STABILITY OF TRANSVERSE COLLECTIVE MODES WITH NONLINEAR SPACE CHARGE

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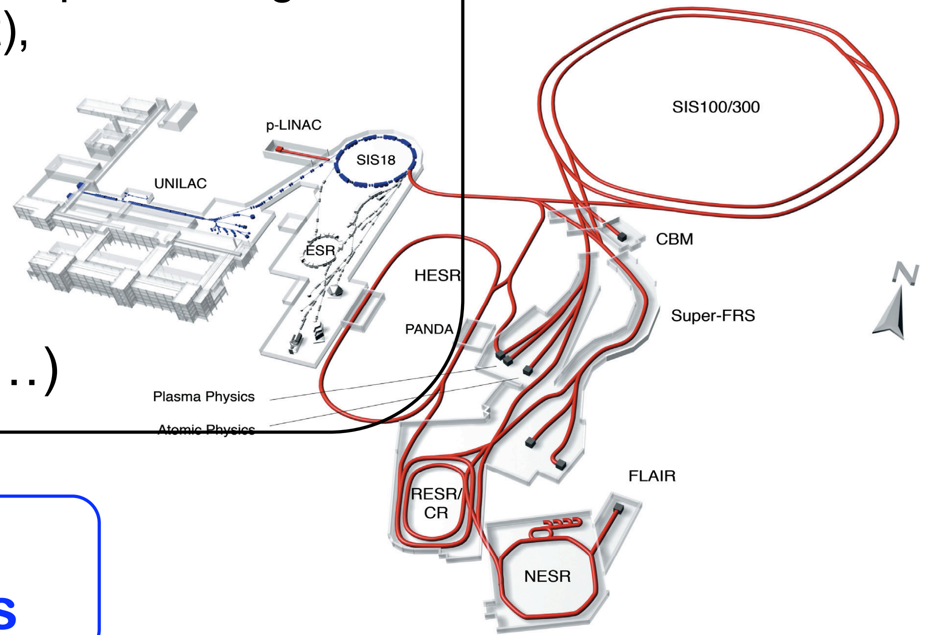
INTRODUCTION

High quality and high intensity operation for FAIR
may be limited by transverse collective instabilities

Transverse collective beam dynamics for
the specific FAIR parameters
(strong space charge, small aperture/radius ratios)

- **analytical**
  still uncertainties about the role of nonlinear space charge (amplitude-dependent incoherent tune shift), 3D effects for bunches
- **numerical**
  particle tracking (PATRIC, HEADTAIL)
- **experimental at SIS18**
  Transv. Schottky and BTF, instabilities
- **impedance calculations** (RW, kicker,…)



impedance budget
how to damp instabilities

DISPERSION RELATION

approach of D.Möhl (1969),
here for the horizontal plane:

$$\int \frac{\Delta Q_{\rm coh} - \Delta Q_{\rm inc}}{\Omega/\omega_0 - (Q_{\rm ex} + \Delta Q_{\rm inc})} \left(-\frac{a^2}{2} \frac{\mathrm{d}\psi_a}{\mathrm{d}a} \right) b\, \psi_b(b)\ \ \psi_p(p)\, \mathrm{d}a\, \mathrm{d}b\, \mathrm{d}p = 1$$

"external" incoherent tune shifts:

$$Q_{\rm ex}(a,b,p) = Q_0 + \Delta Q_{\rm oct}(a,b) + \Delta Q_\xi(p)$$

(no external effects ⇒ no damping)

nonlinear space charge:

$$\Delta Q_{\rm inc}(a,b) = \Delta Q_{\rm kv}\Big[1 + \kappa(a,b)\Big]$$

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a / *b* : horizontal / vertical incoherent amplitude

ΔQ_{KV} : direct space charge tune shift for KV-beam

$$\Delta Q_\xi = \xi Q_0\, \Delta p/p_0$$

STABILITY DIAGRAMS

$$V + iU \propto Z^{\perp}(\Omega)$$

$$V \propto Re(Z^{\perp}) \propto Im(\Delta Q)$$

$$U \propto Im(Z^{\perp}) \propto Re(\Delta Q)$$

normalization here: $$\Delta U = \frac{\mathcal{R}e(\Delta Q_{\mathrm{coh}})}{|\Delta Q_{\mathrm{kv}}|}$$

characteristic tune spreads:

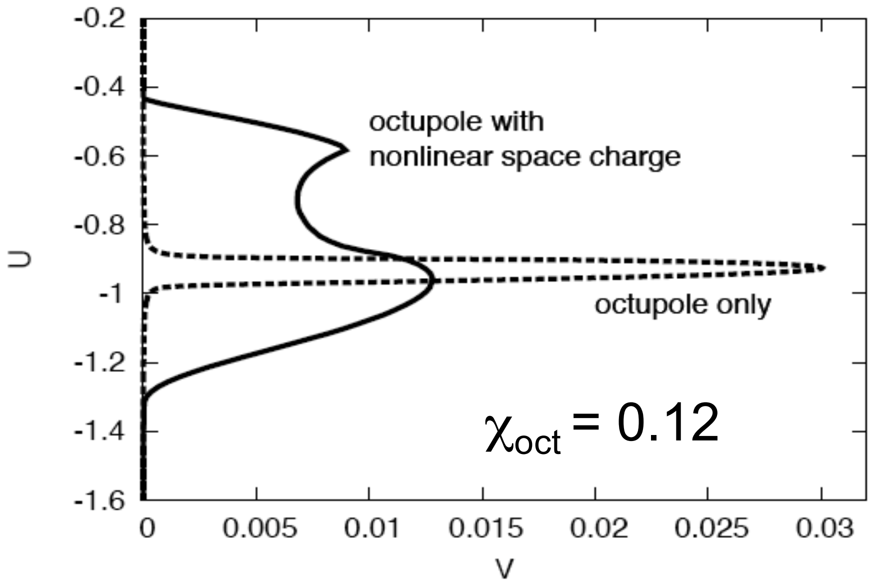
octupole

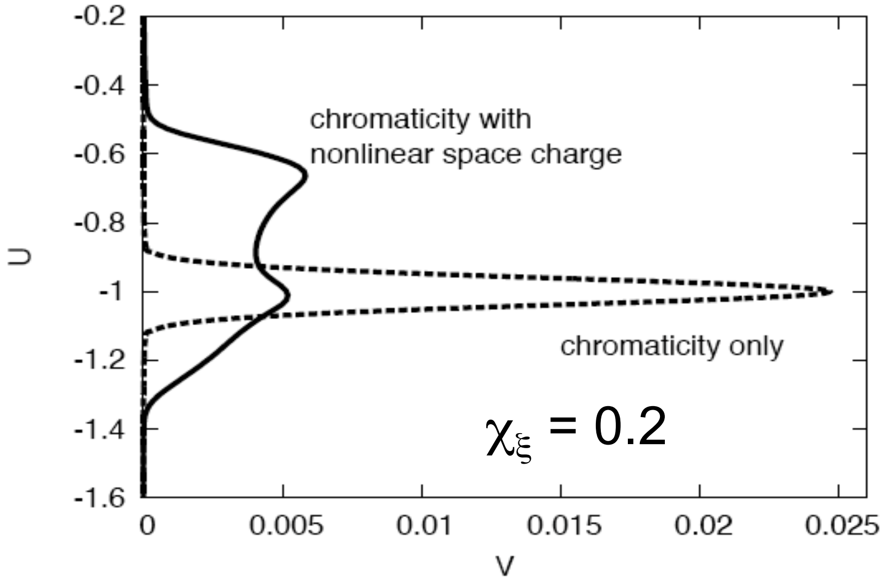
$$\chi_{\mathrm{oct}} = \frac{\delta Q_{\mathrm{oct}}}{\delta Q_{\mathrm{sc}}}$$

chromaticity

$$\chi_{\xi} = \frac{\delta Q_{\xi}}{\delta Q_{\mathrm{sc}}}$$

examples here: strong space charge



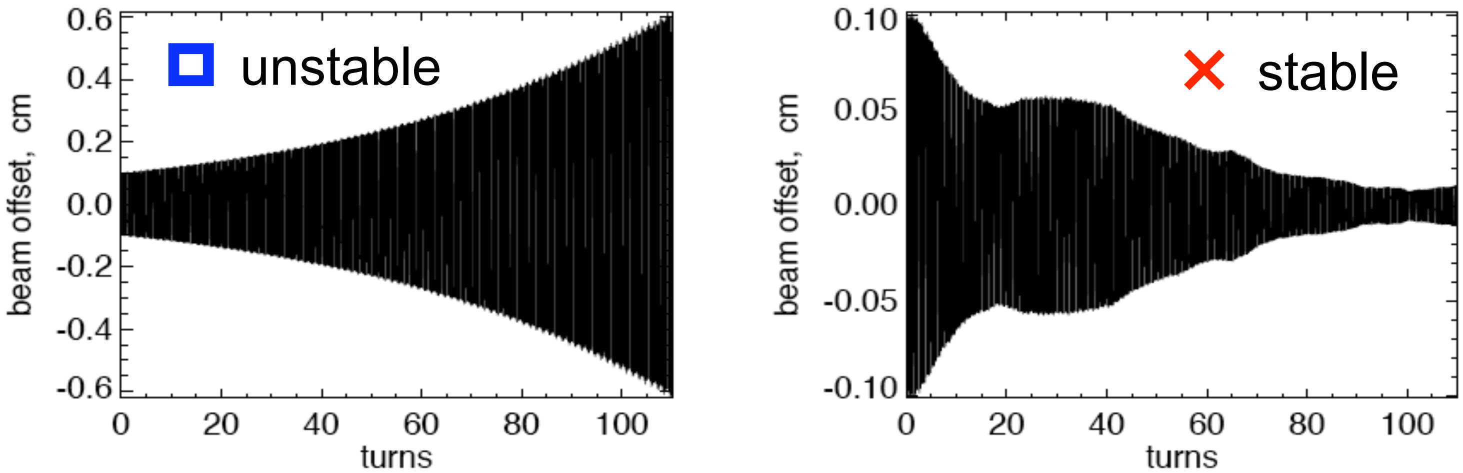


PARTICLE TRACKING

code PATRIC

- particle-in-cell tracking
- coasting and bunched beams
- sliced approach
- self-consistent space charge
- rectangular and elliptic boundaries
- ‘frozen’ space charge (***E*** rigid and follows $\overline{x}$, $\overline{y}$)
- transverse impedance module $\Big\{ Re(Z^{\perp}), Im(Z^{\perp}) \Big\} \longrightarrow \{V, U\}$
- developed at FAIR-AT

an example:
beam oscillations
for two $Im(Z^{\perp})$
and fixed $Re(Z^{\perp})$
for octupole + SC



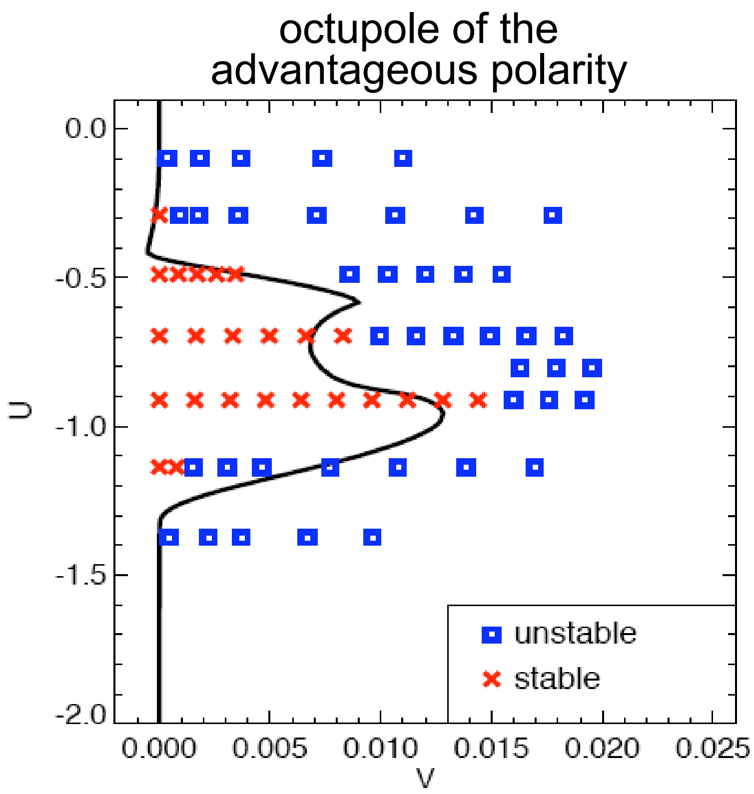
SIMULATIONS VS. DISP. RELATION

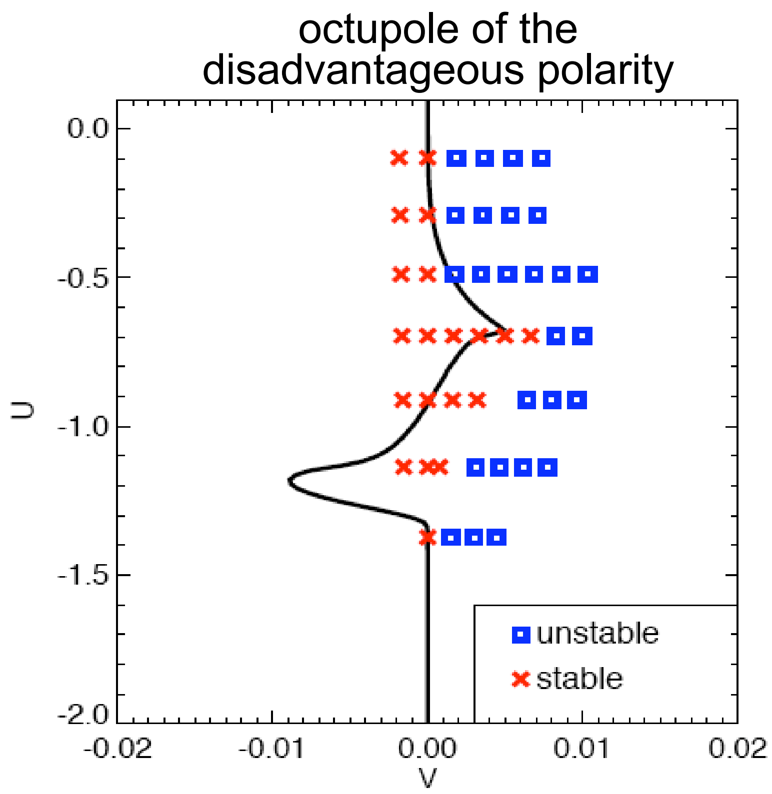
Comparisons of simulations with dispersion relation

PATRIC runs ⇒ symbols,

DR ⇒ lines (stability boundary)

Combination of nonlinear space charge with octupoles, self-consistent electric field





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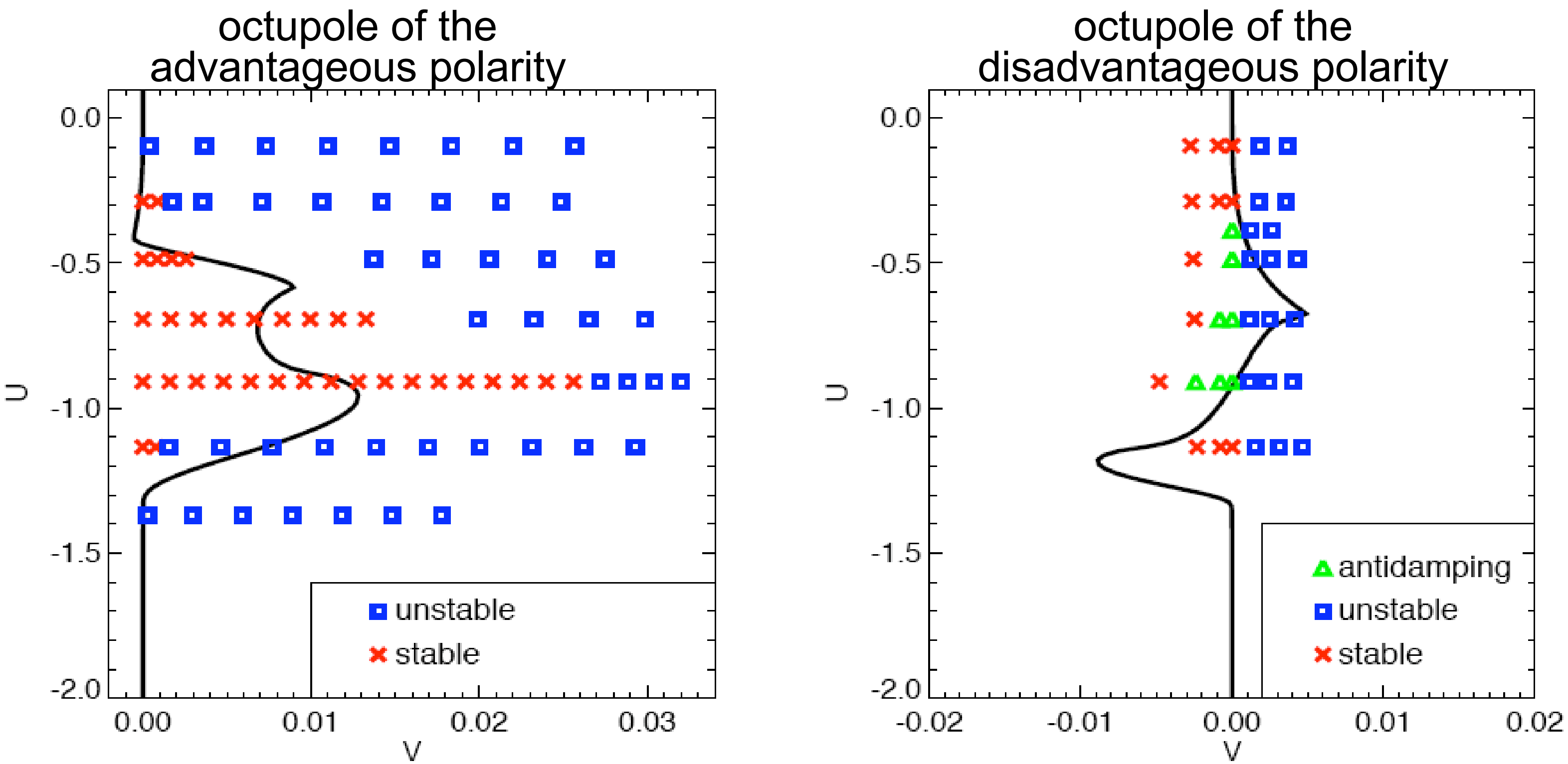


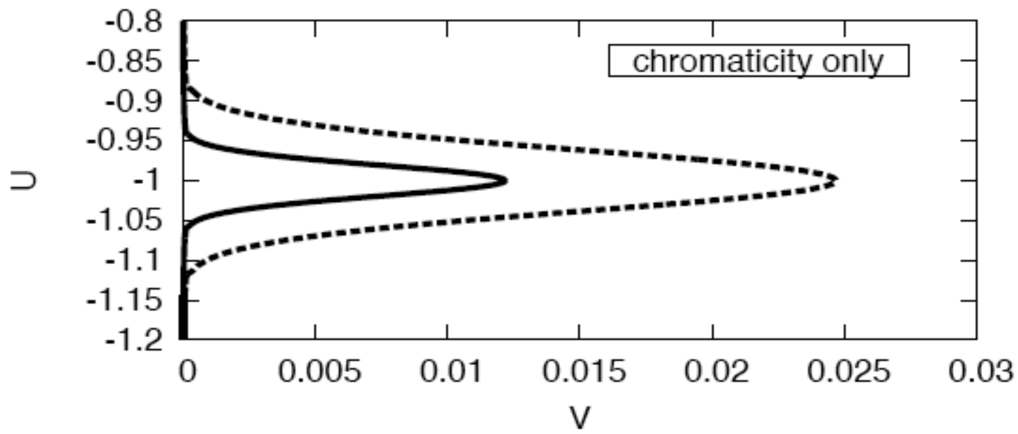
antidamping: an instability for $Re(Z^{\perp}) \leq 0$

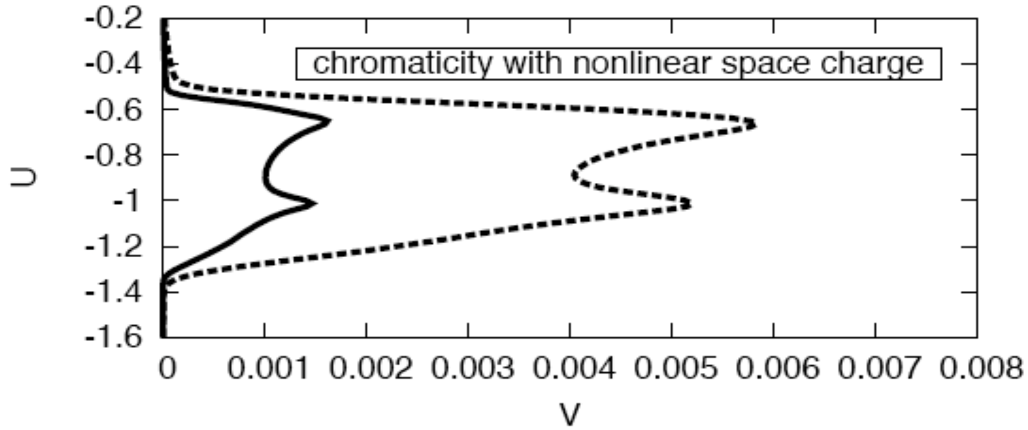
SIMULATIONS VS. DISP. RELATION

Dispersion Relation:
role of nonlinear space charge
for damping due to ξ and δp

linear space charge:
only a shift downwards,
simple scaling

nonlinear space charge:
modifies stability area,
complex scaling for
strong space charge





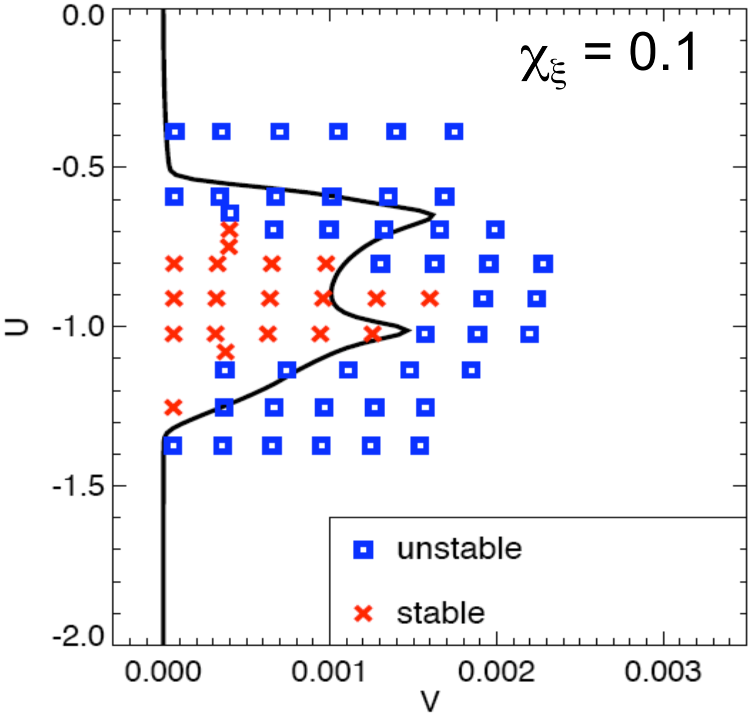
$\delta p_{(dash\ line)} = 2\ \delta p_{(solid\ line)}$

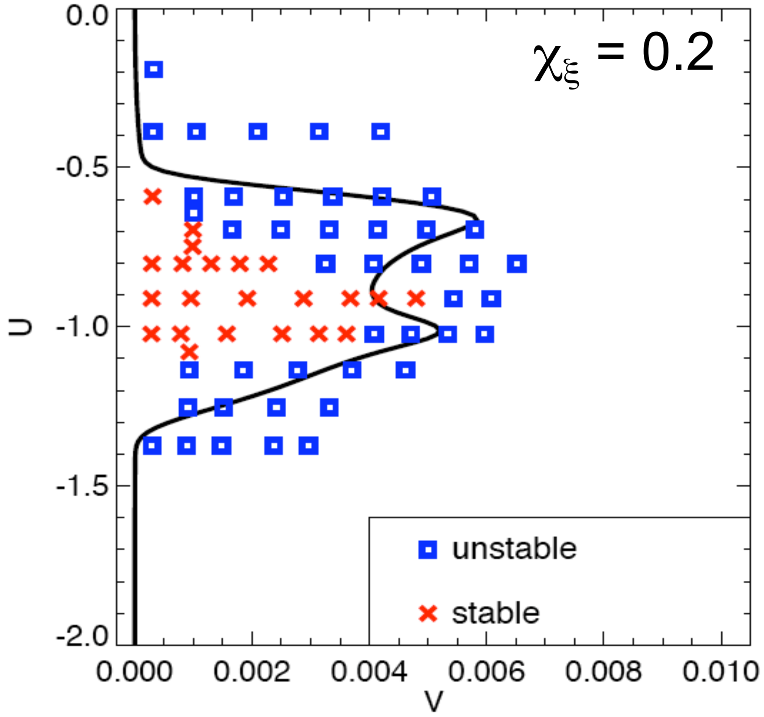
$\chi_\xi = 0.2$ $\chi_\xi = 0.1$

SIMULATIONS VS. DISP. RELATION

Comparisons of simulations with dispersion relation

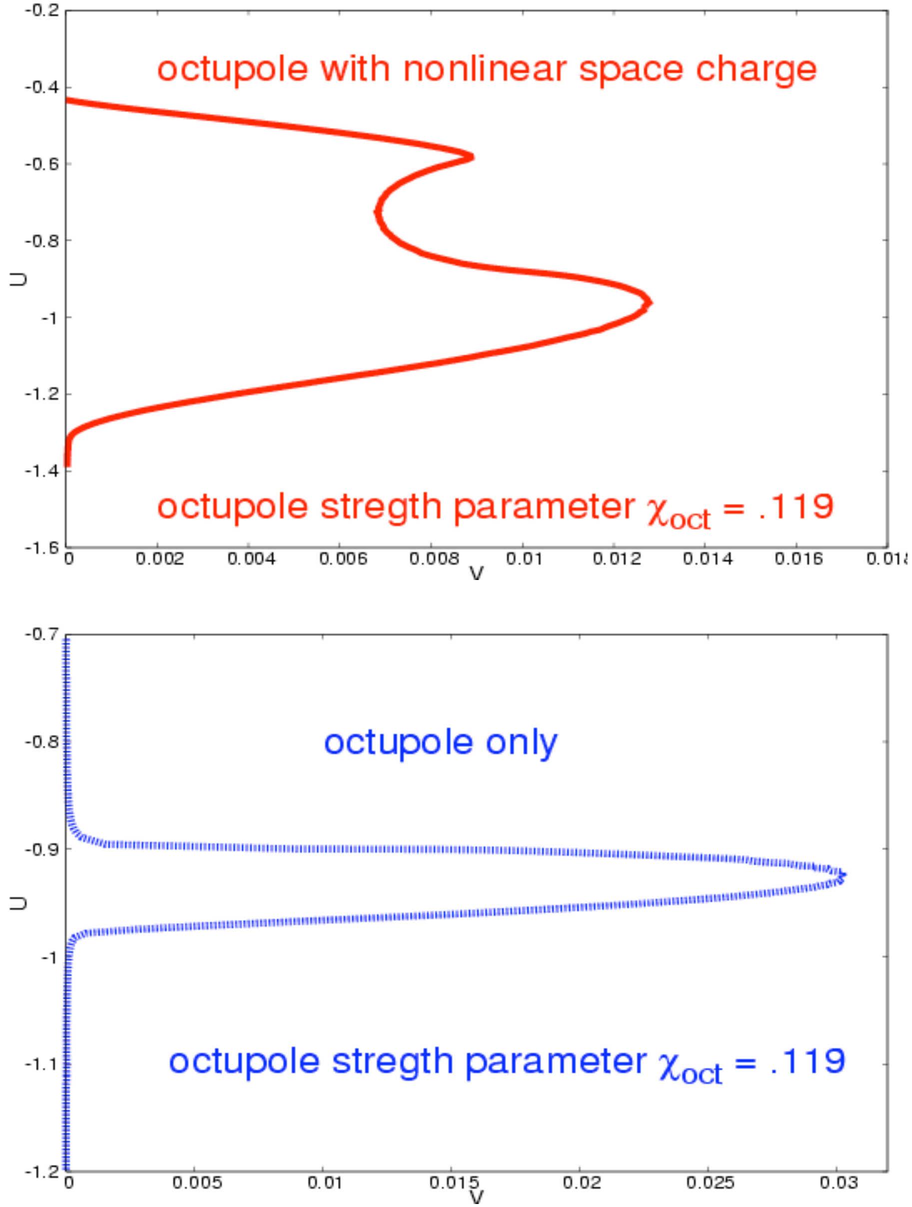
Combination of chromatic effects with nonlinear space charge, self-consistent electric field





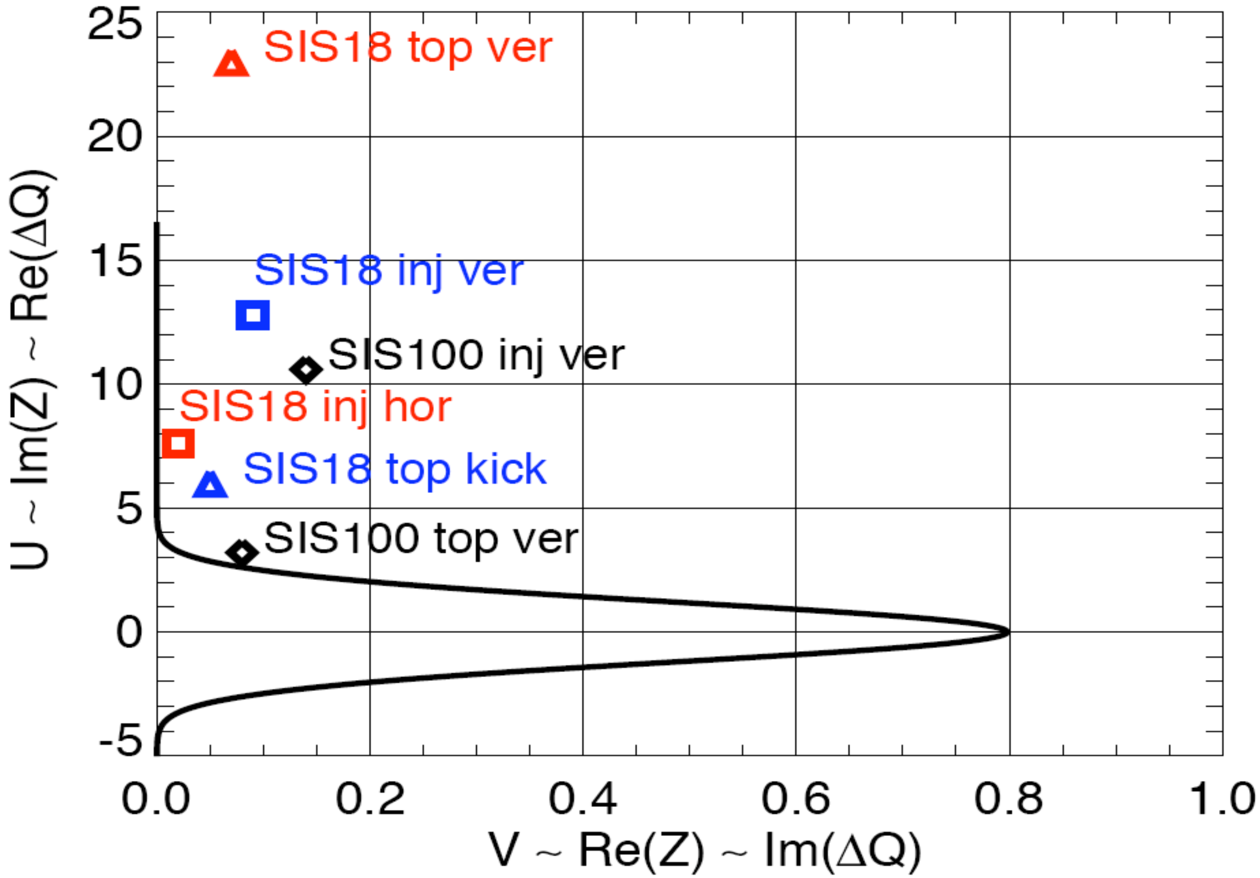
DISPERSION RELATION

Illustration for
the effect of
nonlinear space charge



FAIR: LINEAR DAMPING DUE TO δp

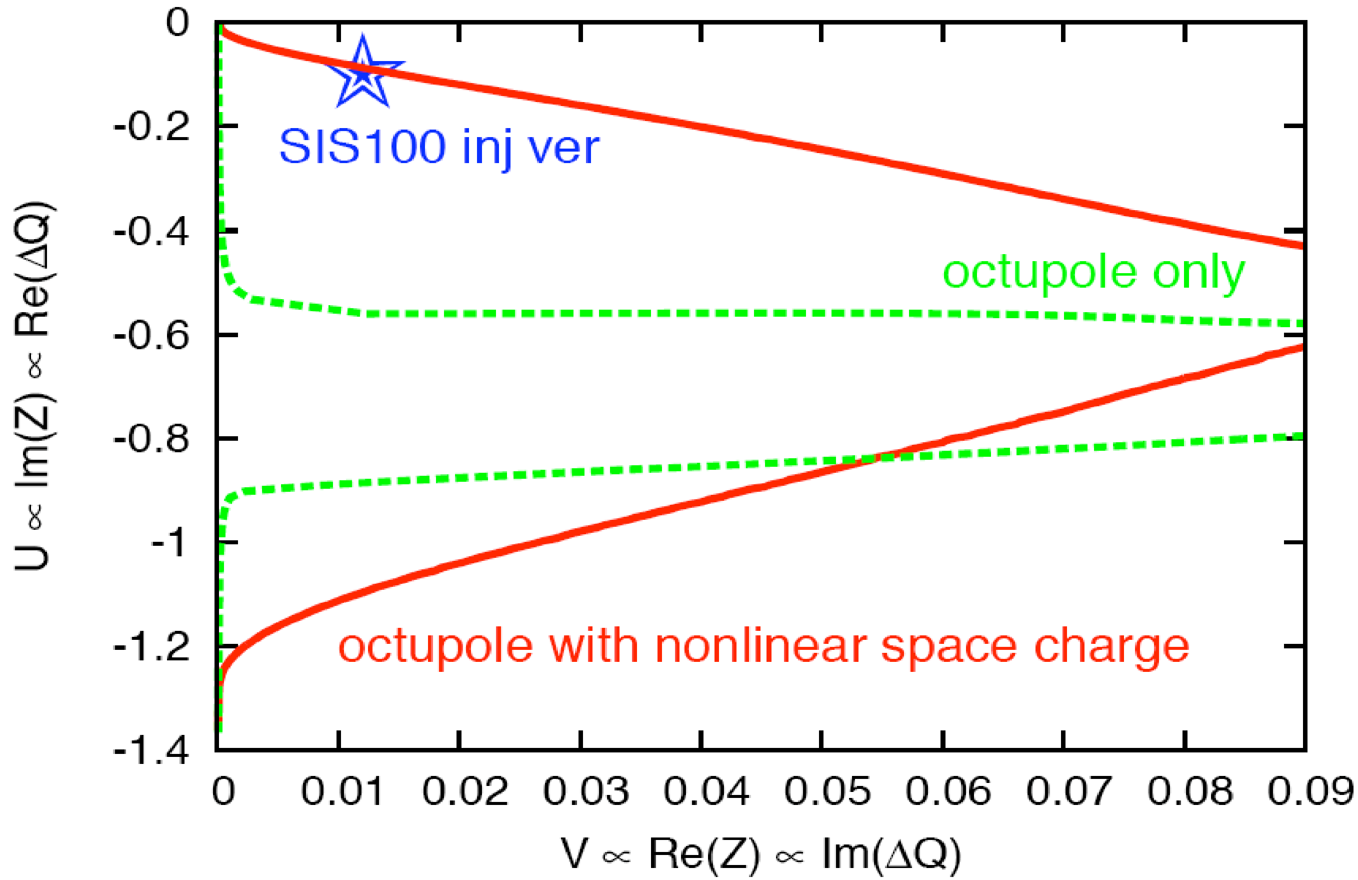
reference U^{28+}
coasting beams
for FAIR synchrotrons,
ver / hor RW, kickers



FAIR: NONLINEAR DAMPING

For SIS100,
injection energy,
vertical RW

SIS100 Magnets:
12 octupoles,
length 75 cm,
max. 2000 T/m^3

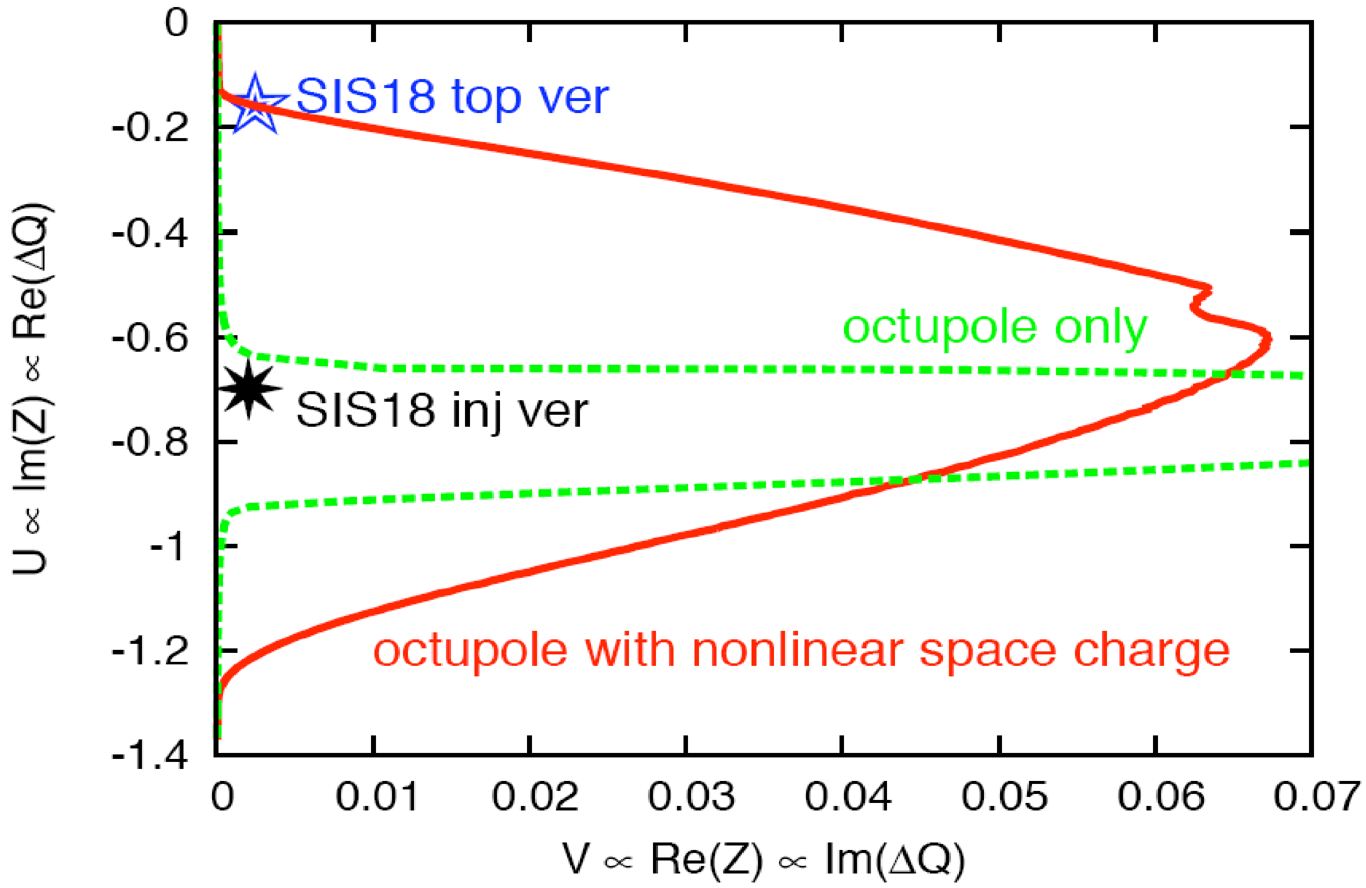


additionally:
δp-damping; non-coasting: 3D effects; transv. distribution; …

FAIR: NONLINEAR DAMPING

For SIS18,
vertical RW,
top energy,
injection energy

Magnets assumed:
12 octupoles,
length 75 cm,
max. 1000 T/m^3



(remember additional effects)

FIRST STEPS TOWARDS
3D STUDIES FOR (LONG) BUNCHES

NUMERICAL CODES

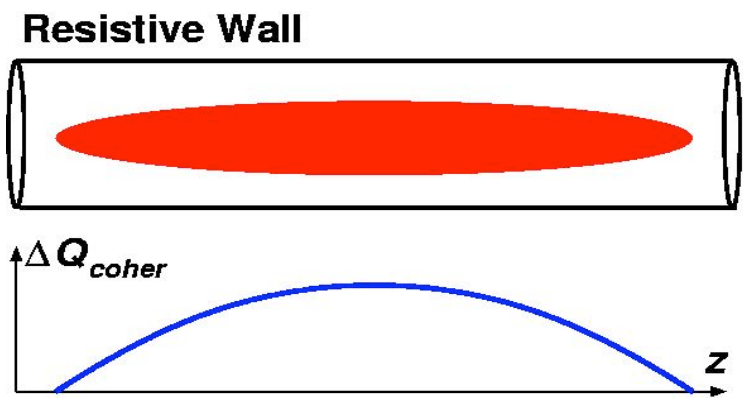
PATRIC	HEADTAIL
impedances $Z_\perp(\Omega)$	wake fields $W_\perp(s)$
self-consistent SC	analytical SC
general energy, ions	ultra relativistic
many steps per turn	once per turn
coasting / long bunches	short bunches

(joint work with G. Rumolo)

3D DAMPING MECHANISMS

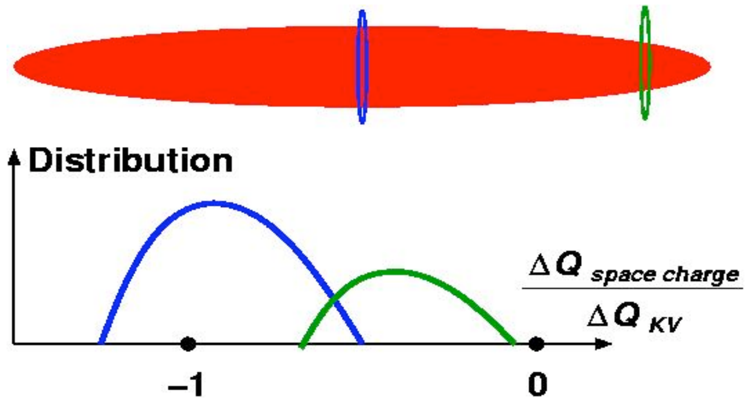
coherent tune spread

Im($Z_\perp$) causes a spread
of coherent eigenfrequency
$\delta\Omega_{coher} = \Delta\Omega_{max}$
$\Downarrow$
decoherence?



incoherent tune spread

different incoherent SC
tune spreads ⇒ affects stability
of the whole bunch?

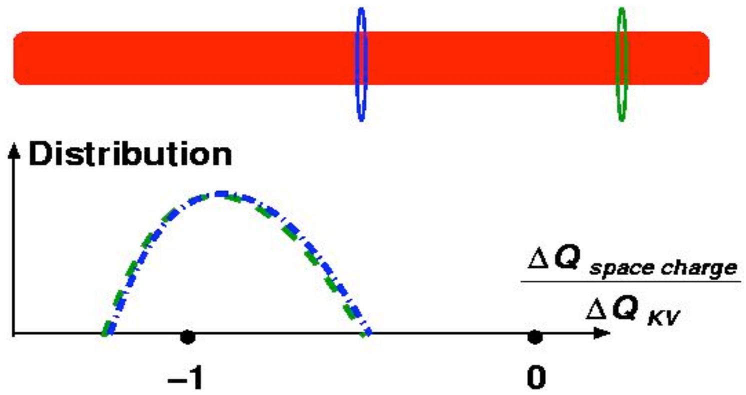


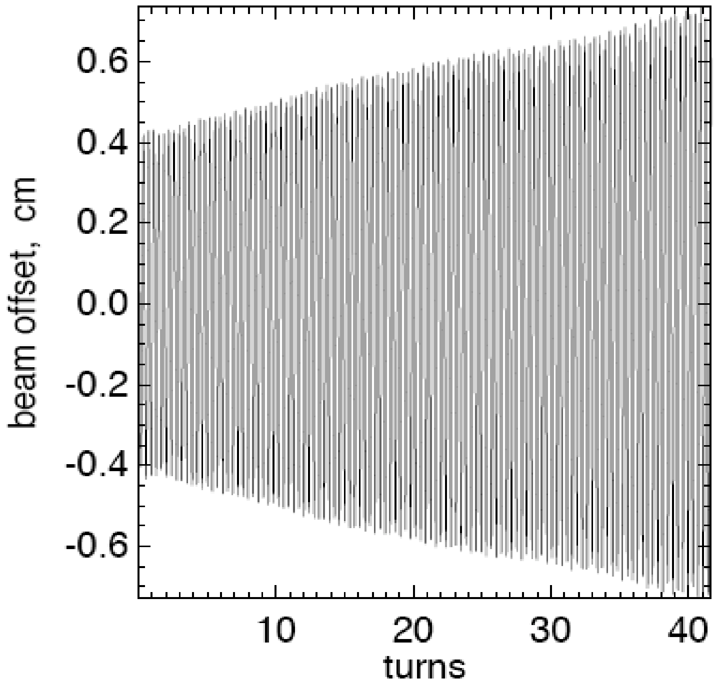
for example: octupoles which damp at ends;
δp-damping should not change.

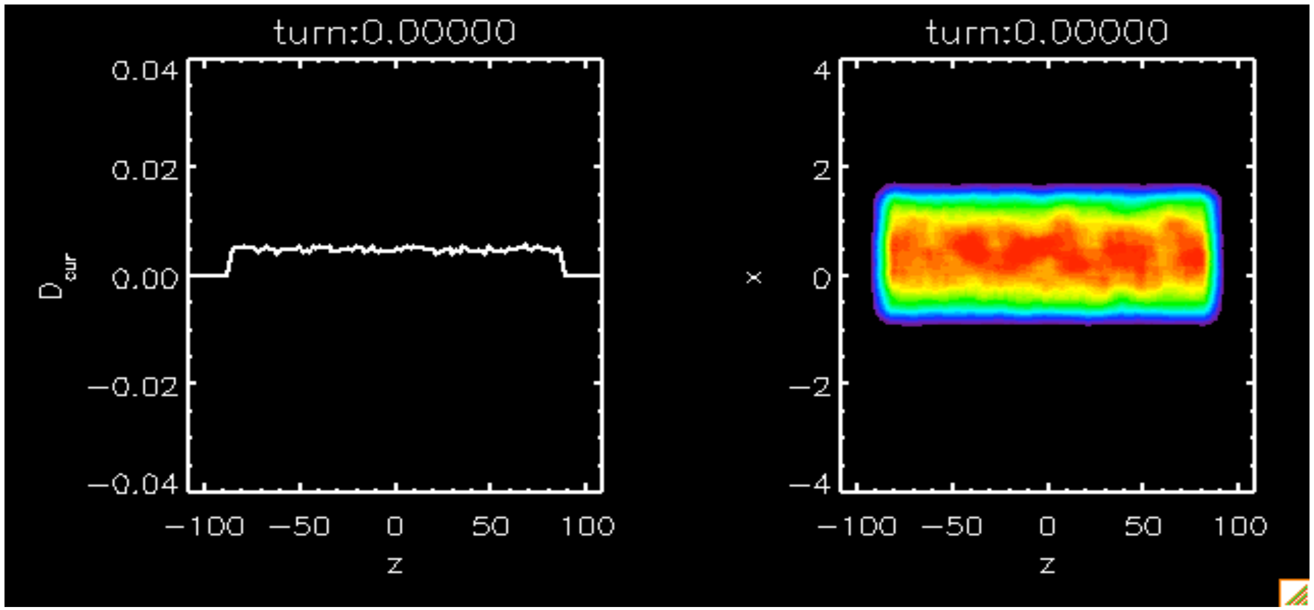
role of other effects: synchrotron dynamics, self-cons. SC,…

PATRIC **SIMULATIONS**

evolution of the *n=0* mode,
barrier bucket

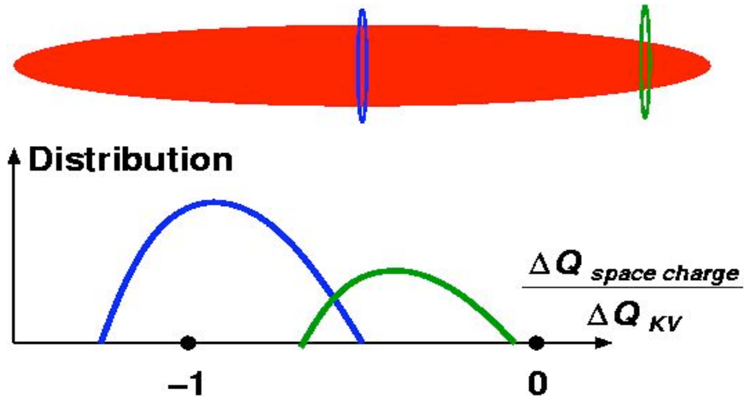


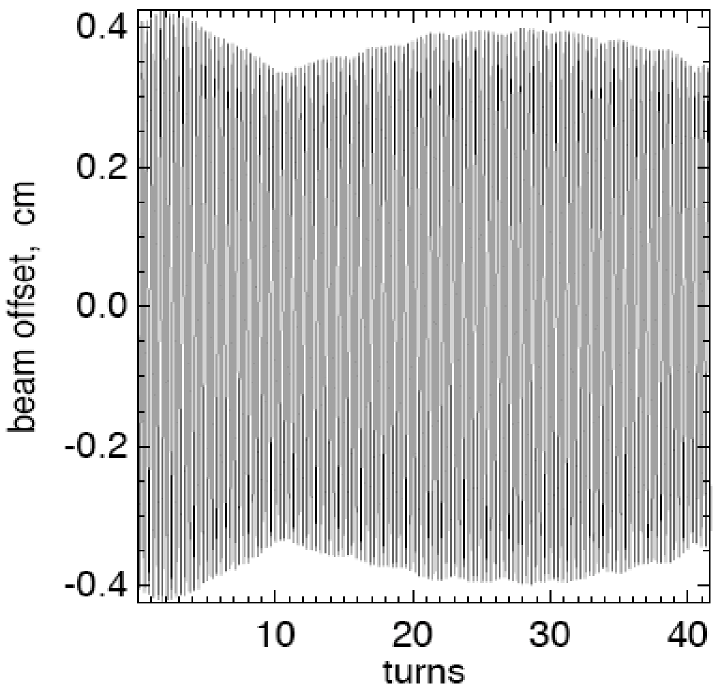


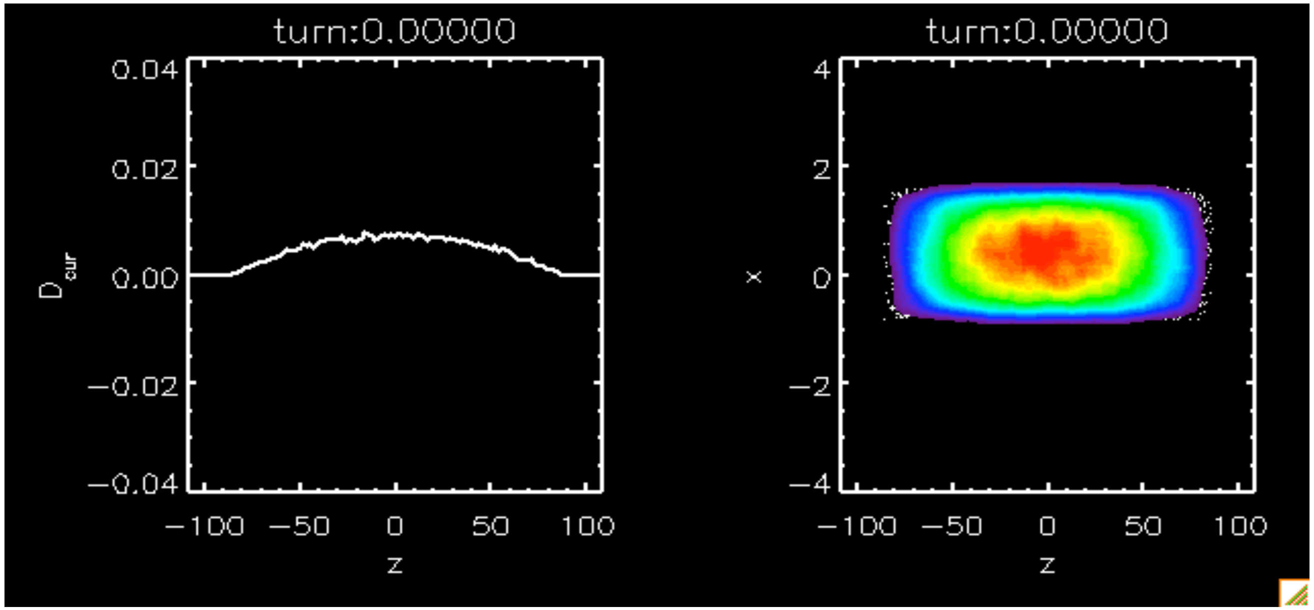


PATRIC **SIMULATIONS**

evolution of the *n=0* mode,
parabolic bunch



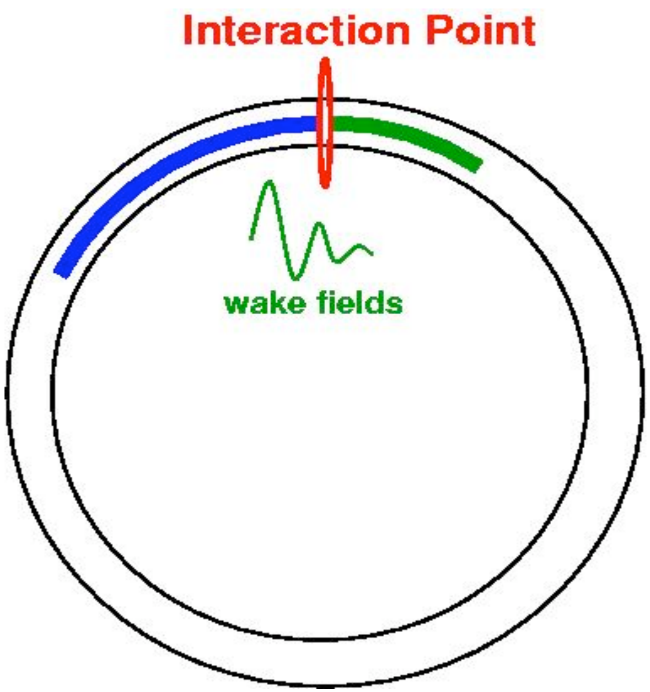




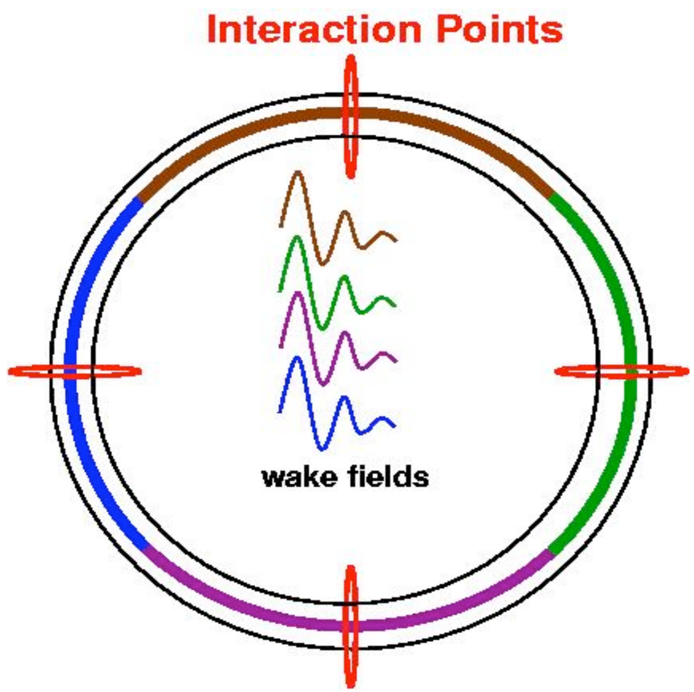
HEADTAIL: COASTING BEAM

for a coasting beam,
it is necessary to resolve oscillations due to
coherent mode number $\omega_{slow} = (n\text{-}Q)\ \omega_0$

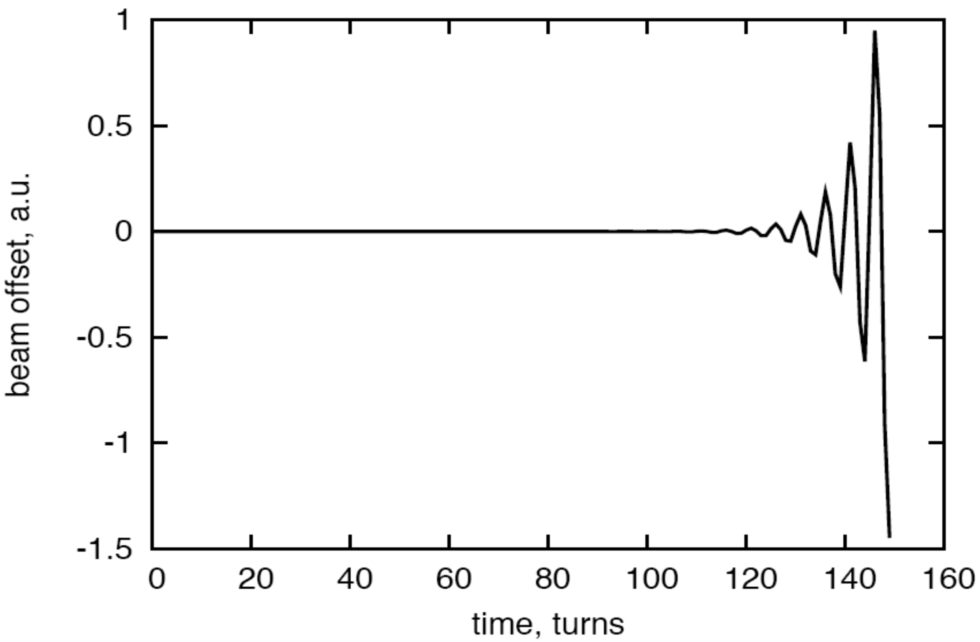
Wake Field model
for bunches

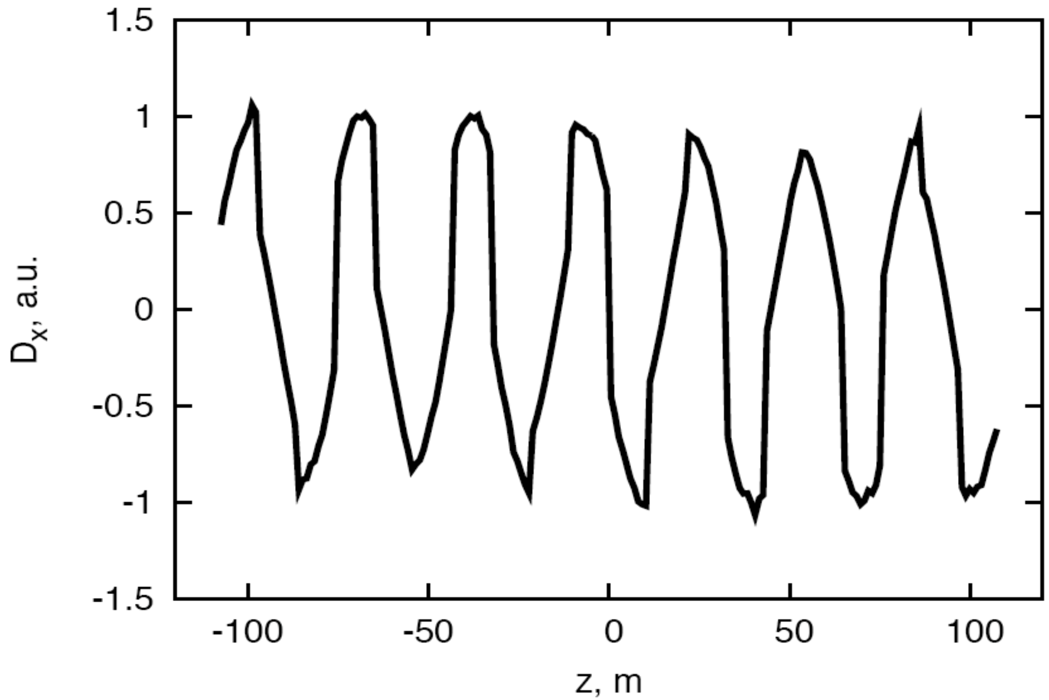


New model of Wake Fields
for coasting (long) beams



expon. growing mode corresponds to
the picture of the slow wave $\omega_{slow} = (n\text{-}Q)\ \omega_0$
(BB Impedance at Ω_Z)
$\Omega_Z\ /\ \omega_0 = 4.2 = n\text{-}2.8$
a slight disagreement in the growth time





SUMMARY

- nonlinear space charge can strongly modify stability properties of an octupole and ξ, confirmed by PATRIC simulation scans
- octupole of disadvantageous polarity reduces stability, different scalings with strong space charge
- non-self-consistent approaches for space charge are not always applicable (e.g. produces antidamping)
- octupoles may be used at FAIR to damp transverse instabilities
- various 3D effects must be investigated to predict stability of (long) bunches, started with PATRIC and HEADTAIL