Collision with a crossing angle
Large Piwinski angle

K. Ohmi

KEK

BEAM'07 CERN Oct. 1-6, 2007

Francesco Ruggiero Memorial symposium

Introduction

- Effect of crossing angle
- Large Piwinski angle (ϕ) collision

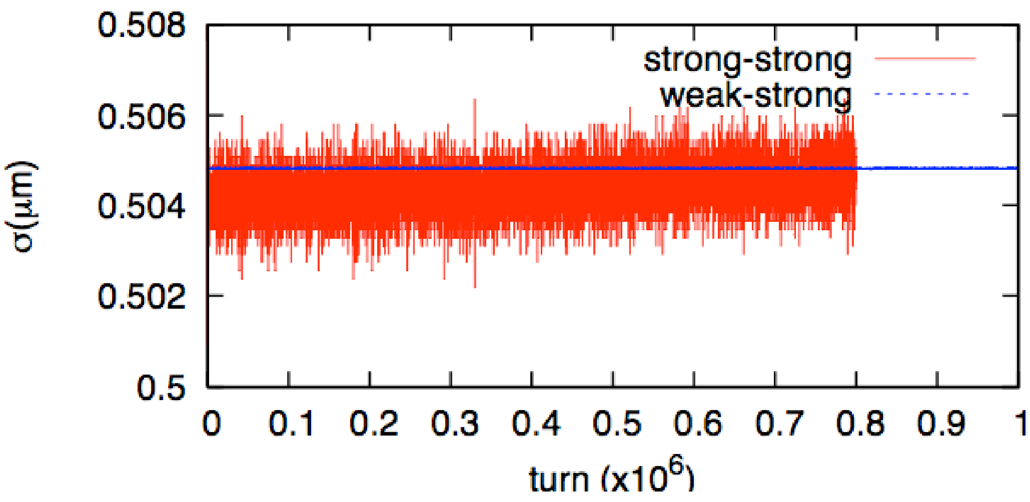
$$\phi = \frac{\theta_{x(y)}\sigma_z}{\sigma_{x(y)}}$$

θ: half crossing angle

- Crossing scheme at two interaction points. Hor.-Hor, Hor.-Ver….
- Crab crossing and crab waist schemes in e^+e^- colliders.

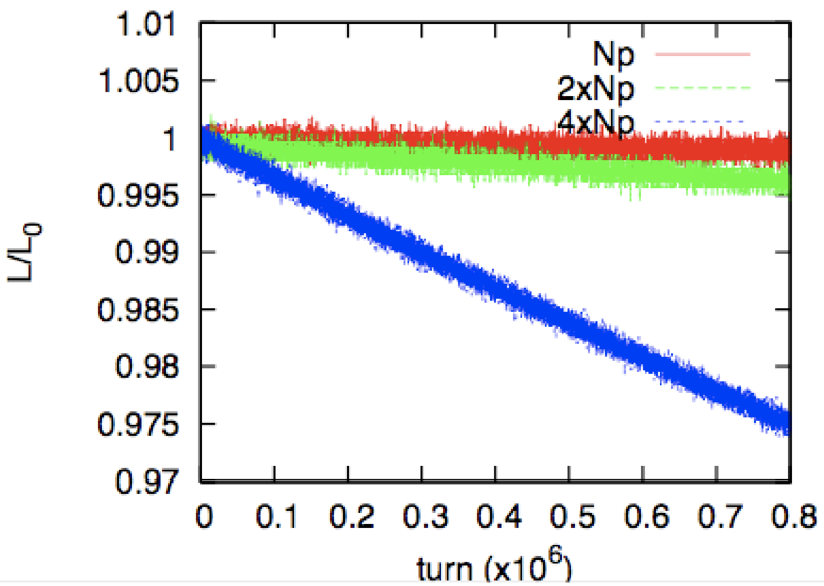
Beam-beam simulation for proton beams

- Weak-strong or strong-strong simulations
- Strong-strong simulation contains statistical noise, for example the dipole position fluctuates $\sigma/N^{1/2}$. Such noise gives artificial emittance growth.
- 1M macro-particles, 0.1% noise, gives one day luminosity life for nominal LHC parameters.
- Weak-strong simulation is reliable and simple.



- Emittance growth for weak-strong and strong-strong simulation

1 day life time=10^{-9}/turn

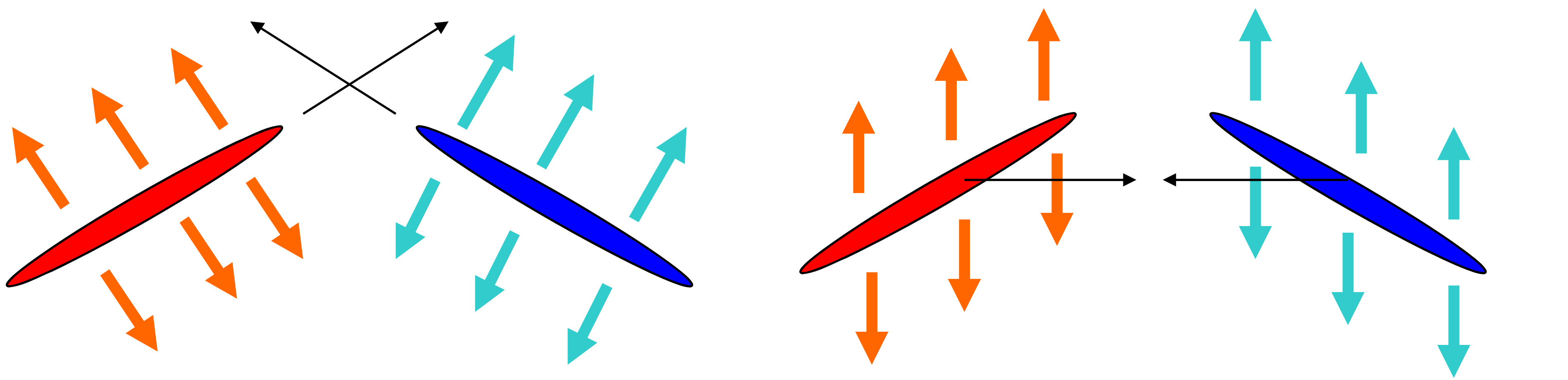


- Luminosity decrement for strong-strong simulation

- Weak-strong simulation did not give Luminosity decrement as shown later.

Crossing angle

- Lorentz boost is used to make perpendicular field for moving direction. (J. Augustin, K. Hirata)
- Lorentz transformation seems to be not sympletic for the accelerator coordinate system $p_x=P_x/p_0$, remember adiabatic damping.
- Lorentz transformation is sympletic in the physical coordinate system.



Crossing angle and crab crossing

- Transformation from Lab. frame to head-on frame.

(θ: half crossing angle)

$$x^* = \tan\theta z + [1 + h_x^* \sin\theta] x$$

$$p_x^* = (p_x - h\tan\theta)/\cos\theta$$

$$y^* = y + h_x^* \sin\theta x$$

$$p_y^* = p_y^* / \cos\theta$$

$$z^* = z/\cos\theta + h_z^* \sin\theta x$$

$$p_z^* = p_z - p_x \tan\theta + h\tan^2\theta$$

$$h = p_z + 1 - \sqrt{(p_z + 1)^2 - p_x^2 - p_y^2}$$

Linear part

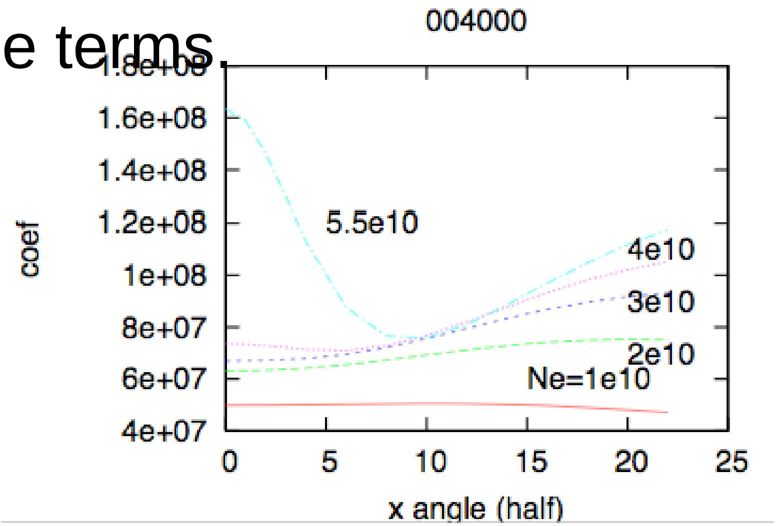
$$\begin{pmatrix} 1 & 0 & 0 & 0 & \tan\theta & 0 \\ 0 & 1/\cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/\cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/\cos\theta & 0 \\ 0 & -\tan\phi & 0 & 0 & 0 & 1 \end{pmatrix}$$

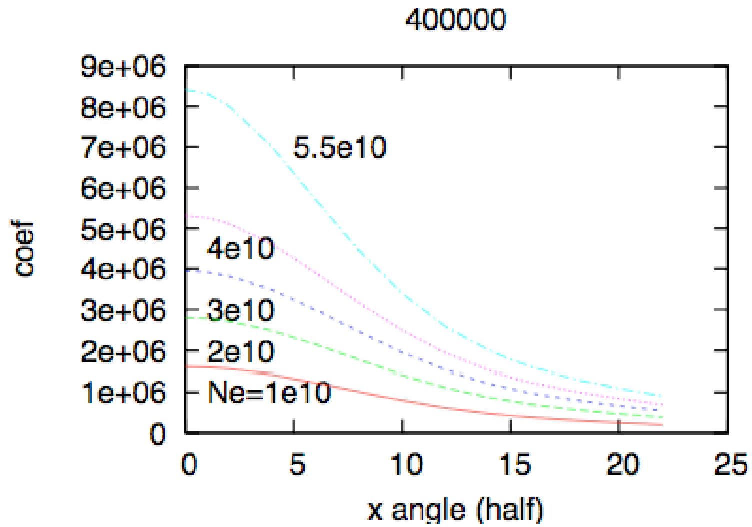
Jacobian matrix and determinant of linear matrix contain $1/\cos^3\theta$ due to Lorentz transformation.

This transformation is sympletic.

Does crossing angle affect the beam-beam performance?

- The beam-beam performance is degraded at a high beam-beam parameter, for example it was degraded a half for KEKB.
- How is in LHC, low beam-beam parameter and no radiation damping?
- Crossing angle induces odd terms in Hamiltonian.
- The odd terms degrade luminosity performance in e^+e^- colliders. Tune scan shows clear resonance lines due to the terms.





Taylor map analysis

- Calculate beam-beam map

$$\mathbf{x} = \mathbf{f}(\mathbf{x}_0)$$

- Remove linear part

$$\mathbf{X} = \mathbf{f}(R^{-1}\mathbf{x}_0) = \mathbf{x}_0 + \sum a_{ij} x_{0,i} x_{0,j} + 3\text{-rd order } .....$$

- Factorization , integrate polynomial

$$\mathbf{X} = \exp\left(-:(H_3 + H_4 + ...):\right)\mathbf{x}_0$$

$$\sum a_{ij} x_{0,i} x_{0,j} = [-H_3,\ \mathbf{x}_0]$$

Coefficients of beam-beam Hamiltonian

- Expression-1 $(k_x,k_p,k_y,k_q,k_z,k_e)$ $p=p_x, q=p_y, e=p_z$
- Expression-2 (n_x,n_y,n_z)

- 4-th order coefficients

C400 (400000), (310000), (220000), (130000), (040000)

C301 (300010), (210010),(120010),(030010)

C220 (202000), (112000), (022000), (201100), (111100), (021100), (200200), (110200), (020200)

C040 (004000), (003100), (002200), (000300), (000400)

C121 (102010), (012010), (101110), (011110), (100210), (010210)

- 3rd order coefficients (except for chromatic terms)

C300 (300000), (210000), (120000), (030000)

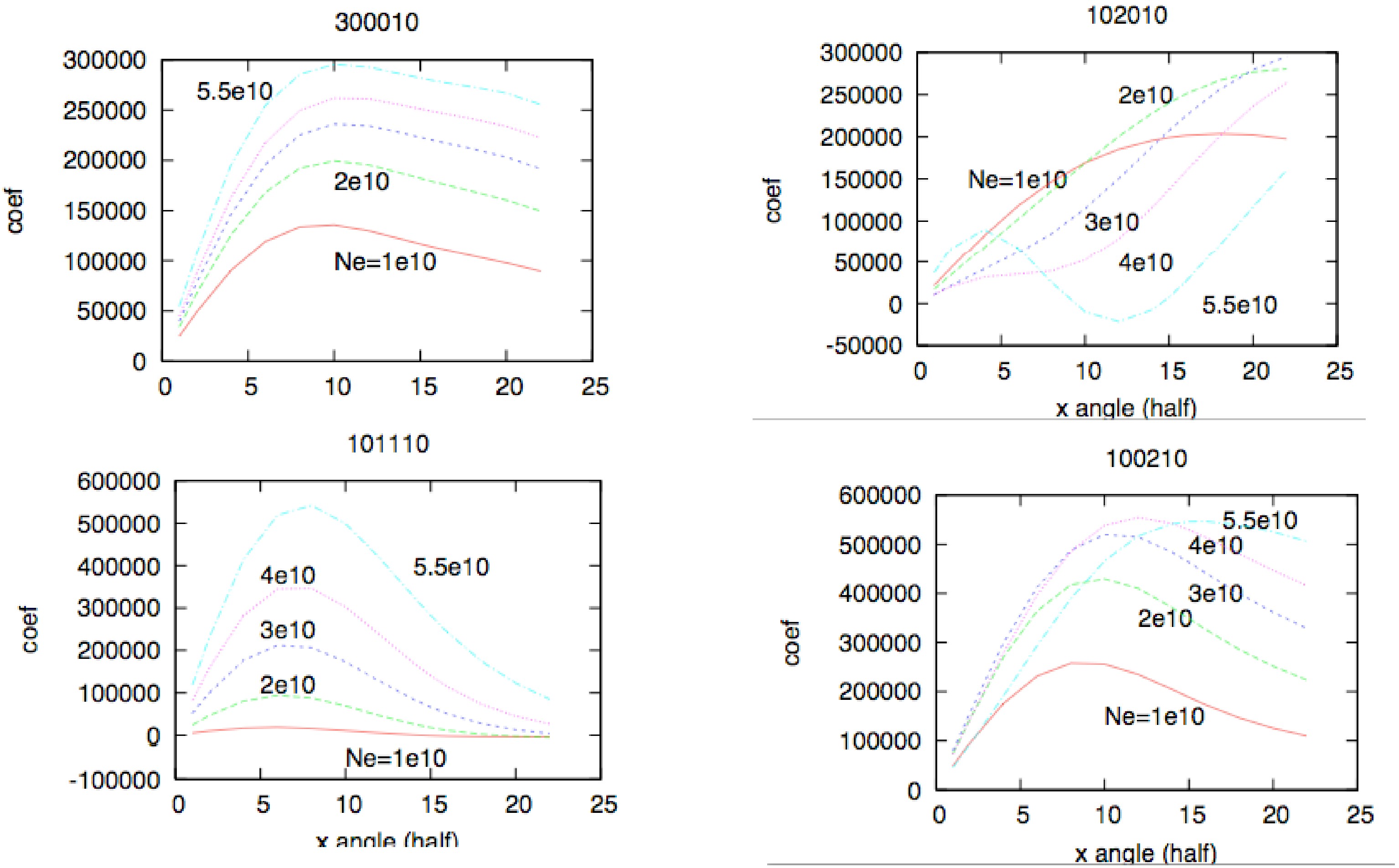
C210 (201000), (111000), (021000), (200100), (110100), (020100)

C120 (102000), (012000), (101100), (011100), (100200), (010200)

- Low order nonlinear terms are efficient in e+e- colliders, while higher order terms are efficient in proton colliders.

Taylor map analysis for KEKB

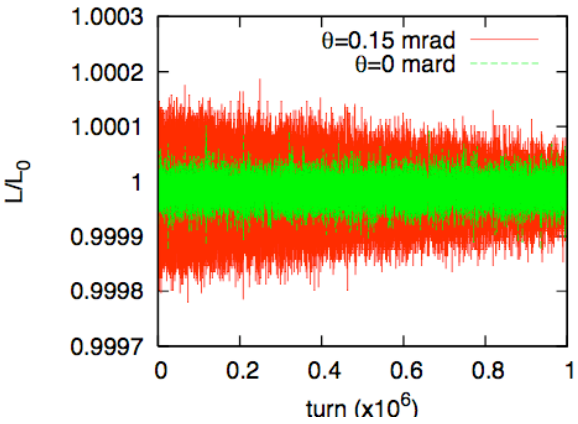
- Resonance line $\nu_x - 2\nu_y = k$ is effective for the beam-beam limit in e^+e^- colliders.

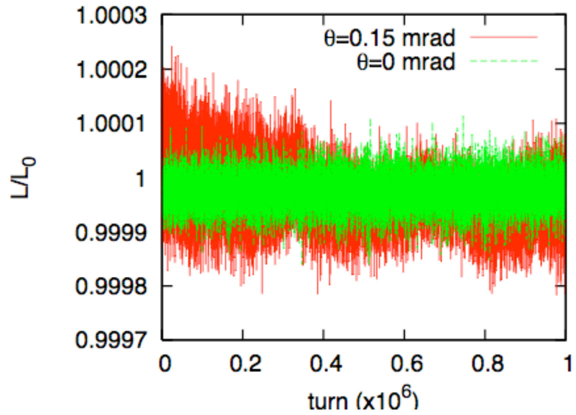


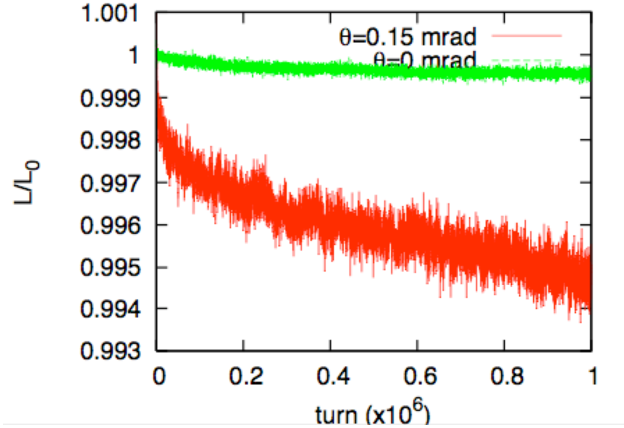
Simulation (weak-strong) for LHC

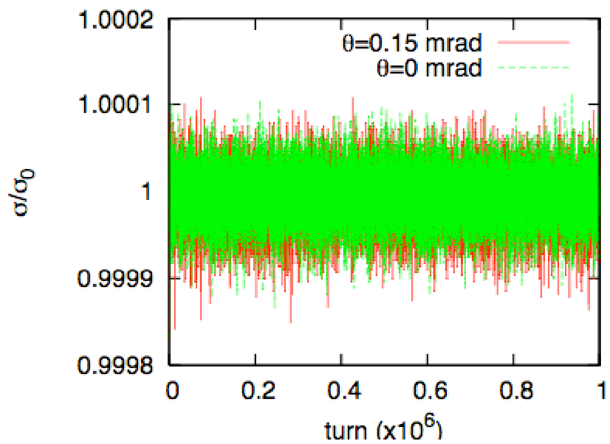
- Simulation for N_p=1.15x10^{11} (nominal), 2xN_p, 4xN_p and 8xN_p.
- The crossing angle affects the luminosity performance at much higher intensity than nominal value, 8xN_p, if there is no noise and other errors.

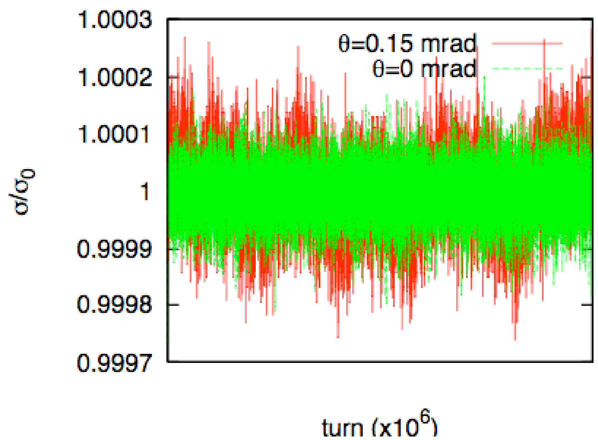
$2xN_p$ $4xN_p$ $8xN_p$

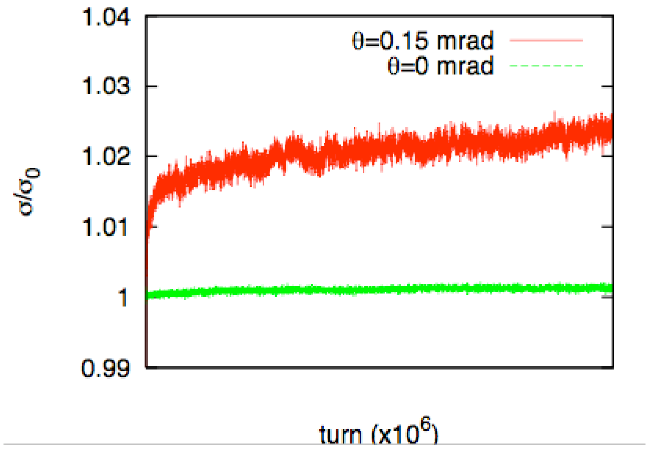












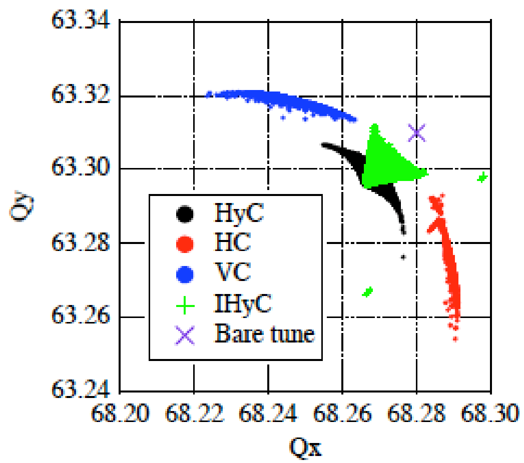
No parasitic collision

Large Piwinki angle scheme for LHC (F. Zimmermann, PAC07)

- Shorter bunch length than that for Superbunch scheme with ϕ>>1.
- Piwinski angle ϕ=2(0.4). Note () is nominal.
- Bunch spacing 50 (25) ns , n_b=1401(2808).
- Uniform longitudinal profile with σ_z=11.8(7.55) cm, L_z=41 cm. θ(half)=190(143) μrad.
- N_p=4.9(1.15)x10^{11}, β^*=0.25 cm
- L=10(1)x10^{34} $cm^{-2}s^{-1}$.

Crossing scheme

- Hor.-Hor.
- Hor.-Vert. (Hybrid)
- Hybrid Incline (slanted col.)



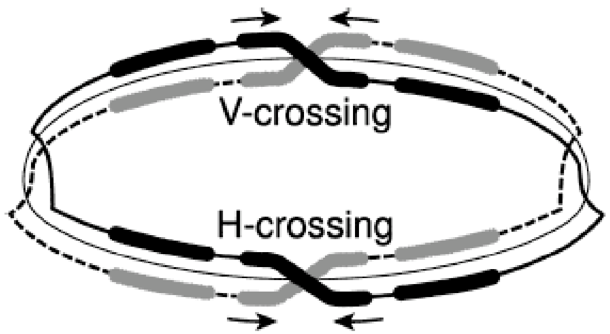


FIG. 1. Schematic view of a superbunch hadron collider.

Y. Shimosaki,

Inclined hybrid: Tune shift is small but how is x-y coupling?

K. Takayama et al., PRL88, 144801 (2002)

F. Ruggiero and F. Zimmermann, PRST,5, 061001 (2002)

Nonlinear term of each collision scheme

- Hor.-Hor.

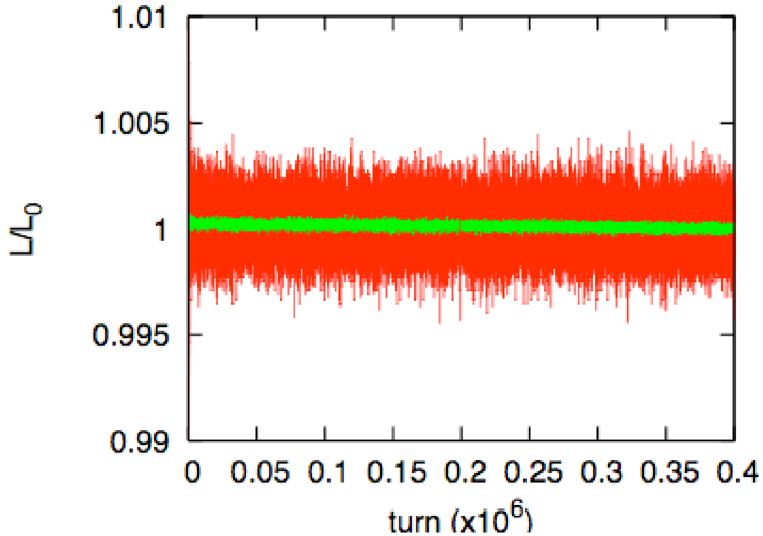
Tune spread is wide range, but terms even for y exists.

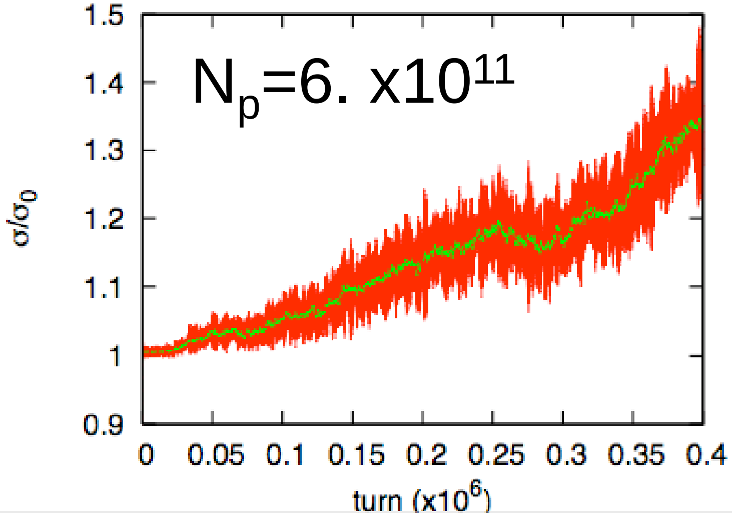
- H-V

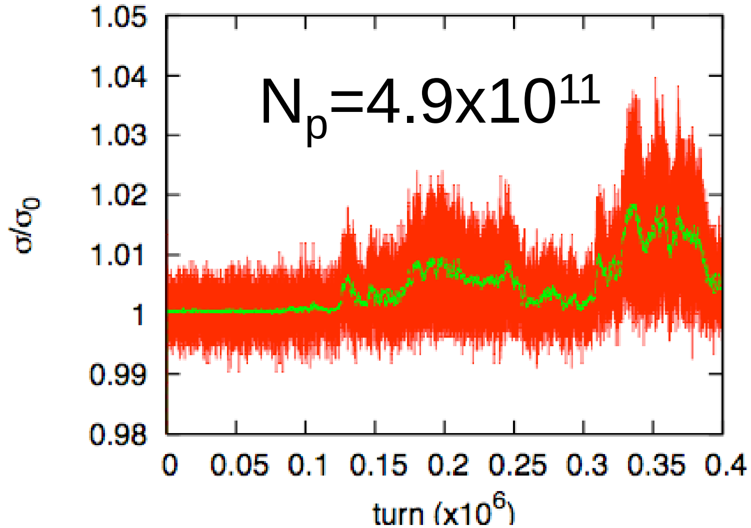
All nonlinear term can be exist. More resonance lines may active than Hor.-Hor.

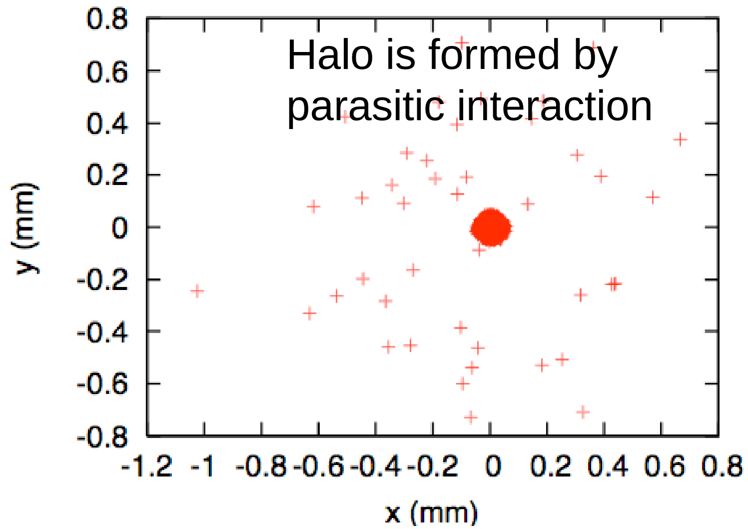
- An example showed H-V crossing is serious for Halo formation. The halo was formed by parasitic interaction.
- H-H with and without and H-V without parasitic interactions was no problem.

An example of simulation result for H-V crossing





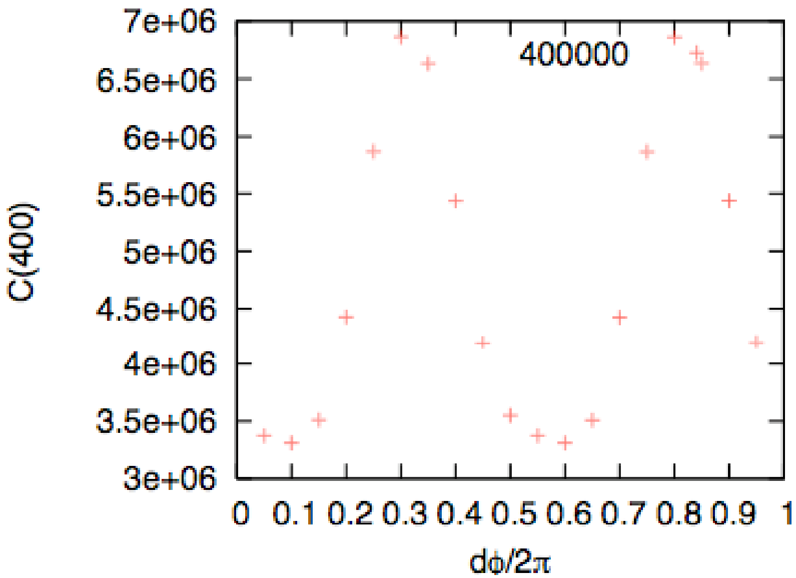




7 parasitic interactions each side.

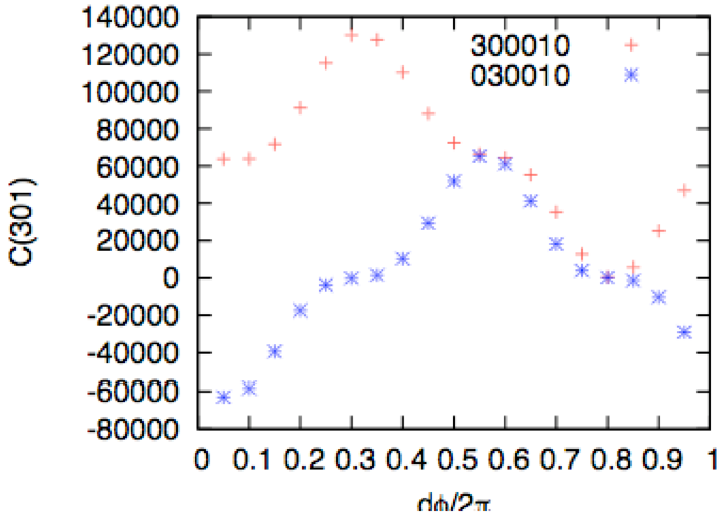
Phase advance between two interaction points

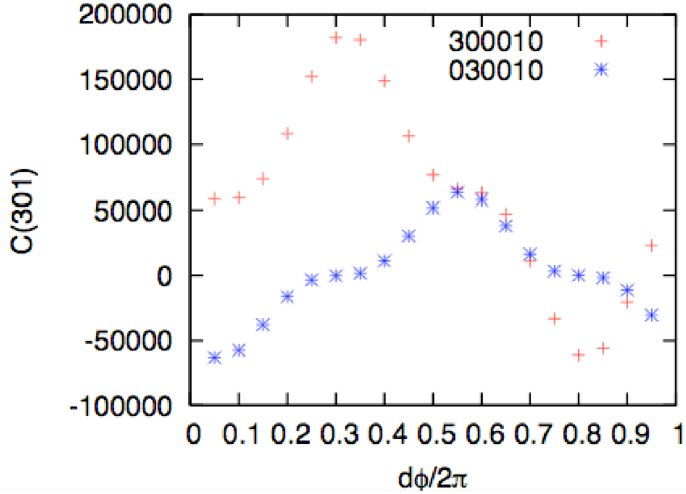
- Nonlinear map can depend on the betatron phase difference between two IP's.
- Preliminary results for Taylor map analysis are presented.

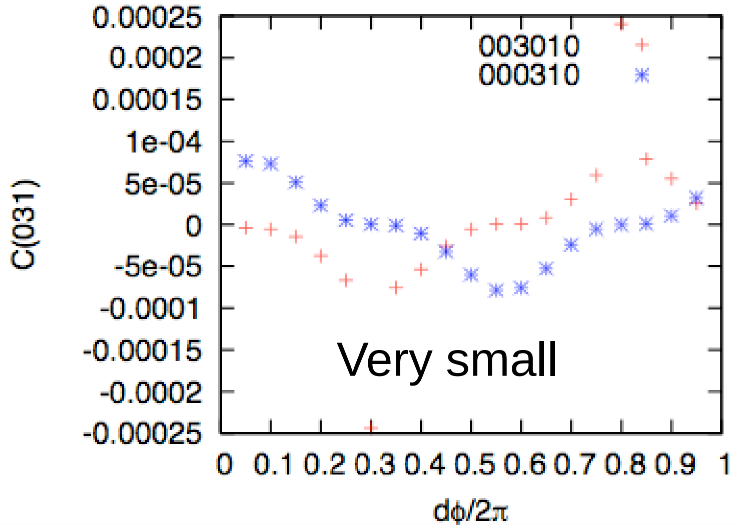


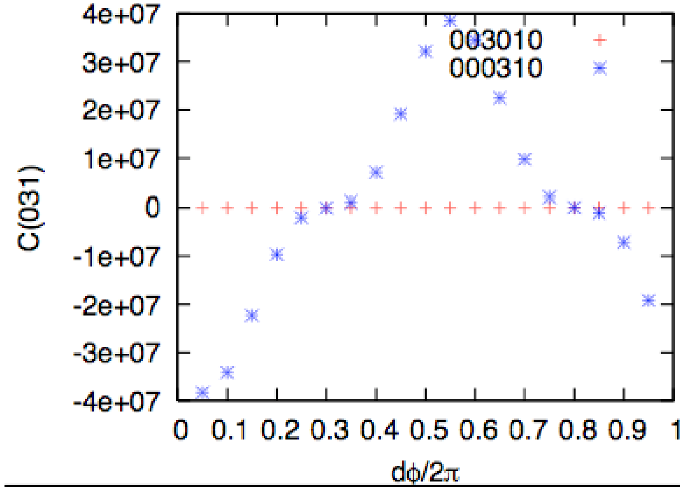
- HH

HV

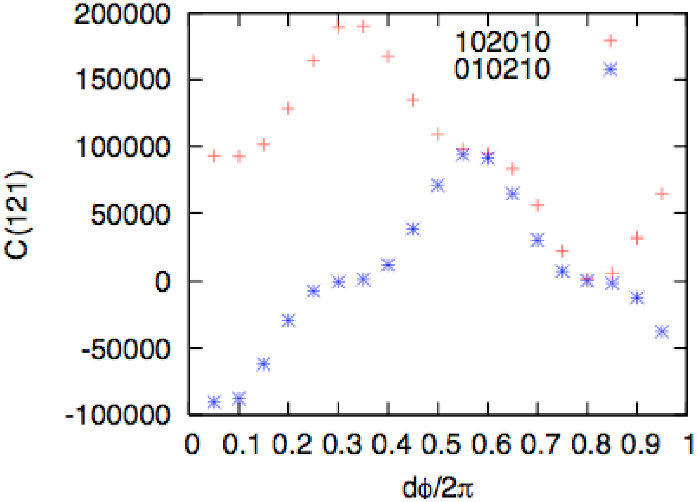

C(301)
dφ/2π
300010
030010

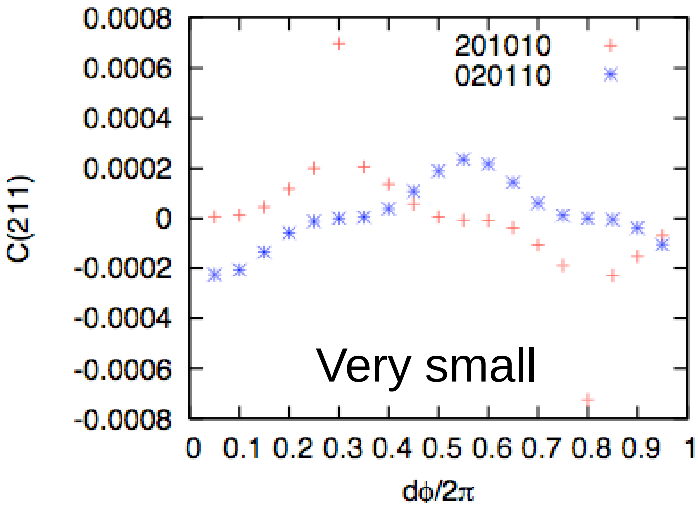

C(301)
dφ/2π
300010
030010


C(031)
dφ/2π
003010
000310
Very small

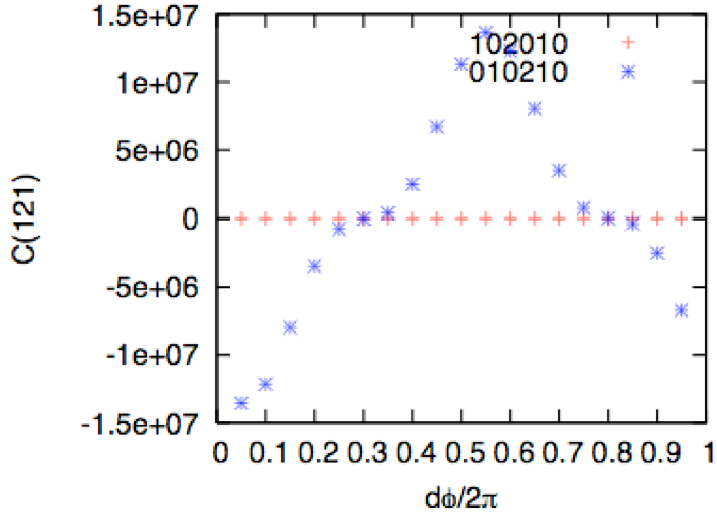

C(031)
dφ/2π
003010
000310

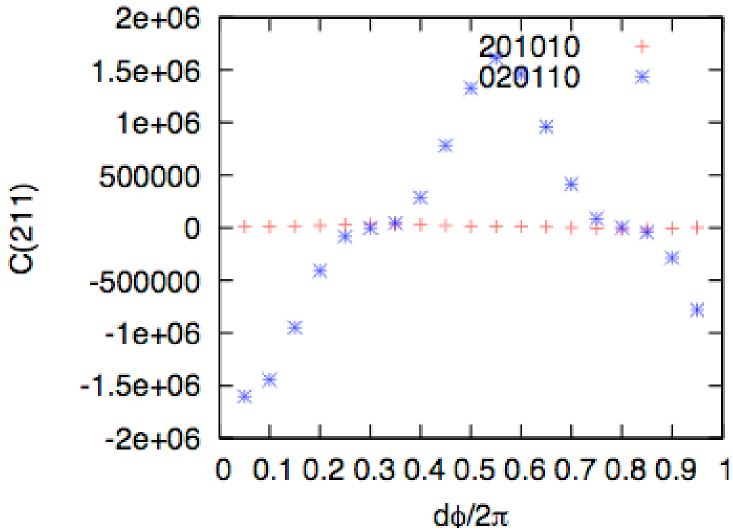
HH


C(121)
dϕ/2π
102010
010210


C(211)
dϕ/2π
201010
020110
Very small

HV


C(121)
dϕ/2π
102010
010210


C(211)
dϕ/2π
201010
020110

Large Piwinski angle design in e^+e^- colliders (Super B)

- Keeping bunch length, σ_z~6 mm.
- Small emittance, ε_x=1nm, ε_y=2pm (similar as ILC damping ring)
- Small IP beta, β_x=20 mm, β_y=0.2 mm.
- Very high Piwinski angle ϕ~34.
- Reasonable beam-beam parameter ξ<0.1.
- Lower current $N_e=2\times10^{10}$, while 8×10^{10} for KEKB and PEPII.

Waist control, Crab waist (P. Raimondi et al.)

$$\mathbf{M} = e^{-:H_I:}\mathbf{M}_0 e^{:H_I:}$$

$$H_I = axp_y^2$$

$$\overline{y} = y + \frac{\partial H_I}{\partial P_y} = y + axP_y \qquad \overline{p_x} = p_x - \frac{\partial H}{\partial x} = p_x - ap_y^2$$

- Take linear part for y, since x is constant during collision.

$$\begin{pmatrix} \overline{\beta} & -\overline{\alpha} \\ -\overline{\alpha} & \overline{\gamma} \end{pmatrix} = T \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} T^t = \begin{pmatrix} \beta + \frac{a^2x^2}{\beta} & \frac{ax}{\beta} \\ \frac{ax}{\beta} & \frac{1}{\beta} \end{pmatrix}$$

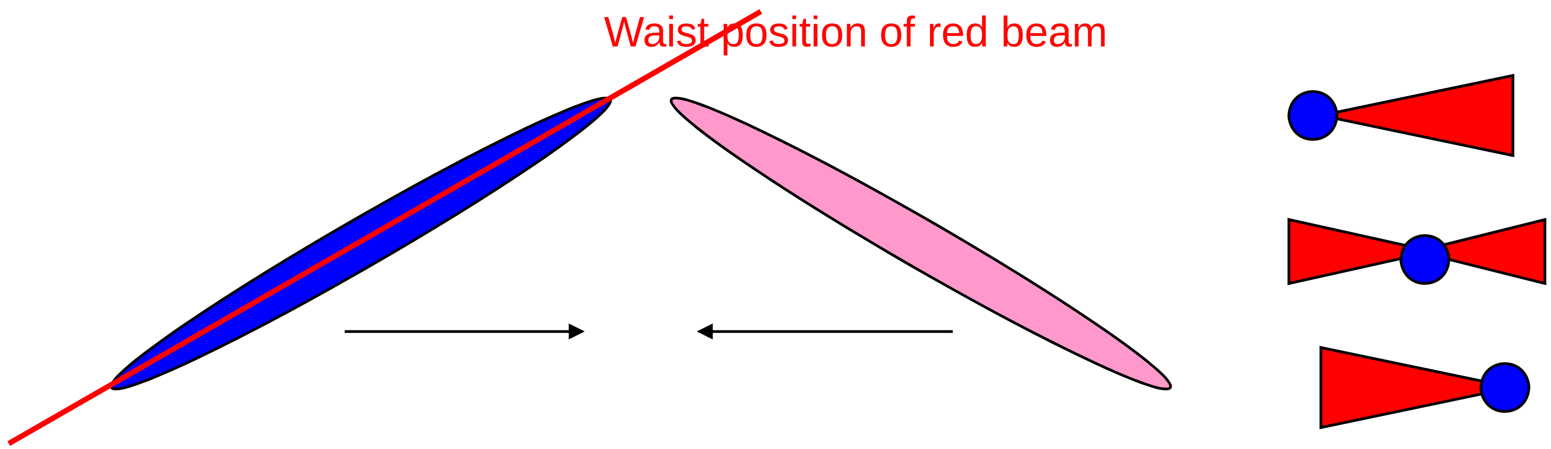
$$T = \begin{pmatrix} 1 & ax \\ 0 & 1 \end{pmatrix}$$

$$M(s)\begin{pmatrix} \beta+\dfrac{a^2x^2}{\beta} & \dfrac{ax}{\beta} \\ \dfrac{ax}{\beta} & \dfrac{1}{\beta} \end{pmatrix} M^t(s) = \begin{pmatrix} \beta+\dfrac{(s+ax)^2}{\beta} & \dfrac{s+ax}{\beta} \\ \dfrac{s+ax}{\beta} & \dfrac{1}{\beta} \end{pmatrix}$$

β waist is shifted to s=-ax

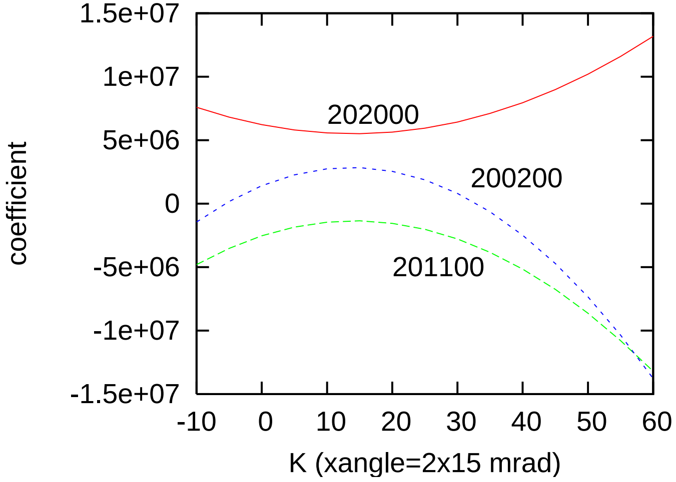
Taking a=1/2θ

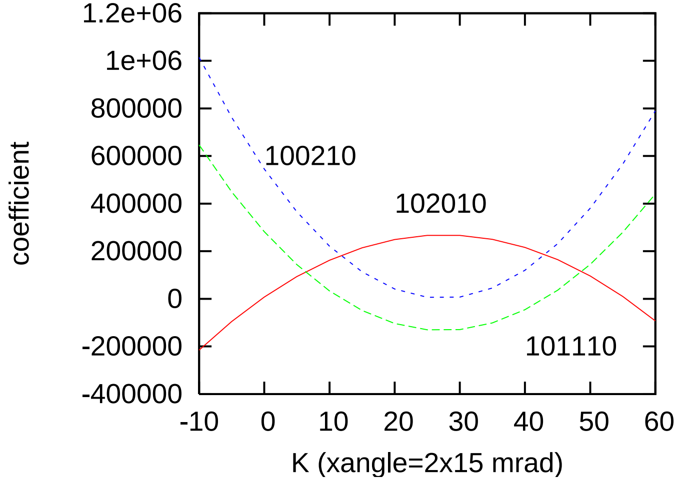
- Beam particles with various x collides with other beam at their waist.

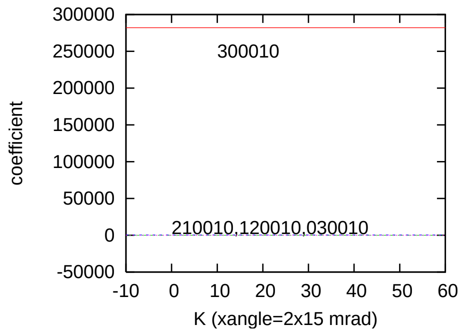


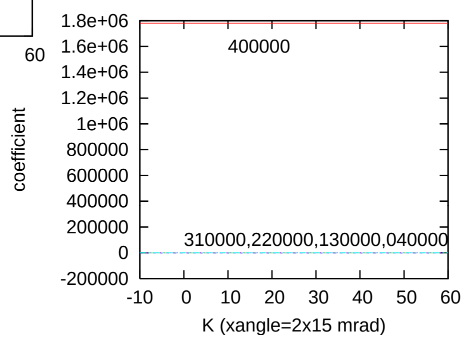
Beam shape on red beam frame

4-th order Coefficients as a function of crab sextupole strength, KEKB









- H=K x $p_y^2/2$, theoretical optimum, K=1/xangle.
- Clear structure- 220,121
- Flat for sextupole strength- 400, 301, 040

Summary

- Crossing angle induces resonance lines related to odd terms for x.
- The effect is not strong for ideal case without noise and errors.
- Collision with a large Piwinski angle was studied by simulation and Taylor map analysis
- H-H collision gives wide tune spread but limited resonance, while H-V collision gives narrow tune spread but more resonances.
- Phase difference between two IP’s.
- Systematic studies have not performed yet.