

# Halo Clustering beyond the Local Bias Model

Tobias Baldauf

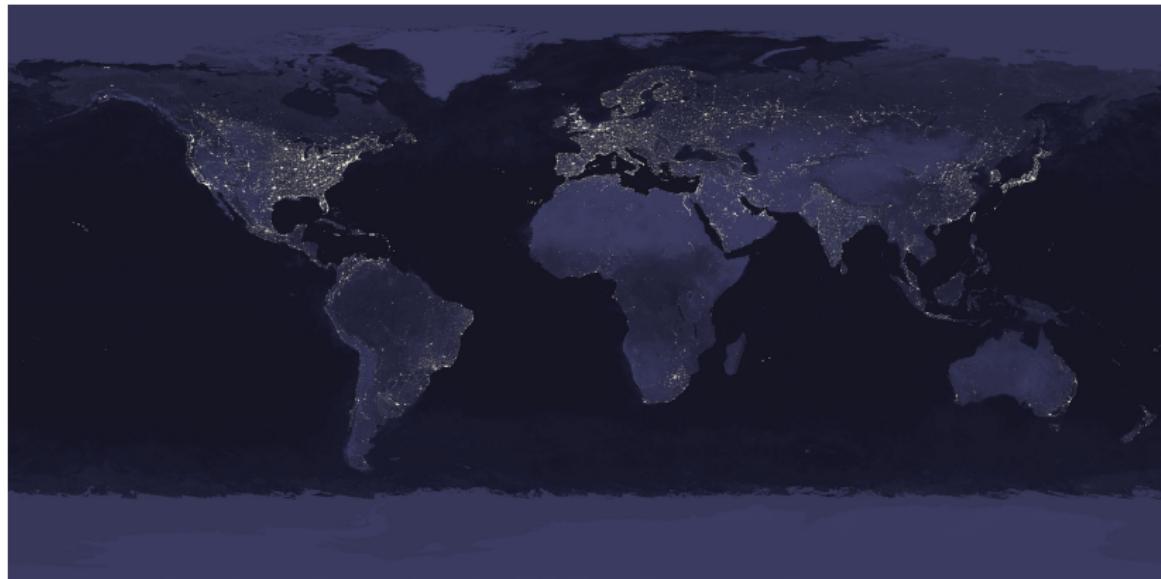
with Vincent Desjacques, Nico Hamaus, Pat McDonald, Uroš Seljak and Robert E. Smith

Institut für Theoretische Physik  
Universität Zürich

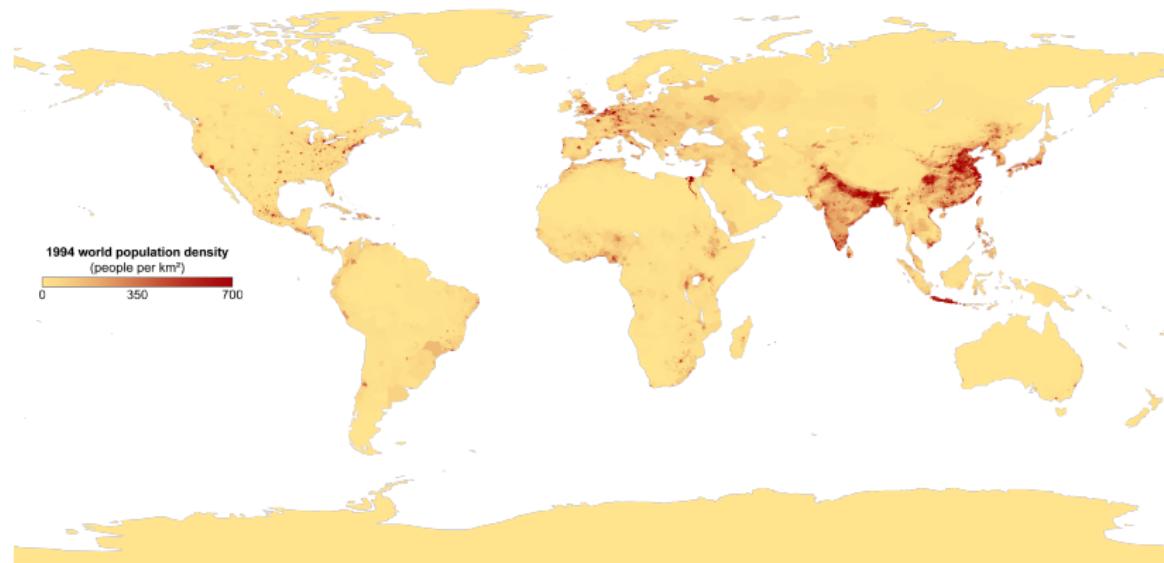
Theoretical Methods for non-linear Cosmology  
CERN, Geneva  
03.09.2012



# Light traces Population Density



# Light traces Population Density Money!



## 1 Introduction

## 2 Non-Local Bias

## 3 Noise Corrections

## 4 Summary & Outlook

## 1 Introduction

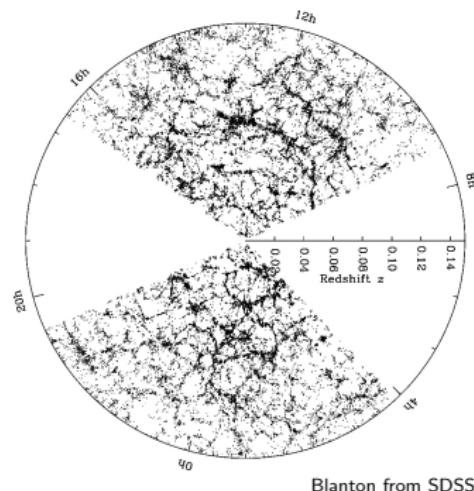
## 2 Non-Local Bias

## 3 Noise Corrections

## 4 Summary & Outlook

# Galaxy Clustering as a Cosmological Probe

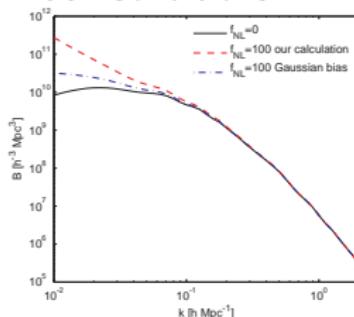
- current and future dark energy surveys  
⇒ better precision and larger scales ⇒  
complementary to CMB
- galaxies as cosmological probes
  - BAO  $k \approx 0.06 h\text{Mpc}^{-1}$
  - primordial non-Gaussianity  
 $k \approx 0.001 h\text{Mpc}^{-1}$
- Large Scale Structure Simulations
  - $10.8 h^{-1}\text{Gpc}$  Horizon Run [Kim et al. 2011]
  - Coyote Universe Emulator [Heitmann et al. 2008]



Blanton from SDSS

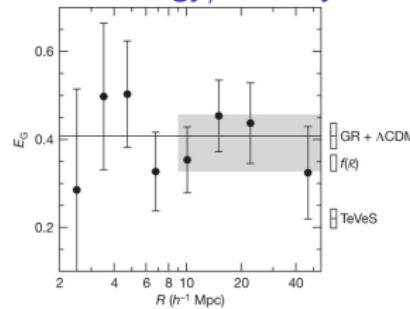
# Why do we need to understand galaxy/halo bias?

## Initial Conditions



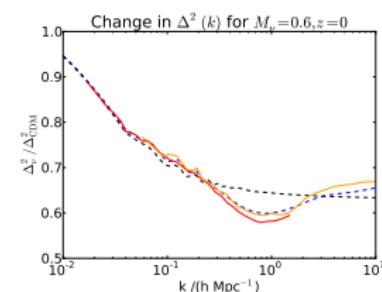
[Baldauf, Seljak & Senatore 2011]

## Dark Energy/Gravity



[Reyes et al. 2010]

## Neutrino Mass



[Bird, Viel & Haehnelt 2011]

⇒ Halo power spectrum is sensitive to fundamental physics

# Bias Models on the Market

## non-Perturbative models

- Kaiser thresholded regions  $\delta > \delta_c$

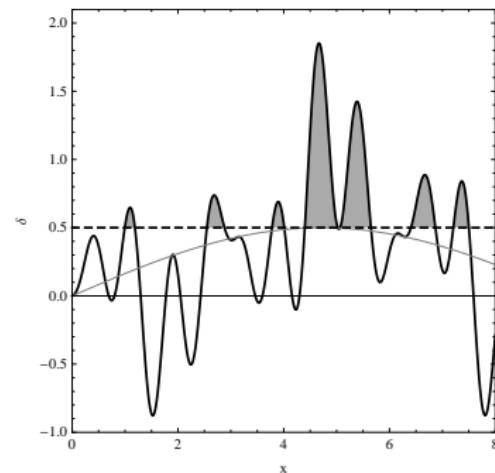
$$\xi_{\text{tr}}(r) \xrightarrow{r \rightarrow \infty} b_1^2 \xi_{\text{lin}}(r)$$

- Peak formalism  $\delta > \delta_c, \delta' = 0, \delta'' < 0$

$$\xi_{\text{pk}}(r) \xrightarrow{r \rightarrow \infty} (b_\nu - b_\zeta \nabla^2)^2 \xi_{\text{lin}}(r)$$

## Perturbative models

- local Lagrangian Bias
- local Eulerian Bias



<sup>0</sup>[Kaiser 1984, BBKS 1986, Fry & Gaztanaga 1993]

# Local Eulerian Bias in Various Statistics

$$\delta_h(\mathbf{x}) = b_1^{(E)} \delta_m(\mathbf{x}) + \frac{1}{2!} b_2^{(E)} \delta_m^2(\mathbf{x}) + \dots$$

Auto Power Spectrum

$$P_{hh}(k) = b_1^2 P_{mm}(k) + b_1 b_2 l_{12}(k) + \frac{1}{2} b_2^2 l_{22}(k) + \frac{1}{\bar{n}}$$

Cross Power Spectrum

$$P_{hm}(k) = b_1 P_{mm}(k) + \frac{1}{2} b_2 l_{12}(k)$$

Cross Bispectrum

$$B_{mmh}(k_1, k_2, k_3) = b_1 B_{mmm}(k_1, k_2, k_3) + b_2 P(k_1) P(k_2)$$

# Local Eulerian Bias in Various Statistics

$$\delta_h(\mathbf{x}) = b_1^{(E)} \delta_m(\mathbf{x}) + \frac{1}{2!} b_2^{(E)} \delta_m^2(\mathbf{x}) + \dots$$

## Auto Power Spectrum

$$P_{hh}(k) = b_1^2 P_{mm}(k) + b_1 b_2 I_{12}(k) + \frac{1}{2} b_2^2 I_{22}(k) + \frac{1}{\bar{n}}$$

## Cross Power Spectrum

$$P_{hm}(k) = b_1 P_{mm}(k) + \frac{1}{2} b_2 I_{12}(k)$$

## Cross Bispectrum

$$B_{mmh}(k_1, k_2, k_3) = b_1 B_{mmm}(k_1, k_2, k_3) + b_2 P(k_1) P(k_2)$$

# Local Eulerian Bias in Various Statistics

$$\delta_h(\mathbf{x}) = b_1^{(E)} \delta_m(\mathbf{x}) + \frac{1}{2!} b_2^{(E)} \delta_m^2(\mathbf{x}) + \dots$$

## Auto Power Spectrum

$$P_{hh}(k) = b_1^2 P_{mm}(k) + b_1 b_2 I_{12}(k) + \frac{1}{2} b_2^2 I_{22}(k) + \frac{1}{\bar{n}}$$

## Cross Power Spectrum

$$P_{hm}(k) = b_1 P_{mm}(k) + \frac{1}{2} b_2 I_{12}(k)$$

## Cross Bispectrum

$$B_{mmh}(k_1, k_2, k_3) = b_1 B_{mmm}(k_1, k_2, k_3) + b_2 P(k_1)P(k_2)$$

# Local Eulerian Bias in Various Statistics

$$\delta_h(\mathbf{x}) = b_1^{(E)} \delta_m(\mathbf{x}) + \frac{1}{2!} b_2^{(E)} \delta_m^2(\mathbf{x}) + \dots$$

## Auto Power Spectrum

$$P_{hh}(k) = b_1^2 P_{mm}(k) + b_1 b_2 I_{12}(k) + \frac{1}{2} b_2^2 I_{22}(k) + \frac{1}{\bar{n}}$$

## Cross Power Spectrum

$$P_{hm}(k) = b_1 P_{mm}(k) + \frac{1}{2} b_2 I_{12}(k)$$

## Cross Bispectrum

$$B_{mmh}(k_1, k_2, k_3) = b_1 B_{mmm}(k_1, k_2, k_3) + b_2 P(k_1)P(k_2)$$

# Local Eulerian Bias in Various Statistics

$$\delta_h(\mathbf{x}) = b_1^{(E)} \delta_m(\mathbf{x}) + \frac{1}{2!} b_2^{(E)} \delta_m^2(\mathbf{x}) + \dots$$

## Auto Power Spectrum

$$P_{hh}(k) = b_1^2 P_{mm}(k) + b_1 b_2 I_{12}(k) + \frac{1}{2} b_2^2 I_{22}(k) + \frac{1}{\bar{n}}$$

## Cross Power Spectrum

$$P_{hm}(k) = b_1 P_{mm}(k) + \frac{1}{2} b_2 I_{12}(k)$$

## Cross Bispectrum

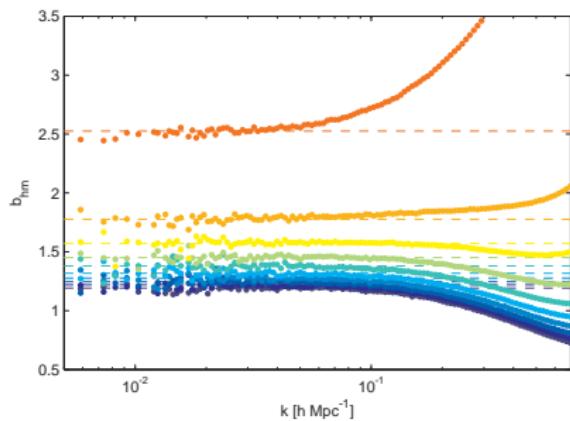
$$B_{mmh}(k_1, k_2, k_3) = b_1 B_{mmm}(k_1, k_2, k_3) + b_2 P(k_1) P(k_2)$$

⇒ Bias parameters should be consistent! (at least in low- $k$  regime)

# Are the biases consistent?

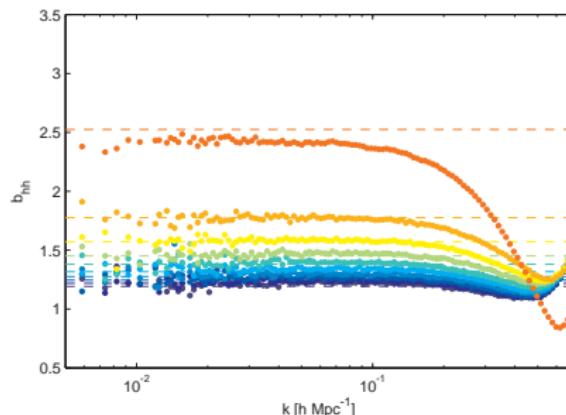
Bias from Cross Power

$$b_{\text{hm}} = \frac{P_{\text{hm}}}{P_{\text{mm}}}$$



Bias from Auto Power

$$b_{\text{hh}} = \sqrt{\frac{P_{\text{hh}}}{P_{\text{mm}}}}$$



## 1 Introduction

## 2 Non-Local Bias

## 3 Noise Corrections

## 4 Summary & Outlook

# The Local Bias Model and its Extensions

## The original Local Model<sup>1</sup>

$$\delta_h(\mathbf{x}, \eta) = \mathcal{F}[\delta(\mathbf{x}', \eta)] \approx b_1 \delta(\mathbf{x}, \eta) + \frac{1}{2!} b_2 \delta^2(\mathbf{x}, \eta) + \dots$$

## Tidal Terms allowed by Symmetry<sup>2</sup>

$$s_{ij}(\mathbf{x}, \eta) = \left[ \frac{\partial_i \partial_j}{\partial^2} - \frac{1}{3} \delta_{ij}^{(K)} \right] \delta(\mathbf{x}, \eta)$$

---

<sup>1</sup>[Fry & Gaztanaga 1993]

<sup>2</sup>[McDonald & Roy 2009]

# The Local Bias Model and its Extensions

The original Local Model<sup>1</sup>

$$\delta_h(\mathbf{x}, \eta) = \mathcal{F}[\delta(\mathbf{x}', \eta)] \approx b_1 \delta(\mathbf{x}, \eta) + \frac{1}{2!} b_2 \delta^2(\mathbf{x}, \eta) + \dots$$

Tidal Terms allowed by Symmetry<sup>2</sup>

$$s_{ij}(\mathbf{x}, \eta) = \left[ \frac{\partial_i \partial_j}{\partial^2} - \frac{1}{3} \delta_{ij}^{(K)} \right] \delta(\mathbf{x}, \eta)$$

---

<sup>1</sup>[Fry & Gaztanaga 1993]

<sup>2</sup>[McDonald & Roy 2009]

# The Local Bias Model and its Extensions

## The original Local Model<sup>1</sup>

$$\delta_h(\mathbf{x}, \eta) = \mathcal{F}[\delta(\mathbf{x}', \eta)] \approx b_1 \delta(\mathbf{x}, \eta) + \frac{1}{2!} b_2 \delta^2(\mathbf{x}, \eta) + \dots$$

## Tidal Terms allowed by Symmetry<sup>2</sup>

$$s_{ij}(\mathbf{x}, \eta) = \left[ \frac{\partial_i \partial_j}{\partial^2} - \frac{1}{3} \delta_{ij}^{(K)} \right] \delta(\mathbf{x}, \eta)$$

$$\delta_h(\mathbf{x}, \eta) = \mathcal{F}[\delta(\mathbf{x}', \eta)] \approx b_1 \delta(\mathbf{x}, \eta) + \frac{1}{2!} b_2 \delta^2(\mathbf{x}, \eta) + b_{s^2} \underbrace{s_{ij}(\mathbf{x}) s^{ij}(\mathbf{x})}_{s^2(\mathbf{x})} + \dots$$

---

<sup>1</sup>[Fry & Gaztanaga 1993]

<sup>2</sup>[McDonald & Roy 2009]

# Coevolution of Haloes and Dark Matter

## Assumptions

- initial bias is local  $\delta_{h,i}(\mathbf{q}) = b_1^{(L)}\delta_{m,i}(\mathbf{q}) + b_2^{(L)}\delta_{m,i}^2(\mathbf{q})$
- haloes flow with the dark matter  $\mathbf{v}_h = \mathbf{v}_m$  (no velocity bias)

## Coevolution<sup>3</sup>

- mapping from Lagrangian to Eulerian space  $\mathbf{x}(\mathbf{q}) = \mathbf{q} + \Psi(\mathbf{q})$

$$\begin{aligned}\delta_h(\mathbf{x}, \eta) &= \left(1 + b_1^{(L)}(\eta)\right) \left({}^{(1)}\delta(\mathbf{x}, \eta) + {}^{(2)}\delta(\mathbf{x}, \eta)\right) \\ &\quad + \left(\frac{4}{21}b_1^{(L)}(\eta) + \frac{1}{2}b_2^{(L)}(\eta)\right) {}^{(1)}\delta^2(\mathbf{x}, \eta) - \frac{2}{7}b_1^{(L)}(\eta)s^2(\mathbf{x}, \eta),\end{aligned}$$

---

<sup>3</sup>[Catelan et al. 2000]

# Coevolution of Haloes and Dark Matter

## Assumptions

- initial bias is local  $\delta_{h,i}(\mathbf{q}) = b_1^{(L)}\delta_{m,i}(\mathbf{q}) + b_2^{(L)}\delta_{m,i}^2(\mathbf{q})$
- haloes flow with the dark matter  $\mathbf{v}_h = \mathbf{v}_m$  (no velocity bias)

## Coevolution<sup>3</sup>

- mapping from Lagrangian to Eulerian space  $\mathbf{x}(\mathbf{q}) = \mathbf{q} + \Psi(\mathbf{q})$

$$\begin{aligned}\delta_h(\mathbf{x}, \eta) &= \left(1 + b_1^{(L)}(\eta)\right) \left(^{(1)}\delta(\mathbf{x}, \eta) + ^{(2)}\delta(\mathbf{x}, \eta)\right) \\ &\quad + \left(\frac{4}{21}b_1^{(L)}(\eta) + \frac{1}{2}b_2^{(L)}(\eta)\right) ^{(1)}\delta^2(\mathbf{x}, \eta) - \frac{2}{7}b_1^{(L)}(\eta)s^2(\mathbf{x}, \eta),\end{aligned}$$

---

<sup>3</sup>[Catelan et al. 2000]

# Coevolution of Haloes and Dark Matter

## Assumptions

- initial bias is local  $\delta_{h,i}(\mathbf{q}) = b_1^{(L)}\delta_{m,i}(\mathbf{q}) + b_2^{(L)}\delta_{m,i}^2(\mathbf{q})$
- haloes flow with the dark matter  $\mathbf{v}_h = \mathbf{v}_m$  (no velocity bias)

## Coevolution<sup>3</sup>

- mapping from Lagrangian to Eulerian space  $\mathbf{x}(\mathbf{q}) = \mathbf{q} + \Psi(\mathbf{q})$

$$\begin{aligned}\delta_h(\mathbf{x}, \eta) = & b_1^{(E)}(\eta) \left( {}^{(1)}\delta(\mathbf{x}, \eta) + {}^{(2)}\delta(\mathbf{x}, \eta) \right) \\ & + \frac{1}{2} b_2^{(E)}(\eta) {}^{(1)}\delta^2(\mathbf{x}, \eta) \underbrace{- \frac{2}{7} b_1^{(L)}(\eta) s^2(\mathbf{x}, \eta)}_{b_s^2},\end{aligned}$$

---

<sup>3</sup>[Catelan et al. 2000]

# Imprint on the Bispectrum<sup>4</sup>

## Unsymmetrized Cross Bispectrum

$$B_{\text{mmh}}^{(\text{unsym})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) (2\pi)^3 \delta^{(\text{D})}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) = \langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta_h(\mathbf{k}_3) \rangle,$$

2<sup>nd</sup> order Bias + Tidal Terms

$$B_{\text{mmh}}^{(\text{unsym})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) - b_1 B_{\text{mmm}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2P(k_1)P(k_2) \left[ b_2 + b_{s^2} \left( \mu^2 - \frac{1}{3} \right) \right].$$

Isolate Angular Dependence

$$M(\mu) = \frac{B_{\text{mmh}}^{(\text{unsym})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) - b_1 B_{\text{mmm}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{2P(k_1)P(k_2)} = b_2 L_0(\mu) + b_{s^2} L_2(\mu).$$

---

<sup>4</sup> see also [Kwan, Scoccimarro & Sheth 2012]

# Imprint on the Bispectrum<sup>4</sup>

## Unsymmetrized Cross Bispectrum

$$B_{\text{mmh}}^{(\text{unsym})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) (2\pi)^3 \delta^{(\text{D})}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) = \langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta_h(\mathbf{k}_3) \rangle,$$

## 2<sup>nd</sup> order Bias + Tidal Terms

$$B_{\text{mmh}}^{(\text{unsym})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) - b_1 B_{\text{mmm}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2P(k_1)P(k_2) \left[ b_2 + b_{s^2} \left( \mu^2 - \frac{1}{3} \right) \right].$$

## Isolate Angular Dependence

$$M(\mu) = \frac{B_{\text{mmh}}^{(\text{unsym})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) - b_1 B_{\text{mmm}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{2P(k_1)P(k_2)} = b_2 L_0(\mu) + b_{s^2} L_2(\mu).$$

---

<sup>4</sup> see also [Kwan, Scoccimarro & Sheth 2012]

# Imprint on the Bispectrum<sup>4</sup>

## Unsymmetrized Cross Bispectrum

$$B_{\text{mmh}}^{(\text{unsym})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) (2\pi)^3 \delta^{(\text{D})}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) = \langle \delta(\mathbf{k}_1) \delta(\mathbf{k}_2) \delta_h(\mathbf{k}_3) \rangle,$$

## 2<sup>nd</sup> order Bias + Tidal Terms

$$B_{\text{mmh}}^{(\text{unsym})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) - b_1 B_{\text{mmm}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2P(k_1)P(k_2) \left[ b_2 + b_{s^2} \left( \mu^2 - \frac{1}{3} \right) \right].$$

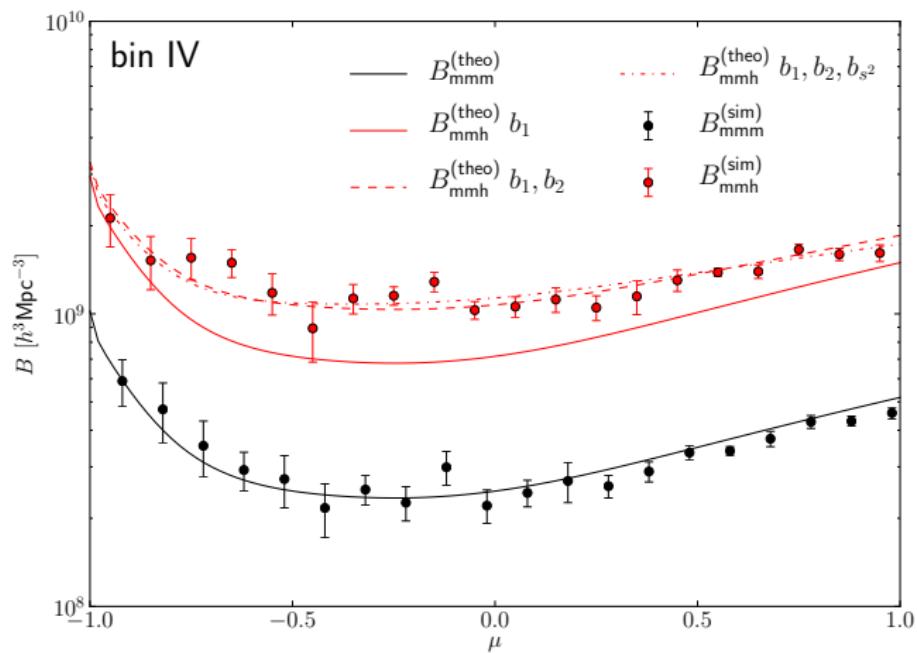
## Isolate Angular Dependence

$$M(\mu) = \frac{B_{\text{mmh}}^{(\text{unsym})}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) - b_1 B_{\text{mmm}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{2P(k_1)P(k_2)} = b_2 L_0(\mu) + b_{s^2} L_2(\mu).$$

---

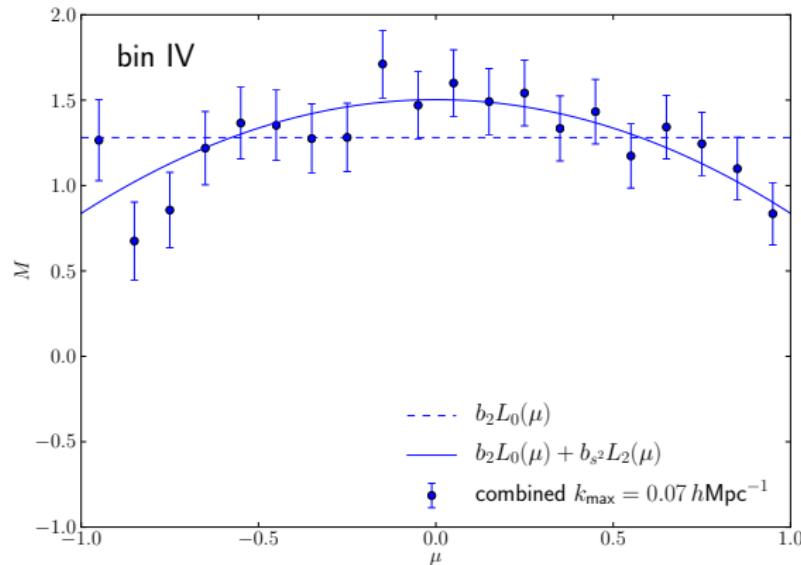
<sup>4</sup> see also [Kwan, Scoccimarro & Sheth 2012]

# Imprint on the Bispectrum



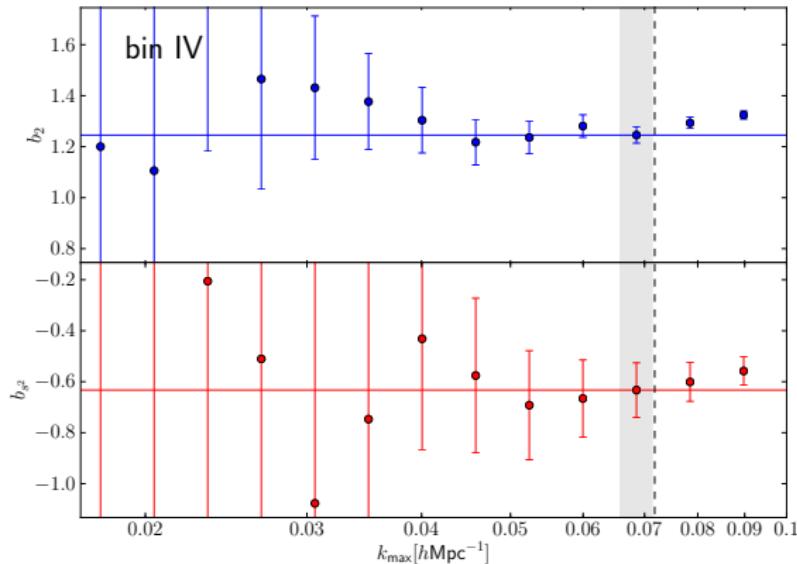
## Imprint on the Bispectrum

$$\overline{M}(\mu) = \sum_{k_1, k_2}^{k_{\max}} \frac{\hat{M}(k_1, k_2, \mu)}{\Delta M^2(k_1, k_2, \mu)} \left( \sum_{k_1, k_2} \frac{1}{\Delta M^2(k_1, k_2, \mu)} \right)^{-1}$$

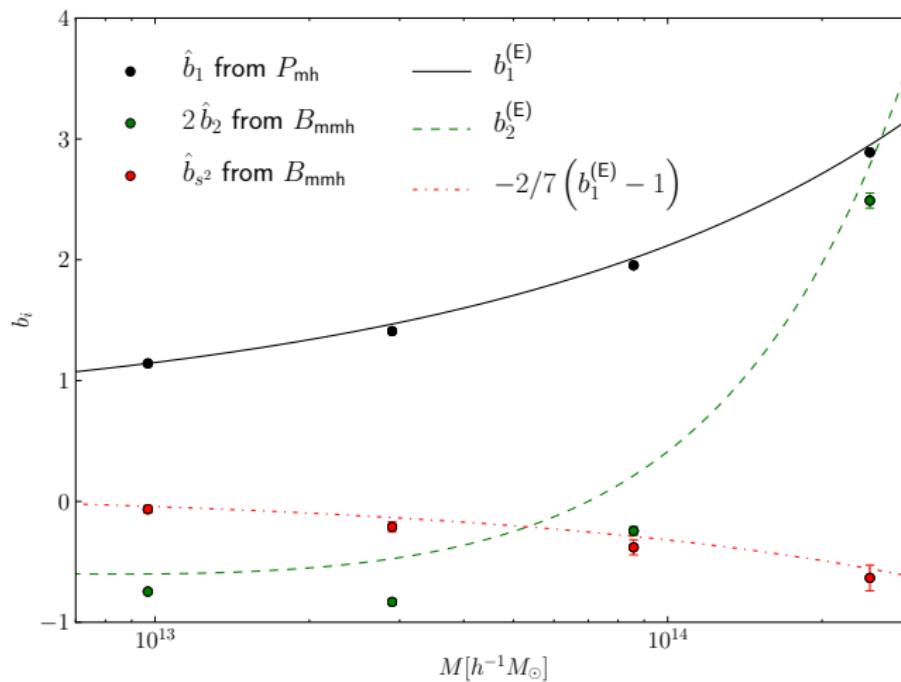


# Imprint on the Bispectrum

$$\overline{M}(\mu) = \sum_{k_1, k_2}^{k_{\max}} \frac{\hat{M}(k_1, k_2, \mu)}{\Delta M^2(k_1, k_2, \mu)} \left( \sum_{k_1, k_2} \frac{1}{\Delta M^2(k_1, k_2, \mu)} \right)^{-1}$$



# Imprint on the Bispectrum



# Non-local Cubic Term

## Quadratic Term - Influences Bispectrum

$$\begin{aligned} {}^{(2)}\delta_h(\mathbf{k}, \eta) = \prod_{i=1}^2 \int \frac{d^3 q_i}{(2\pi)^3} \left\{ \frac{b_2^{(L)}(\eta)}{2} + b_1^{(E)}(\eta) F_2(\mathbf{q}_1, \mathbf{q}_2) \right. \\ \left. - \frac{2}{7} b_1^{(L)}(\eta) S_2(\mathbf{q}_1, \mathbf{q}_2) \right\} {}^{(1)}\delta(\mathbf{q}_1, \eta) {}^{(1)}\delta(\mathbf{q}_2, \eta) \delta^{(D)} \left( \mathbf{k} - \sum \mathbf{q}_j \right) \end{aligned}$$

## Cubic Term - Influences Trispectrum

$$\begin{aligned} {}^{(3)}\delta_h(\mathbf{k}, \eta) = \prod_{i=1}^3 \int \frac{d^3 q_i}{(2\pi)^3} \left\{ b_1^{(E)}(\eta) F_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) + \frac{b_2^{(L)}(\eta)}{2} \alpha(\mathbf{q}_1, \mathbf{q}_2 + \mathbf{q}_3) + \frac{b_3^{(L)}(\eta)}{3!} \right. \\ \left. + b_1^{(L)}(\eta) K(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) \right\} {}^{(1)}\delta(\mathbf{q}_1, \eta) {}^{(1)}\delta(\mathbf{q}_2, \eta) {}^{(1)}\delta(\mathbf{q}_3, \eta) \delta^{(D)} \left( \mathbf{k} - \sum \mathbf{q}_j \right) \end{aligned}$$

# Non-local Cubic Term

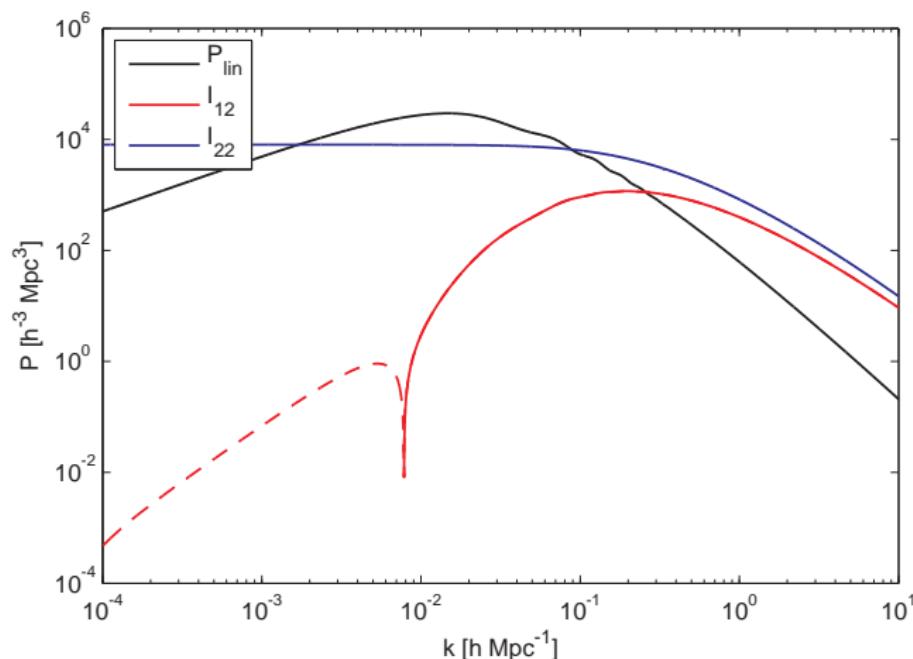
## Quadratic Term - Influences Bispectrum

$$\begin{aligned} {}^{(2)}\delta_h(\mathbf{k}, \eta) = \prod_{i=1}^2 \int \frac{d^3 q_i}{(2\pi)^3} \left\{ \frac{b_2^{(L)}(\eta)}{2} + b_1^{(E)}(\eta) F_2(\mathbf{q}_1, \mathbf{q}_2) \right. \\ \left. - \frac{2}{7} b_1^{(L)}(\eta) S_2(\mathbf{q}_1, \mathbf{q}_2) \right\} {}^{(1)}\delta(\mathbf{q}_1, \eta) {}^{(1)}\delta(\mathbf{q}_2, \eta) \delta^{(D)} \left( \mathbf{k} - \sum \mathbf{q}_j \right) \end{aligned}$$

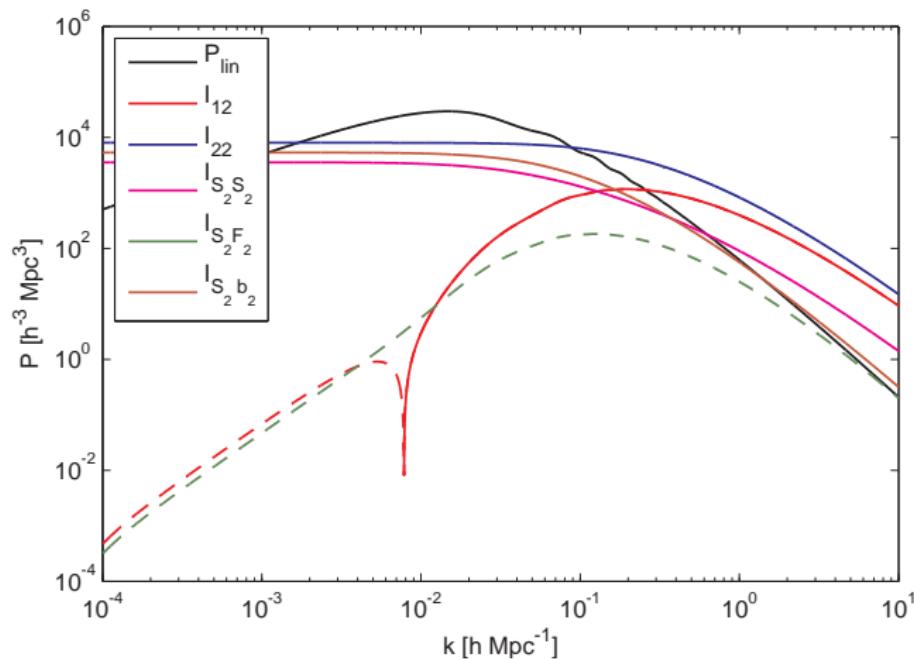
## Cubic Term - Influences Trispectrum

$$\begin{aligned} {}^{(3)}\delta_h(\mathbf{k}, \eta) = \prod_{i=1}^3 \int \frac{d^3 q_i}{(2\pi)^3} \left\{ b_1^{(E)}(\eta) F_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) + \frac{b_2^{(L)}(\eta)}{2} \alpha(\mathbf{q}_1, \mathbf{q}_2 + \mathbf{q}_3) + \frac{b_3^{(L)}(\eta)}{3!} \right. \\ \left. + b_1^{(L)}(\eta) K(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) \right\} {}^{(1)}\delta(\mathbf{q}_1, \eta) {}^{(1)}\delta(\mathbf{q}_2, \eta) {}^{(1)}\delta(\mathbf{q}_3, \eta) \delta^{(D)} \left( \mathbf{k} - \sum \mathbf{q}_j \right) \end{aligned}$$

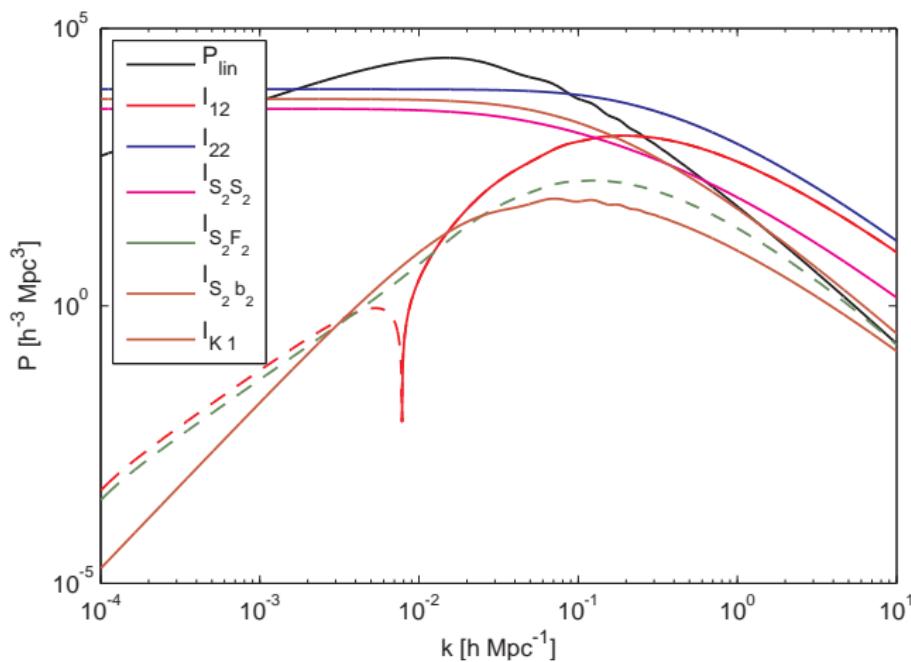
# Scale Dependence in the Power Spectrum



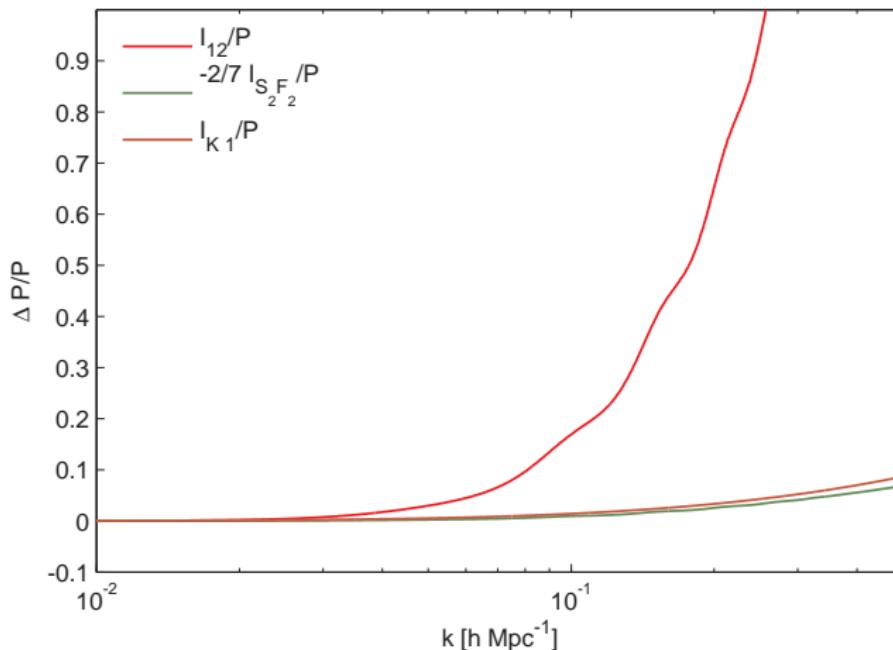
# Scale Dependence in the Power Spectrum



# Scale Dependence in the Power Spectrum



# Scale Dependence in the Power Spectrum



## 1 Introduction

## 2 Non-Local Bias

## 3 Noise Corrections

## 4 Summary & Outlook

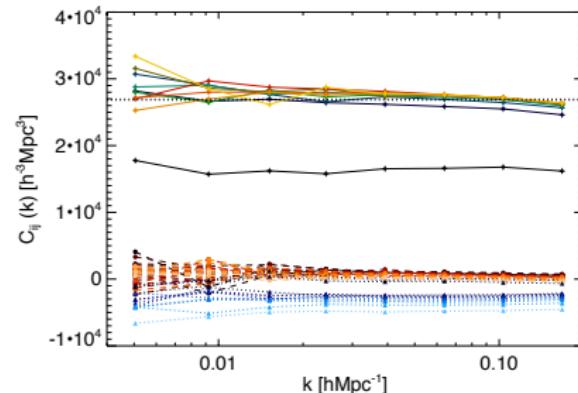
# Motivation & Approach

## Motivation

- sub-Poissonian shotnoise in [Hamaus et al. 2009]
- discrepancies between  $\hat{b}_{hh}$  and  $\hat{b}_{hm}$  [Okumura et al. 2012]
- presence of large scale corrections in the perturbative local bias model

## Approach

- noise: all deviations from linear bias power spectrum in the  $k \rightarrow 0$  limit
- small scale correlation function  
↔ large scale power spectrum



[Hamaus et al. 2009]

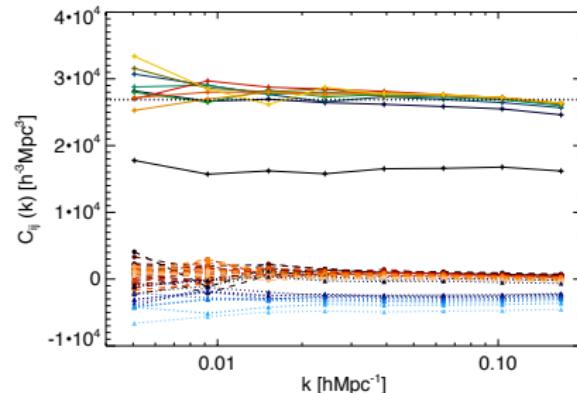
# Motivation & Approach

## Motivation

- sub-Poissonian shotnoise in [Hamaus et al. 2009]
- discrepancies between  $\hat{b}_{hh}$  and  $\hat{b}_{hm}$  [Okumura et al. 2012]
- presence of large scale corrections in the perturbative local bias model

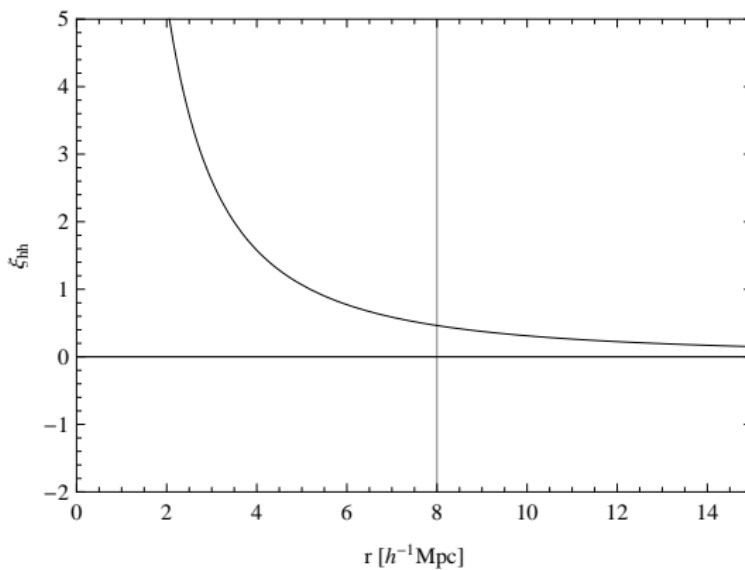
## Approach

- noise: all deviations from linear bias power spectrum in the  $k \rightarrow 0$  limit
- small scale correlation function  
 $\leftrightarrow$  large scale power spectrum



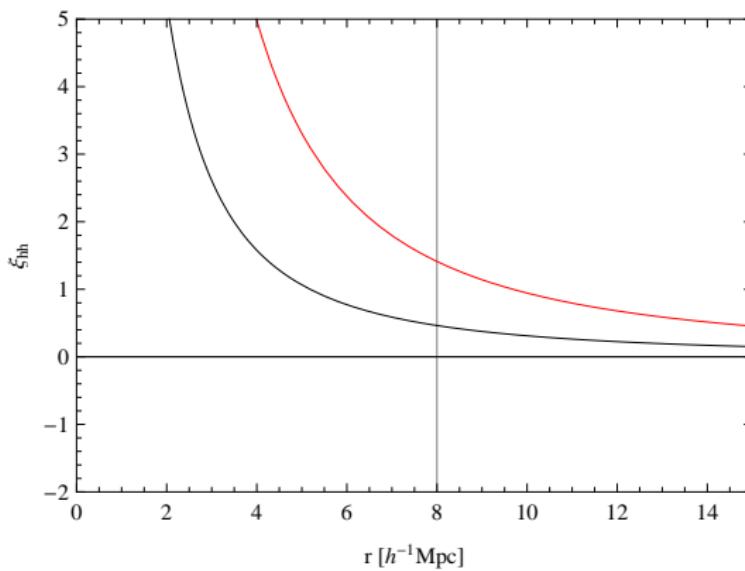
[Hamaus et al. 2009]

# Phenomenology of the Small Scale Correlation Function



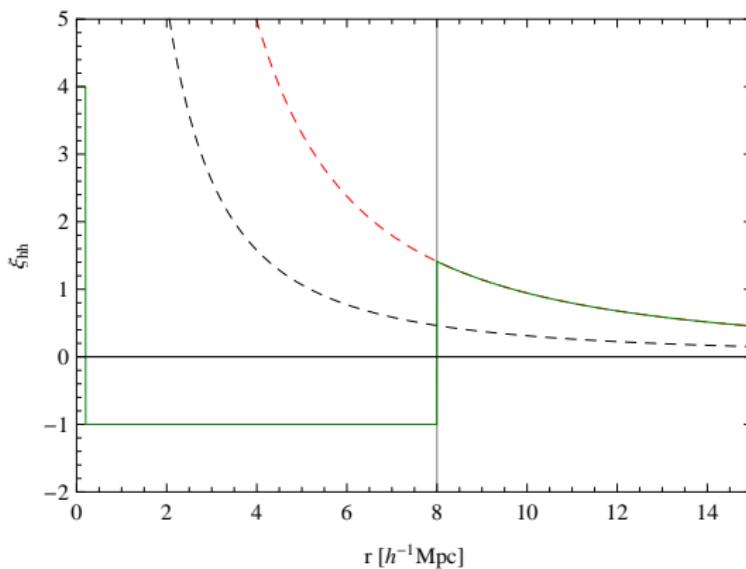
$\xi_{\text{hh}} = \text{linear bias}$

# Phenomenology of the Small Scale Correlation Function



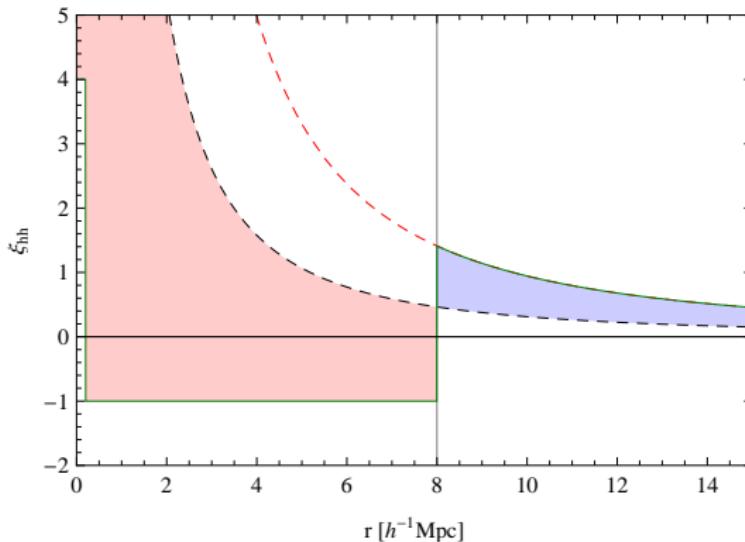
$$\xi_{\text{hh}} = \text{linear bias} + \text{non-linear bias}$$

# Phenomenology of the Small Scale Correlation Function



$$\xi_{\text{hh}} = \text{linear bias} + \text{non-linear bias} - \text{exclusion} + \text{shot noise}$$

# Phenomenology of the Small Scale Correlation Function



$$P_{hh} = \text{linear bias} + \text{shot noise} + \text{non-linear bias correction} - \text{exclusion correction}$$

# Power Spectrum of Discrete Tracers

Density Perturbation - real-space

$$\delta^{(d)}(\mathbf{r}) = \frac{n(\mathbf{r})}{\bar{n}} - 1 = \frac{1}{\bar{n}} \sum_i \delta^{(D)}(\mathbf{r} - \mathbf{r}_i) - 1,$$

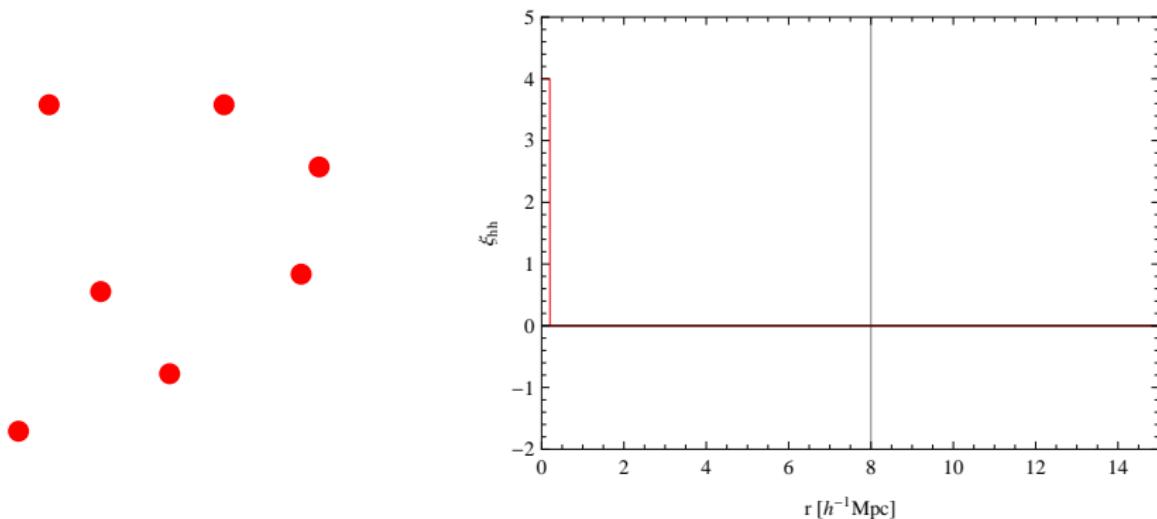
Density Perturbation -  $k$ -space

$$\delta^{(d)}(\mathbf{k}) = \int d^3r \exp[i\mathbf{k} \cdot \mathbf{r}] \delta(\mathbf{r}) = \frac{1}{\bar{n}} \sum_i \exp[i\mathbf{k} \cdot \mathbf{r}_i] - V\delta_{\mathbf{k},0}^{(K)}.$$

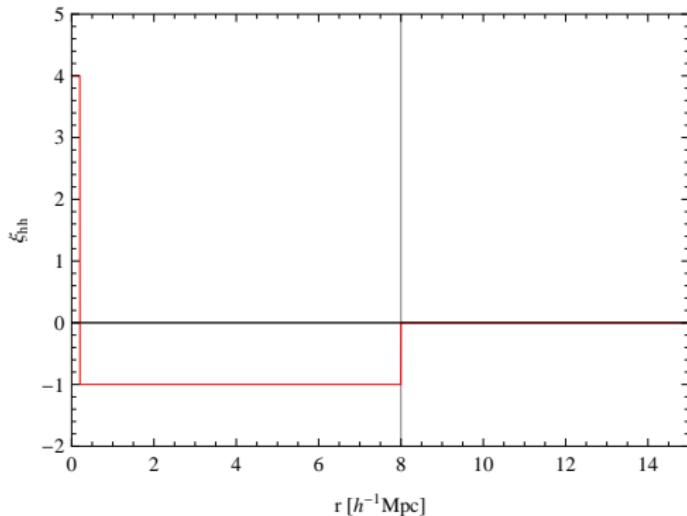
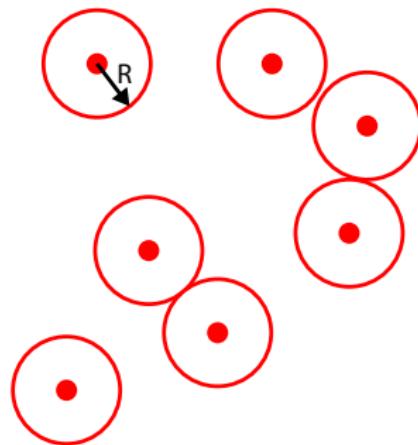
Power Spectrum

$$P^{(d)}(\mathbf{k}) = \frac{1}{\bar{n}} + \frac{V}{N^2} \sum_{i \neq j} \langle \exp[i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)] \rangle - V\delta_{\mathbf{k},0}^{(K)}$$

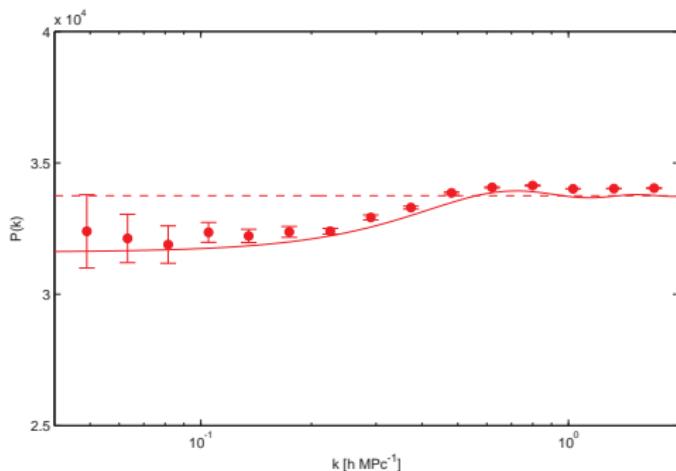
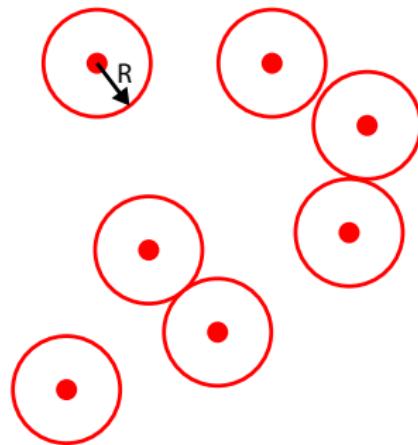
# Toy Model: Random Sample with Exclusion



# Toy Model: Random Sample with Exclusion

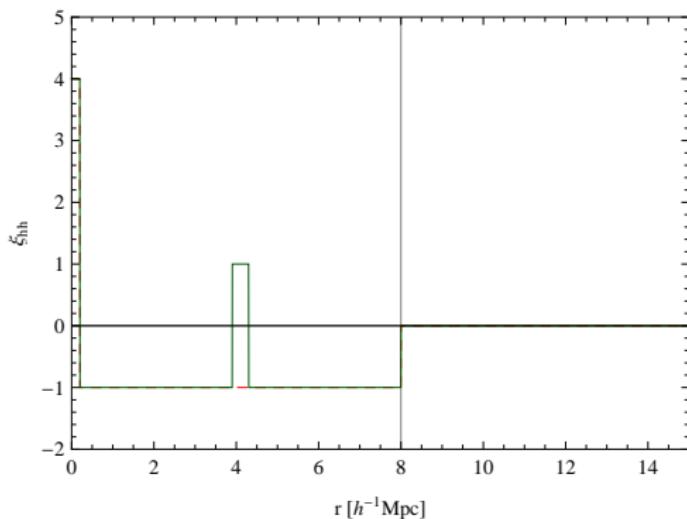
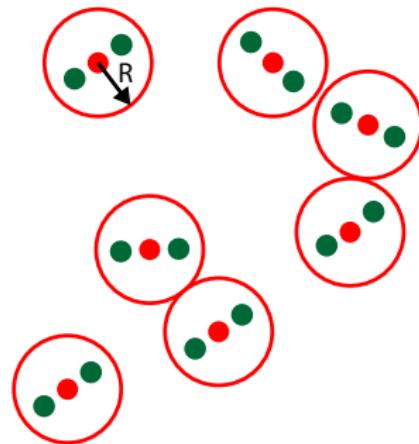


# Toy Model: Random Sample with Exclusion

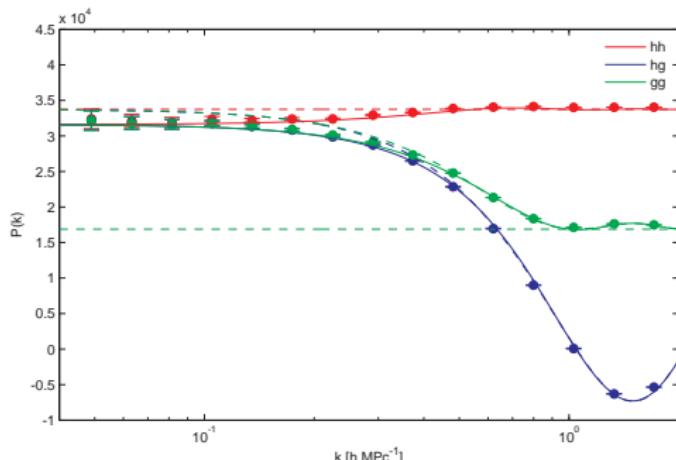
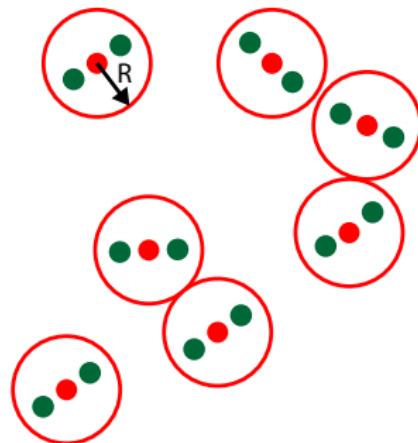


$$P_{hh}(k) = \frac{1}{\bar{n}} - \frac{4\pi R^3}{3} W_R(k)$$

# Toy Model: Random Sample with Exclusion



# Toy Model: Random Sample with Exclusion



$$P_{gg}(k) = \frac{1}{\bar{n}_g} \left( 1 + (N_{g,h} - 1) \frac{\sin(kR_g)}{kR_g} \right) - \frac{4\pi R^3}{3} W_R(k)$$

# Small Scale Exclusion

## Random Sample

$$\xi_{\text{hh}}^{(\text{d})}(r) = \begin{cases} -1 & r < R \\ 0 & r \geq R \end{cases} \quad P_{\text{hh}}^{(\text{d})}(k) = \frac{1}{\bar{n}} - V_{\text{excl}} W_R(k)$$

## Correlated Sample

$$\xi_{\text{hh}}^{(\text{d})}(r) = \begin{cases} -1 & r < R \\ \xi_{\text{hh}}^{(\text{c})}(r) & r \geq R \end{cases} \quad P_{\text{hh}}^{(\text{d})}(k) = \frac{1}{\bar{n}} - V_{\text{excl}} W_R(k) + P_{\text{hh}}^{(\text{c})}(k) - V_{\text{excl}} [P_{\text{hh}}^{(\text{c})} * W_R](k)$$

# Small Scale Exclusion

## Random Sample

$$\xi_{\text{hh}}^{(\text{d})}(r) = \begin{cases} -1 & r < R \\ 0 & r \geq R \end{cases} \quad P_{\text{hh}}^{(\text{d})}(k) = \frac{1}{\bar{n}} - V_{\text{excl}} W_R(k)$$

## Correlated Sample

$$\xi_{\text{hh}}^{(\text{d})}(r) = \begin{cases} -1 & r < R \\ \xi_{\text{hh}}^{(\text{c})}(r) & r \geq R \end{cases} \quad P_{\text{hh}}^{(\text{d})}(k) = \frac{1}{\bar{n}} - V_{\text{excl}} W_R(k) + P_{\text{hh}}^{(\text{c})}(k) - V_{\text{excl}} [P_{\text{hh}}^{(\text{c})} * W_R](k)$$

$$W_R(k) = 3 \frac{\sin(kR) - kR \cos(kR)}{(kR)^3}$$

# Halo Power Spectra in the Local Bias Model<sup>5</sup>

## Halo-Halo

$$\begin{aligned} P_{\text{hh}}^{(c)}(k) = & b_{1,E}^2 P_{\text{1-loop}}(k) \\ & + b_{1,E} b_{2,E} \int \frac{d^3 q}{(2\pi)^3} F_2(\mathbf{q}, \mathbf{k} - \mathbf{q}) P_{\text{lin}}(q) P_{\text{lin}}(|\mathbf{k} - \mathbf{q}|) \\ & + \frac{1}{2} b_{2,E}^2 \int \frac{d^3 q}{(2\pi)^3} P_{\text{lin}}(q) P_{\text{lin}}(|\mathbf{k} - \mathbf{q}|) \end{aligned}$$

## Halo-Matter

$$\begin{aligned} P_{\text{hm}}^{(c)}(k, \eta_i) = & b_{1,E} P_{\text{1-loop}}(k) \\ & + b_{2,E} \int \frac{d^3 q}{(2\pi)^3} F_2(\mathbf{q}, \mathbf{k} - \mathbf{q}) P_{\text{lin}}(q) P_{\text{lin}}(|\mathbf{k} - \mathbf{q}|) \end{aligned}$$

---

<sup>5</sup>[McDonald 2006]

# Halo Power Spectra in the Local Bias Model<sup>5</sup>

## Halo-Halo

$$P_{\text{hh},i}^{(c)}(k) = b_{1,L}^2 P_{\text{lin},i}(k)$$

$$+ b_{1,L} b_{2,L} \int \frac{d^3 q}{(2\pi)^3} F_2(\mathbf{q}, \mathbf{k} - \mathbf{q}) P_{\text{lin},i}(q) P_{\text{lin},i}(|\mathbf{k} - \mathbf{q}|)$$

$$+ \frac{1}{2} b_{2,L}^2 \underbrace{\int \frac{d^3 q}{(2\pi)^3} P_{\text{lin},i}(q) P_{\text{lin},i}(|\mathbf{k} - \mathbf{q}|)}_{I_{22}(k)}$$

## Halo-Matter

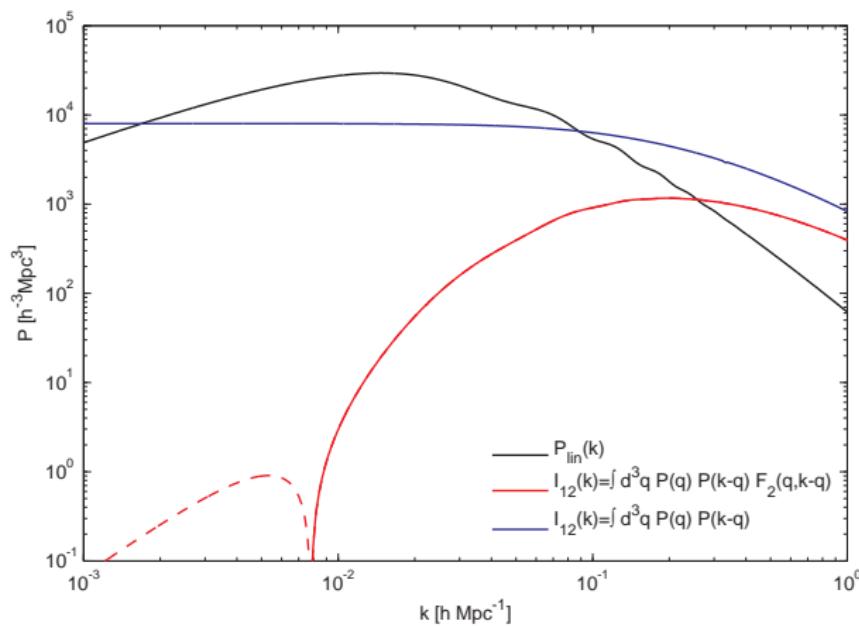
$$P_{\text{hm},i}^{(c)}(k) = b_{1,L} P_{\text{lin},i}(k)$$

$$+ b_{2,L} \int \frac{d^3 q}{(2\pi)^3} F_2(\mathbf{q}, \mathbf{k} - \mathbf{q}) P_{\text{lin},i}(q) P_{\text{lin},i}(|\mathbf{k} - \mathbf{q}|)$$

---

<sup>5</sup>[McDonald 2006]

# Scale Dependence of the Terms



# Halo Power Spectra Including Exclusion

## Perturbation Theory + Exclusion

$$\begin{aligned} P_{\text{hh},i}^{(\text{d})}(k) = & \frac{1}{\bar{n}} + b_1^2 P_{\text{lin},i}(k) + \frac{1}{2} b_2^2 I_{22}(k) \\ & - V_{\text{excl}} W_R(k) - b_1^2 V_{\text{excl}} [P_{\text{lin},i} * W_R](k) - \frac{1}{2} b_2^2 V_{\text{excl}} [I_{22} * W_R](k) \end{aligned}$$

$k \rightarrow 0$  limit

$$P_{\text{hh},i}^{(\text{d})}(k \rightarrow 0) = \frac{1}{\bar{n}} + \frac{1}{2} b_2^2 \int_R^\infty d^3r \xi_{\text{lin},i}^2(r) - V_{\text{excl}} - b_1^2 \int_0^R d^3r \xi_{\text{lin},i}(r)$$

# Halo Power Spectra Including Exclusion

## Perturbation Theory + Exclusion

$$\begin{aligned} P_{\text{hh},i}^{(\text{d})}(k) = & \frac{1}{\bar{n}} + b_1^2 P_{\text{lin},i}(k) + \frac{1}{2} b_2^2 I_{22}(k) \\ & - V_{\text{excl}} W_R(k) - b_1^2 V_{\text{excl}} [P_{\text{lin},i} * W_R](k) - \frac{1}{2} b_2^2 V_{\text{excl}} [I_{22} * W_R](k) \end{aligned}$$

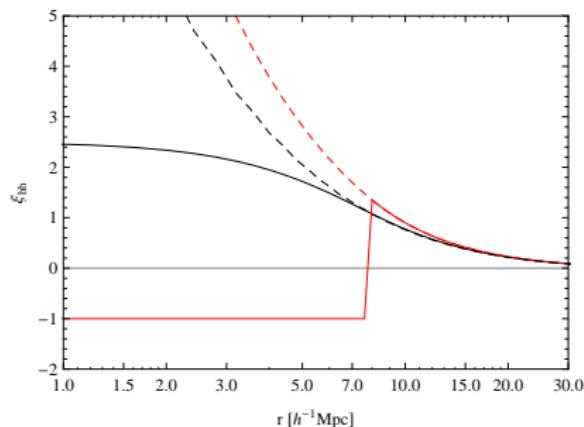
$k \rightarrow 0$  limit

$$P_{\text{hh},i}^{(\text{d})}(k \rightarrow 0) = \frac{1}{\bar{n}} + \frac{1}{2} b_2^2 \int_R^\infty d^3 r \xi_{\text{lin},i}^2(r) - V_{\text{excl}} - b_1^2 \int_0^R d^3 r \xi_{\text{lin},i}(r)$$

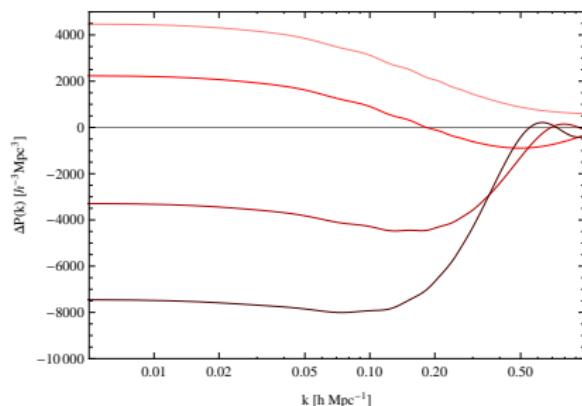
# Correlation of Thresholded Regions<sup>6</sup>

$$1 + \xi_{\text{tr}}(r) = \frac{1}{n_{\text{tr}}^2} \int_{\nu_{\min}}^{\nu_{\max}} d\nu_1 \int_{\nu_{\min}}^{\nu_{\max}} d\nu_2 \frac{1}{(2\pi)^3 \sqrt{1 - \xi_0^2(r)/\sigma_0^4}} \exp \left[ -\frac{1}{2} \frac{\nu_1^2 + \nu_2^2 - 2\nu_1\nu_2\xi_0(r)/\sigma_0^2}{1 - \xi_0^2(r)/\sigma_0^4} \right]$$

Correlation Function



Power Spectrum Correction

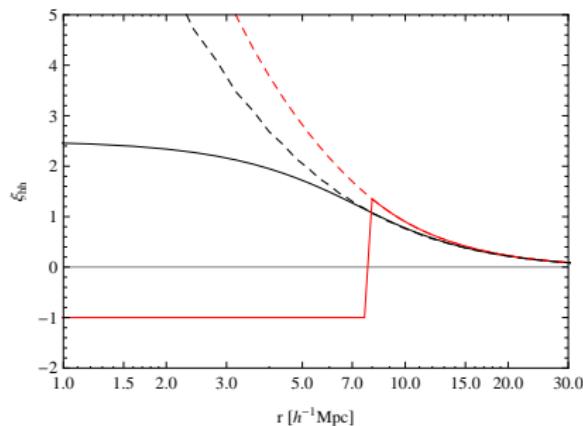


<sup>6</sup>[Kaiser 1984, Beltran & Durrer 2011]

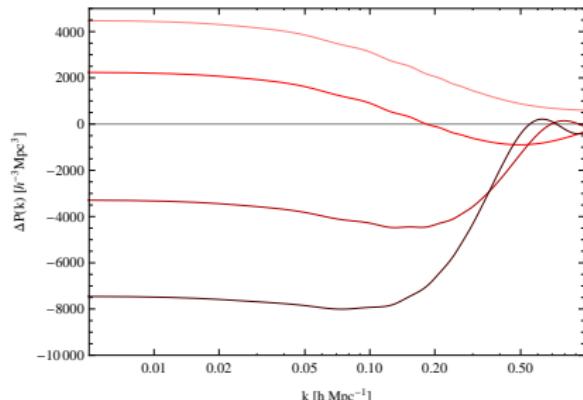
# Correlation of Thresholded Regions<sup>6</sup>

$$\Delta P_{\text{tr}}(k) = \text{FT}[\xi_{\text{tr}}](k) - b^2 P_{\text{lin}}(k)$$

Correlation Function



Power Spectrum Correction



<sup>6</sup>[Kaiser 1984, Beltran & Durrer 2011]

# Estimating Exclusion in Lagrangian Space

Why should the noise be the same in Lagrangian and Eulerian Space?

- Peebles: mass and momentum conservation → gravity can only generate  $k^2$  terms in  $\delta(\mathbf{k})$
- low  $k$ -limit  $P_{22}(k) \propto k^4$  and  $P_{13}(k) \propto k^2 P(k)$

## Methodology

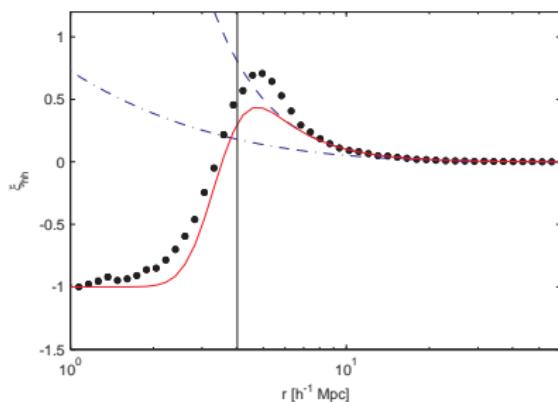
- 10 equal number density mass bins
- trace back the particles that form halo at  $z_f = 0$  to initial conditions  $z_i = 50$

# Correlation Function from Simulations

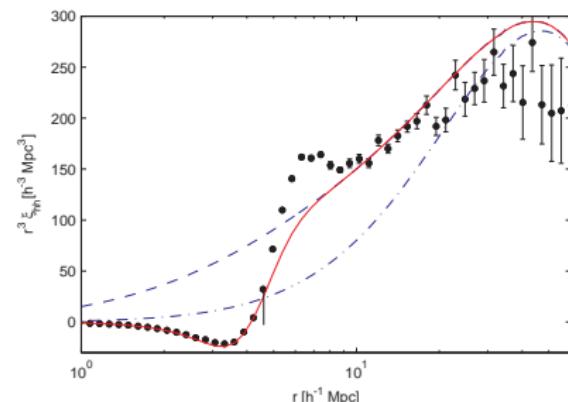
## Local Lagrangian Bias

$$\xi_{hh}^{(c)}(r) = b_1^2 \xi_{mm,lin}(r) + \frac{1}{2} b_2^2 \xi_{mm,lin}^2(r) + \dots$$

Initial Conditions -  $\xi_{hh}(r)$



Initial Conditions -  $r^3 \xi_{hh}(r)$

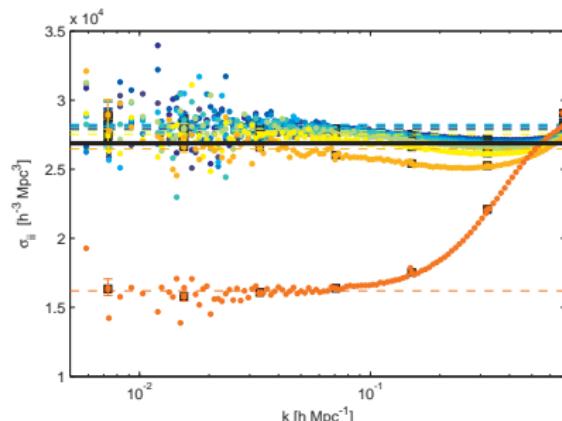


# SN Matrix from Simulations

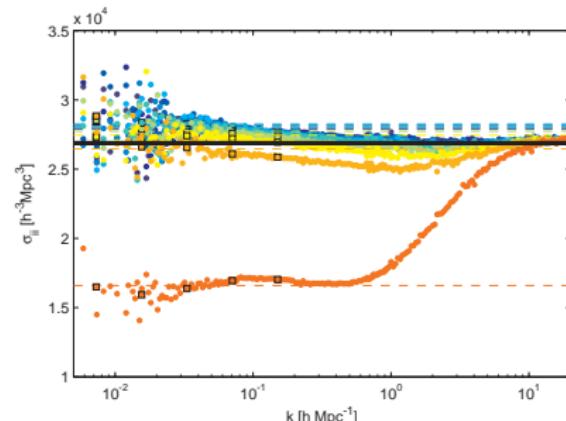
## Shotnoise Matrix<sup>7</sup>

$$\begin{aligned}\sigma_{ij}(k) &= \langle (\delta_i - b_{1,i}\delta)(\delta_j - b_{1,j}\delta) \rangle \\ &= P_{ij}(k) - b_{1,i}P_{j\delta}(k) - b_{1,j}P_{i\delta}(k) + b_{1,i}b_{1,j}P_{\text{mm}}(k)\end{aligned}$$

Initial Conditions  $z_i = 50$



Final Distribution  $z_f = 0$



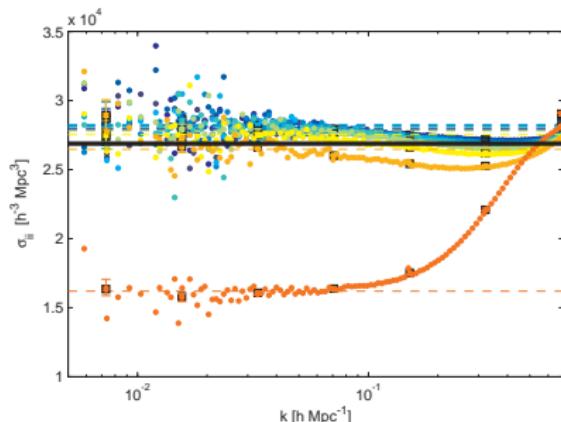
<sup>7</sup>[Hamaus et al. 2009]

# SN Matrix from Simulations

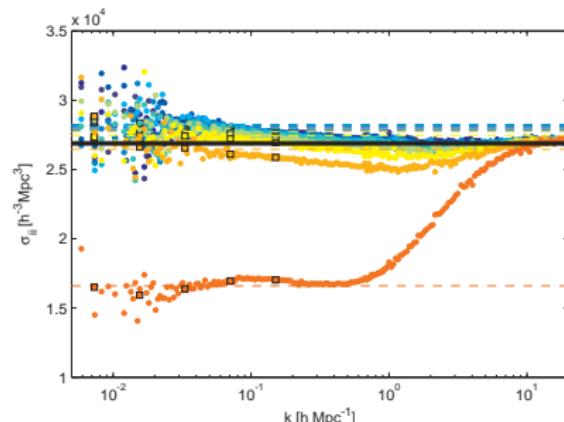
Shotnoise Matrix<sup>7</sup>

$$\sigma_{ii}(k) = P_{ii}(k) - 2b_{1,i}P_{i\delta}(k) + b_{1,i}^2P_{\text{mm}}(k)$$

Initial Conditions  $z_i = 50$



Final Distribution  $z_f = 0$



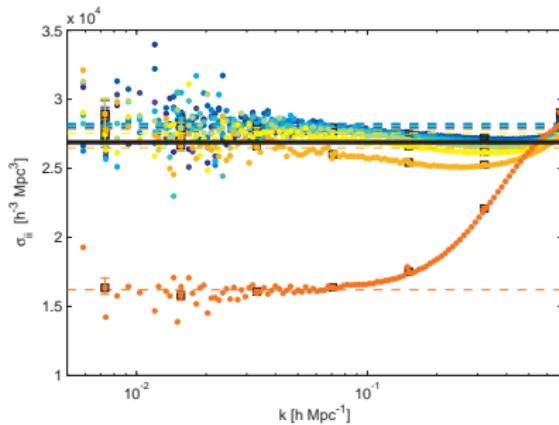
<sup>7</sup>[Hamaus et al. 2009]

# SN Matrix from Simulations and Theory

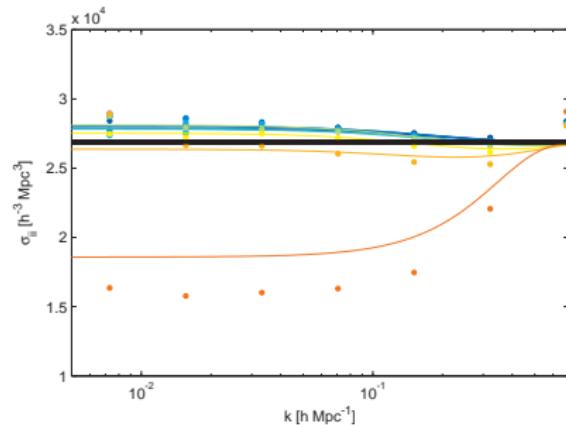
## Shotnoise Matrix

$$\sigma_{ii}(k) = P_{ii}(k) - 2b_{1,i}P_{i\delta}(k) + b_{1,i}^2P_{\text{mm}}(k)$$

## Initial Conditions - Simulation



## Initial Conditions - Model

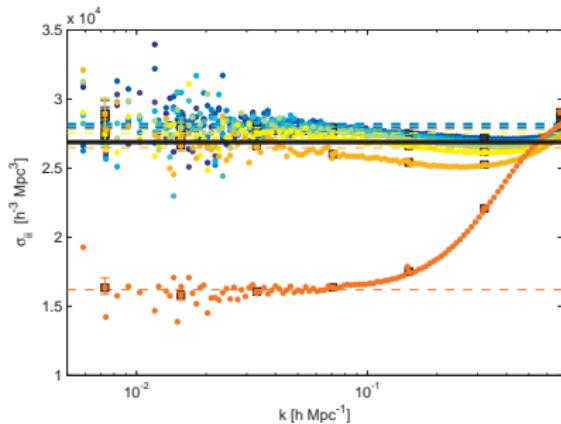


# SN Matrix from Simulations and Theory

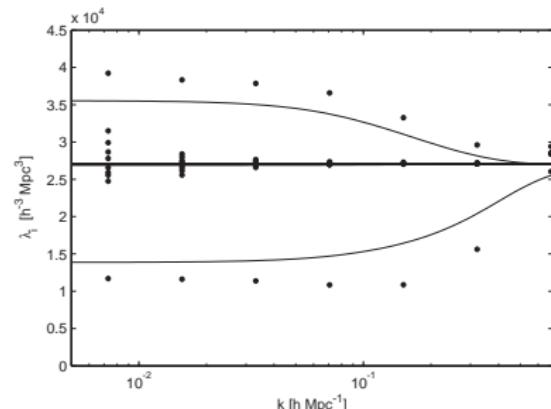
## Shotnoise Matrix

$$\sigma_{ii}(k) = P_{ii}(k) - 2b_{1,i}P_{i\delta}(k) + b_{1,i}^2P_{\text{mm}}(k)$$

## Initial Conditions - Simulation



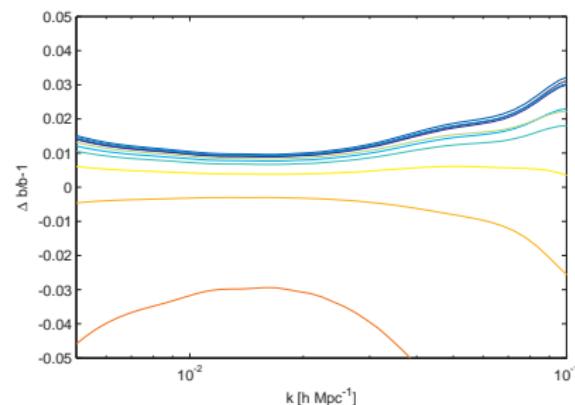
## Initial Conditions - Model



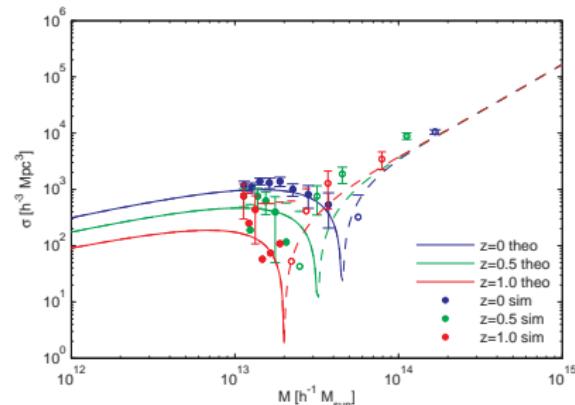
# Effect on Bias and Redshift Dependence

$$\hat{P}_{hh}(k) - \frac{1}{\bar{n}} = \Delta P_{hh}(k) + b_1^2 P_{mm,\text{lin}}(k) = \hat{b}_{1,hh}^2 \hat{P}_{mm}$$

Error on  $\hat{b}_{hh}$



Mass & Redshift Dependence



# Summary & Outlook

## Findings

- non-local terms in the late time halo bias at 2<sup>nd</sup> and 3<sup>th</sup> order
- noise component is not white and different from  $1/\bar{n}$
- amplitude of noise component is unaffected by evolution in  $k \rightarrow 0$  and  $k \rightarrow \infty$  limits
- qualitative explanation of the effects based on a model of  $\xi_{hh}$ 
  - positive correction from non-linear clustering of haloes
  - negative correction from exclusion

## Open Questions

- non-Local bias in the ICs (peak bias, velocity bias, tidal terms)
- accurate model for small scale correlation function & exclusion
- combine with perturbation theory for late time full  $P_{hh}$

# Summary & Outlook

## Findings

- non-local terms in the late time halo bias at 2<sup>nd</sup> and 3<sup>th</sup> order
- noise component is not white and different from  $1/\bar{n}$
- amplitude of noise component is unaffected by evolution in  $k \rightarrow 0$  and  $k \rightarrow \infty$  limits
- qualitative explanation of the effects based on a model of  $\xi_{hh}$ 
  - positive correction from non-linear clustering of haloes
  - negative correction from exclusion

## Open Questions

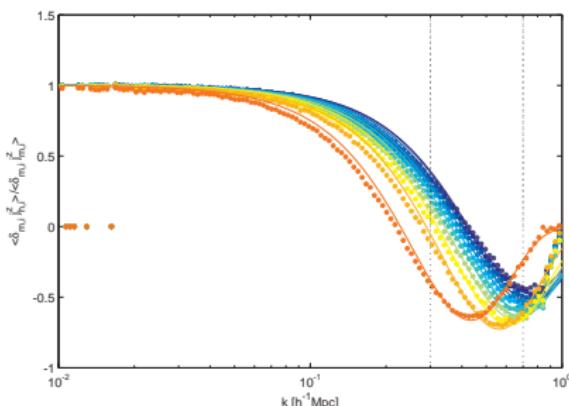
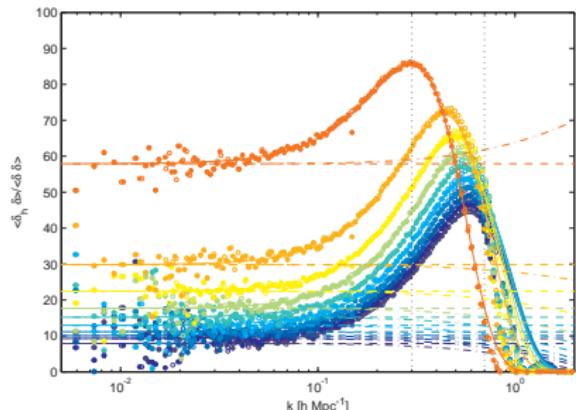
- non-Local bias in the ICs (peak bias, velocity bias, tidal terms)
- accurate model for small scale correlation function & exclusion
- combine with perturbation theory for late time full  $P_{hh}$

# Coevolution: Coupling & Velocity Bias

$$j_h^z(\mathbf{x}) = (1 + \delta_h(x)) \mathbf{v}_h(\mathbf{x}) \cdot \mathbf{e}_z$$

$$P_{01,hm} = \langle \delta_m(\mathbf{k}) j_h^z(-\mathbf{k}) \rangle$$

# Coevolution: Coupling & Velocity Bias



# Coevolution: Coupling & Velocity Bias

