The Fate
of the
Quantum

Gerard 't Hooft

Institute for Theoretical Physics
Spinoza Institute
Utrecht University, the Netherlands

The Cellular Automaton Interpretation of QM

Axioms:

i CA-States evolve classically:
$t = t_1 \to t_2, \to \cdots , \quad \vec{Q}_1 \to \vec{Q}_2 \to \cdots$
Write these states as a *basis* for a *Hilbert space*.

ii Write evolution operator U : $\quad |\vec{Q}(t+\delta t)\rangle = U(\delta t)|\vec{Q}(t)\rangle$;

Example: $U = \begin{pmatrix} 0 & 0 & \cdots & 1 \\ 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ & & \cdots & \end{pmatrix}$

iii Find operator H such that $\quad U = \exp(-iH\,\delta t)$.

iv Make unitary transformation to *any* other basis. One finds a "quantum mechanical" system, obeying Schrödinger equation
$\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = -iH|\psi(t)\rangle$.

This is nearly quantum mechanics, except:

the expectation value of an operator is not: $\langle\,\mathcal{O}\,\rangle \stackrel{?}{=} \langle\psi|\mathcal{O}|\psi\rangle$,
but:

$$\langle\,\mathcal{O}\,\rangle = \sum_{\vec{Q}} |\langle\vec{Q}|\psi\rangle|^2\,\langle\vec{Q}|\,\mathcal{O}\,|\vec{Q}\rangle = \mathrm{Tr}\,\varrho\,\mathcal{O}\ \ ;\ \ \varrho = \sum_{\vec{Q}} \varrho_{\vec{Q}}|\vec{Q}\rangle\langle\vec{Q}|\ .$$

This means that we have to restrict ourselves to density matrices diagonal in the CA basis.

The wave function first "collapses" before any macroscopic measurement can take place,
since classical states are diagonal in $|\vec{Q}\rangle$.

But this happens automatically – the collapse problem is solved in this framework.

Therefore axiom # v (just a formality):

- *v* Measurement: "classical" states are CA states: planets, people, indicators of detectors. Therefore, at the end of an experiment, all observed quantities are diagonal in $|\vec{Q}\rangle$.

Work was motivated when attempting to formulate the quantum rules of compact universes.

*) SUPERSTRINGS may provide for the best scenario for Cellular Automaton - determinism!

**) But one has to address *NO - GO* theorems !

Can one escape no-go theorems (Bell's inequalities)?

Theorem (Bell):
In any deterministic theory intended to reproduce quantum behavior, (for instance when Einstein-Podolsky-Rosen photons are observed through two spacelike separated filters, $\vec{a}$ and $\vec{b}$), one will have to allow superluminal signals between $\vec{a}$ and $\vec{b}$.

... since we should be allowed to modify the settings $\vec{a}$ and/or $\vec{b}$ any time, at free will...

But there is no "free will" in a deterministic theory (Super-determinism).

Superdeterminism?

Theorem: even so, you cannot avoid Bell's inequalities!

unless you accept "ridiculous fine-tuning", or "conspiracy"

Today's claim: we never need actual signals going backwards in time or faster than light. All we need is non-locally correlated vacuum fluctuations.

Vacuum fluctuations are ubiquitous in QFT vacua.

This does *not* invalidate Bell's theorem completely: the correlations at any given Cauchy surface must "anticipate" what Aice and Bob are going to decide.

This form of superdeterminism requires: *conspiracy*.

"That's disgusting".

But:

is "disgusting" a sound mathematical argument?

The CA is classical, how can it be quantum at the same time?

The CA is a universal computer. To calculate $\varrho(\vec{Q})$ at atomic scales requires $> 10^{4\times19}$ computations. If you could do that, you would obtain fine-tuned results.

Only the CA $\rightarrow$ quantum mapping will produce a QFT. With that, you can employ the Renormalization Group to bridge 19 orders of magnitude.

Causality is not violated:

in QFT, commutators vanish outside the light cone.

This is the CA interpretation of QM. Should we believe it?

Dogma: Leave Bell's (important) inequalities aside; worry about them later.

litmus test:

Find a good example of a CA $\rightarrow$ quantum mapping!

Now introduce the Superstring

Superstring theory appears to be complicated and counter intuitive. Some physicists insist that physics at smaller distance scales will be strange and counter intuitive.

"*stranger even than quantum mechanics*" (D. Gross) .

should we believe this ?

Gell-Mann: the world seems to become more and more complex, *until you reach new understanding.* Then things are simple again.

I now want to present my theory:

Superstring theory is even simpler than classical mechanics!

(because there is not even chaos. . .)

If you understand what String Theory really is ...

this idea is conceptually simple, but mathematically hard . . .

The Superstring can be mapped onto a CA

QFT works with operator fields whose eigenvalues are real numbers
CA works with observables that are discrete, or integers

Theorem:

There are unitary transformations between

- Hilbert spaces spanned by the eigenstates of real number operators

and

- those spanned by pairs of integer valued operators.

And indeed, in some special cases, the integers may evolve *classically* while the real number operators can only evolve *quantum mechanically*.

Real numbers and integers

Imagine that, in contrast to appearances, the real world, at its most fundamental level, were *not* based on real numbers at all. We here consider systems where *only* integers describe what happens at a deeper level. Can one understand *why* our world *appears* to be based on real numbers?

A *mapping* exists of

deterministic physics of a set of $2N$ integers Q_i, P_i	onto	quantum physics on N real observables q_i with N associated momenta p_i

Canonical Variables. Our mapping replaces quantum operator sets p_i and q_i (with usual commutation relations) by sets of universally commuting integers P_i and Q_i.

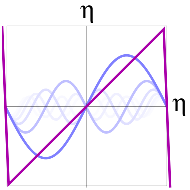
Operators

Even if we have a cellular automaton processing the numbers P_i and Q_i deterministically, we can still introduce *operators*.

Define $\epsilon^{iN\eta}|Q\rangle = |Q-N\rangle$, and Fourier transform the function η on the interval $-\frac{1}{2} < \eta < \frac{1}{2}$: $\epsilon = e^{2\pi} = 535.5$

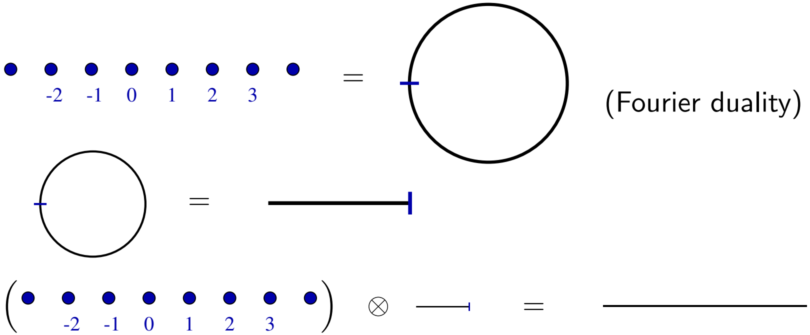
$$\eta = \sum_N \epsilon^{iN\eta} \int_{-\frac{1}{2}}^{\frac{1}{2}} \eta \, \mathrm{d}\eta \, \epsilon^{-iN\eta} = \sum_{N\neq 0} \frac{i(-1)^N}{2\pi N} \epsilon^{iN\eta} ,$$

$$\langle Q_1|\eta|Q_2\rangle = \frac{i}{2\pi}(1-\delta_{Q_1Q_2})\frac{(-1)^{Q_1-Q_2}}{Q_1-Q_2} .$$



$$Q_1|[\eta_Q, Q]|Q_2\rangle = \frac{i}{2\pi}\left(\delta_{Q_1Q_2} - (-1)^{Q_2-Q_1}\right) = \frac{i}{2\pi}(\mathbb{I} - |\psi\rangle\langle\psi|) .$$

$|\psi\rangle$ is an *edge state* : $\eta|\psi\rangle = \delta(\eta - \frac{1}{2})$.



Make real number operators $-\infty < q < \infty$ as follows: $q = Q + \eta_P$

There is a unitary transformation of states from one basis to another: $\langle Q,\, \eta_P | \psi \rangle \;=\; \langle q | \psi \rangle$.

Then transform $\langle Q,\, \eta_P | \psi \rangle = \displaystyle\sum_{P=-\infty}^{\infty} \epsilon^{-iP\eta_P} \langle Q,\, P | \psi \rangle \quad = \; \langle q | \psi \rangle$

Alternatively, find the p basis: $\quad \langle q | p \rangle = \epsilon^{ipq}$

Matrix elements

(mathematical detail is skipped here: make the mapping P-Q symmetric)

In Hilbert space $\{|Q,\, P\rangle\}$, we have

$$q = Q + a_Q \quad , \quad p = P + a_P \ ,$$

$$\langle Q_1, P_1|a_Q|Q_2, P_2\rangle = \frac{(-1)^{P+Q+1}\, iP}{2\pi(P^2+Q^2)}$$

$$\langle Q_1, P_1|a_P|Q_2, P_2\rangle = \frac{(-1)^{P+Q}\, iQ}{2\pi(P^2+Q^2)} \ .$$

From these:

$$\langle Q_1, P_1|\,[q,p]\,|Q_2, P_2\rangle = \frac{i}{2\pi}(-1)^{Q_1-Q_2+P_1-P_2}(\delta_{Q_1\,Q_2}\delta_{P_1\,P_2} - 1)\ .$$

$$= \frac{i}{2\pi}(1 - |\psi_{\rm edge}\rangle\langle\psi_{\rm edge}|), \qquad \text{with} \qquad (\langle Q, P|\psi_{\rm edge}\rangle = (-1)^{Q+P}$$

How does this work in QFT?

It only works in 1 space, 1 time dimension

(exactly what's needed in string theory)

Free massless bosons in $1+1$ dimensions

$$(\partial_x^2 - \partial_t^2)\phi(x,t) = (\partial_x + \partial_t)(\partial_x - \partial_t)\phi(x,t) = 0 \ \rightarrow$$
$$\phi(x,t) = \phi_L(x+t) + \phi_R(x-t) \ .$$

$$[\phi(x,t), p(y,t)] = \tfrac{i}{2\pi}\delta(x-y) \ ; \qquad H = \pi \int \mathrm{d}x (p(x)^2 + (\partial_x\phi)^2) \ .$$

Temporary: put x and t on a lattice.

Discrete p_x, ϕ_x : $\phi_{x,t} \equiv \phi(x,t)$; $[\phi_{x,t},\, p_{x',t}] = \tfrac{i}{2\pi}\delta_{x,x'}$.

We have: $\phi(x,t+a) + \phi(x,t-a) = \phi(x-a,t) + \phi(x+a,t)$.

We would like to *map* this model one-to-one on the cellular automaton:

$$Q(x,t+a) + Q(x,t-a) = Q(x-a,t) + Q(x+a,t) \ ,$$

where Q are integers.

In ordinary QFT, the splitting $\phi(\vec{x},t) \;\to\; Q(x,t)+\eta_P(x,t)$ does not survive the field equations, because "integer part" and "fractional part" are non-linear procedures! It *does* work with real numbers, if the field equations just *interchange* them.
In 1+1 dimensions, we have *left movers* and *right movers*:

$$\phi(x,t)=\phi^L(x+t)+\phi^R(x-t)\;;\;p(x,t)=\tfrac{1}{2}a^L(x+t)+\tfrac{1}{2}a^R(x-t)\;.$$

$$a^L=p+\partial_x\phi\;;\quad a^R=p-\partial_x\phi\;.$$

Now, $$H=\tfrac{1}{2}(p^2+(\partial_x\phi)^2)\;=\;\tfrac{1}{4}({a^L}^2+{a^R}^2)\;,$$

$$[a^L,\,a^R]=0\;;\quad [a^L(x),\,a^L(y)]=\tfrac{i}{\pi}\partial_x\delta(x-y)\quad;$$

Our cellular automaton will be on a lattice: $(x,t)\in\mathbb{Z}$. Therefore, replace commutator by

$$[\phi(x),\,p(y)]=\tfrac{i}{2\pi}\delta_{x,y} \tag{1}$$

$$[a^L(x),\,a^L(y)]=\pm\tfrac{i}{2\pi}\ \text{ if }\ y=x\pm1\quad .$$

Now, we modify the hamiltonian of the continuum in such a way that its action over one time unit is a pure replacement.
Write $H = H^L + H^R$. In *momentum space* :

$$H^L = \tfrac{1}{2}\int_0^{1/2} \mathrm{d}k\, a^L(k)a^L(-k)\, M(k) \quad ; \quad M(\kappa) = \frac{\pi\kappa}{\sin(2\pi\kappa)} \ .$$

This hamiltonian turns $a^L(x)$ into a pure left-mover, and $a^R(x)$ into a right-mover.

Next, we can try to express $a^L(x)$ in terms of the left-movers of the cellular automaton:

$$A^L(x+t) = Q(x, t+1) - Q(x-1, t)$$

Demanding $[a^L(x),\ a^L(y)] = \pm\frac{i}{2\pi}$ if $y = x \pm 1$

Disregarding periodicity:

$$a^L(x) = A^L(x) + \eta^L_A(x-1)\ .$$

More elegant procedure: in η space,

$$a^L(x) = A^L(x) + \frac{\partial}{\partial\eta^L_A(x)}\Big(\phi(\eta^L_A(x+1), \eta^L_A(x)) - \phi(\eta^L_A(x-1), \eta^L_A(x))\Big)$$

This gives the mapping.

ϕ is a conveniently chosen phase factor.

Edge states: if two consecutive $\eta^L_A(x)$ are in the corner: $\pm\frac{1}{2}$

The (super) string is a 1+1 dimensional theory.

Here, the quantized field is the set of (super) string coordinates. They are now replaced by the integer valued left- and right-movers $A^{L,R}(x \pm t)$.

Re-inserting the units gives a surprise: these coordinates form a discrete lattice with lattice length a that is independent of the lattice chosen on the world sheet. Even if you send the world sheet to a continuum, the space-time lattice length a is

$$a = 2\pi\sqrt{\alpha'} \ .$$

Furthermore, as we will see later, the string constant ρ is not freely adjustable.

Lorentz transformations ?

1. The deterministic theory only has manifest $O(D-2,\mathbb{Z})$ invariance (the transverse modes)
2. The quantum theory has manifest $O(D-2,\mathbb{R})$ invariance.
3. In the quantum theory, we impose the constraints to obtain the longitudinal coordinates and the remaining parts of $D(D-1,1,\mathbb{R})$ invariance, as usual.

Note that Lorentz invariance is only needed in the quantum formalism. In terms of the CA variables, Lorentz transformations are now quite complicated operators.

Fermions

A fermionic system can be handled the same way. Assume a Majorana fermionic field ψ_A with $\psi_A = \psi_A^\dagger$, $A = 1, 2$ (or, $A = L, R$). Dirac equation: $(\gamma_+\partial_- + \gamma_-\partial_+)\psi = 0$.

$$\text{One finds that} \quad \psi_A^\mu(x,t) = \begin{pmatrix} \psi_L^\mu(x+t) \\ \psi_R^\mu(x-t) \end{pmatrix} \ .$$

The corresponding classical theory now has Boolean degrees of freedom, $\sigma(x,t) = \pm 1$, obeying the equations:

$$\sigma(x,\ t+1) = \sigma(x-1,\ t)\ \sigma(x-1,\ t)\ \sigma(x,\ t-1)\ .$$

This also splits up into left- and right-movers:

$$\sigma(x,t) = \sigma_L(x+t)\ \sigma_R(x-t)\ .$$

Superstring theories contain $D-2$ independent bosonic fields (coordinates) and $D-2$ Majorana fermion species. All these can be mapped onto deterministic models processing integers classically.

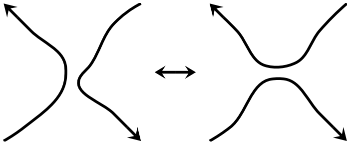
So-far, we only handled strings of infinite length. We need to add: (periodic) boundary conditions, interactions, and *constraints.* The constraints give us the remaining two *longitudunal* coordinates, needed to investigate *Lorentz invariance*.

The constraints only need to be imposed on the quantum side of the theory, as is done in superstrings.
As is standard in Superstring theory, this restricts us to $D=10$.

In Superstring Theory, both bosons and fermions obey gauge conditions and constraints, which should determine $\psi_A^\pm$ in terms of $a_{L,R}^{\mathrm{tr}}$, and so also $\sigma_{L,R}^0$ and $\sigma_{L,R}^{D-1}$ should be determined by the transverse $\sigma_{L,R}^a$

String Interactions

One can write down a *classical and unique* interaction among these classical strings: if two strings hit the same spacetime point Q^μ, two arms are exchanged:



This is also deterministic if the string coupling constant g_s is fixed to $g_s = 1$, and the strings must be oriented.

This generates closed, interacting, oriented strings.
We automatically have exact invariance under $O(D-2, \mathbb{Z})$.

After the mapping to QM in $D-2=8$, the constraints confirm that we have 10-dimensional Lorentz invariance.

Conclusions

Superstring theory is a quantum theory that can be mapped onto a cellular automaton. The automaton puts the system on a space-time lattice with lattice length $a = 2\pi\sqrt{\alpha'}$. Obviously, this is finite.
We do have to caution that the lattice on the *world sheet* was not yet sent to zero, but this seems to be a question of gauge-fixing rather than a physical limit, since we should have conformal invariance on the world sheet.

According to the CA interpretation, the collapse of the wave function and the Born rule are **automatic consequences of the Schrödinger equation itself**. They need not be put in by hand afterwards.

THE END

arXiv: 1204.4926
arXiv: 1205.4107
arXiv: 1207.3612
and to be published.