# PRECISION HIGGS THEORY

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Northwestern U. & ANL

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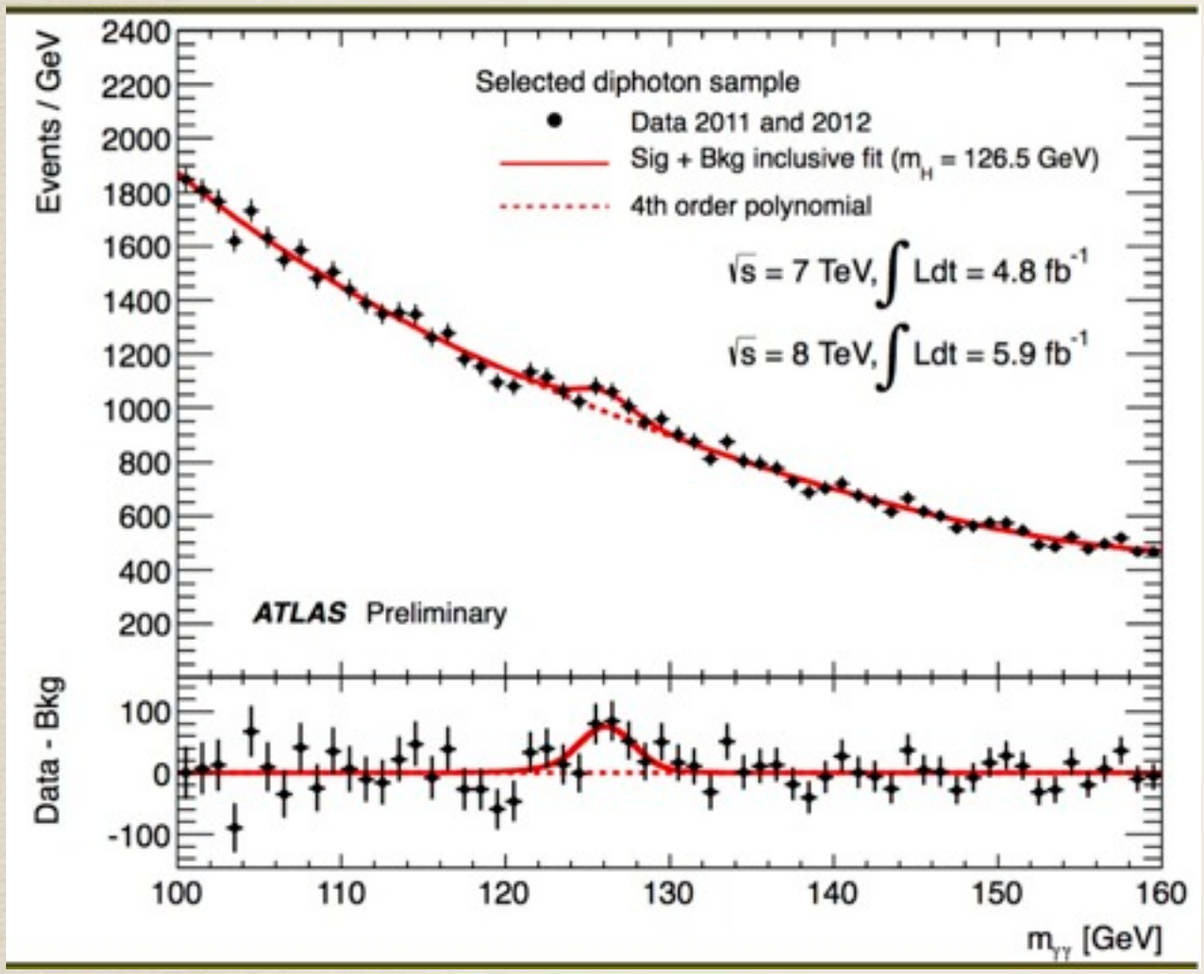
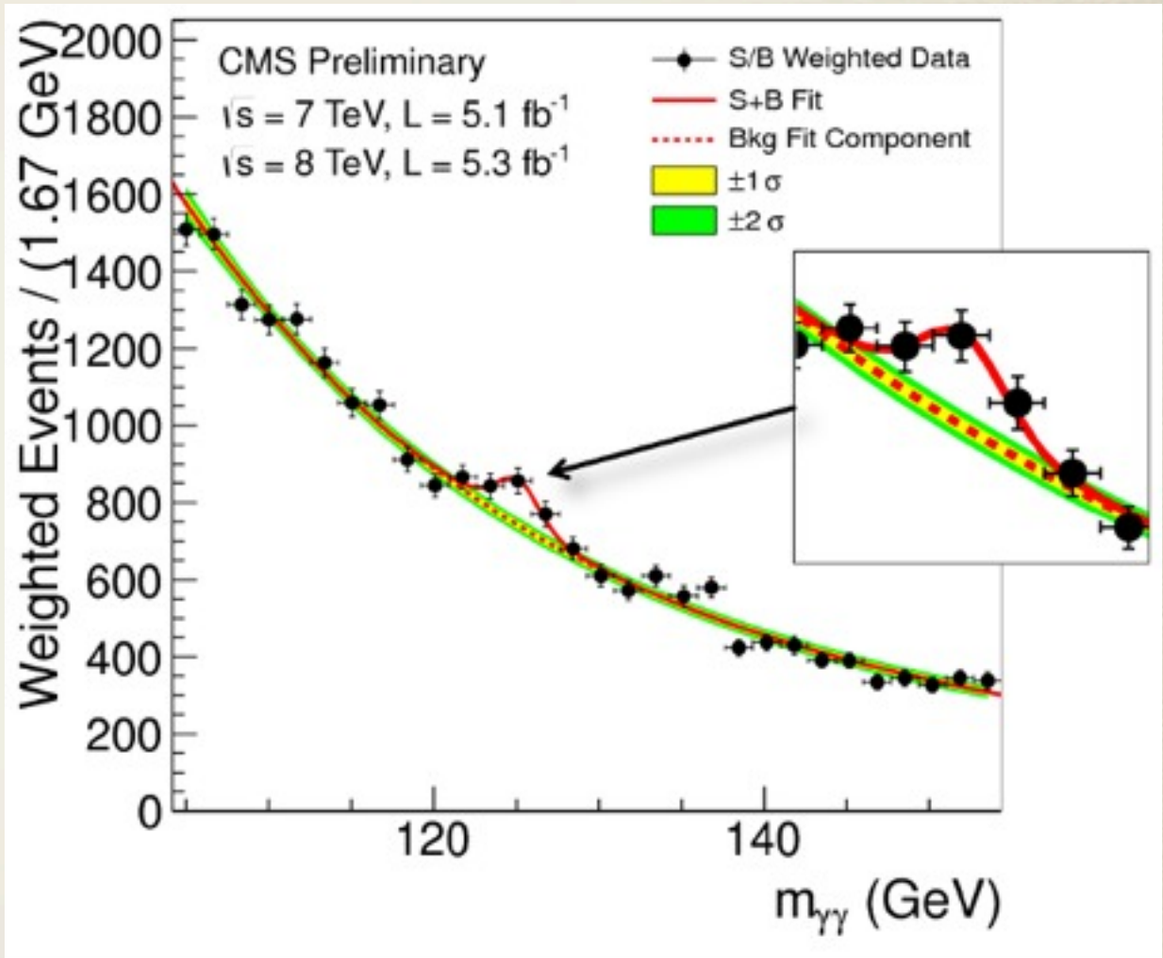
# Higgs Discovery?

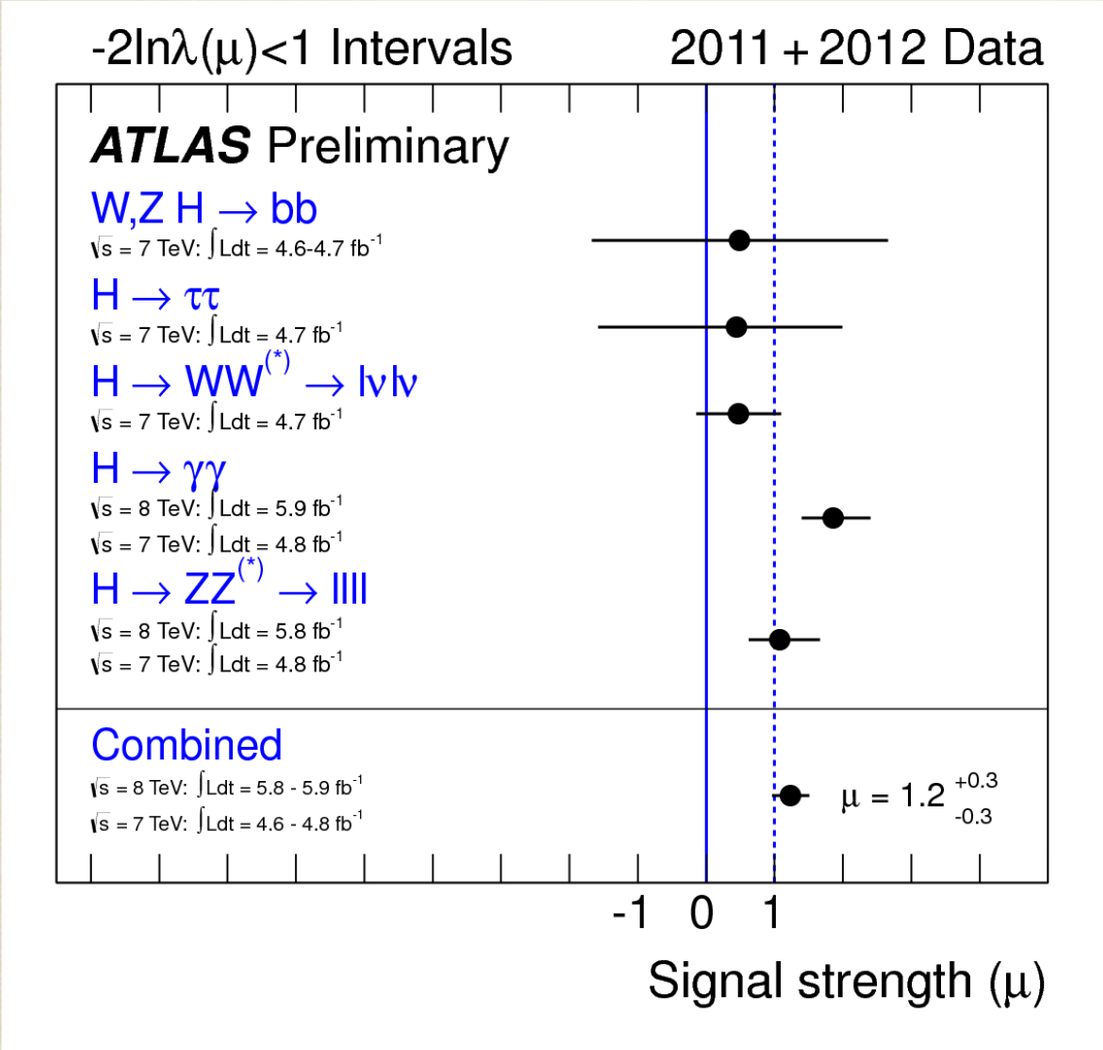


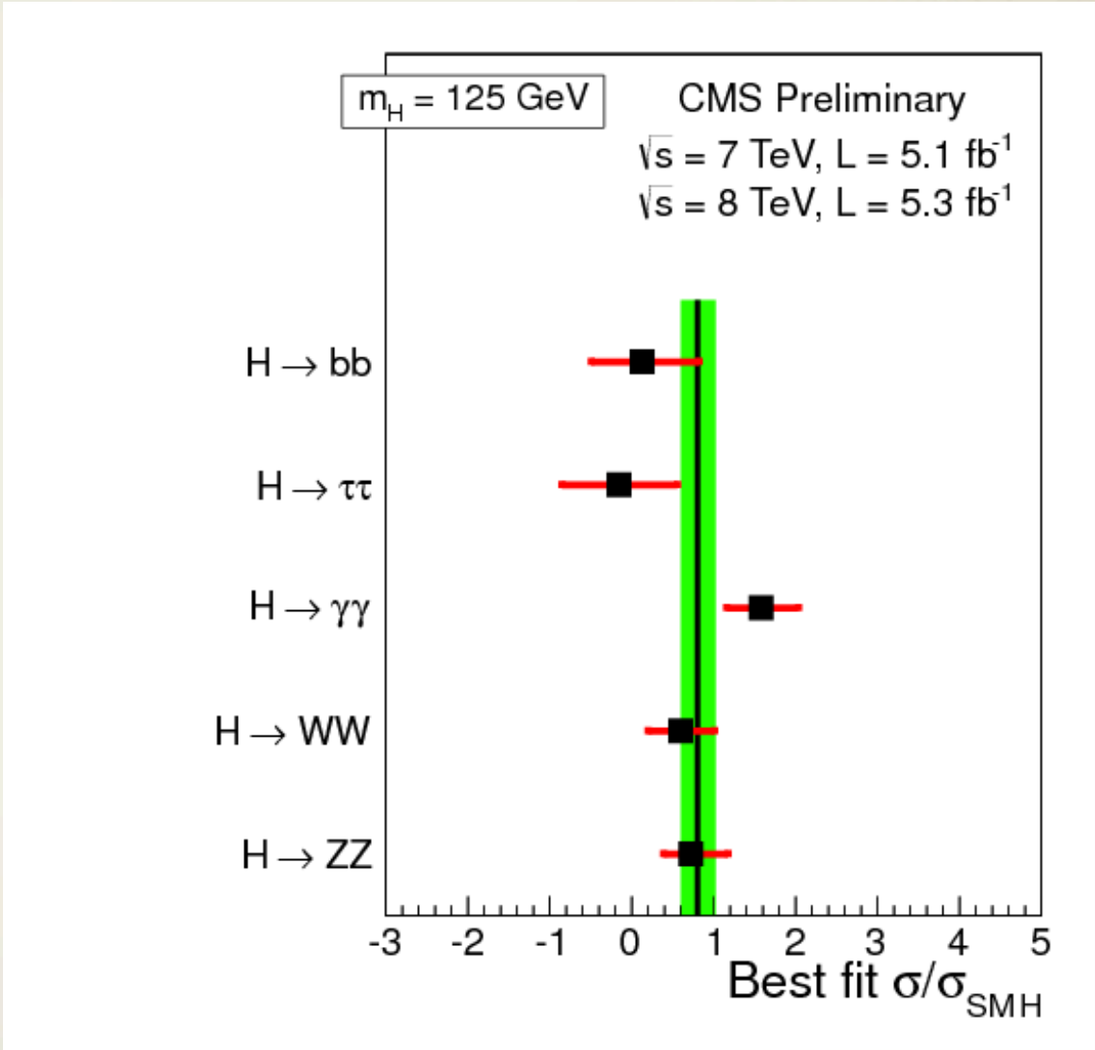




Combined significance roughly 5σ for each experiment

# What we know so far

- Gross properties of the new state roughly indicate SM-like couplings

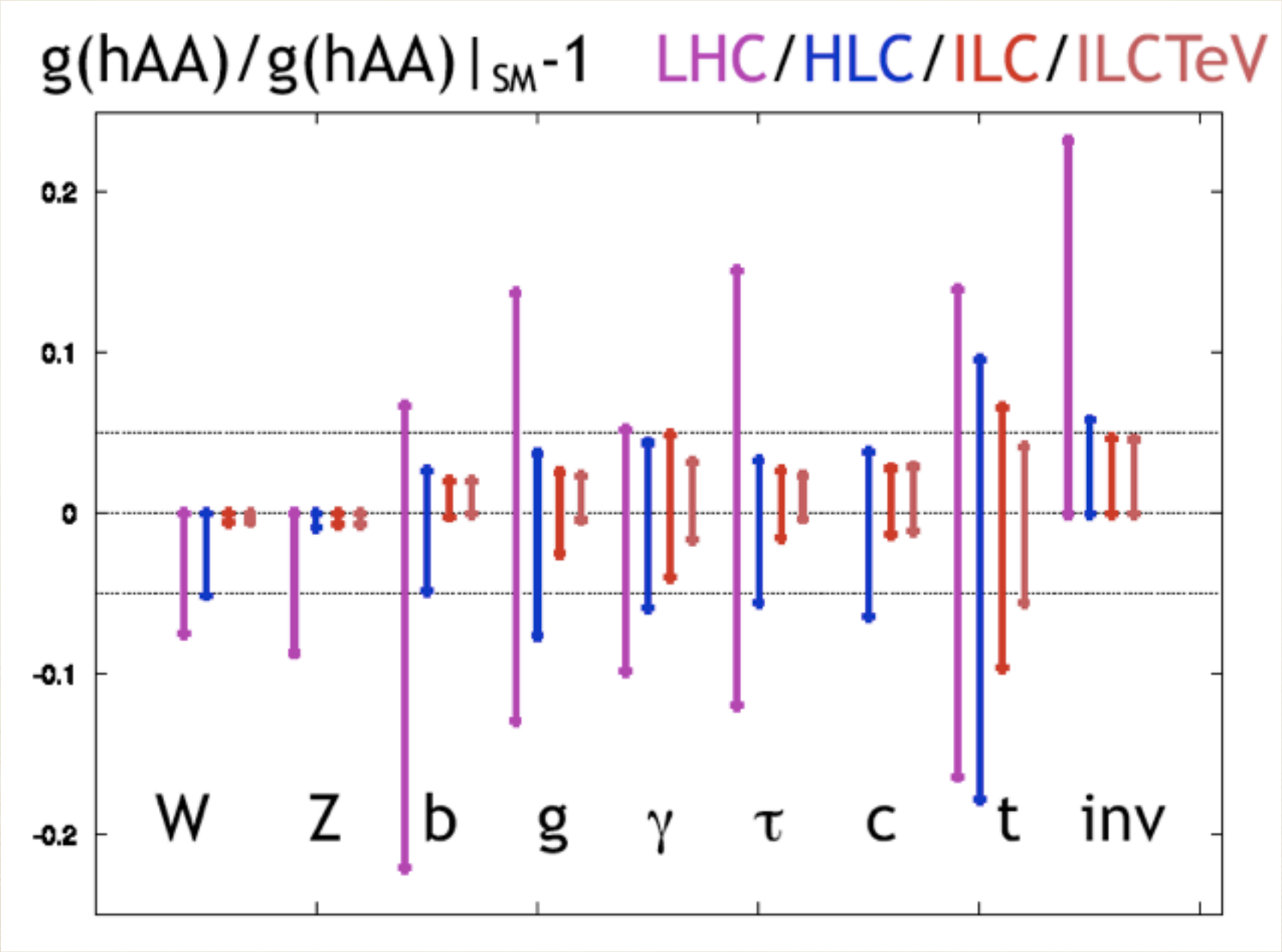



Spin, parity of the state unknown as of yet

# The future

- Expect 3-4 times more data by the end of the year
- This discovery motivates future experiments to definitively determine the properties of this state




Peskin, 1207.2516

# The future

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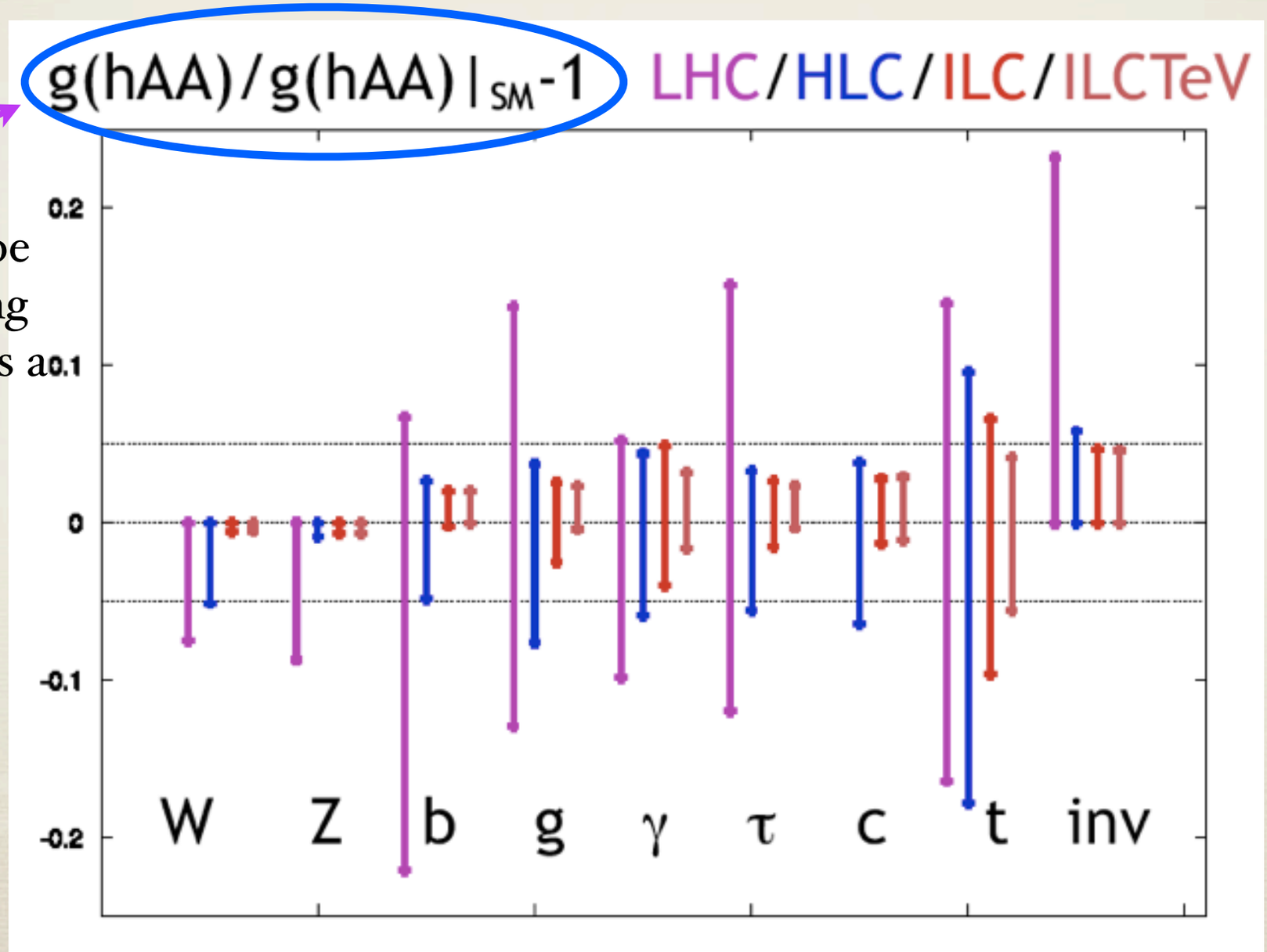
All results will be interpreted using the SM Higgs as a benchmark




Peskin, 1207.2516

# Outline

- The goals of these lectures are:

  (1) Introduce you to the phenomenology of the SM Higgs
  (2) Provide some sense of how precisely we can calculate Higgs properties, and mention the current tricky issues
  (3) Introduce you to calculational tools that should be useful both within and beyond the SM
- Lightning review: the SM Higgs mechanism  Howie's lectures
- Prehistory (and maybe future): searches at $e^+e^-$ colliders
- A phenomenological profile: decays of the Higgs boson
- LHC phenomenology of the SM Higgs
- Step-by-step calculation of the gluon-fusion process
- Current issues in Higgs physics

# Problems with mass

- The Lagrangian of the SM:

$$\mathcal{L}_{gauge+ferm} = -\frac{1}{4}\overbrace{B_{\mu\nu}B^{\mu\nu}}^{U(1)_Y} - \frac{1}{4}\overbrace{W^a_{\mu\nu}W^{\mu\nu}_a}^{SU(2)_L} - \frac{1}{4}\overbrace{G^a_{\mu\nu}G^{\mu\nu}_a}^{SU(3)_C}$$
$$+ \underbrace{\sum_f i\bar{f}\,\not{D} f}_{f=Q_L,u_R,d_R,L_L,e_R}$$

- We know the $W^{\pm}$, Z bosons have mass, but this is not allowed by gauge symmetry

$$\mathcal{L}^{SU(2)}_{mass} = \frac{1}{2}m^2 W^a_\mu W^\mu_a \Rightarrow \Delta\mathcal{L}^{SU(2)}_{mass} \neq 0 \text{ under G.T.}$$

- Similarly, fermion mass terms are not allowed by $SU(2)_L$ or $U(1)_Y$

$$\mathcal{L}^{ferm}_{mass} = -m\,\underbrace{\left[\bar{f}_R f_L + \bar{f}_L f_R\right]}_{\text{transforms as } SU(2)_L \text{ doublet, } \sum Y \neq 0}$$

# Spontaneous symmetry breaking

- The solution: Lagrangian is symmetric, ground state isn't ⇒ *spontaneous symmetry breaking*
- Complex scalar transforming as (1,2,1/2) under $SU(3)_C \times SU(2)_L \times U(1)_Y$

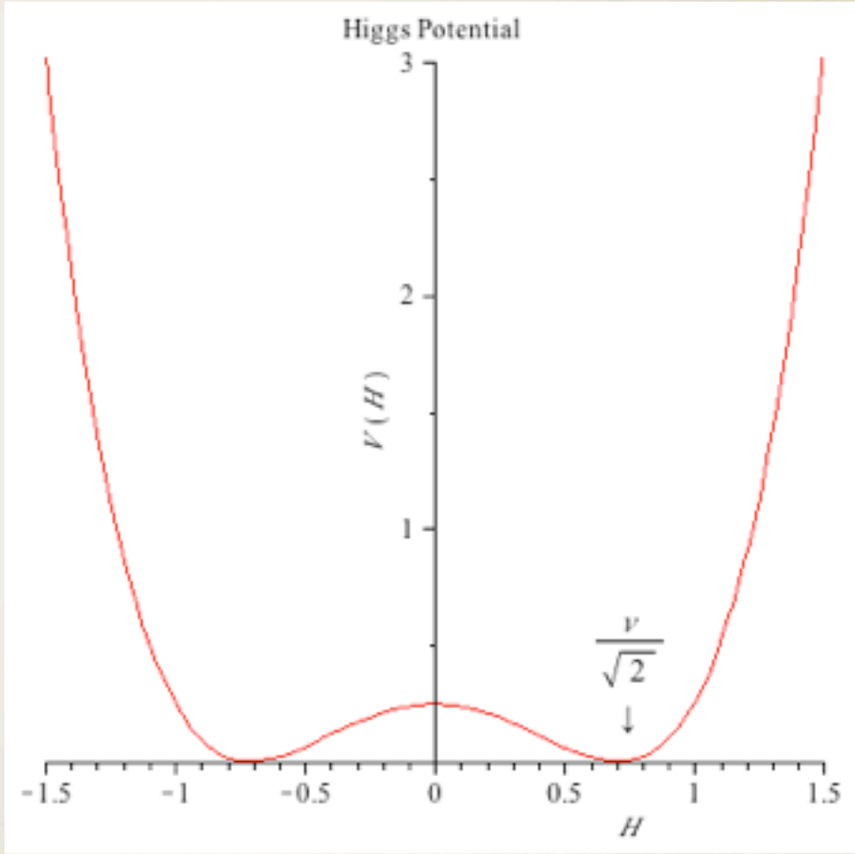
$$\mathcal{L}_{Higgs} = (D_\mu H)^\dagger D^\mu H - \overbrace{\lambda \left( H^\dagger H - \frac{v^2}{2} \right)^2}^{V(H)}$$

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$$D^\mu = \partial^\mu - igW_a^\mu \frac{\sigma^a}{2} - ig' B^\mu \frac{1}{2}$$

Vacuum expectation value: $< H > = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

Expand around vev: $H = \begin{pmatrix} \phi^+ \\ \frac{v + h + i\chi}{\sqrt{2}} \end{pmatrix}$




($\varphi^+$,$\chi$ can be removed by G.T., set to zero)

# The Higgs mechanism

- Work out the kinetic part of Higgs Lagrangian

$$D_\mu H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} - \frac{i}{2}\left[\frac{v+h}{\sqrt{2}}\right]\begin{pmatrix} \sqrt{2}gW_\mu^+ \\ \sqrt{g^2+g'^2}Z_\mu \end{pmatrix}$$

$$(D^\mu H)^\dagger D_\mu H = \frac{1}{2}\partial_\mu h \partial^\mu h + \left(1+\frac{h}{v}\right)^2 \left( \underbrace{\frac{g^2v^2}{4}}_{M_W^2} W^{\mu+}W_\mu^- + \frac{1}{2}\underbrace{\frac{(g^2+g'^2)v^2}{4}}_{M_Z^2} Z_\mu Z^\mu \right)$$

$$Z_\mu = c_W W_\mu^3 - s_W B_\mu, A_\mu = s_W W_\mu^3 + c_W B_\mu, \quad W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}$$

$$c_W = \frac{g}{\sqrt{g^2+g'^2}}, \quad s_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

- $W^\pm$, Z acquire mass by "eating" $\varphi^+, \chi$

# Fermion masses

- Yukawa interactions with Higgs doublets give fermions mass

$$\begin{aligned}\mathcal{L}_{Yuk} &= -\lambda_d \bar{Q}_L H d_R - \lambda_u \bar{Q}_L (i\sigma_2 H^*) u_R - \lambda_e \bar{L}_L H e_R + \text{h.c.} \\ &\Rightarrow -\left(1+\frac{h}{v}\right) \sum_{f=u,d,e} m_f \bar{f} f \quad \text{with} \quad m_f = \frac{\lambda_f v}{\sqrt{2}}\end{aligned}$$

  (matrix in generation space, implicitly diagonalized at price of $V_{CKM}$ in charged currents)

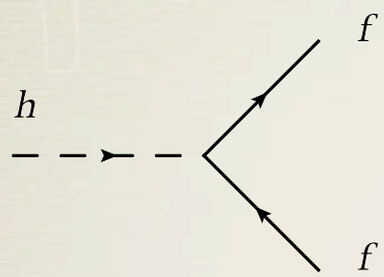
- Sum of all pieces so far give the SM Lagrangian:

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge+ferm} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk}$$

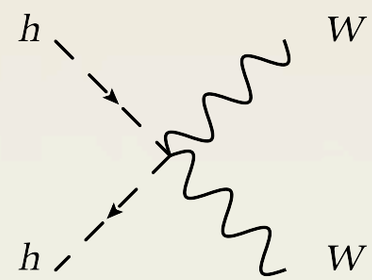
- The single Higgs doublet is just the simplest way to break $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$; EWSB could be more intricate.
  But this is the benchmark to compare other theories against.

# Feynman rules

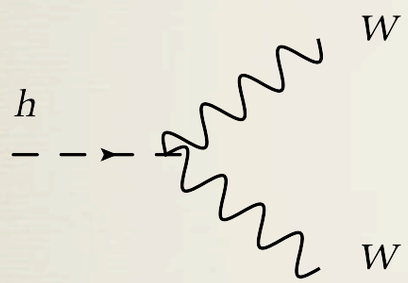
- Work out the experimental predictions with Feynman rules:




$$= -i\frac{m_f}{v}$$




$$= 2i\frac{M_W^2}{v^2}g_{\mu\nu}$$

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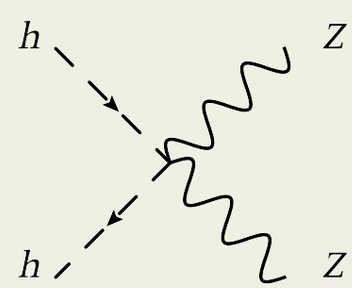


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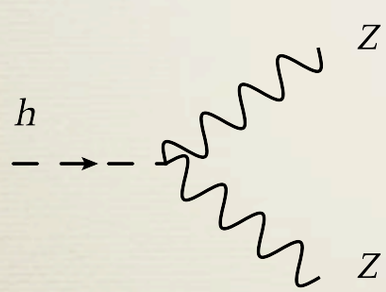



$$= 2i\frac{M_Z^2}{v}g_{\mu\nu}$$

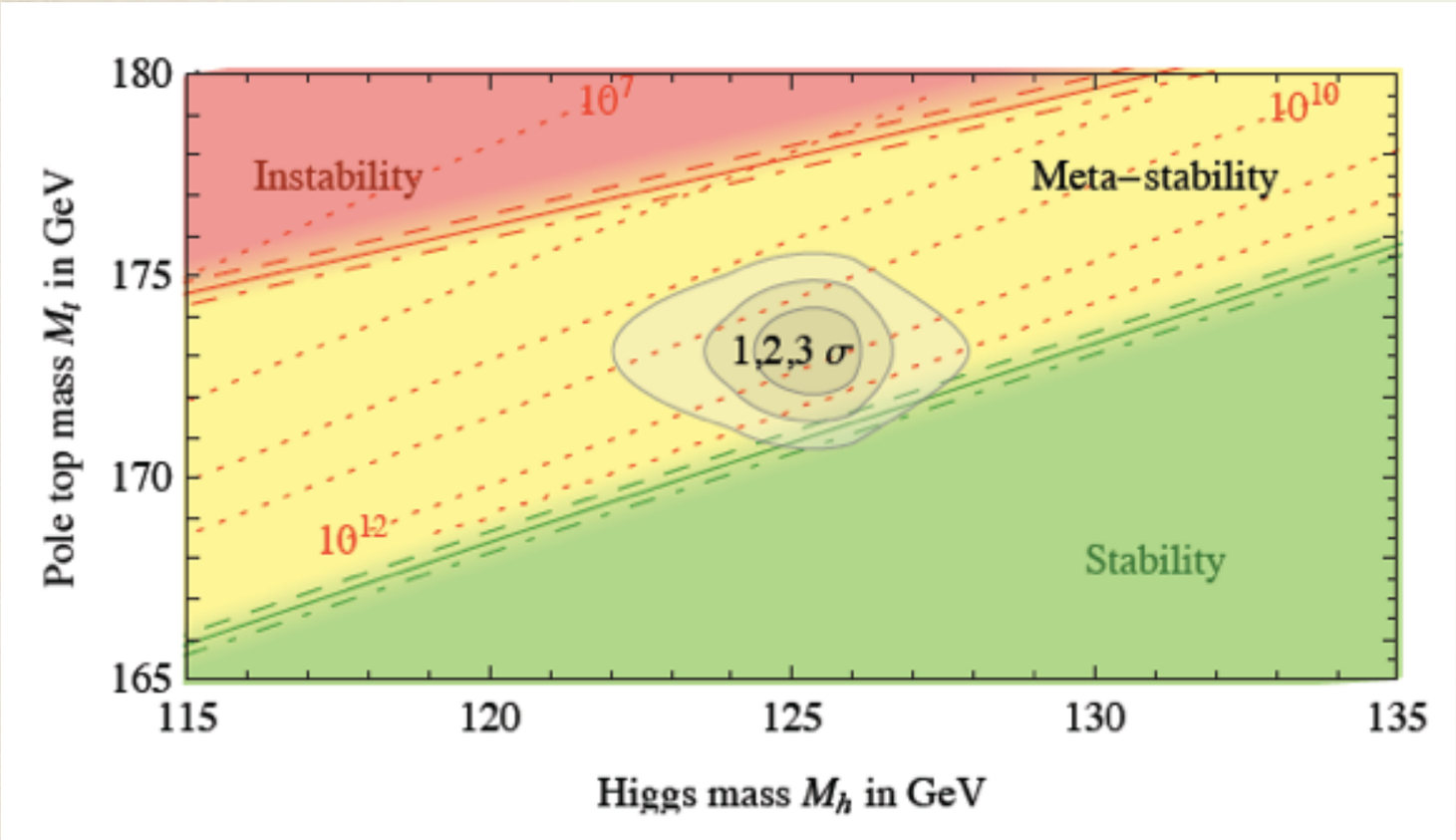
From muon decay,
$v^2=1/(G_F\sqrt{2}) \Rightarrow v\approx 246$ GeV

- Only scalars with vevs have linear HVV couplings

Test the consequences of the Higgs mechanism

# Where do we look?

- Only unknown parameter in the theory: $M_H$
- Consistency of the theory gives us some clues where to look
  - Perturbative unitarity of WW scattering
  - Landau pole of $\lambda h^4$ coupling
  - Stability of the vacuum



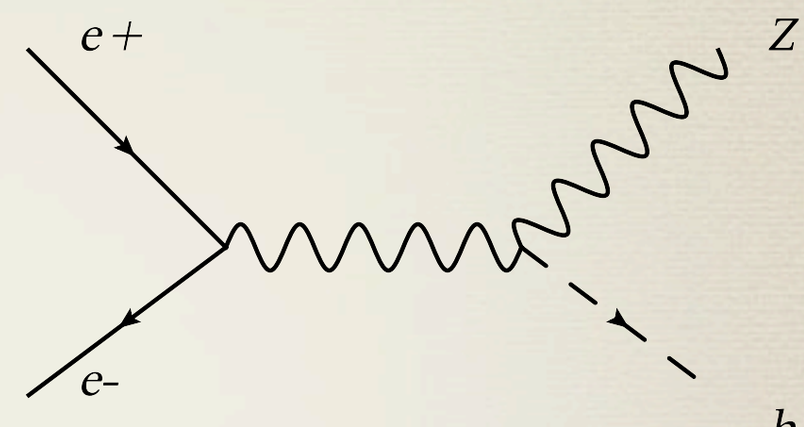

More in Howie's lectures

Degrassi et al., 1205.6497

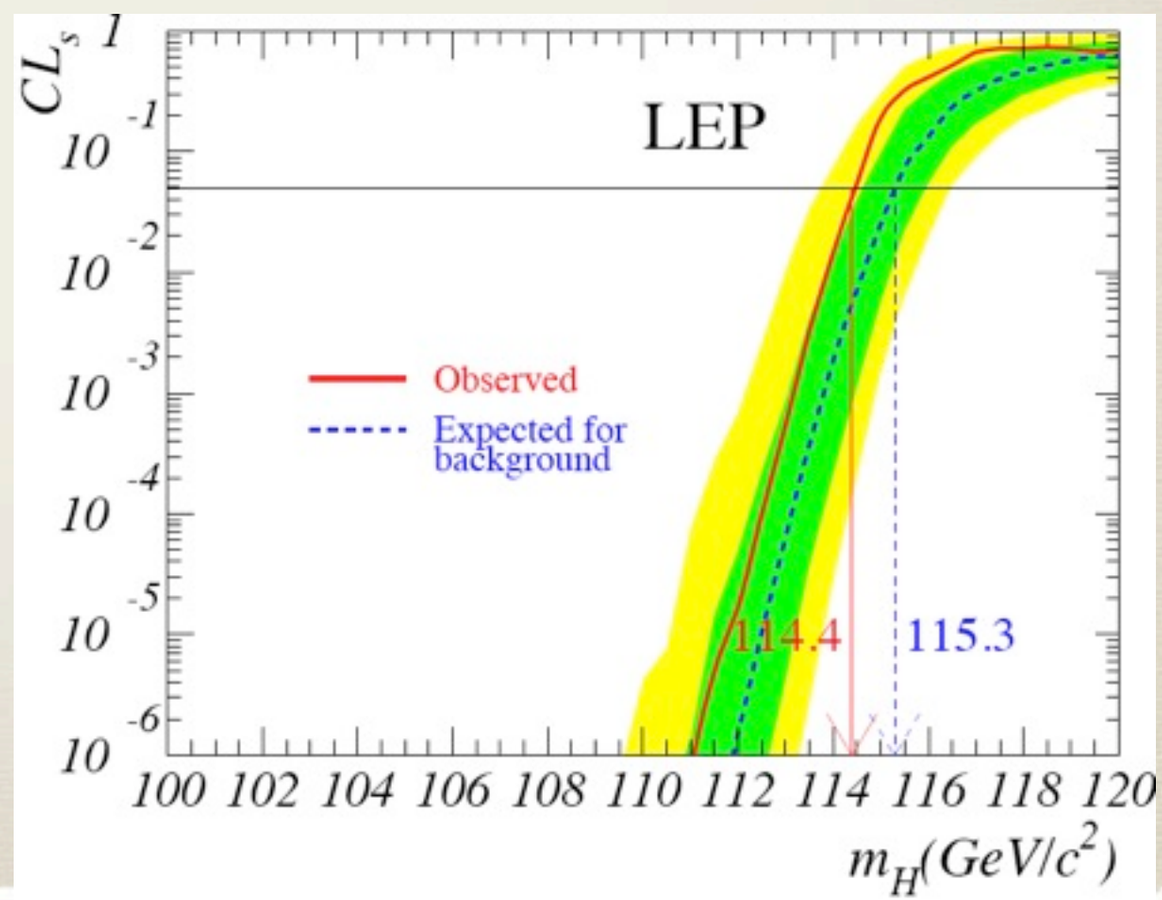
# Searches at $e^+e^-$ colliders

# Direct searches at LEP

- LEP2 ran until 2001 at energies reaching $\sqrt{s} \leq 209$ GeV
- Dominant production process: $e^+e^- \rightarrow HZ$
- SM analysis utilizes the following channels:
  - $h \rightarrow bb$, $Z \rightarrow qq$
  - $h \rightarrow bb$, $Z \rightarrow \nu\nu$
  - $h \rightarrow bb$, $Z \rightarrow ll$ ($l=e,\mu$)
  - $h \rightarrow bb$, $Z \rightarrow \tau\tau$
  - $h \rightarrow \tau\tau$, $Z \rightarrow qq$

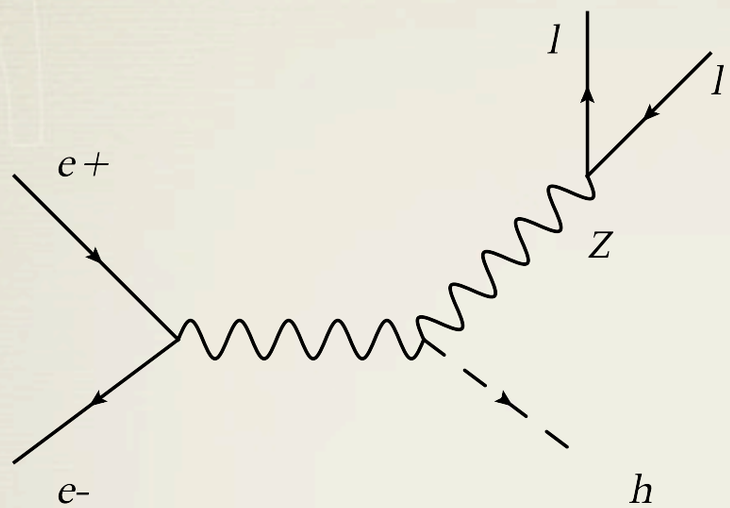


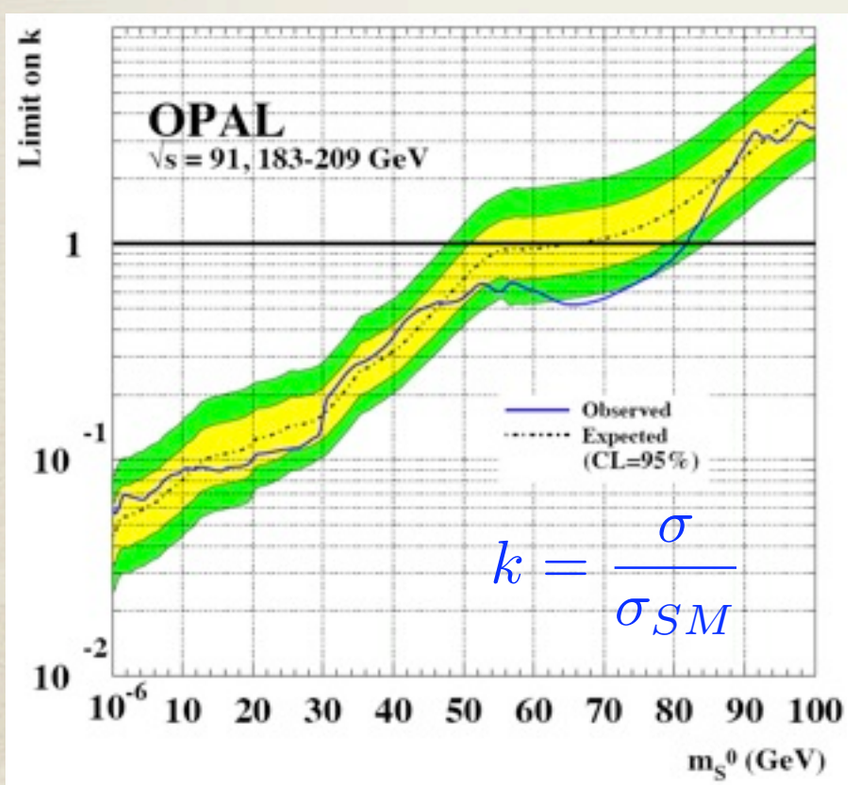

**$M_H > 114.4$ GeV**



# Model-independent search

- This is optimized for SM decays, any way to remove this bias?




Measure two leptons in final state, demand they reconstruct to Z mass

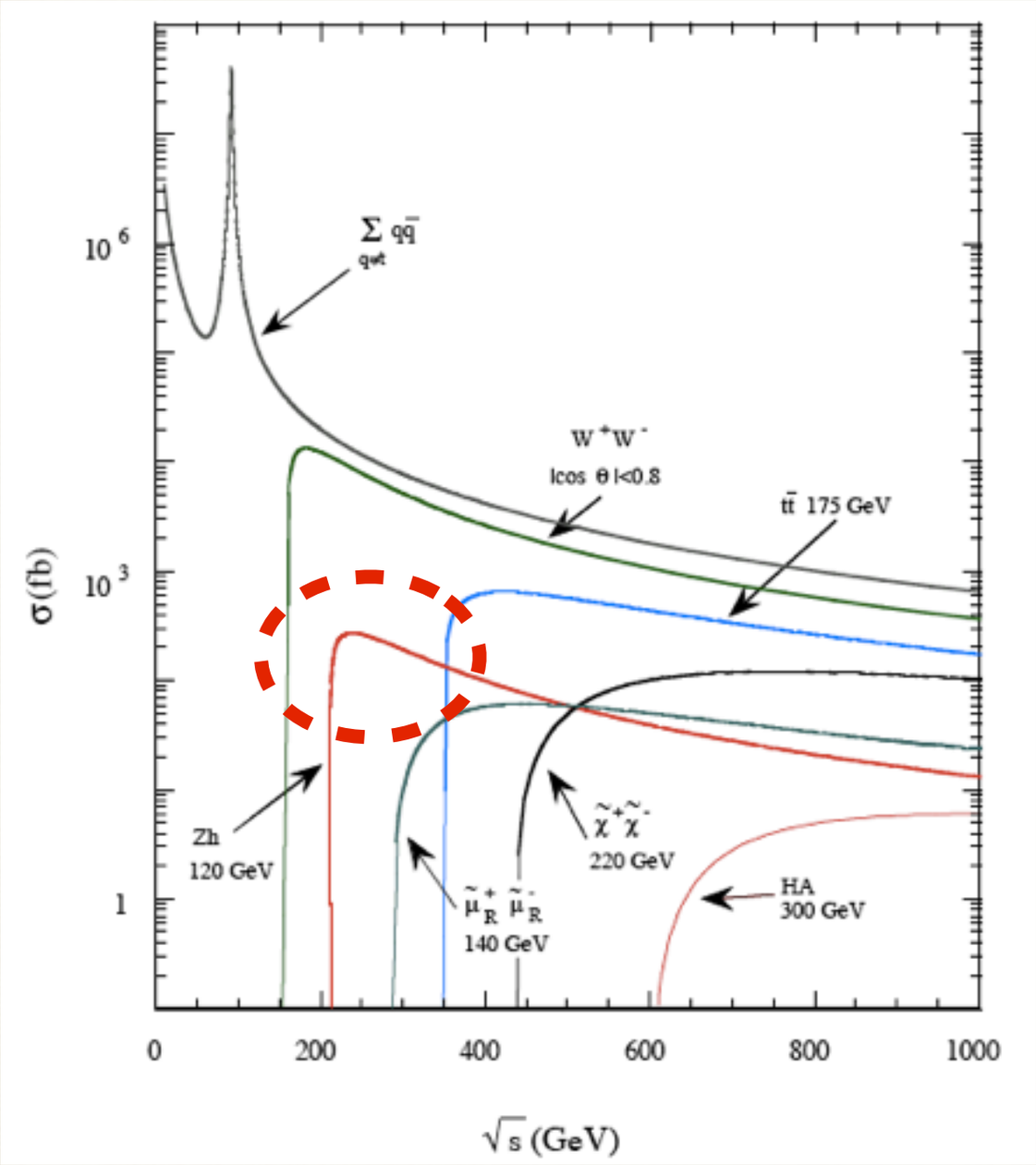
$$
\begin{aligned}
p_{e^+} + p_{e^-} &= p_{l^+} + p_{l^-} + p_X \\
&= p_{ll}^{rec} + p_X \\
\Rightarrow M_X^2 &= s - 2E_{ll} + M_{ll}^2 \\
\text{Predicted peak} &: M_X^2 = M_H^2
\end{aligned}
$$

Limits hold for *any* decay mode

Many other searches designed for specific models

# Future $e^+e^-$ possibilities

- With the mass known, can investigate future possibilities




Build a machine (LEP3) with CM energy ~240 GeV, near maximum of Zh cross section

http://indico.cern.ch/conferenceDisplay.py?confId=193791

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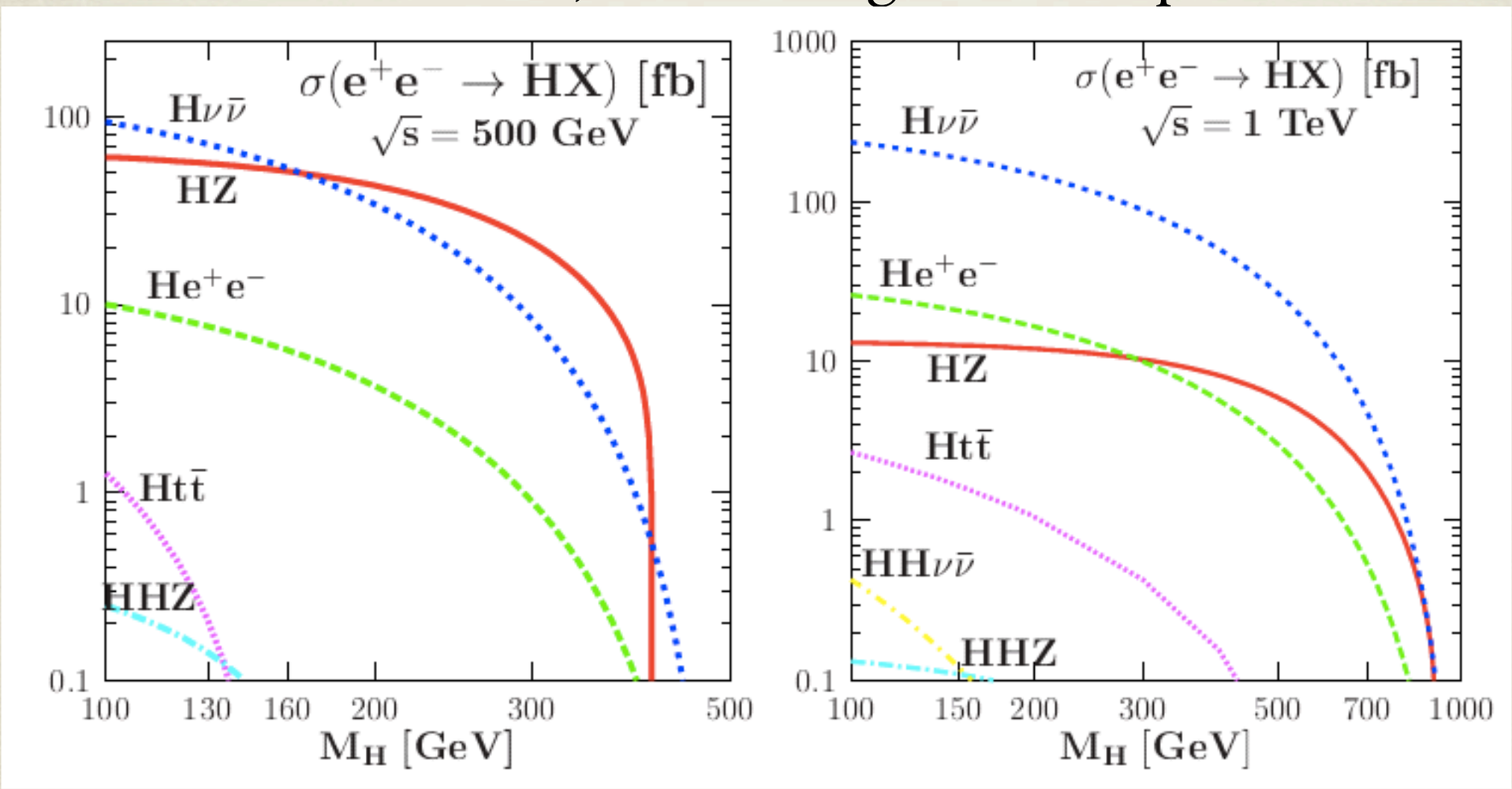



- Access to additional production modes with a 500 GeV, 1 TeV ILC

Useful reference: 0709.1893

# Electroweak precision

- Can experimentally probe properties of the Higgs indirectly
- LEP+SLC: millions of $e^+e^- \to Z \to f\bar{f}$, high-precision measurements of SM electroweak parameters; CDF+D0: $M_W$ measurement $\Rightarrow$ effect of Higgs?
- Compare predictions of SM to data

Useful references:
PDG review by Erler & Langacker
TASI 1990 lectures by Jegerlehner
TASI 2004 lectures by J. Wells

# The EW global fit

- Basic idea in renormalizable theory: fix most precisely known quantities, calculate others in terms of them
- Typical choice: $G_F$, $M_Z$, $\alpha$; also need $M_H$, $m_f$
- Renormalization scheme: for example, on-shell takes $s_W^2=1-(M_W/M_Z)^2$

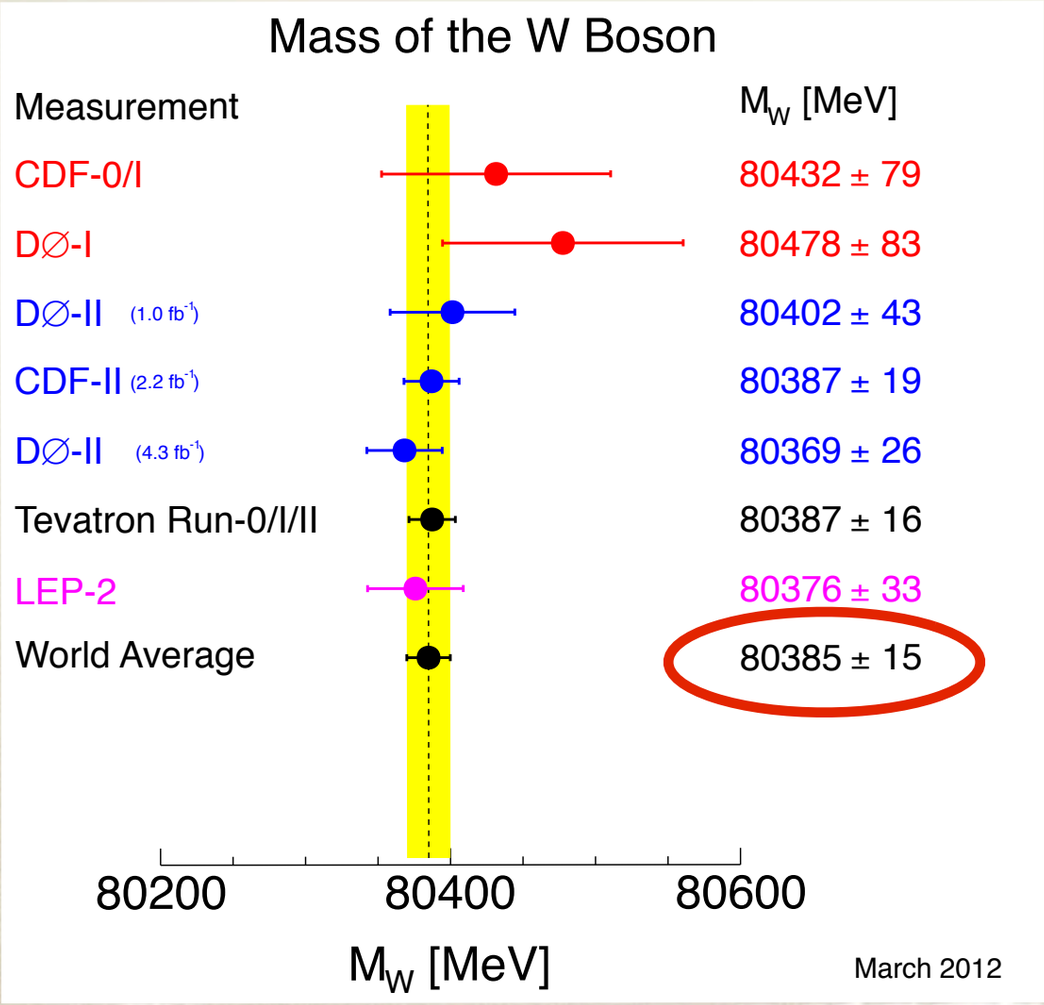

$$\Delta\chi^2(G_F, M_H, \ldots) = \sum_j \frac{(\mathcal{O}_j^{exp} - \mathcal{O}_j^{th}(G_F, M_H, \ldots))^2}{\Delta\mathcal{O}_j^2}$$

- Only unknown in SM is $M_H$; use statistical tests to determine whether a given $M_H$ value is allowed

# The Standard Model at 1-loop

- Let's calculate the W mass at tree-level. Muon decay defines $G_F$, solve:

$$M_W^2 = \frac{M_Z^2}{2}\left\{1 + \left[1 - \frac{2\sqrt{2}\alpha}{G_F M_Z^2}\right]^{1/2}\right\} \approx 80.94\,\mathrm{GeV}$$




- Extraordinary experimental precision necessitates 1-loop study of SM... thankfully, otherwise we wouldn't get any information on the Higgs from this analysis

Exercise: work through approximate calculation of $M_W$ in Appendix I

# The blue-band plot

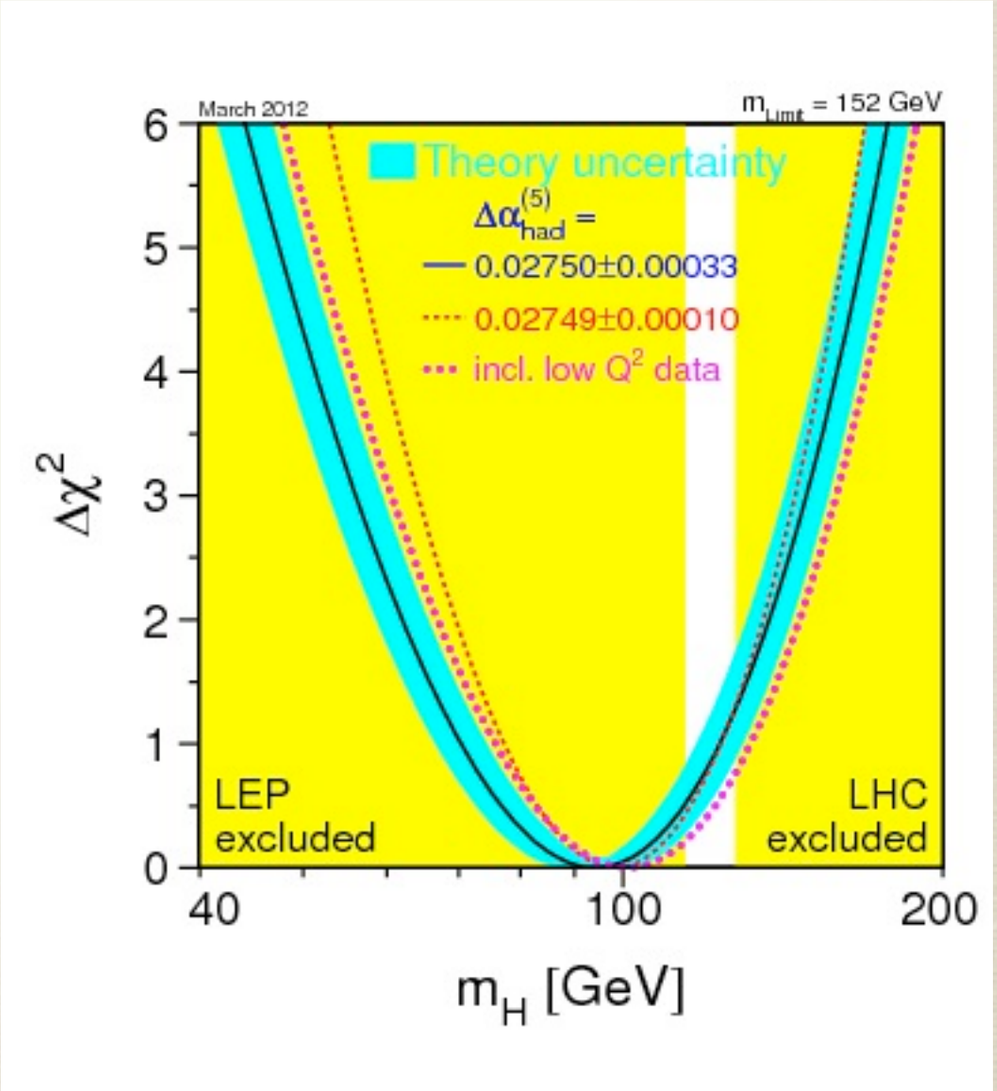
- Logarithmic dependence on $M_H$ allows $M_W$, and other precision observables, to bound it

$$M_W^2 = \frac{M_Z^2}{2}\left\{1 + \left[1 - \frac{2\sqrt{2}\alpha(1+\Delta r)}{G_F M_Z^2}\right]^{1/2}\right\}$$

$$\Delta r \sim \ln\frac{M_H}{M_Z}$$

Best fit: $M_H = 94^{+29}_{-24}$ GeV (68% CL)

The observed state is right at the upper range of the 1σ band



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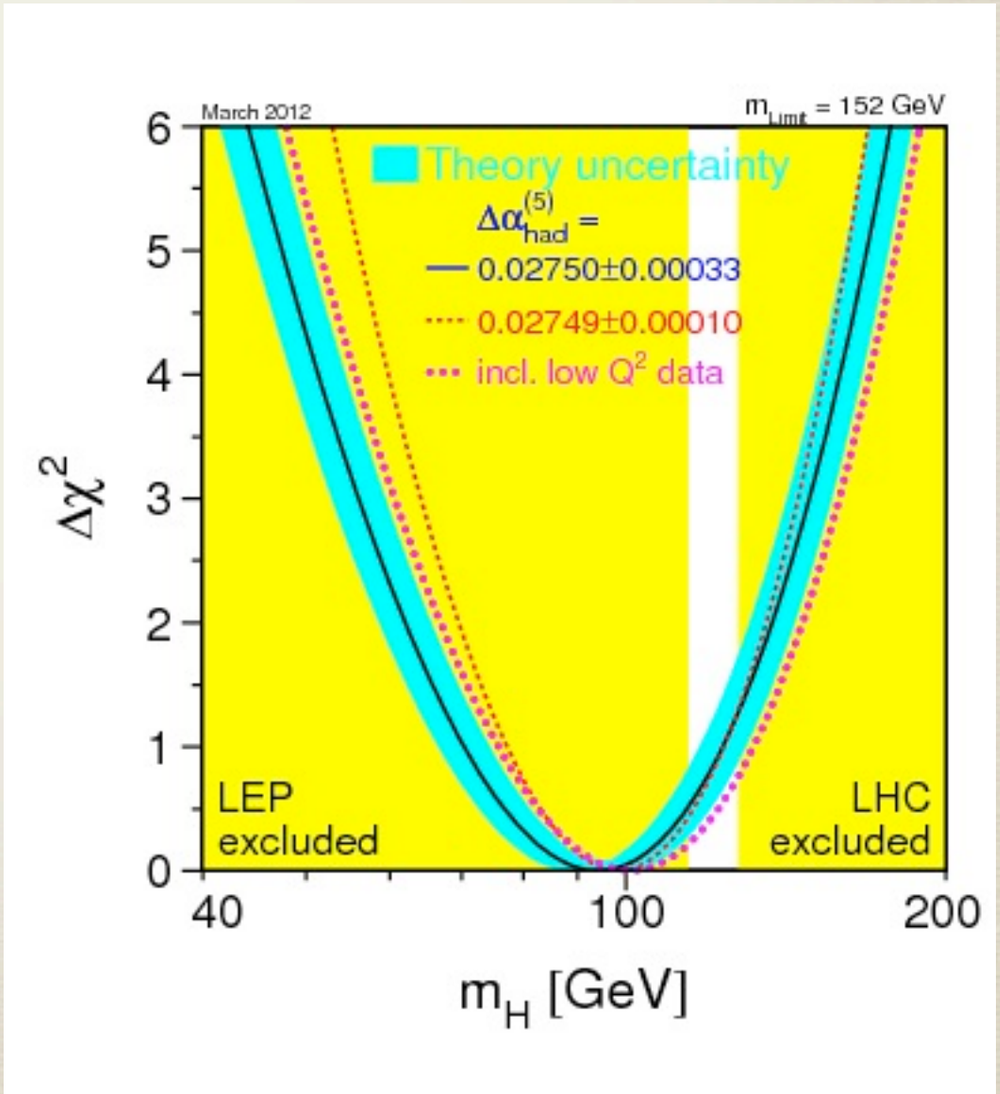
Q: since we've found the state, why do we care about indirect constraints?



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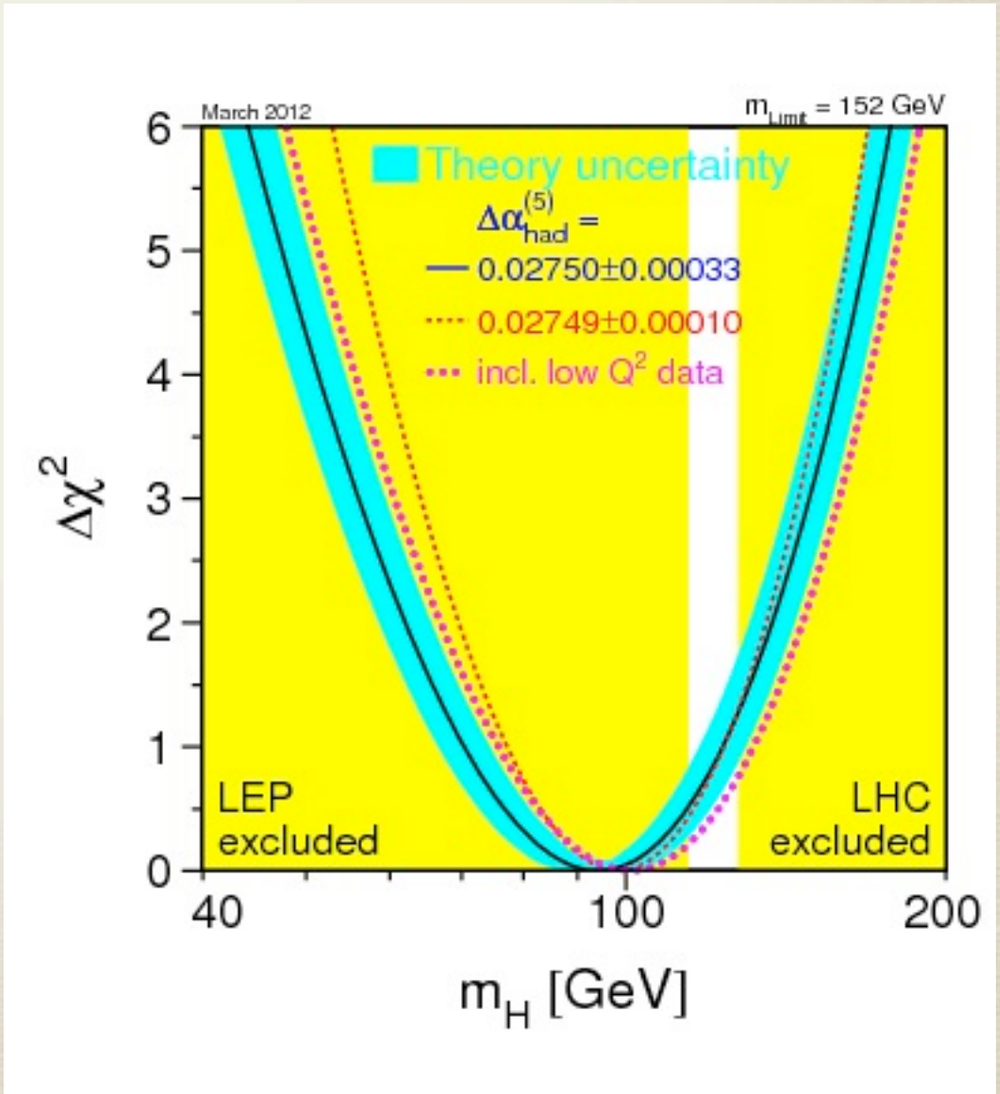
A: when we measure its couplings, we must test consistency with the EW data



# Profiling the Higgs boson: decays

# Higgs decays

- Since $g_{Hxx} \sim m_x$, Higgs tends to decay to heaviest kinematically accessible states (with many important caveats...)
- Tree-level decays to various massive final states:

$$\Gamma_{qq} = N_c \frac{G_F}{4\sqrt{2}\pi} M_H m_f^2 \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}, \quad \Gamma_{ll} = \frac{G_f}{4\sqrt{2}\pi} M_H m_f^3 \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$$\Gamma_{VV} = \frac{G_F}{8\sqrt{2}\pi n_V} M_H^3 \left(1 - 4x\right)^{1/2} \left(1 - 4x + 12x^3\right) \text{ with } x = \frac{M_V^2}{M_H^2}, n_W = 1, n_Z = 2$$

- Threshold structure depends on spin, CP (3/2→1/2 for CP-odd A)
- Note $\Gamma_{ff} \sim M_H$, while $\Gamma_{VV} \sim (M_H)^3 \Rightarrow$ when W, Z channels open, Higgs becomes very broad

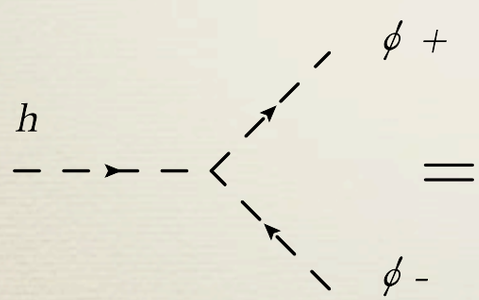
Exercise: if you've never done so before, calculate these widths

# Equivalence theorem

- Growth of VV width comes from longitudinal gauge modes

$$\begin{aligned}
\mathcal{A}(h \to W_L^+ W_L^-) &= 2\frac{M_W^2}{v}\epsilon_L^+ \cdot \epsilon_L^-, \quad \epsilon_L^\pm = \frac{E}{M_W}\left(\pm\beta_W, \vec{0}, 1\right) \\
\mathcal{A}(h \to W_L^+ W_L^-) &\to -\frac{M_H^2}{v} + \mathcal{O}\left(\frac{M_V^2}{M_H^2}\right) \\
\Gamma_{WW} &= \frac{1}{16\pi M_H}|\mathcal{A}|^2 \to \frac{G_F M_H^3}{8\pi\sqrt{2}} + \mathcal{O}\left(\frac{M_V^2}{M_H^2}\right)
\end{aligned}$$

- In the high energy limit, longitudinal mode interactions equivalent to those of eaten scalar ⇒ *Goldstone boson equivalence theorem*

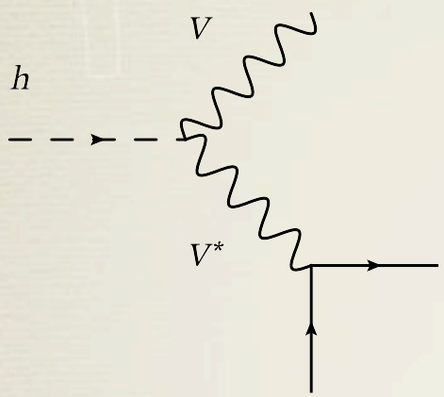



$$= -i\frac{M_H^2}{v}$$

$$\mathcal{A}(h \to \phi^+\phi^-) = -\frac{M_H^2}{v}$$

# Three-body decays

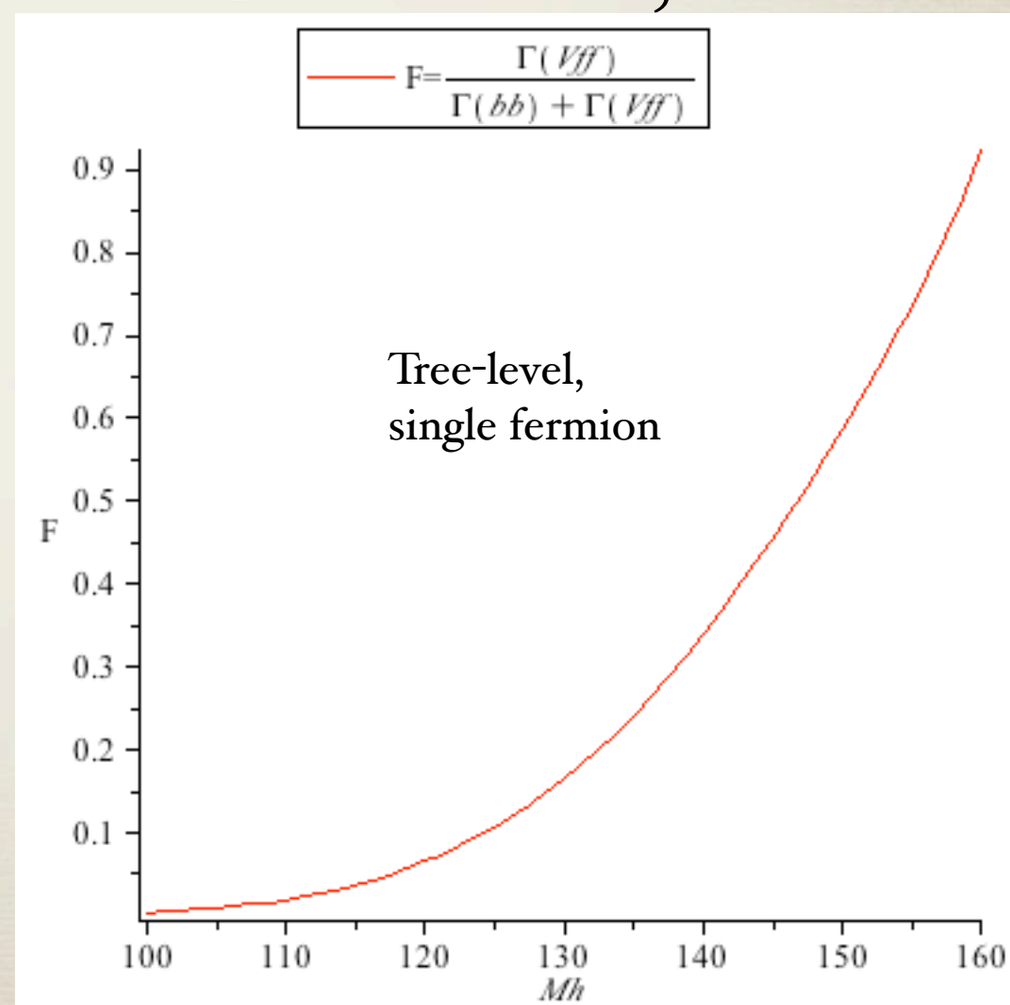
- Since $M_{W,Z} >> m_{b,c,\tau}$, $H \to VV^* \to Vff$ important for $M_H < 2M_{W,Z}$




$$\Gamma_{W\bar{f}f} = \frac{3G_F^2 M_W^4}{16\pi^3} M_H \left\{ \frac{3(1-8x+20x^2)}{\sqrt{4x-1}} \arccos\left(\frac{3x-1}{2x^{3/2}}\right) - \frac{1-x}{2x}(2-13x+47x^2) - \frac{3}{2}(1-6x+4x^2)\ln x \right\}$$

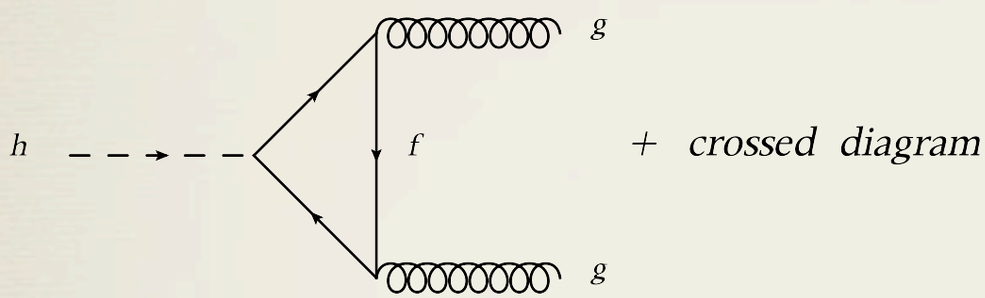
$$x = M_W^2/M_H^2$$

- Important mode even down at $M_H \approx 130$ GeV since f=e,μ



# Loop-induced H→gg

- Can we leverage the large Htt, HVV couplings at low $M_H$?
- Two important cases: h→gg (production more important), h→γγ




$$\Gamma_{gg} = \frac{G_F \alpha_s^2 M_H^3}{36\pi^3\sqrt{2}} \left| \frac{3}{4} \sum_Q \mathcal{F}_{1/2}(\tau_Q) \right|^2 \quad \text{with } \tau_Q = \frac{M_H^2}{4m_Q^2}$$

$$\mathcal{F}_{1/2}(\tau) = \frac{2}{\tau^2}\left[\tau + (\tau-1) f(\tau)\right]$$

$$f(\tau) = \begin{cases} \arcsin^2\sqrt{\tau} & \tau \le 1 \\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right]^2 & \tau > 1 \end{cases}$$

$$\tau \to 0 \quad \Rightarrow \quad \mathcal{F}_{1/2} \to \frac{4}{3}$$

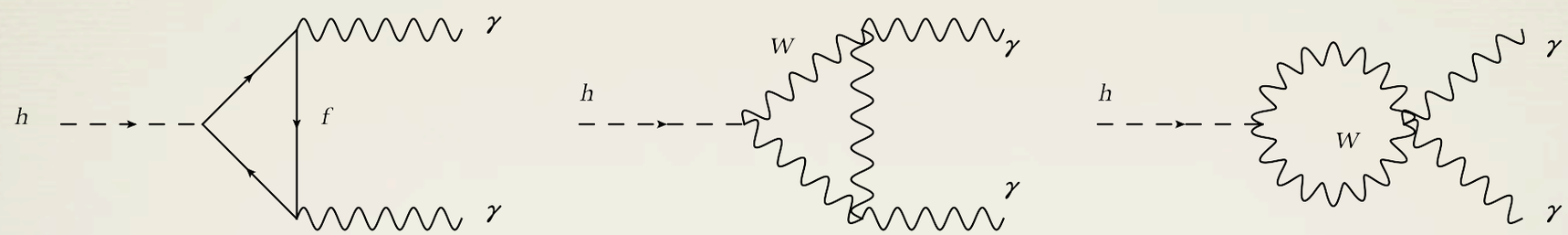
$$\tau \to \infty \quad \Rightarrow \quad \mathcal{F}_{1/2} \to -\frac{2m_Q^2}{M_H^2} \ln\frac{M_H^2}{m_Q^2}$$

- Independent of $m_f$ when $m_f \to \infty$ ⇒ true for *any* heavy fermion that gets its mass entirely from Higgs

Exercise: Derive $m_t \to \infty$ result from direct integration

# Loop-induced H→γγ

- Crucial for low-mass Higgs search at LHC




$$\Gamma_{\gamma\gamma} = \frac{G_F \alpha^2 M_H^3}{128\pi^3\sqrt{2}} \left| \sum_f N_c Q_f^2 \mathcal{F}_{1/2}(\tau_f) + \mathcal{F}_1(\tau_W) \right|^2$$

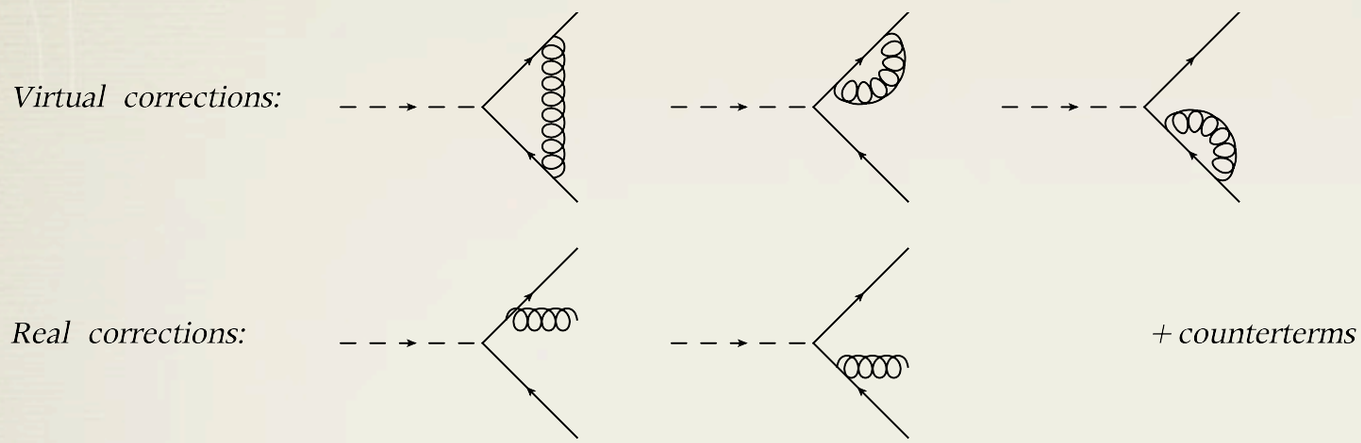
$$\mathcal{F}_1(\tau) = -\frac{1}{\tau^2}\left[2\tau^2 + 3\tau + 3(2\tau - 1) f(\tau)\right]$$

$\tau \to 0 \Rightarrow \mathcal{F}_1 \to -7$

W contribution larger than top-quark, they interfere destructively

# QCD and decays to heavy quarks

- Which mass to use in $\Gamma_{bb,cc}$; pole mass, MS-bar?




- Pole scheme calculation (on-shell counterterm used):

$$\Gamma_{qq}^{NLO} = N_c \frac{G_F}{4\sqrt{2}\pi} M_H m_q^2 \left[ 1 + \frac{4}{3}\frac{\alpha_s}{\pi} \left( \frac{9}{4} + \frac{3}{2} \underbrace{\ln \frac{m_q^2}{M_H^2}}_{large} \right) \right] + \mathcal{O}(m_q^2/M_H^2)$$

Negative for $m_q$~10 MeV

- Log comes only from counterterms (KLN theorem applied to $\mathrm{Im}[\Pi(M_H)]$ requires this)

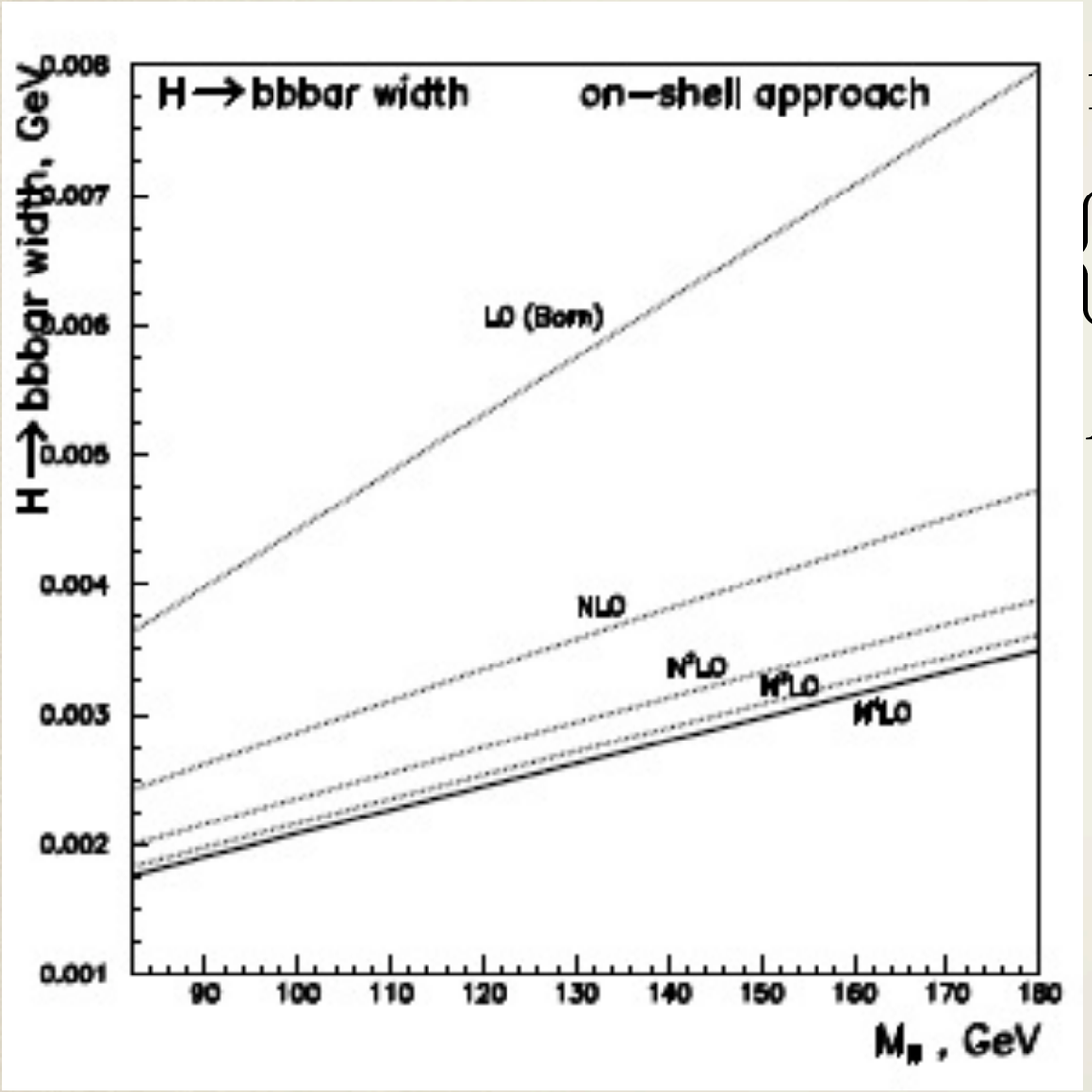
# Translation to running mass

- Translate from pole→MSbar scheme (leading terms only)

$$
\begin{aligned}
m_q &= \bar{m}(m_q)\left\{1+\frac{4}{3}\frac{\alpha_s}{\pi}\right\} \quad \text{(derive this)} \\
\bar{m}(m_q) &= \bar{m}(M_H)\left\{1-\frac{\alpha_s}{\pi}\ln\frac{M_H^2}{m_q^2}\right\} \quad \text{(standard RGE)} \\
\Rightarrow \Gamma_{qq}^{NLO} &= N_c\frac{G_F}{4\sqrt{2}\pi}M_H\bar{m}_q^2(M_H)\left[1+\frac{17}{3}\frac{\alpha_s}{\pi}\right]
\end{aligned}
$$

# Translation to running mass

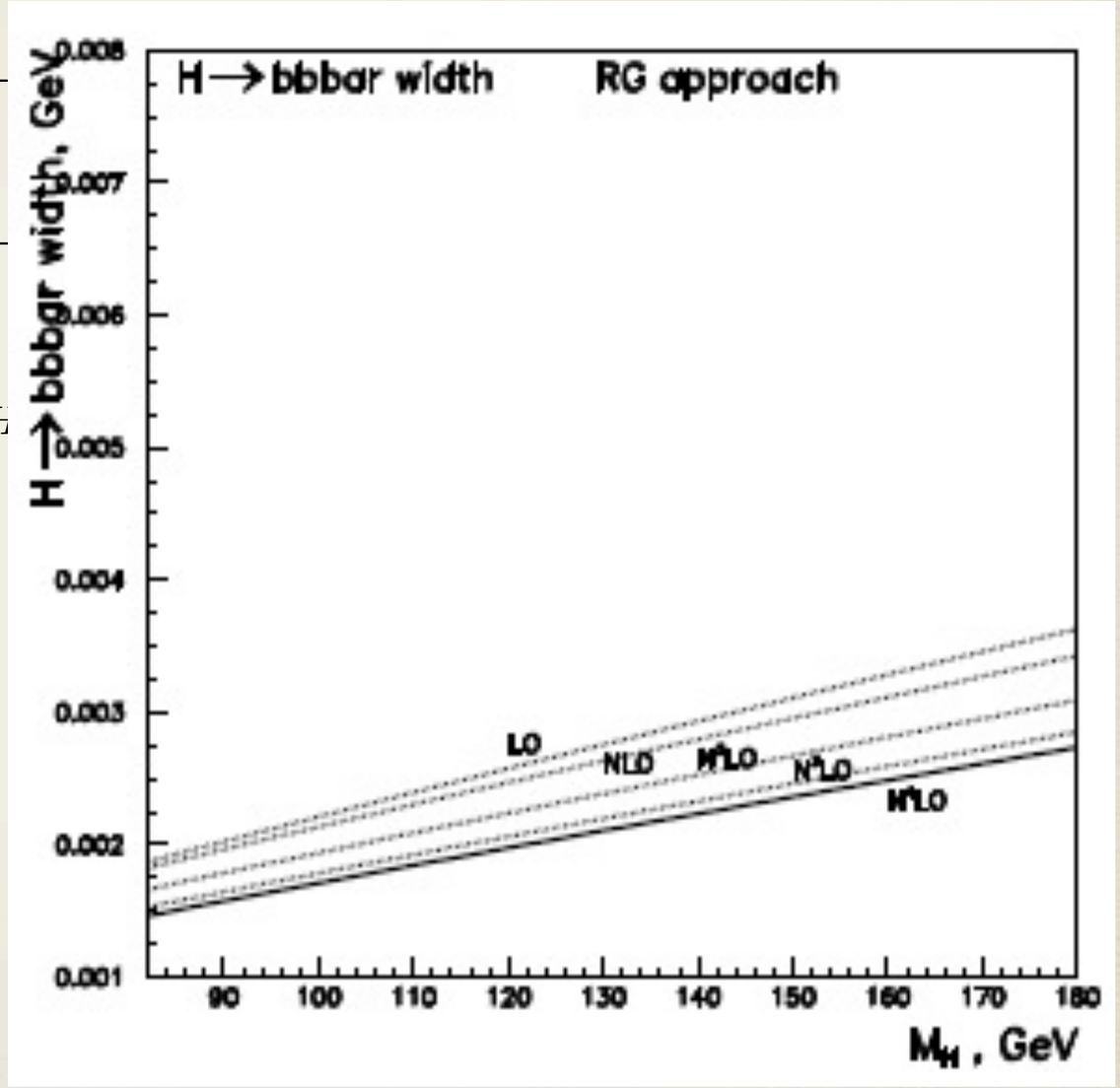
- Translate from pole→MSbar scheme (leading terms only)




$1 +$

$1 -$

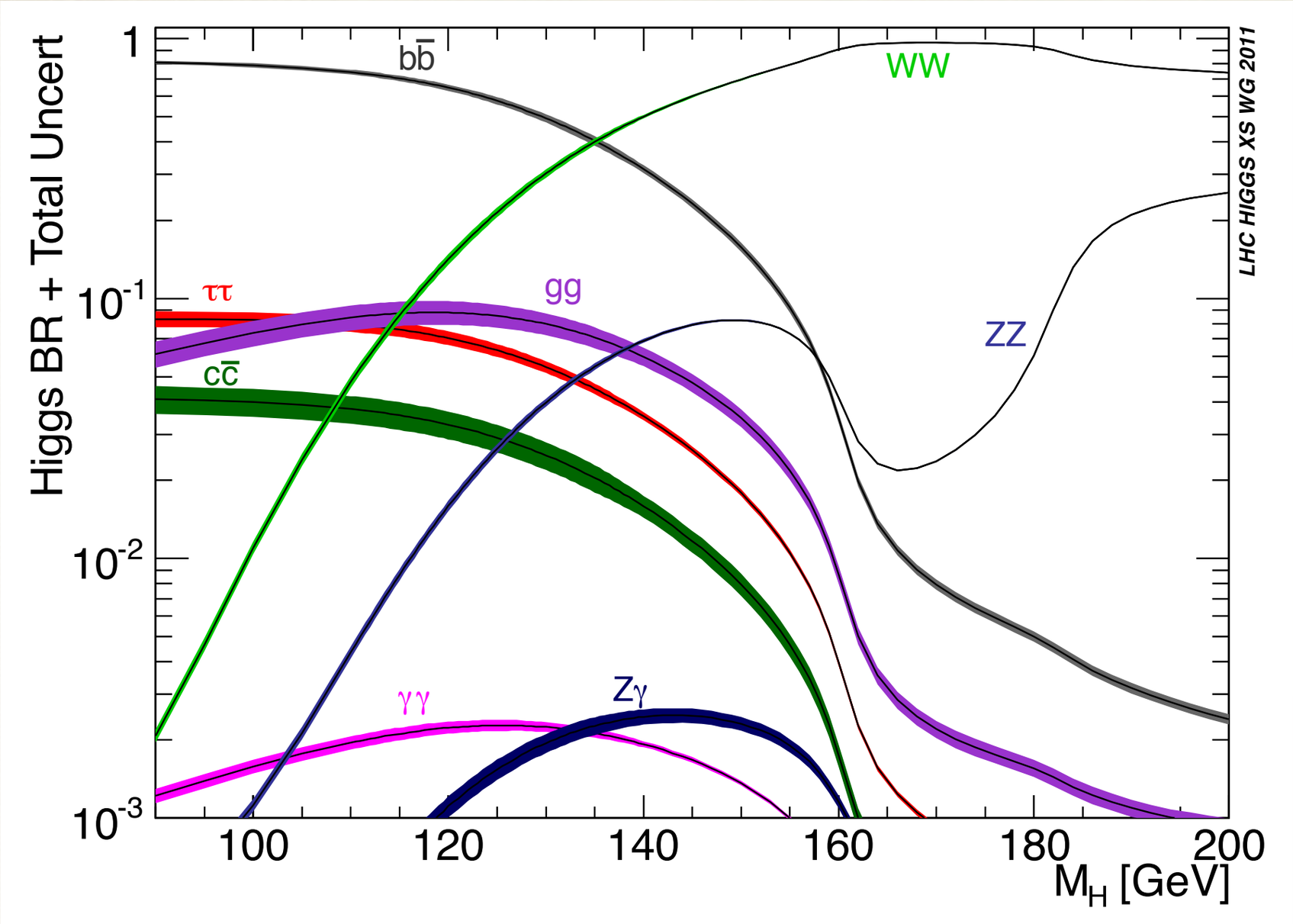
$M_H$




First example that proper QCD is crucial for Higgs phenomenology

(from Kataev, Kim 0902.1442, can get other literature there)

# Putting it all together



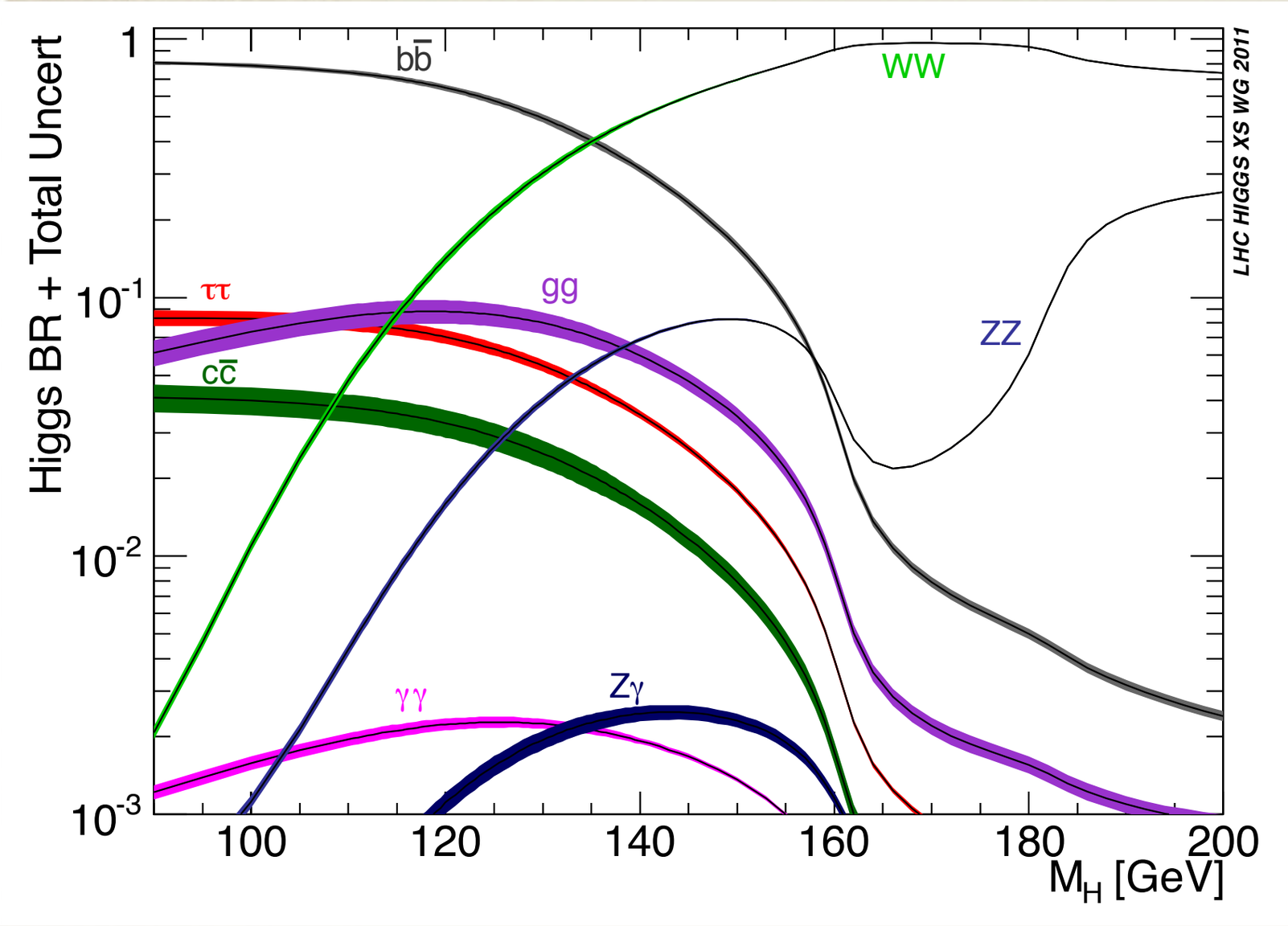

Useful reference: LHC Higgs cross section working group reports 1101.0593, 1201.3084

Available general-purpose code: HDECAY: M. Spira, http://people.web.psi.ch/spira

https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CrossSections

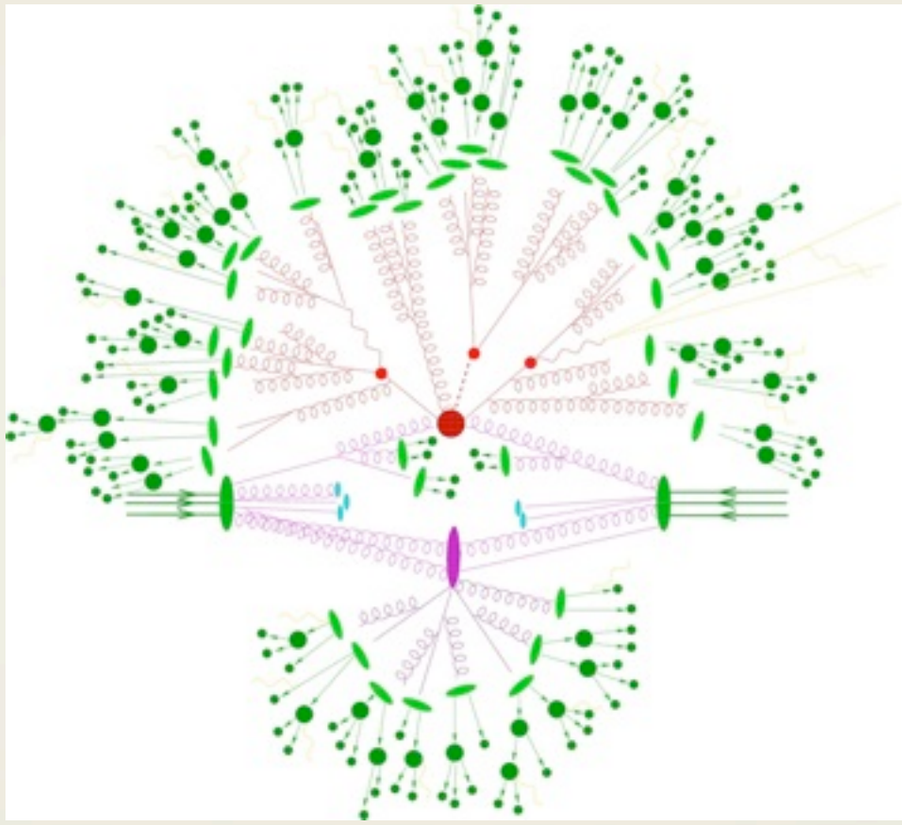
- 125-127 GeV is an optimal mass for the Higgs; experiment has access to most SM decay modes

# Putting it all together




- cc uncertainty: 12%, $m_c$, $\alpha_S$ parametric uncertainties
- gg: 10%, $\alpha_S$, higher-order QCD (more later)
- γγ: 5% uncertainty, combination of missing higher orders and $m_b$
- ττ: 6% uncertainty, missing EW corrections, $m_b$

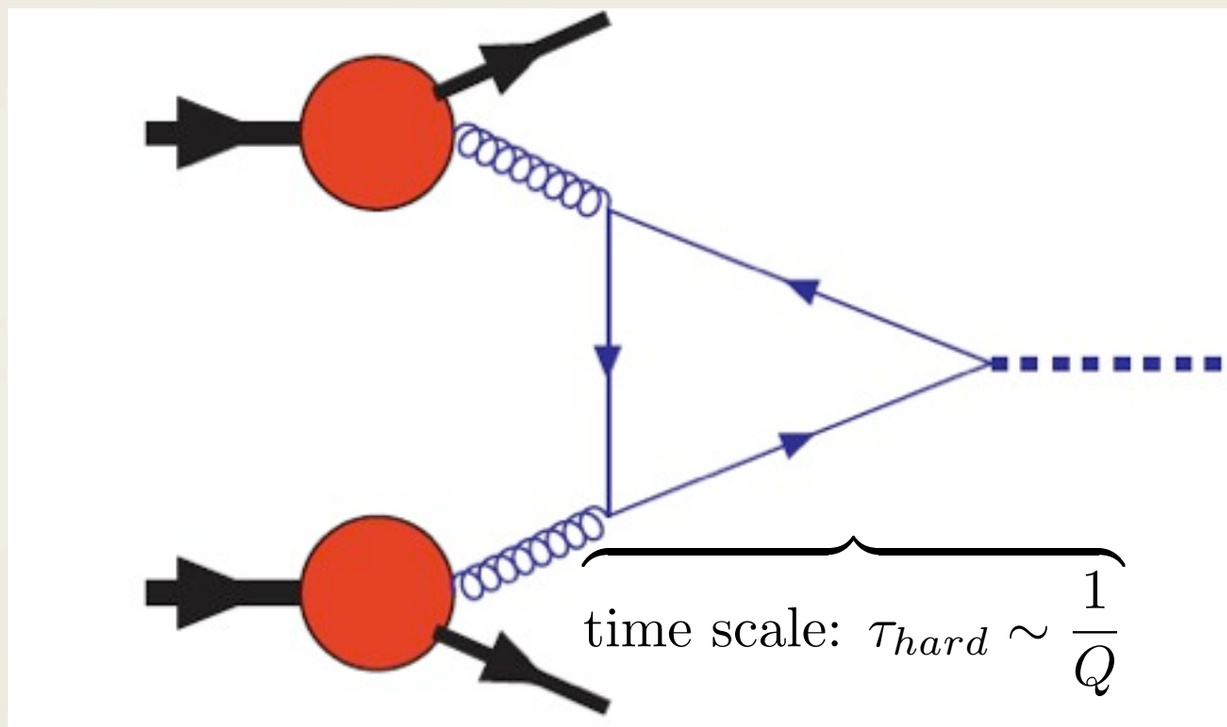
# Hadron-collider basics



# Hadron collider basics

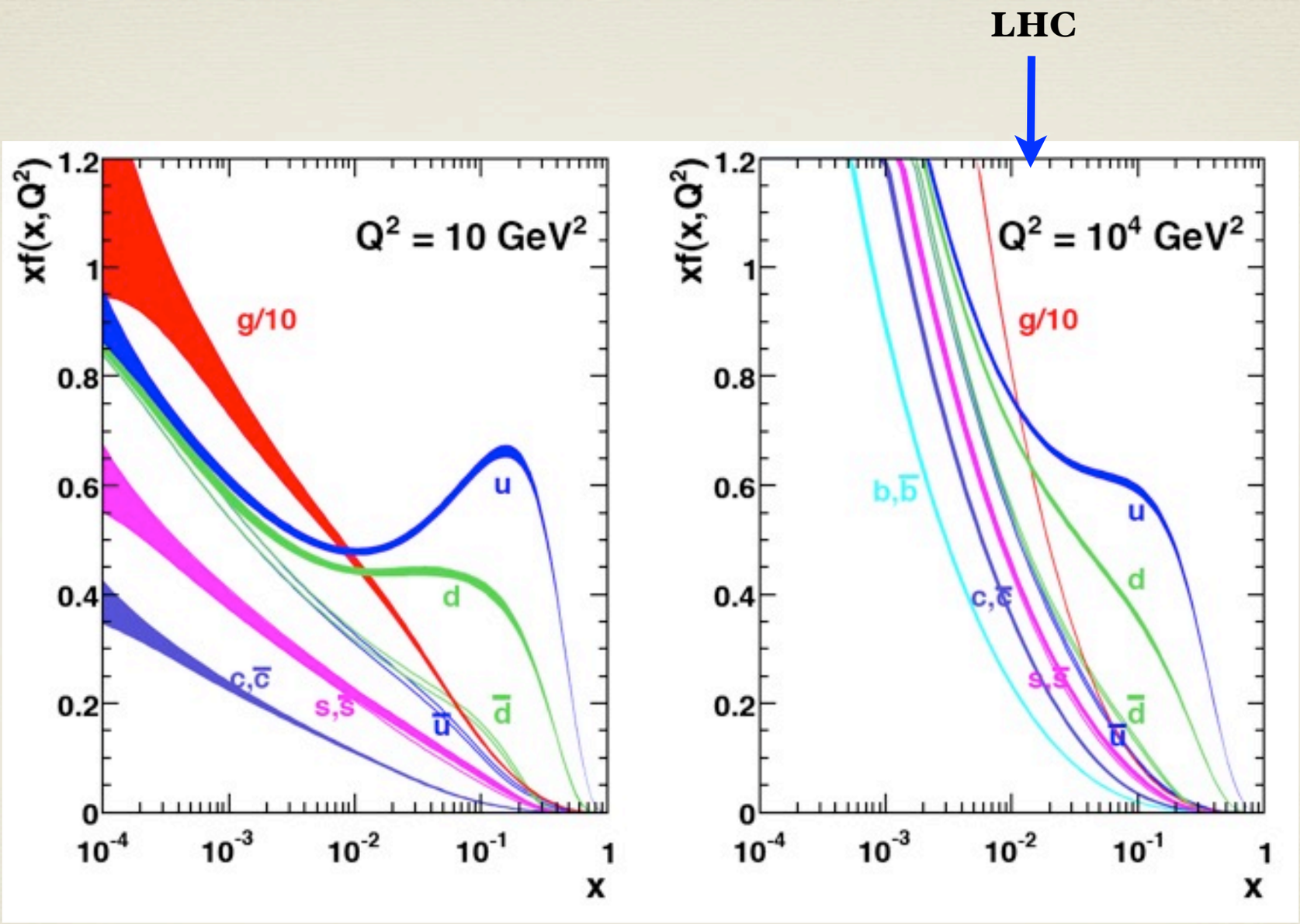
- The basic picture of hadronic collisions: factorize long and short time processes

$$\underbrace{\text{time scale: } \tau_{proton} \sim \frac{1}{\Lambda_{QCD}}}$$




$$\sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1; \overbrace{\mu_F^2}^{\text{factorization scale}}) f_{h_1/j}(x_2; \mu_F^2)}_{PDFs} \underbrace{\sigma_{ij \to X}(x_1, x_2, \mu_F^2, \{q_k\})}_{\text{partonic cross section}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^n}_{\text{power corrections}}$$

# Parton distribution functions

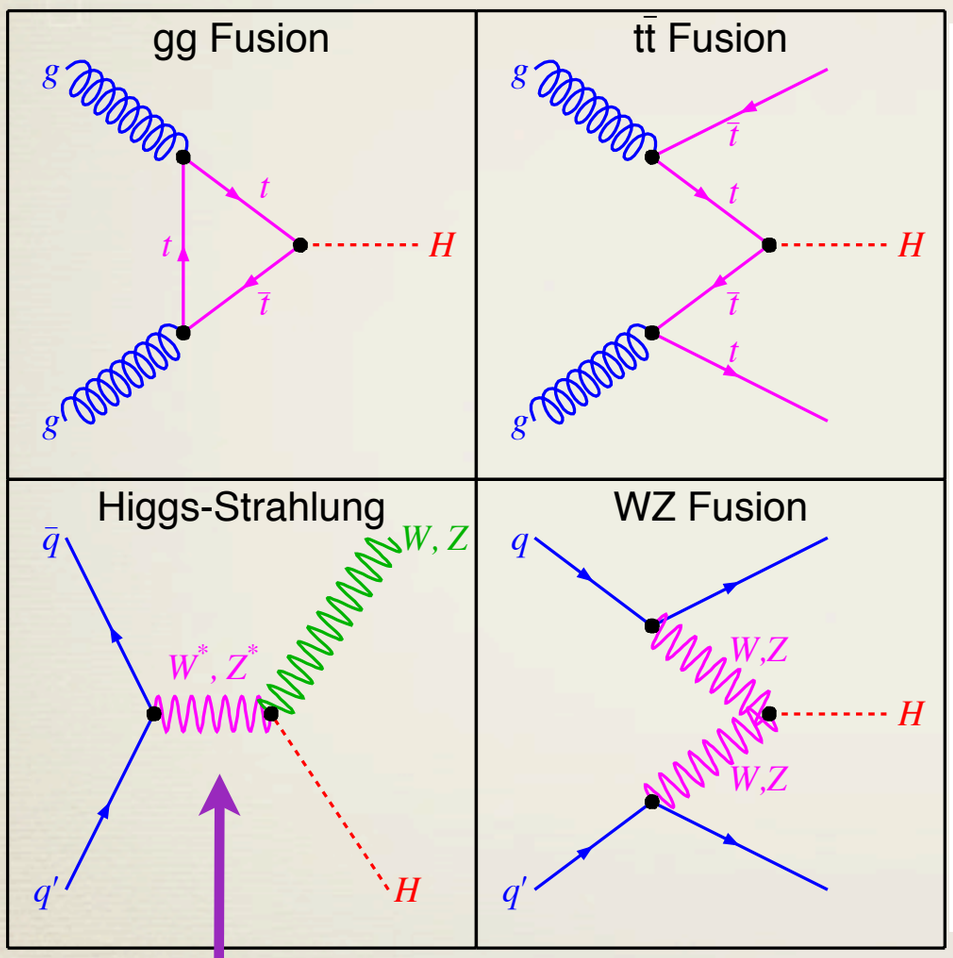


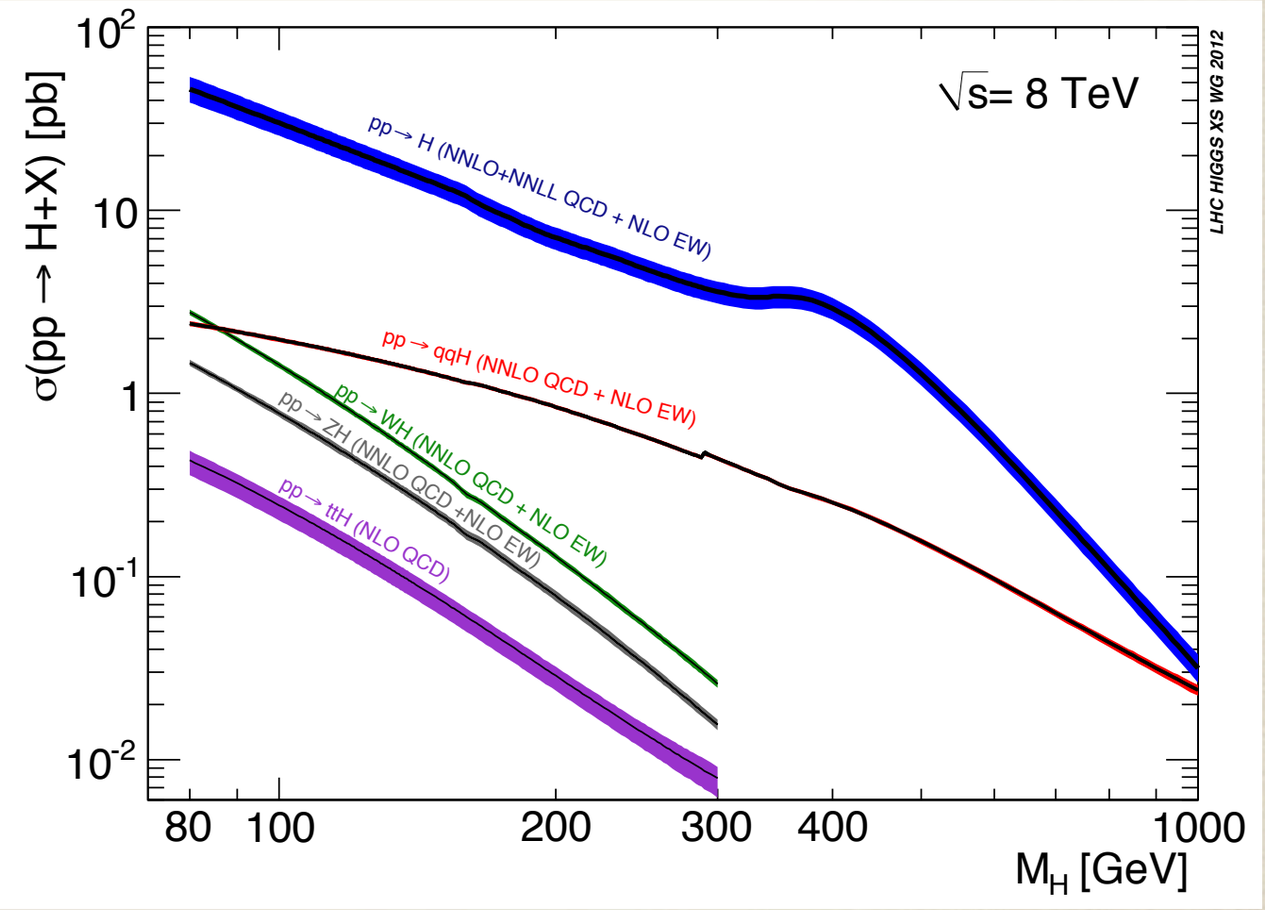

$$x \sim M_H/\sqrt{s}$$

Lots of gluons at the LHC!

# Summary of production

- Clearly want to use large gluon luminosity; W, Z assisted production another option




Can't do model-independent LEP search, √s not fixed at hadron machine

# Summary of production

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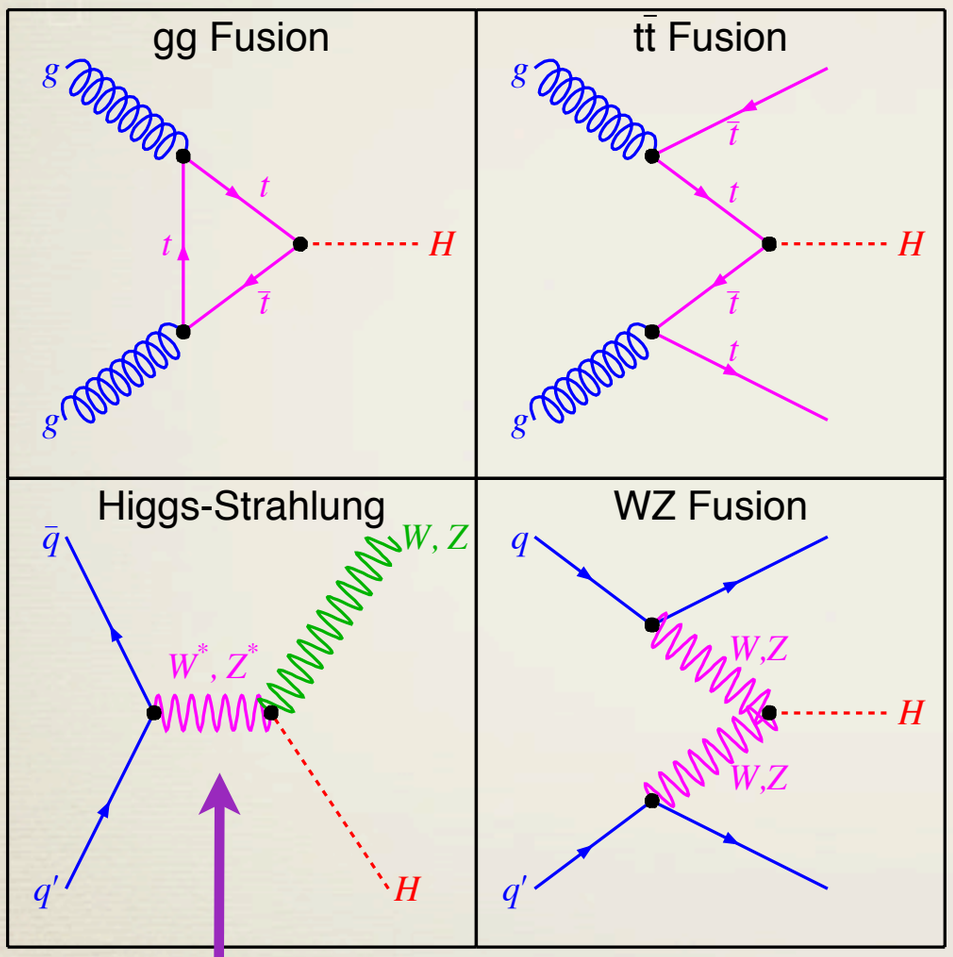
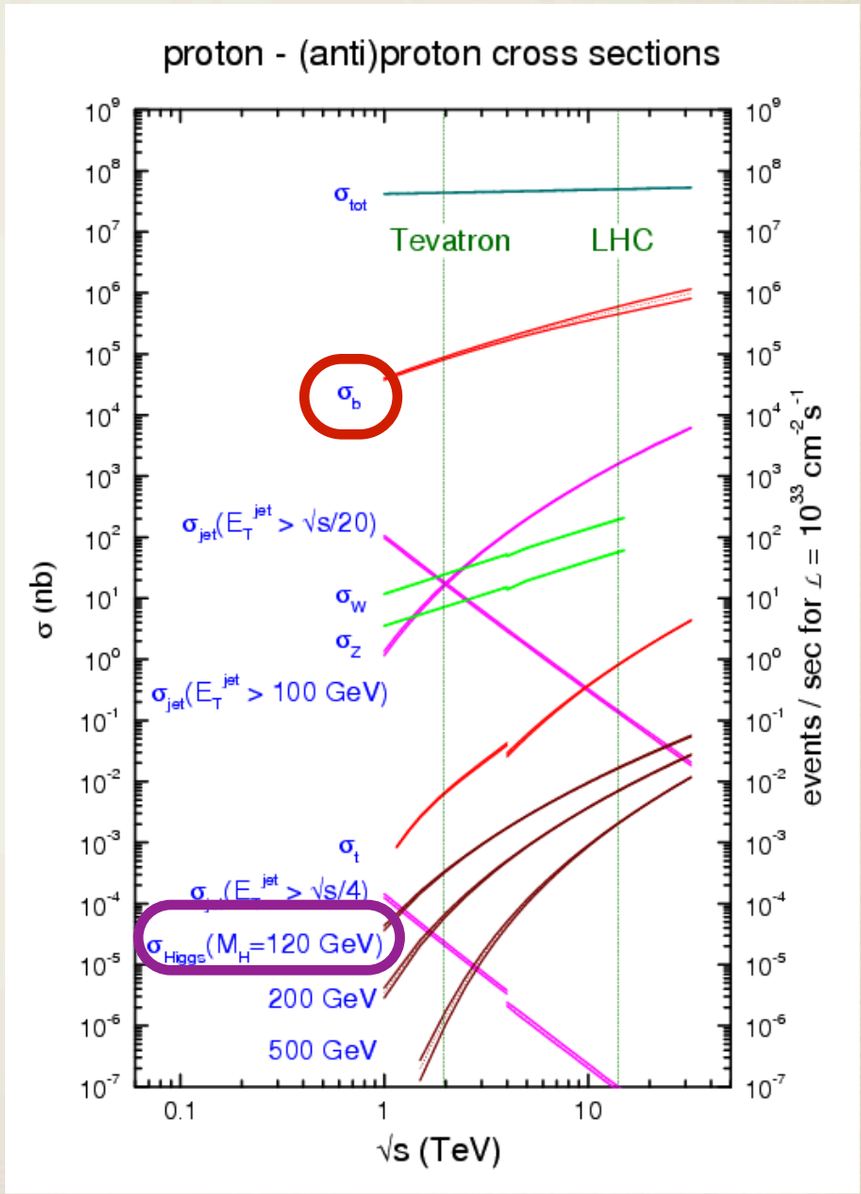



Can't do model-independent LEP search, √s not fixed at hadron machine

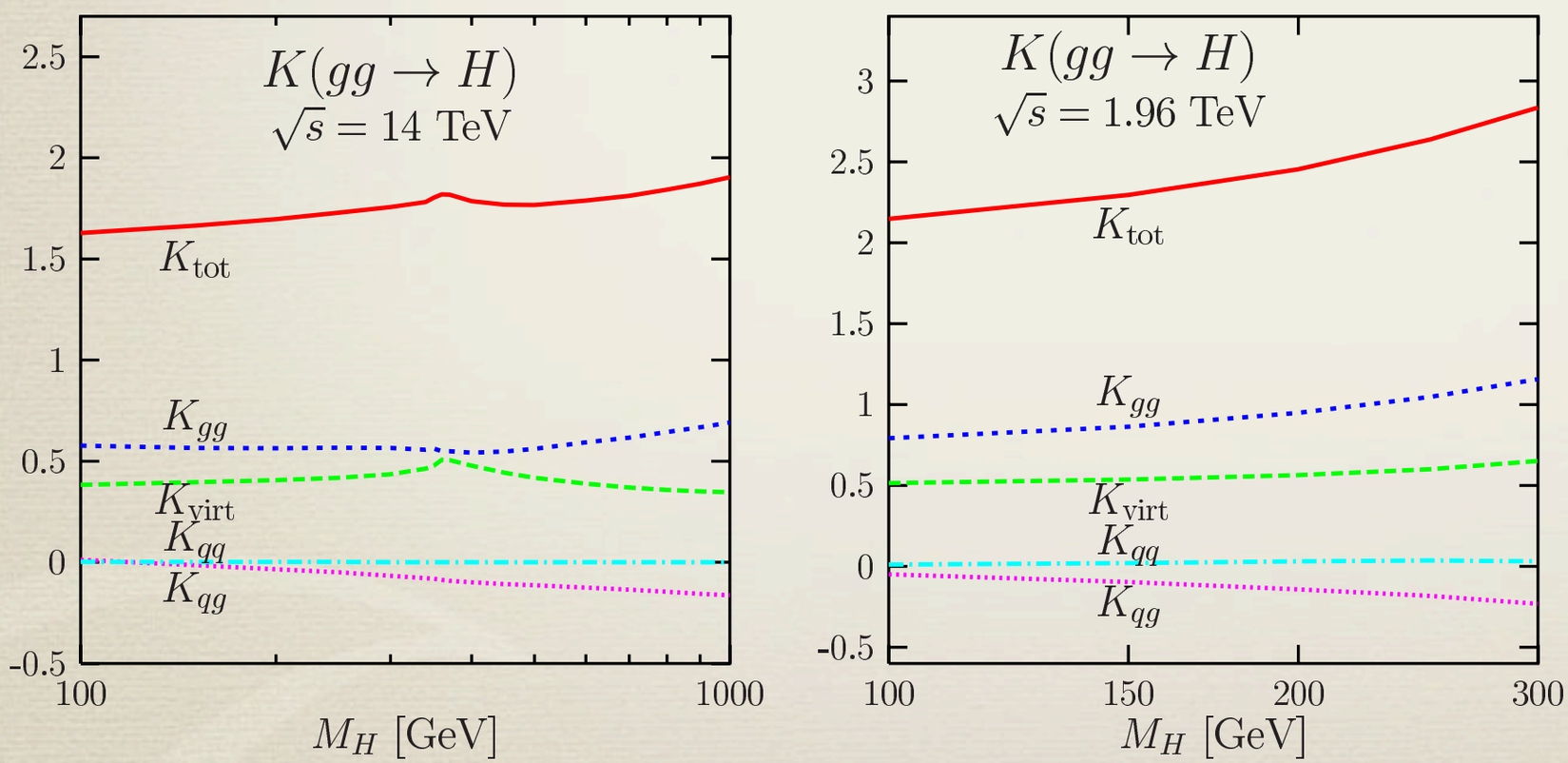
Unfortunately, must confront backgrounds

# Gluon fusion production

- Largest mode at Tevatron and LHC; through top-quark loops (reuse the calculation of the width we did before)

$$\sigma^{LO}_{gg\to h} = \frac{G_F \alpha_s^2}{288\pi\sqrt{2}} \left| \frac{3}{4} \sum_Q \mathcal{F}_{1/2}(\tau_Q) \right|^2 \delta(1-z), \quad \tau_Q = \frac{M_H^2}{4m_Q^2}. \quad z = \frac{M_H^2}{\hat{s}}$$

- NLO QCD corrections require 2-loop virtual, 1-loop real-virtual

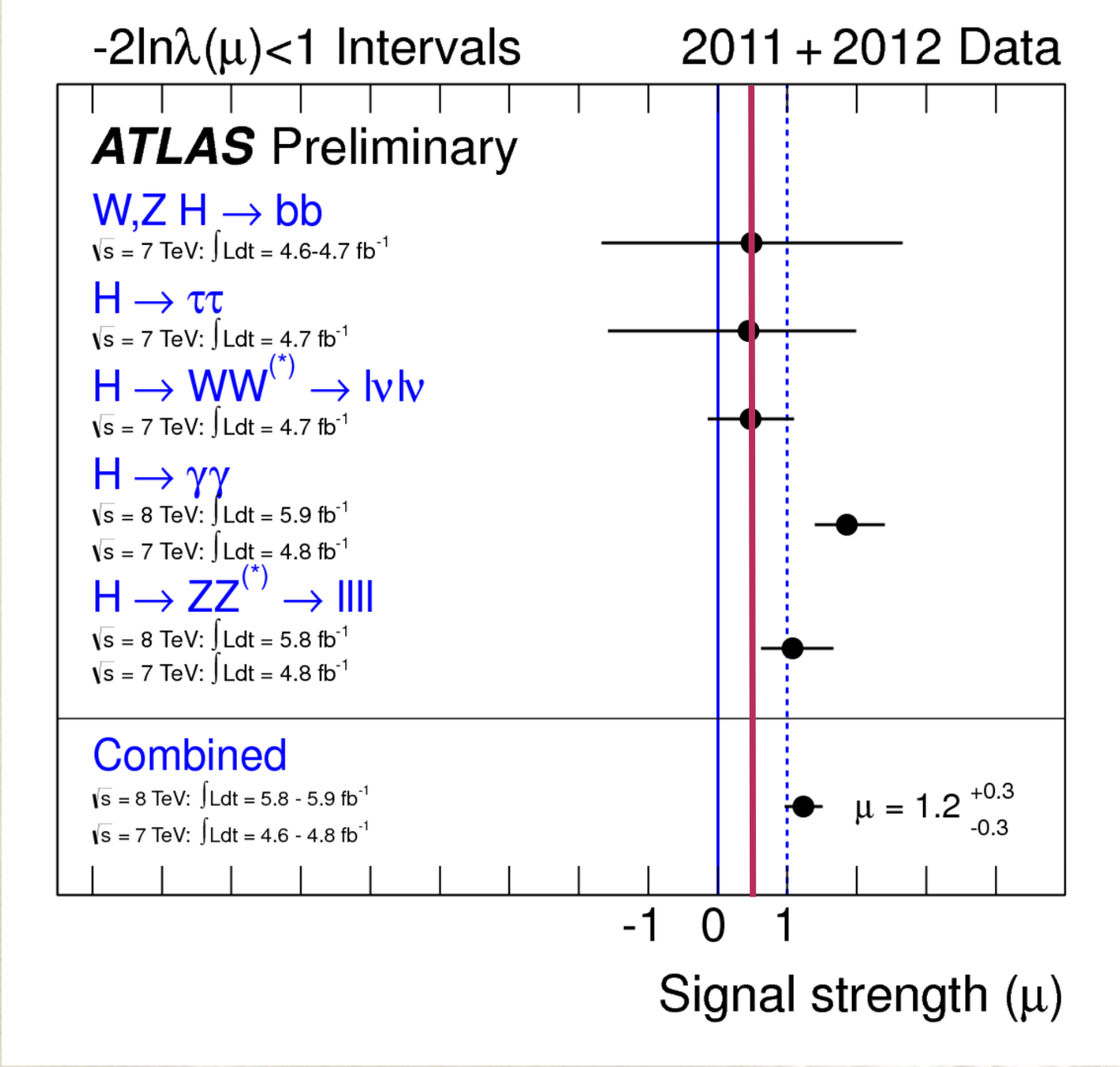



- This is the largest, most important mode at the LHC and has large QCD corrections... we're going to study it in detail

Dawson; Djouadi, Graudenz, Spira, Zerwas 1991, 1995

# Gluon fusion production

- Largest mode at Tevatron and LHC; through top-quark loops (reuse the calculation of the width we did before)

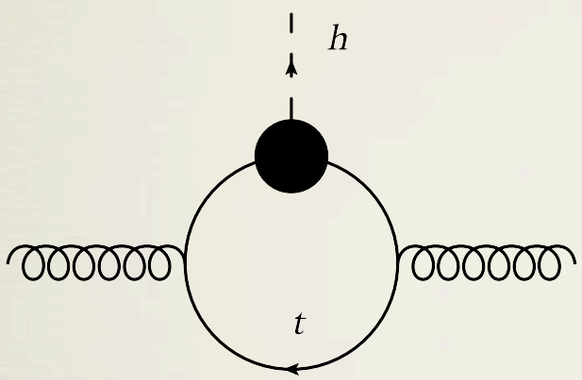



- Without a detailed understanding of QCD, we would have a factor of 3 excess in the γγ channel... and even more theoretical frenzy about beyond the SM physics

# The Higgs effective field theory

# Low-energy theorems

- We've already calculated exactly the loop diagrams relevant for Higgs decay to gluons
- Useful, illuminating alternative approach for $2m_t > M_H$




$$\frac{i}{\not{k} - m_t} \rightarrow \frac{i}{\not{k} - m_t}\frac{-im_t}{v}\frac{i}{\not{k} - m_t} = i\frac{m_t}{v}\left(\frac{1}{\not{k} - m_t}\right)^2$$
$$= \frac{m_t}{v}\frac{\partial}{\partial m_t}\frac{i}{\not{k} - m_t}$$

Generates both diagrams in the $M_H \rightarrow 0$ limit

- Diagrammatically, clear that Higgs interaction comes from derivatives of the top part of the gluon self-energy:

$$\mathcal{M}(hgg) \underbrace{=}_{p_H \rightarrow 0} \frac{m_t}{v}\frac{\partial}{\partial m_t}\mathcal{M}(gg)$$

# The Higgs effective Lagrangian

- Integrate out the top quark to produce an effective Lagrangian

$$\mathcal{L}_{full} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \mathcal{L}_{top}$$

$$\underbrace{G^{a'}_\mu}_{\text{EFT field}} = \underbrace{\sqrt{\zeta_3}}_{\text{decoupling constant}} \underbrace{G^a_\mu}_{\text{QCD field}}$$

$$\mathcal{L}_{EFT} = -\frac{\zeta_3}{4} G^{a'}_{\mu\nu} G^{\mu\nu'} a$$

(remember to amputate external legs)

- Matching calculation: equate full and EFT propagators

$$-\frac{i g_{\mu\nu}}{p^2}\zeta_3 = -\frac{i g_{\mu\nu}}{p^2} \underbrace{[1 + \Pi_t(0)]}_{m_t^2 \gg p^2}$$

top-quark contribution to gluon self-energy

$$\Rightarrow \zeta_3 = 1 + \Pi_t(0)$$

$$\Rightarrow \mathcal{L}_{EFT} = -\frac{[1 + \Pi_t(0)]}{4} G^{a\prime}_{\mu\nu} G^{\mu\nu\prime}_a$$

# The Higgs effective Lagrangian

- Now apply the low energy theorem to derive HGG operator:

$$\mathcal{L}_{EFT}^{hgg} = -\frac{m_t}{4v}\left(\frac{\partial}{\partial m_t}\Pi_t(0)\right) h\, G_{\mu\nu}^{a\prime} G_a^{\mu\nu\prime}$$

$$\Rightarrow \Pi_t(0) = \frac{\alpha_s}{6\pi}\left[\frac{\bar{\mu}^2}{m_t^2}\right]^{\epsilon} \frac{\Gamma(1+\epsilon)}{\epsilon}$$

$$\Rightarrow \mathcal{L}_{EFT}^{hgg} = \frac{\alpha_s}{12\pi}\frac{h}{v} G_{\mu\nu}^{a\prime} G_a^{\mu\nu\prime}$$

- Numerous nice features of this formulation...

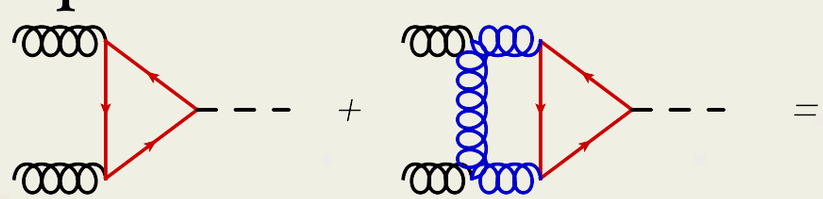
# The Higgs effective Lagrangian

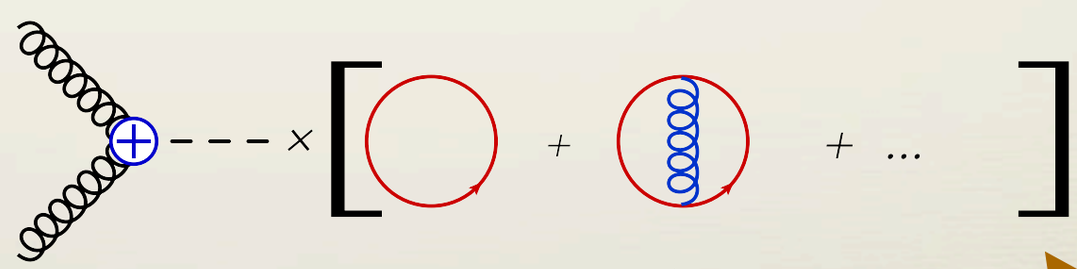
- Systematically, simply extendable to higher orders in QCD

Useful references: Kniehl, Spira hep-ph/9505225; Steinhauser hep-ph/0201075

- Reduces calculations by one loop order; 1-loop becomes tree, etc.
- Turns a two-scale problem into two one-scale problems

Two scales:
$M_{Higgs}$, $m_{top}$



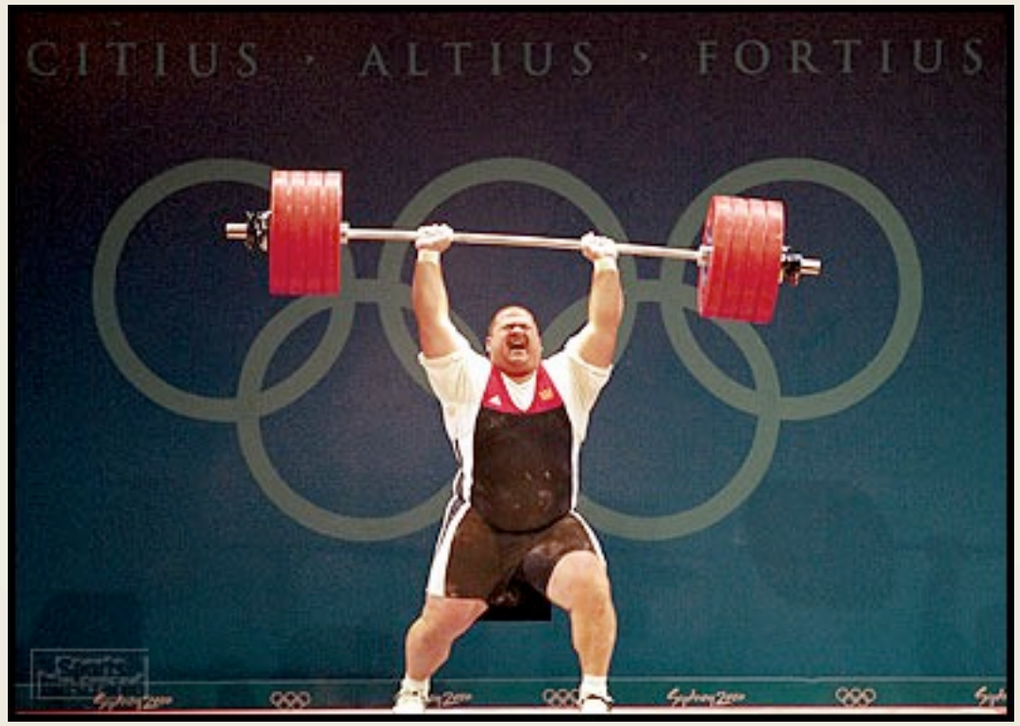


Only $M_{Higgs}$ Only $m_{top}$ $O(M^2_{Higgs}/4m^2_{top})$

# The Higgs effective Lagrangian

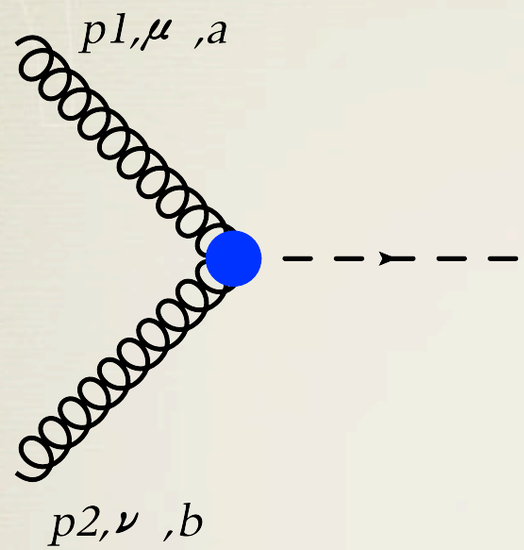
- Factorizes QCD effects (dynamics of gluons, light quarks from $L_{EFT}$) from new physics (heavy particles into Wilson coefficients)
- Applicable to the $h\gamma\gamma$ coupling also
- Can be used when a particle does not obtain all its mass from the Higgs (for a recent formulation, see Carena et al. 1206.1082)
- Valid much beyond the expected region of validity; forms the basis for much of Tevatron/LHC phenomenology
- Let's try it out...

# Exercise: gg→H at NLO

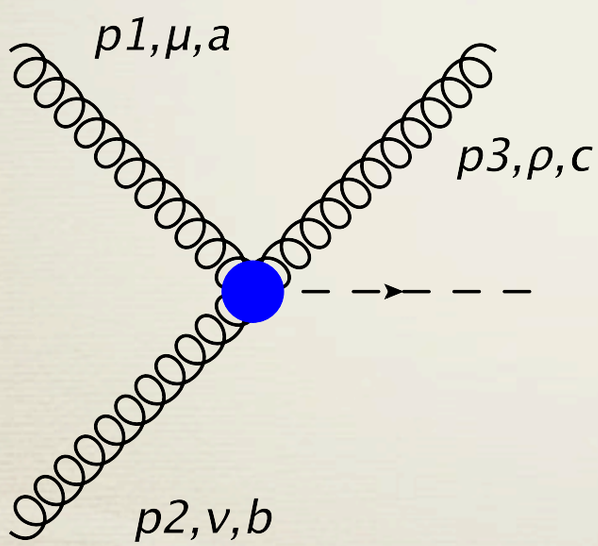


# Setup

- Our Feynman rules are 5-flavor QCD plus the EFT vertices:




$$= -i\frac{\alpha_s}{3\pi v}\left\{1+\frac{11}{4}\frac{\alpha_s}{\pi}\right\}\delta^{ab}\left[p_1\cdot p_2 g^{\mu\nu}-p_1^\nu p_2^\mu\right]$$




$$= g_s\frac{\alpha_s}{3\pi v}f^{abc}\left\{g_{\mu\nu}(p_1-p_2)_\rho\right.$$
$$\left.+g_{\nu\rho}(p_2-p_3)_\mu+(p_3-p_1)_\nu\right\}$$

# Steps

- Pick a regularization scheme (dimensional regularization for us)
- Get the tree-level result
- Calculate 1-loop diagrams as a Laurent series in $\varepsilon$
- Perform the ultraviolet renormalization
- Calculate the real emission diagrams, extract singularities that appear in soft/collinear regions of phase space
- Absorb initial-state collinear singularities into PDFs
- Get numbers

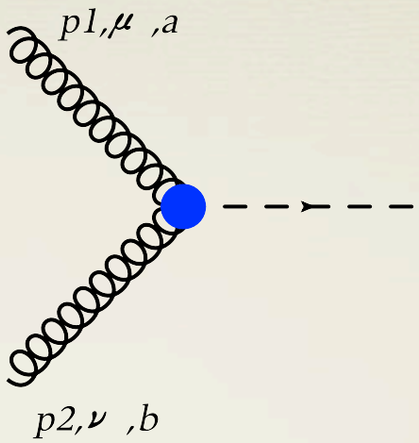
Work through steps in detail as an exercise if you haven't done so before

# Tree-level

$$\sigma_{h_1 h_2 \to h} = \int dx_1\, dx_2 f_g(x_1) f_g(x_2)\, \hat{\sigma}(z)$$

$$+ \quad \text{smaller partonic channels}$$

$(z = M_H^2/x_1 x_2 s)$




- Calculate the spin-, color-averaged matrix element squared

$$|\bar{\mathcal{M}}|^2 = \underbrace{\frac{1}{256(1-\epsilon)^2}}_{8 \text{ colors, } 2(1-\epsilon) \text{ spins}} \times |\mathcal{M}|^2 = \frac{\hat{s}^2}{576 v^2 (1-\epsilon)} \left(\frac{\alpha_s}{\pi}\right)^2$$

- Get the phase space and flux factor

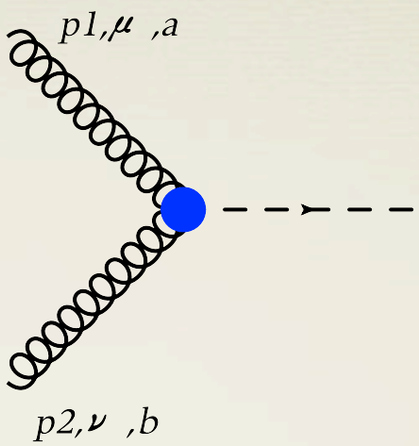
$$\frac{1}{2\hat{s}} \int \frac{d^d p_h}{(2\pi)^d}\, 2\pi \delta(p_H^2 - M_H^2)\, (2\pi)^d \delta^{(d)}(p_1 + p_2 - p_H) = \frac{\pi}{\hat{s}^2} \delta(1-z)$$

# Tree-level

$$\sigma_{h_1 h_2 \to h} = \int dx_1\, dx_2 f_g(x_1) f_g(x_2)\, \hat{\sigma}(z)$$

$$+ \quad \text{smaller partonic channels}$$

$(z = M_H^2/x_1 x_2 s)$




- Combine to get the LO result:

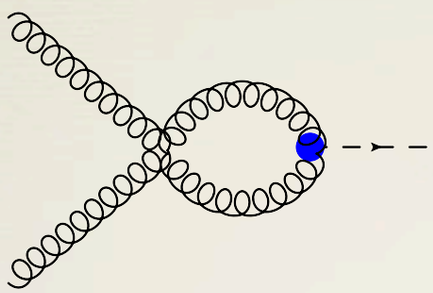
$$\hat{\sigma}_0(z) = \sigma_0 \delta(1-z) = \frac{\pi}{576 v^2} \left(\frac{\alpha_s}{\pi}\right)^2 \delta(1-z)$$

- We will later need the full d-dimensional tree-level result:

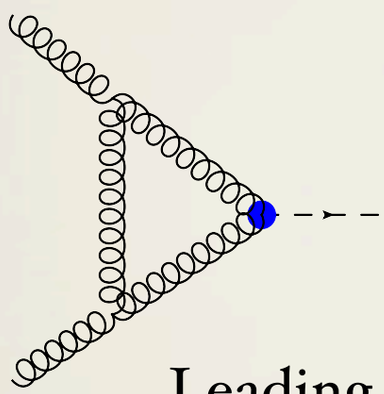
$$\sigma_0^{(d)} = \frac{\sigma_0}{1-\epsilon}$$

# Virtual corrections

- Calculate $2\times\mathrm{Re}[(M_0)^* M_1]$, which appears in the cross section



$$= \sigma_0^{(d)} \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{\hat{s}}{\mu^2}\right)^{-\epsilon} \left\{ -\frac{13}{4\epsilon} - \frac{11}{3} \right\} \delta(1-z)$$



$$= \sigma_0^{(d)} \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{\hat{s}}{\mu^2}\right)^{-\epsilon} \left\{ -\frac{3}{\epsilon^2} + \frac{13}{4\epsilon} + \frac{11}{3} + 2\pi^2 \right\} \delta(1-z)$$

Leading soft+collinear singularity; emitting gluons from gluons gives color factor $C_A=3$

- External leg corrections *scaleless:* $\int d^d k \, (k^2)^n = 0$

# UV renormalization

- LO dependence on $\alpha_S$ gives the counterterm:

$$\sigma_0^{(d)} \frac{\alpha_s}{\pi} \frac{1}{\epsilon} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \left\{ -\frac{11}{2} + \frac{N_F}{3} \right\}$$

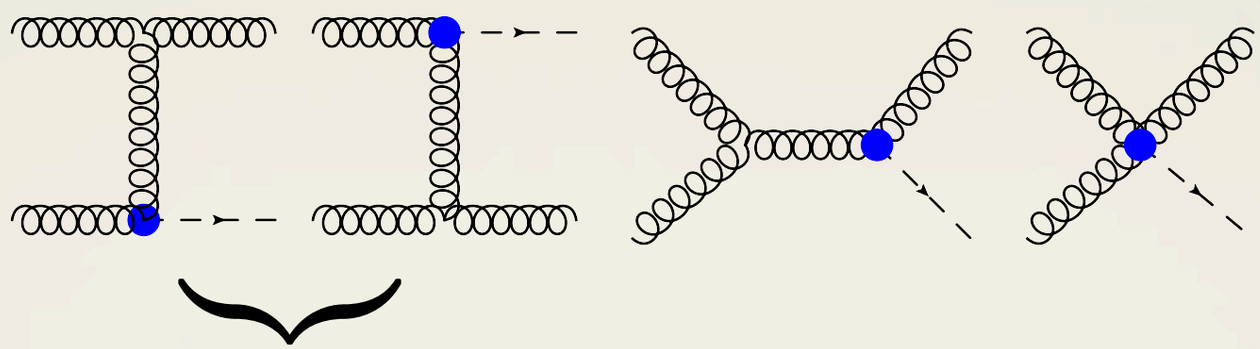
- The remaining singularities are of soft/collinear origin; summing what we have so far yields

$$\sigma_0^{(d)} \frac{\alpha_s}{\pi} \left\{ -\frac{3}{\epsilon^2} + \frac{3}{\epsilon} \ln \frac{\hat{s}}{\mu^2} - \frac{1}{\epsilon} \left( \frac{11}{2} - \frac{N_F}{3} \right) + \text{finite} \right\} \delta(1-z)$$

- The pole structure can be checked to be correct: Catani, hep-ph/9802439

# Real radiation corrections

- Get the corrections coming from emission of an additional gluon



singular

$$|\bar{\mathcal{M}}|^2 = 24\,\alpha_s\sigma_0\left\{\frac{(1-2\epsilon)}{(1-\epsilon)}\frac{M_H^8+\hat{s}^4+\hat{t}^4+\hat{u}^4}{\hat{s}\hat{t}\hat{u}}+\frac{\epsilon}{2(1-\epsilon)^2}\frac{(M_H^4+\hat{s}^2+\hat{t}^2+\hat{u}^2)^2}{\hat{s}\hat{t}\hat{u}}\right\}$$

- This can vanish when either $p_g \to 0$ (soft), or $p_g \parallel p_1$, $p_g \parallel p_2$ (collinear)
- Need a parameterization of phase space to extract these singularities appropriately

$$\hat{s} = (p_1+p_2)^2$$
$$\hat{t} = (p_1-p_g)^2$$
$$\hat{u} = (p_2-p_g)^2$$

# Real radiation corrections

$$\frac{1}{2\hat{s}}\int\frac{d^dp_g}{(2\pi)^d}\int\frac{d^dp_H}{(2\pi)^d}(2\pi)\delta(p_g^2)(2\pi)\delta(p_H^2-M_H^2)(2\pi)^d\delta^{(d)}(p_1+p_2-p_g-p_H)$$

- Introduce the following parameterization of $p_g$:

$$p_g=\frac{\hat{s}(1-z)}{2}\left(1,2\sqrt{\lambda(1-\lambda)},0,1-2\lambda\right)$$

- Obtain: $$\frac{1}{16\pi\hat{s}}\left(\frac{s}{4\pi}\right)^{-\epsilon}\frac{1}{\Gamma(1-\epsilon)}(1-z)^{1-2\epsilon}\int_0^1 d\lambda\,[\lambda(1-\lambda)]^{-\epsilon}$$

- When we combine matrix elements and phase space, get terms of the following form:

$$(1-z)^{-1-2\epsilon}[\lambda(1-\lambda)]^{-1-\epsilon}$$

singular regulator

**λ→0,1**: collinear
**z→1**: soft

# Real radiation corrections

- The integrals over λ can be done in terms of Gamma functions, while the soft singularities as z→1 can be extracted using *plus distributions*:

$$(1-z)^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta(1-z) + \left[\frac{1}{1-z}\right]_+ - 2\epsilon\left[\frac{\ln(1-z)}{1-z}\right]_+ + \mathcal{O}(\epsilon^2)$$

$$\int_0^1 dz\, f(z)\left[\frac{g(z)}{1-z}\right]_+ = \int_0^1 dz \frac{g(z)}{1-z}[f(z)-f(1)]$$

- Arrive at the following contribution to the cross section:

$$\sigma_0^{(d)}\frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{\hat{s}}{\mu^2}\right)^{-\epsilon}\Bigg\{\overbrace{\frac{3}{\epsilon^2}\delta(1-z)}^{\text{cancels virtual poles}} - \frac{6}{\epsilon}\left[\frac{1}{1-z}\right]_+ + \frac{6z(z^2-z+2)}{\epsilon}$$

$$-\frac{3\pi^2}{2}\delta(1-z) + 12\left[\frac{\ln(1-z)}{1-z}\right]_+ - 12z(z^2-z+2)\ln(1-z) - \frac{11}{2}(1-z)^3\Bigg\}$$

# Remaining terms

- Absorb remaining initial-state collinear singularities into PDFs, which amounts to adding the following counterterm:

One for each PDF

$$2 \times \frac{\alpha_s}{2\pi}\frac{1}{\epsilon}\frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} P_{gg} \otimes \hat{\sigma}_0(z) \qquad f \otimes g(z) = \int_0^1 dx\, dy\, f(x)\, g(y)\, \delta(z - xy)$$

Arrive at the contribution:

$$\sigma_0^{(d)} \frac{\alpha_s}{\pi}\frac{1}{\epsilon}\left\{\left(\frac{11}{2} - \frac{N_F}{3}\right)\delta(1-z) + \frac{6}{[1-z]_+} - 6z(z^2 - z + 2)\right\}$$

- This cancels all remaining poles, but we need to add on the NLO correction to the Wilson coefficient in the EFT:

$$\sigma_0^{(d)} \frac{\alpha_s}{\pi}\frac{11}{2}\delta(1-z)$$

# Final result

- Arrive at the final NLO result for the inclusive cross section:

$$\Delta\sigma = \sigma_0 \frac{\alpha_s}{\pi} \left\{ \left( \frac{11}{2} + \pi^2 \right) \delta(1-z) + 12 \left[ \frac{\ln(1-z)}{1-z} \right]_+ - 12z(-z+z^2+2)\ln(1-z) \right.$$
$$\left. - \frac{11}{2}(1-z)^3 + 6\ln\frac{\hat{s}}{\mu^2} \left[ \frac{1}{[1-z]_+} - z(z^2-z+2) \right] \right\} \quad (M^2/s \le z \le 1)$$

(integration over PDFs⇒integration over z)

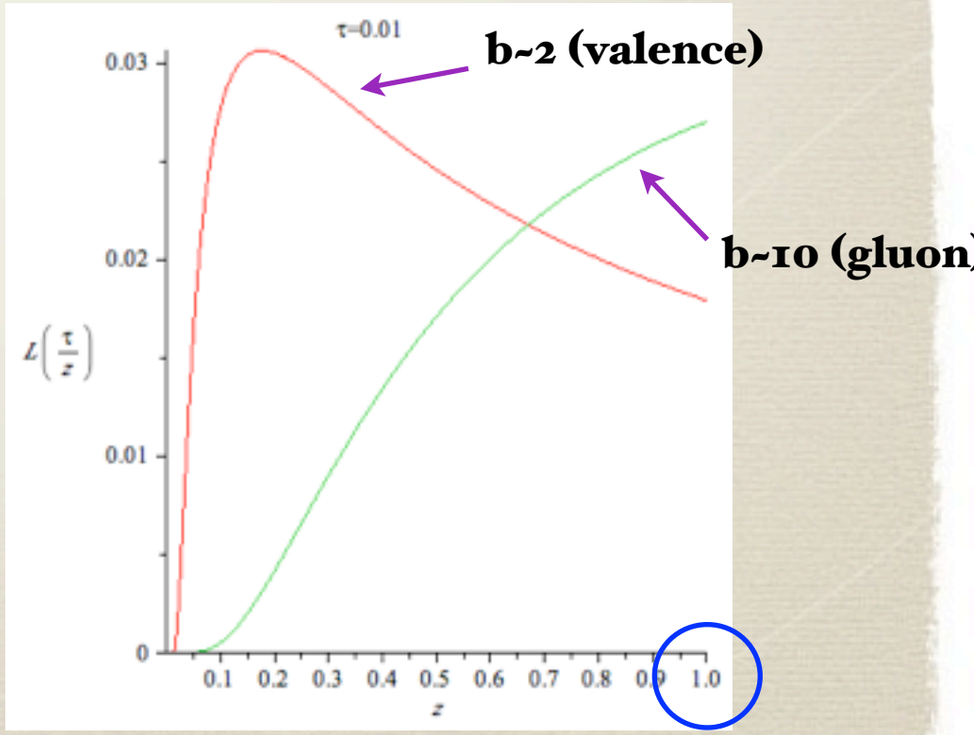
- First source of large correction: $11/2+\pi^2 \Rightarrow 50\%$ increase
- Second source: shape of PDFs enhances *threshold* logarithm

$$\sigma_{had} = \tau \int_\tau^1 dz \frac{\sigma(z)}{z} \mathcal{L}\left(\frac{\tau}{z}\right)$$

$$\mathcal{L}(y) = \int_y^1 dx \frac{y}{x} f_1(x) f_2(y/x) \quad \text{(partonic luminosity)}$$

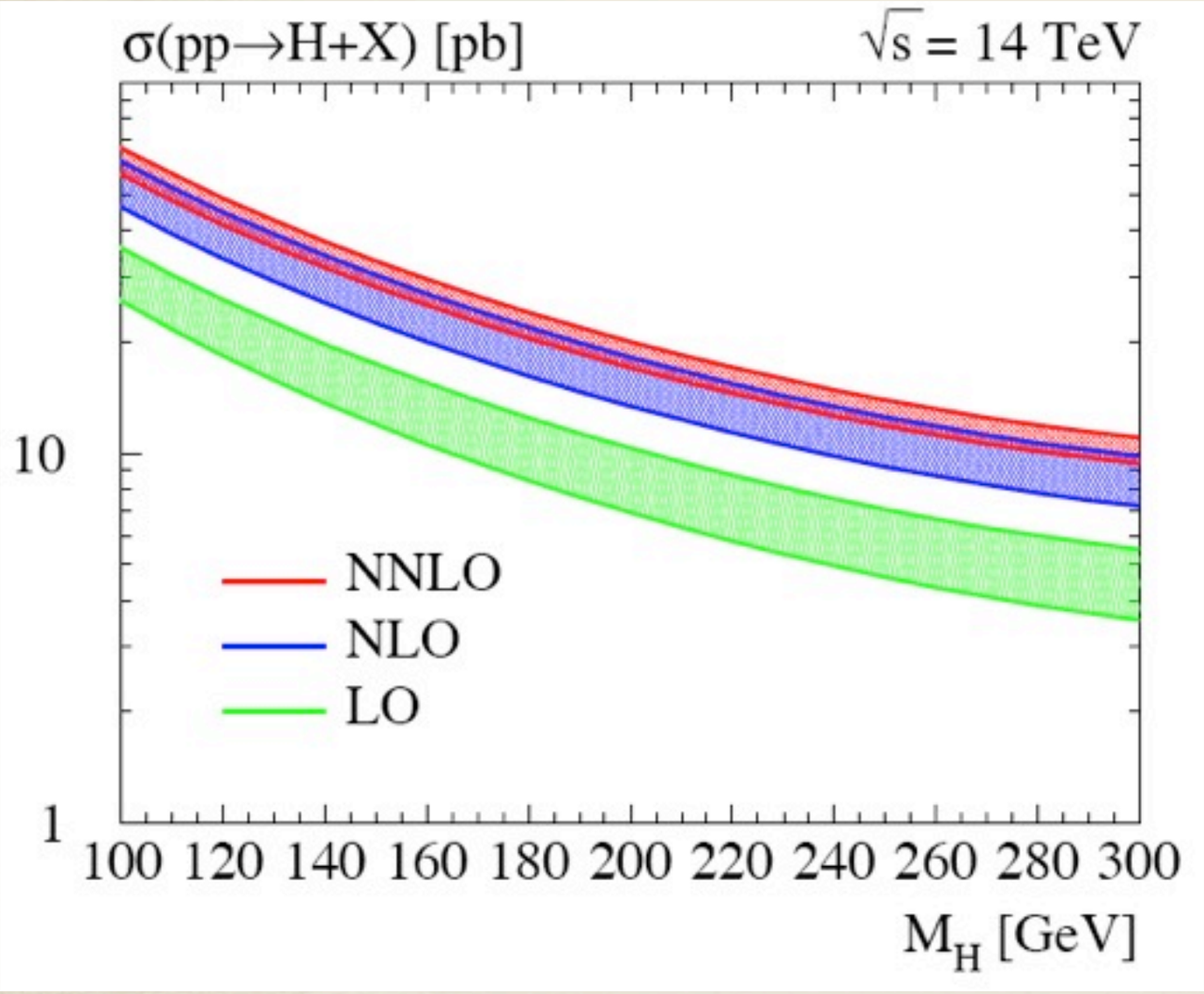
Assume $f_i \sim (1-x)^b$; plot L for various b
Look for peak near $z \approx 1$

⇒Sharp fall-off of gluon PDF enhances correction



# NNLO in the EFT

- Use of the EFT allows the NNLO cross section to be obtained




Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith van Neerven '03

- The left-over μ is associated with the factorization scale of the PDFs, and the renormalization scale of $\alpha_s$
- Must cancel in the all-orders result; use variation as an estimate of theoretical uncertainty
- Scale variation, especially at LO, can badly underestimate error!

# Unreasonably effective EFT

- NLO in the EFT:

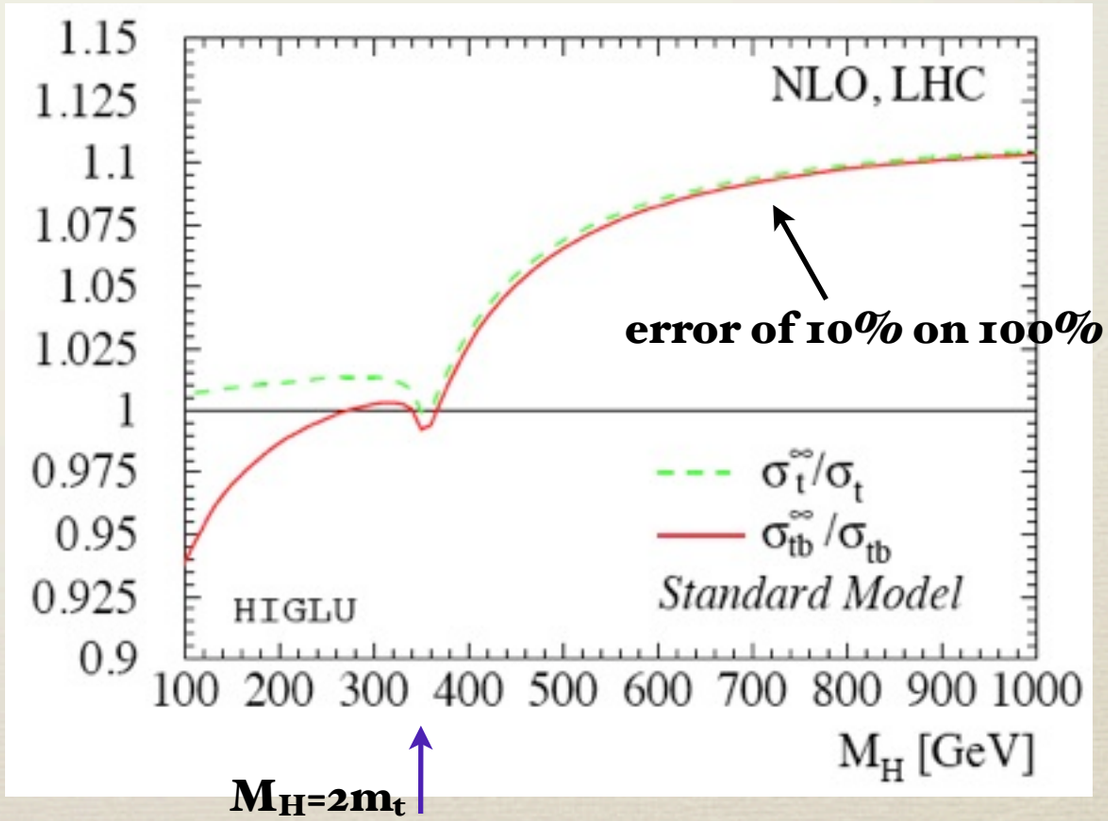
analytic continuation to time-like form factor

$$\Delta\sigma = \sigma_0 \frac{\alpha_s}{\pi}\left\{\left(\frac{11}{2}+\pi^2\right)\delta(1-z)+12\left[\frac{\ln(1-z)}{1-z}\right]_+ - 12z(-z+z^2+2)\ln(1-z) -6\frac{(z^2+1-z)^2}{1-z}\ln(z)-\frac{11}{2}(1-z)^3\right\}$$

eikonal emission of soft gluons

Identical factors in full theory with $\sigma_0 \rightarrow \sigma_{LO,\ \text{full theory}}$

$$\sigma_{NLO}^{approx} = \left(\frac{\sigma_{NLO}^{EFT}}{\sigma_{LO}^{EFT}}\right)\sigma_{LO}^{QCD}$$




NNLO study of $1/m_t$ suppressed operators, matched to large s-hat limit, large indicates this persists
Harlander, Mantler, Marzani, Ozeren; Pak, Rogal, Steinhauser 2009

# Summary of gluon fusion

- Serves as a very accurate framework for all LHC phenomenology
- Current uncertainty estimates: roughly 10% from uncalculated higher orders, 10% from PDFs, a few percent from other effects (use of EFT, bottom-quark effects, EW effects)

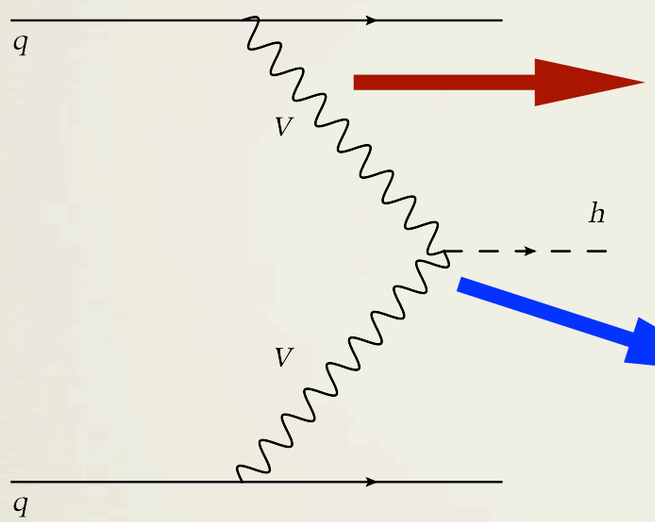
Useful references: S. Dawson, NPB359 (1991) 283-300 and *QCD and Collider Physics* by Ellis, Stirling, Webber (detailed NLO calculation);
1101.0593 (detailed discussion of uncertainties)

Available codes: http://theory.fi.infn.it/grazzini/hcalculators.html
http://www.phys.ethz.ch/~pheno/ihixs/index.html
http://particle.uni-wuppertal.de/harlander/software/ggh@nnlo/
HIGLU: http://people.web.psi.ch/spira/higlu/

# Phenomenology of the other modes

# Weak boson fusion: effective W/Z

- Important throughout large region of Higgs mass and in many decay modes; forward jets give experimental handle
- First approximation: inclusive cross section for $M_H >> M_{W,Z}$




$$\frac{1}{(p_1 - p_3)^2 - M_V^2} \quad \Rightarrow \quad \text{peaked at } p_1 \cdot p_3 = 0$$

$$\Rightarrow \tau_{q \to V} \quad \sim \quad 1/M_V$$

$$\tau_{VV \to h} \sim 1/M_H \ll \tau_{q \to V}$$

- Should be able to factorize, think of V as a parton in q

$$\sigma_{qq \to VV \to h} = \int dz_1 dz_2 \, f_{q/V_1}(z_1) f_{q/V_2}(z_2) \, \sigma_{VV \to h}$$

# VBF + the equivalence theorem

- Can derive when $M_V \ll \sqrt{s}$ (small angle scattering dominated)

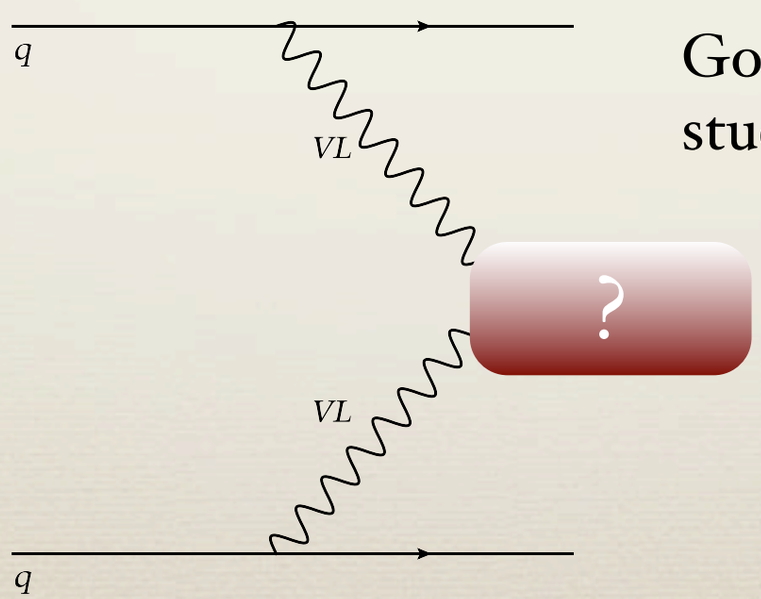
$$\sigma_{q_1 q_2 \to VV \to h} = \int_{2M_V/\sqrt{\hat{s}}}^{1} dz_1 \int_{2M_V/\sqrt{\hat{s}}}^{1} dz_2\, f_{q/V_L}(z_1) f_{q/V_L}(z_2)\, \sigma_{V_L V_L \to h}(z_1 z_2 \hat{s})$$

$$\sigma_{V_L V_L \to h}(x) = \frac{\pi}{36}\, g_{HVV}^2 \frac{x}{M_V^2} \delta(x - M_H^2)$$

$$f_{q/V_L}(z) = \frac{g_v^2 + g_a^2}{4\pi^2} \frac{1-z}{z}$$

Exercise: Derive this

- Angular momentum cons. prevents emission of transverse boson with forward quark: $\bar{u}^{\pm}(p\hat{z})\, \not{\epsilon} u^{\pm}(p'\hat{z}) \Rightarrow \text{Set} \quad \not{\epsilon} = \gamma^{1,2} \Rightarrow \xi_{\pm}^{\dagger} \sigma^{1,2} \xi_{\pm} = 0$

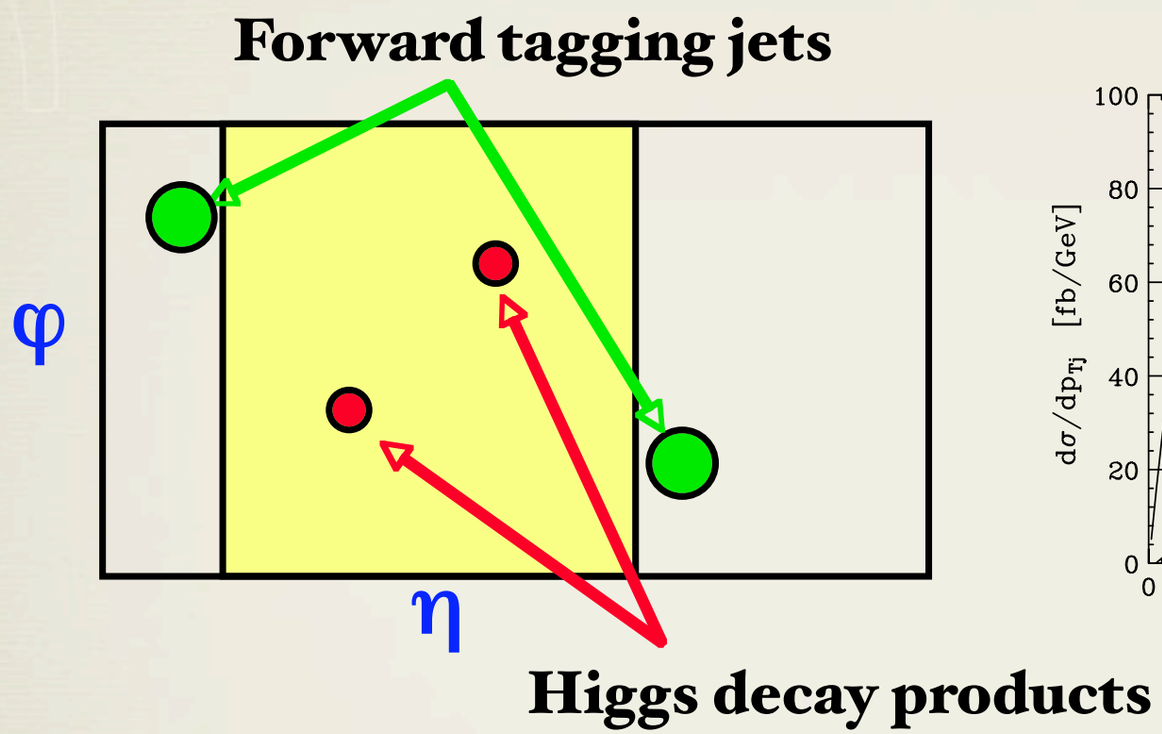


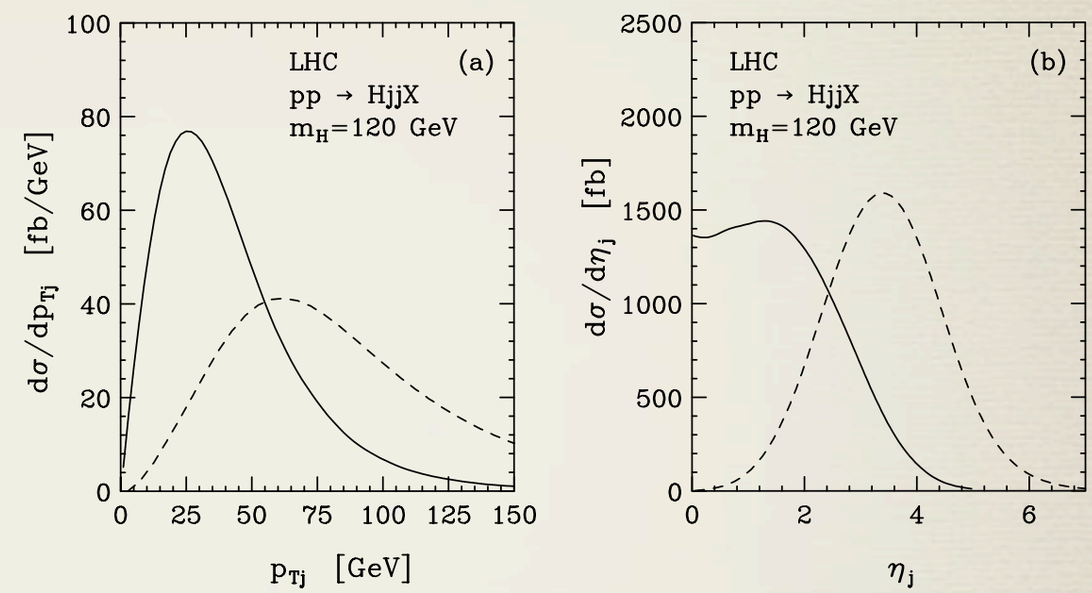

Good channel to study strong EWSB

Dawson 1984; Chanowitz, Gaillard 1985

# Kinematics of VBF

- Two energetic ($p_T$~40 GeV) jets with large rapidity separation




Rainwater, Zeppenfeld 1999 and many others... check refs+citations

- Extra gluon emission suppressed; impose central jet veto

$$\mathcal{M}(q_1 q_2 \to q_3 q_4 h + g) \quad \propto \quad \mathcal{M}(q_1 q_2 \to q_3 q_4 h)\, T^a \left\{ \frac{p_3 \cdot \epsilon_g^a}{p_3 \cdot p_g} + \frac{p_4 \cdot \epsilon_g^a}{p_4 \cdot p_g} - \frac{p_1 \cdot \epsilon_g^a}{p_1 \cdot p_g} - \frac{p_2 \cdot \epsilon_g^a}{p_2 \cdot p_g} \right\}$$

$$\to \quad 0 \;\text{ since }\; p_1 \parallel p_3, \;\; p_2 \parallel p_4$$

Exercise: Derive this

# VH associated production

- With bb decay of Higgs, most important low-mass mode at Tevatron
- Boosted analysis promising at LHC Butterworth, Davison, Rubin, Salam 2008

- Inclusive NLO QCD: +30% (Han, Wllenbrock 1990)
- NLO EW: +5-10% (Ciccolini, Dittmaier, Denner 2003)
- NNLO QCD: 1-2% in bulk of phase space (Ferrera, Grazzini, Tramontano 2011)

| Variable | $W(\ell\nu)H$ | $Z(\ell\ell)H$ | $Z(\nu\nu)H$ |
|---|---|---|---|
| $p_T(j_1)$ | $> 30$ GeV | $> 20$ GeV | $> 80$ GeV |
| $p_T(j_2)$ | $> 30$ GeV | $> 20$ GeV | $> 20$ GeV |
| $p_T(jj)$ | $> 150$ (165) GeV | $> 100$ GeV | $> 160$ GeV |
| $p_T(V)$ | $> 150$ (160) GeV | $> 100$ GeV | – |
| $E_T^{miss}$ | $> 35$ GeV [for $W(e\nu)H$] | – | $> 160$ GeV |
| $\Delta\phi(V, H)$ | – ($> 2.95$) rad | – ($> 2.90$) rad | – ($> 2.90$) rad |
| $CSV_{max}$ | $> 0.40$ (0.90) | $> 0.244$ (0.90) | $> 0.50$ (0.90) |
| $CSV_{min}$ | $> 0.40$ | $> 0.244$ (0.50) | $> 0.50$ |
| $N_{al}$ | $= 0$ | – | $= 0$ |
| $N_{aj}$ | – ($= 0$) | – ($< 2$) | – |
| $\Delta\phi(E_T^{miss}, jet)$ | – | – | $> 0.5$ (1.5) rad |

CMS, 1202.4195

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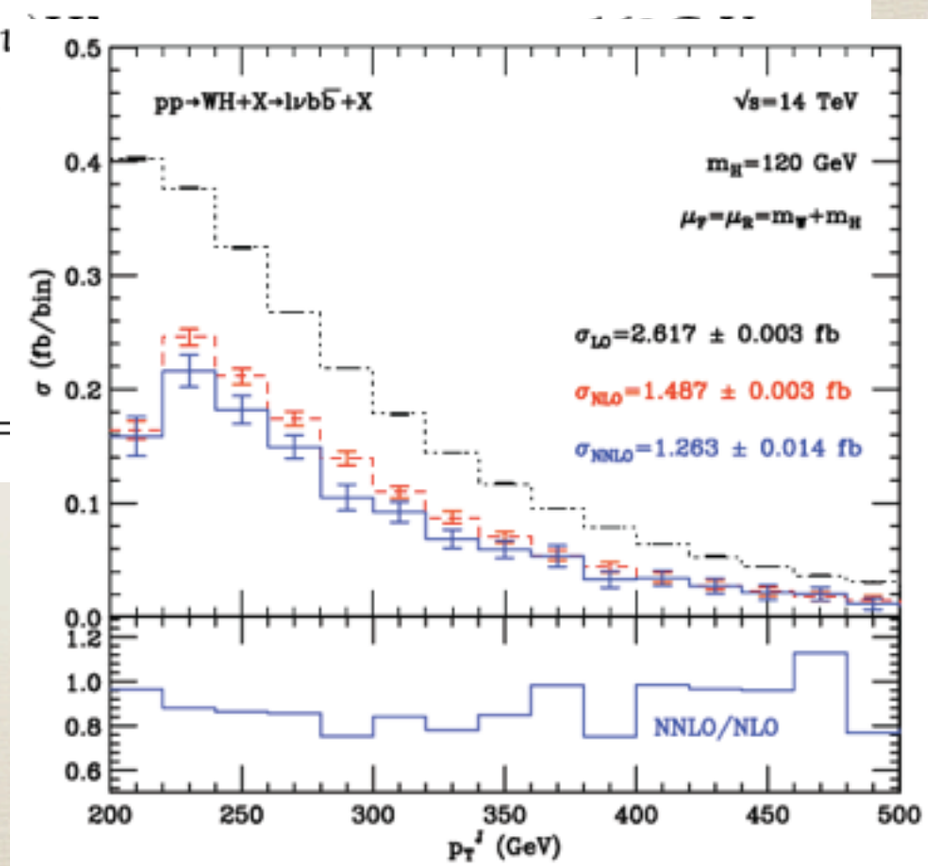
| $CSV_{min}$ | $> 0.40$ | | |
| $N_{al}$ | $= 0$ | | |
| $N_{aj}$ | – ($= 0$) | | |
| $\Delta\phi(E_T^{miss}, jet)$ | – | | |

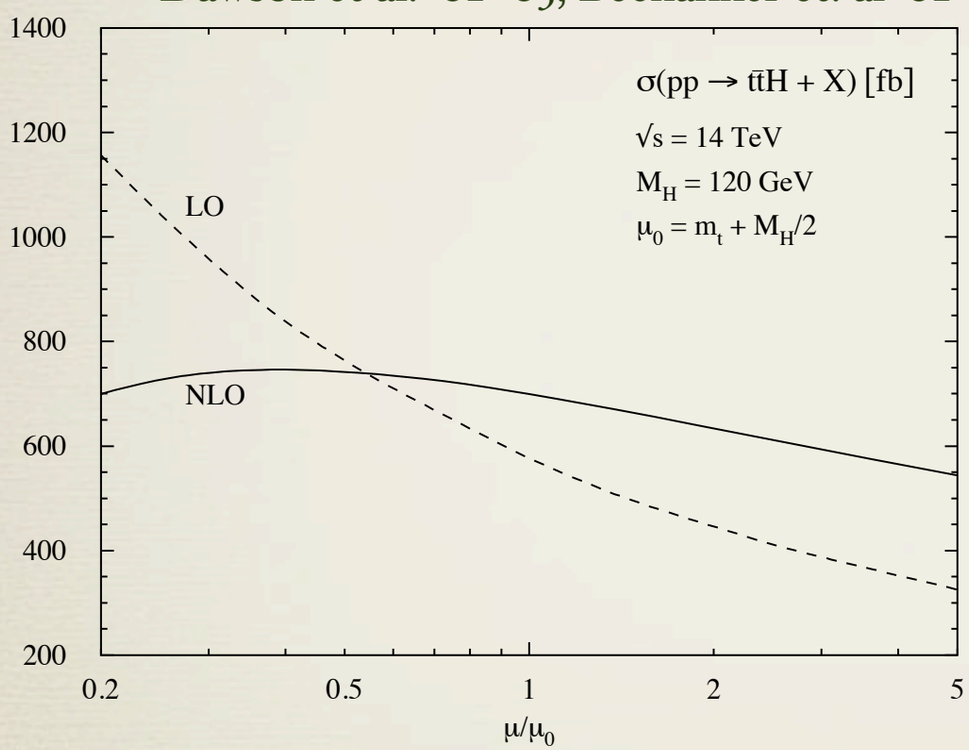



- Special care must be taken with predictions when analysis imposes a jet veto!

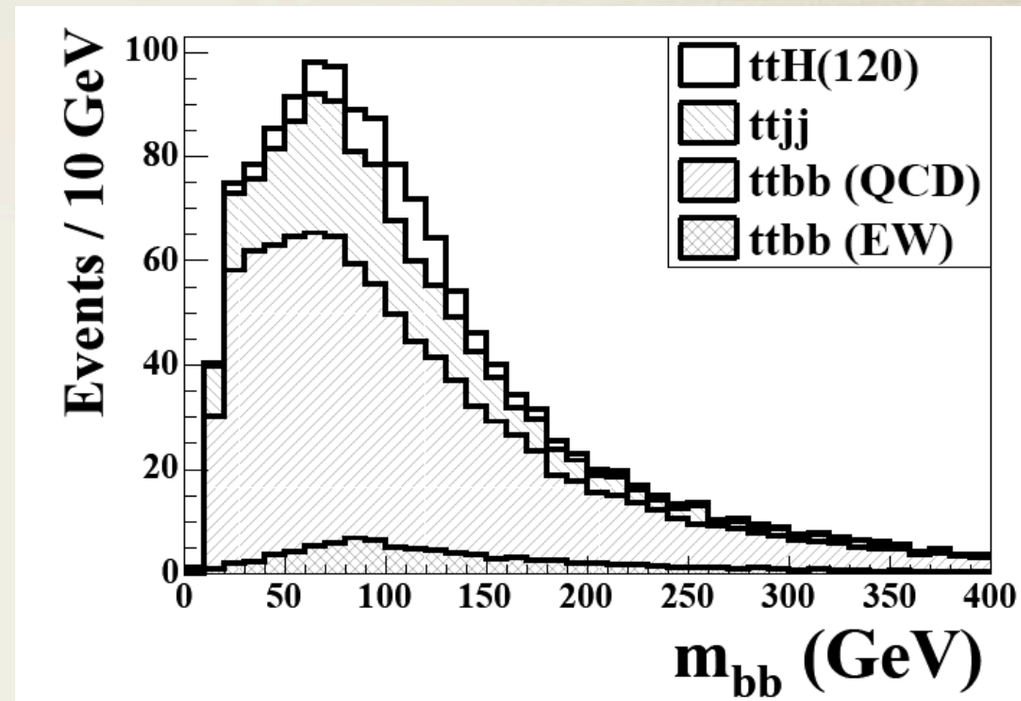
# tth associated production

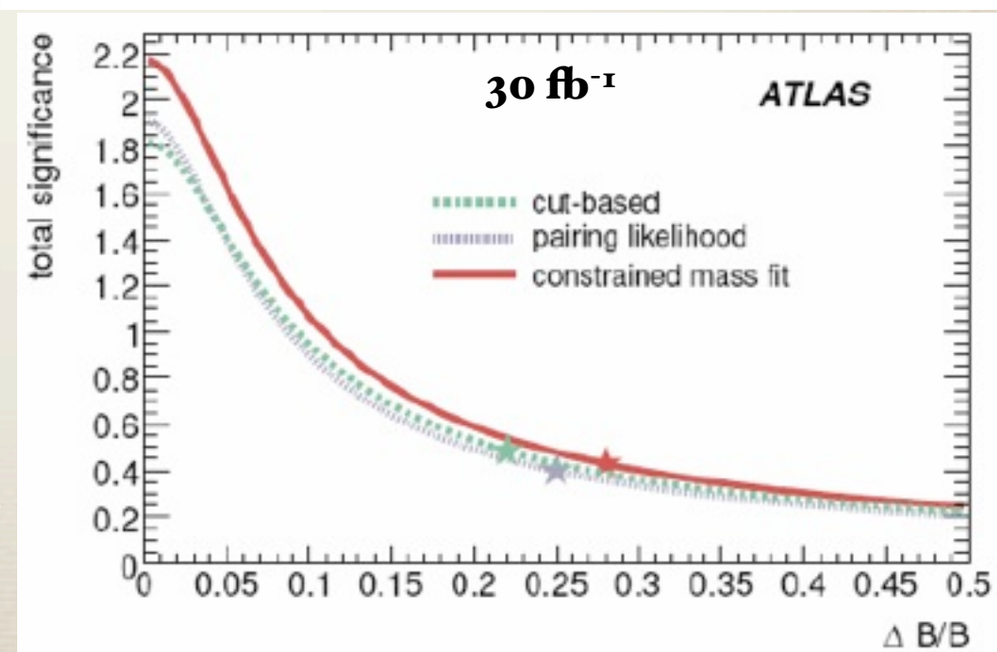
- Allows measurement of htt Yukawa coupling, and also hbb coupling through h→bb decay

Dawson et al. '02-'03; Beenakker et. al '02




NLO corrections reduce scale dependence




- Large SM ttbb background that cuts shape to look just like signal; high luminosity only

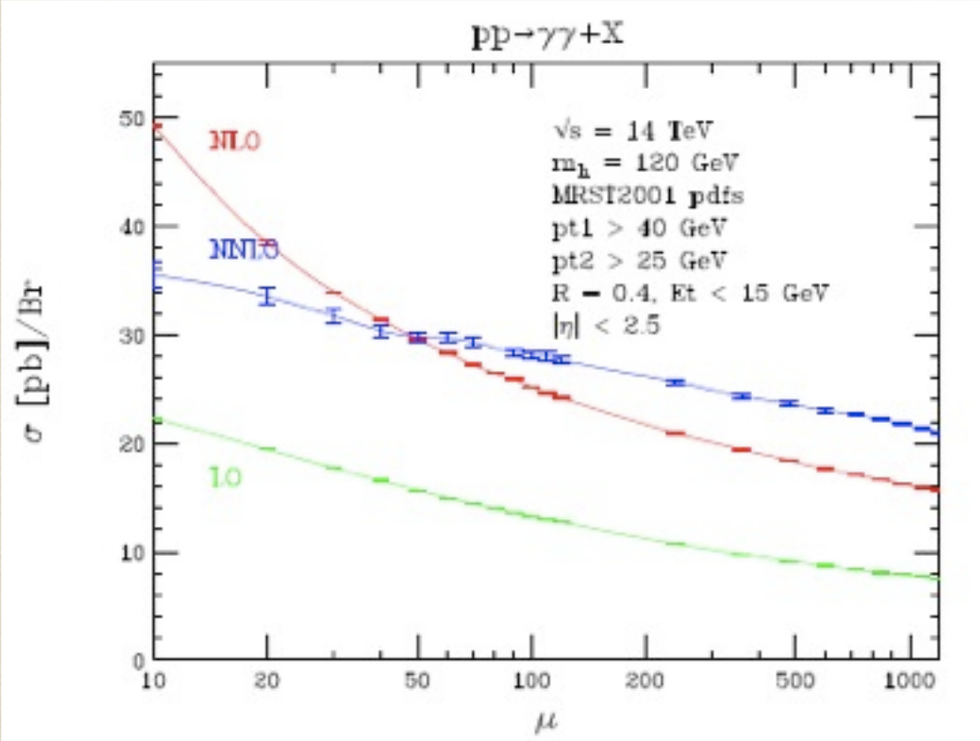
# References

The current detailed status of Higgs production in the Standard Model and the MSSM is reviewed in two CERN Yellow Reports: 1101.0593, 1205.4465
An older but still useful review is: Djouadi, hep-ph/0503172, hep-ph/0503173

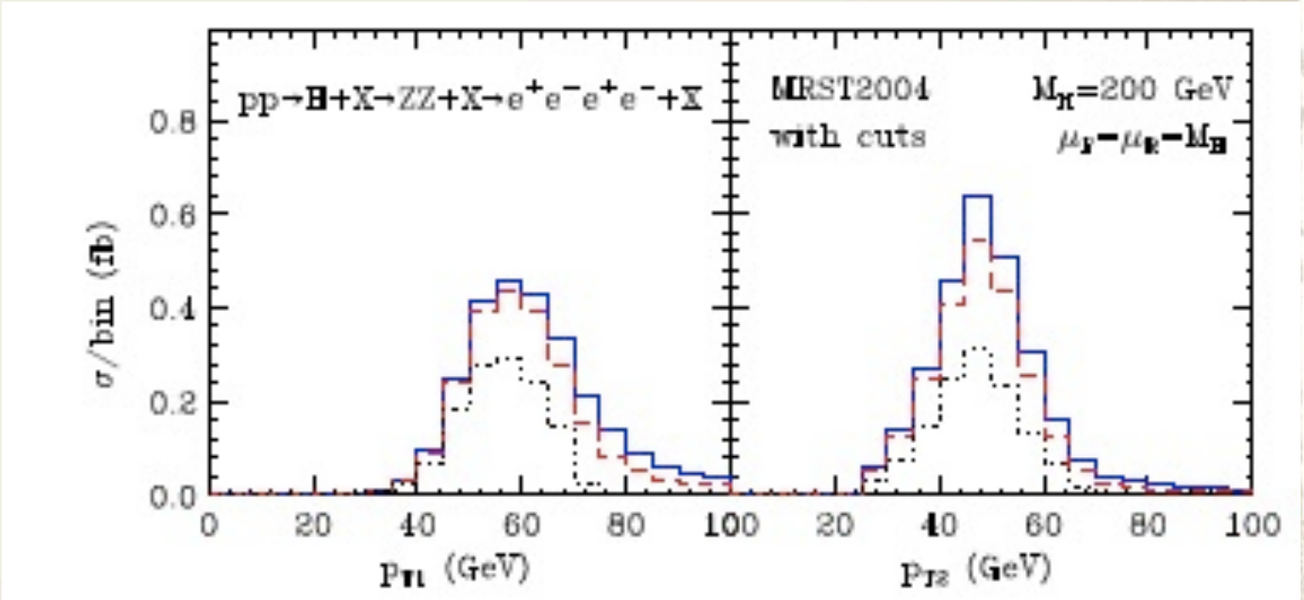
# Current issue: differential cross sections and jet vetos

# Confronting reality

- Unfortunately, the overwhelming backgrounds at the LHC require that significant cuts are imposed on the final state.
- For gluon fusion, two NNLO parton-level simulation codes exist



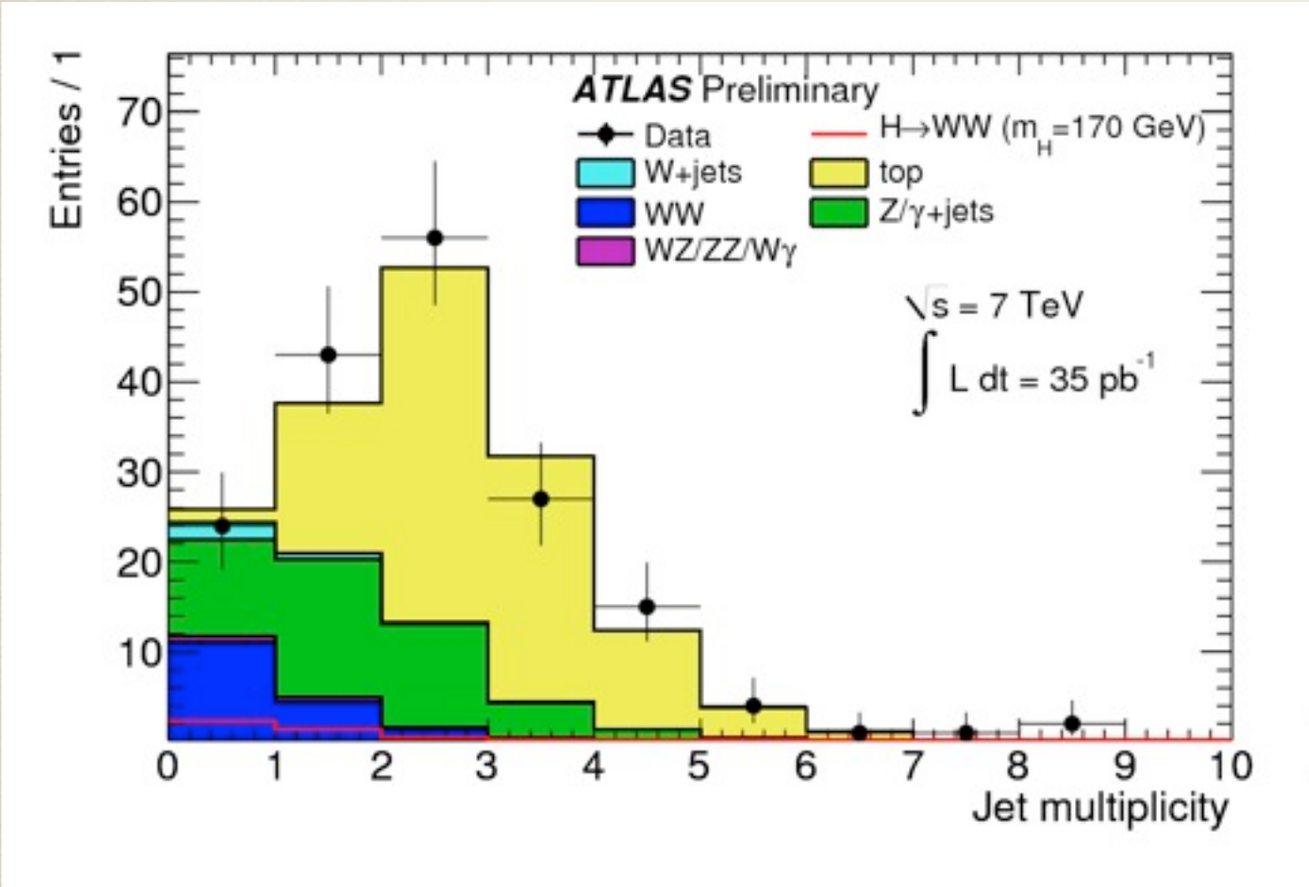

**FEHiP**: Anastasiou, Melnikov, FP 2005



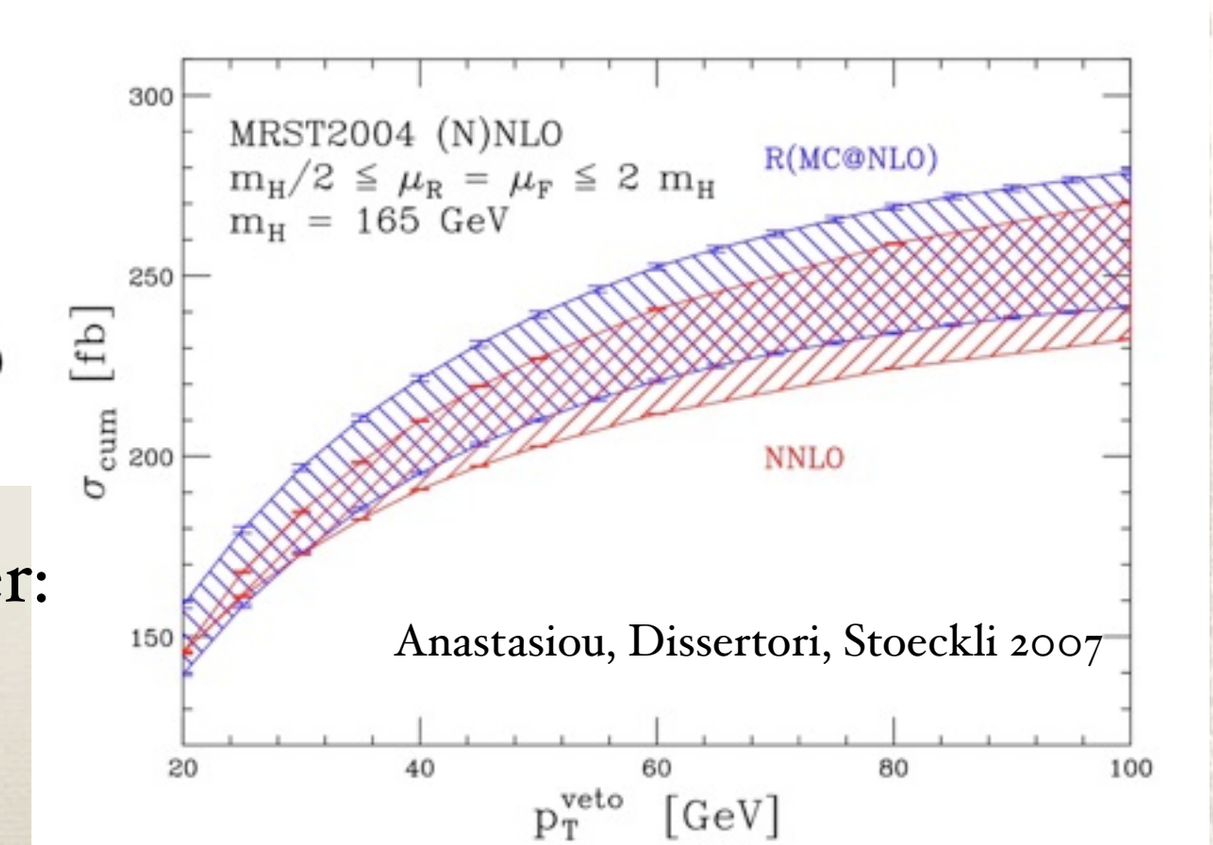

**HNNLO**: Catani, Grazzini 2007-2008

# The jet veto

- A typical cut is to divide the final state into bins of differing jet multiplicity




- Required in the WW channel to reduce top-quark background
- 25-30 GeV jet cut used

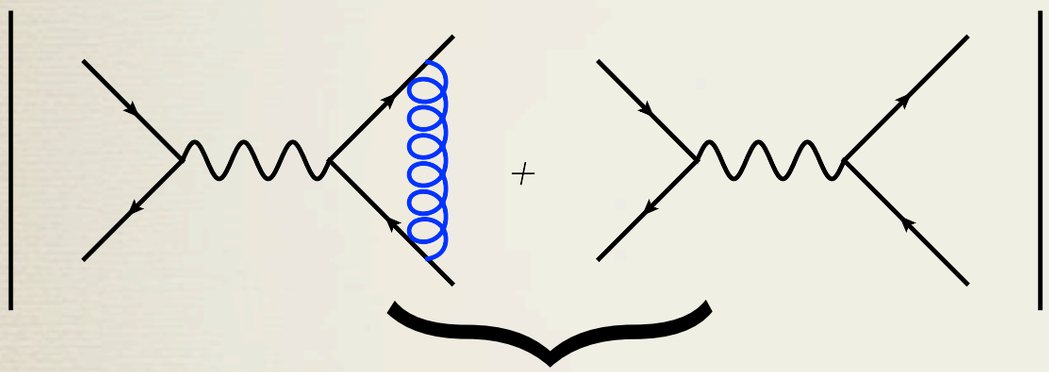



Anastasiou, Dissertori, Stoeckli 2007

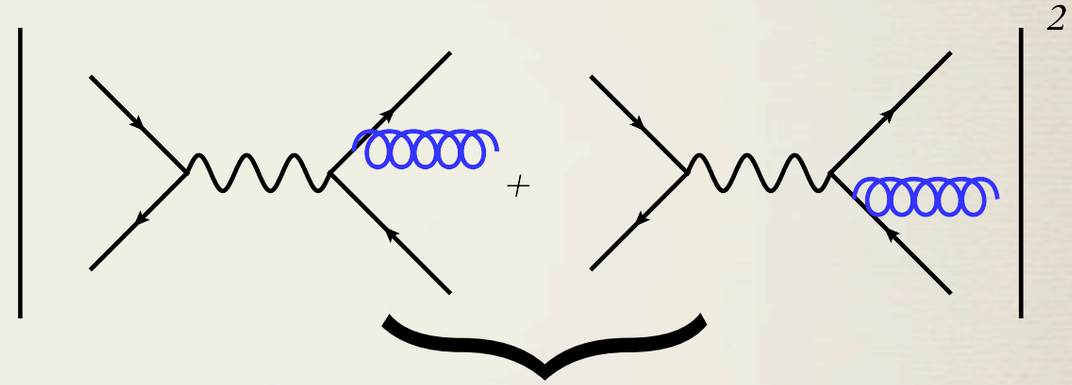
- When we try to compute at fixed order:
- Does the uncertainty really become smaller with a stricter veto?

# The jet veto

- Significant interest in trying to understand the impact of jet vetos on Higgs searches Stewart, Tackmann 1107.2117; Banfi, Salam, Zanderighi 1203.5773
- We also saw this in VH, although we'll focus on gluon-fusion here
- Why are jet vetos dangerous?




Virtual corrections: $-1/\varepsilon_{IR}^2$




Real corrections: $1/\varepsilon_{IR}^2 - \ln^2(Q/p_{T,cut})$

- Relevant log term for Higgs searches: $6(\alpha_S/\pi)\ln^2(M_H/p_{T,veto}) - 1/2$

$\Rightarrow$ should be *resummed* to all orders, fixed-order breaks down

# The jet veto

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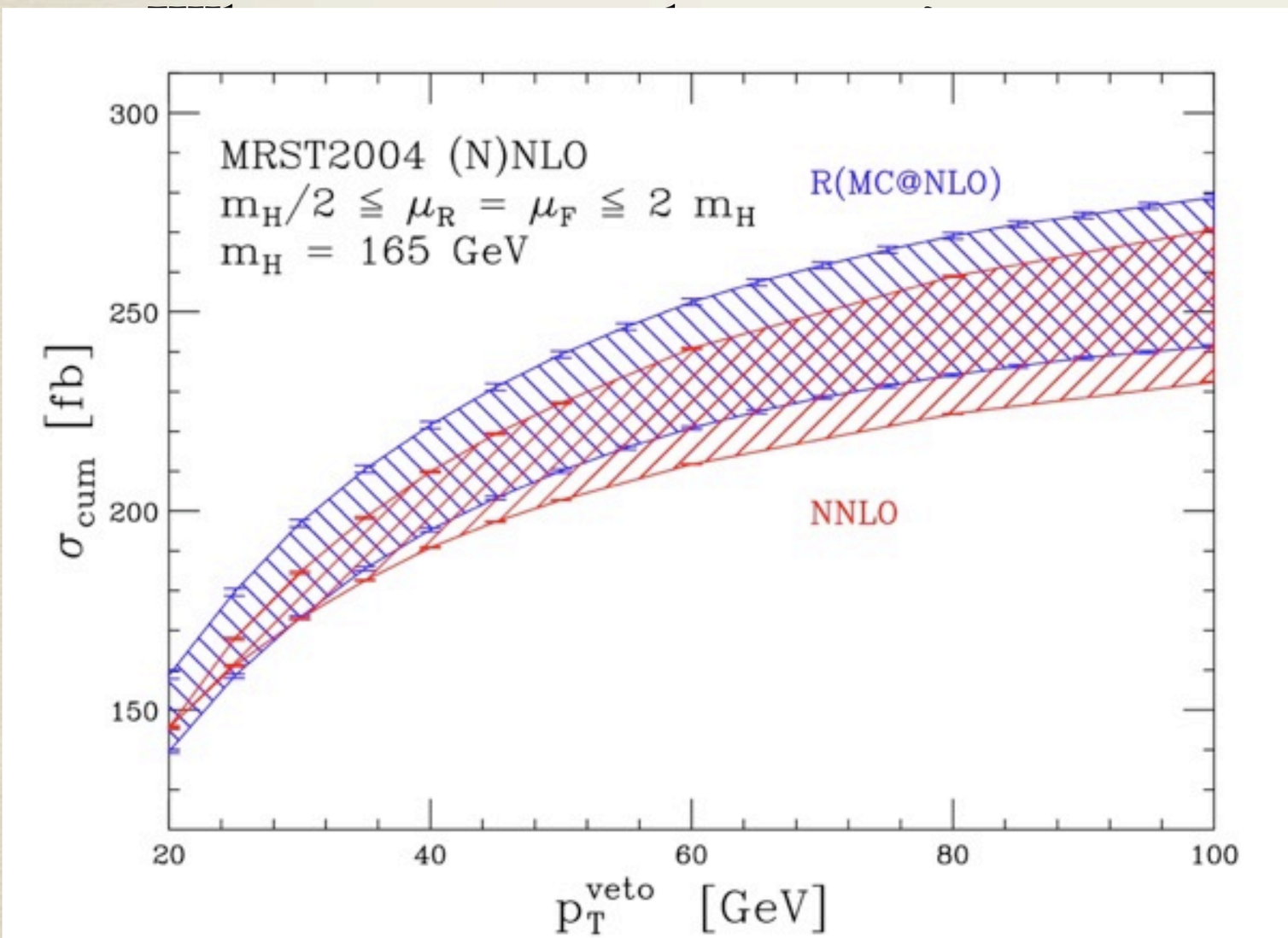
- We also saw this in VH, although we'll focus on gluon-fusion here




- Arises from an accidental cancellation between these logs and the large corrections to the inclusive cross section... no reason to persist at higher orders

# The jet veto

- Significant interest in trying to understand the impact of jet vetos on Higgs searches Stewart, Tackmann 1107.2117; Banfi, Salam, Zanderighi 1203.5773

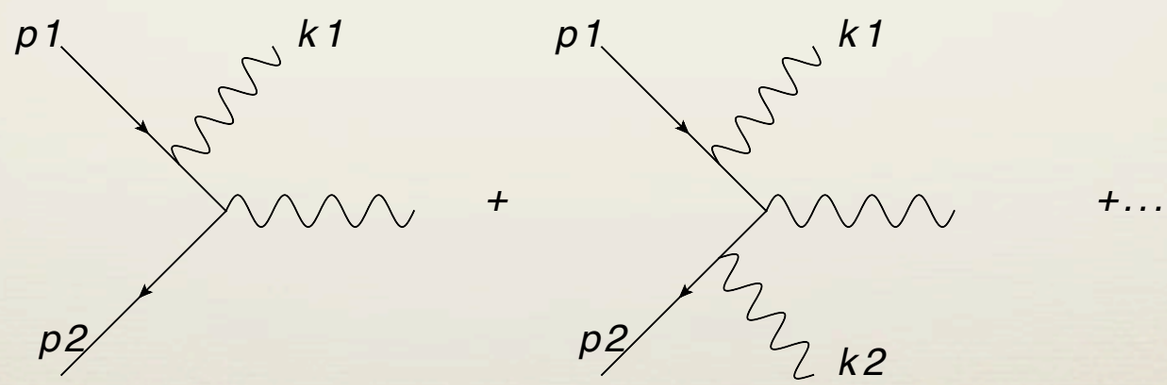
$$\sigma_0(p^{\rm cut}) = \sigma_{\rm total} - \sigma_{\geq 1}(p^{\rm cut})$$
$$\simeq \sigma_B\Big\{\big[1+\alpha_s+\alpha_s^2+\mathcal{O}(\alpha_s^3)\big] - \big[\alpha_s(L^2+L+1)+\alpha_s^2(L^4+L^3+L^2+L+1)+\mathcal{O}(\alpha_s^3 L^6)\big]\Big\}$$

$$\sigma_{\rm total} = (3.32\,{\rm pb})\big[1+9.5\,\alpha_s+35\,\alpha_s^2+\mathcal{O}(\alpha_s^3)\big]\,,$$
$$\sigma_{\geq 1}\big(p_T^{\rm jet}\geq 30\,{\rm GeV}, |\eta^{\rm jet}|\leq 3.0\big) = (3.32\,{\rm pb})\big[4.7\,\alpha_s+26\,\alpha_s^2+\mathcal{O}(\alpha_s^3)\big]\,.$$

- Arises from an accidental cancellation between these logs and the large corrections to the inclusive cross section... no reason to persist at higher orders

# Resumming jet-veto logs

- Option 1: directly resum the logs in the presence of a jet algorithm. This is complicated, and is the subject of 'healthy debate' in the literature Banfi, Monni, Salam, Zanderighi, 1206.4998; Tackmann, Walsh, Zuberi 1206.4312; Becher, Neubert 1205.3806
- Option 2: build intuition from simpler but closely related variables
- Typical choice is $p_T$ of the Higgs; equivalent to a jet veto through $O(\alpha_S)$. Other choices possible Berger et al. 1012.4480
- Toy example of $\ln(p_T)$ resummation: $e^+e^- \to \gamma^*$, multiple soft-photon effects



# Soft emissions in b-space

- Both matrix elements and phase space simplify in this limit

Eikonal approximation for n-photon matrix-elements:

$$\mathcal{M}_n \propto g^n \mathcal{M}_0 \left\{ \frac{p_1 \cdot \epsilon_1 \ldots p_1 \cdot \epsilon_n}{p_1 \cdot k_1 \ldots p_1 \cdot k_n} + (-1)^n \frac{p_2 \cdot \epsilon_1 \ldots p_2 \cdot \epsilon_n}{p_2 \cdot k_1 \ldots p_2 \cdot k_n} \right\}$$

Phase-space for n-photon emission:

$$d\Pi_n \propto \nu(k_{T1})\, d^2k_{T1} \ldots \nu(k_{Tn})\, d^2k_{Tn}\; \delta^{(2)} \Bigg( \overbrace{\vec{p}_T - \sum_i \vec{k}_{Ti}}^{\text{sum to Higgs } p_T} \Bigg)$$

$$\nu(k_T) = k_T^{-2\epsilon} \ln\left(\frac{s}{k_T^2}\right)$$

- Would be independent emissions if not for phase-space constraint
- Fourier transform:

$$\int \frac{d^2b}{(2\pi)^2}\; \mathrm{e}^{-i\vec{b}\cdot\vec{p}_T} \int d^2k_{T1}\, f(k_{T1}) \ldots d^2k_{Tn}\, f(k_{Tn})\, \delta^{(2)} \left( \vec{p}_T - \sum_i \vec{k}_{Ti} \right)$$

$$= \int \frac{d^2b}{(2\pi)^2}\, \mathrm{e}^{-i\vec{b}\cdot\vec{p}_T} \left[\tilde{f}(b)\right]^n, \quad \tilde{f}(b) = \int d^2k_T\, \mathrm{e}^{i\vec{b}\cdot\vec{k}_T} f(k_T)$$

# Exponentiation

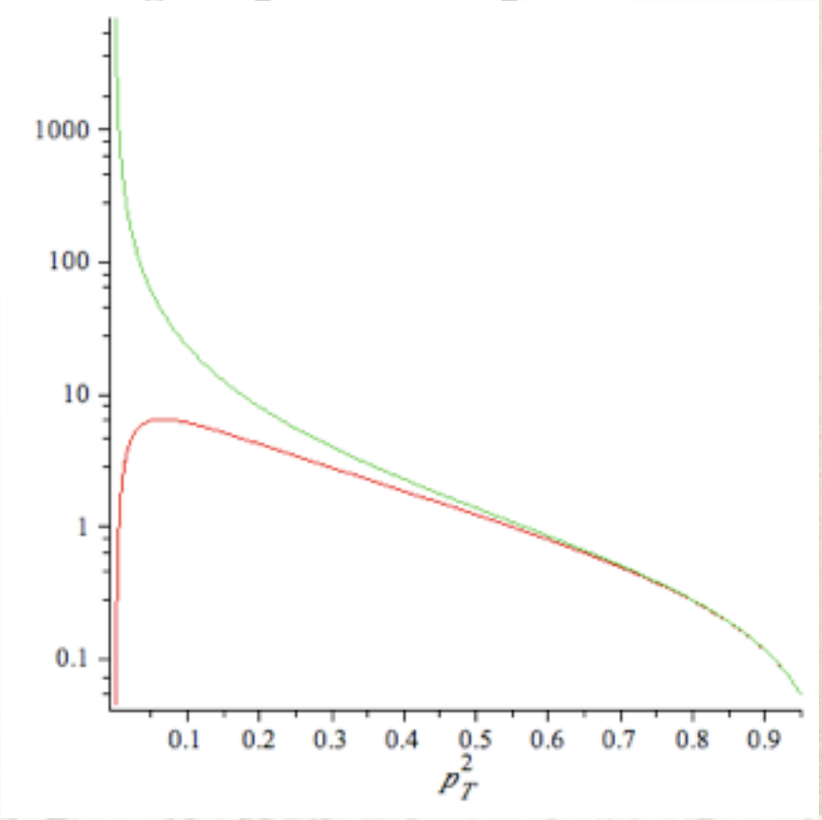
- Product of matrix elements and phase space now exponentiates

$$\frac{d\sigma}{d^2p_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2}\, \mathrm{e}^{-i\vec{b}\cdot\vec{p}_T}\, \tilde{\sigma}(b)$$

$$\tilde{\sigma}(b) = \exp\left\{ \frac{g^2}{4\pi^2} \int d^2k_T\, \mathrm{e}^{i\vec{b}\cdot\vec{k}_T} \left[ \frac{\ln(s/k_T^2)}{k_T^2} \right]_+ \right\}$$

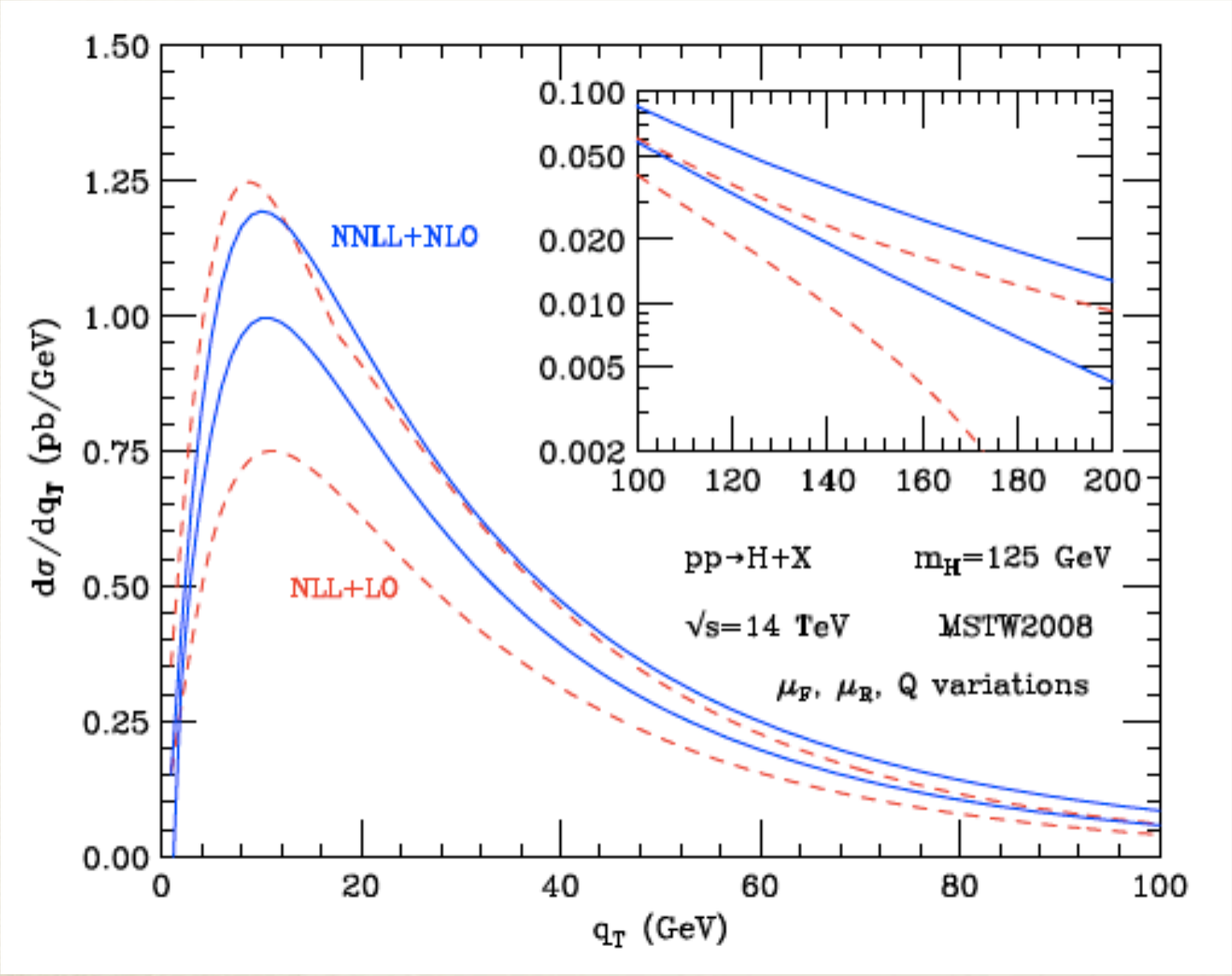
- Large b ⇔ small $p_T$; inverse transform keeping leading terms

$$\frac{d\sigma}{dp_T^2} = \frac{\alpha}{\pi}\sigma_0 \frac{1}{p_T^2} \ln\frac{s}{p_T^2} \exp\left\{ -\frac{\alpha}{2\pi} \ln^2 \frac{s}{p_T^2} \right\}$$



# $P_T$ resummation for Higgs

- Known to the next-to-next-to-leading logarithmic level



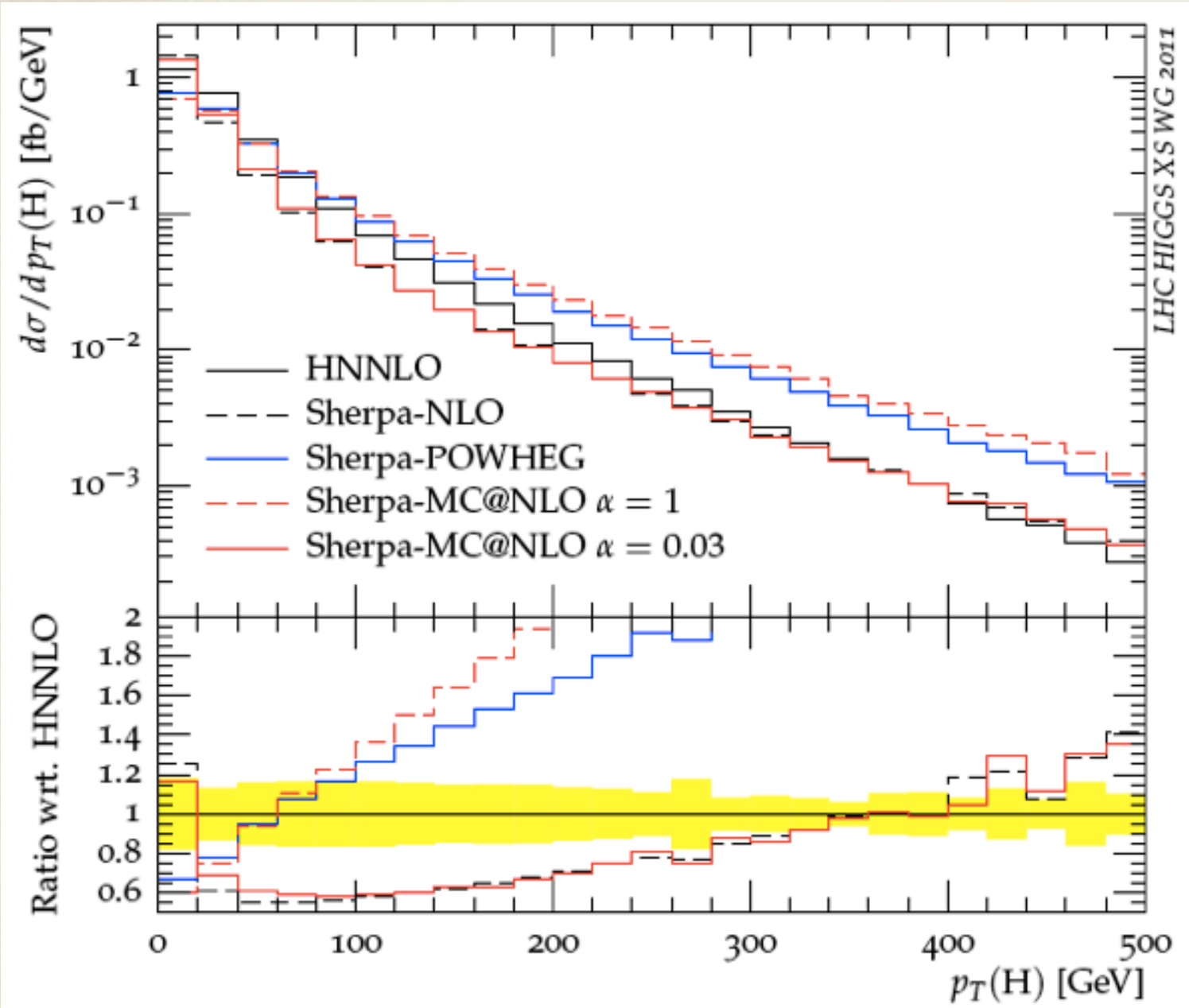

**HqT**: de Florian, Ferrera, Grazzini, Tommasini 2011

- Used to reweight Monte-Carlo simulation programs such as POWHEG, MC@NLO to properly model Higgs kinematics and describe the jet veto

Classic ref for low $p_T$ resummation: Collins, Soper, Sterman NPB250 (1985)
b-space: Parisi, Petronzio NPB154 (1979)

# $P_T$ resummation for Higgs

- This reweighting of Monte Carlos is necessary!




- What exactly is stuck up in the exponent in the various codes modifies the pT spectrum dramatically
- Matching to resummed calculation needed to ameliorate these differences

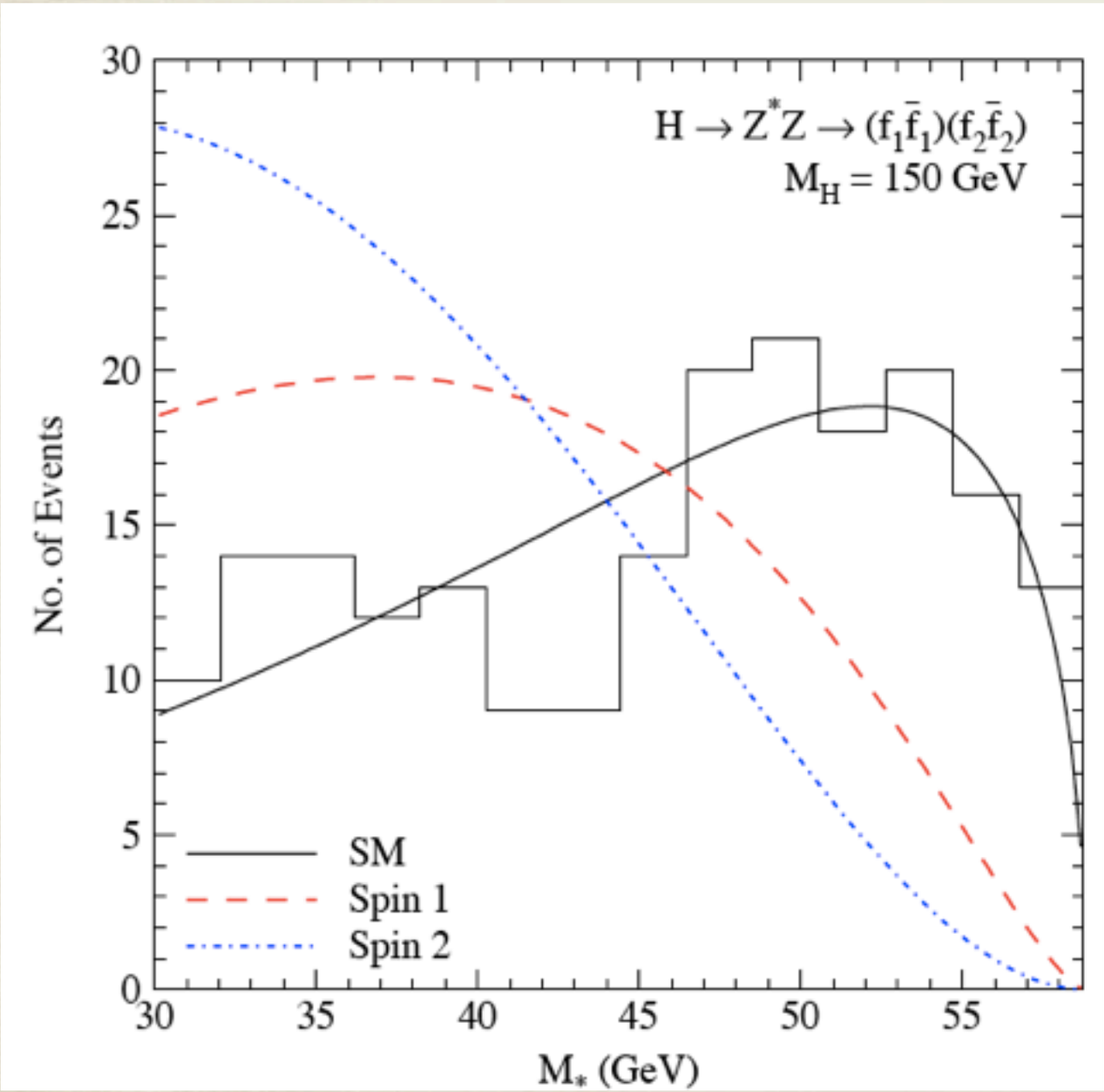
# Current issue: analyzing the discovery

# What we want to know

- Now that a new state has been found, what properties do we want to measure
- Clearly the spin; the Landau-Yang theorem tells us that it's either spin-0 or spin-2, not spin-1
- Assume spin-0 for now: is it CP-even, CP-odd, or a mixture?
- What are the values of the couplings to the other SM states? This will point toward whether it's a SM Higgs, a composite one, or something else

# Spin determination in ZZ*

- Four lepton final state offers several kinematic handles




Choi et al. hep-ph/0210077

Decay distribution of $M_*$, the invariant mass of the off-shell Z, has different behavior near the kinematic limit for spin-0, spin-2

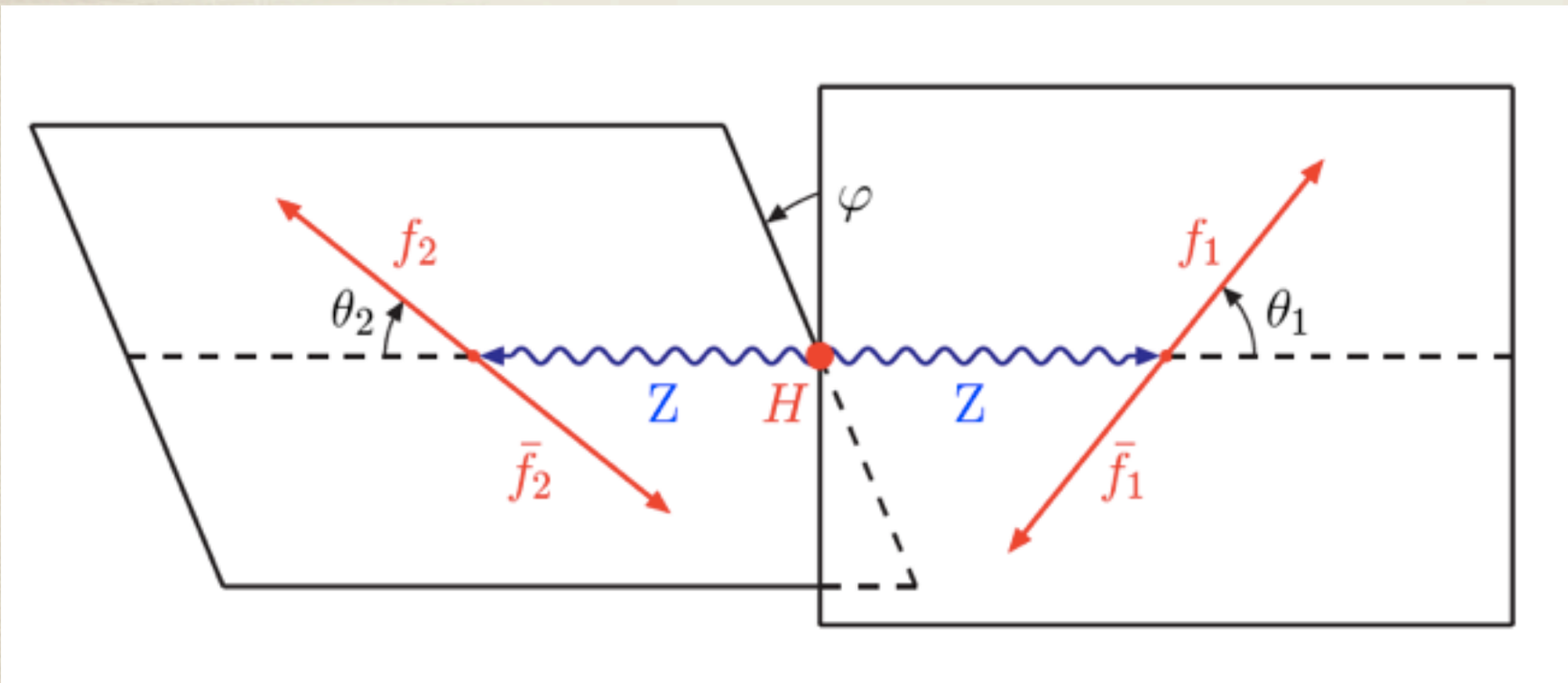
$$\frac{d\Gamma_0}{dM_*^2} \sim \beta$$

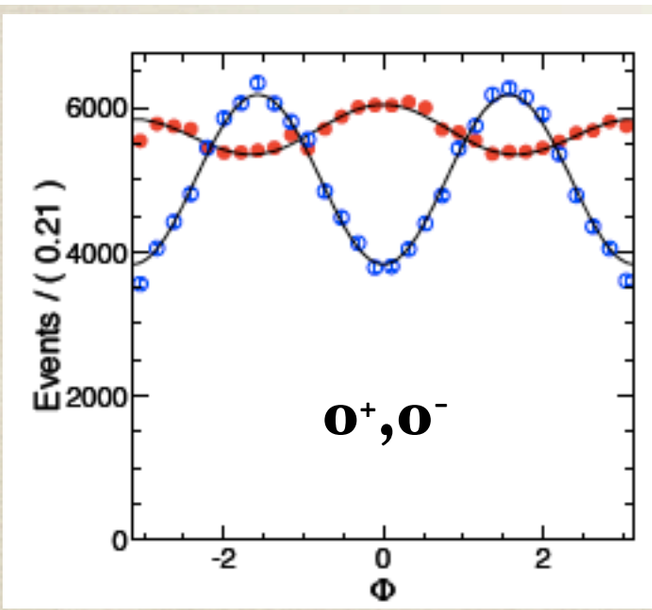
$$\frac{d\Gamma_2}{dM_*^2} \sim \beta^5$$

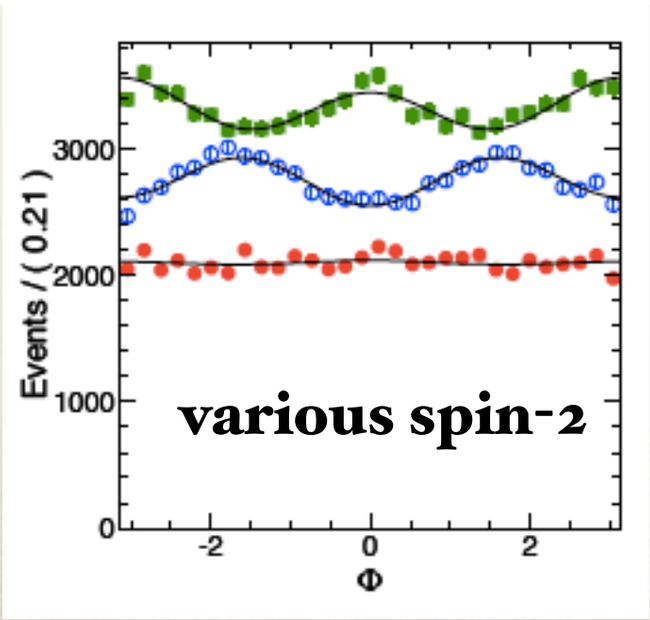
$$\beta \sim \sqrt{(M_H - M_Z)^2 - M_*^2}$$

# Spin, parity determination in ZZ*

- Four lepton final state offers several kinematic handles



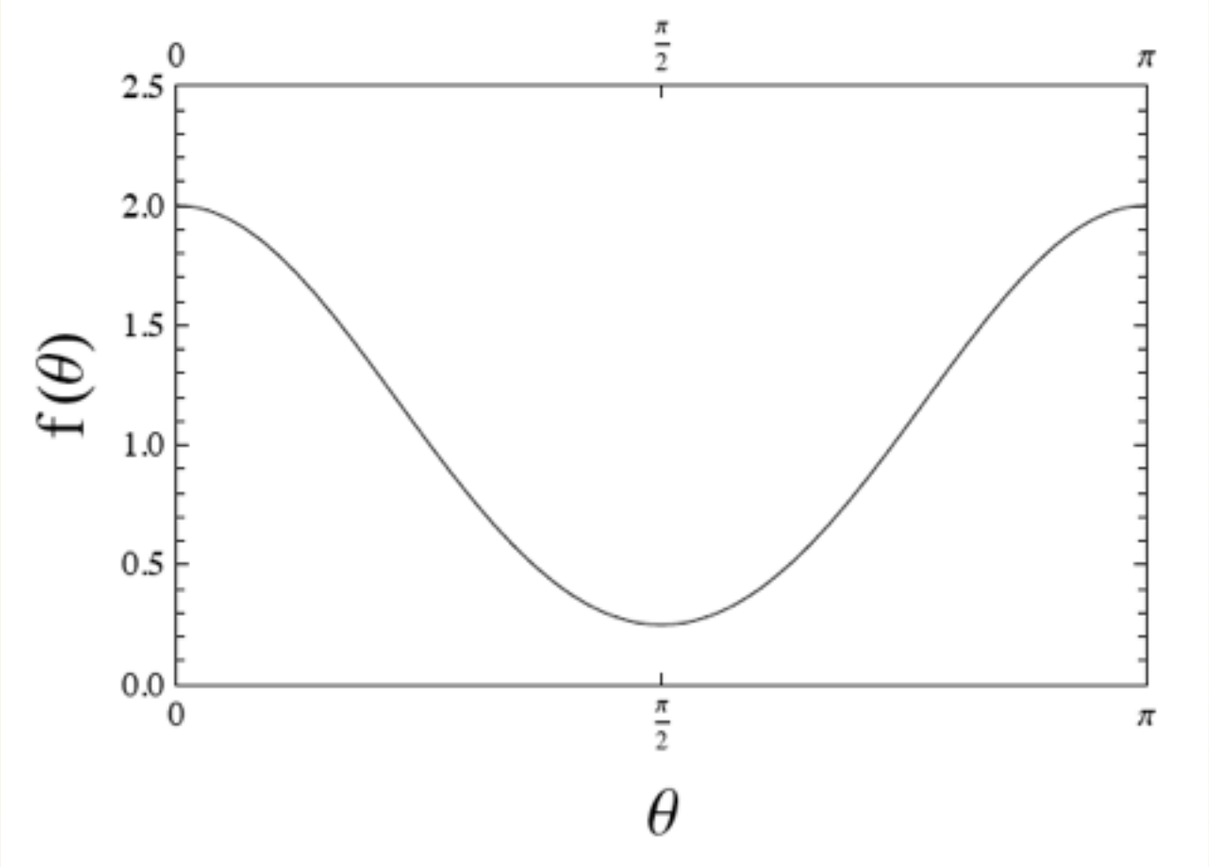

Can perform multi-variate analysis including all angular information to discriminate spins

Gao et al., 1001.3396

# Spin determination in γγ

- Polar angle distribution of photons is flat for spin-0, not for spin-2

$$\frac{d\sigma}{d\cos\theta} \propto 1 + 6\cos^2\theta + \cos^4\theta$$




Ellis, Hwang 1202.6660

- Background is large, but its angular distribution is measurable in sidebands; the large fraction from prompt photon production is also calculable

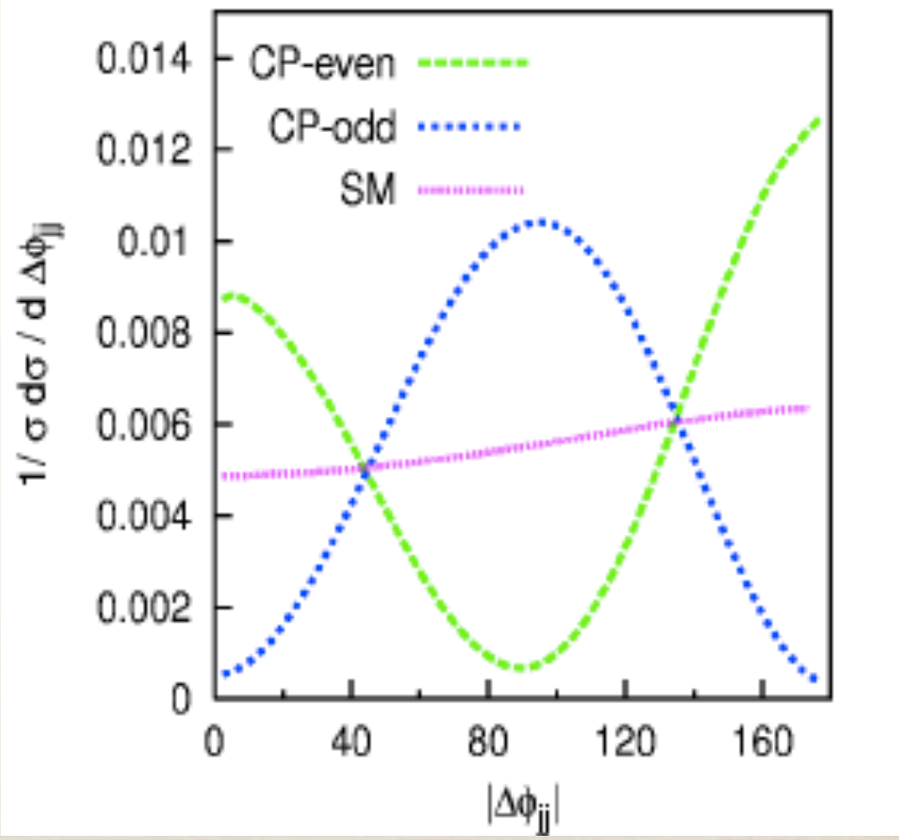
# CP determination in H+jets

- Angular distributions in both the VBF and gg production modes give a handle on the CP properties of the state

- General structure of VV→Φ tensor :

$$T^{\mu\nu}(q_1,q_2) = a_1(q_1,q_2)\, g^{\mu\nu} + a_2(q_1,q_2)\,[q_1\cdot q_2 g^{\mu\nu} - q_2^{\mu} q_1^{\nu}] + a_3(q_1,q_2)\,\varepsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma}$$

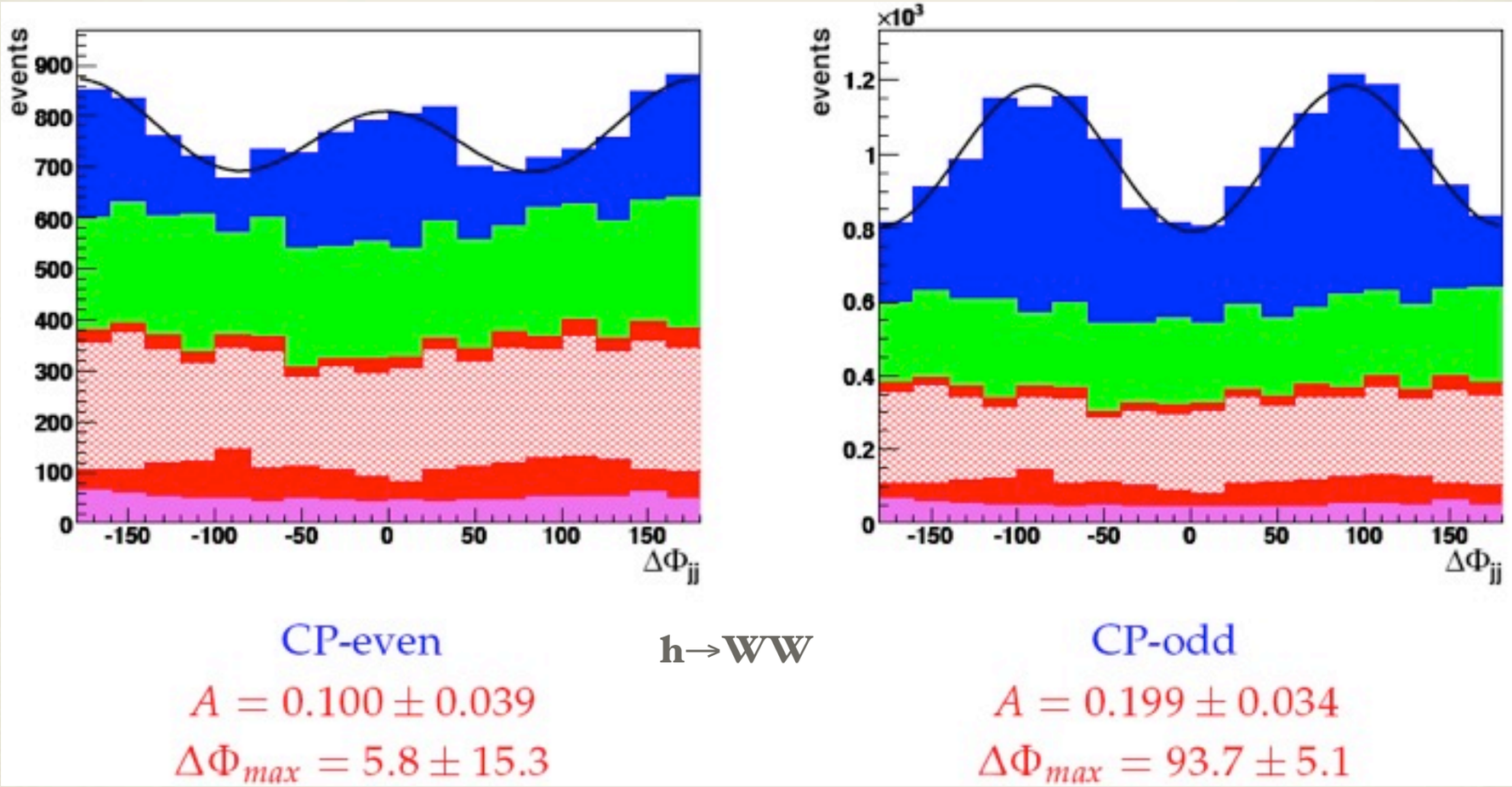
$a_1$=const : SM  $a_2$ : CP-even  $a_3$ : CP-odd




from M. Duehrssen

# CP determination in H+jets

- Angular distributions in both the VBF and gg production modes give a handle on the CP properties of the state




CP-even: $$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi}\frac{h}{v}G^a_{\mu\nu}G^{\mu\nu}_a$$

CP-odd: $$\mathcal{L}_{eff} = \frac{\alpha_s}{8\pi}\frac{a}{v}G^a_{\mu\nu}G^a_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}$$

# Measuring Higgs couplings

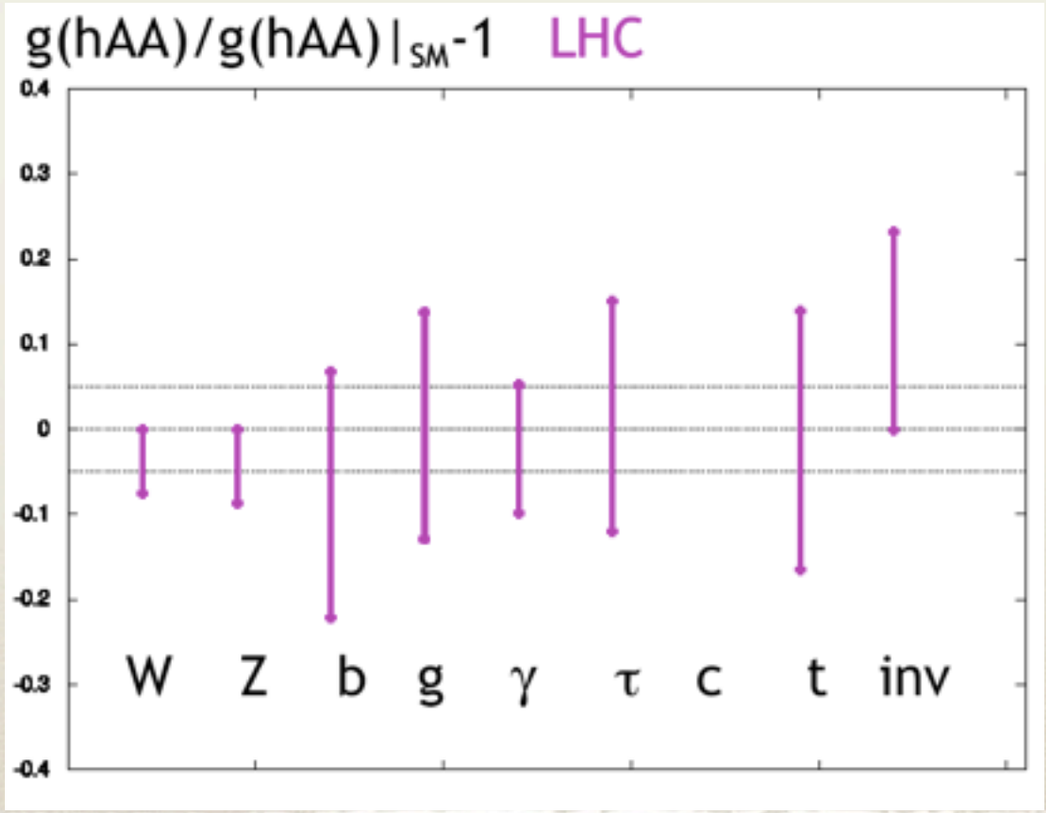
- Measurements at LHC of AA→H→BB measure the combination

$$\frac{g^2(hAA)\, g^2(hBB)}{\Gamma_{tot}}$$

Scaling degeneracy if total width unknown:

$$g^2 \to f\, g^2, \Gamma_{tot} \to f^2\, \Gamma_{tot}$$

- Total width is unmeasurable, but mild theoretical assumptions valid in models with a CP-even Higgs and no doubly-charged scalar states, together with VBF WW measurement, can tightly bound $\Gamma_{tot}$




From Peskin, 1207.2516; see also Duhrssen et al. hep-ph/0406323

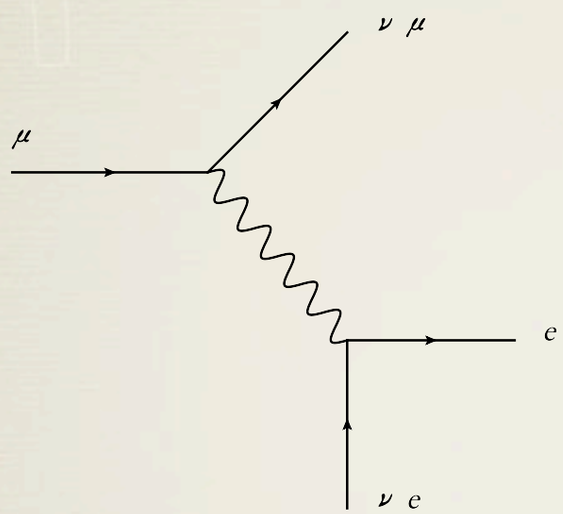
# Conclusions

- It's an exciting time to be doing high energy physics, and an especially prescient choice by the SSI organizers to focus on the Higgs this year...
- Just the beginning; we don't yet know much about the new state discovered. Is it a Higgs, the SM Higgs, ...?
- I hope I conveyed in these lectures the framework in which the data from the LHC will be evaluated: the SM Higgs
- Crucial to control QCD to pin down Higgs properties
- If the branching fractions aren't SM-like, can we explain by extending the Higgs EFT to contain new states? (pay attention to the excess in the VBF component of $\gamma\gamma$)
- Enjoy your weekend!

# Appendix I: $M_W$ calculation in SM

# Muon decay

- Muon-decay at tree-level:




$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2} \quad (m_{e,\mu} = 0)$$

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2} \quad \text{(on-shell scheme)}$$

$$\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2\left(1 - M_W^2/M_Z^2\right)}$$

$$\Rightarrow M_W^2 = \frac{M_Z^2}{2}\left\{1 + \left[1 - \frac{2\sqrt{2}\pi\alpha}{G_F M_Z^2}\right]^{1/2}\right\}$$

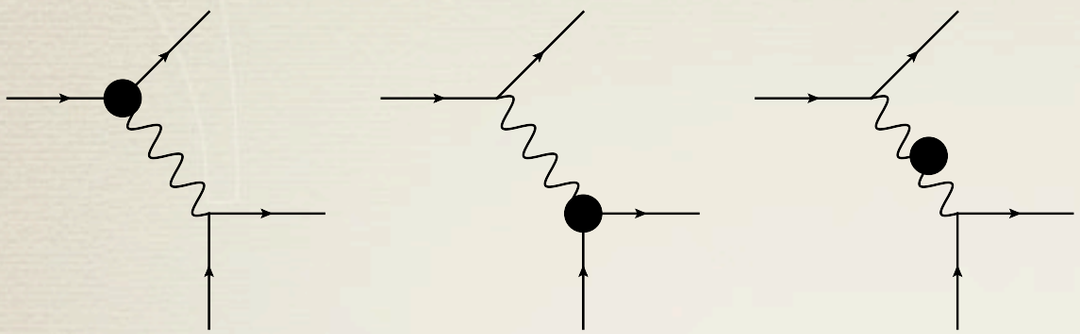
$$\approx 80.94 \text{ GeV} \qquad \Rightarrow \text{experiment gets 80.4 GeV!}$$

- Keep only leading corrections ($m_t$, $M_H$, running of $\alpha$; others defined as 'small')

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2 s_W^2}(1 + \Delta r)$$

$$\Rightarrow M_W^2 = \frac{M_Z^2}{2}\left\{1 + \left[1 - \frac{2\sqrt{2}\pi\alpha\,(1 + \Delta r)}{G_F M_Z^2}\right]^{1/2}\right\}$$

# Muon-decay at one loop



+ *boxes, counterterms*

No vertex, box can depend on $m_t$,$M_H$ ($m_{e,\mu}\approx 0$) $\Rightarrow$ only self-energy, counterterms

$$
\begin{aligned}
e_0^2 &= e^2 - \delta e^2 \\
M_{W0}^2 &= M_W^2 + \delta M_W^2 \\
M_{Z0}^2 &= M_Z^2 + \delta M_Z^2 \\
s_{W0}^2 &= 1 - \frac{M_{W0}^2}{M_{Z0}^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ig_{\mu\rho}}{M_W^2}\left[i\Pi_{WW}(0)\right]\frac{ig_{\rho\nu}}{M_W^2} \\
&= \frac{ig_{\mu\nu}}{M_W^2}\left[1 - \frac{\Pi_{WW}(0)}{M_W^2}\right] \\
\Rightarrow\ & \Delta r_1 = -\frac{\Pi_{WW}(0)}{M_W^2}
\end{aligned}
$$

$$
\frac{e_0^2}{s_{W0}^2 M_{W0}^2} = \frac{e^2}{s_W^2 M_W^2}\left\{1 - \frac{\delta e^2}{e^2} - \frac{c_W^2}{s_W^2}\left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right] - \frac{\delta M_W^2}{M_W^2}\right\}
$$

$$
\Delta r_2 = -\frac{\delta e^2}{e^2} - \frac{c_W^2}{s_W^2}\left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right] - \frac{\delta M_W^2}{M_W^2}
$$

$$
\Delta r = \Delta r_1 + \Delta r_2 + \Delta r_{rem}
$$

Useful reference for SM renormalization: Denner, 0709.1075

# Muon decay at one-loop

on-shell mass renormalization : $\delta M_V^2 = \Pi_{VV}(M_V^2)$

charge renormalization : $\delta e^2/e^2 = \Pi_{\gamma\gamma}(0)$

$$\Pi_{VV}(M_V^2) = \Pi_{VV}(0) + \underbrace{\ldots}_{small}$$

$$\begin{aligned}\Pi_{\gamma\gamma}(0) &= -\left[\Pi_{VV}(M_Z^2) - \Pi_{VV}(0)\right] + \underbrace{\Pi_{VV}(M_Z^2)}_{small} \\ &\approx -\frac{\alpha(M_Z^2) - \alpha(0)}{\alpha(0)} \equiv -\Delta\alpha \quad \text{(non-perturbative; light quarks)}\end{aligned}$$

$$\left[\Pi_{VV}(M_Z^2) - \Pi_{VV}(0)\right] \sim \ln \frac{M_Z^2}{m_f^2}$$

Combine all terms to obtain the following for Δr (drop 'small' terms)

$$\boxed{\begin{aligned}\Delta r &= \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho \\ \Delta\rho &= \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\end{aligned}}$$

Use optical theorem to relate hadronic vacuum polarization to $e^+e^- \to$ hadrons

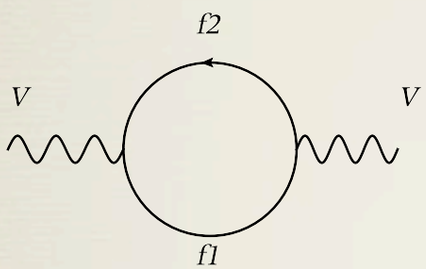
$\Delta\alpha = 0.06649(12)$ (PDG)

# Δρ and non-decoupling

Δr receives important contribution from gauge-boson self-energies

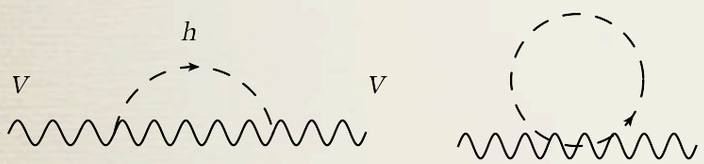
$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho$$

$$\Delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$




$$\Delta\rho_{ferm} = \overbrace{\frac{3G_F m_t^2}{8\pi^2\sqrt{2}}}^{\text{quadratic in } m_t} + \text{subleading terms}$$

Exercise: Derive these




$$\Delta\rho_{Higgs} = -\frac{3G_F M_Z^2 s_W^2}{4\pi^2\sqrt{2}} \underbrace{\ln\frac{M_H}{M_Z}}_{\text{logarithmic in } M_H} + \text{subleading terms}$$

Decoupling theorem holds only if dimensionful parameters made large

$$m_t = \frac{\lambda_t v}{\sqrt{2}} \quad \Rightarrow \quad m_t \to \infty, \;\; v \text{ fixed} \Rightarrow \lambda_t \to \infty$$

$$M_H^2 = 2\lambda v^2 \quad \Rightarrow M_H \to \infty, \;\; v \text{ fixed} \Rightarrow \lambda \to \infty$$