Depletion of the Longitudinal Coupling Impedance by Uniform Electrons Clouds

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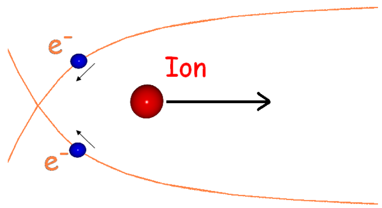
- Introduction – Motivation – Problems – Importance
- Electromagnetic fields in a cylindrical pipe
  - Electron Cloud Contribution to the Sources
  - Solution with boundary conditions
- Longitudinal space charge and resistive wall impedance
- Examples: Low energy GSI SIS-18, SIS-100, High Energy Rings
- Evanescent and surface-wave sustained modes
- Summary and Outlook

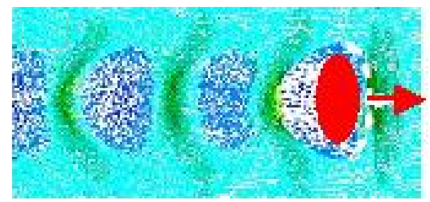
Introduction

In high energy rings (CERN, KEK, RHIC, Los Alamos) unwanted electron clouds
density: $n_{\mathrm{ec}} = 10^{11} \cdots 10^{12}\ \mathrm{m}^{-3}$ (few % of beam charge)
GSI SIS18: $1.5 \cdots 3.5 \times 10^{11}\ \mathrm{m}^{-3}$ (Rumolo et al. ECLOUD'04)
Protons: Generated by impact of protons on the pipe walls in the dipoles ...
Heavy ions: rest gas ionization (larger cross sections)

$\Longrightarrow$ Wake-field, fast head-tail
two-stream instability, linearized dispersion relation: $\frac{m_i}{\omega_e^2} = \frac{m_e}{\omega^2} + \frac{m_i}{(\omega - k v_0)^2}$
dynamical friction, density enhancement behind ?



$\Longrightarrow$



Important for design of accelerators and electron coolers !

Are the EC's localized, lifetime, equilibrium,longitudinal – transverse ?
Are electrons trapped ?
e.g. transverse: K. Ohmi, F. Zimmermann, E. Perevedentsev 2001...
Collective response of free electrons ?

Here electron plasma frequency: $\omega_{\mathrm{ec}} = \sqrt{\frac{n_{\mathrm{ec}} e^2}{m_e \epsilon_0}}$

Introduction...

Simple & solvable description via accessible quantity:

Longitudinal coupling impedance between beam and pipe (transverse ?)
from monopole excitation and linear response

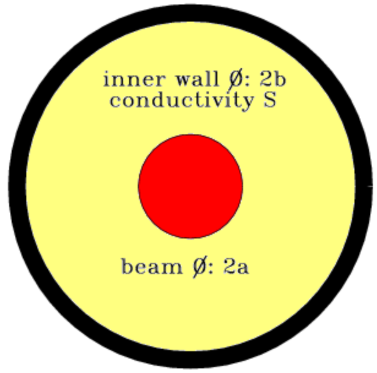
$$\boldsymbol{Z}_{||}(\boldsymbol{\omega}) = \frac{1}{\boldsymbol{Q}^2} \int_{\mathrm{V}_{\mathrm{beam}}} \mathrm{d}^3\boldsymbol{x}' \, \vec{\boldsymbol{E}}(\boldsymbol{r}', \boldsymbol{\omega}) \, \vec{\boldsymbol{j}}^*(\boldsymbol{r}', \boldsymbol{\omega})$$

$\Longrightarrow$ $\omega_{\mathrm{ec}} < \omega$: EC reduces $Z_{||}(\omega)$
$\omega_{\mathrm{ec}} \approx \omega$: impedance goes to zero $\Longrightarrow$ May prevent longitudinal instabilities
Very high electron densities: $\omega_{\mathrm{ec}} > \omega$:
$\Longrightarrow$ overdamped evanescent and surface-wave sustained modes

Exact solutions of model – no approximations !

GSI

Prelude



Homogeneous EC of density n_{ec} spreading all over the interior of the aperture (radius b, conductivity S)

$n_{ec} = 10^{11} \cdots 10^{12}\ \mathrm{m}^{-3} \Longrightarrow f_{ec} = \omega_{ec}/2\pi$ = 1· · ·10 MHz.

Charged particle beam (radius a, velocity $\vec{v}_b = \beta c \hat{z}$)

EC acts like shielding the beam which, in turn, depletes the coupling impedance

$\Longrightarrow$ Effect is governed by the ratio ω_{ec}/ω (ω: excitation frequency)

Rotationally symmetric disc

Total (effective) current $\vec{J}$ and charge $\rho(\vec{r}, t)$: $\quad \vec{J}(\vec{r}, t) = \vec{j}_0 + \vec{j}_e, \quad \rho = \rho_0 + \rho_e$

Mulsers oscillator model (P. Mulser, *High Power Laser-Matter Interaction*, 2006)

$\vec{j}_0$ and ρ_0 associated with the streaming motion of the beam particles

$\vec{j}_e$ and ρ_e : collective current and charge densities due to the coupling between the background electrons and the excited electric fields in the cylindrical pipe.

Field equations

Cold, collisionless, unmagnetized electron background
Electrons do not oscillate by field of beam $\longrightarrow$ no equilibrium
Electron equation of motion in the effective electromagnetic field :

$$\frac{d\vec{j}_e}{dt} = \frac{e^2 n_{0ec}}{m_e}\vec{E} - \frac{e}{m_e}\vec{j}_e \times \vec{B}$$

Axially symmetric monopole solution

$$\vec{j}_e = \frac{i\omega e^2 n_{0ec}}{m_e \omega^2}\vec{E}$$

Faraday's and Ampere's laws (Maxwells equations): $\Longrightarrow$

$$E_r(r,\omega) = \frac{-i}{k_z\left(1-\beta^2\varepsilon_{ec}\right)}\frac{\mathrm{d}E_z}{\mathrm{d}r},$$

$$B_\theta(r,\omega) = -\frac{i\omega\mu_0\epsilon_0\epsilon_{ec}}{k_z^2\left(1-\beta^2\varepsilon_{ec}\right)}\frac{dE_z}{\mathrm{d}r},$$

Longitudinal dielectric function ($k_z = \omega/v$) : $\varepsilon_{ec} = 1 - \frac{\omega_{ec}^2}{\omega^2}$

at $\beta^2\varepsilon_{ec} = 1$ and not $\varepsilon_{ec} = 1$!

Differential equations

Within the conducting wall of coductivity S:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{k_z^2}{\underline{\gamma}^2}\right] E_z(r,\omega) = 0\ , \quad b \leq r < \infty,$$

Modified relativistic factor due to the conductivity reads

$$\underline{\gamma} = \frac{\gamma_0}{\sqrt{1 - i\mu_0 \omega S \gamma_0^2 / k_z^2}}$$

Solution (I_ν, K_ν modified cylindrical Bessel functions of 1^{st} and 2^{nd} kind).

$$E_z(r,\omega) = \begin{cases} A_1\, r^{-\nu}\, I_\nu(\sigma_{\mathrm{ec}} r) - i\dfrac{Q}{\pi a^2 \epsilon_0 \varepsilon_{\mathrm{ec}} k_z \beta c} & 0 \leq r \leq a \\ A_2\, r^{-\nu}\, I_\nu(\sigma_{\mathrm{ec}} r) + A_3\, r^{-\nu}\, K_\nu(\sigma_{\mathrm{ec}} r) & a \leq r \leq b \\ A_4\, K_0(\underline{\sigma} r) & b \leq r < \infty \end{cases}$$

whith $\sigma_{\mathrm{ec}}^2 = \frac{k_z^2}{\gamma_0^2}\frac{\varepsilon_{\mathrm{ec}}}{1-\alpha}$, $\quad \underline{\sigma} = k_z/\underline{\gamma}$, $\quad \alpha = \frac{\omega_{\mathrm{ec}}^2}{\omega^2}\frac{1}{1-\beta^2\varepsilon_{\mathrm{ec}}}$, $\quad$ order: $\nu = \frac{\alpha/2}{1-\alpha}$

$A_1 \cdots A_4$ constants determined from boundary conditions

$E_r(r,\omega)$ similar

Coupling impedance

Longitudinal coupling impedance for the beam monopole source term:

$$Z_{\|}(\omega) = -\frac{2L}{Qa^2}\int_0^a \mathrm{d}r\, r\left[A_1\frac{I_\nu(\sigma_{\mathrm{ec}} r)}{r^\nu} - i\frac{Q}{\pi a^2\epsilon_0\varepsilon_{\mathrm{ec}} k_z \beta c}\right]$$

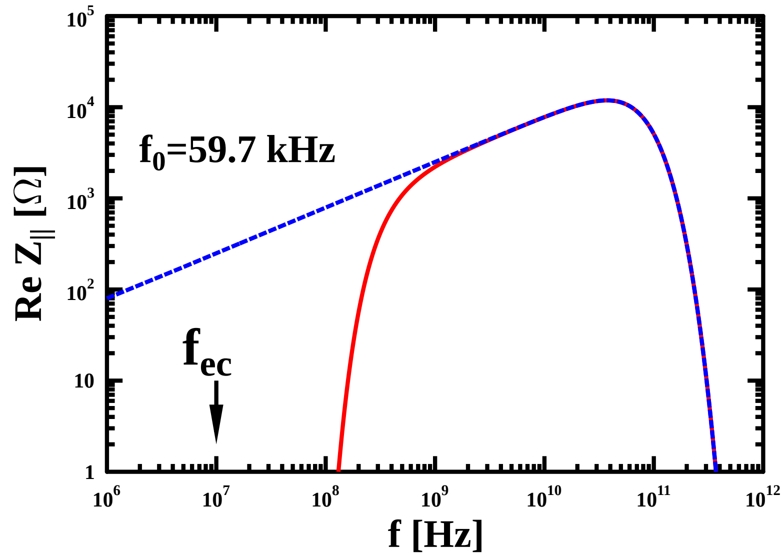
Solve integral in closed form:

$$Z_{\|}(\omega) = -\frac{2L}{Qa^2}\left[A_1\left(\frac{I_{\nu-1}(\sigma_{\mathrm{ec}} a)}{\sigma_{\mathrm{ec}} a^{\nu-1}} - \frac{\sigma_{\mathrm{ec}}^{\nu-2}}{2^{\nu-1}\Gamma(\nu)}\right) - i\frac{Q}{\pi a^2\epsilon_0\varepsilon_{\mathrm{ec}} k_z \beta c}\frac{a^2}{2}\right]$$

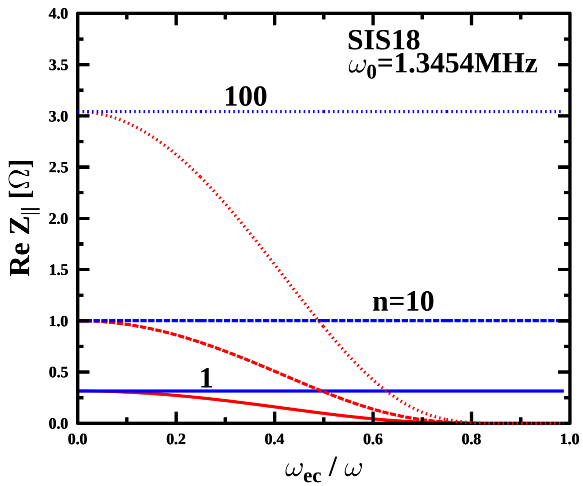
For $\omega_{\mathrm{ec}} = 0$ one recovers: $Z_{\|}(\omega) = i\,\frac{nZ_0}{2\beta\gamma^2}\,\frac{4\gamma_0^2}{k_z^2 a^2}\left[1 - 2I_1^2(\sigma_0 a)F\right]$

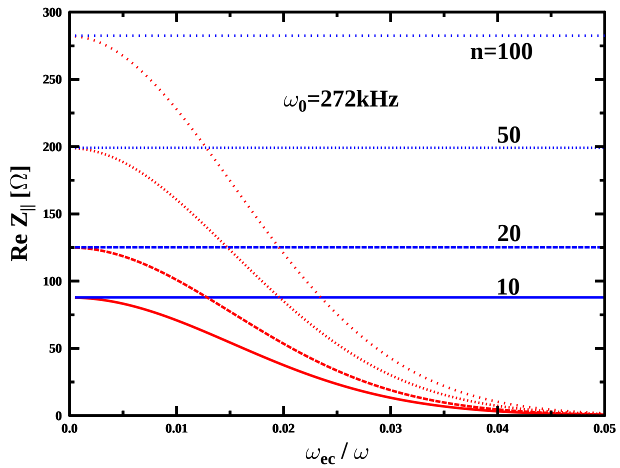
constants A_1, F

Results



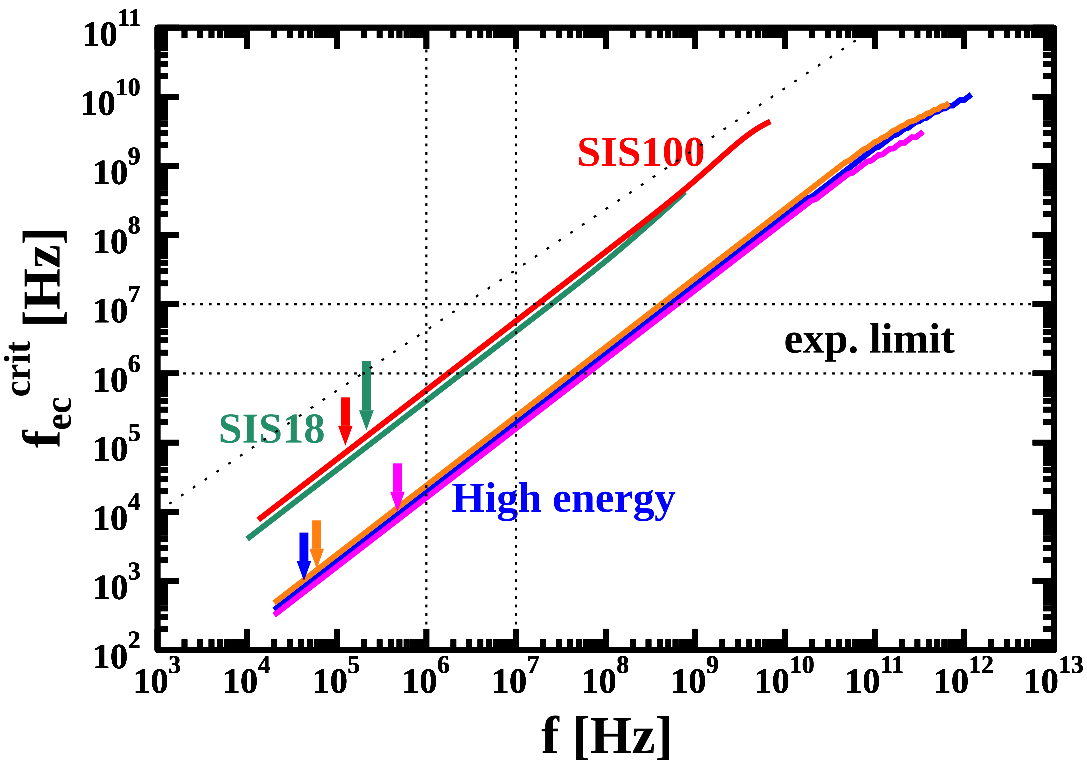
Real part of the longitudinal resistive wall impedance on the excitation frequency for $f_{ec} = 10$ MHz ($n_{ec} = 10^{12}\ m^{-3}$, KEKB LER).
Blue line: impedance without electron cloud
Red line: with EC





Left: low energy ring GSI-SIS18
right: high energy ring
n : harmonic number

Results ...



Fraction of critical electron cloud frequency where the longitudinal resistive wall impedance is half depleted as compared to the excitation frequency.

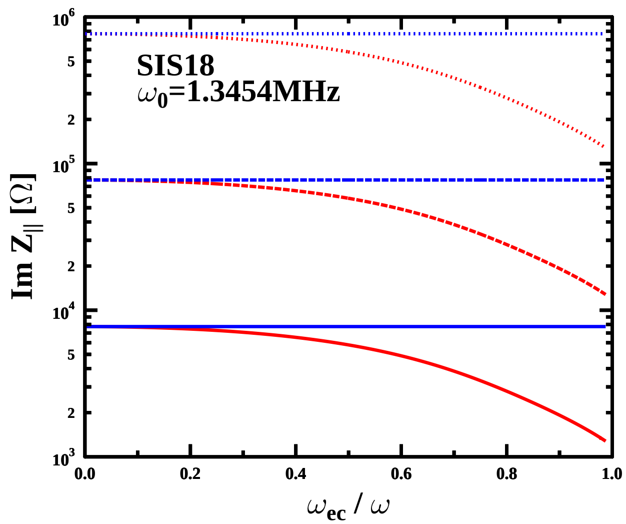
Dashed lines: measured values of electron cloud density of 10^{12} m^{-3} and 10^{11} m^{-3} in high energy rings.

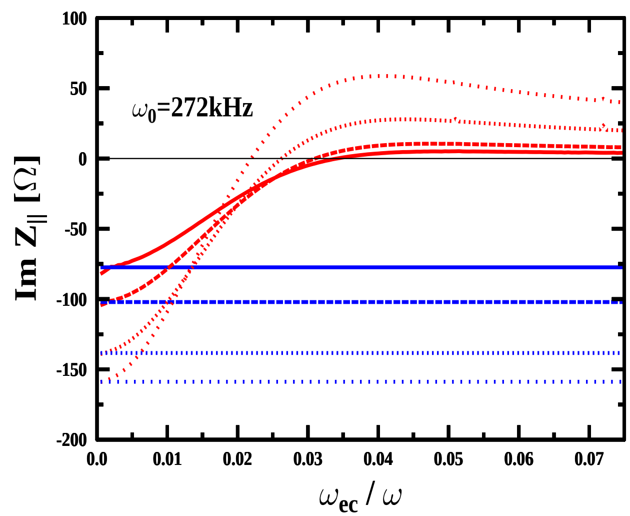
The arrows point to the respective revolution frequencies.

Dotted line: measured $f_{\rm ec}^{\rm crit}/f = 1$

Imaginary part

Two parts: Resistive wall and the space charge $Z_{||}(\omega_{ec}=0, S=\infty)$
Total: positive, EC is a capacitive medium for all frequencies $\omega > \omega_{ec}$.
Resistive part: negative, inductive.
$\omega < \omega_{ec}$ EC will act as an inductive medium with negative imaginary part.
High energy rings: imaginary part is negative and turns capacitive only in the presence of strong electron fields.



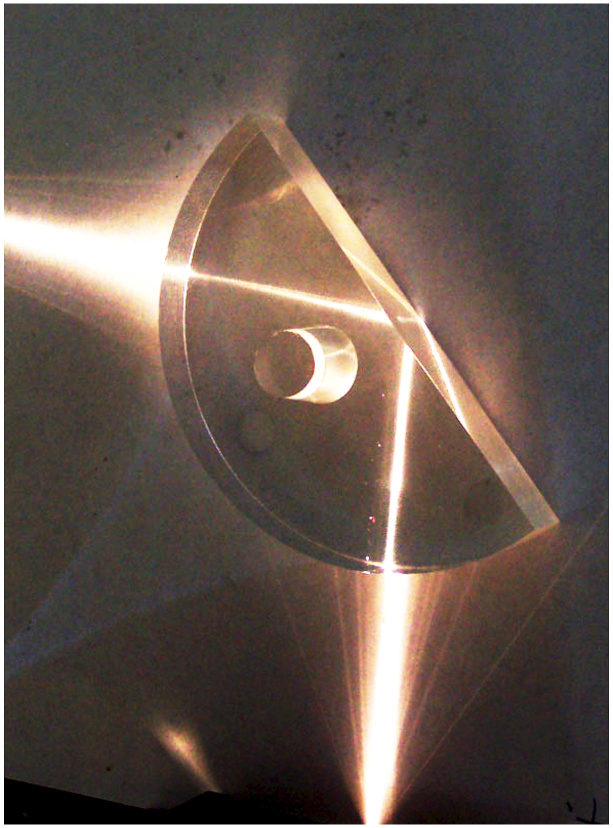


Imaginary part of the longitudinal resistive wall impedance (SIS18 left, high energy ring right)
bottom to top: for n=1, 10, 100 (left) or 100, 50, 20, 10 (right) (normally plot Im $Z_{||}/n$)

Evanescent & surface-wave-sustained modes

$\omega_{\rm ec} > \omega$:
Beam is completely shielded by the electromagnetic field of the electron cloud
Electromagnetic waves cannot propagate in an over-dense plasma
Waves are reflected at conducting surface due to skin effect and become evanescent
Penetration depth ≈ skin depth $\delta_s \approx \frac{c}{\sqrt{\omega_{\rm ec}^2 - \omega^2}}$.

Instead of traversing the plasma, waves creep along the plasma surface.



Evanescent & surface-wave-sustained modes...

May give rise to heating a plasma rather than damping
Waves then do not travel any more in the radial direction but rather propagate along the plasma surface.
The wave energy is transferred to the plasma by the evanescent wave
Wave enters the plasma perpendicular to its surface and decays exponentially with the skin depth
Heating: real part of the impedance becomes negative (feedback).
This transfer mechanism allows to support over-dense plasmas
Alternative way of heating a plasma:
Wave energy is then transferred to the plasma by an evanescent wave
Wave enters the plasma perpendicular to its surface and decays exponentially with the skin depth. Allows to generate over-dense plasmas with electron densities beyond the critical density
Even higher excitation frequencies, $\omega_{\mathrm{ec}}/\omega > \sqrt{2}$:
Waves do not propagate any more along the surface but are rather overdamped in the longitudinal direction.

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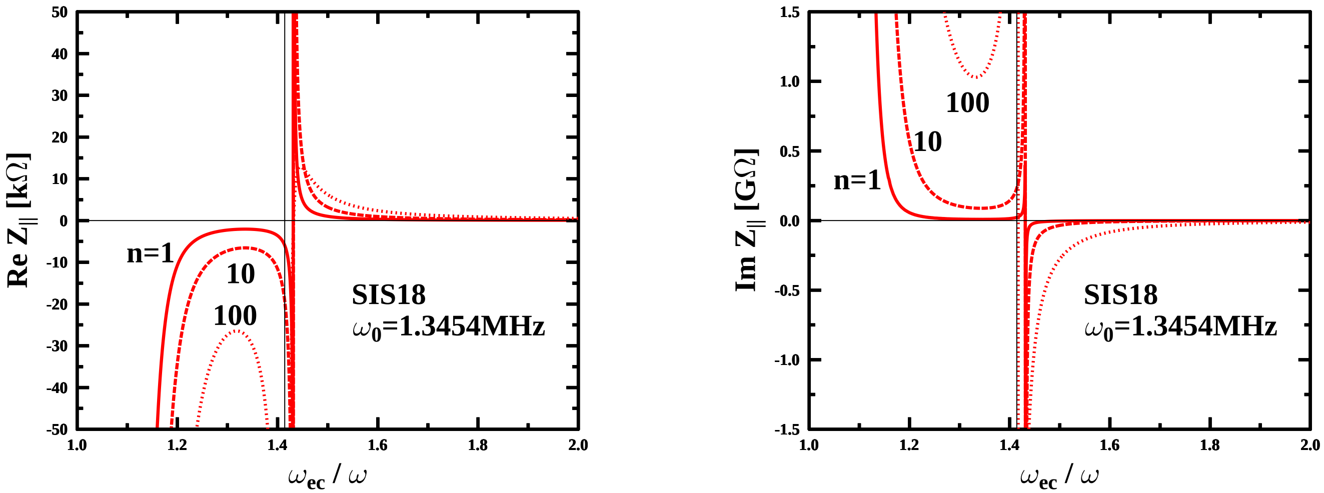
$\omega_{ec} > \omega$



Vertical line points to the lower limit of surface waves $\omega_{ec}/\omega = \sqrt{2}$
3 regions of a beam embedded in an over-damped plasmas:

- (i) No waves exist for $1 < \omega_{ec}/\omega \lesssim 1.05$
- (ii) In the intermediate region of evanescent waves, $1.05 \lesssim \omega_{ec}/\omega \lesssim \sqrt{2}$ the real part of the impedance is negative (feedback) and the imaginary part is positive, thus capacitive due to the strong magnetic field
- (iii) Overdamped surface waves $\omega_{ec}/\omega > \sqrt{2}$ with positive real part and negative imaginary part exist for $\omega_{ec}/\omega > \sqrt{2}$ in analogy to the previous case of $\omega > \omega_{ec}^{crit}$
- The latter two regions are separated by a resonance transition.
- Presence of the weak beam shifts critical value $\sqrt{2}$ slightly.
  More intense beams shift substantially

Epilogue, Summary and Outlook

- Effect of shielding of a beam by a homogeneous electron cloud on the coupling impedance has been calculated in dependence on electron cloud density
- $\beta^2 \left(1 - \frac{\omega_{ec}^2}{\omega^2}\right)$ is decisive quantity
- Low energy rings: Depletion of the coupling impedance is complete at the excitation frequency corresponding to the plasma frequency of the EC
- High energy rings: not as expected but already for much lower electron cloud densities.
- Theory does not yet account for resonance effects at the singularity at $\omega = \omega_{ec}$.
- Extend to include Joule heating by losses from the imaginary part of ω
- Extend to small excitation frequencies $\omega < \omega_{ec}$.
- Apply theory due to the presence of an electron cooler The electron beams of present electron coolers generate amperes of current and, hence, shield the beam substantially.

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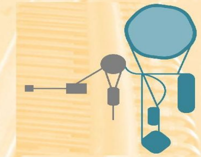
Some references



[1] F. Zimmermann, *Electron-Cloud Studies for the Low Energy Ring of KEKB*, CERN-SL-Note-2000-004 AP.

[2] M. Giovannozzi, Métral, G. Rumolo, and F. Zimmermann, Phys. Rev. ST Accel. Beams **6** (2003) 010101.

[3] G. Rumolo, A. Z. Ghalam, T. Katsouleas, C. K. Huang, V. K. Decyk, C. Ren, W. B. Mori F. Zimmermann, and F. Ruggiero, Phys. Rev. ST Accel. Beams **6** (2003) 081002.

[4] W. Fischer, J.M. Brennan, M. Blaskiewicz, and T. Satogata, Proc. ECLOUD'02 Workshop, CERN, 15-18 April 2002, CERN Yellow Report CERN-2002-001.

[5] F. Zimmermann, Phys. Rev. ST Accel. Beams **7** (2004) 124801.

[6] *http://ab-abp-rlc.web.cern.ch/ab-abp-rlc-ecloud*.

[7] G. Rumolo and F. Zimmermann, Phys. Rev. ST Accel. Beams **5** (2002) 121002.

[8] G. Rumolo and Métral, Proc. 9th Int. Computational Accelerator Physics Conference, Oct. 2-6, 2006, Chamonix Mont-Blanc, France, ICAP06, *http://accelconf.web.cern.ch, MOA2IS02*.

[9] P. Mulser, *High Power Laser-Matter Interaction*, 2006

[10] A.M. Al - Khateeb, O. Boine-Frankenheim, R.W. Hasse, and I. Hofmann, Phys. Rev. E **71**, 026501-1 (2005).

[11] A.W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, Wiley, New York, (1993).

[12] V. Balbekov, *Space Charge Impedance and Transverse Beam Instability*, FERMILAB report FN-0782-AD (2006).

[13] G. V. Stupakov and P. Chen, Phys. Rev. Lett. **76** (1996) 3715.

[14] A. Boardman, *Electromagnetic Surface Waves*, Wiley, N.Y. 1982.

[15] P. Mulser, *High Power Laser-Matter Interaction*, Springer, Berlin 2006