PAUL SCHERRER INSTITUT

Space IT

Paul Scherrer Institut

SpaceIT

G4 Biasing
Weight correction, energy dependent cross sections, and test with MC toy

Laurent Desorgher

Needed weight corrections for:

- Cross section biasing
- Forced interaction
- Free flight

Treatment of energy dependent cross section in biasing

MC Toy validation of theory

Some definitions

- Probability to have no interaction over length x
  - Process i $P_i^{NI}(x)$
  - All processes $P^{NI}(x) = \prod_i P_i^{NI}(x)$
- *Effective cross section to get the probabilty of occurrence over dx when the particle has reached x*
  - Process i $P_i^{I}(x, dx) = \sigma_i^{eff}(x)\,dx$
  - All process $\sigma^{eff}(x) = \sum_i \sigma_i^{eff}(x)\,dx$
- Probability density function to give the probability to have occurence of processes over [x,x+dx]
  - Process i $pdf_i(x)\,dx = P_i^{NI}(x)\,\sigma_i^{eff}(x)\,dx$
  - All processes $pdf(x)\,dx = P^{NI}(x)\,\sigma^{eff}(x)\,dx$

Non biased and biased process

- <u>Non biased processes</u>

$$P_i^{NI}(x) = \exp(-\sigma_i x) \qquad \sigma_i^{eff}(x) = \sigma_i$$

- *Biased process*

$$P'^{NI}_i(x) \qquad \sigma'^{eff}_i(x)$$

  - Along_step_do_it weight correction factor

$$w_{along} = \frac{P_i^{NI}(x)}{P'^{NI}_i(x)} = \frac{\exp(-\sigma_i x)}{P'^{NI}_i(x)}$$

  - Post_step_do_it weight correction

$$w_{post} = \frac{\sigma_i}{\sigma'^{eff}_i(x)}$$

All processes can be considered independently for correction of weights. Possibility to mix non biased and biased processes (rather) easily!!!

Non biased and biased process

- Non biased processes

$$P_i^{NI}(x)=\exp(-\sigma_i x) \qquad \sigma_i^{eff}(x)=\sigma_i$$

- *Biased process*

$$P'^{NI}_i(x) \qquad \sigma'^{eff}_i(x)$$

  - Along step do it weight correction factor for difference in survival probability

$$w_{along}=\frac{P_i^{NI}(x)}{P'^{NI}_i(x)}=\frac{\exp(-\sigma_i x)}{P'^{NI}_i(x)}$$

  - Post step do it weight correction factor at process occurrence to correct for difference in effective cross section

$$w_{post}=\frac{\sigma_i}{\sigma'^{eff}_i(x)}$$

All processes can be considered independently for correction of weights.
Possibility to mix non biased and biased processes (rather) easily!!!

Cross section biasing

- Change in process cross section

$$\sigma'^{eff}_i(x) = \sigma'_i \qquad P'^{NI}_i(x) = \exp(-\sigma'_i x)$$

- *Along step weight correction factor*

$$w_{along} = \exp(-(\sigma_i - \sigma'_i)x)$$

- *Post step weight correction factor at occurrence of a biased process*

$$w_{post} = \frac{\sigma_i}{\sigma'_i}$$

1D MC toy

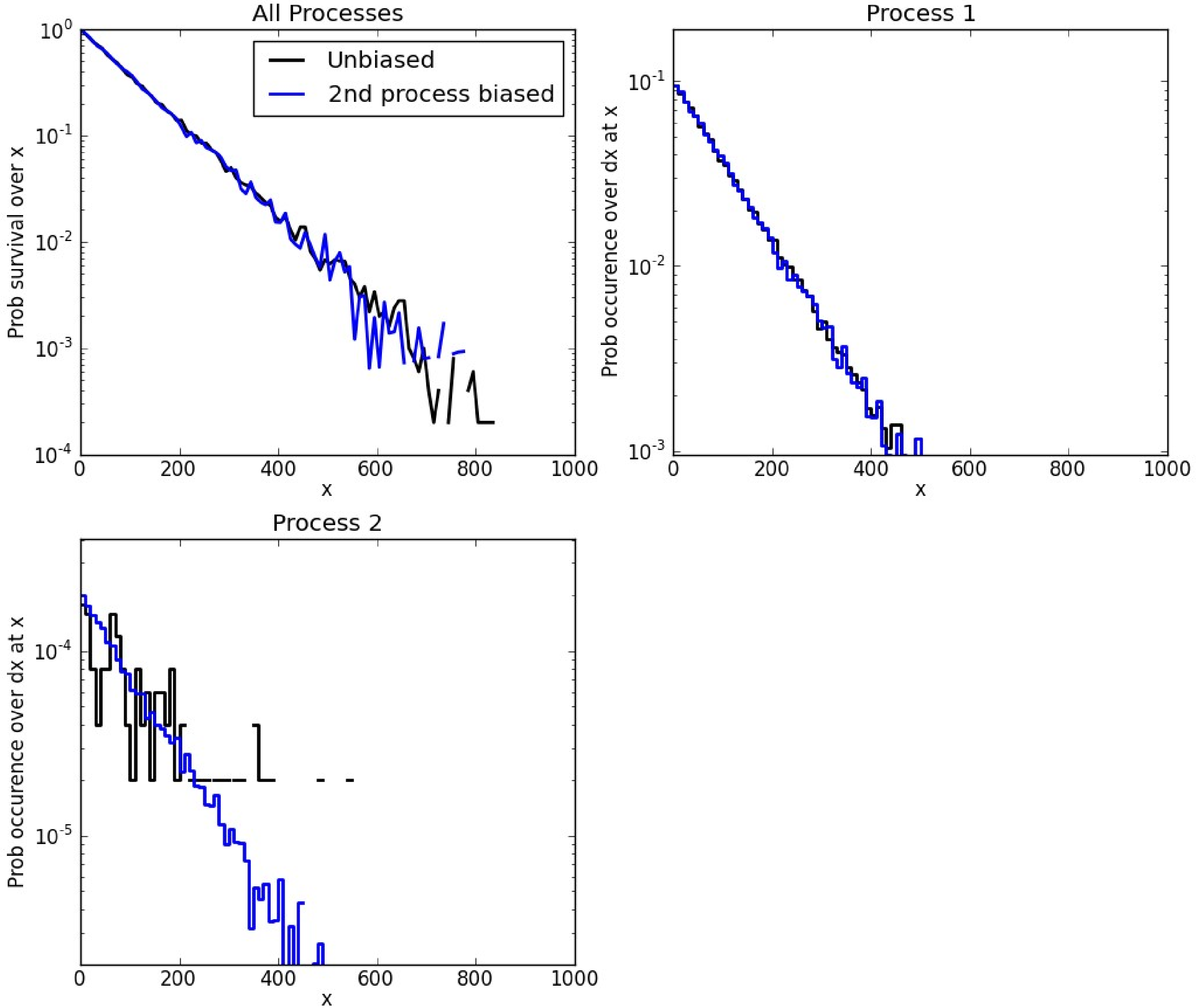
- PYTHON code to test the weight corrections in the different biasing modes

- Take into account different processes with different cross sections (varying or not) and different biasing modes

- Compute :
  - the position of first occurence of a process
  - Compute the weight correction
- Register:
  - (weigthed)probability of occurrence of a process vs position
  - Survival probability vs position

- *Post step weight correction factor at occurrence of a biased process*

Test of cross section biasing



Forced interaction

- Probability to interact over [l1,l1+dx] $$\frac{\exp(-\sigma'_i l_1)\sigma_i' dx}{1-\exp(-\sigma'_i L)}$$

- Probability to interact between [0,l1] $$\frac{1-\exp(-\sigma'_i l_1)}{1-\exp(-\sigma'_i L)}$$

- The probability for a particle to survive over [0,l1]

$$1-\frac{1-\exp(-\sigma'_i l_1)}{1-\exp(-\sigma'_i L)}=\frac{\exp(-\sigma'_i l_1)-\exp(-\sigma'_i L)}{1-\exp(-\sigma'_i L)}$$

- The probability for a particle to survive over [l1,l2]

$$\frac{\exp(-\sigma'_i l_2)-\exp(-\sigma'_i L)}{\exp(-\sigma'_i l_1)-\exp(-\sigma'_i L)}$$

- Effective cross section:

$$\frac{\sigma'_i}{1-\exp(-\sigma'_i(L-l))}$$

Forced interaction over L

- Along step do it weight correction

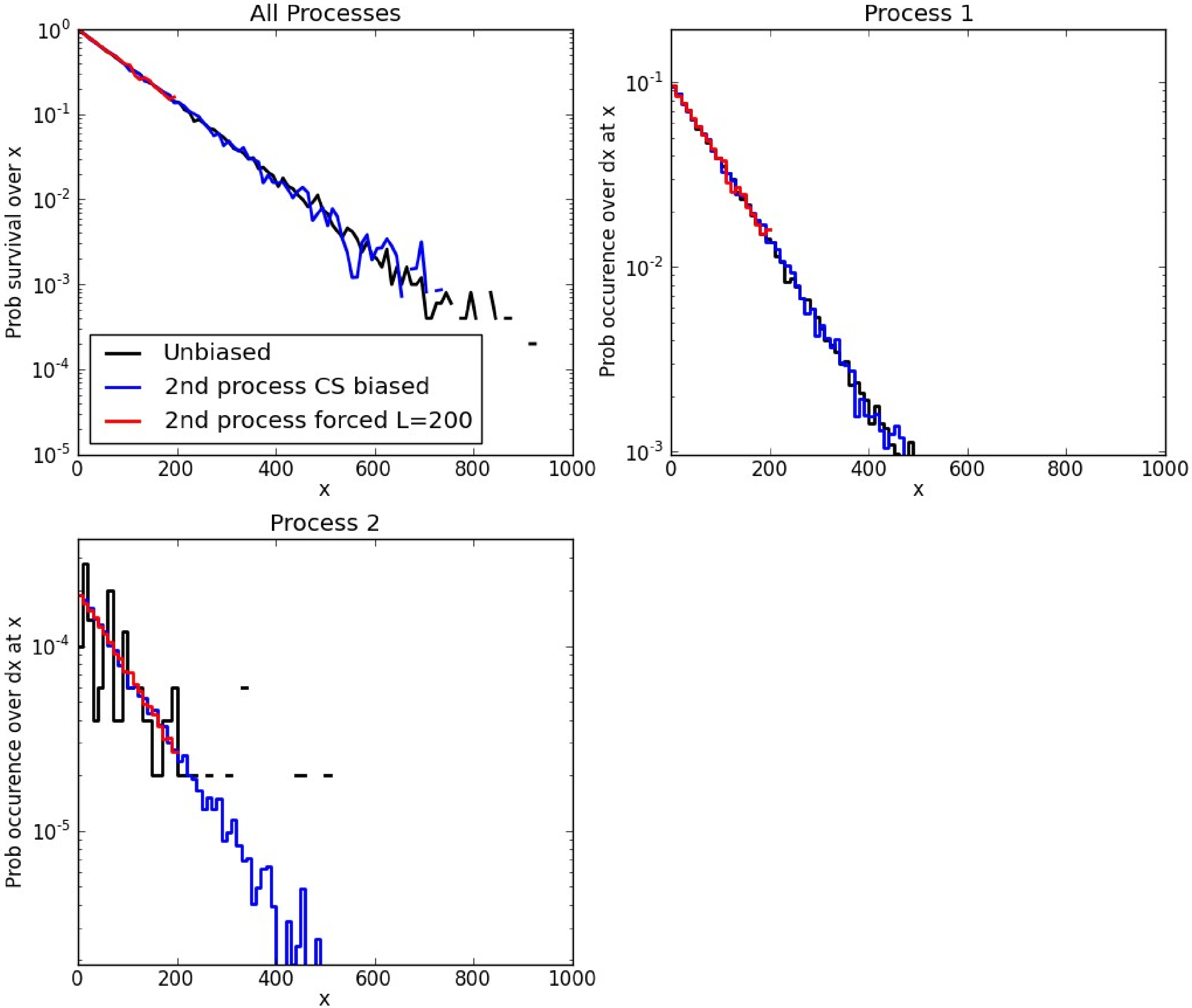
$$\frac{\exp\left(-\sigma_i\left(l_2-l_1\right)\right)\left(\exp\left(-\sigma'_i l_1\right)-\exp\left(-\sigma'_i L\right)\right)}{\exp\left(-\sigma'_i l_2\right)-\exp\left(-\sigma'_i L\right)}$$

- Post step do it weight correction: $\dfrac{\sigma_i\left(1-\exp\left(-\sigma'_i\left(L-l\right)\right)\right)}{\sigma'_i}$

- Sampling of *l*: *random variable x* $\exp\left(-\sigma_i' L\right)\leq x\leq 1$

$$l=-\log\left(x\right)$$

Forced interaction Test



Forced flight, no interaction over l

- Change in process cross section

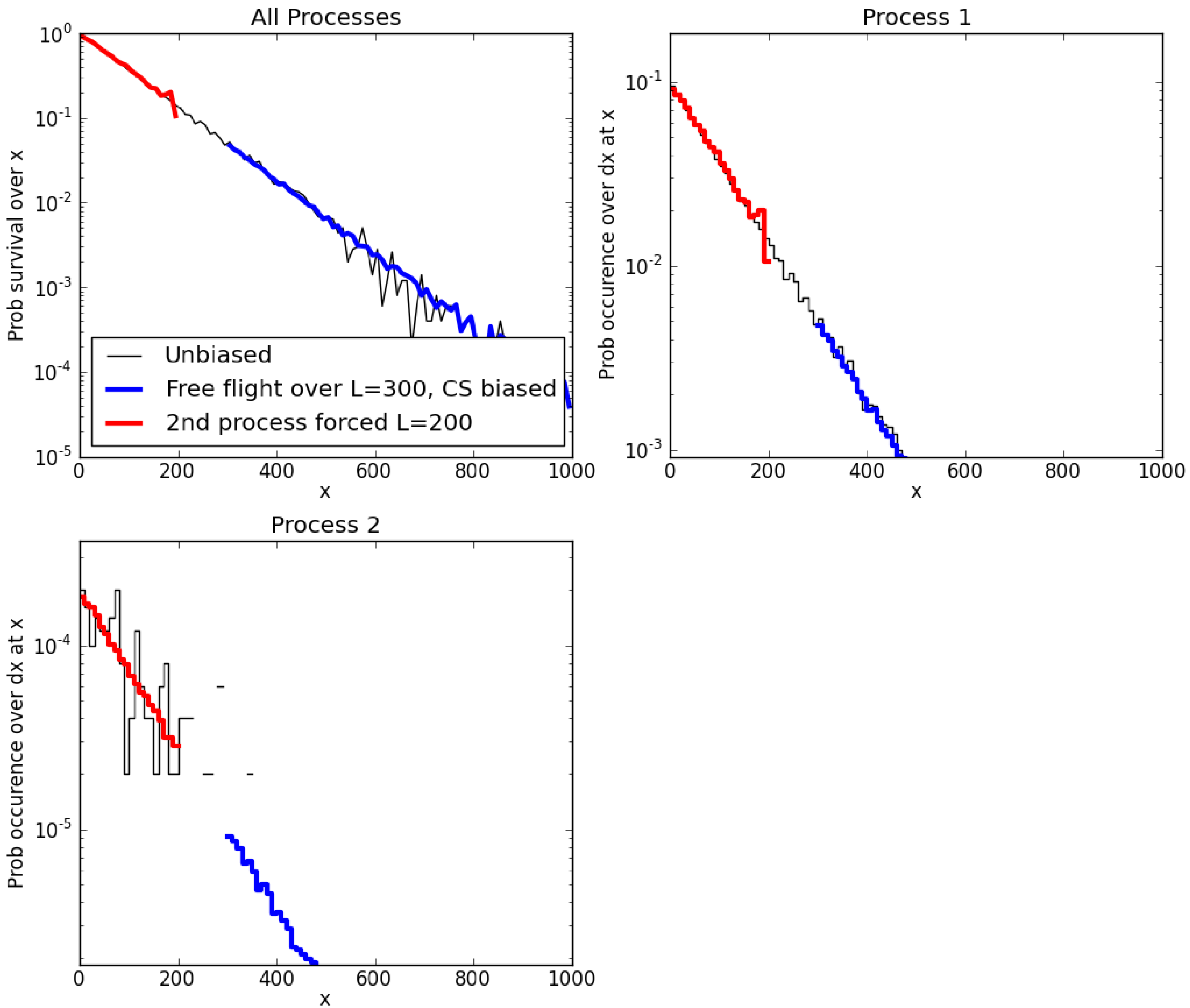
$$\sigma'^{eff}_i(x) = 0 \qquad P'^{NI}_i(x) = 1$$

- *Along step weight correction factor*

$$w_{along} = \exp(-\sigma_i x)$$

- *No need of post step correction because no process occurs*

Forced Flight, Forced interaction



Varying Cross sections

- Cross sections for charged particles are varying over a tracking step
- A technical note were this issue is adressed (and we hope solved) for biasing in Geant4 has been produced
- Varying cross section can be taken into account by a rejection test at the occurrence of a process for CS biasing and for forced interaction as it is already applied in EM models for Analogue MC.
- For the flight free biasing this rejection method can not be used as the process by definition will not occur and an extra weight correction is needed

Rejection test for varying CS

- The cross section used to sample the next occurrence of process *i* is taken as a maximum cross section over the next step σ^{i}_{max}
- When a process occurs at x the following rejection test is applied

$$ran(0,1) \geq \frac{\sigma^{i}(x)}{\sigma^{i}_{max}}$$

- It consist into correcting locally the cross section by the factor $\sigma^{i}(x)/\sigma^{i}_{max}$
- As it is done at any location where the process occurs it is integrated all over a step and it induces a natural correction of the survival probability by the expected factor

$$e^{\int \sigma^{i}(x) - \sigma^{i}_{max}\, dx}$$

- This can be proved also analytically and the proof is given in the technical note
- This rejection technique can be applied to CS biasing and forced interaction as in the analogue mode

Varying cross sections for free flight

- By definition a process does not occur over the step L when the free flight is simulated

- Therefore the rejection technique can not be applied for the "free flight"

- An additional weight correction factor $e^{\int \sigma^i(x)-\sigma^i_{max}\,dx}$ is needed
  –
- Instead of computing analytically the weight correction factor it can be Monte Carlo integrated

- The Monte Carlo integration consists in:
  - Determining randomly the positions $l_1, l_2, l_3, \ldots l_j$, where a process would have been rejected in analogue MC before reaching the position L
  - For each position where the process is considered as rejected add a weight term proportional to

$$(\sigma^i_{max})^j L(L-l_1)(L-l_2)\ldots(L-l_{j-1})$$

Test of varying cross section