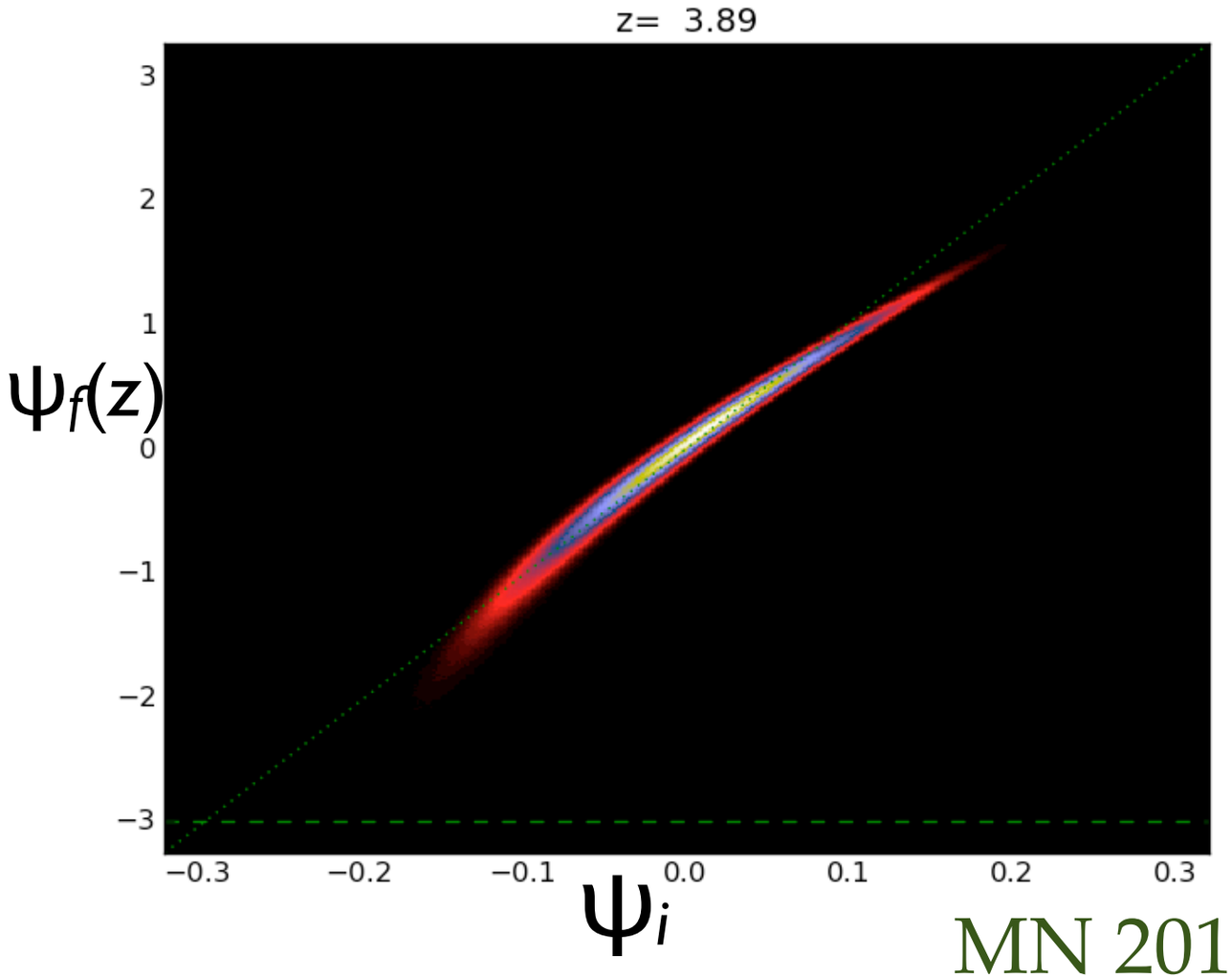

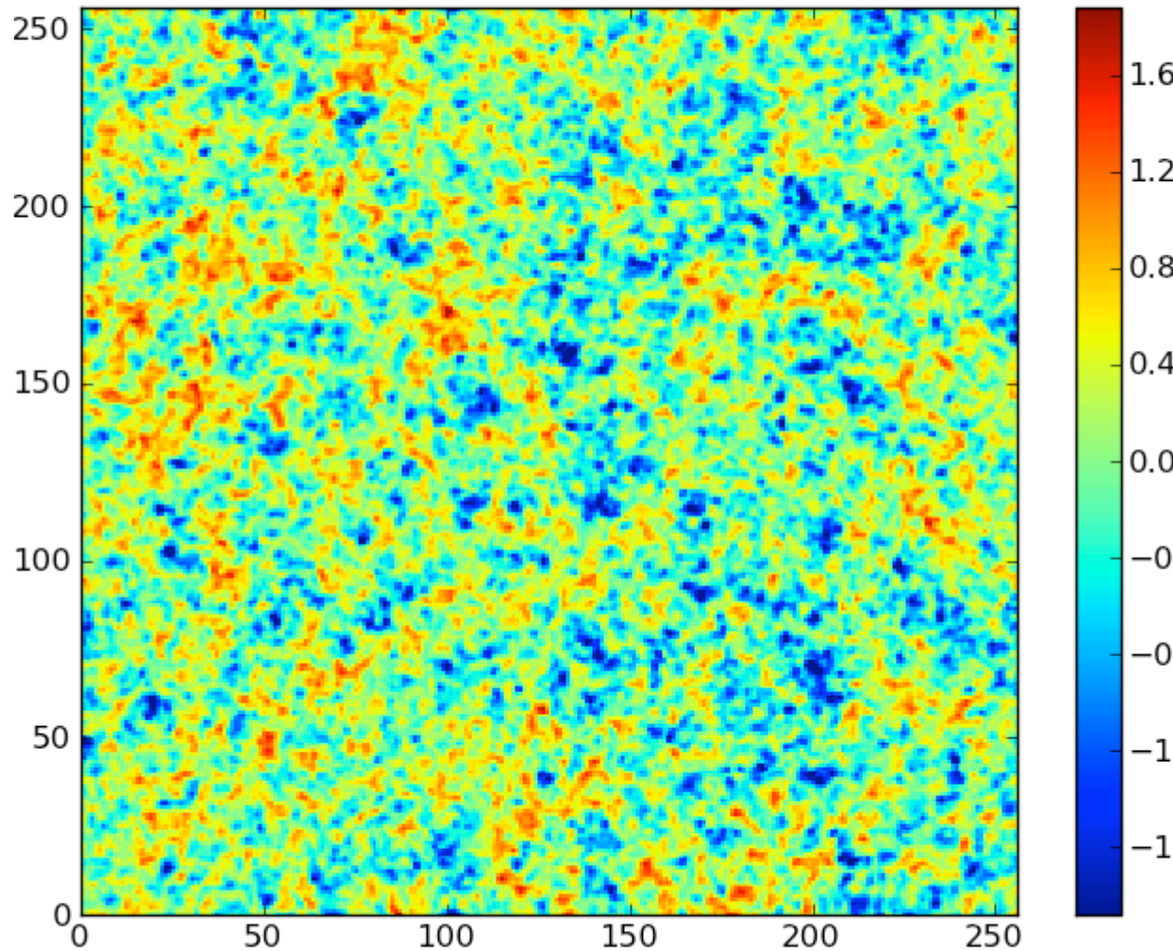

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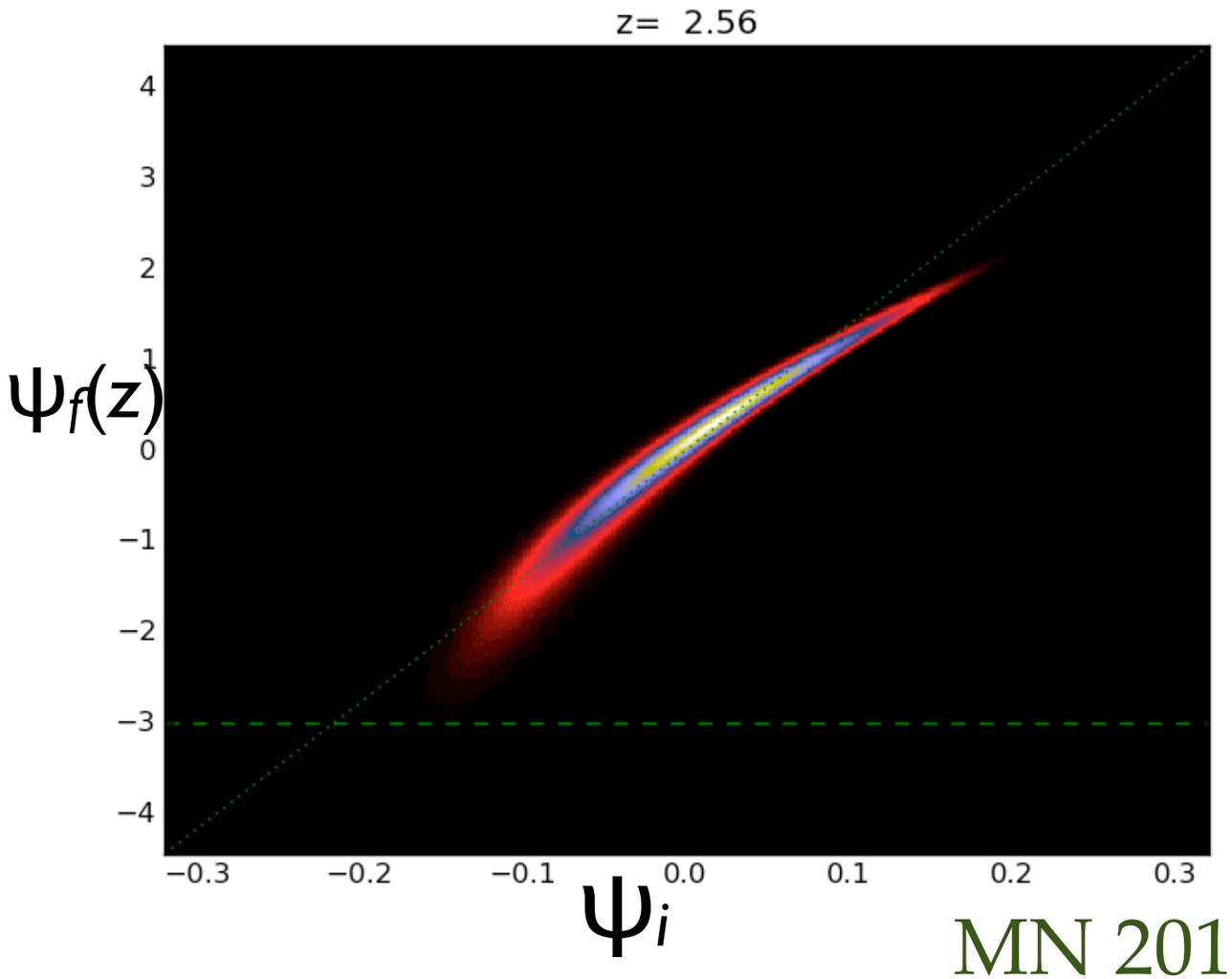

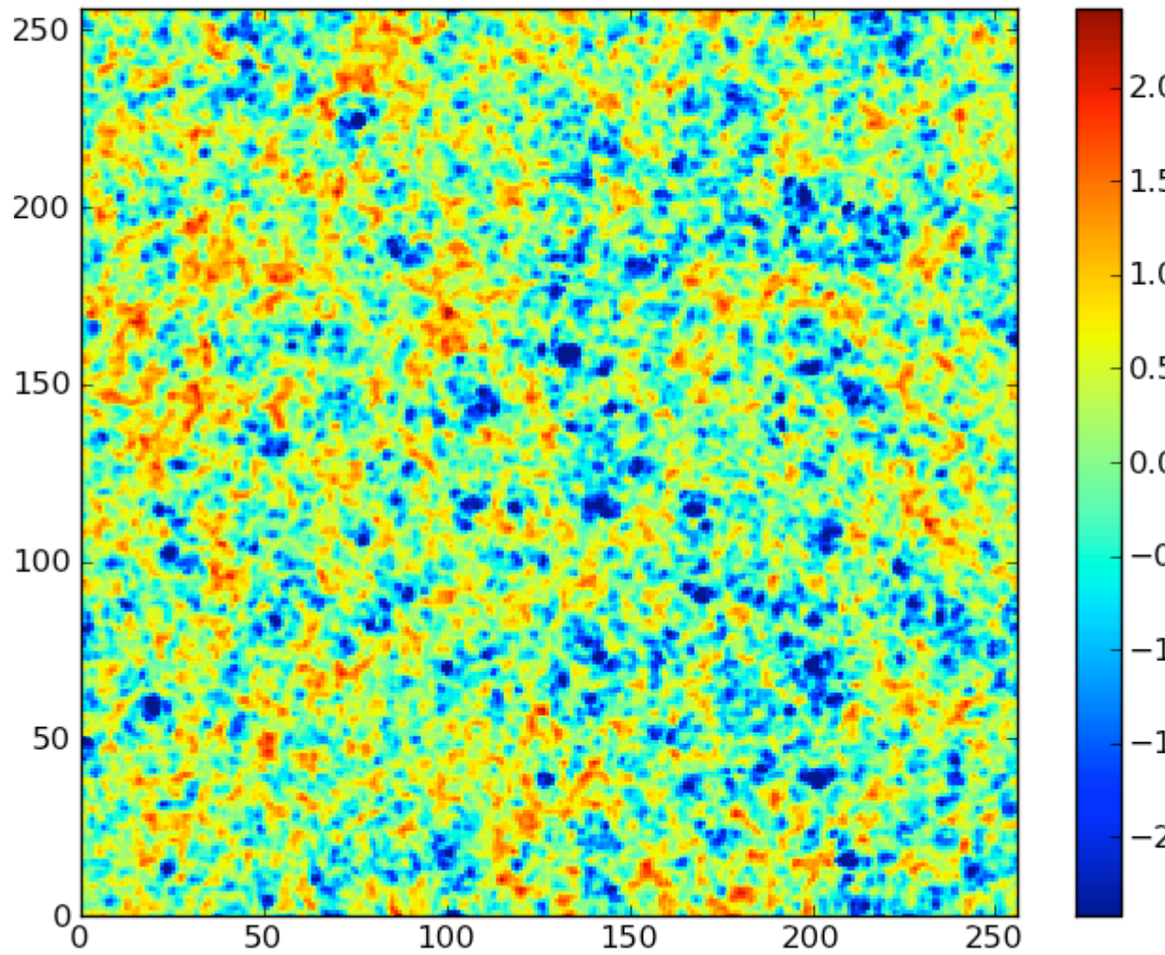

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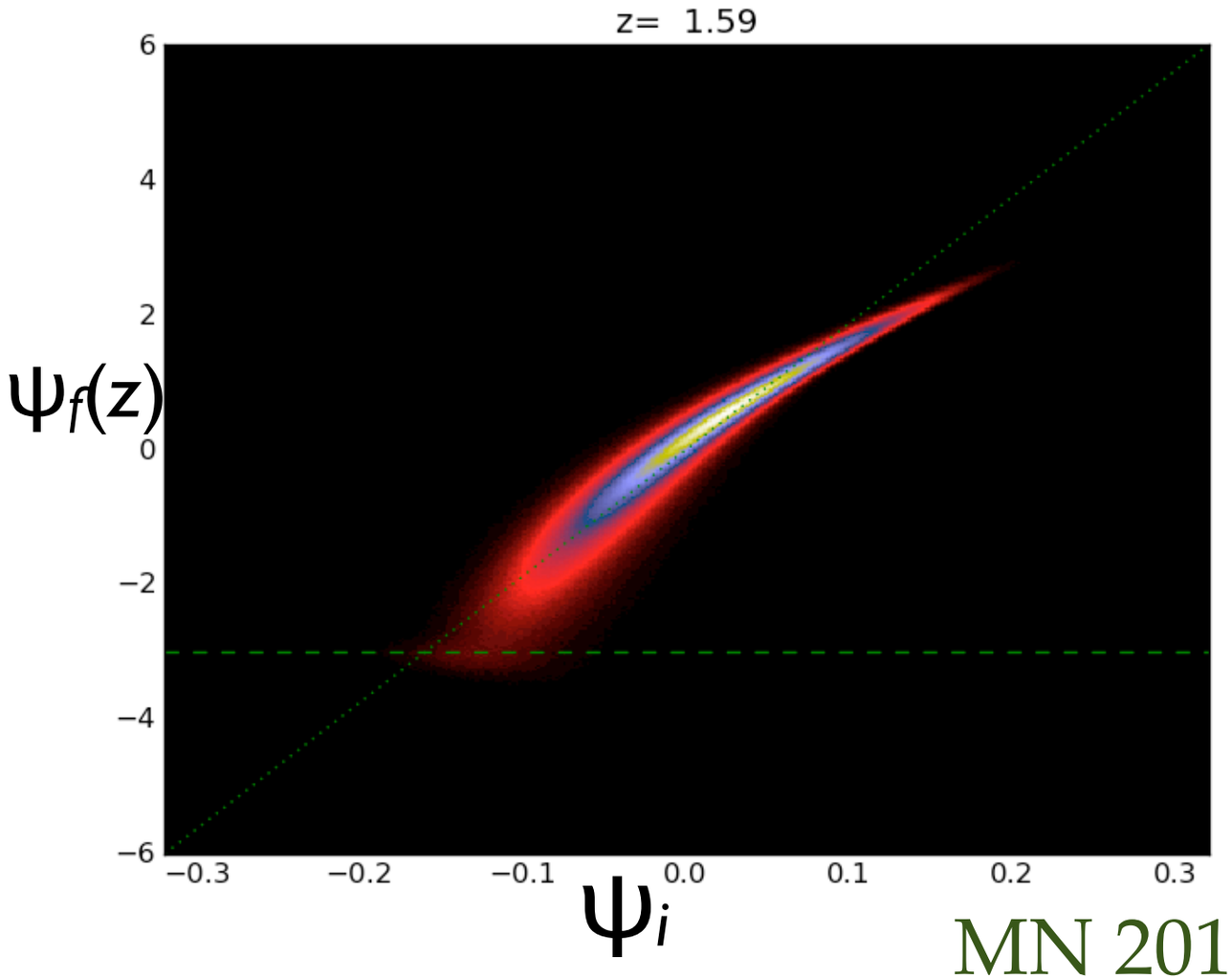

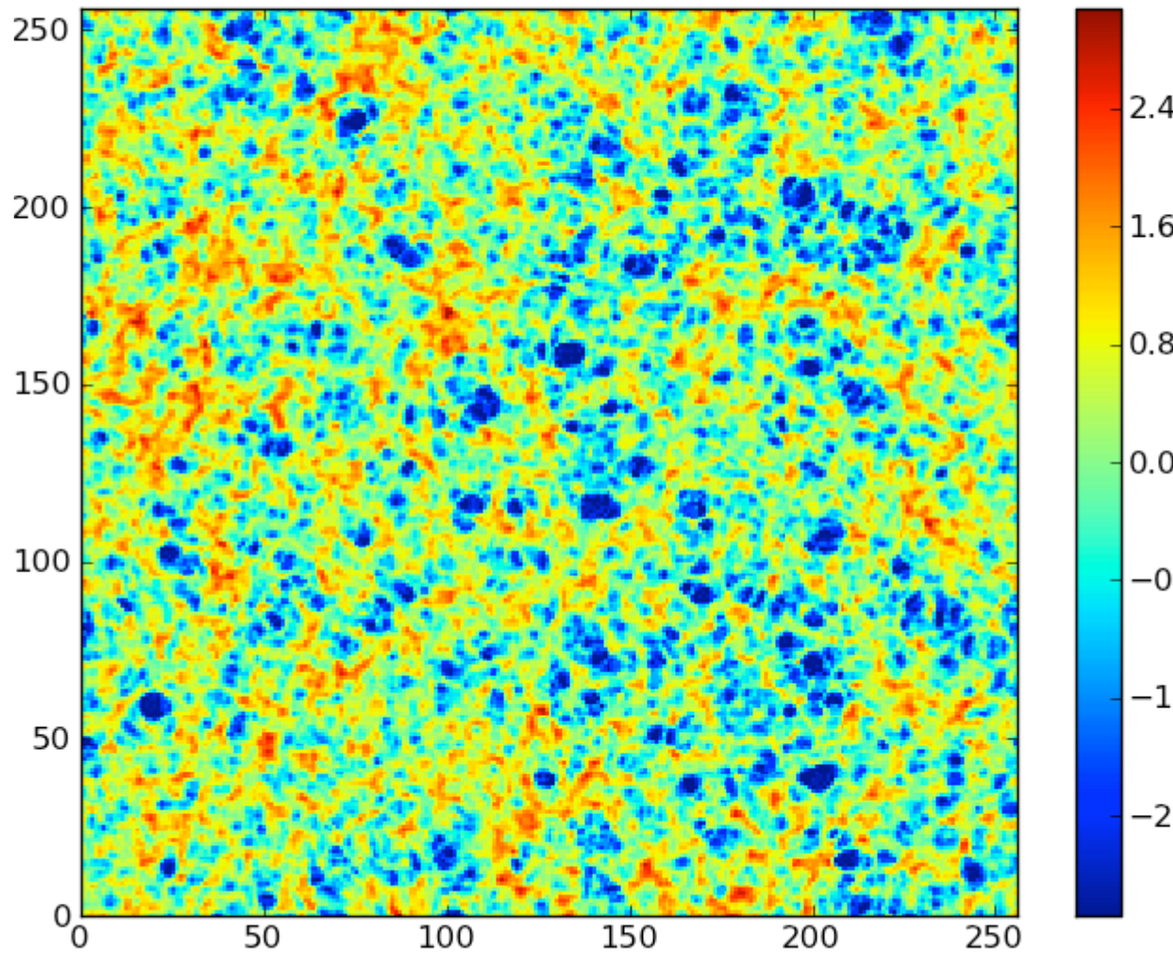

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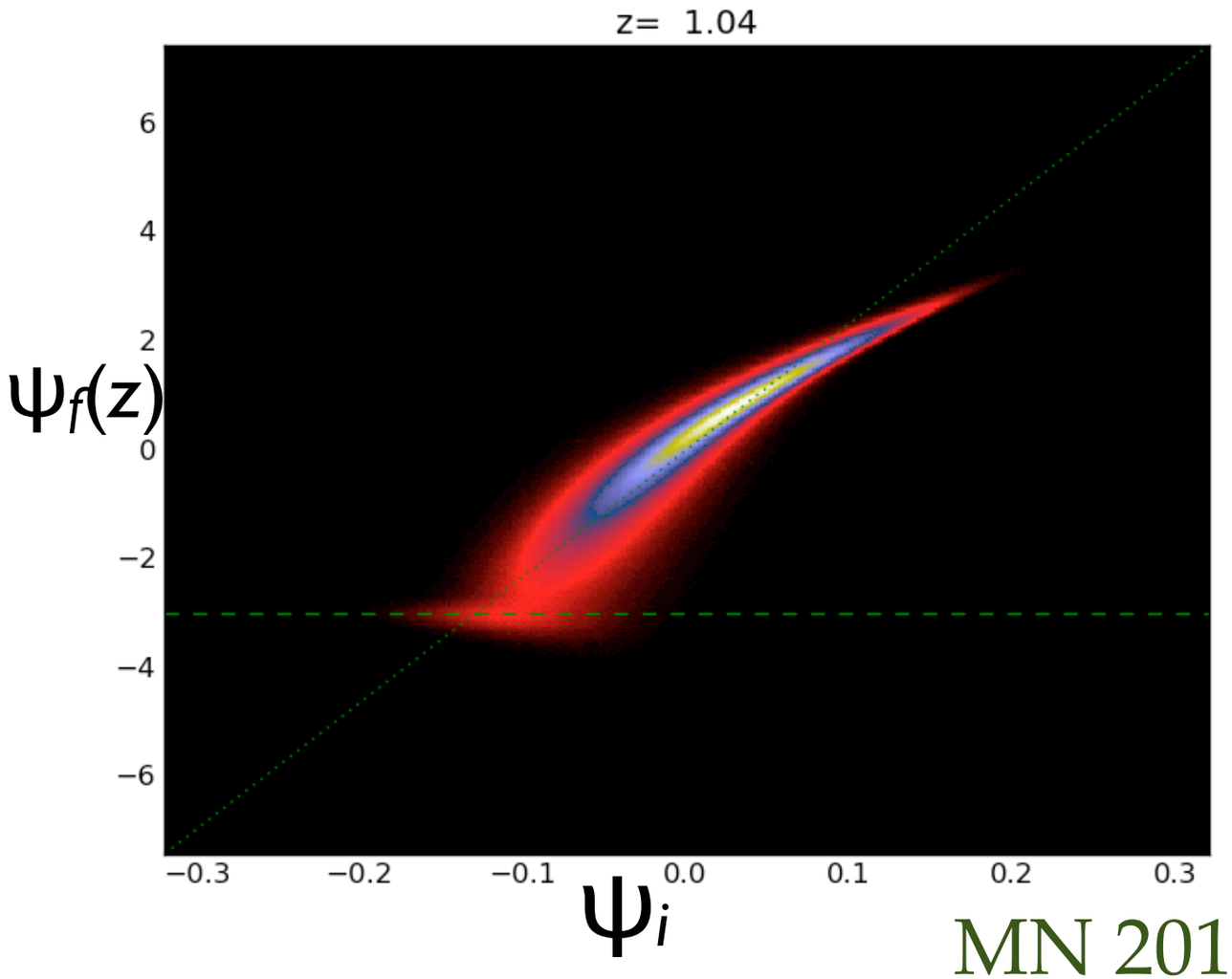

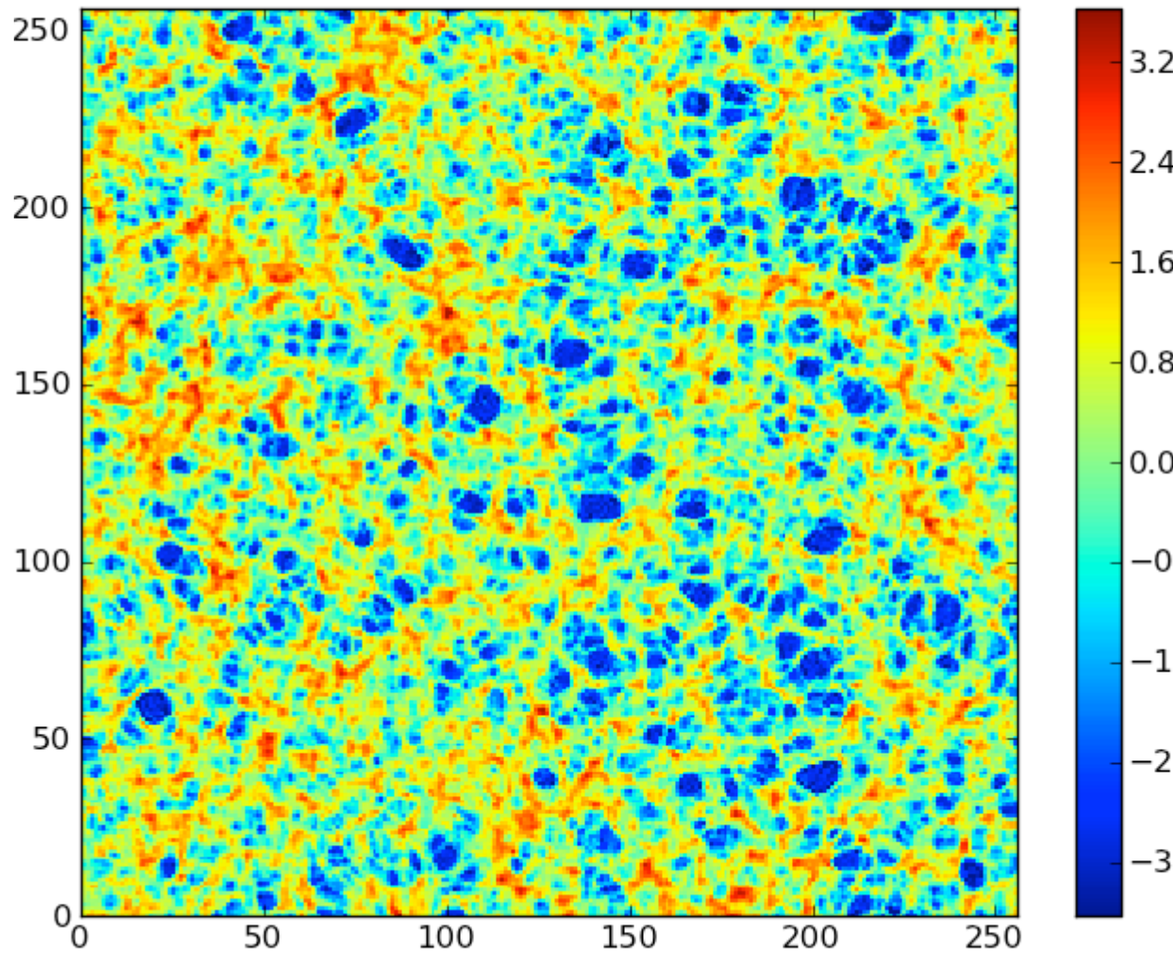

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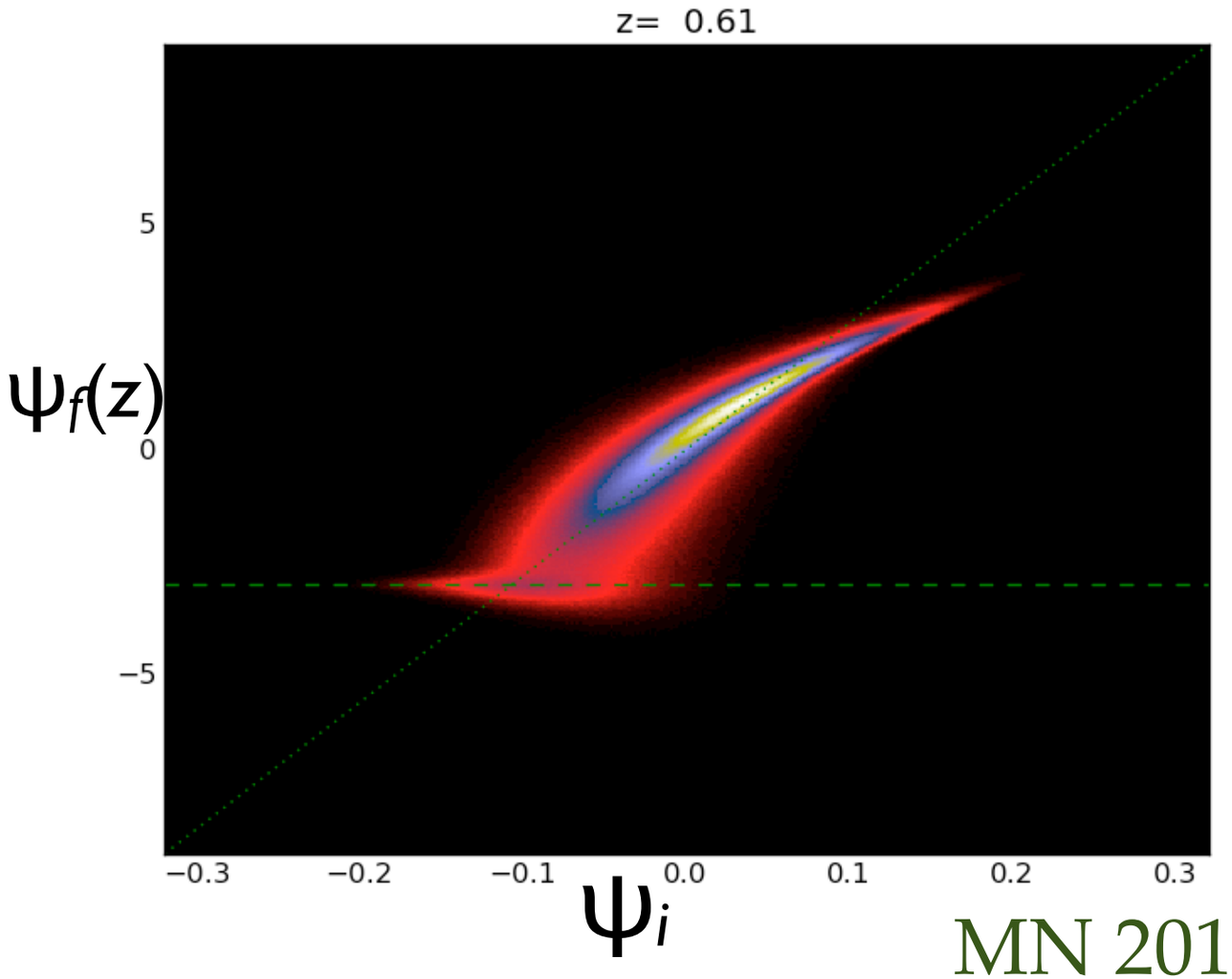

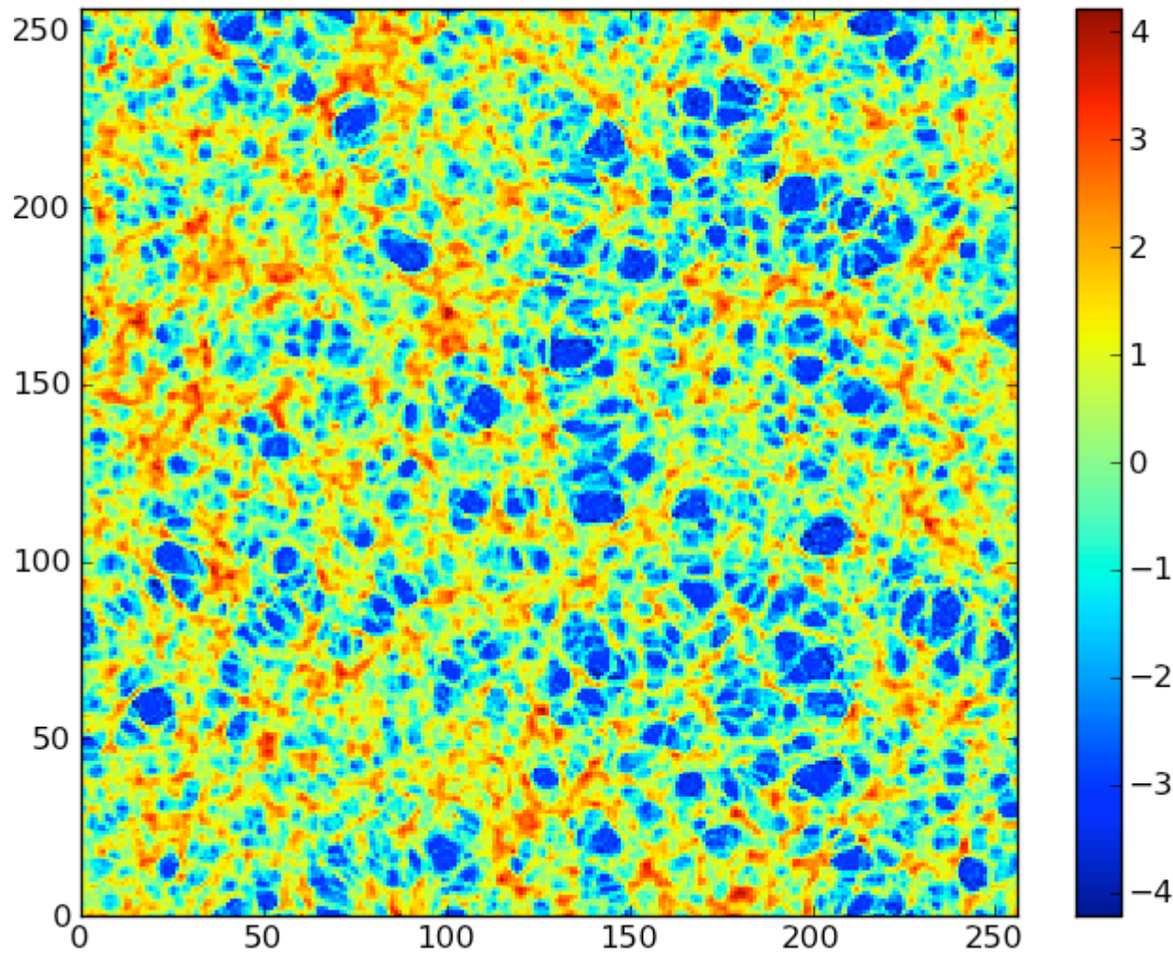

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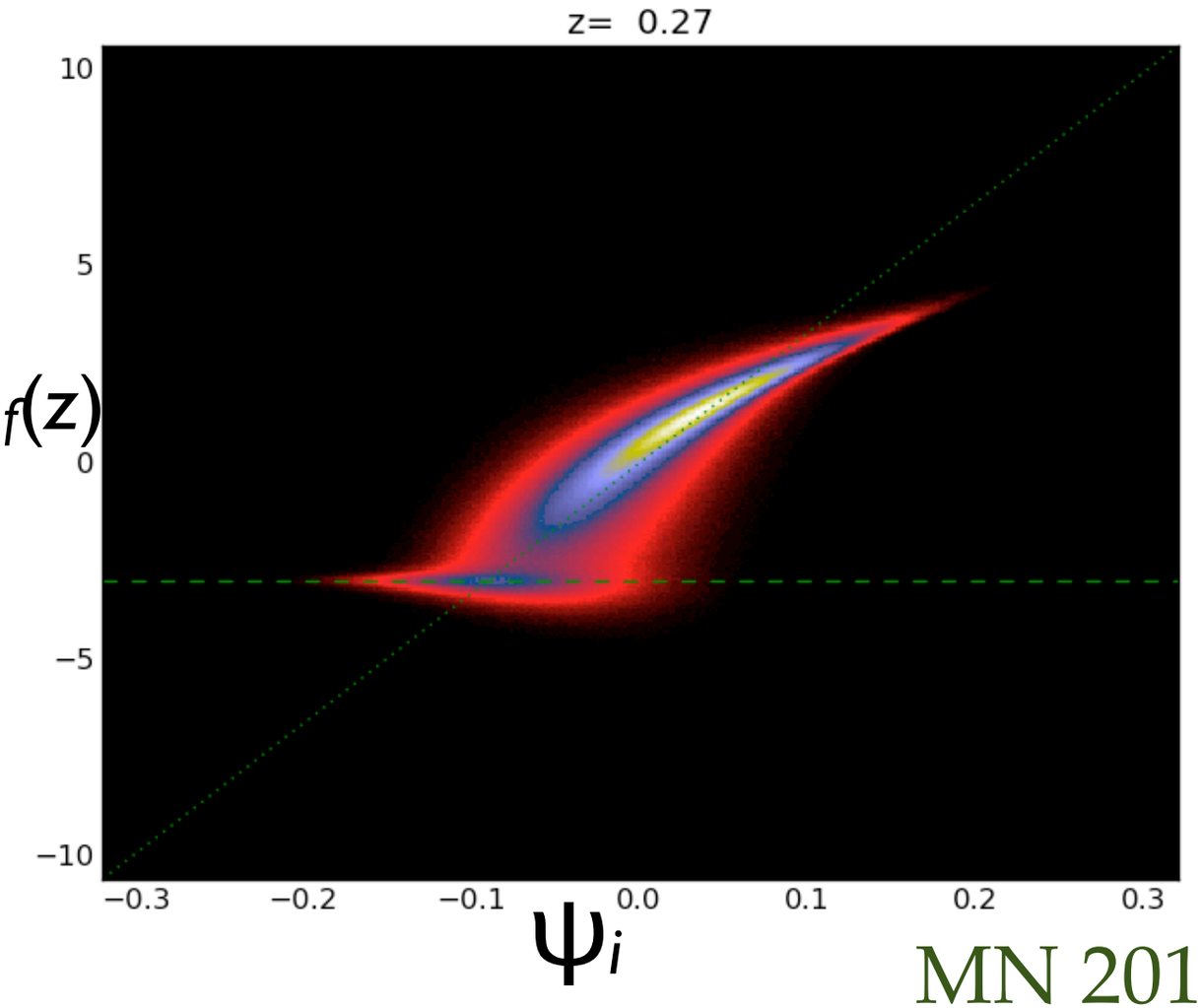

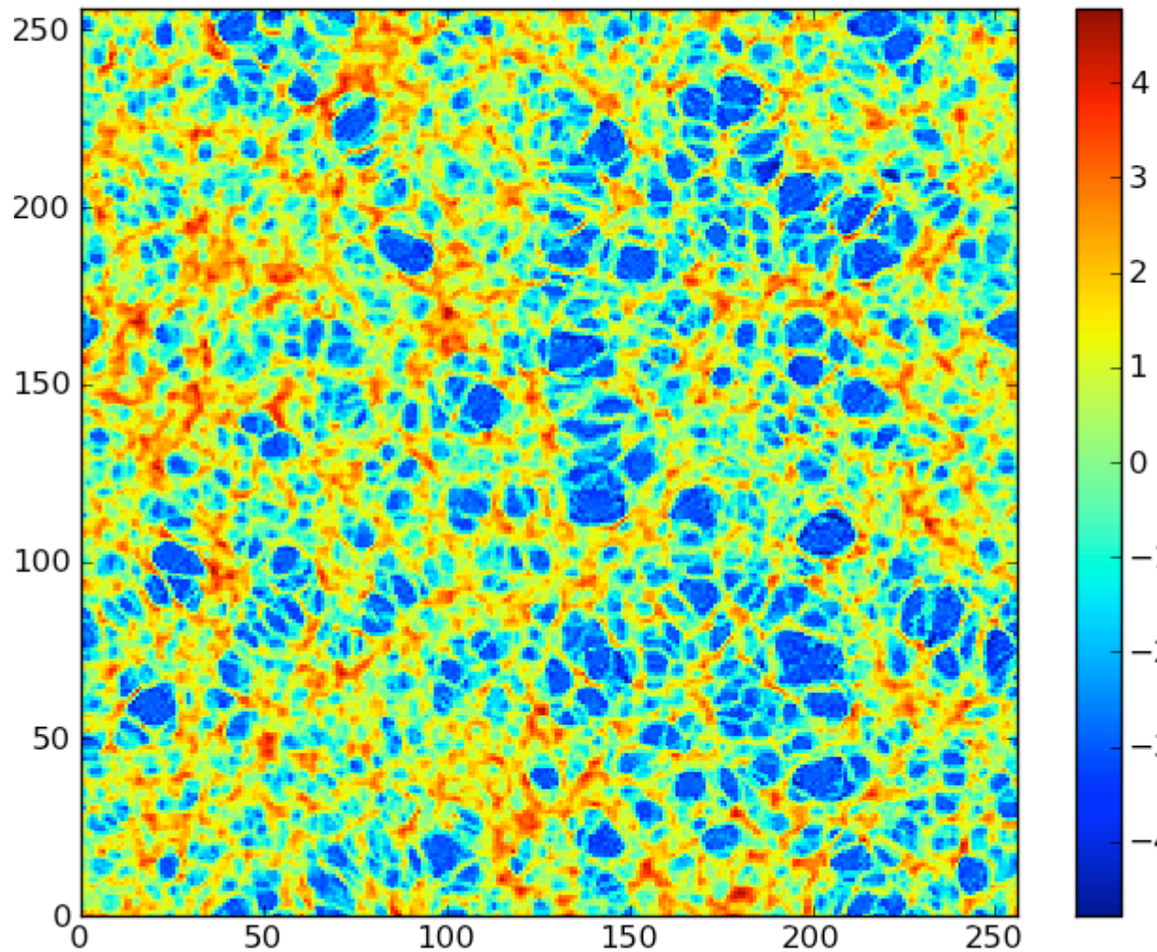

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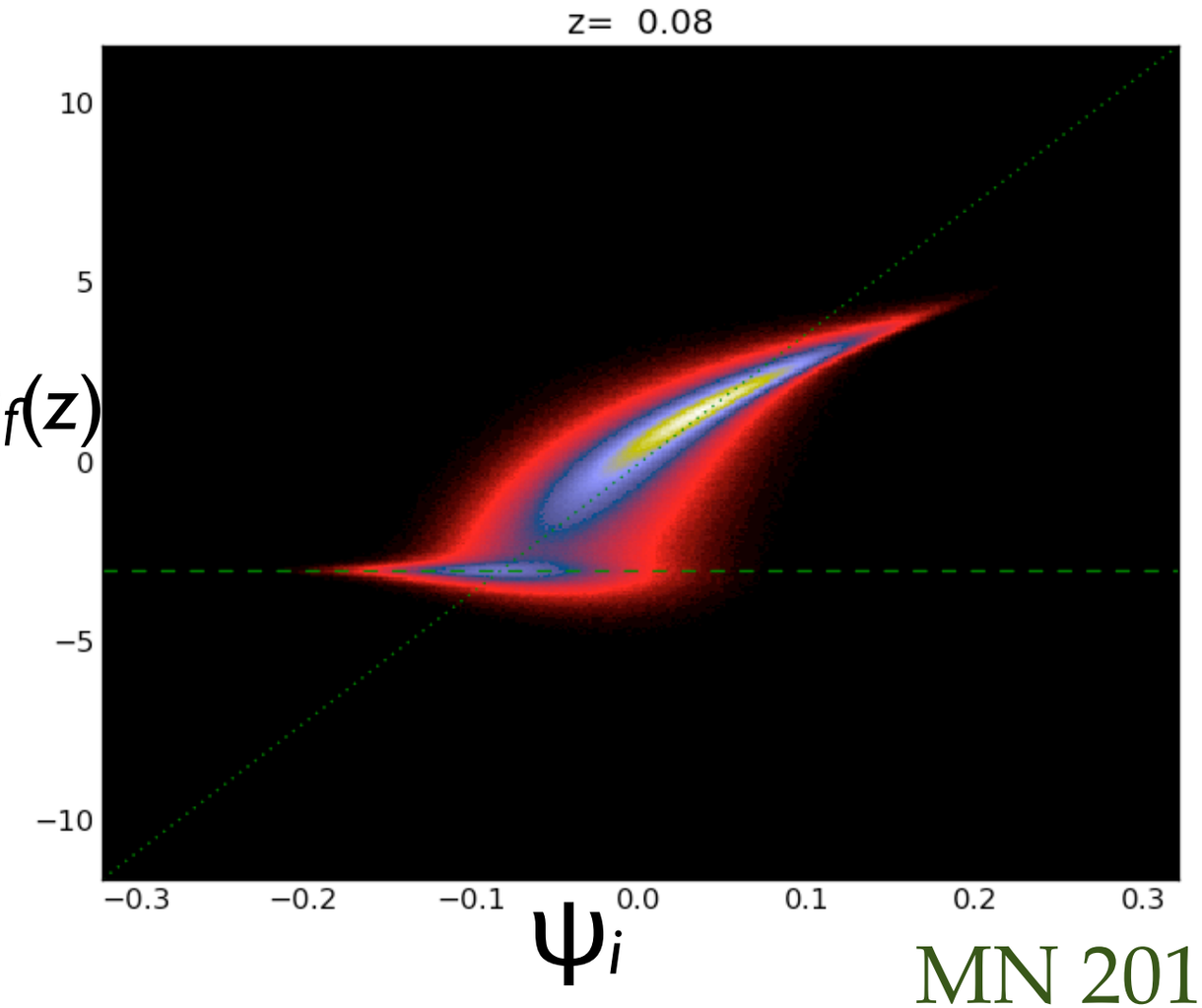

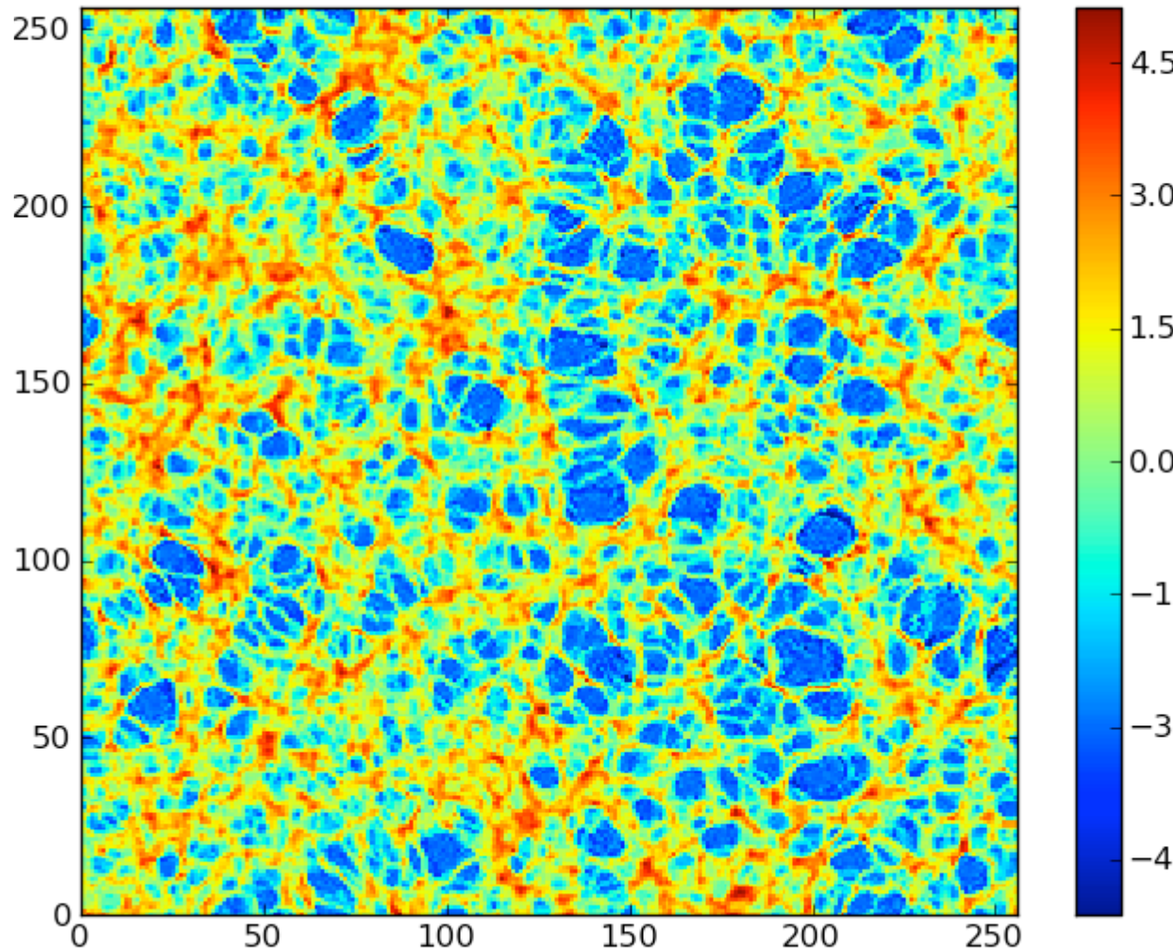

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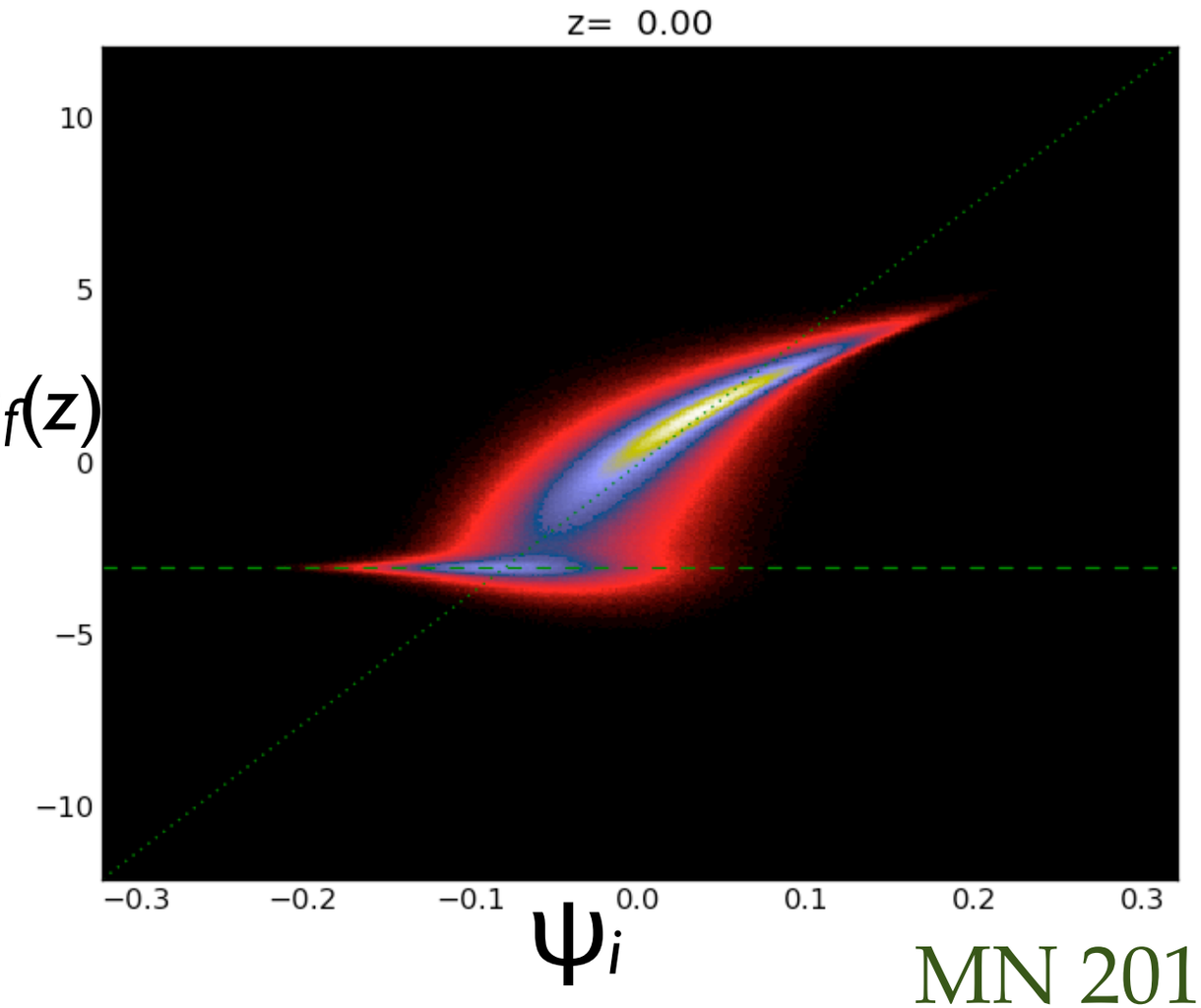

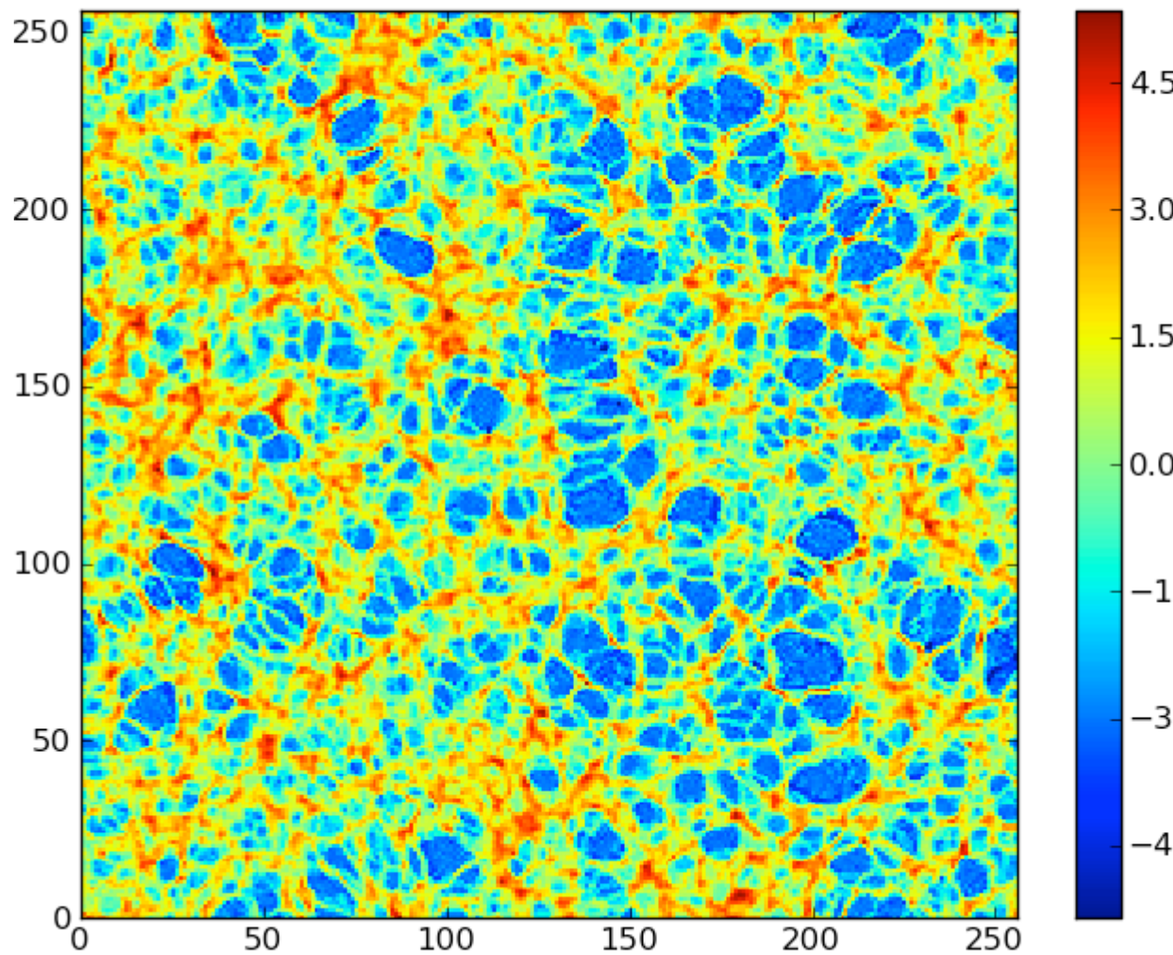

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Can we get a good approximate particle realization from this?

- Use this formula to predict $\nabla_L \cdot \psi$. Where collapse, set $\nabla_L \cdot \psi$=-3.

- Should give accuracy in voids, and little stream-crossing.

- Poor void behavior in voids for 2LPT (as in Sahni & Shandarin 1996)

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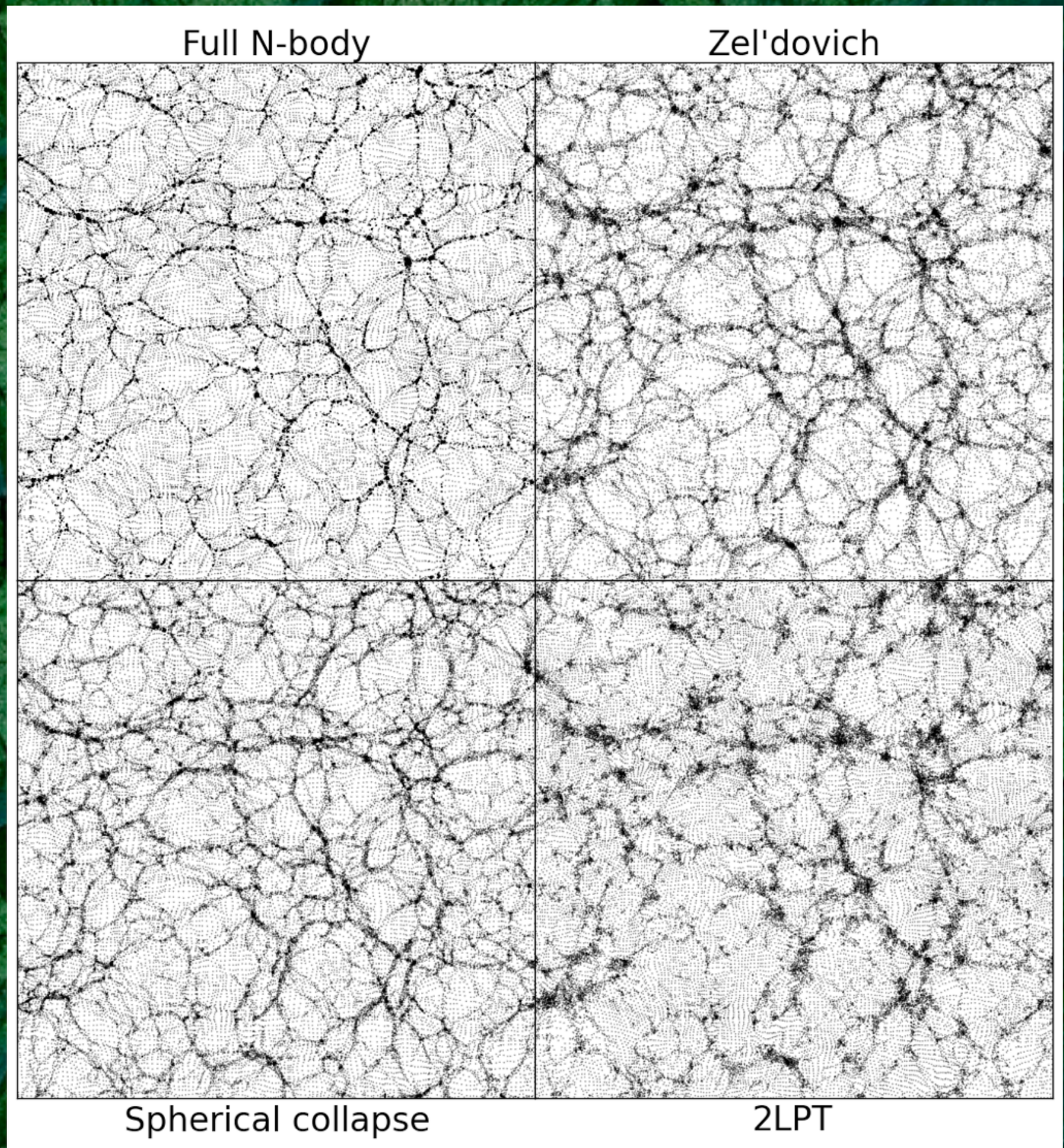

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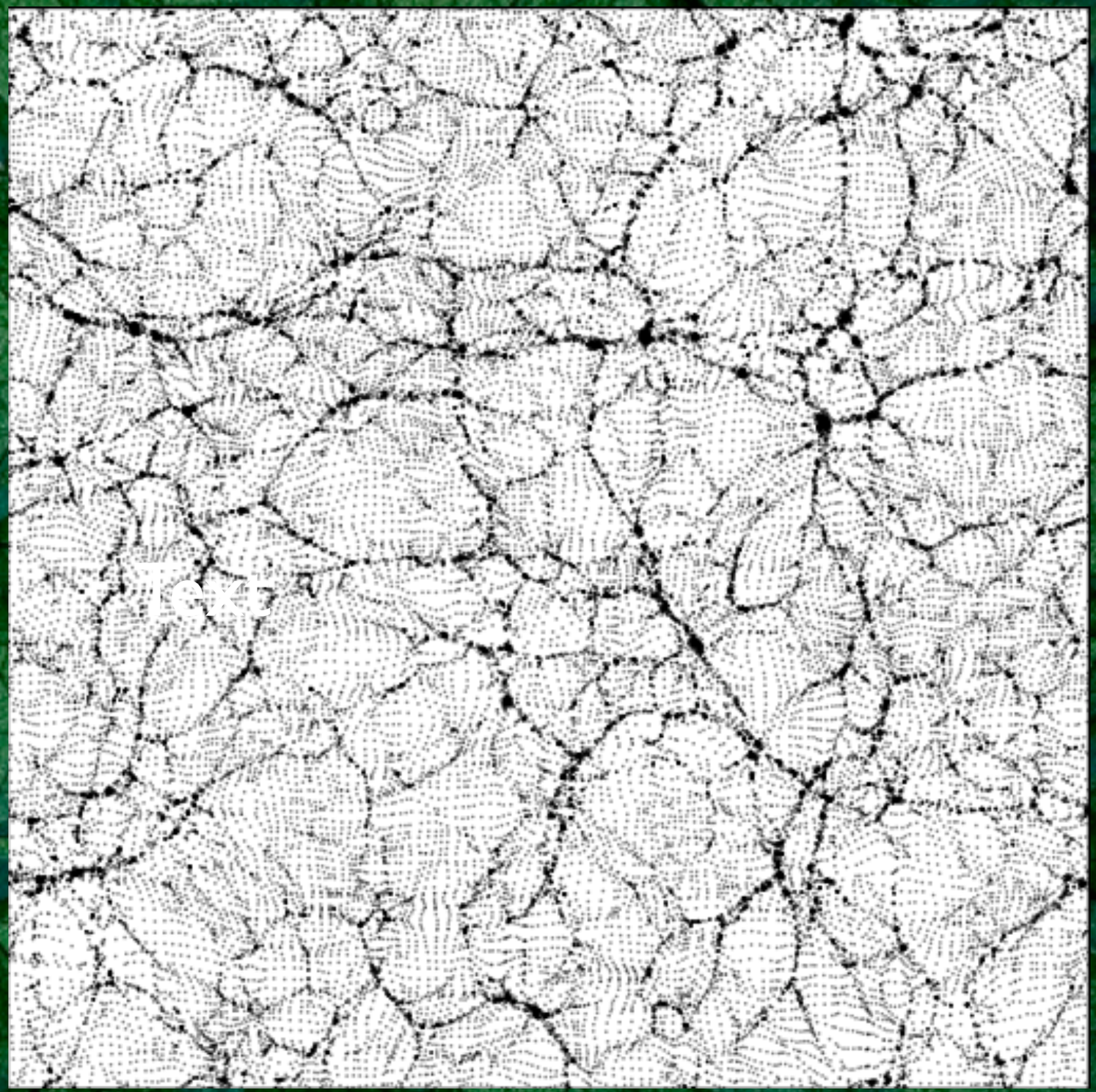

(suggests caution in using 2LPT at high resolution and low redshift -- obvious problem with using perturbation theory outside its range of validity)

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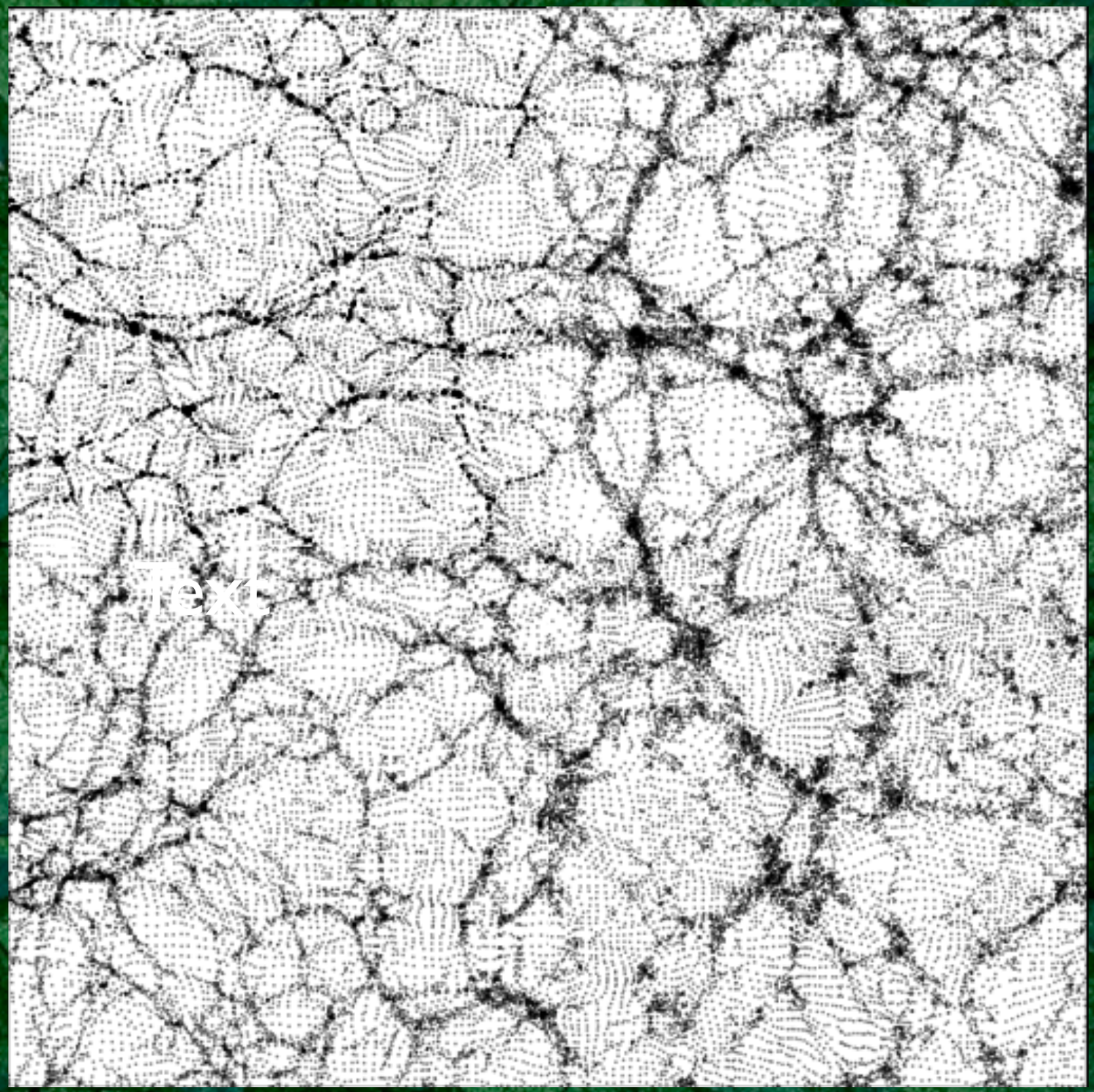
(suggests caution in using 2LPT at high resolution and low redshift -- obvious problem with using perturbation theory outside its range of validity)



MN 2012, arXiv: 1204.1326

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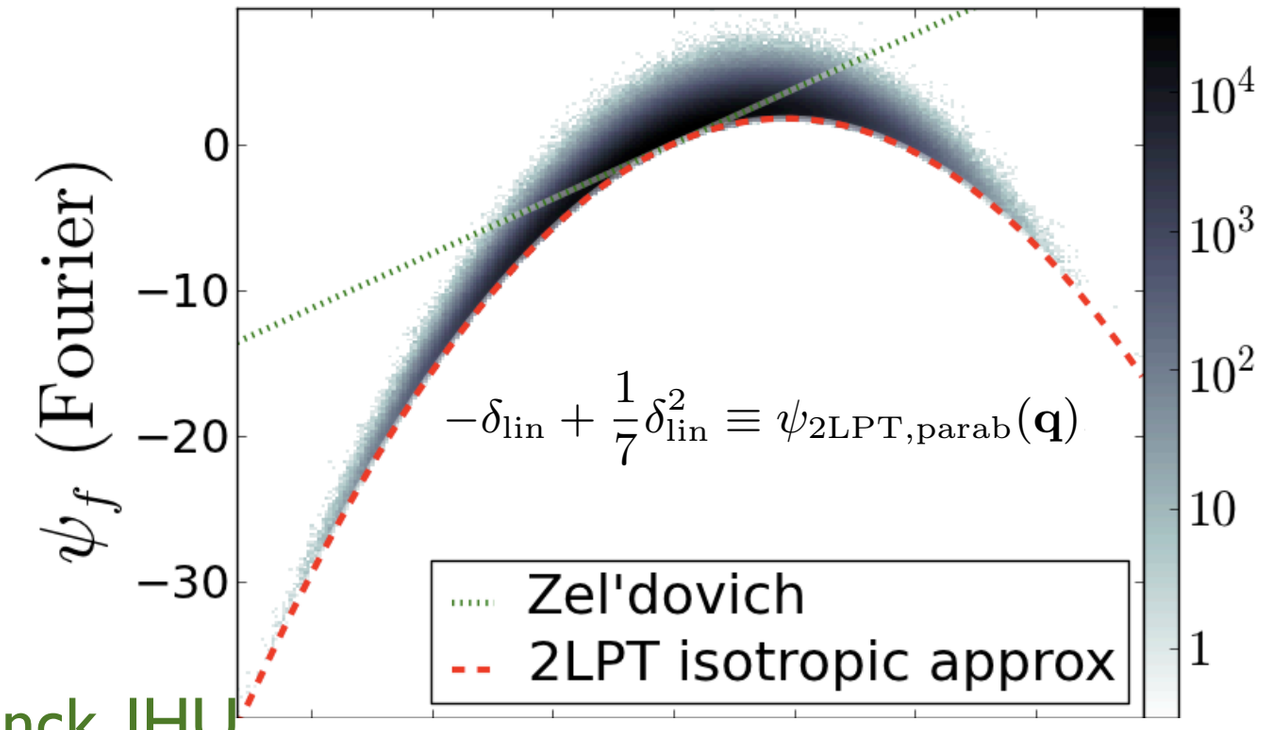
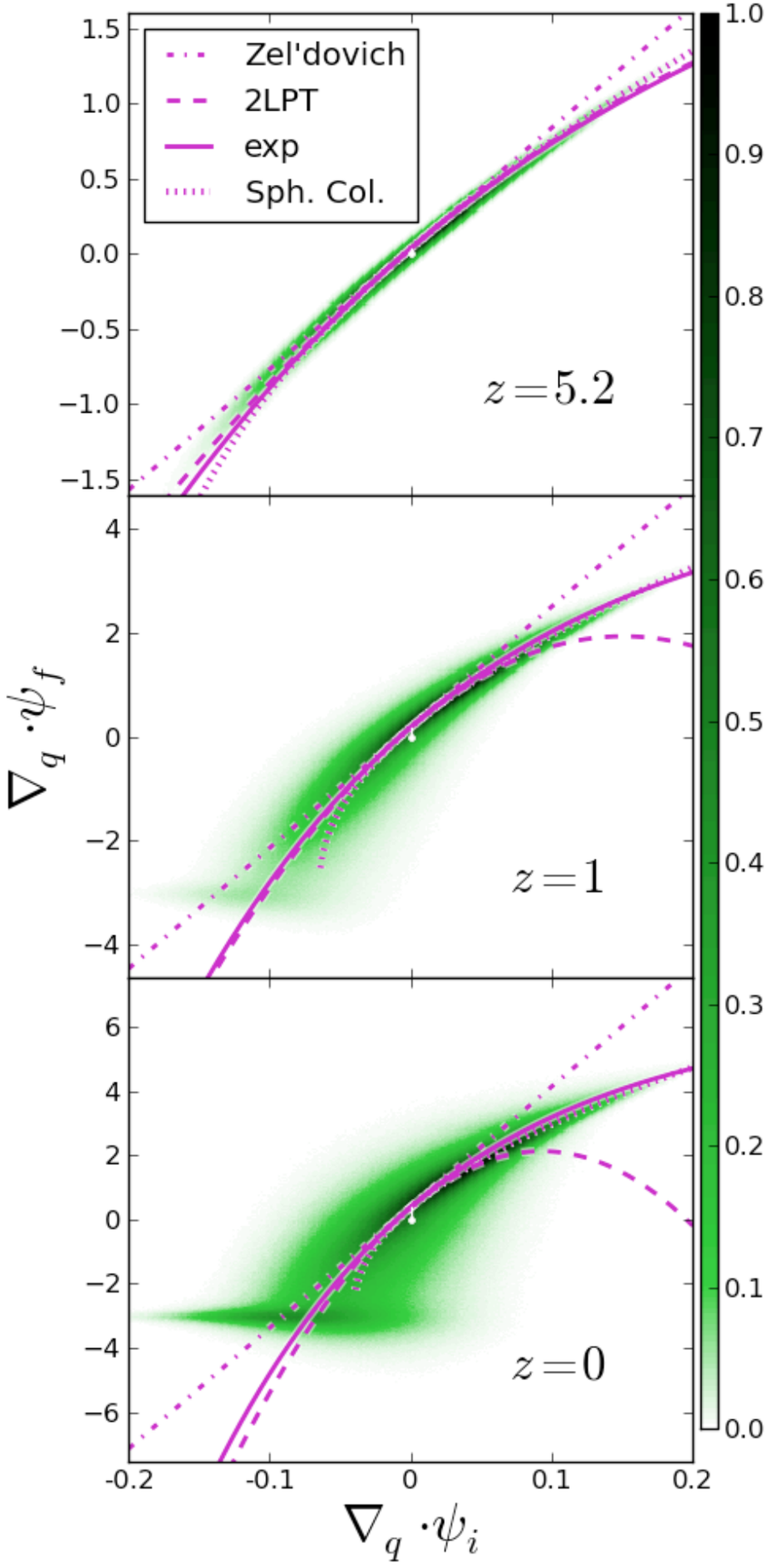
"Local Lagrangian" approximations work quite well (Protogeros & Scherrer 1997)

- Bernardeau's (1992, 1994) "spherical-collapse" fit (also see Mohayaee et al. 2006) works quite well:

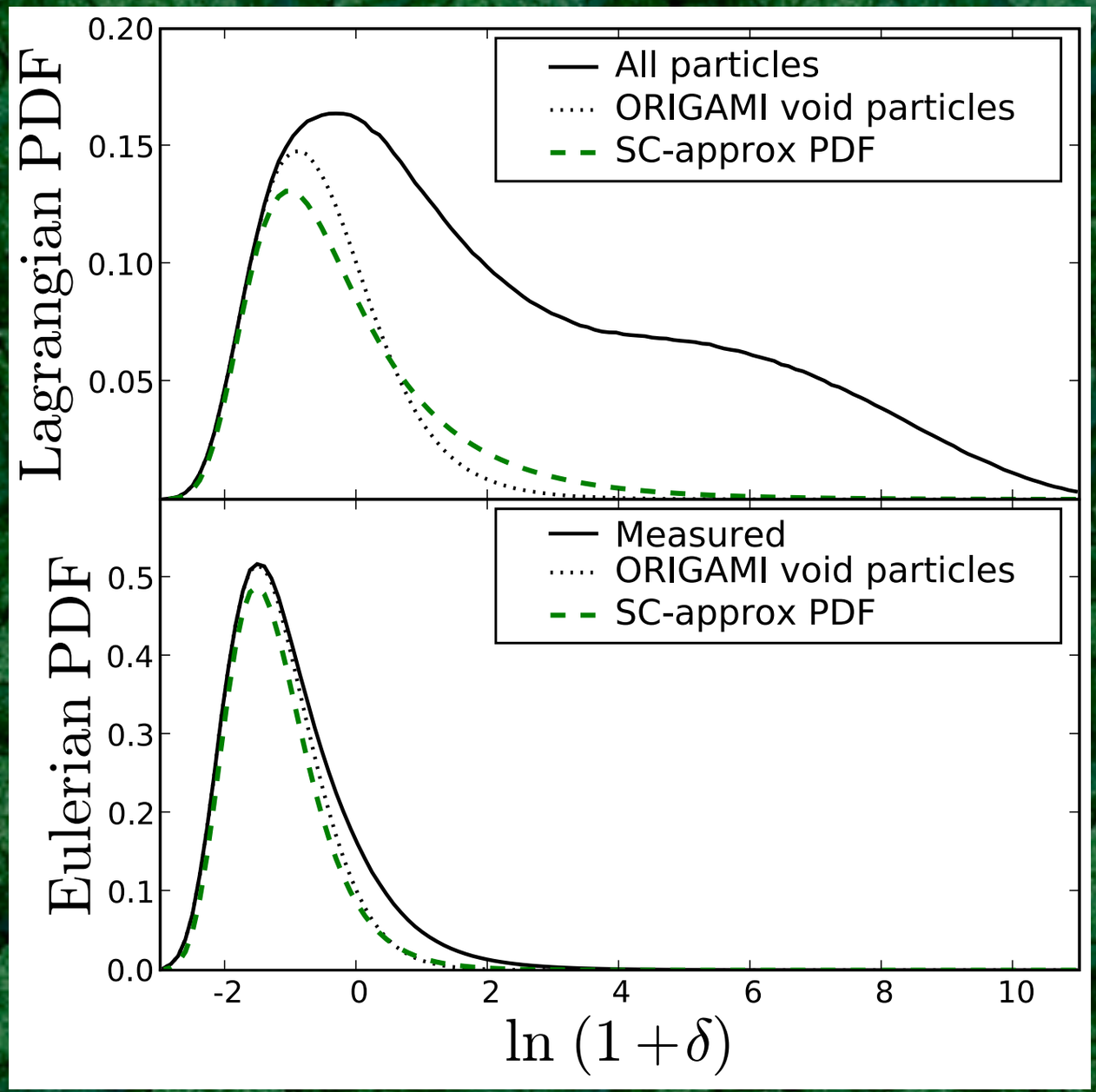
$$\psi_{\rm sc} \quad = \quad 3\left[\left(1+\frac{2}{3}\psi_{\rm lin}\right)^{1/2}-1\right]$$

- 2LPT on the other hand, behaves oddly for highly nonlinear density fields





Allows a good description of the pdf in voids.



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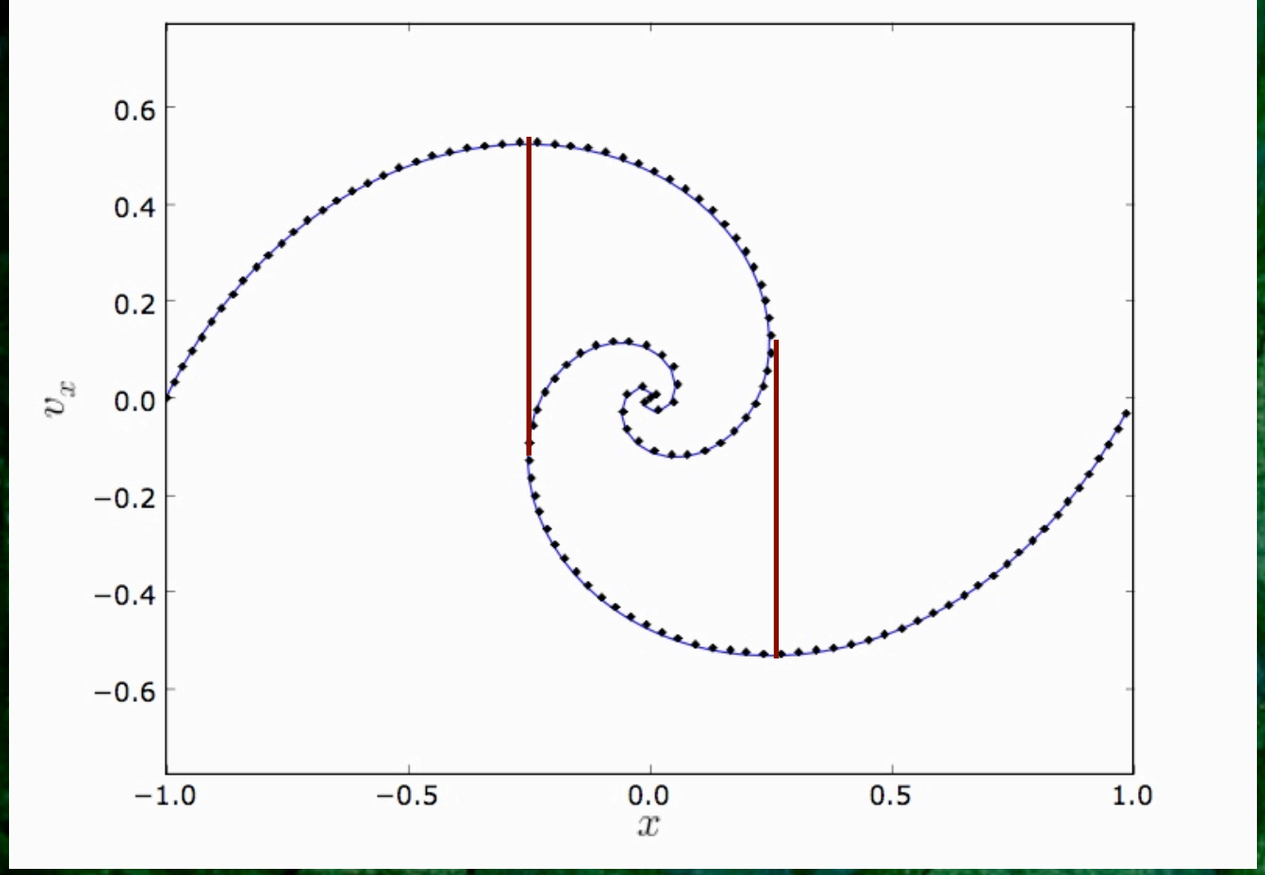

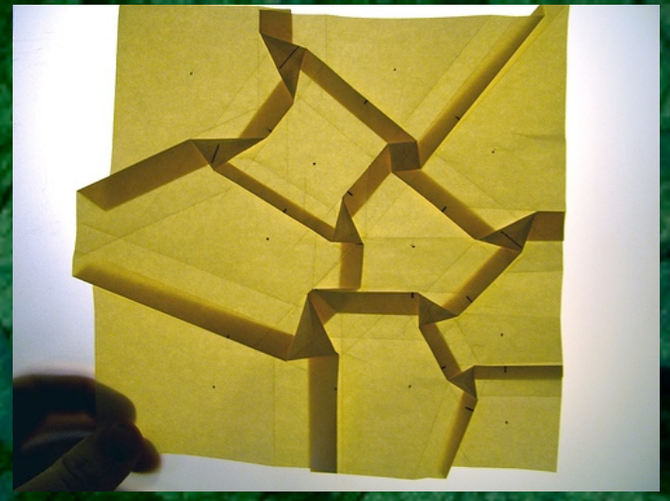

ORIGAMI

Order-ReversIng Gravity, Apprehended Mangling Indices

Bridget Falck, MN, & Alex Szalay,
arXiv:1201.2353

- *Objective, dynamic, geometric* definition of halo boundaries
- 1d: particle in a halo if its order wrt *any* another particle is swapped compared to the original Lagrangian ordering
- 3d: particle in a halo if this condition holds along 3 orthogonal axes (2 axes=filament, 1 axis=wall, 0 axes=void)
- Finds places where streams have crossed

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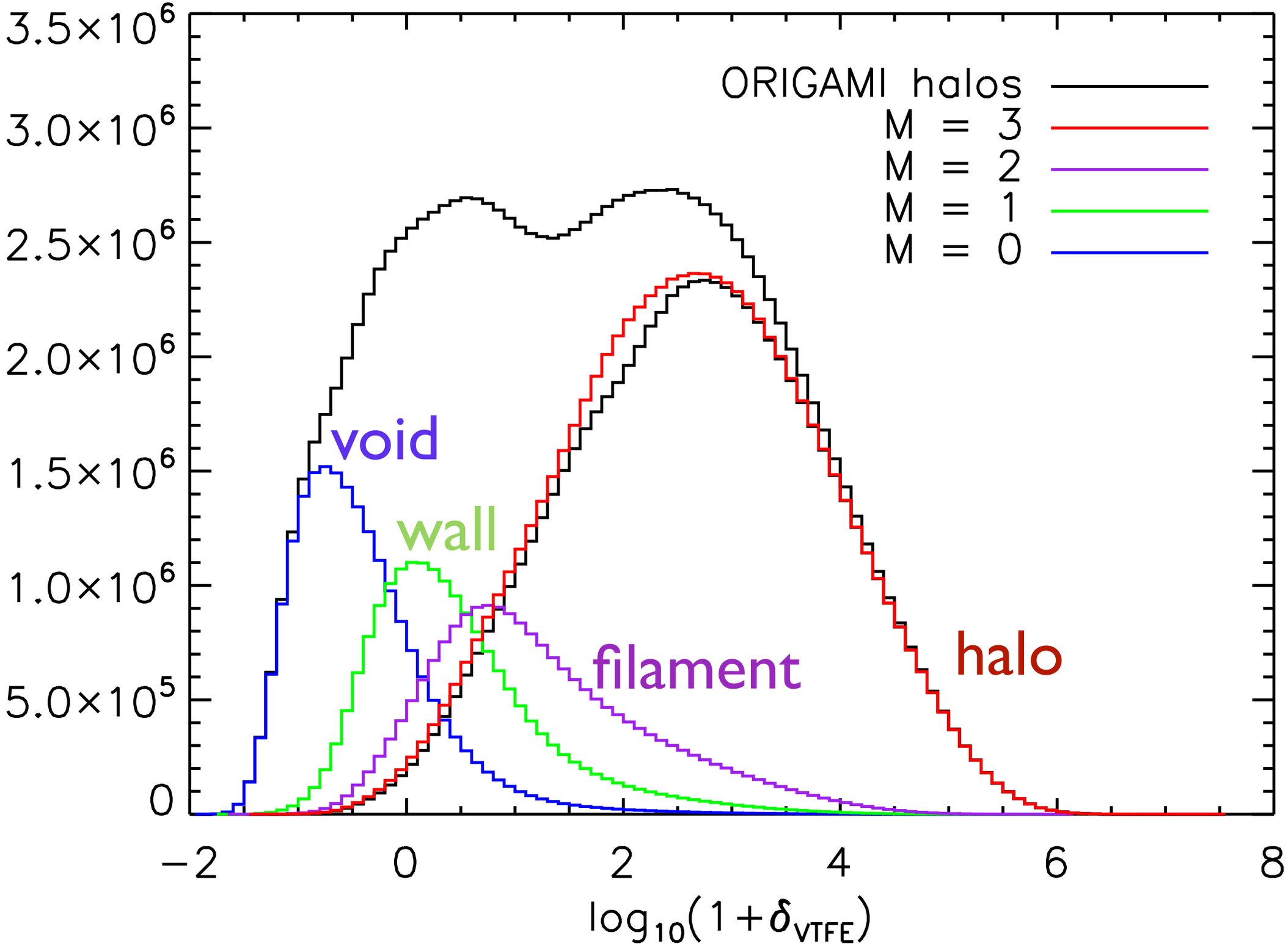
Mass-weighted histogram of particle density



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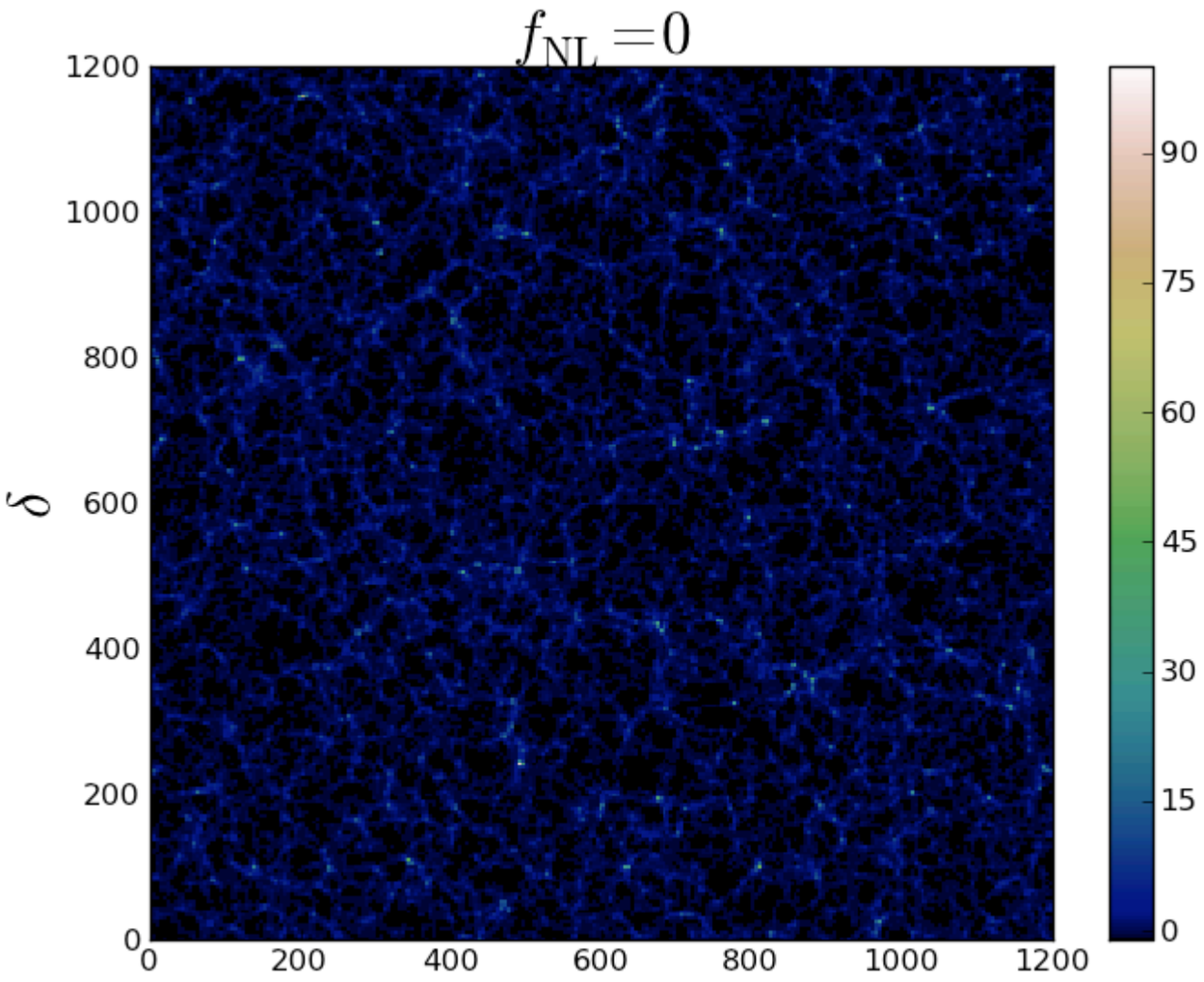
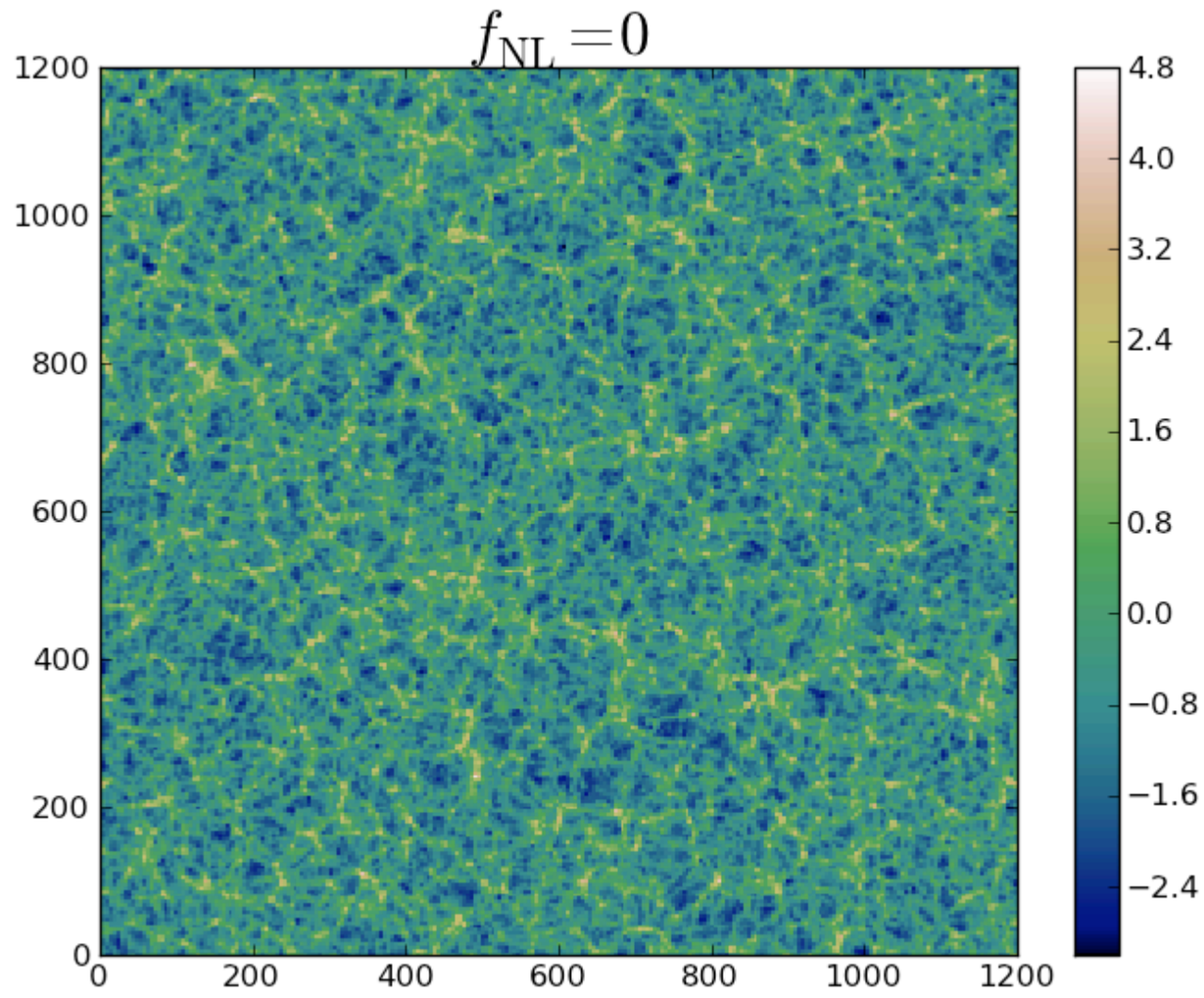
Example of simplicity in single-stream regions: f_{NL}





Simulations of Giannantonio & Porciani (2010), collaboration w/ many incl. Toni Riotto

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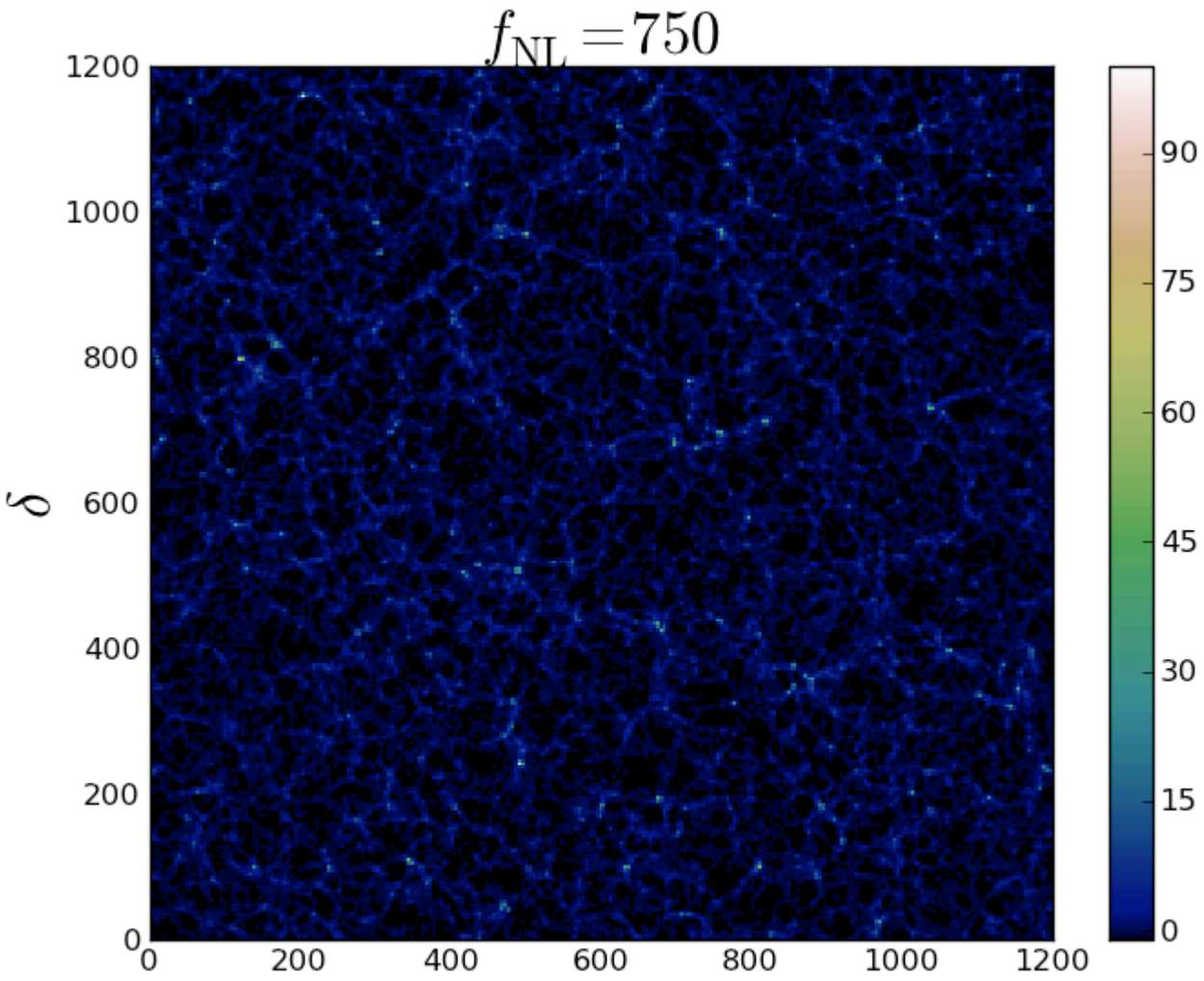

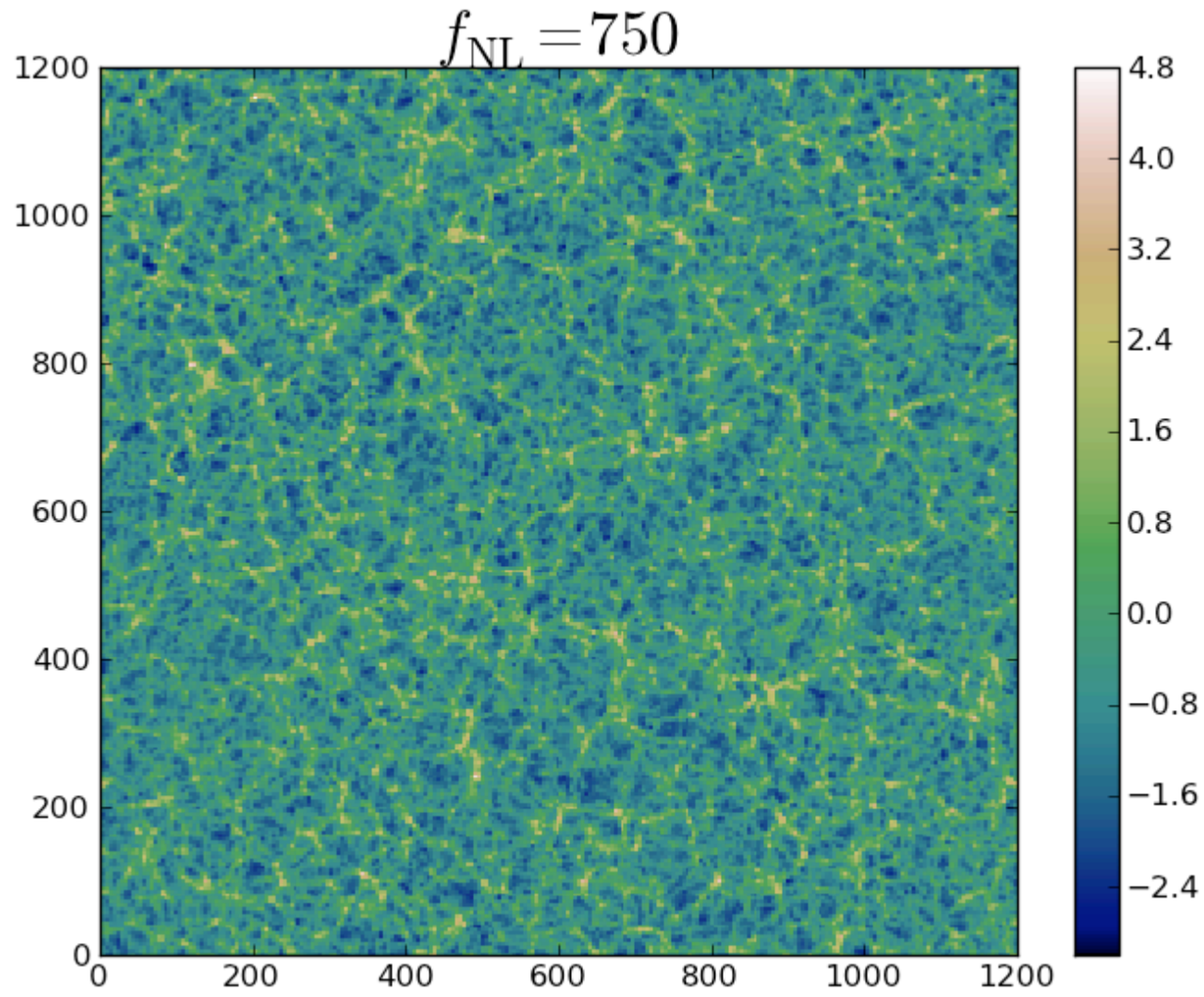

Simulations of Giannantonio & Porciani (2010), collaboration w/ many incl. Toni Riotto

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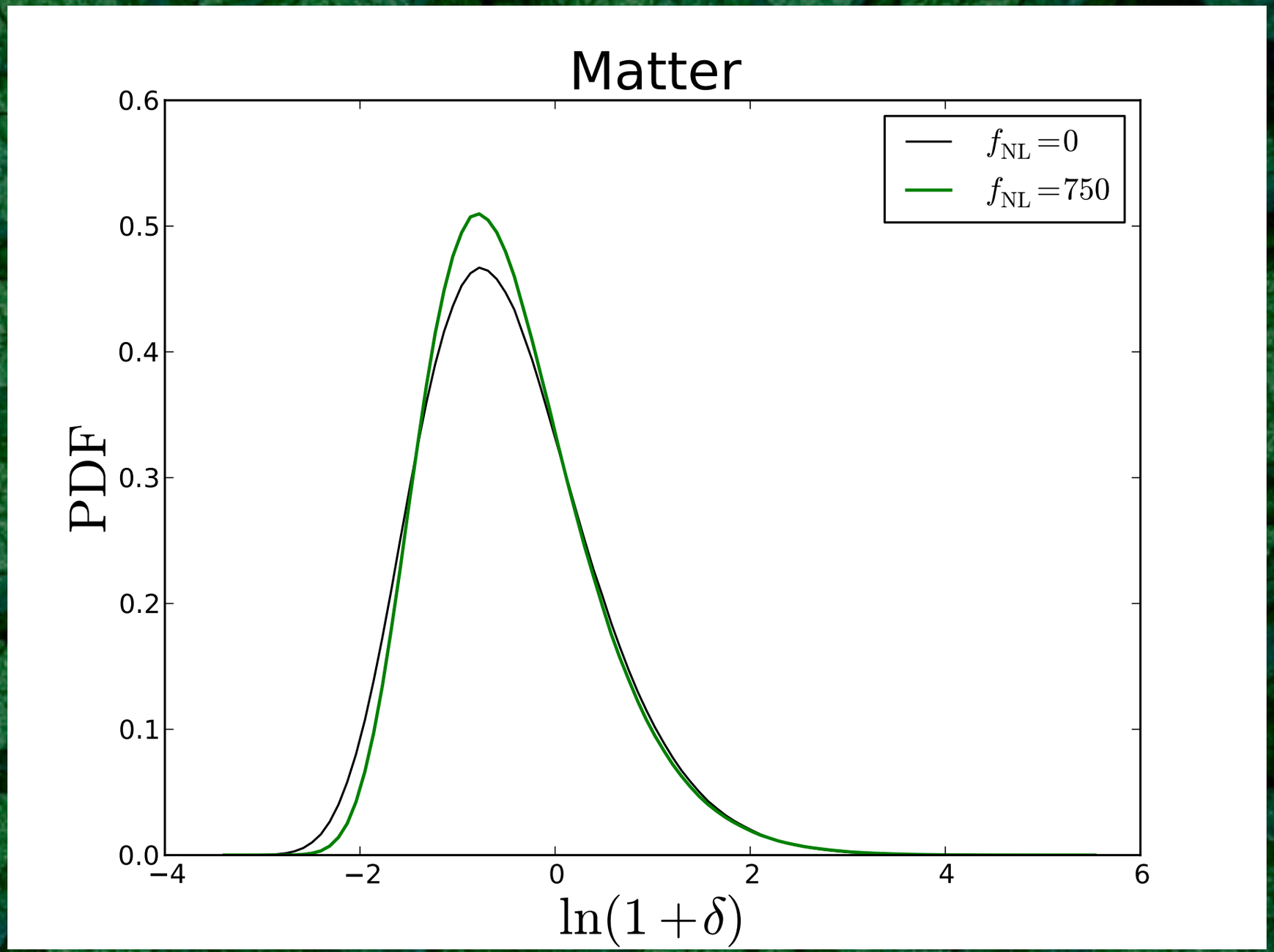
Example of simplicity in single-stream regions: f_{NL}

Difference mainly for low δ



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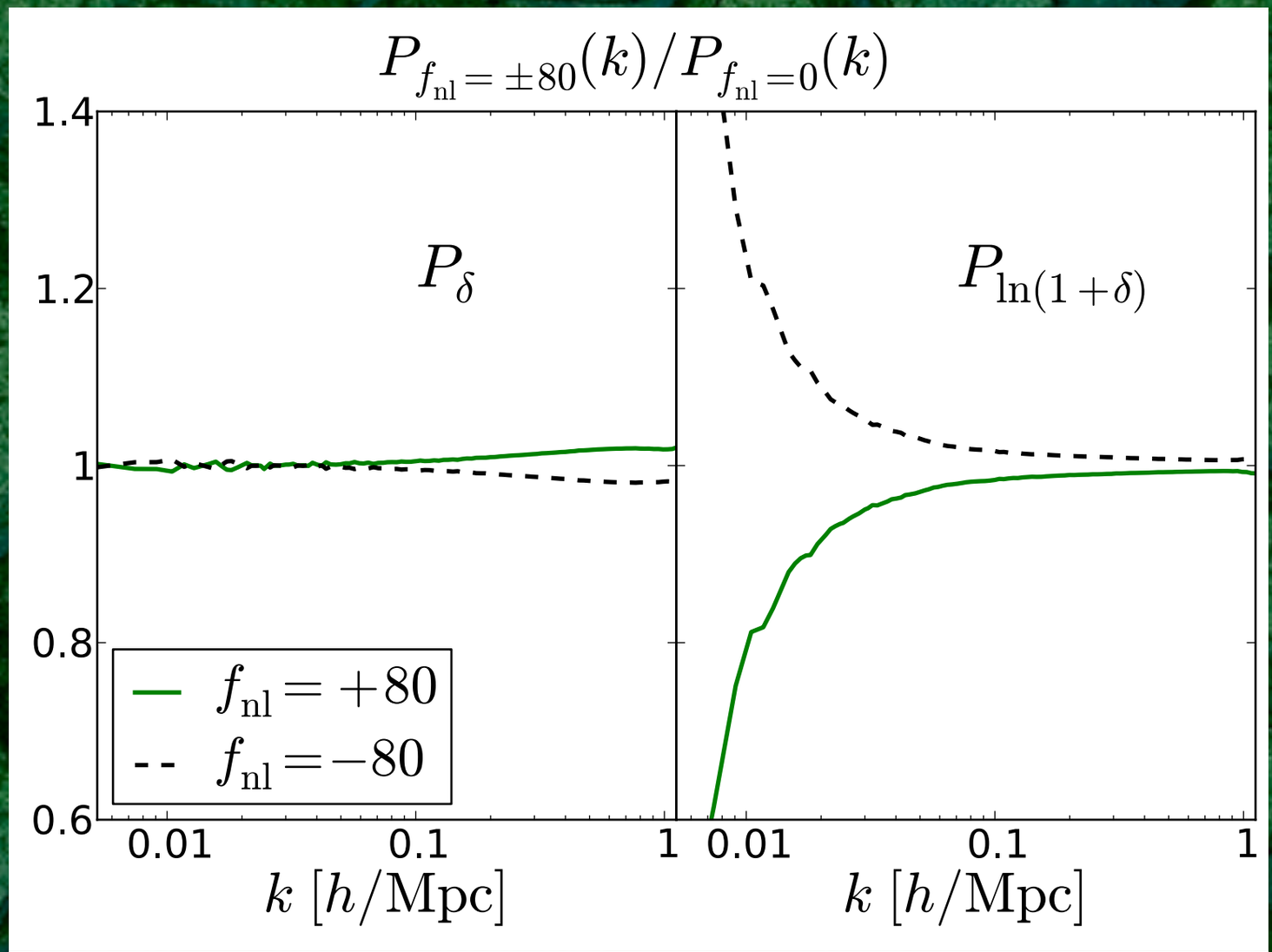
Power spectra: similar effect as large-scale halo biasing (Dalal et al. 2008)



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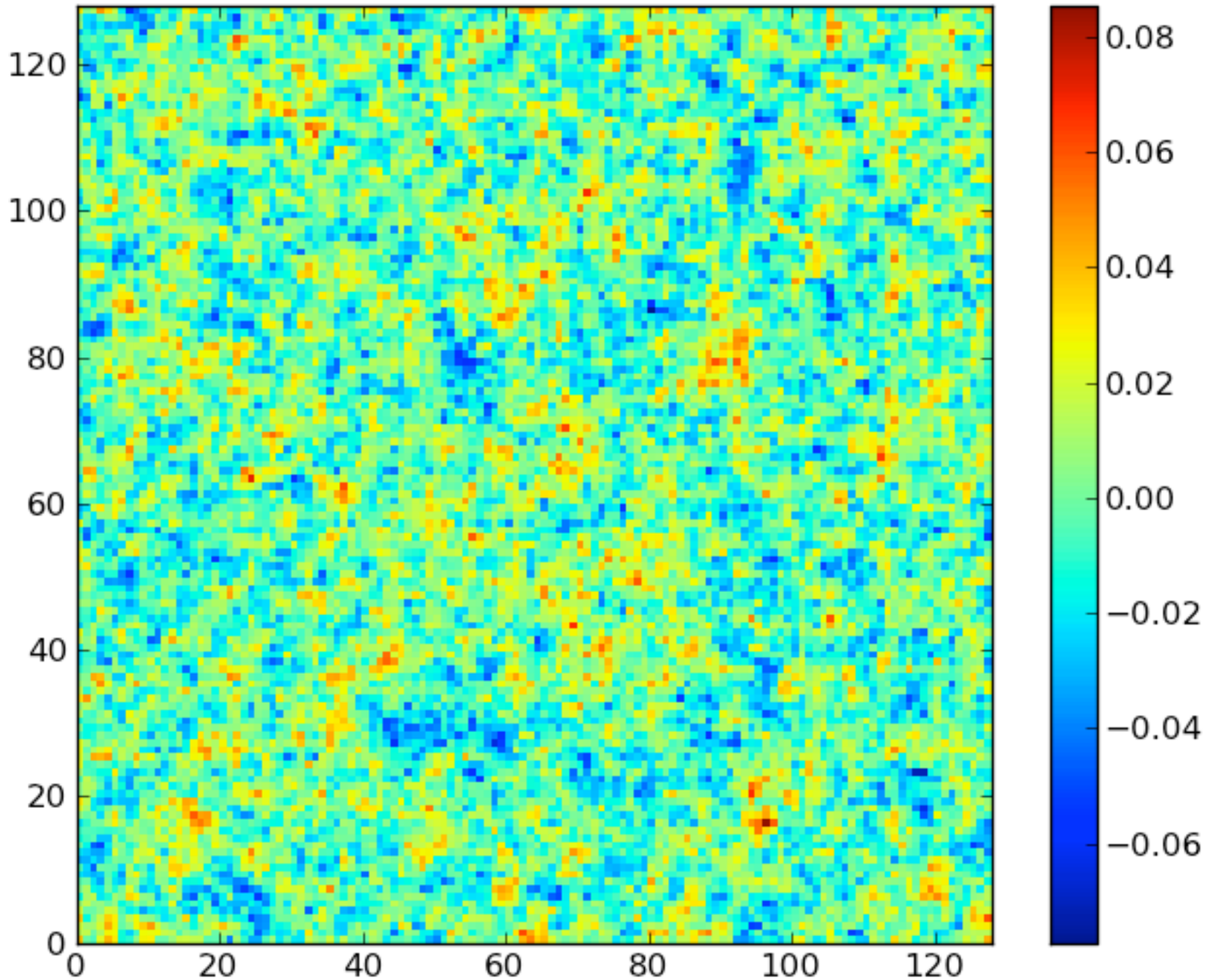
Now, back to analytic modeling, shifting around the density, not the mass ...



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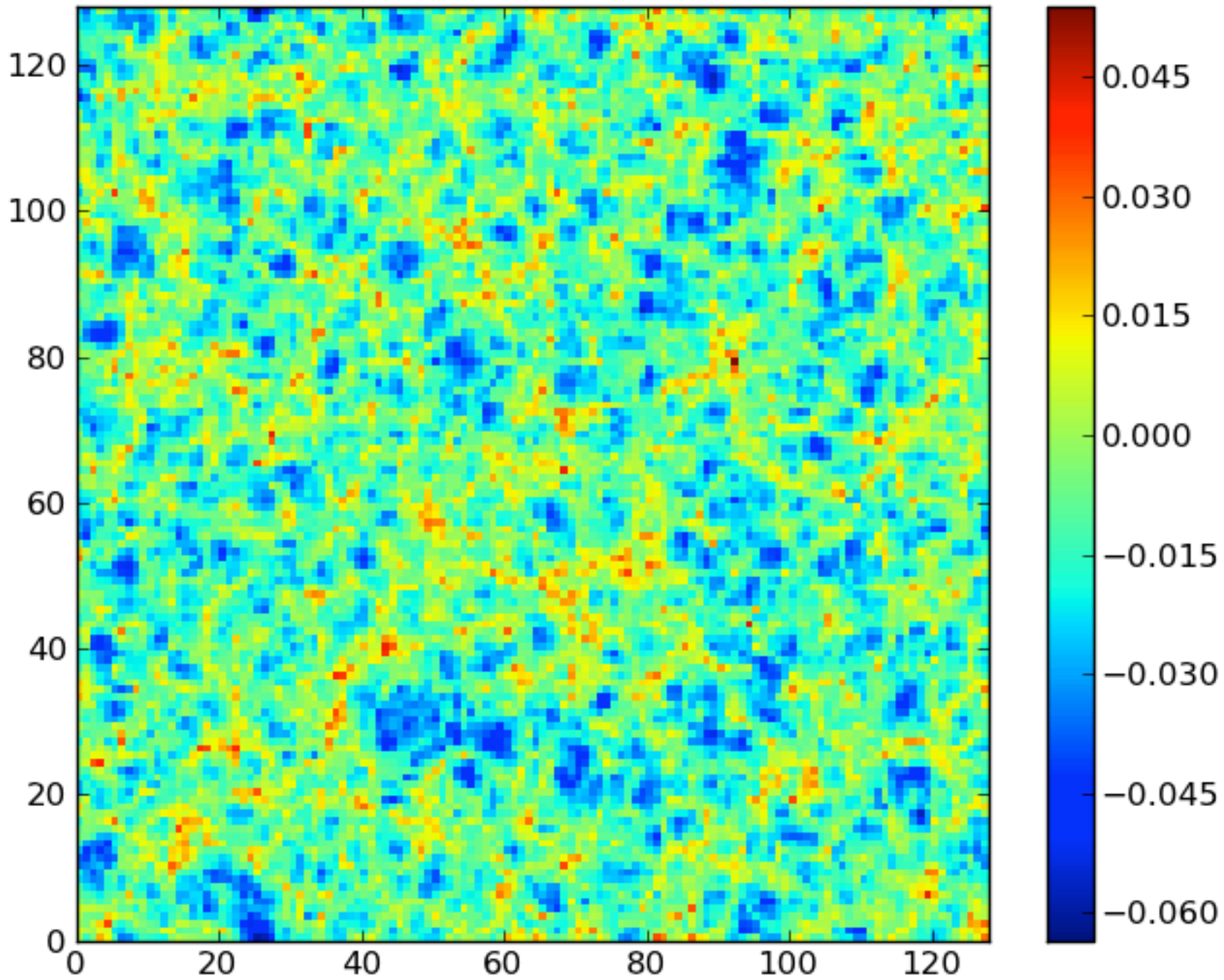

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One motivation: BAO

- Sharp peaks move slightly together →
slight (-0.15%) bias in peak position from large-scale flows (e.g. Eisenstein et al. 2007, Matsubara 2008, Smith, Scoccimarro & Sheth 2008, Seo et al. 2008)

- Upweight smaller objects, even voids?
~volume-weighting, not ~mass weighting

- Hope: might this remove bias?, increase S/N?

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(w/ Nuala McCullagh)

- Build on McCullagh & Szalay (2012)'s Zel'dovich-approx correlation function, where δ_{NL} is the Zel'dovich density:

$$\begin{aligned}\delta_x(\vec{x},t) &= \delta_{NL}(\vec{x} + D(t)\nabla_q\phi(\vec{q}(\vec{x}))) \\ &= (\delta_{NL}(\vec{q}) + D(t)\nabla_q\phi\cdot\nabla_q\delta_{NL}(\vec{q}) + ...)\,|_{\vec{q}=\vec{x}}\end{aligned}$$

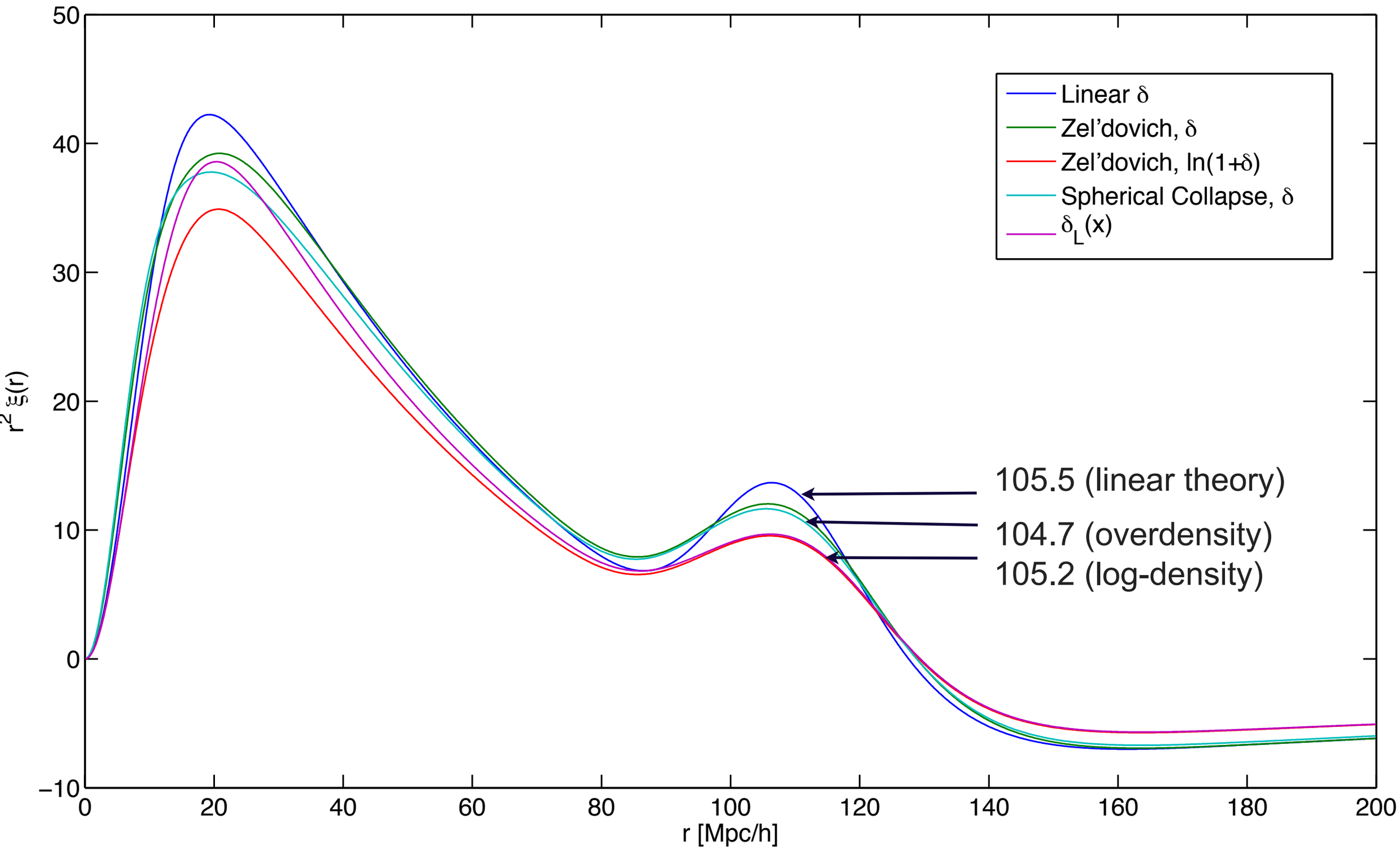
Try substituting δ_L for δ_{NL} instead. Could also try a (Taylor-expanded) nonlinear function of δ_L. Simple prescription for clustering of a nonlinearly-transformed field!
(difficult to model in general -- Wang et al. 2011). Try SC function:

$$1 + \delta_{NL}(\vec{q},t) = \frac{V_0}{V} = \left(1 - \frac{2}{3}\delta_L(\vec{q})D(t)\right)^{-3/2}$$

or the Zel'dovich log-density

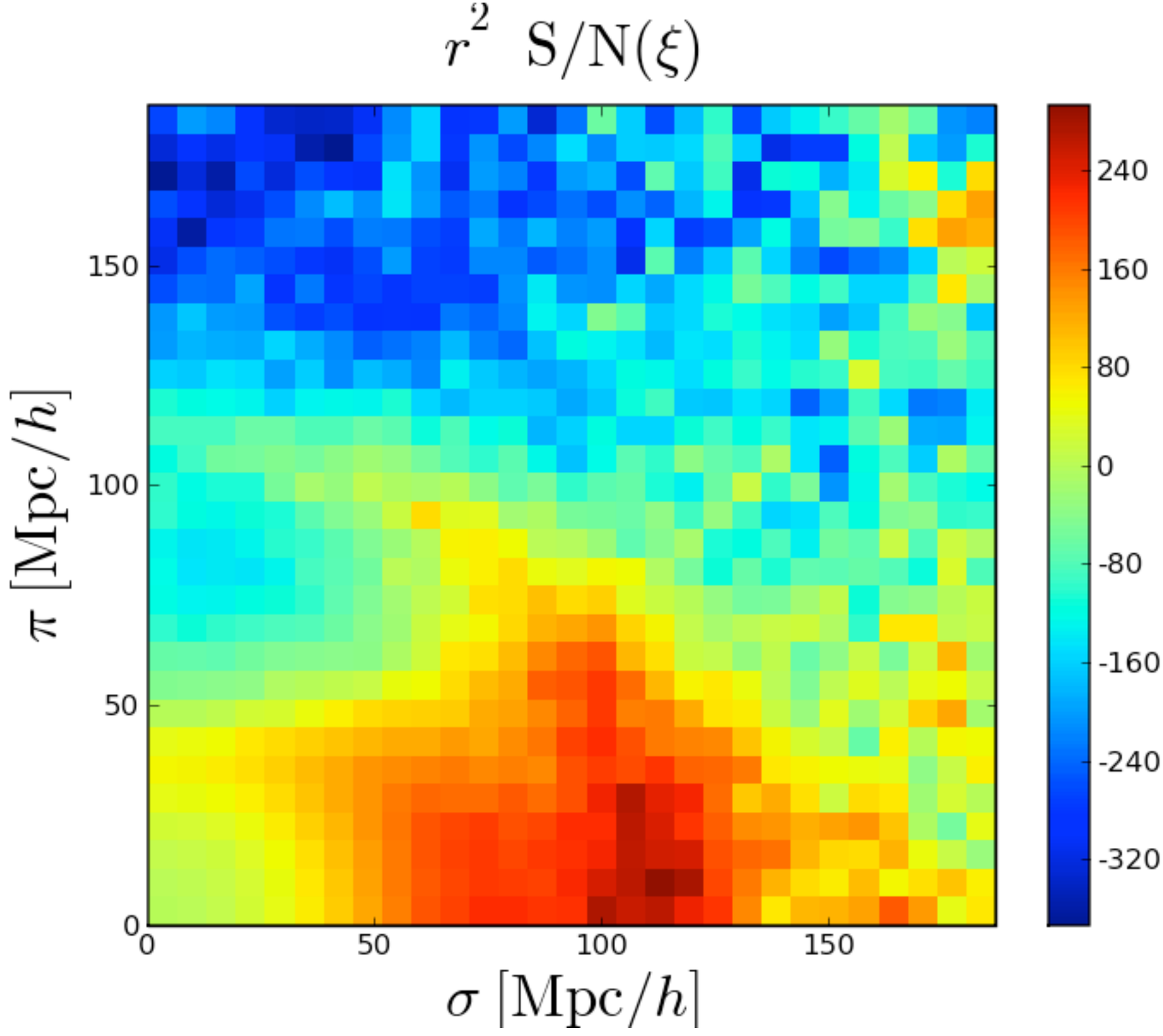
$$\begin{aligned}1 + \delta_q(\vec{q},t) &= \frac{1}{J(\vec{q},t)} \\ \ln\left(1+\delta_q(\vec{q},t)\right) &= -\ln J(\vec{q},t) \\ &= -\ln(1-D\lambda_1) - \ln(1-D\lambda_2) - \ln(1-D\lambda_3)\end{aligned}$$

“Volume-weighted” ξ’s largely debiased!



What about "reality?"

Horizon-1 LRG-like subhaloes, binned in 6.6 Mpc/h cells



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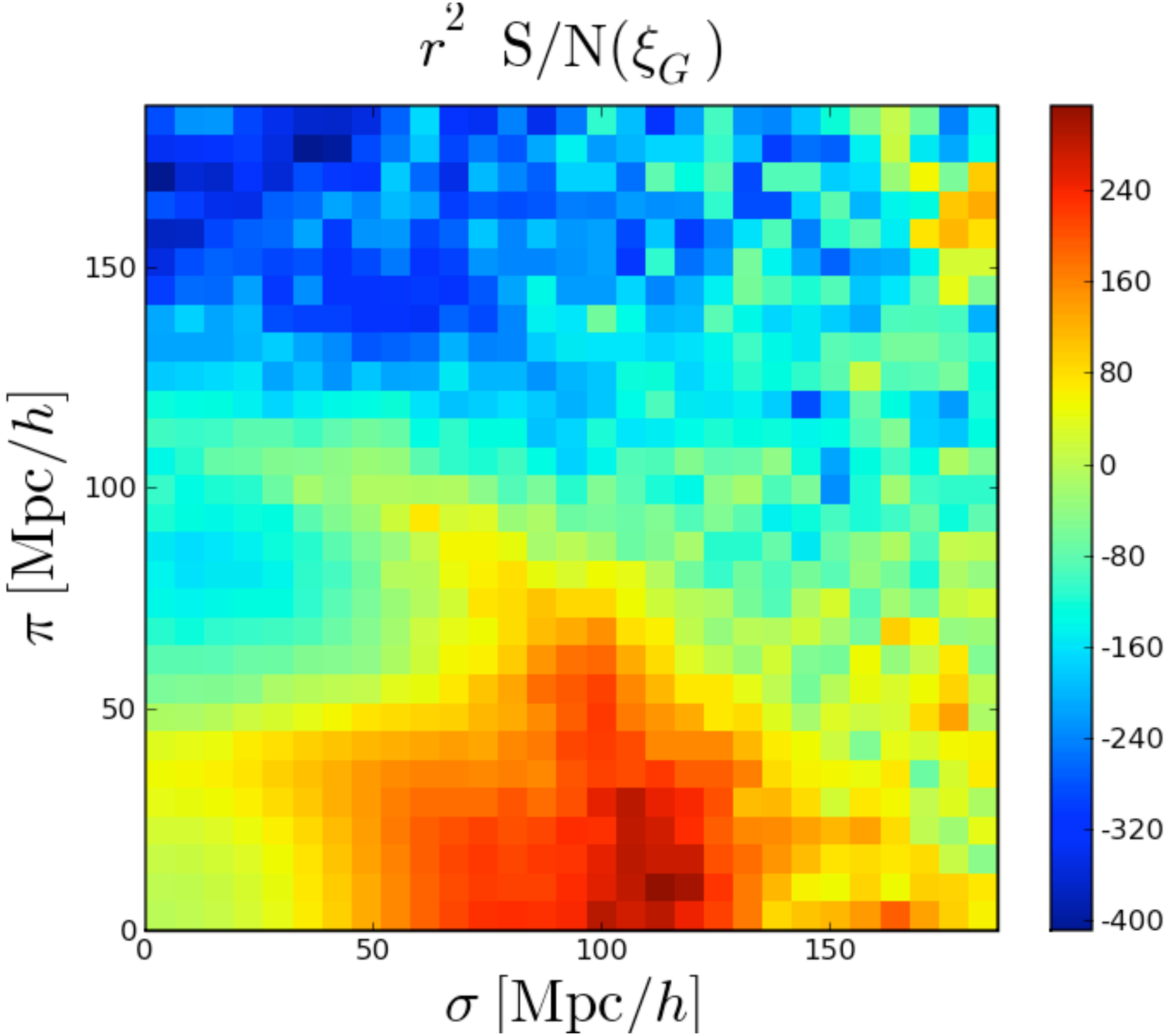

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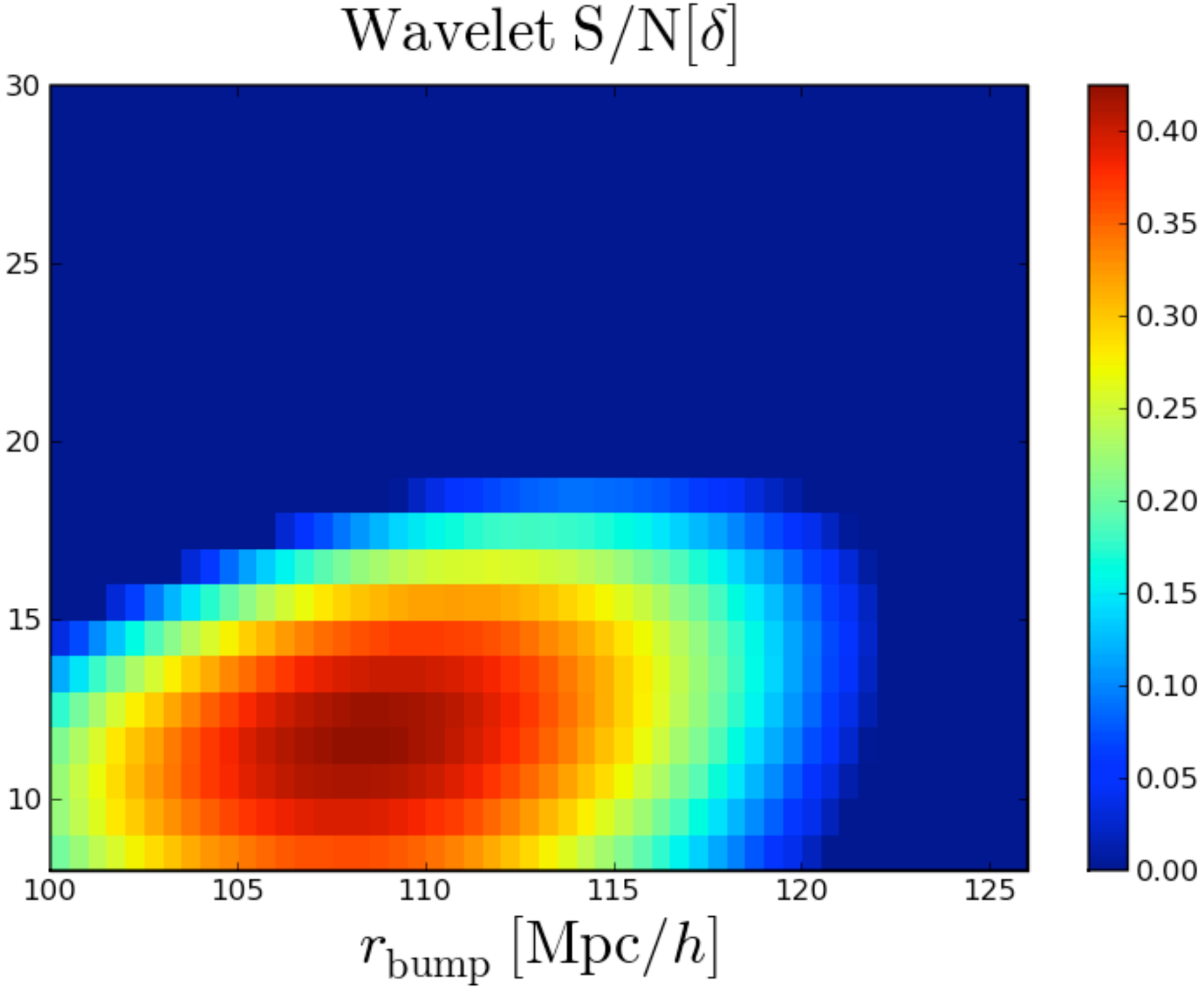
- Small shift: +0.15% BAO bump shift in Gaussianized case. Close to negative bias in ξ_δ!

- The Gaussianized-density peak seems a bit broader



- $\Omega(r_{\rm bump}, s)$ -- Mexican-hat wavelet applied to correlation function

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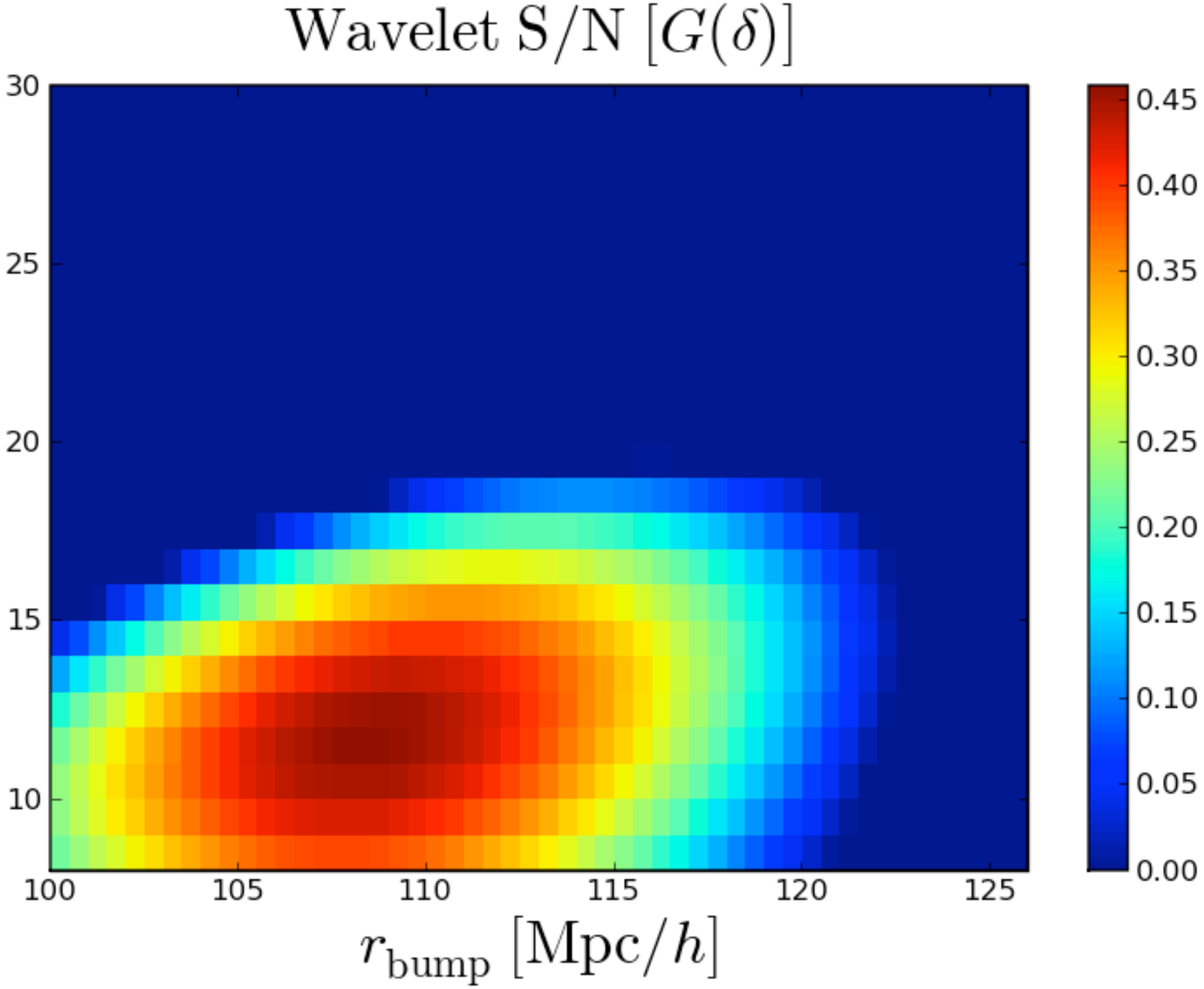

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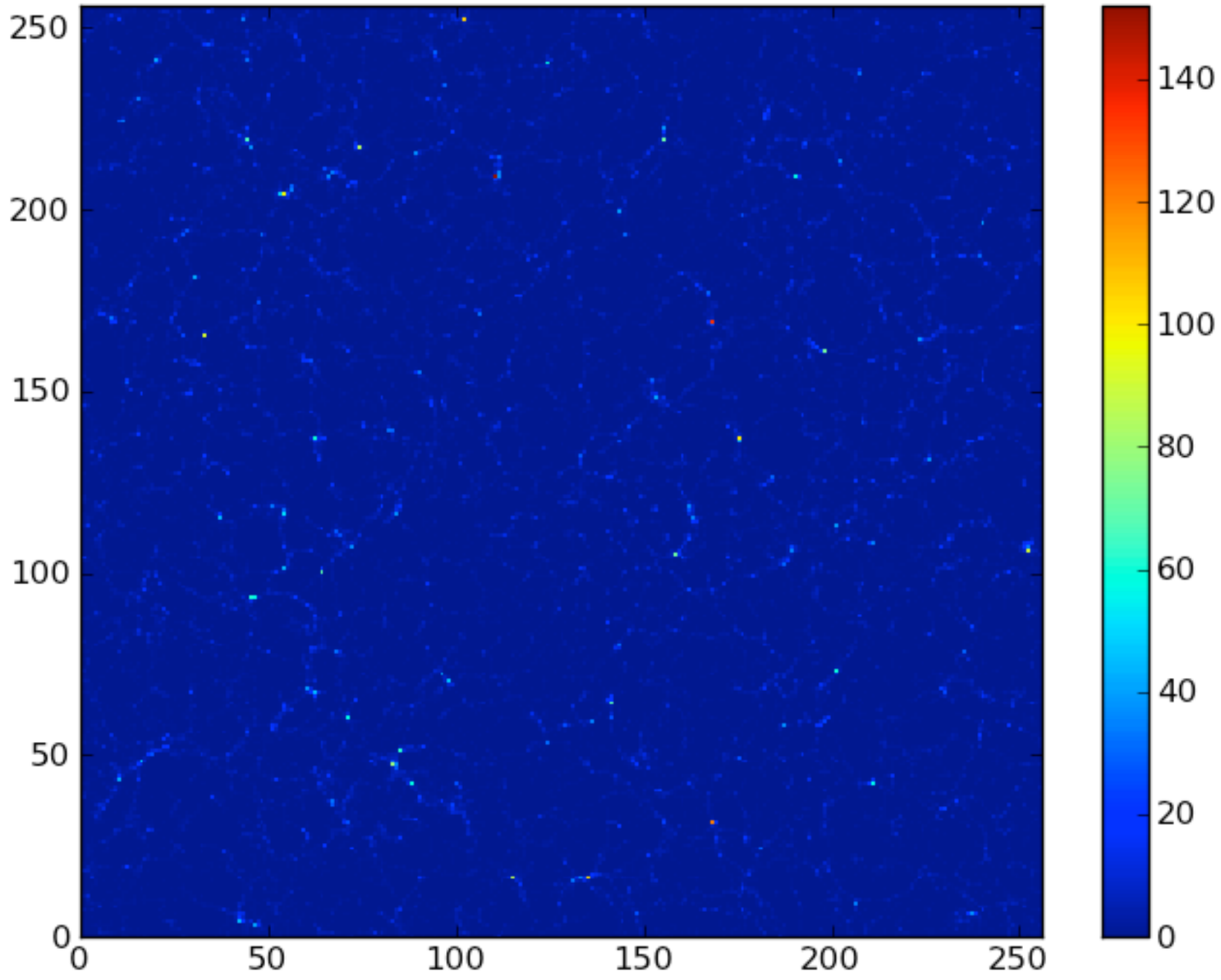
Benefits of Gaussianization in redshift space

Benefits of Gaussianization in redshift space

Real space



512^3 particles
1 h^{-1} Gpc box
4-h^{-1}Mpc cells

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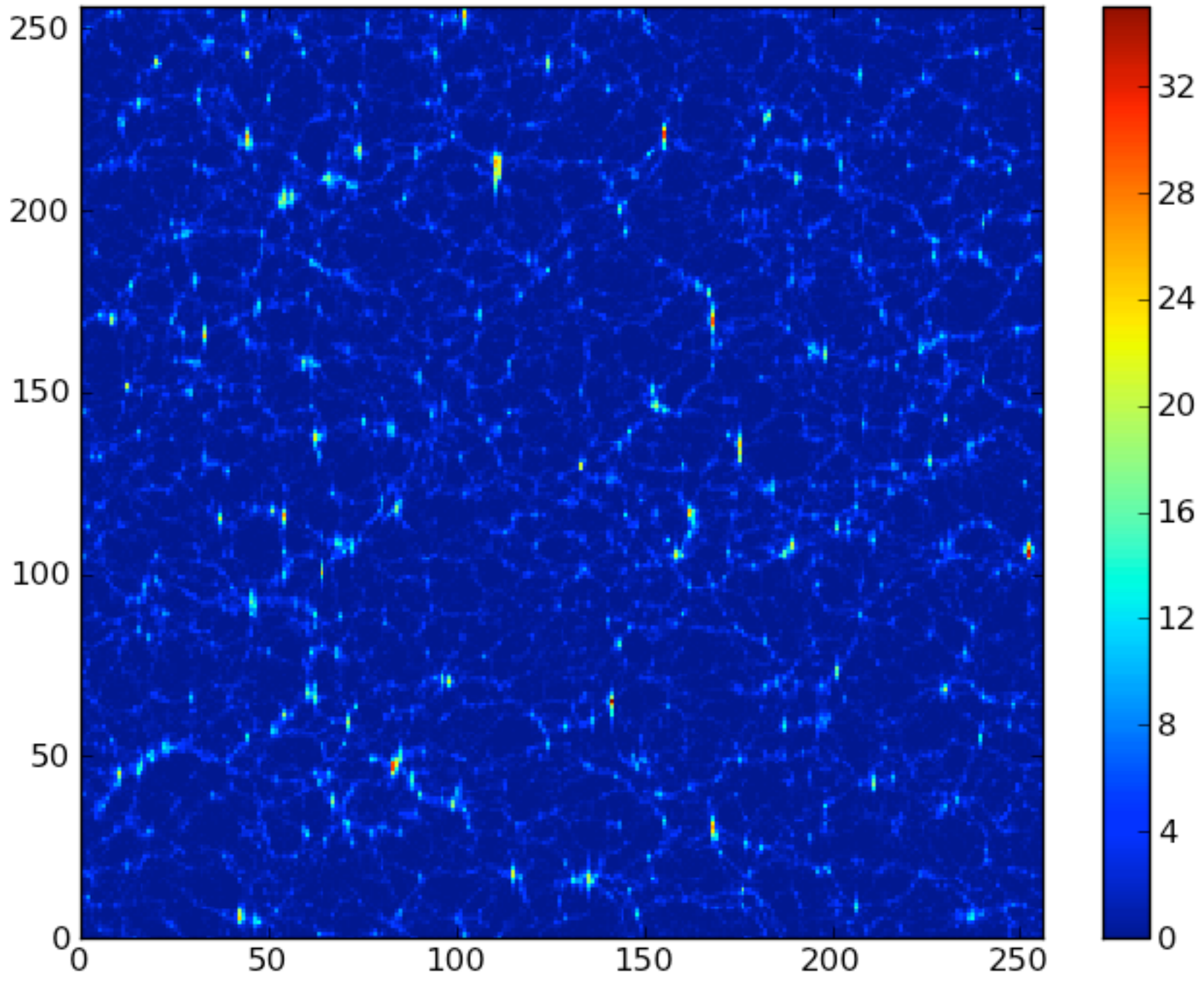
Redshift space

Sharp peaks become fingers of god -- tail of PDF contracts.

Does this mean Gaussianization will be useless?



1 h^{-1} Gpc box

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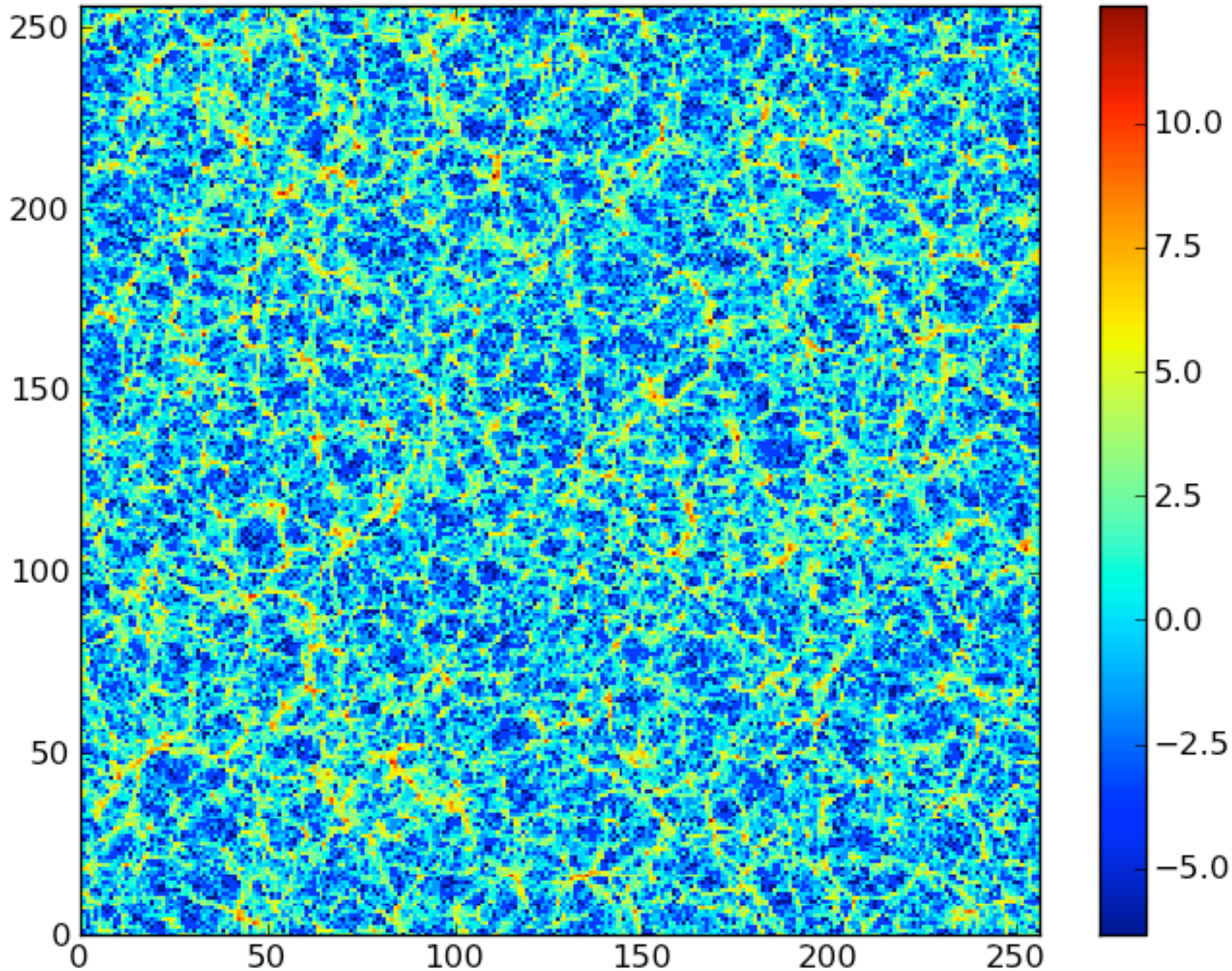
Redshift space



1 h^{-1} Gpc box

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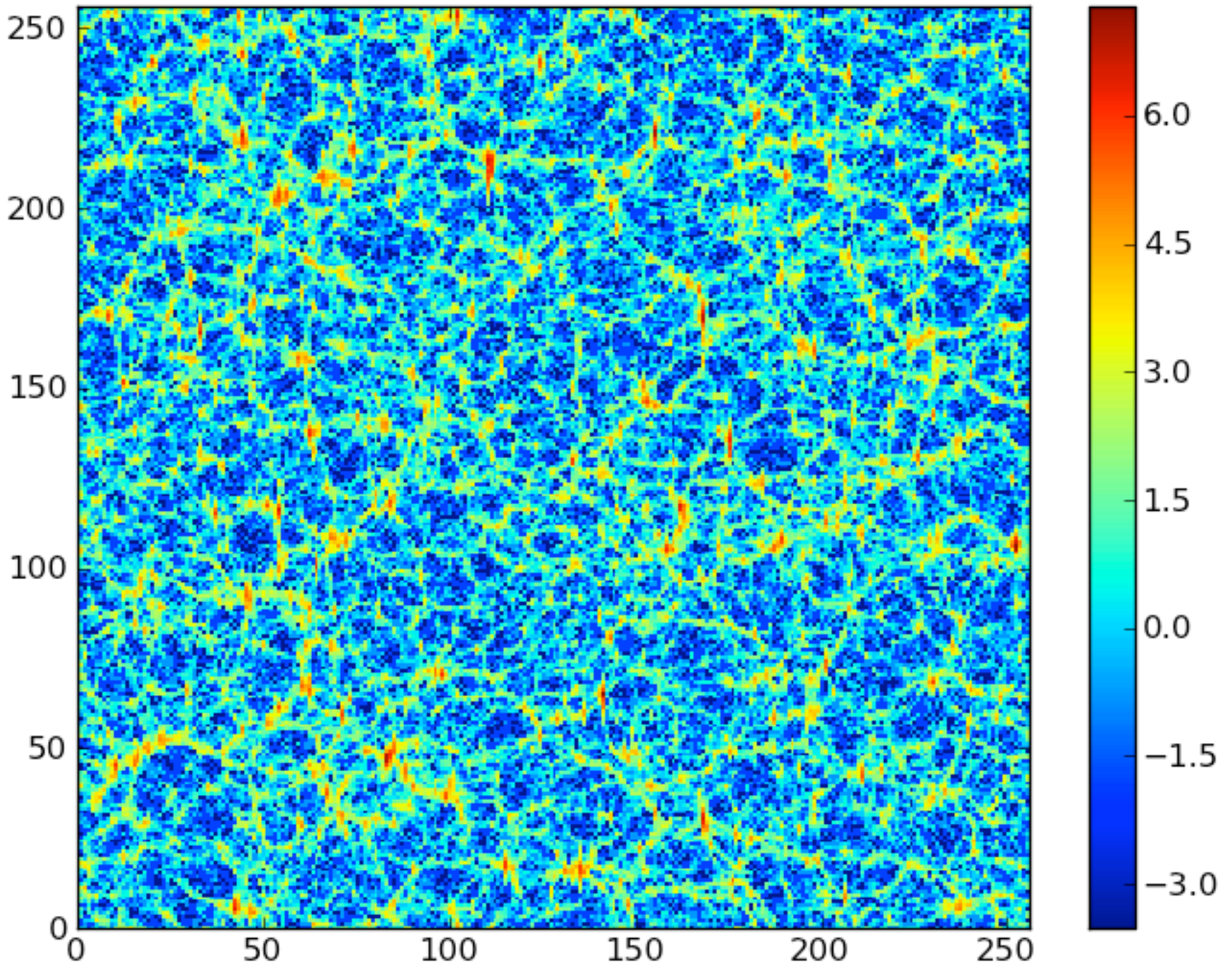
Gaussianized redshift space

- in δ, the most visible behavior is the development of FoG's.

- in G(δ), on the other hand, *linear* squashing is the most visible effect.



1 h^{-1} Gpc box

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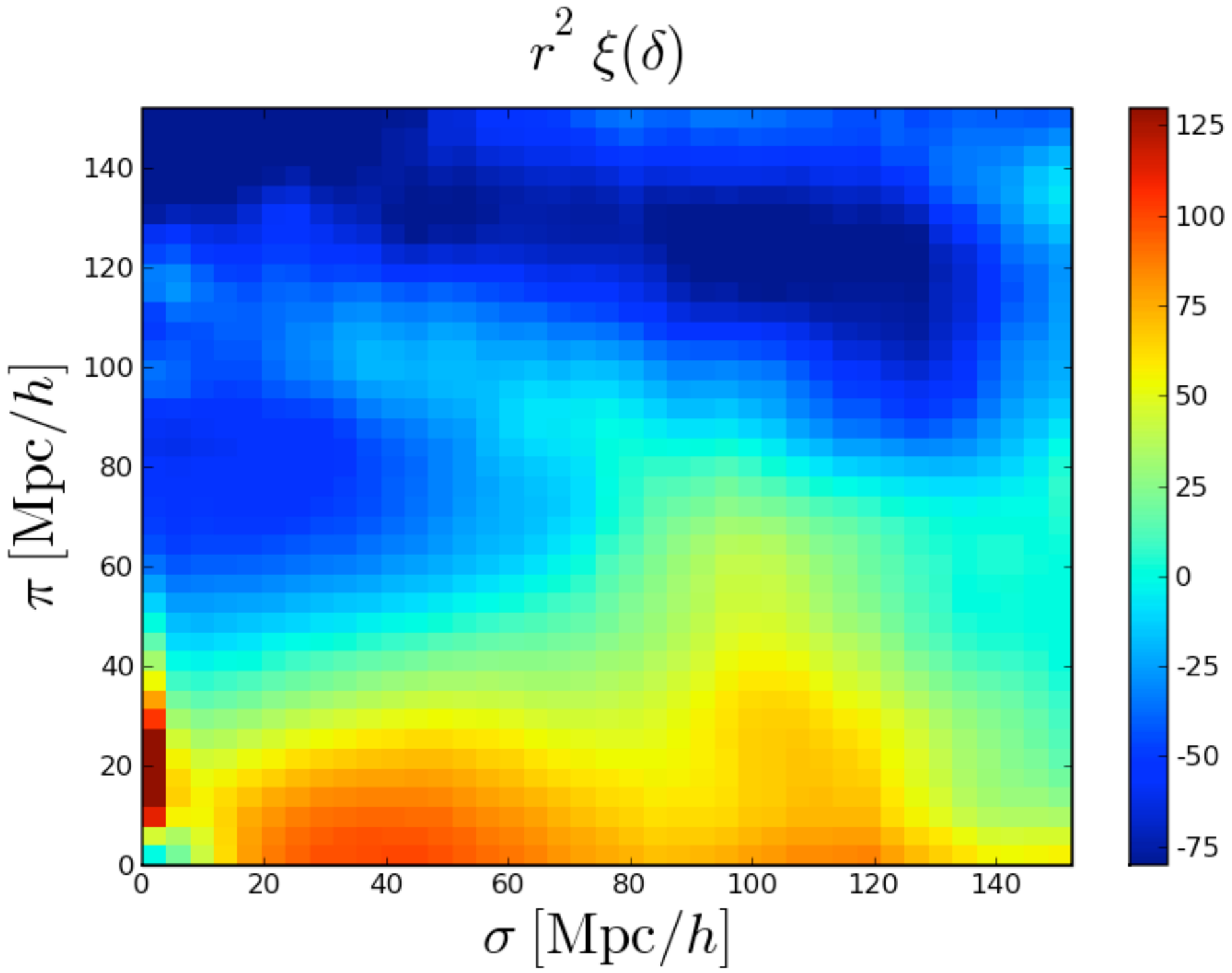
Correlation functions



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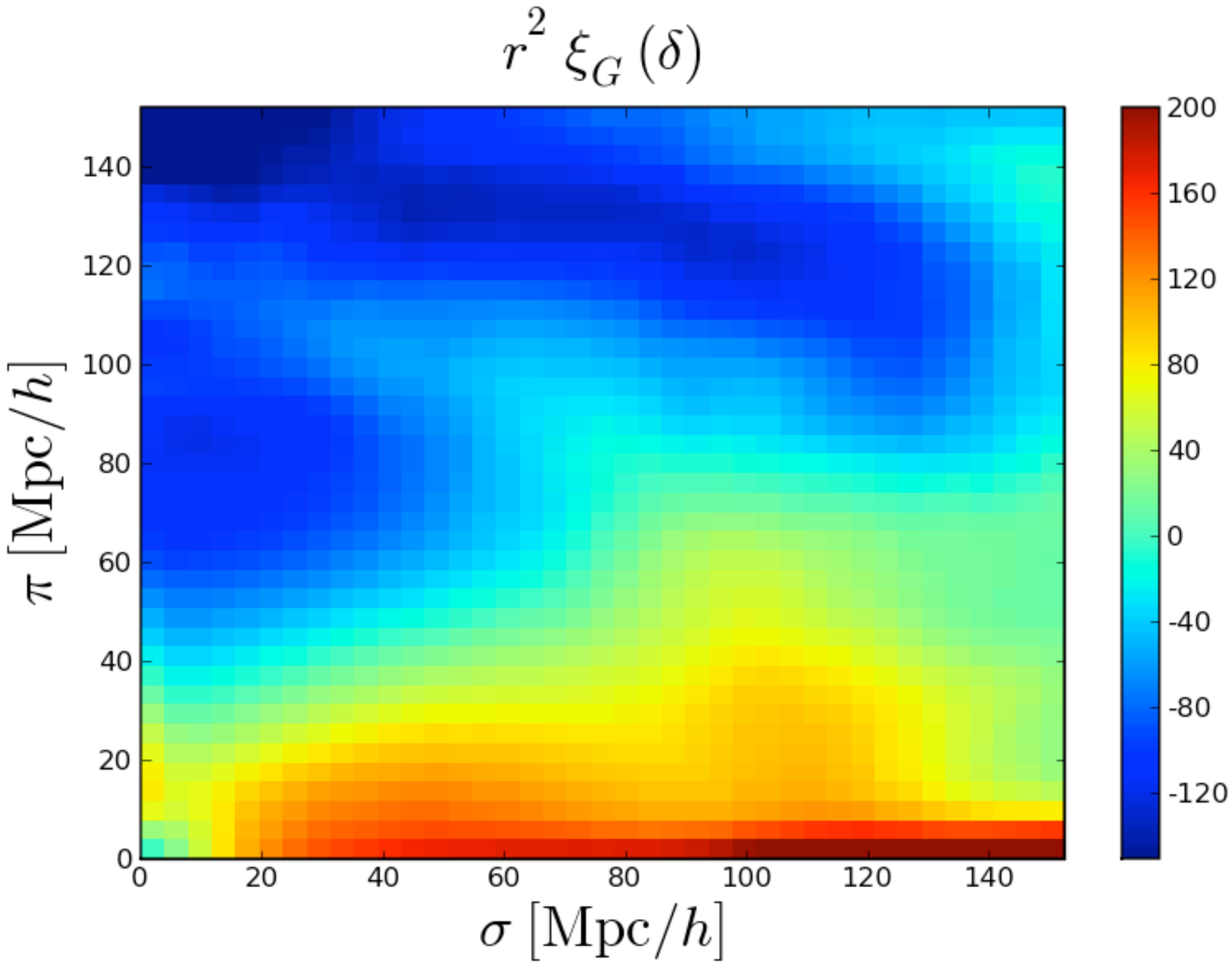
Correlation functions

FoG's reduced compared to linear-type distortions in Gaussianized case.



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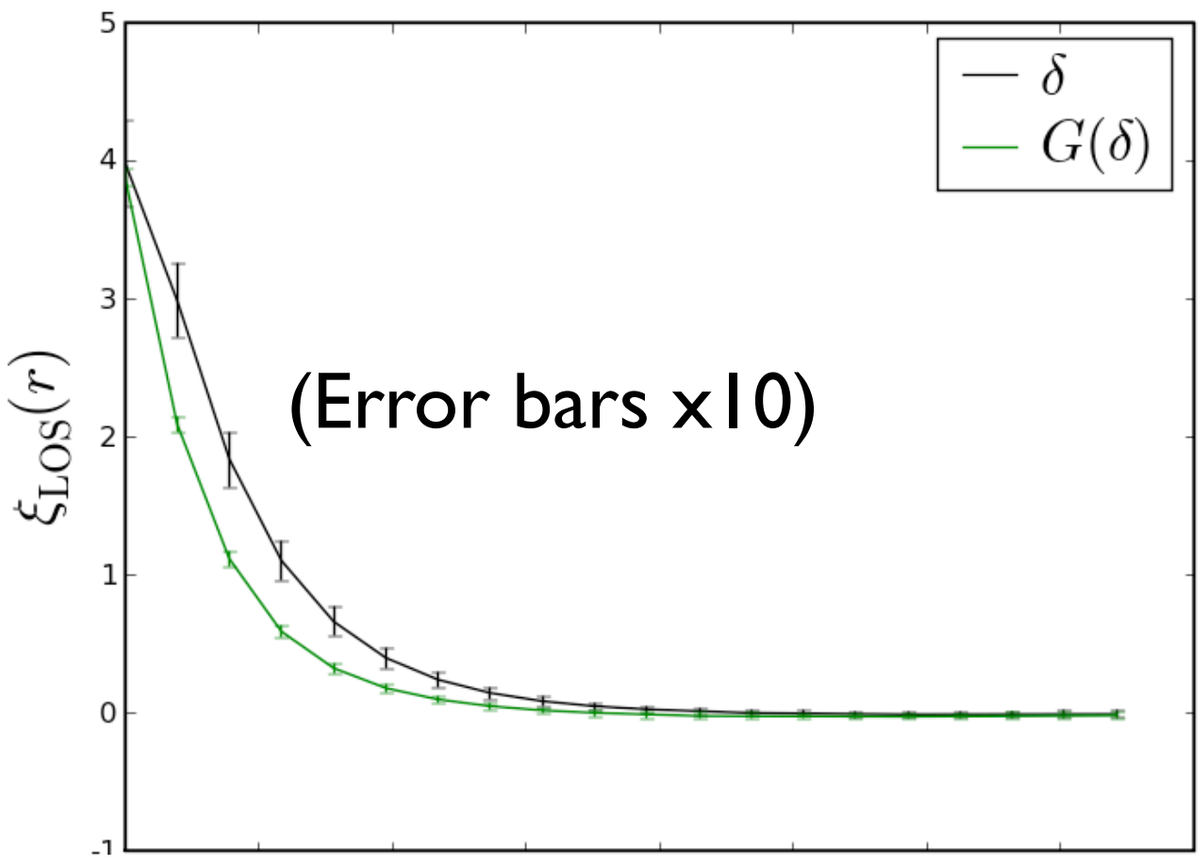
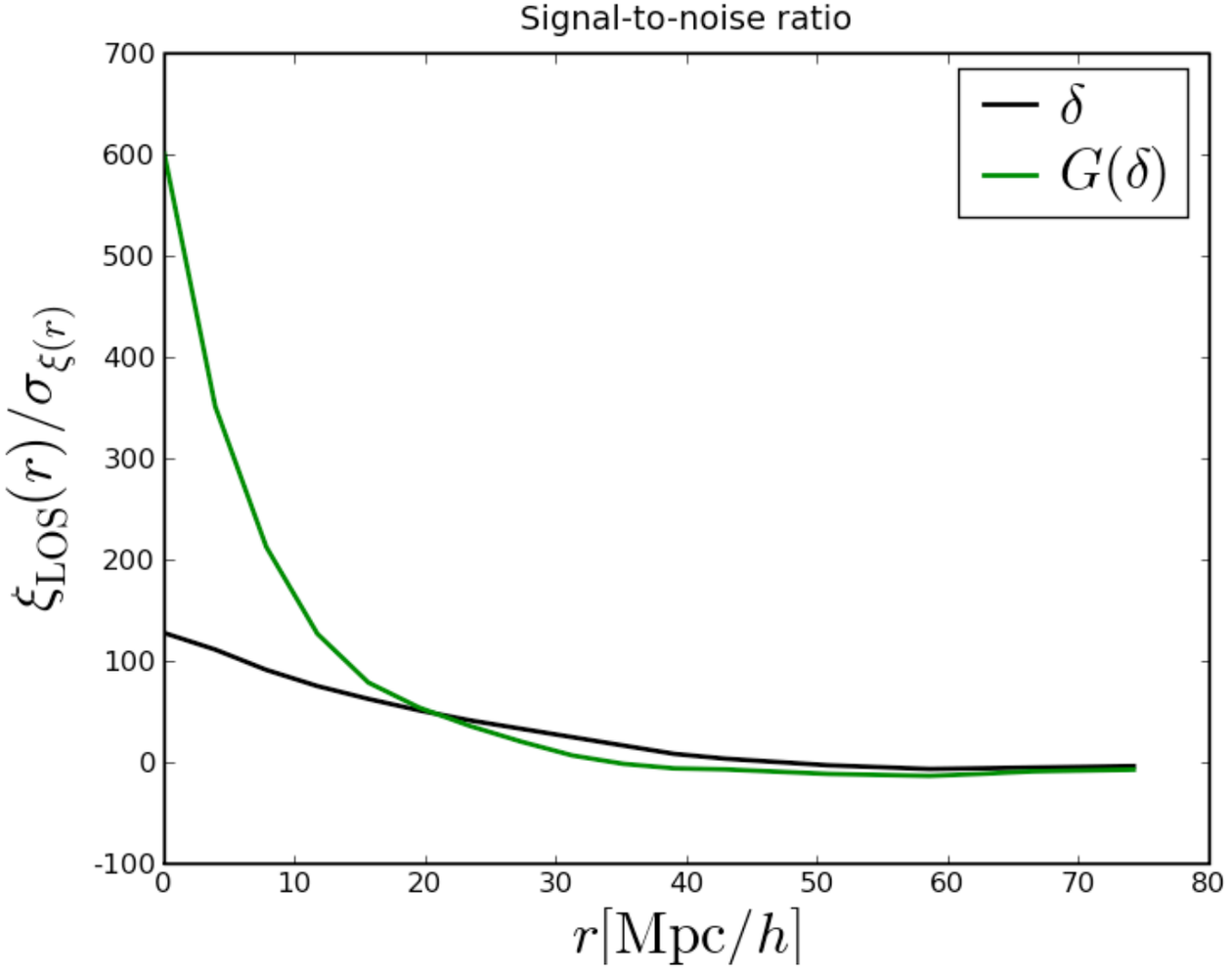
Line-of-sight correlation functions:

FoG's slightly less prominent in G(δ).

Error bars hugely reduced.





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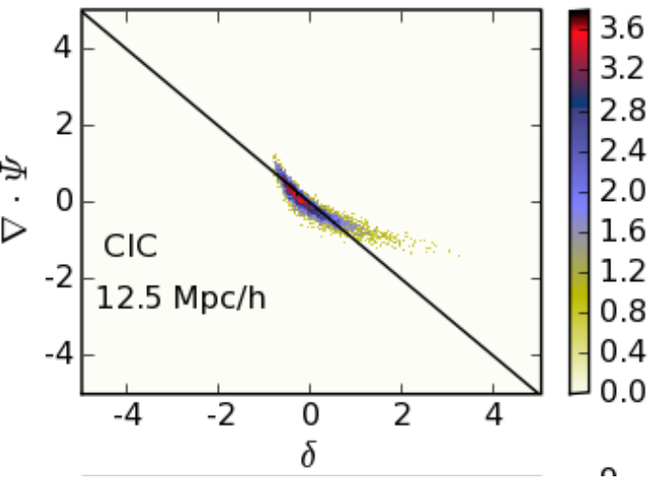
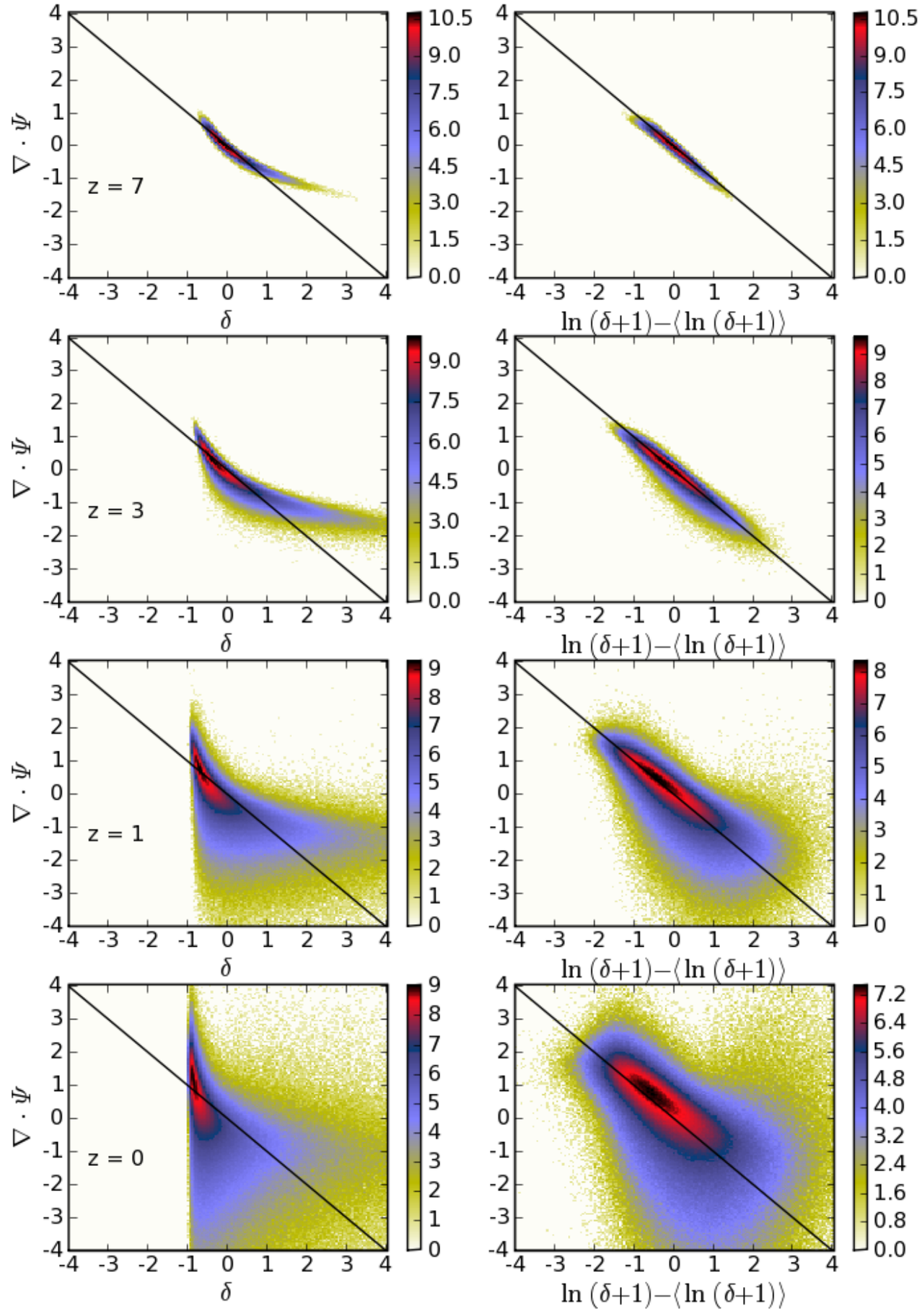
Displacement field better-estimated using log-density (Eulerian divergence here)

Highly nonlinear smoothing scale! (1.6 Mpc/h)





Falck, MN, et al 2012, ApJ, arXiv:1111.4466

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Scales		Stats
Hubble radius	Many Gpc	Cosmological parameters
Linear, Gaussian scales	x > ~100 Mpc/h k < ~0.1 h/Mpc	Power spectrum
Nonlinear scales	x < ~50 Mpc/h	Morphological statistics?

Scales		Stats
Hubble radius	Many Gpc	Cosmological parameters
Linear, Gaussian scales	x > ~100 Mpc/h k < ~0.1 h/Mpc	Power spectrum
Translinear scales	~ 0.1 h/Mpc < k < ~ 0.8 h/Mpc	Gaussianized power spectrum (my claim)
Fully nonlinear scales	x < few Mpc	Morphological statistics?

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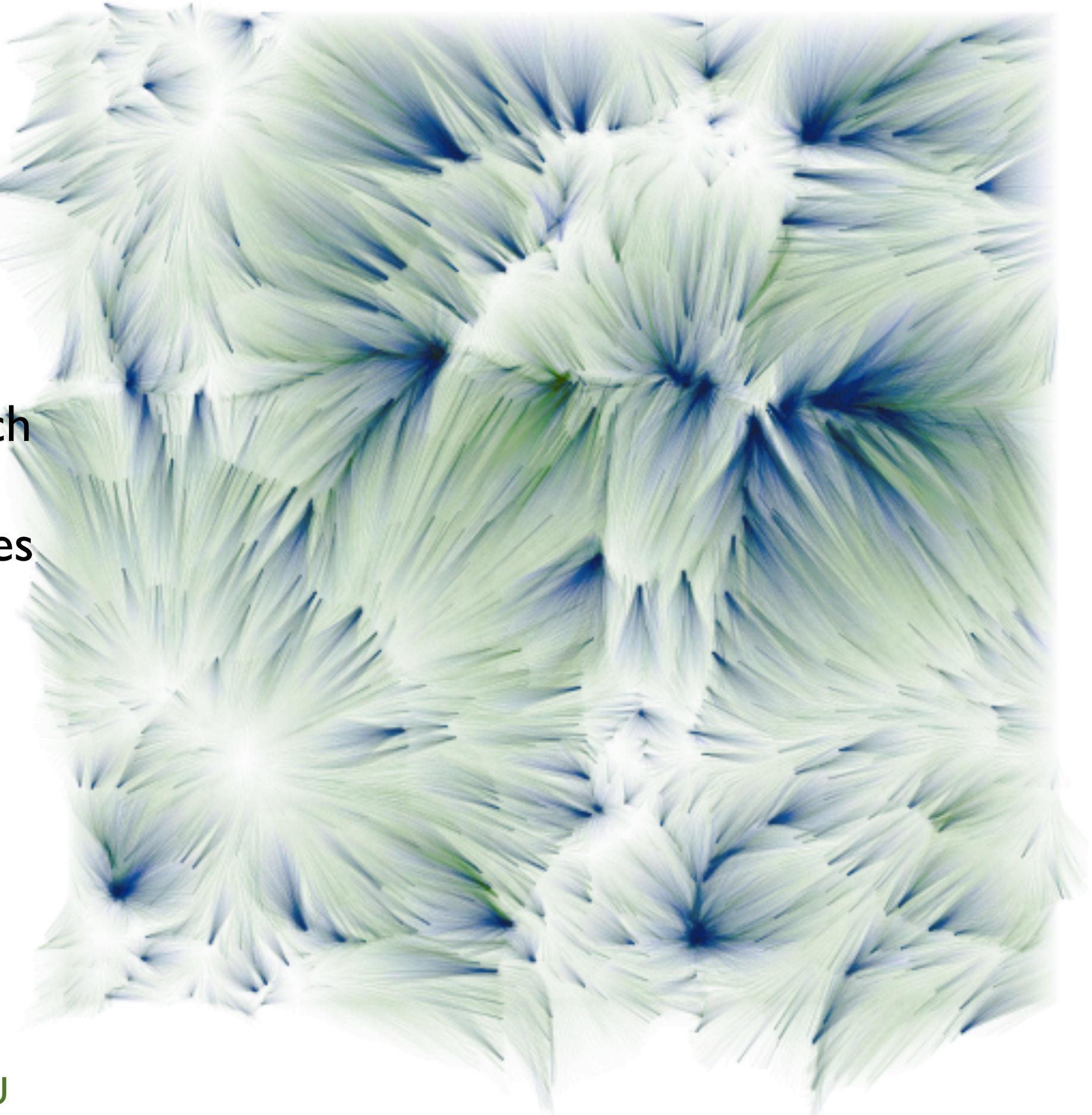
Conclusions

- Wrinkles more friendly in a Lagrangian approach

- When the wrinkles are not too bad, they can largely be Gaussianized away



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