

Leonardo Senatore (CERN & Stanford)

The Effective Field Theory of Large Scale Structures

with Carrasco, Hertzberg
1206.2926 [astro-ph.CO]

with Carrasco, Foreman
in completion [astro-ph.CO]

Cosmological non-Linearities as an Effective Fluid

with Baumann, Nicolis and Zaldarriaga, **1004.2488** (JCAP)

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The Universe is Chocolate

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The Universe is Melted Chocolate



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Cosmological non-Linearities as an Effective Fluid

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We always knew: the Universe is a piece of cake

with Carrasco, Hertzberg
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Cosmological non-Linearities as an Effective Fluid

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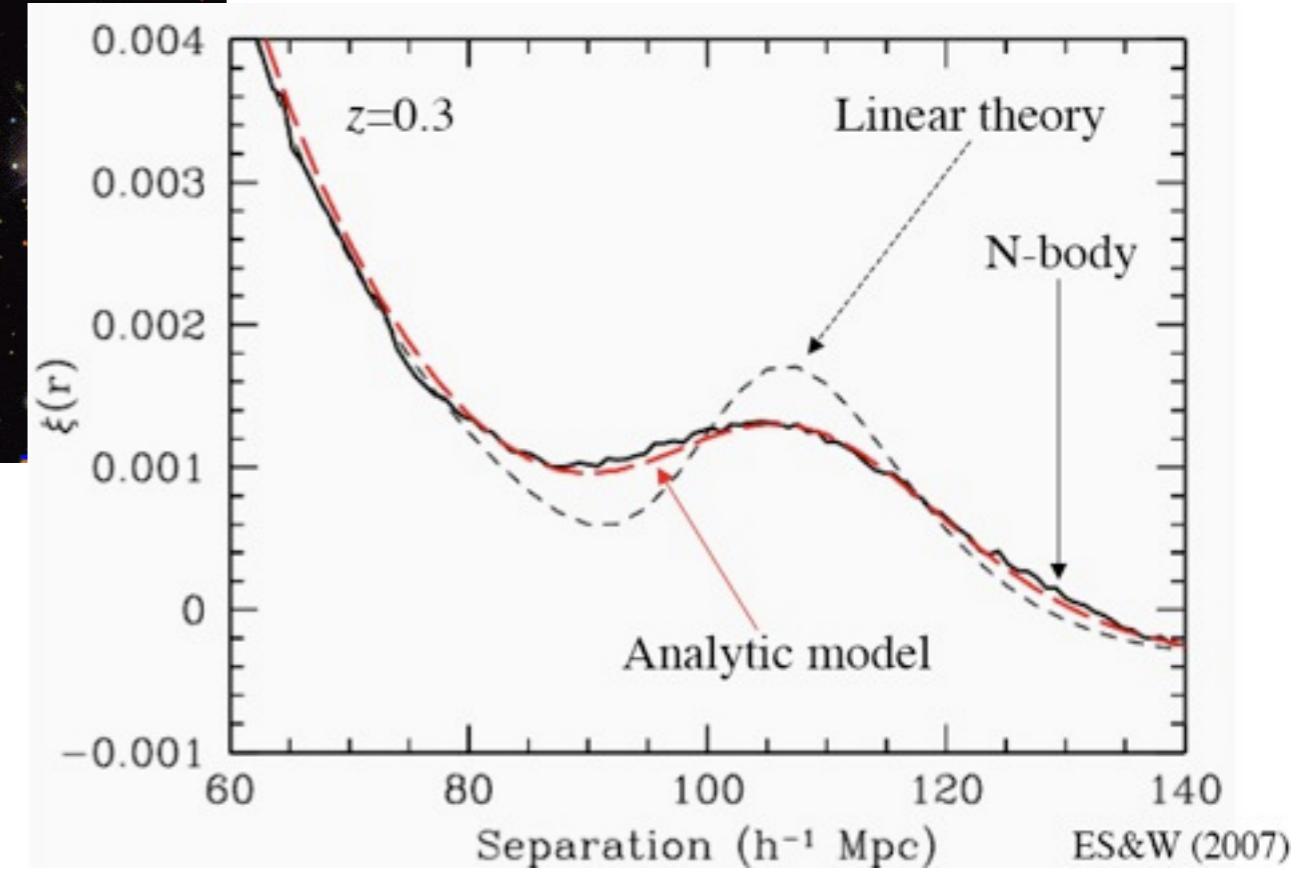
A well defined perturbation theory for LSS Surveys

- Observe the correlation of Galaxies



Analogous of CMB peaks

- Information about Dark Energy,
Non-Gauss,

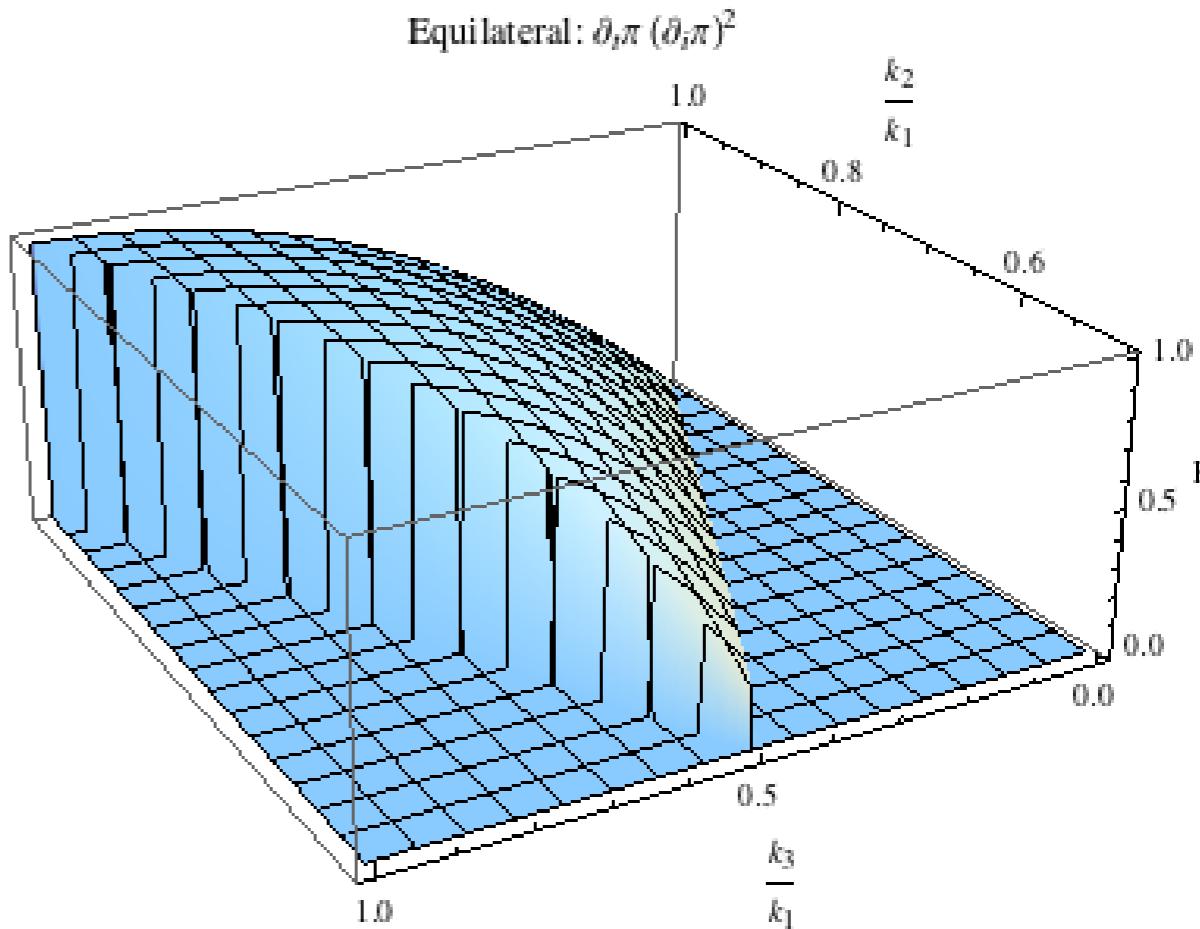


Exploring Inflation: the Effective Field Theory of Inflation

with C. Cheung, P. Creminelli,
L. Fitzpatrick, J. Kaplan
JHEP 0803:014,2008

$$S_\pi = \int d^4x \sqrt{-g} \left[M_{\text{Pl}}^2 \dot{H} (\dot{\pi}^2 - (\partial_i \pi)^2) + M_2^4 (\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} (\partial_i \pi)^2) - M_3^4 \dot{\pi}^3 + \dots \right]$$

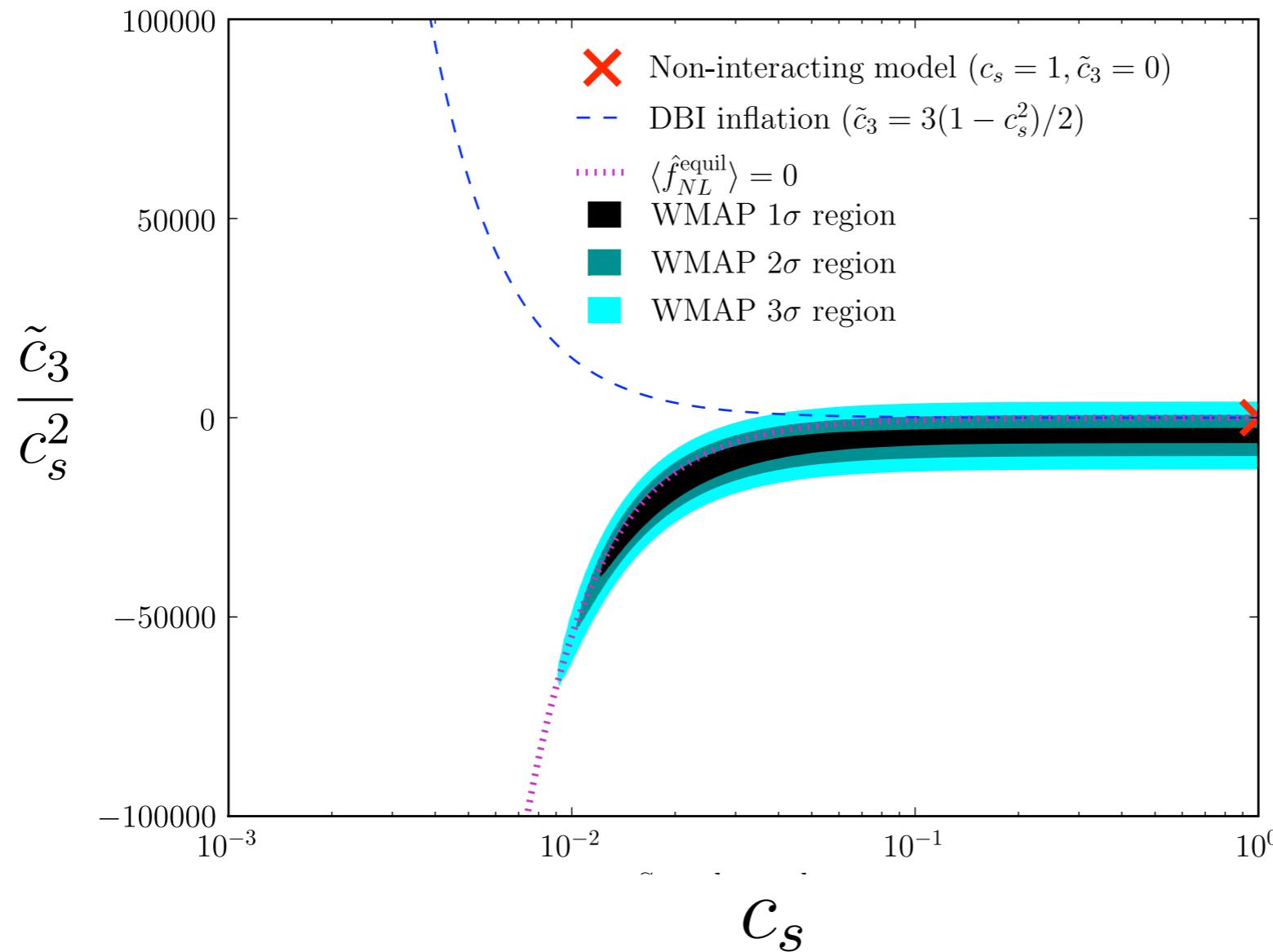
- Inflation as the **Theory of a Goldstone Boson**
- Unique Lagrangian for all possible models
- New Signatures: possible large interactions
 - Ex: 3-point function
 - Similar to Scattering Amplitude
- Large Scale Structure has *information* than CMB



Our true Knowledge of Inflation

$$S_\pi = \int d^4x \sqrt{-g} \left[M_{\text{Pl}}^2 \dot{H} (\dot{\pi}^2 - (\partial_i \pi)^2) + M_2^4 (\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} (\partial_i \pi)^2) - M_3^4 \dot{\pi}^3 + \dots \right]$$

- Limits on f_{NL} 's get translated into limits on the universal parameters



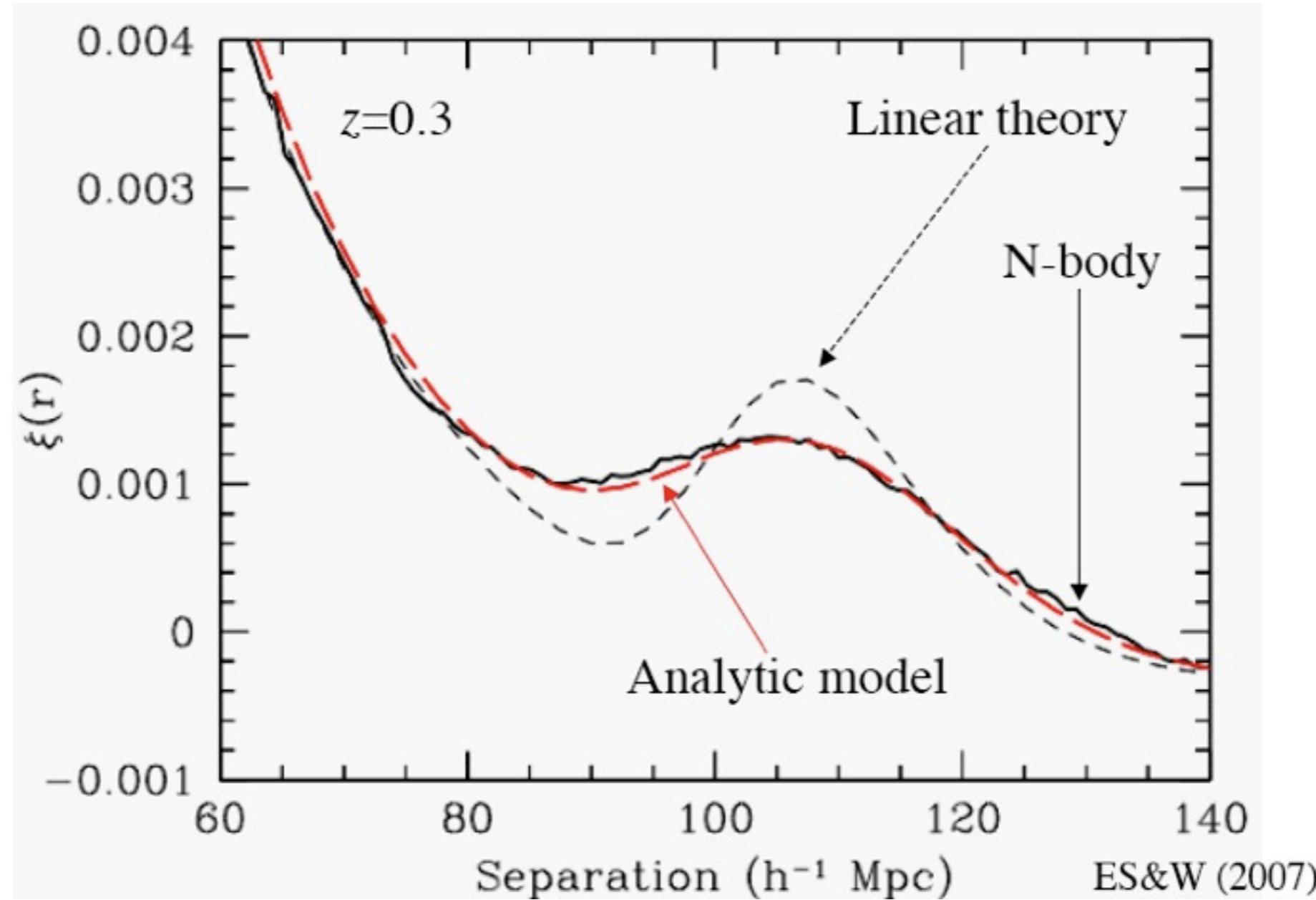
With Smith and Zaldarriaga,
JCAP1001:028,2010

Very similar in spirit to
Peskin and Takeuchi
PRD46:381,1992
**(Complete Connection to
Particle Physics)**

$$\frac{1}{c_s^2} \dot{\pi} (\partial_i \pi)^2 + \frac{\tilde{c}_3}{c_s^2} \dot{\pi}^3$$

A well defined perturbation theory is needed

- Baryon Acoustic Oscillations scale is close to non-linear scale (factor of ~ 10)

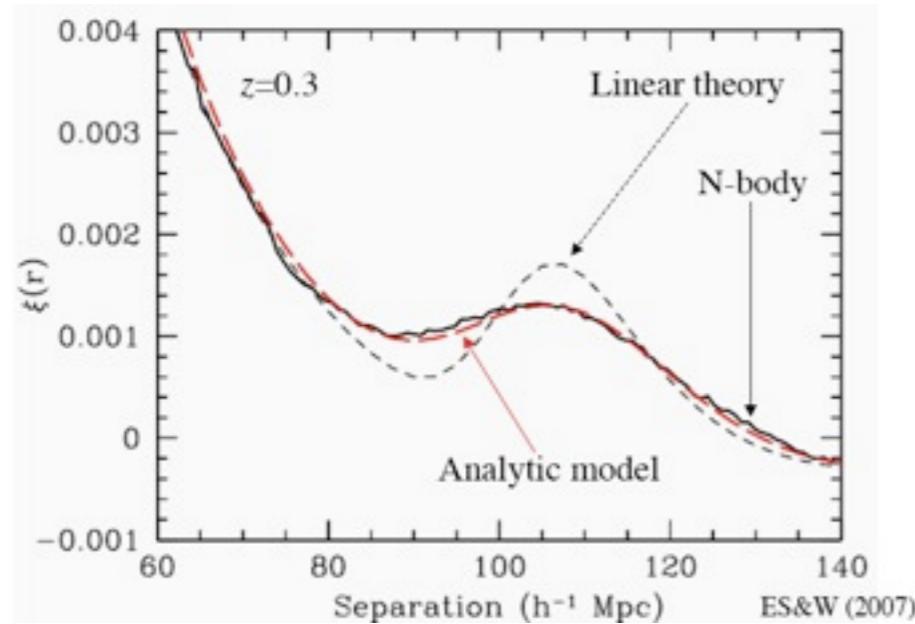


- Fitted with damping and stochasticity

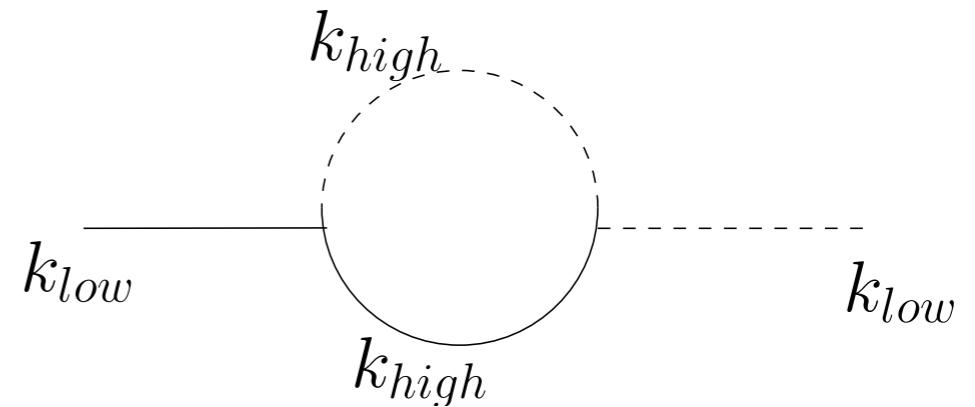
$$P_{\text{obs}}(k) = e^{-\frac{1}{2}k^2\Sigma^2} P_{\text{L}}(k) + P_{\text{mc}}(k)$$

A well defined perturbation theory

- Baryon Acoustic Oscillations scale is close to non-linear scale (factor of ~ 10)

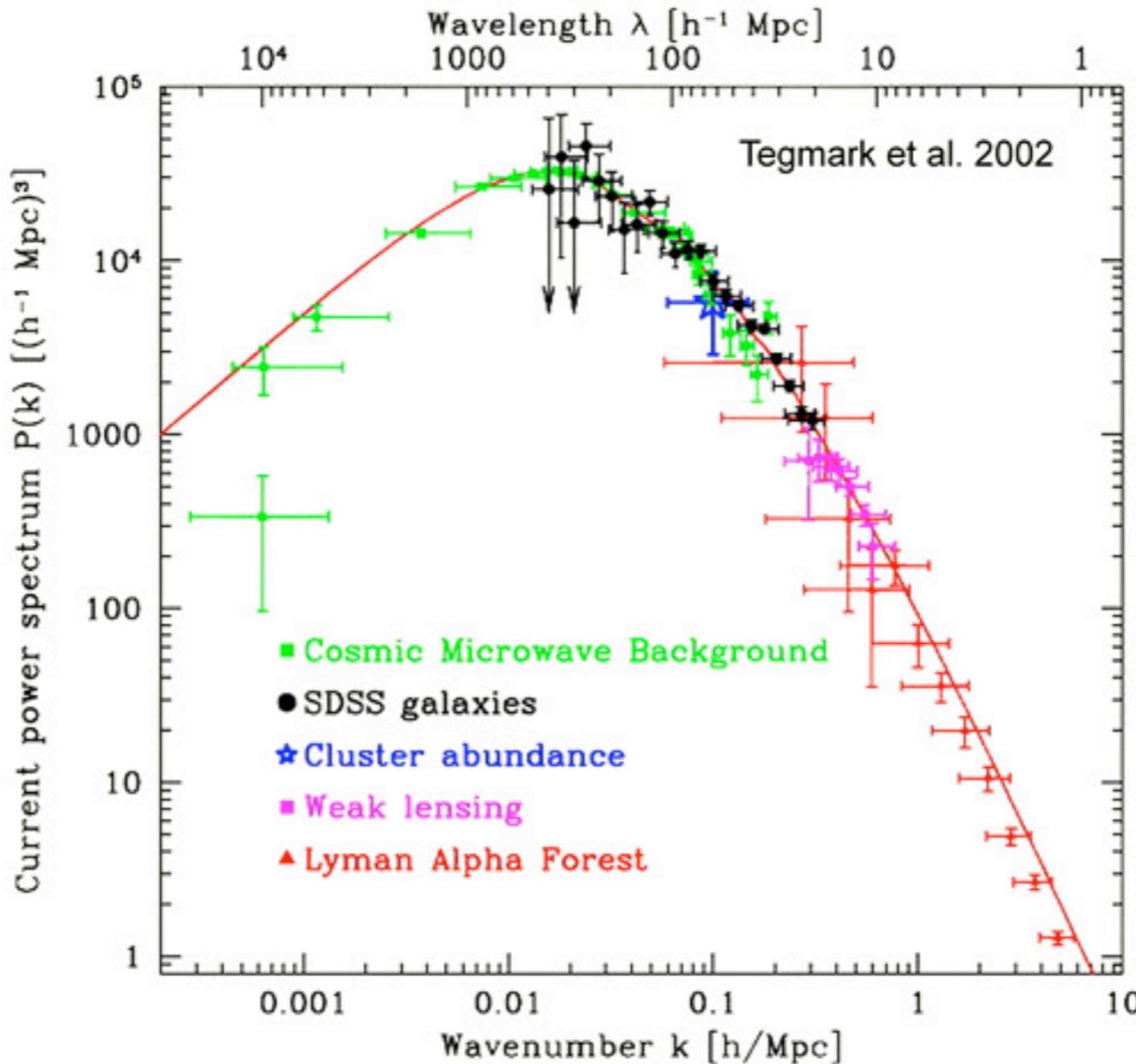


- It is very unclear if current perturbation theory is well defined (at 1% level ?!)
- Standard techniques
 - perfect fluid $\dot{\rho} + \partial_i (\rho v^i) = 0$,
 - expand in $\delta\rho/\rho$
- Equations break in the UV for two reasons
 - $\delta\rho/\rho \gg 1$
 - no fluid



A well defined perturbation theory

- A lucky help in our universe



- In a only dark matter universe, all standard Perturbation theories are doomed
 - Accuracy depends on initial conditions. Unacceptable.

with Carrasco, Foreman, Tashev and Zaldarriaga, **in progress**

Idea of the Effective Field Theory

Our universe

- How does our universe looks like?

- Non-linear on short scales

$$\lambda_{NL} \sim 1 - 10 \text{ Mpc}$$

$$\delta\rho/\rho \gg 1$$

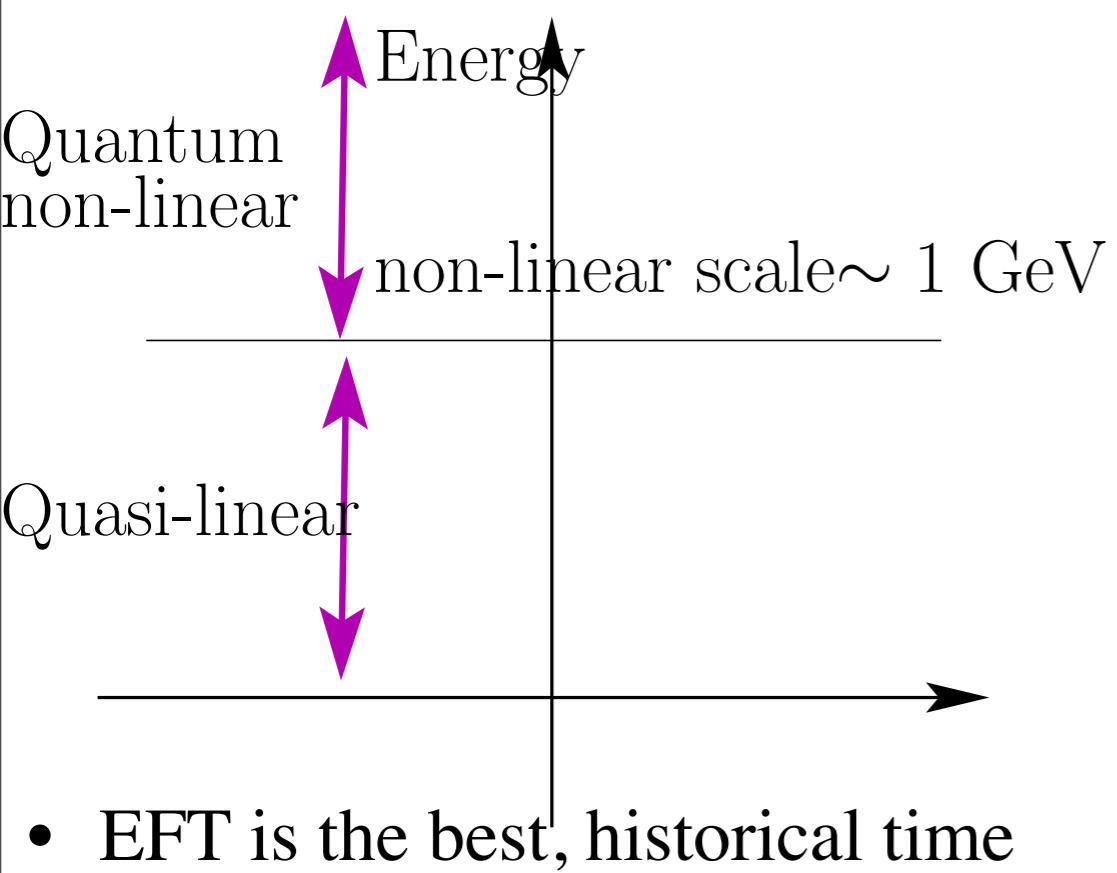
- Linear on large-scales

$$H^{-1} \sim 14000 \text{ Mpc}$$

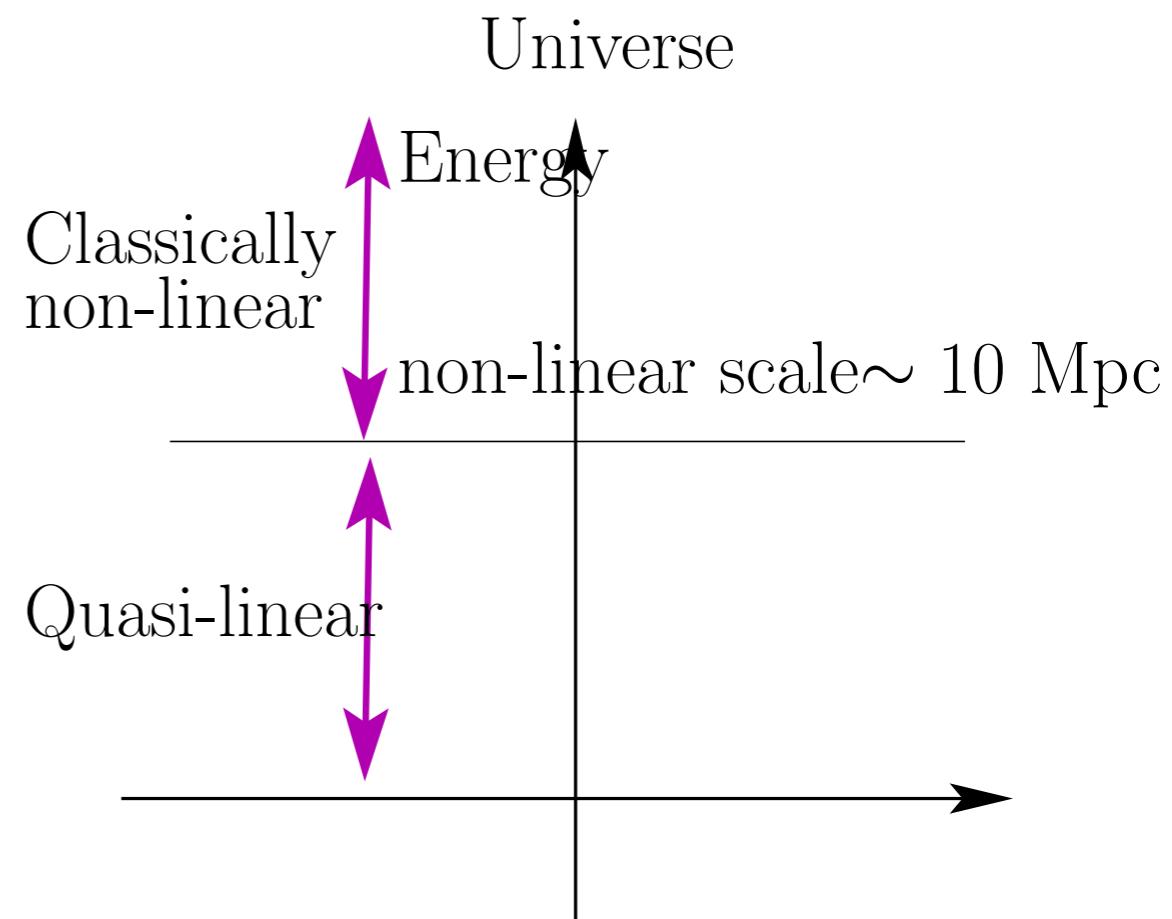
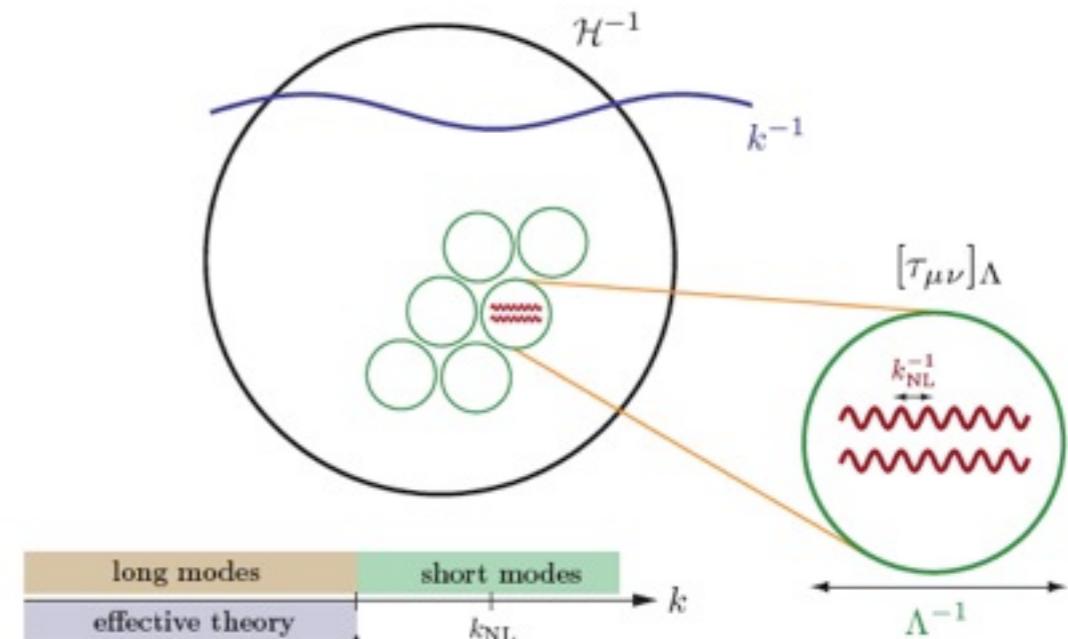
$$\delta\rho/\rho \ll 1$$

- Similar to QCD Chiral Lagrangian

QCD



$$S = \int d^4x \left[(\partial\pi)^2 + \frac{1}{F_\pi^2}\pi^2(\partial\pi)^2 + \frac{1}{\tilde{F}_\pi^2}(\partial\pi)^4 + \dots \right]$$

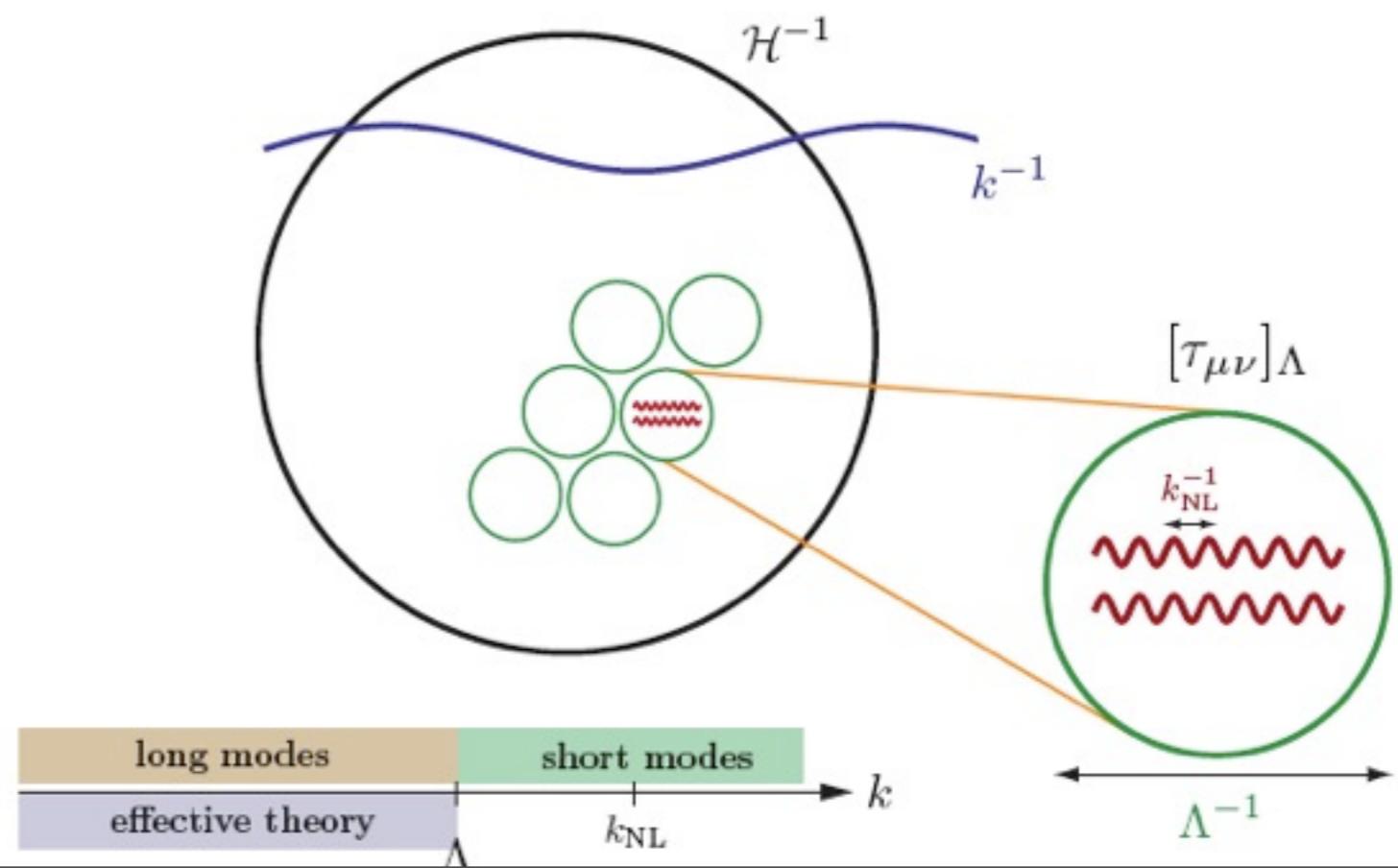


A well defined perturbation theory

- We will define a manifestly convergent perturbation theory

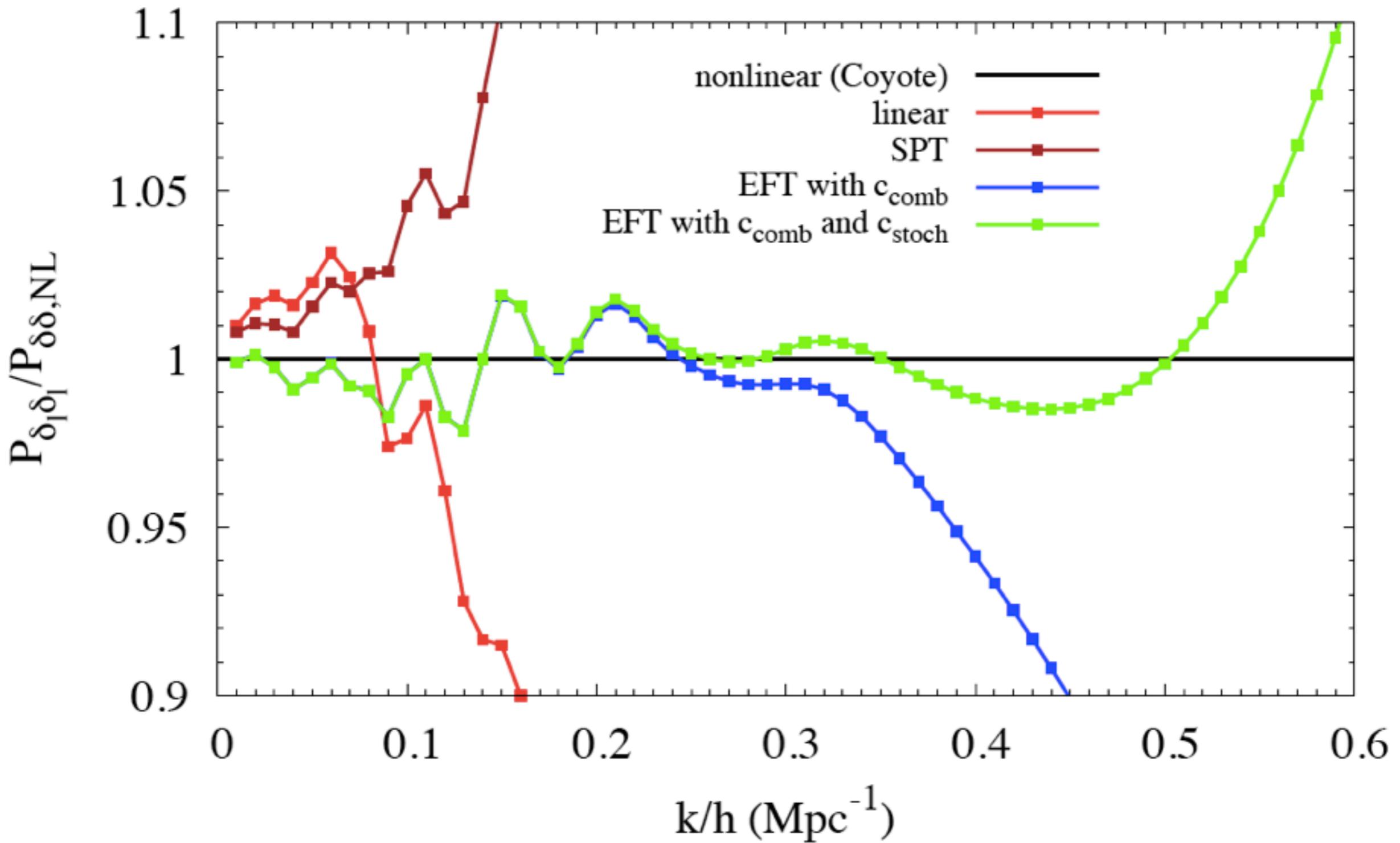


– where the ingredient is
an imperfect stochastic fluid with
 $\delta_\ell, v_\ell, \Phi_\ell \ll 1$



Bottom line result

- 1.5 loops in EFT perturbation theory



- Percent agreement up to $k \sim 0.55$ (data go as k^3)

Construction of the Effective Field Theory: from UV to IR

From Dark Matter Particles to Cosmic Fluid

- UV
- Dark Matter described by distribution $f(\vec{x}, \vec{p}) = \sum \delta^{(3)}(\vec{x} - \vec{x}_n) \delta^{(3)}(\vec{p} - m a \vec{v}_n)$
- Boltzmann equation $\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{ma^2} \cdot \frac{\partial f}{\partial \vec{x}} - m \sum_{n, \bar{n}; \bar{n} \neq n} \frac{\partial \phi_{\bar{n}}}{\partial \vec{x}} \cdot \frac{\partial f_n}{\partial \vec{p}} = 0$.
- and Newtonian gravity $\partial^2 \phi = 4\pi G a^2 (\rho - \rho_b)$
- Smoothing the fields $W_\Lambda(\vec{x}) = \left(\frac{\Lambda}{\sqrt{2\pi}}\right)^3 e^{-\frac{1}{2}\Lambda^2 x^2}$
 $\mathcal{O}_l(\vec{x}, t) = [\mathcal{O}]_\Lambda(\vec{x}, t) = \int d^3x' W_\Lambda(\vec{x} - \vec{x}') \mathcal{O}(\vec{x}')$
- Smooth Boltzmann equation
 $\left[\frac{Df}{Dt} \right]_\Lambda = \frac{\partial f_l}{\partial t} + \frac{\vec{p}}{ma^2} \cdot \frac{\partial f_l}{\partial \vec{x}} - m \sum_{n, \bar{n}, n \neq \bar{n}} \int d^3x' W_\Lambda(\vec{x} - \vec{x}') \frac{\partial \phi_n}{\partial \vec{x}'}(\vec{x}') \cdot \frac{\partial f_{\bar{n}}}{\partial \vec{p}}$.
- and take moments
 $\int d^3p p^{i_1} \dots p^{i_n} \left[\frac{Df}{Dt} \right]_\Lambda(\vec{x}, \vec{p}) = 0$,
- Boltzmann hierarchy perturbative by powers of $\frac{k}{k_{NL}}$, $k_{NL} \sim v_{DM} H^{-1} \sim 10^{-3} H^{-1}$

From Dark Matter Particles to Cosmic Fluid

- We get a fluid!

- First two moments: $\dot{\rho}_l + 3H\rho_l + \frac{1}{a}\partial_i(\rho_l v_l^i) = 0$,

$$\dot{v}_l^i + Hv_l^i + \frac{1}{a}v_l^j\partial_j v_l^i + \frac{1}{a}\partial_i\phi_l = -\frac{1}{a\rho_l}\partial_j [\tau^{ij}]_\Lambda$$

- It is really the momentum that counts

$$v_l^i = \frac{\pi_l^i}{\rho_l}$$

$$\rho(\vec{x}, t) = \frac{m}{a^3} \int d^3p f(\vec{x}, \vec{p}) = \frac{m}{a^3} \sum_n \delta^{(3)}(\vec{x} - \vec{x}_n) ,$$

$$\pi^i(\vec{x}, t) = \frac{1}{a^4} \int d^3p p^i f(\vec{x}, \vec{p}) = \frac{m}{a^3} \sum_n v_n^i \delta^{(3)}(\vec{x} - \vec{x}_n)$$

- Short distance fluctuations appear as enhanced stress tensor for long modes

$$[\tau^{ij}]_\Lambda = \kappa_l^{ij} + \Phi_l^{ij} \sim \text{kinetic + potential : } \kappa \sim \rho v_s^2 , \quad \Phi \sim \rho_s \Phi_s$$

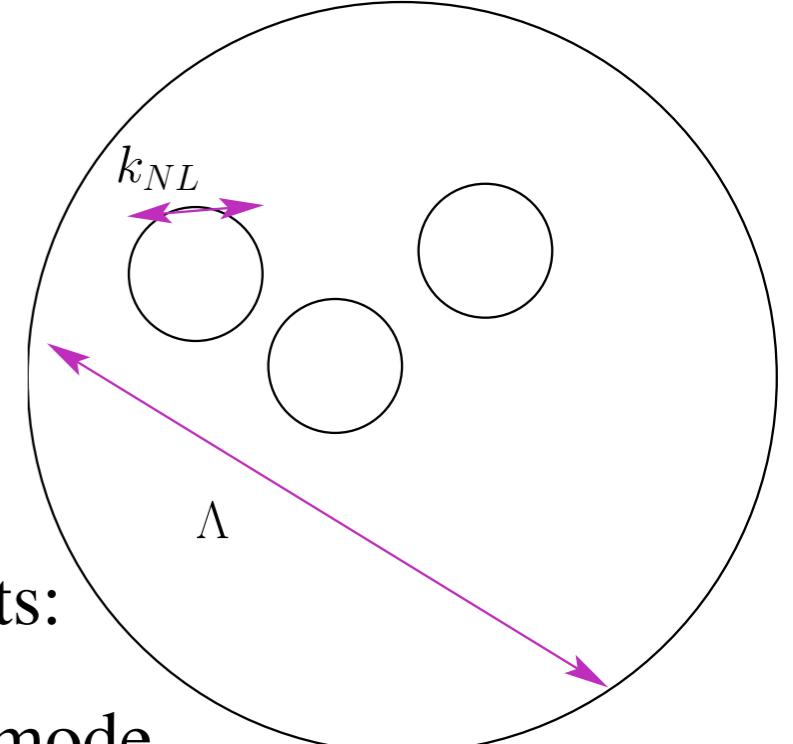
» So far, this theory still contains short distance fluctuations:

» this is not yet a long wavelength, well defined, EFT.

Integrate out
UV modes

Integrating out UV modes

- Integrate out short modes: i.e. solve equations of motion
- This is true realization by realization
- Good approximation:
 - For effect on large scales $\Lambda \ll k_{NL}$, take first two moments:
 - . $\langle [\tau_s^{\mu\nu}]_\Lambda \rangle_{\phi_\ell}$ space-dependence from background long-mode
 - . $\text{Var}([\tau_{\mu\nu}]_\Lambda) \equiv \langle [\tau_{\mu\nu}]_\Lambda^2 \rangle - \langle [\tau_{\mu\nu}]_\Lambda \rangle^2$: random statistical fluctuations (check later)
- Taylor expand: $\langle [\tau^{ij}]_\Lambda \rangle_{\delta_l} = \langle [\tau^{ij}]_\Lambda \rangle_0 + \frac{\partial \langle [\tau^{ij}]_\Lambda \rangle_{\delta_l}}{\partial \delta_l} \Big|_0 \delta_l + \dots$
- Obtain generic fluid stress tensor + stochastic piece (could have been guessed)

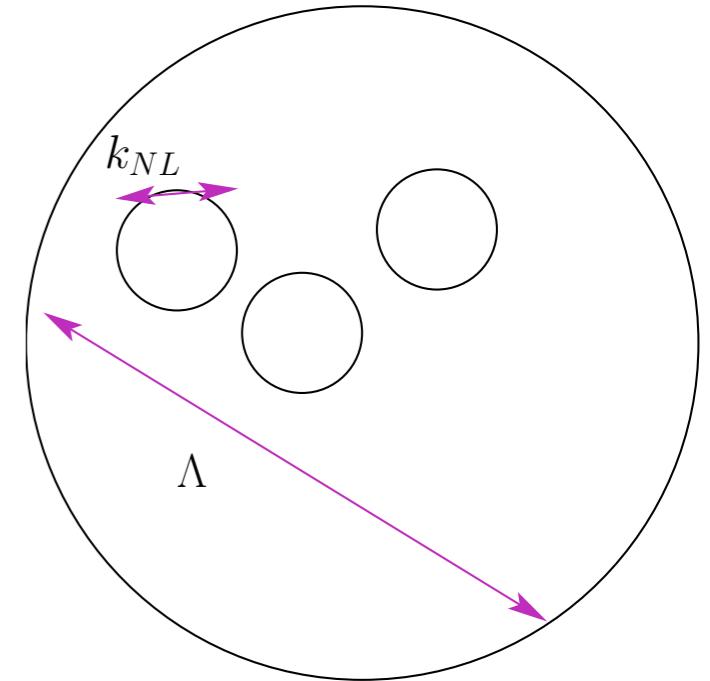


$$\langle [\tau^{ij}]_\Lambda \rangle_{\delta_l} = p_b \delta_l^{ij} + \rho_b \left[c_s^2 \delta_l \delta_l^{ij} - \frac{c_{bv}^2}{Ha} \delta_l^{ij} \partial_k v_l^k - \frac{3}{4} \frac{c_{sv}^2}{Ha} \left(\partial^j v_l^i + \partial^i v_l^j - \frac{2}{3} \delta_l^{ij} \partial_k v_l^k \right) \right] + \Delta \tau^{ij} + \dots$$

- Now effective theory has only long-wavelength modes. We made it!
- Similar to Chiral Lagrangian F_π : UV physics in higher derivative terms

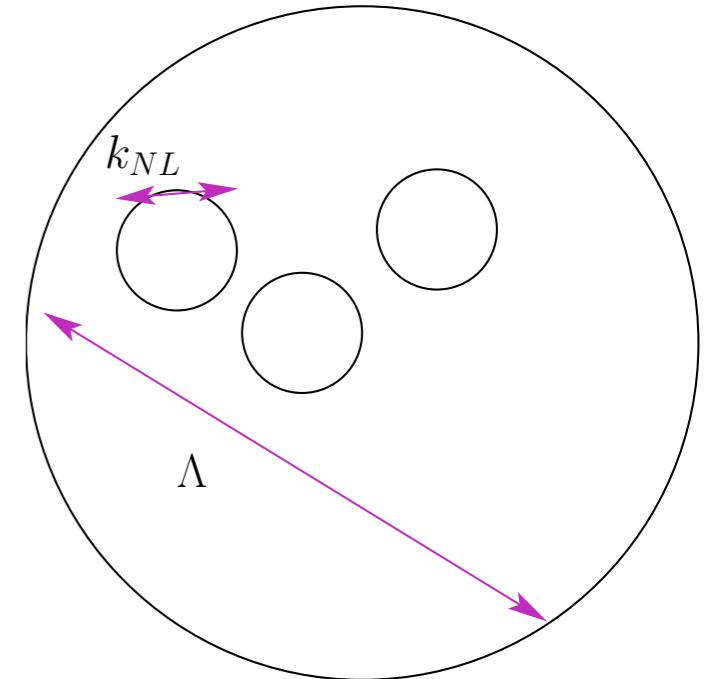
A peculiar fluid: its Chocolate!

- $\langle [\tau^{ij}]_\Lambda \rangle_{\delta_l} = p_b \delta^{ij} + \rho_b \left[c_s^2 \delta_l \delta^{ij} - \frac{c_{bv}^2}{Ha} \delta^{ij} \partial_k v_l^k - \frac{3}{4} \frac{c_{sv}^2}{Ha} \left(\partial^j v_l^i + \partial^i v_l^j - \frac{2}{3} \delta^{ij} \partial_k v_l^k \right) \right] + \Delta \tau^{ij} + \dots$
- What allows truncation of Boltzmann hierarchy?
- In a standard fluid, higher moments are suppressed by $k v_p \tau_c$
 - fluid valid for $k v_p \tau_c \ll 1$
- Here scaling is $k v_p \mathcal{H}^{-1} \lesssim \frac{k}{k_{NL}}$:
 - there is a finite time for the particles to travel,
 - and since they are non-relativistic, we have a derivative expansion inside the horizon.
 - Gravitational fluid!
- It turns out that $c_s^2 \sim c_{vis}^2 \sim \langle v_s^2 \rangle \sim 10^{-6}$
 - very unusual fluid (it needs to damp oscillations)
 - similar to melted Chocolate!



A peculiar fluid: its Chocolate!

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Homogeneous mode EFT

No Large Backreaction

- Look at homogeneous mode $\langle [\tau_s^{\mu\nu}]_\Lambda \rangle$

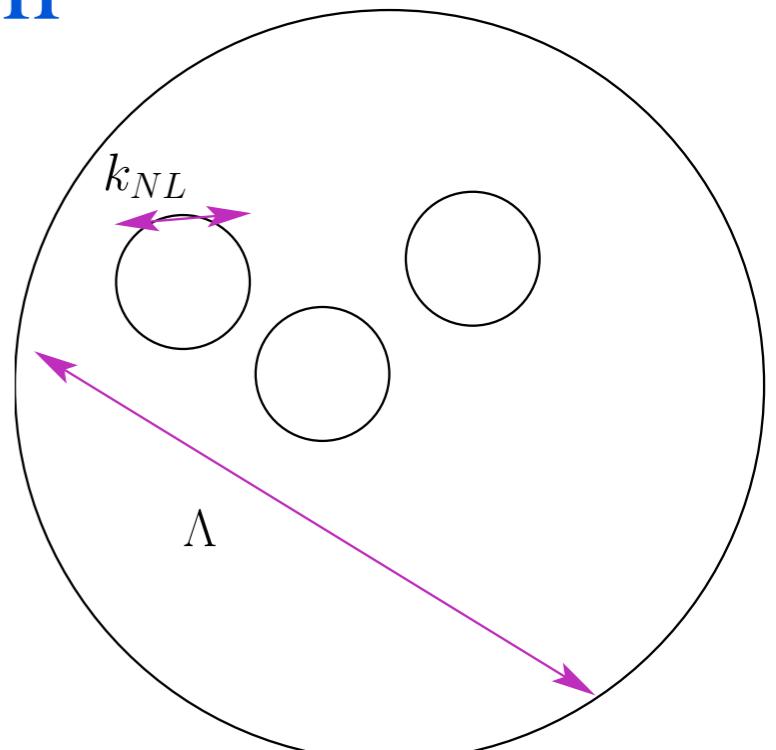
$[\tau_{ij}]_\Lambda = \kappa + w = \text{kinetic energy} + \text{potential energy}$

$$\kappa \sim (1 + \delta) v_s^2, \quad w \sim \delta_s \Phi_s$$

- Renormalized density and pressure

$$\bar{\rho}_{\text{eff}} = \bar{\rho}_m(1 + \kappa + \omega)$$

$$\bar{w}_{\text{eff}} \equiv \frac{\bar{p}_{\text{eff}}}{\bar{\rho}_{\text{eff}}} = \frac{1}{3}(2\kappa + \omega)$$



- Virialized objects decouple! (each term is large... huge)

- \Rightarrow No large backreaction.

- Small difference in equation of state

$$w_{\text{eff}} = \frac{\bar{p}_{\text{eff}}}{\bar{\rho}_{\text{eff}}} \sim \langle \delta \Phi \rangle \sim +\mathcal{O}(\Phi \sim 10^{-5})$$

EFT for fluctuations

Extracting the UV coefficients

- $\langle [\tau^{ij}]_\Lambda \rangle_{\delta_l} = p_b \delta^{ij} + \rho_b \left[c_s^2 \delta_l \delta^{ij} - \frac{c_{bv}^2}{Ha} \delta^{ij} \partial_k v_l^k - \frac{3}{4} \frac{c_{sv}^2}{Ha} \left(\partial^j v_l^i + \partial^i v_l^j - \frac{2}{3} \delta^{ij} \partial_k v_l^k \right) \right] + \Delta \tau^{ij} + \dots$
- EFT: UV physics is encoded in coefficients of higher-derivative operators
- Can extract them from the UV physics (N-body simulations)
- Look at particular correlation functions (not the usual stuff)

$$P_{A\delta}(x) = \langle A_l(\vec{x}' + \vec{x}) \delta_l(\vec{x}') \rangle , \quad A_l \sim \partial_i \partial_j [\tau^{ij}]_l , \quad \Theta \sim \frac{\partial_i v_l^i}{H}$$

$$P_{A\Theta}(x) = \langle A_l(\vec{x}' + \vec{x}) \Theta_l(\vec{x}') \rangle ,$$

$$P_{A^{ki}\Theta_{ki}}(x) = \langle A_l^{ki}(\vec{x}' + \vec{x}) \Theta_{lki}(\vec{x}') \rangle ,$$

$$P_{B\Theta}(x) = \langle B_l(\vec{x}' + \vec{x}) \Theta_l(\vec{x}') \rangle , \quad B_l = \frac{1}{a^2 \rho_b} (\partial_i \partial_j - \delta_{ij} \partial^2) [\tau^{ij}]_\Lambda$$

$$P_{\delta\delta}(x) = \langle \delta_l(\vec{x}' + \vec{x}) \delta_l(\vec{x}') \rangle ,$$

$$P_{\delta\Theta}(x) = \langle \delta_l(\vec{x}' + \vec{x}) \Theta_l(\vec{x}') \rangle ,$$

$$P_{\Theta\Theta}(x) = \langle \Theta_l(\vec{x}' + \vec{x}) \Theta_l(\vec{x}') \rangle ,$$

$$P_{\Theta^{ji}\Theta_i^k}(x) = \langle \Theta_l^{ji}(\vec{x}' + \vec{x}) \Theta_{l i}^k(\vec{x}') \rangle ,$$

- Extract parameters $c_s^2 = a^2 \frac{P_{A\Theta}(x) \partial^2 P_{\delta\Theta}(x) - P_{A\delta}(x) \partial^2 P_{\Theta\Theta}(x)}{(\partial^2 P_{\delta\Theta}(x))^2 - \partial^2 P_{\delta\delta}(x) \partial^2 P_{\Theta\Theta}(x)}$,

$$c_v^2 = a^2 \frac{P_{A\delta}(x) \partial^2 P_{\delta\Theta}(x) - P_{A\Theta}(x) \partial^2 P_{\delta\delta}(x)}{(\partial^2 P_{\delta\Theta}(x))^2 - \partial^2 P_{\delta\delta}(x) \partial^2 P_{\Theta\Theta}(x)} ,$$

- (Very!) similar to measuring F_π from lattice QCD

Perturbation Theory with the EFT

Perturbation Theory within the EFT

- In the EFT we can solve iteratively (loop expansion) $\delta_\ell, v_\ell, \Phi_\ell \ll 1$

$$\nabla^2 \phi_l = \frac{3}{2} H_0^2 \Omega_m \frac{a_0^3}{a} \delta_l + \dots ,$$

$$\dot{\delta}_l = -\frac{1}{a} \partial_i ((1 + \delta_l) \vec{v}_l^i) ,$$

$$\dot{v}_l^i + H v_l^i + \frac{1}{a} v_l^j \partial_j v_l^i + \frac{1}{a} \partial^i \phi_l = -\frac{1}{a} c_s^2 \partial^i \delta_l + \frac{3}{4} \frac{c_{sv}^2}{H a^2} \partial^2 v_l^i + \frac{4c_{bv}^2 + c_{sv}^2}{4 H a^2} \partial^i \partial_j v_l^j - \Delta J^i + ..$$

- Organization

- estimates $\phi_l \sim 10^{-5}$, $\delta_l \sim \frac{k^2}{k_{NL}^2}$, $v_l \sim \frac{H k}{k_{NL}^2}$

- Loop corrections:

$$\frac{\text{non-linear terms}}{\text{Hubble friction}} \sim \frac{\delta_l v_l^j \partial_j v_l^i}{H v_l^i} \sim \frac{H \delta_l^2 v_l^i}{H v_l^i} \sim \delta_l^2$$

- max δ_l within theory

- Pressure and viscosity terms $\frac{\text{Pressure , Viscosity}}{\text{Hubble friction}} \sim \frac{c_s^2 \partial \delta_l}{H v_l^i} \sim c_s^2 \frac{\partial^2 \delta_l}{H^2 \delta_l} \sim \frac{c_s^2}{10^{-5}} \delta_l$

- we expect $c_s^2 \sim \delta(k_{NL}) \phi(k_{NL}) \sim 10^{-5}$ from virialization decoupling

Perturbation Theory within the EFT

- Stochastic terms

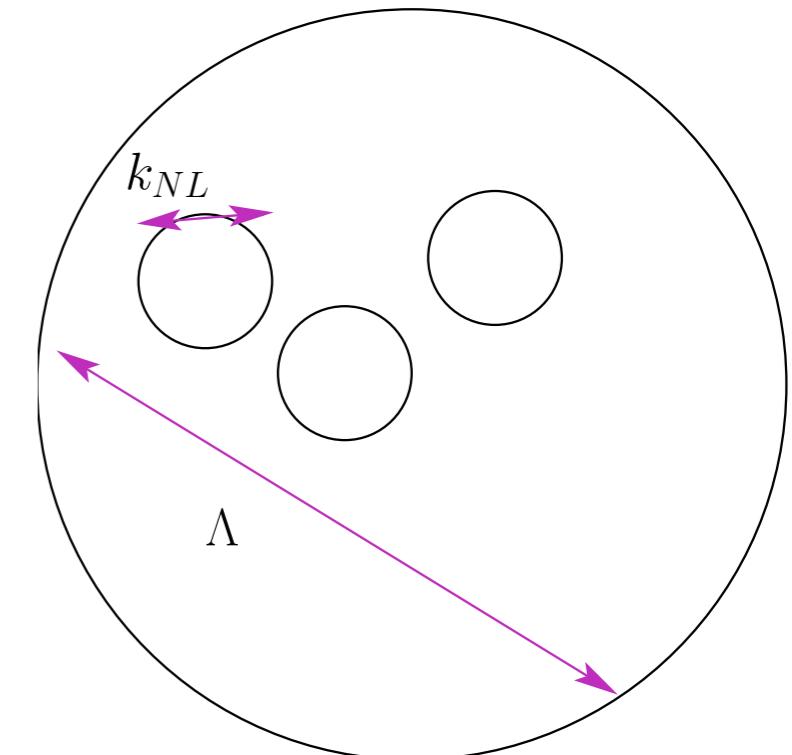
$$\delta_{l,\text{non-lin.}} \sim \delta_{l,\text{lin}} + c_s^2 \frac{\partial^2}{H^2} \delta_{l,\text{lin}} + \frac{\partial^2}{H^2} \frac{\Delta\tau}{\rho_b}$$

- Power spectrum contribution $\langle \delta_l \delta_l \rangle_{\text{1-loop}} \sim c_s^2 \frac{k^2}{H^2} \langle \delta_l^2 \rangle + \left(\frac{k^2}{H^2 \rho_b} \right)^2 \langle \Delta\tau^2 \rangle$
- Poisson like on pixels $\sim k_{NL}^{-1}$

$$\langle \Delta\tau^2 \rangle \sim \langle \tau^2 \rangle \left(\frac{k}{k_{NL}} \right)^3 \sim (c_s^2 \rho_b)^2 \left(\frac{k}{k_{NL}} \right)^3$$

$$-\cdot \frac{\text{Stochastic}}{\text{Pressure}} \sim \frac{k}{k_{NL}}, \quad \Rightarrow \quad \frac{\text{Stochastic}}{\text{Friction}} \sim \frac{k}{k_{NL}} \delta_l \sim \delta_l^{3/2}$$

- 1.5 loops



Perturbation Theory within the EFT

- Higher derivative terms
 - . $k^2/k_{NL}^2 \sim \delta_l \Rightarrow$ higher loops
 - k^2/Λ^2 for finite Λ
- Cutoff (in)dependence and effective expansion parameter
 - Naive $\delta_l(k \sim \Lambda) \sim \Lambda^2/k_{NL}^2$
 - Cutoff dependence in parameters $c_s(\Lambda), \dots$, plus loops cutoff
 - Sum of all must be Λ -independent: thanks to the 1-loop counterterms $c_s(\Lambda), \dots$
 - at finite Λ need also higher-derivative counterterms k/Λ
 - To resolve with least effort, take limit $\Lambda \rightarrow \infty$ by keeping result finite
 - with counterterms
- Effective Expansion parameter (counting with the counterterms):
 - . $\delta_l \sim k^2/k_{NL}^2$ at scale of external mode
- Similar to Chiral Lagrangian E/F_π and E/Λ

Perturbation Theory within the EFT

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1.5-loop Calculation

Perturbation Theory within the EFT

- Non-linear equations

$$a\mathcal{H}\delta' + \theta_l = - \int \frac{d^3q}{(2\pi)^3} \alpha(\vec{q}, \vec{k} - \vec{q}) \delta_l(\vec{k} - \vec{q}) \theta_l(\vec{q}) ,$$

$$a\mathcal{H}\theta'_l + \mathcal{H}\theta_l + \frac{3}{2} \frac{\mathcal{H}_0^2 \Omega_m}{a} \delta_l - c_s^2 k^2 \delta_l + \frac{c_v^2 k^2}{\mathcal{H}} \theta_l = - \int \frac{d^3q}{(2\pi)^3} \beta(\vec{q}, \vec{k} - \vec{q}) \theta_l(\vec{k} - \vec{q}) \theta_l(\vec{q})$$

- Iterative solutions

$$\begin{aligned} \delta^{(2)}(\vec{k}, a) = & \frac{1}{16\pi^3 D(a_0)^2} \\ & \left[\left(\int_0^a d\tilde{a} G(a, \tilde{a}) \tilde{a}^2 \mathcal{H}^2(\tilde{a}) D'(\tilde{a})^2 \right) \left(2 \int d^3q \beta(\vec{q}, \vec{k} - \vec{q}) \delta s_1(\vec{k} - \vec{q}) \delta s_1(\vec{q}) \right) \right. \\ & + \left(\int_0^a d\tilde{a} G(a, \tilde{a}) \left(2\tilde{a}^2 \mathcal{H}^2(\tilde{a}) D'(\tilde{a})^2 + 3\mathcal{H}_0^2 \Omega_m \frac{D(\tilde{a})^2}{\tilde{a}} \right) \right) \\ & \left. \times \left(\int d^3q \alpha(\vec{q}, \vec{k} - \vec{q}) \delta s_1(\vec{k} - \vec{q}) \delta s_1(\vec{q}) \right) \right] . \end{aligned}$$

– where $\delta^{(1)}(k, a) = \frac{D(a)}{D(a_0)} \delta s_1(\vec{k})$

– and $-a^2 \mathcal{H}^2(a) \partial_a^2 G(a, \tilde{a}) - a (2\mathcal{H}^2(a) + a\mathcal{H}(a)\mathcal{H}'(a)) \partial_a G(a, \tilde{a}) + 3 \frac{\Omega_m \mathcal{H}_0^2}{2a} G(a, \tilde{a}) = \delta(a - \tilde{a})$

$G(a, \tilde{a}) = 0$ for $a < \tilde{a}$

- This part similar to SPT done right, but with cutoff.

Perturbation Theory within the EFT

- Counterterm

$$\delta_{c_{\text{comb}}}^{(3)}(\vec{k}, a) = -\frac{k^2}{D(a_0)} \int_0^a d\tilde{a} G(a, \tilde{a}) \bar{c}_{\text{comb}}^2(\tilde{a}) D(\tilde{a}) \delta s_1(\vec{k}) ,$$

- One-loop power spectrum

$$\begin{aligned} \langle \delta_l(\vec{k}, a) \delta_l(\vec{q}, a) \rangle &= \\ (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{q}) &\left(P_{11}(k, a) + P_{22}(k, a) + P_{13}(k, a) + P_{13, c_{\text{comb}}^2}(k, a) + P_{22, \text{stoch}}(k, a) \right) \end{aligned}$$

- with

$$P_{11}(k, a_0) = \langle \delta(\vec{k}, a_0) \delta(\vec{q}, a_0) \rangle' ,$$

$$P_{22}(k, a_0) = \langle \delta^{(2)}(\vec{k}, a_0) \delta^{(2)}(\vec{q}, a_0) \rangle' ,$$

$$P_{13}(k, a_0) = 2 \langle \delta^{(3)}(\vec{k}, a_0) \delta^{(1)}(\vec{q}, a_0) \rangle' ,$$

$$P_{13, c_{\text{comb}}^2}(k, a_0) = 2 \langle \delta_{c_{\text{comb}}}^{(3)}(\vec{k}, a_0) \delta^{(1)}(\vec{q}, a_0) \rangle' ,$$

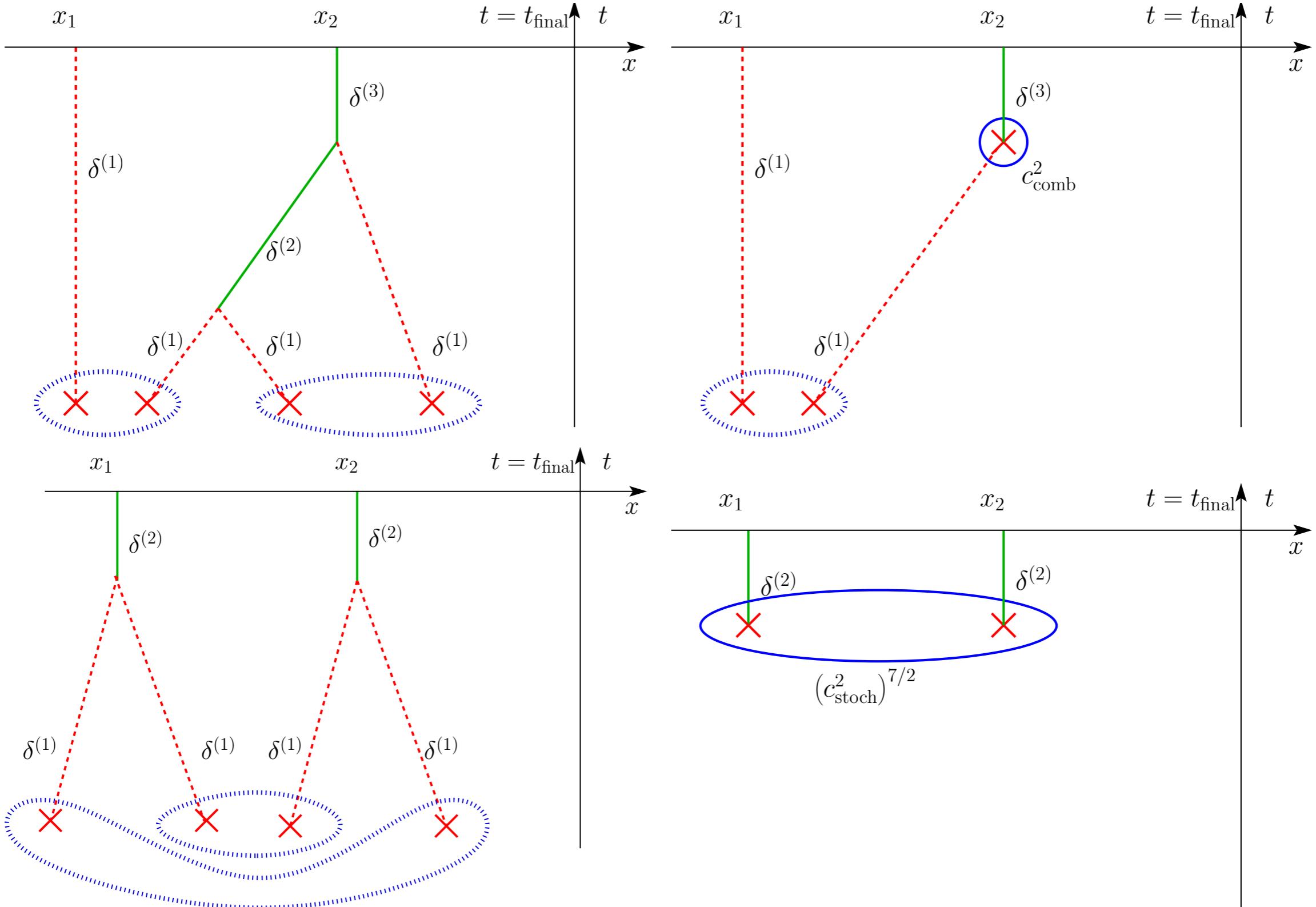
$$P_{22, \text{stoch}}(k, a) \sim \frac{c_s^7}{H^7} k^4$$

Perturbation Theory within the EFT

- One-loop power spectrum

$$\langle \delta_l(\vec{k}, a) \delta_l(\vec{q}, a) \rangle =$$

$$(2\pi)^3 \delta^{(3)}(\vec{k} + \vec{q}) \left(P_{11}(k, a) + P_{22}(k, a) + P_{13}(k, a) + P_{13, c_{\text{comb}}^2}(k, a) + P_{22, \text{stoch}}(k, a) \right)$$

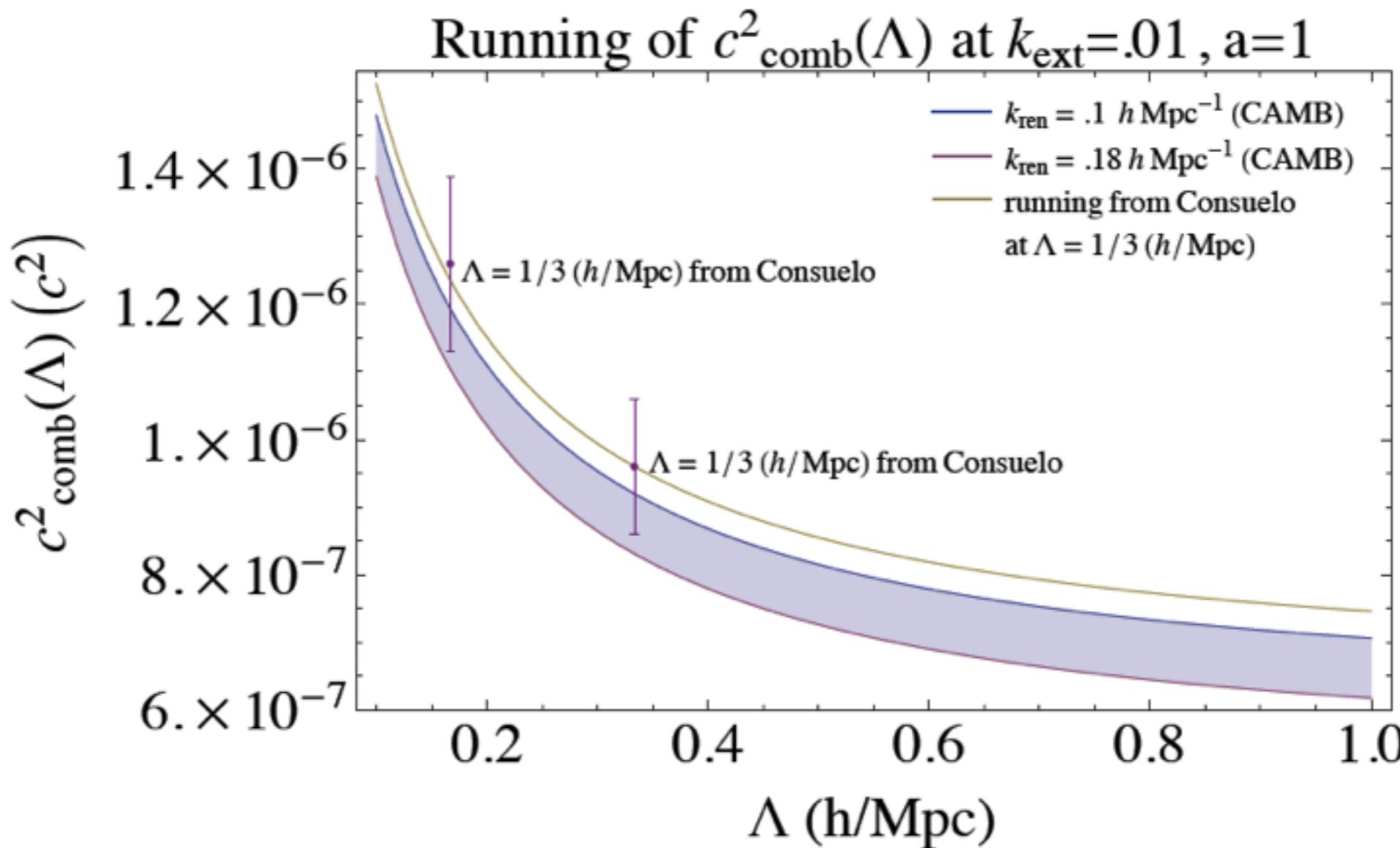


Perturbation Theory within the EFT

- Renormalization

$$c_{\text{comb}}^2(a, \Lambda) = c_{\text{comb;ren.}}^2(a, k_{\text{ren.}}) + c_{\text{comb;ctr.}}^2(a, \Lambda)$$

- Choose $c_{\text{comb}}^2(a, \Lambda)$ to fit observation at $k = k_{\text{ren.}}$, and then extrapolate to $\Lambda \rightarrow \infty$
- Running of $c_{\text{comb}}^2(a, \Lambda)$



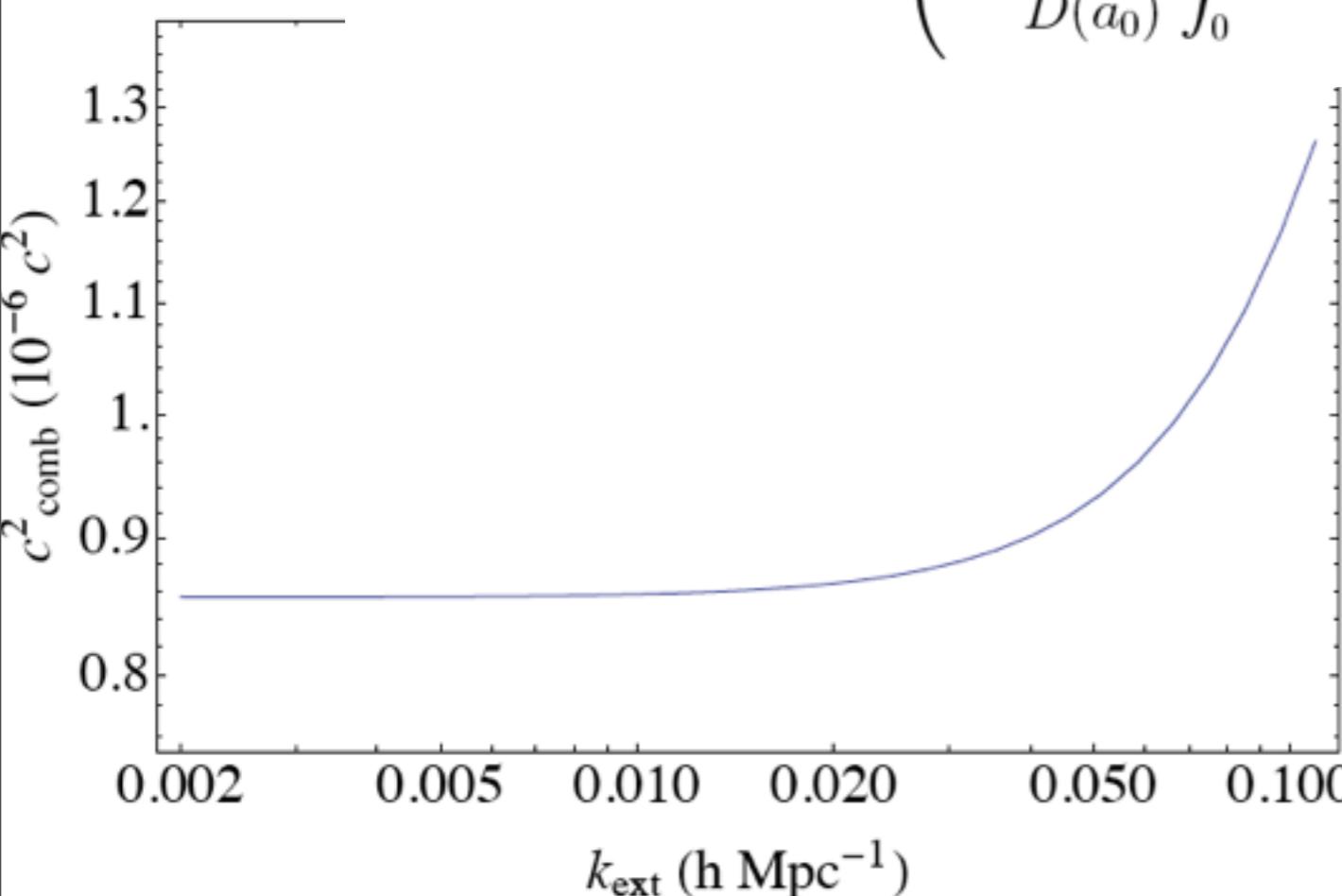
Perturbation Theory within the EFT

- Renormalization

$$c_{\text{comb}}^2(a, \Lambda) = c_{\text{comb;ren.}}^2(a, k_{\text{ren.}}) + c_{\text{comb;ctr.}}^2(a, \Lambda)$$

- Choose $c_{\text{comb}}^2(a, \Lambda)$ to fit observation at $k = k_{\text{ren.}}$, and then extrapolate to $\Lambda \rightarrow \infty$
- Do it at low k_{ext} to reduce higher derivative terms

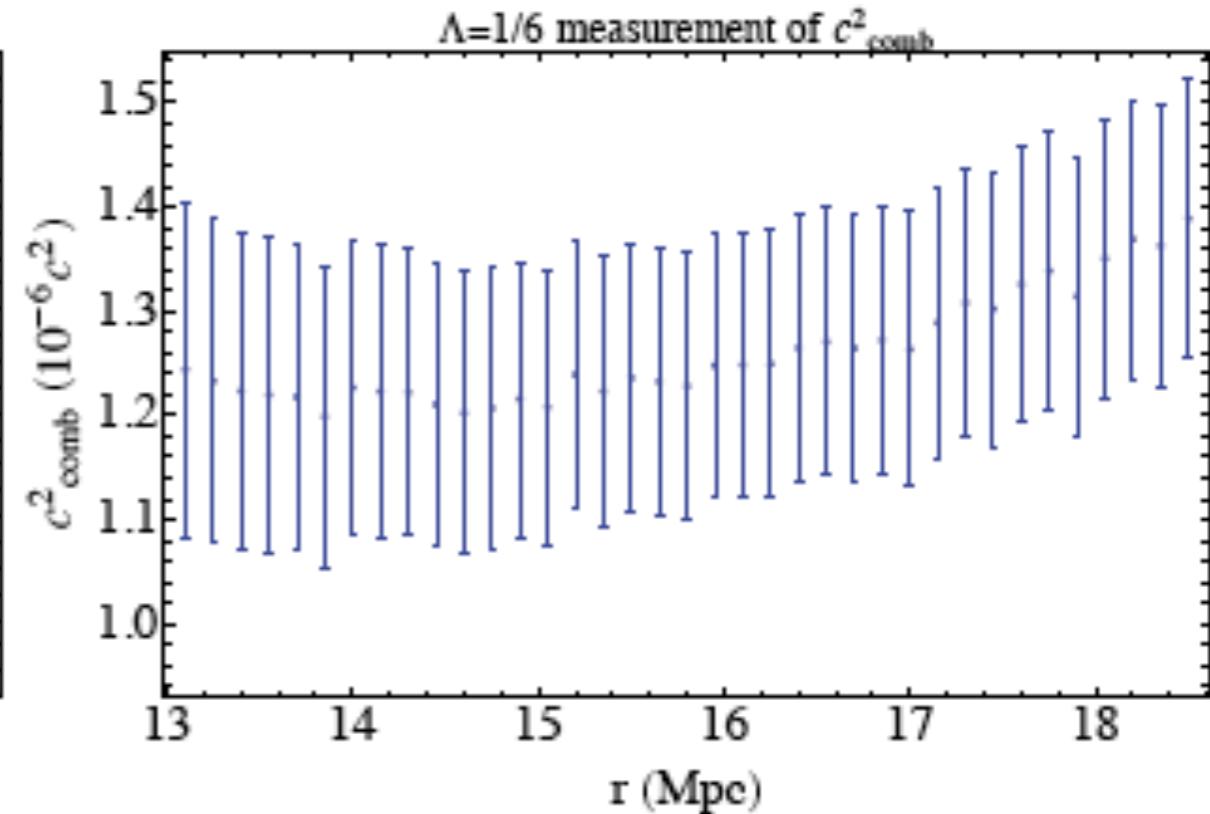
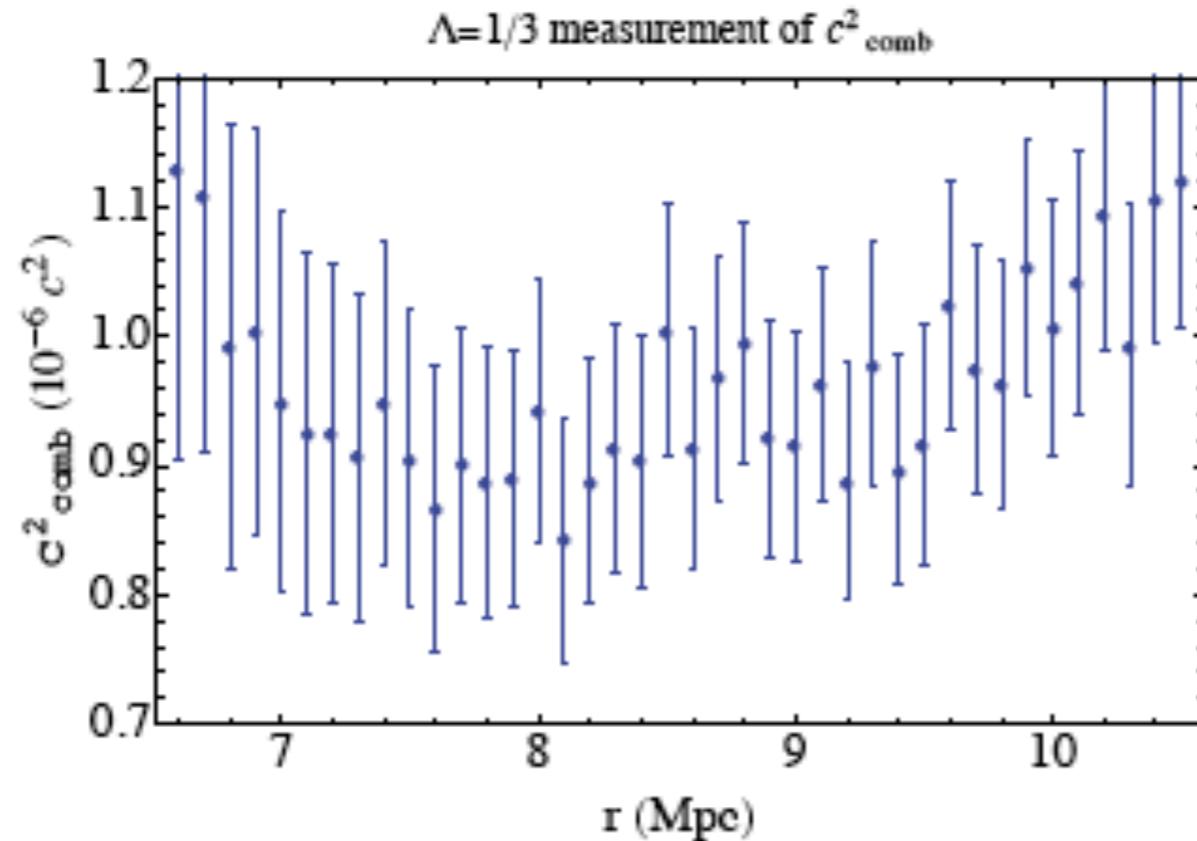
$$\begin{aligned} c_{\text{comb}}^2(a_0, \Lambda \neq \infty) &= c_{\text{comb}}^2(a_0, \Lambda = \infty) + \lim_{k_{\text{ext}} \rightarrow 0} \left[\left(P_{13}(k_{\text{ext}}, a_0, \Lambda = \infty) - P_{13}(k_{\text{ext}}, a_0, \Lambda) \right) \times \right. \\ &\quad \left. \left(-2 \frac{k_{\text{ext}}^2}{D(a_0)} \int_0^{a_0} d\tilde{a} G(a_0, \tilde{a}) \frac{D_{c_{\text{comb}}^2}(\tilde{a})}{D_{c_{\text{comb}}^2}(a_0)} D(\tilde{a}) P_{11,l}(k_{\text{ext}}, \Lambda) \right)^{-1} \right]. \quad (62) \end{aligned}$$



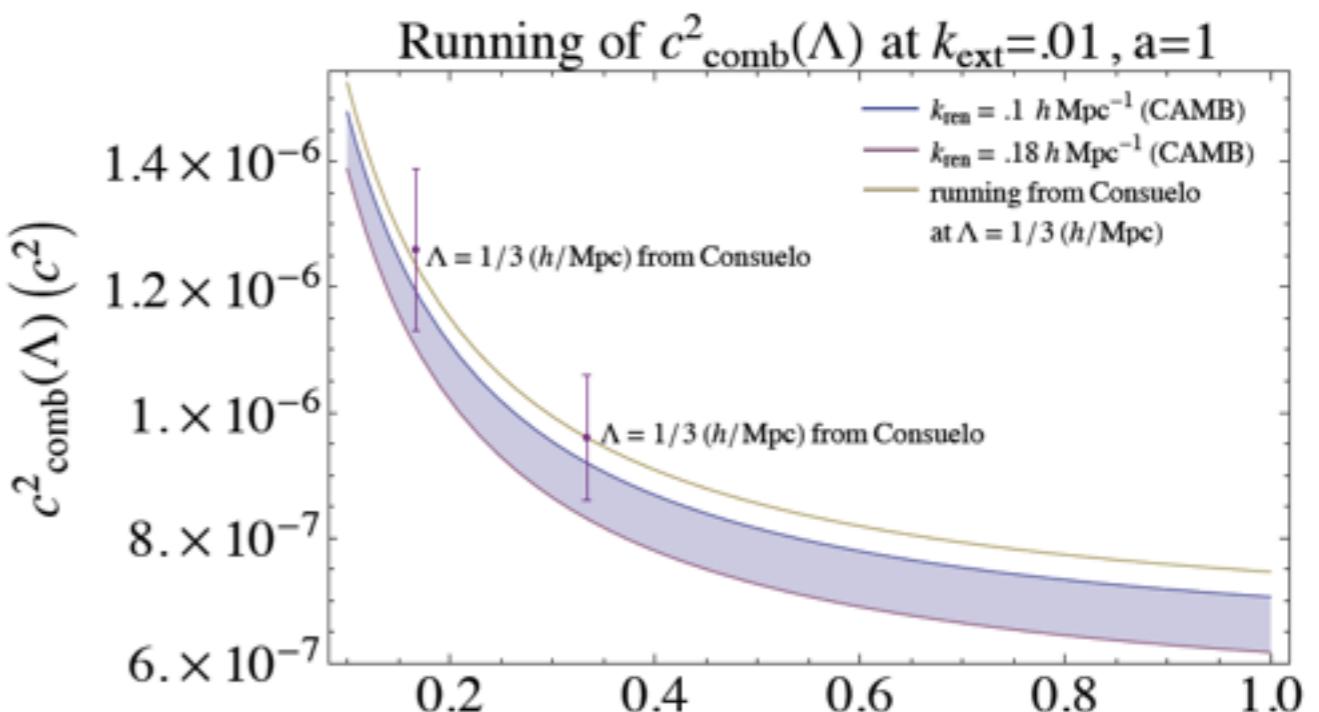
Parameters from
small N-body simulations
(our UV theory, our QCD lattice)

Extract correlation functions

- $$c_s^2 = a^2 \frac{P_{A\Theta}(x)\partial^2 P_{\delta\Theta}(x) - P_{A\delta}(x)\partial^2 P_{\Theta\Theta}(x)}{(\partial^2 P_{\delta\Theta}(x))^2 - \partial^2 P_{\delta\delta}(x)\partial^2 P_{\Theta\Theta}(x)}$$

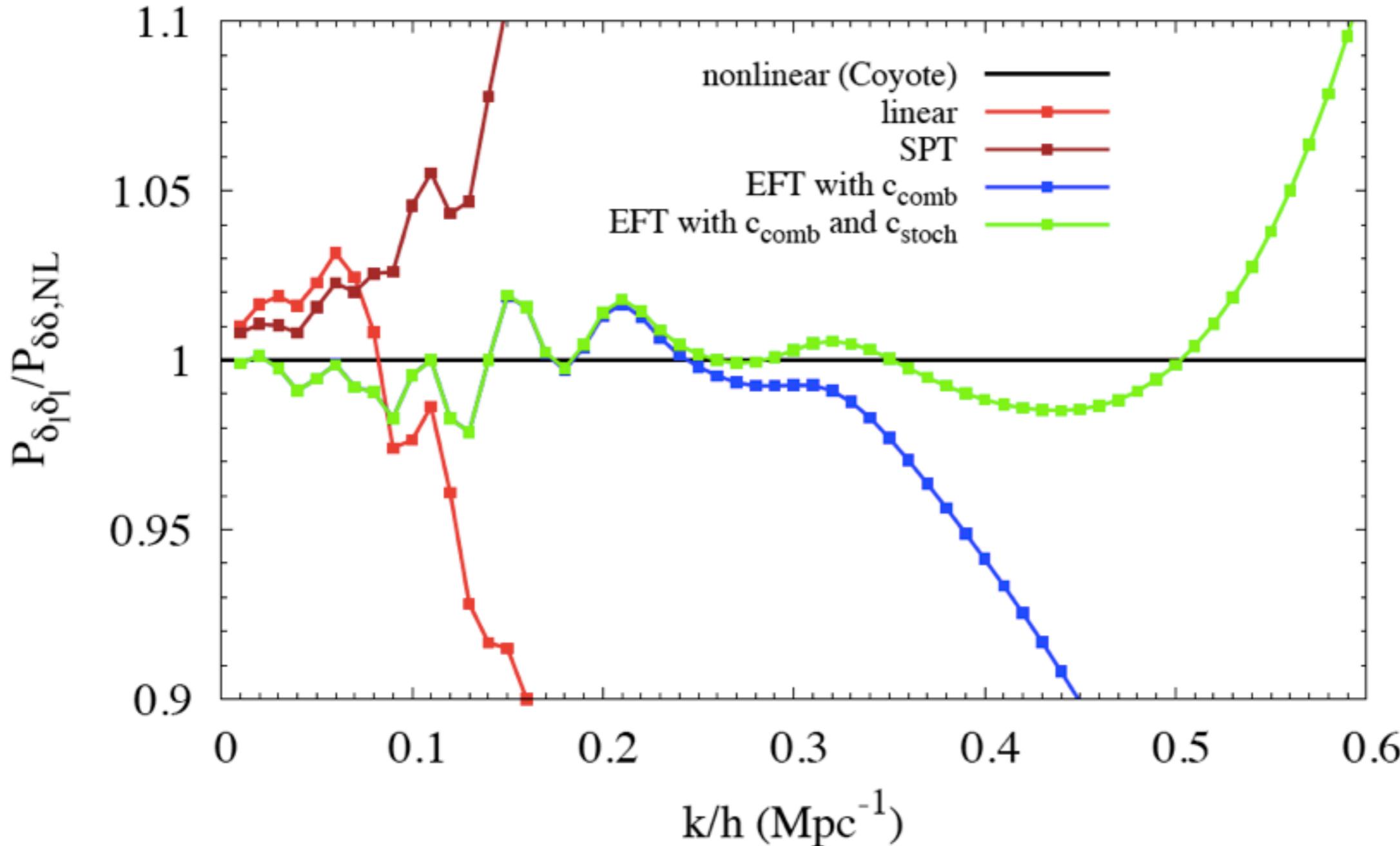


- Plot
- This is melted chocolate viscosity!
- Agrees with fitting to power spectrum
 - notice value and running
- Amazing agreement!!!
 - with values obtained by fitting to obs.



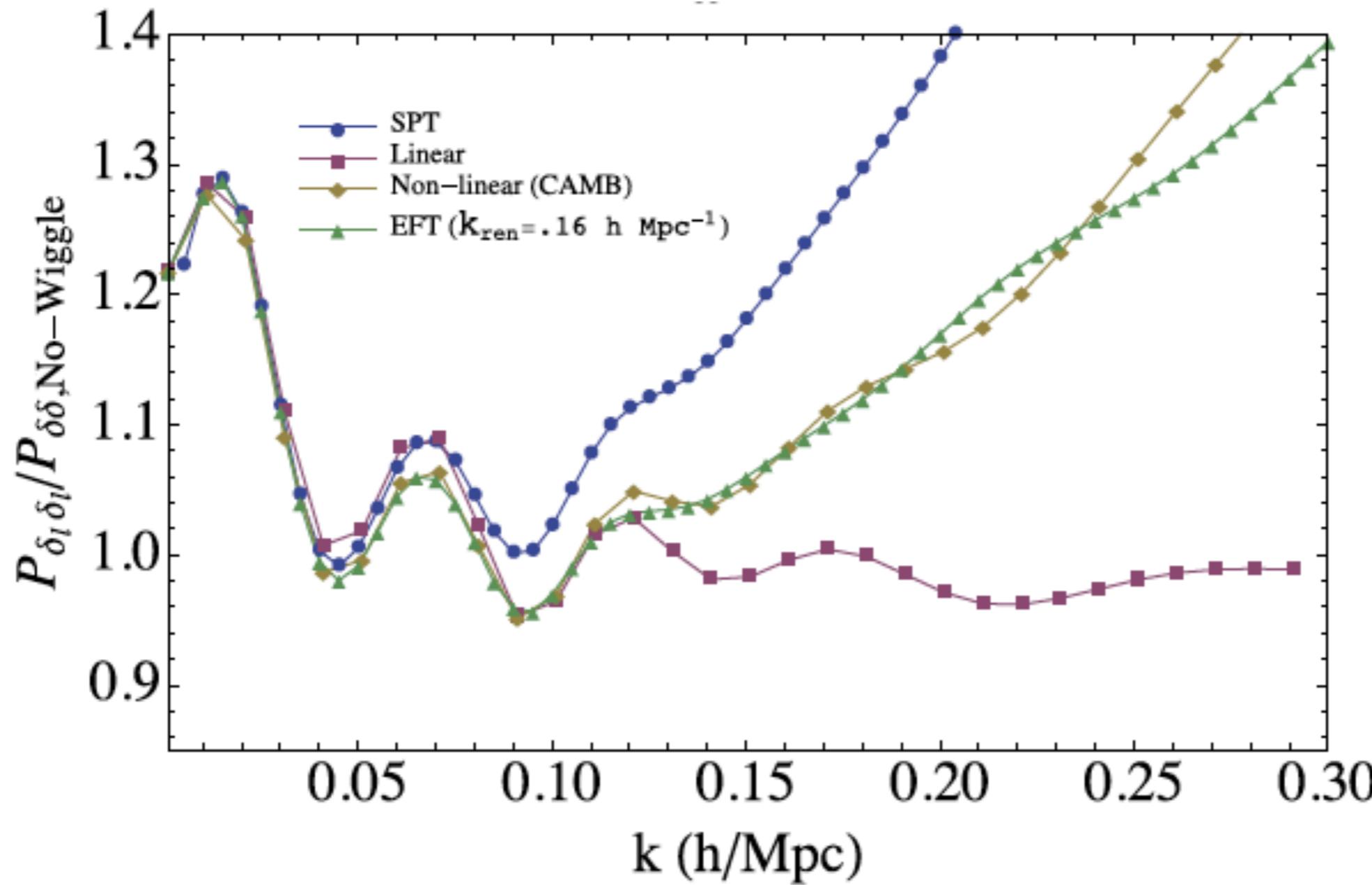
Results

Power Spectrum: z=0



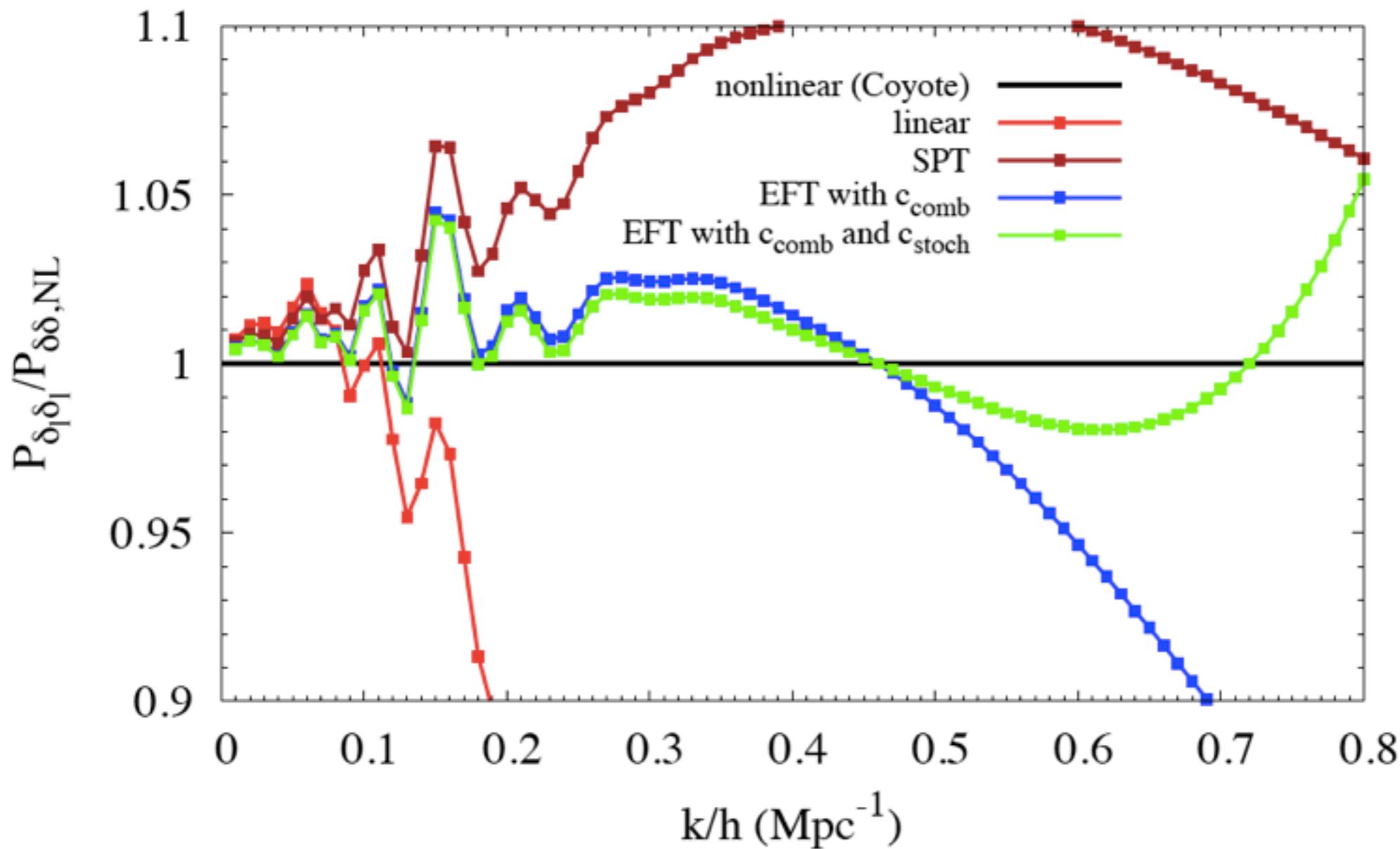
- Much better than SPT, RPT. Most importantly, it is well defined and manif. converg.
- More and more steep as higher order corrections added $\left(\frac{k}{k_{NL}}\right)^N$: ok!
- Values of parameters is both $c_{comb}^2 \sim c_{stoch}^2 \sim 10^{-6}$: ok!
- Adding c_{stoch}^2 does not change relevantly c_{comb}^2 : ok!

Power Spectrum: z=0



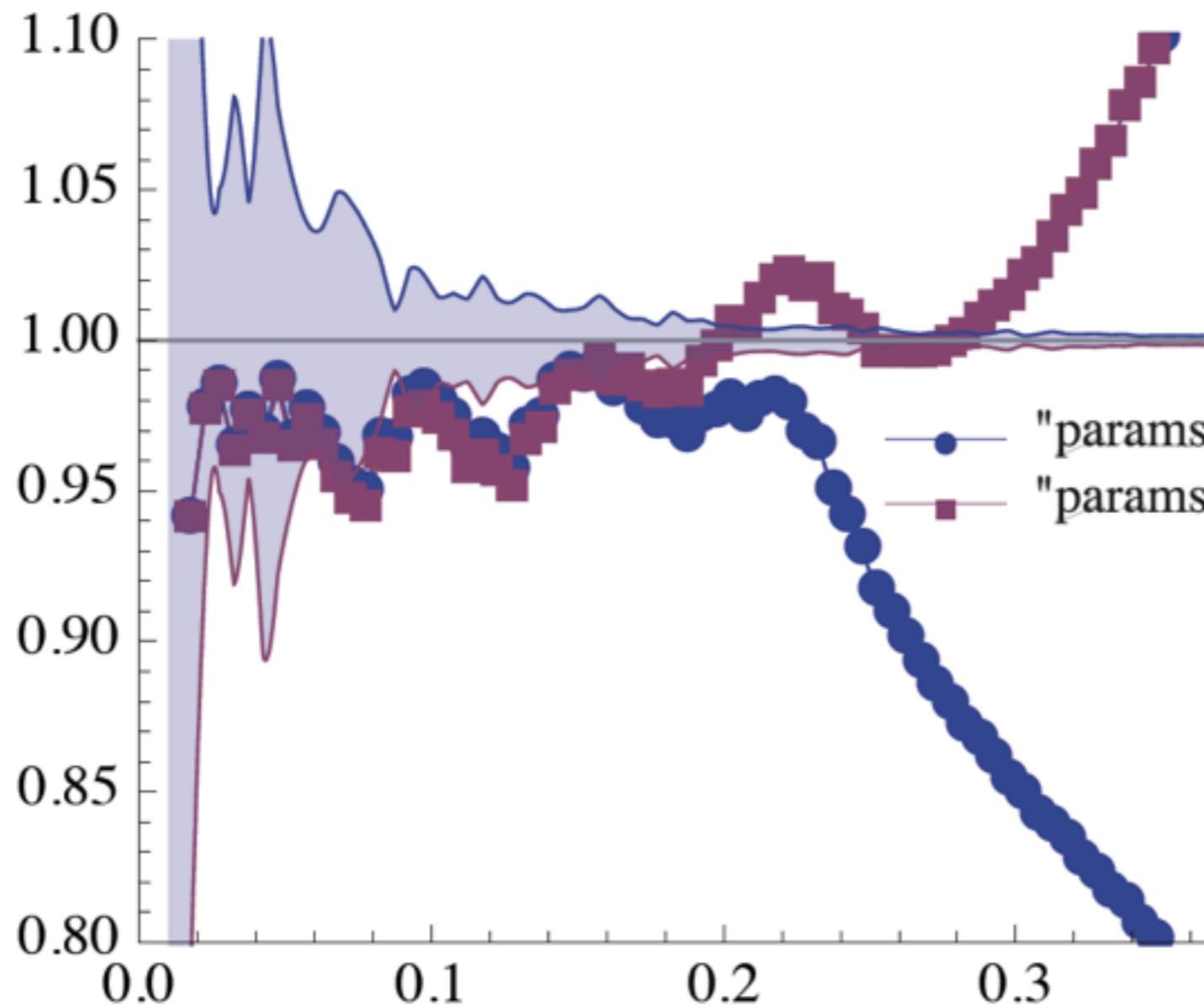
- Large corrections in UV
- (this is only 1-loop)

Power Spectrum: z=1



- Once we fix two parameters at two k 's, all the rest is predicted
- with consistently 2% dependence on renormalization scale.

Momentum Power Spectrum: z=0



- Preliminary
- Totally fixed by the matter power spectrum
- Cutoff dependence cancels only for momentum

Applications & Summary

- A convergent perturbation theory

- an effective fluid with

- pressure, anisotropic stress, and random fluctuations

- coeff. measured from simulation

- Extremely good results in PT $k_{\max} \sim 0.5 h \text{ Mpc}^{-1}$

