Resonance analysis for the electron cooler of SIS–18 using MAD-X

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Motivation and introduction

- **Within FAIR, beams with high requirements to beam quality**
  $\longrightarrow$ need for electron cooling particularly for secondary beams in storage rings
- **Besides cooling, electron cooler acts as nonlinear optical element leading to resonance excitation what may yield emittance growth (effective "heating")**
  $\longrightarrow$ CELSIUS (V. Ziemann, TSL-note 98-43)
  - **Resulting beam blow up yields reduction of electron cooling, so, may lead to particle loss because of beam heating due to interaction with target**
  - **In high current devices, large eigen space charge tune spread**
    $\longrightarrow$ resonance crossing, possible reduction of space charge limit
- **Calculations with parameters of SchwerIonenSynchrotron (SIS) – 18**

- Is a Synchrotron already existing at GSI.

  circumference $C = 216.720$ m

- It will be upgraded for usage within FAIR project.

- There is an electron cooler

  - Radius of electron beam can be decreased below particle beam radius

  - Particle beam can feel non-linear field of electron beam

- Possibly experiment on this topic in near future

Parameter used in the calculations

	SIS–18[1]
Particle	U^{73+}
Particle energy E / (MeV/u)	11.4 (injection)
Relativistic factors $\beta_0, \ \gamma_0$	0.15, 1.01
Cooling length L_C / m	3
Electron current I_e / A	0.3
Cathode radius r_{Cath} / mm	12.7
Adiabatic expansion factor f_E: used, (range)	3, (1 ... 8)
Electron beam radius $(b = r_{Cath}\sqrt{f_E})$ / mm	22
Beta function in the cooler $(\hat{\beta}_x, \ \hat{\beta}_y)$ / m	8, 15

[1] B. Franczak, SIS Parameter list; L. Groening, GSI-Dissertation 98-20, (1998)

Model for tracking

- Model system for tracking: rotational matrix + transverse momentum kick

$$\begin{pmatrix} x_{n+1} \\ x'_{n+1} \end{pmatrix} = \begin{pmatrix} \cos 2\pi\nu_{0,x} & \hat{\beta}_x \sin 2\pi\nu_{0,x} \\ -\frac{1}{\hat{\beta}_x} \sin 2\pi\nu_{0,x} & \cos 2\pi\nu_{0,x} \end{pmatrix} \begin{pmatrix} x_n \\ x'_n + \Delta x'(x_n, y_n) \end{pmatrix}$$

with:

– x_i, x'_i – phase space variables in ith step

– $\hat{\beta}$ – beta function in the cooler

- Matrix gives phase advance of the lattice $2\pi\nu_0$
- Single non-linear transverse momentum kick $\Delta x'(x_n, y_n)$ for electron beam in thin lens approximation

Transverse momentum kick

Kick with effective focal strength for a round electron beam with arbitrary radial current density profile

$$K_1(x,y) := \frac{\Delta x'(x,y)}{x} = \frac{qq'N'}{2\pi\varepsilon_0 m_0 c^2 \beta_0^2 \gamma_0^3} \frac{1}{R^2} \int_0^R \mathrm{d}r\ r\ n_0(r)$$

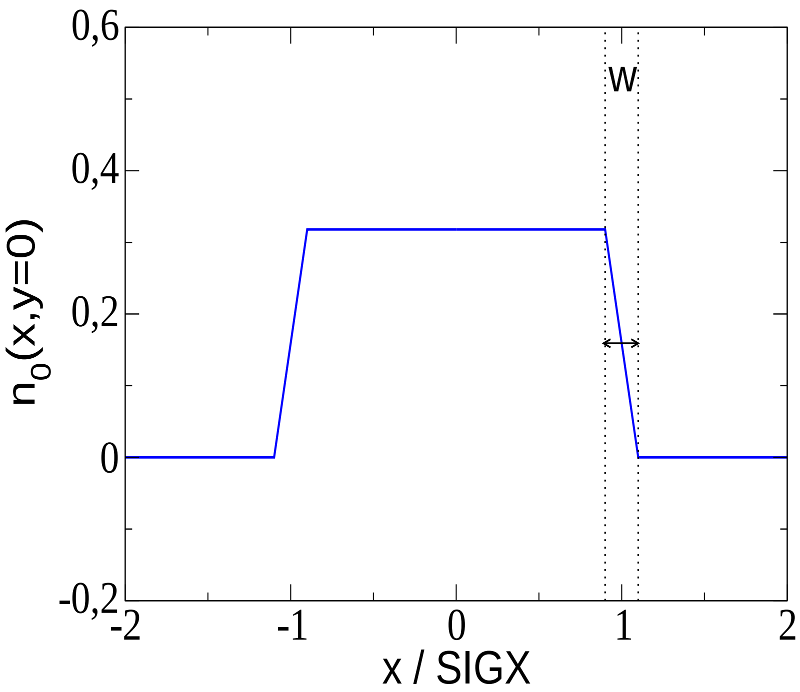
- q, q' – particle charge in considered beam (protons), opposite beam (electrons)
- $N' = \left|\frac{I_e L_{\mathrm{cool}}}{\beta_0 c q'}\right|$ – number of electrons in the cooler
- $R = \sqrt{x^2 + y^2}$ – distance from the centre of the electron beam

  $\longrightarrow$ Kick depends on both perpendicular directions

- Radial electron current density, is normalised by

$$\int_0^\infty \mathrm{d}r\ r\ n_0(r) = 1$$

Electron current profile used



- beambeam kick elements in MAD-X with
  - Gaussian profile
  - Flat top profile
  - Hollow-parabolic profile
- Flat top profile for electron cooler important
- Here, trapezoidal profile used with beam radius b and width of edge layer w
- Analytical expressions for resulting force

Consider core of electron beam with constant current density

- Effective (linear) focal strength

$$K_1 = \frac{\Delta x'}{x} = \frac{qeN'}{2\pi\varepsilon_0 m_0 c^2 \beta_0^2 \gamma_0^3 b^2}$$

U^{73+}: $K_1 = -0.010\ \mathrm{m}^{-1}$

protons: $K_1 = -0.031\ \mathrm{m}^{-1}$

- Resulting linear tune shift

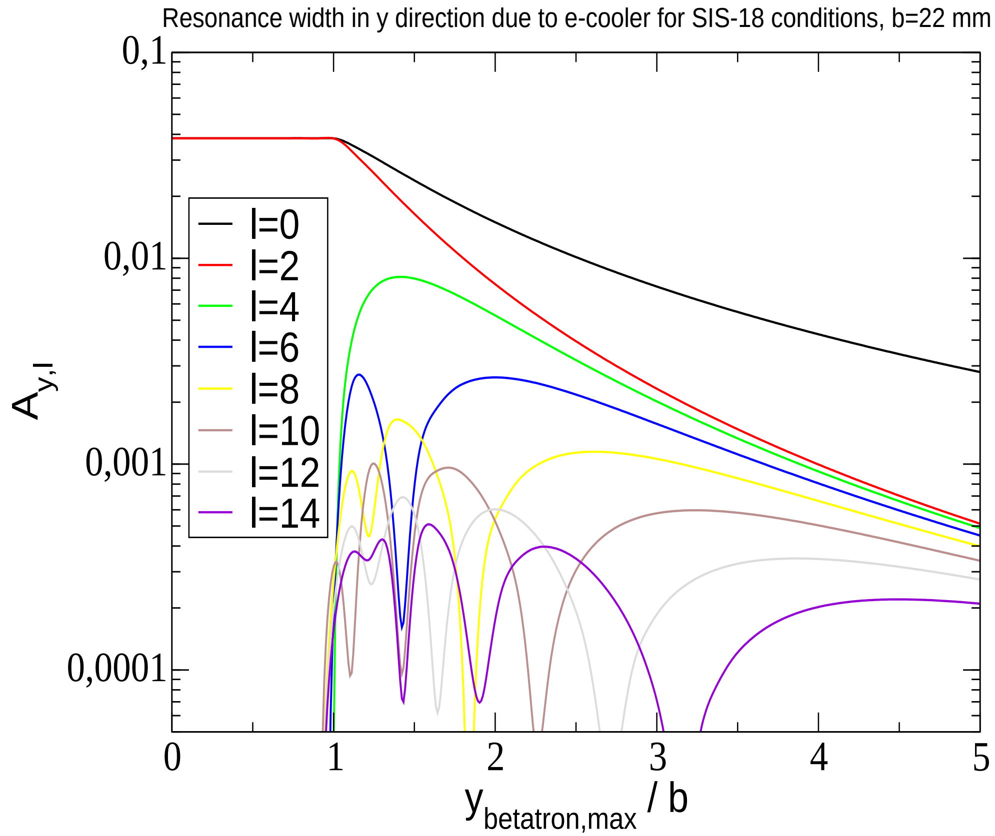
$$\Delta\nu_z = -\frac{\hat{\beta}_z K_1}{4\pi}$$

U^{73+}: $\Delta\nu_x = 0.0066,\ \Delta\nu_y = 0.012$

protons: $\Delta\nu_x = 0.020,\ \Delta\nu_y = 0.037$

Resonance analysis

Use analytic model to find relevant resonances.



- **Flat top electron current profile (trapezoidal)**
- **Resonance strength strongly decreases with increasing order**
- **Only resonances with even order**

  $\Downarrow$

  Search only for resonances of order 4 and 6

Model developed by E. Keil (Proc. of CERN Accelerator school 95-06)

and extended by V. Ziemann, (TSL-notes 98-43 and 2004-60)

Resonance analysis , $\nu_x - \nu_y$ – scan

To detect resonances, the relative beam width $w_{z,\mathrm{rel}} = \sigma_{f,z}/\sigma_{i,z}$.

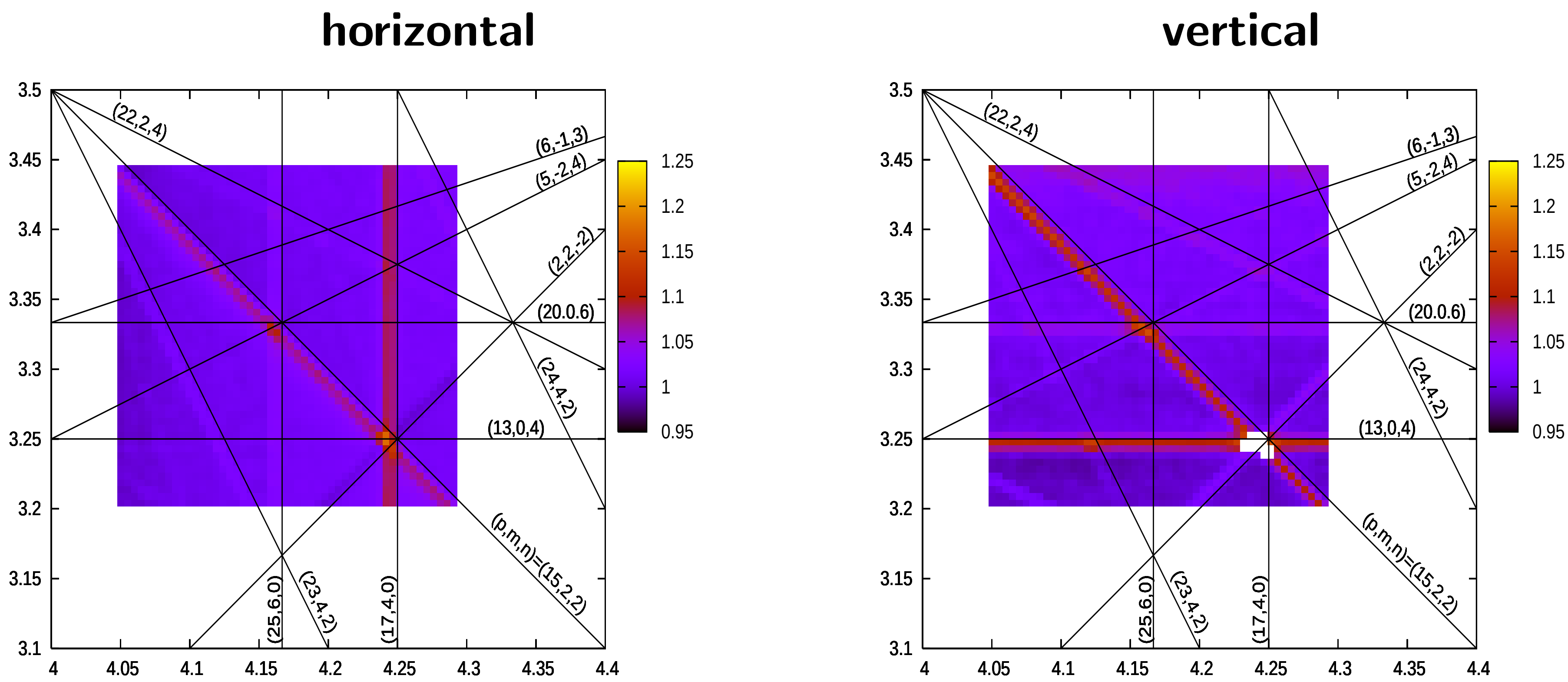
That was calculated with a variable horizontal and vertical tune.

- Tune varied in range $\nu_x \in [4.05, 4.3]$, $\nu_y \in [3.2, 3.49]$
- Range includes nominal working point $(\nu_x, \nu_y) = (4.2, 3.4)$[1]
- No half-integer resonance included
- Round Gaussian particle beam with initial width $\sigma_{i,x} = \sigma_{i,y} = b$
- 51 points in every tune direction $\longrightarrow$ 2601 runs
  - 5000 particles
  - 1000 revolutions

[1] B. Franczak, SIS Parameter list

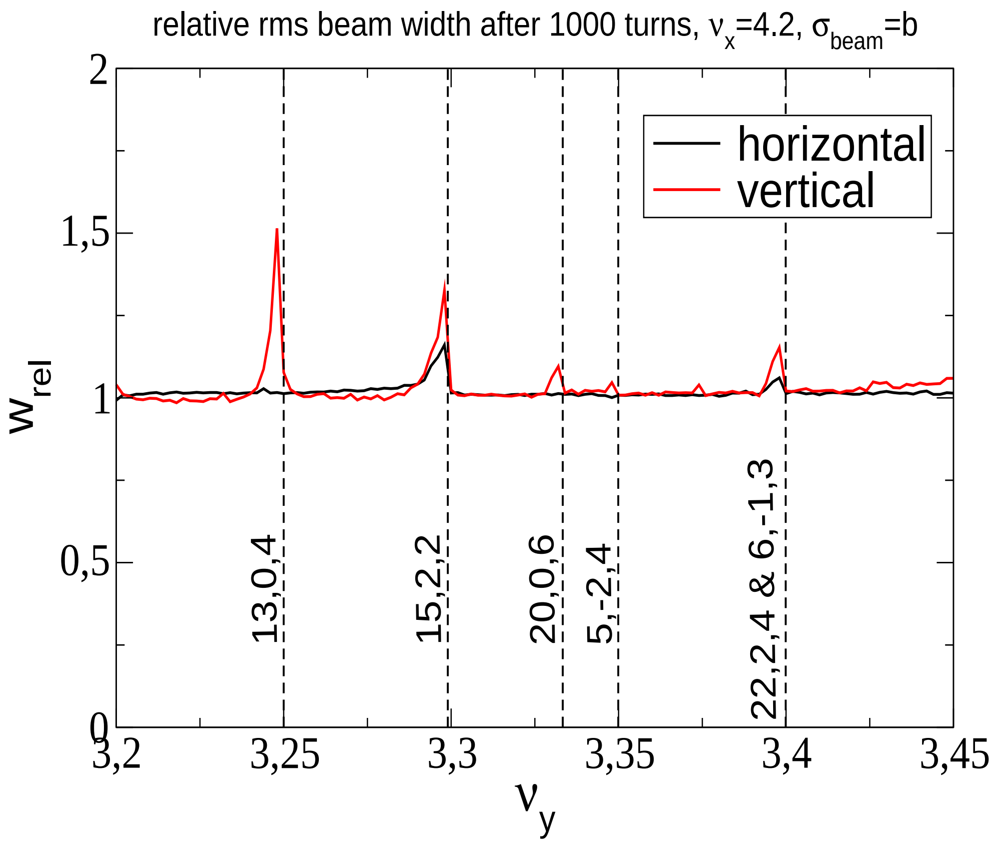
Resonance analysis , $\nu_x - \nu_y$ – scan

Relative beam width $w_{z,\mathrm{rel}} = \sigma_{f,z}/\sigma_{i,z}$ for variable horizontal and vertical tune



- Resonances of order 4 and 6 found could be identified.
- Sum resonances and resonances depending only on 1 tune → beam blow up
- Latter cause beam blow up only in according direction.
- Resonances are shifted because beam-beam interaction yields tune shift.

Relative rms beam width $w_{z,\mathrm{rel}}$ for variable ν_y, $\nu_x = 4.2$, $\sigma_{i,x} = \sigma_{i,y} = b$



- **Stronger vertical beam blow up because of $\hat{\beta}_y > \hat{\beta}_x$**
- **Quantitative verification of resonance widths with analytic model desirable but generally not possible**
- **Only exception: half-integer resonance at $\nu_y = 3.5$**

Half-integer resonance

- Is driven by gradient error, i.e. by linear error. Such an error does not cause emittance growth.
- Resonance width = full width of tune range, where beta function is enhanced by factor 2 or more, is given by half-integer stopband integral

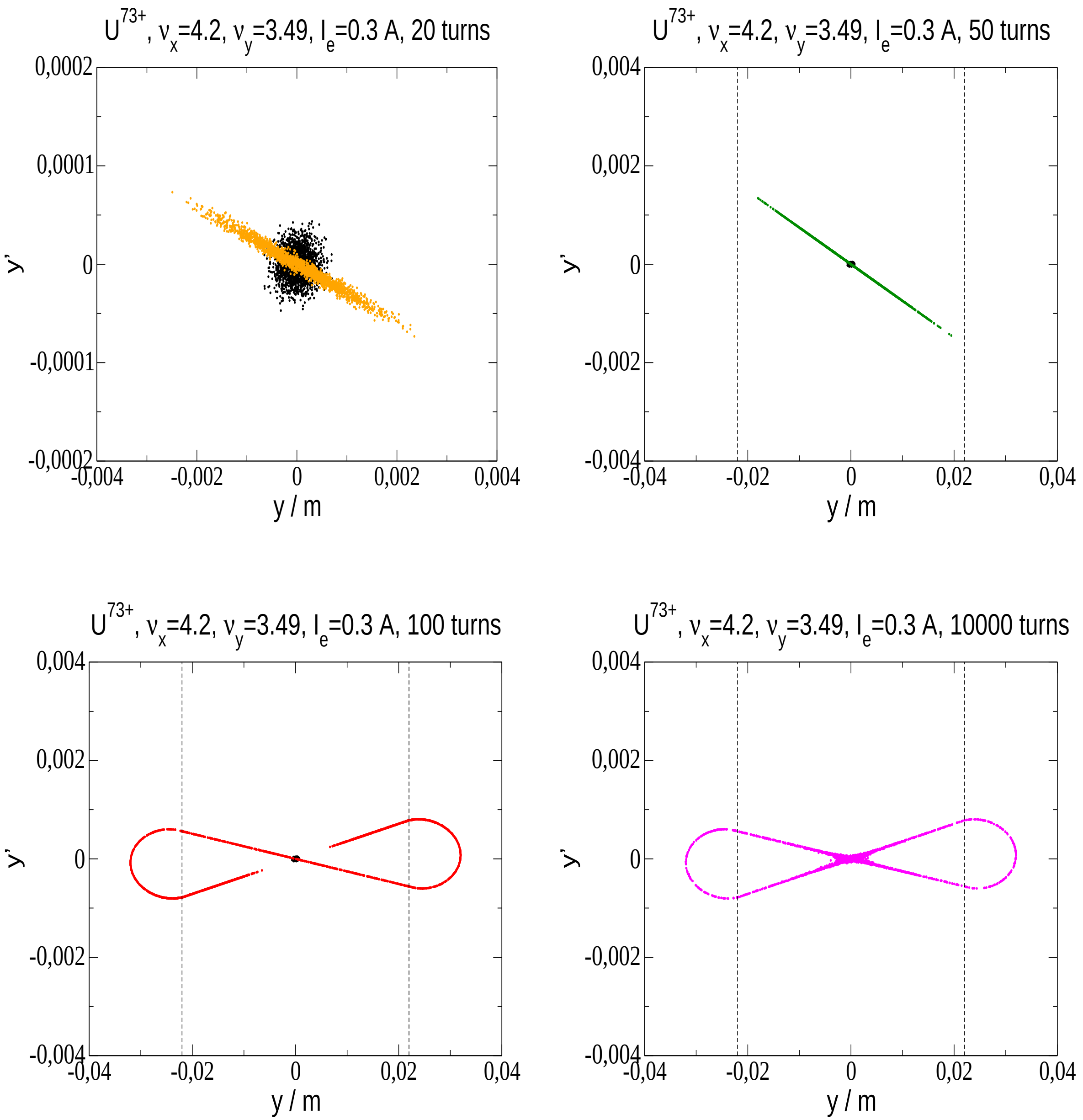
$$J_p = \frac{1}{2\pi} \oint \hat{\beta}\; k_1(s)\; \mathrm{e}^{-\mathrm{i}p\phi}\; \mathrm{d}s,$$

  i.e. where beam width is enhanced by factor of $\sqrt{2}$.
- To prepare such a situation, performed calculation with initial beam width

$$\sigma_{i,x} = \sigma_{i,y} = 0.01\; b \ll b.$$

Resonance analysis

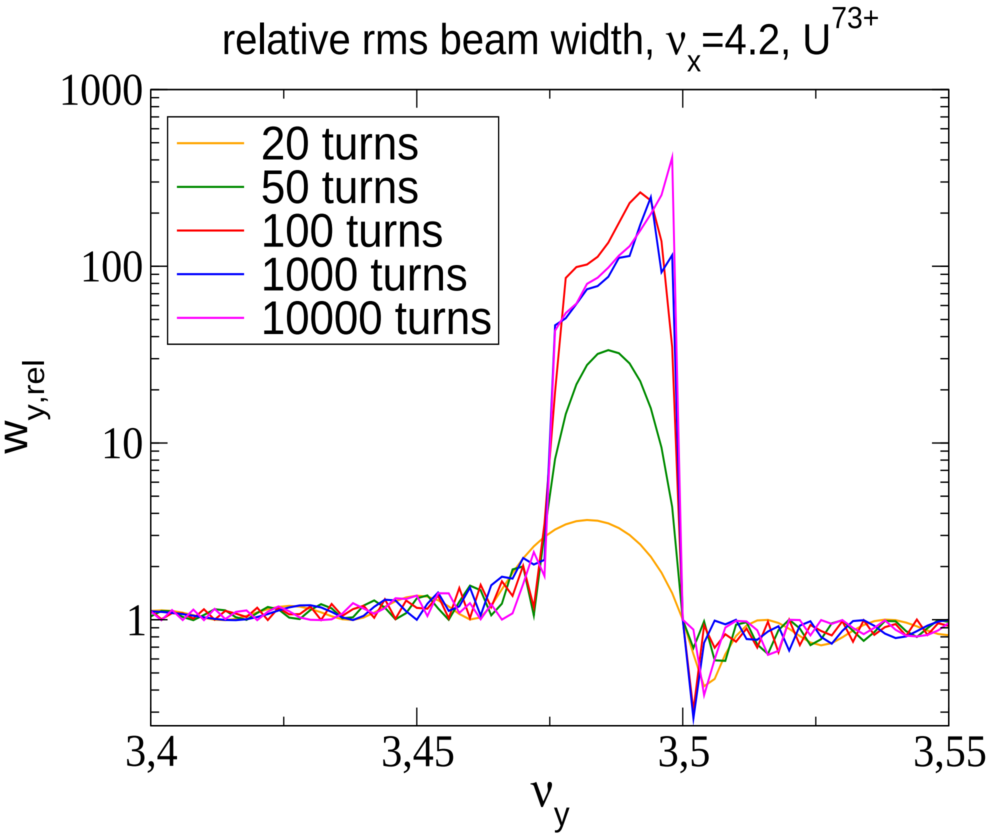
Time evolution of the vertical phase space distribution



- $\nu_x = 4.2, \nu_y = 3.49$
- $\sigma_{i,x} = \sigma_{i,y} = 0.01\ b$
- **Particles reach edge of electron beam after very short time.**
- **Emittance starts to grow**
- **Particles stay near electron beam, i.e. growth of beam width is limited.**

Time evolution of the vertical relative rms beam width

$$\nu_x = 4.2, \sigma_{i,x} = \sigma_{i,y} = 0.01\ b$$



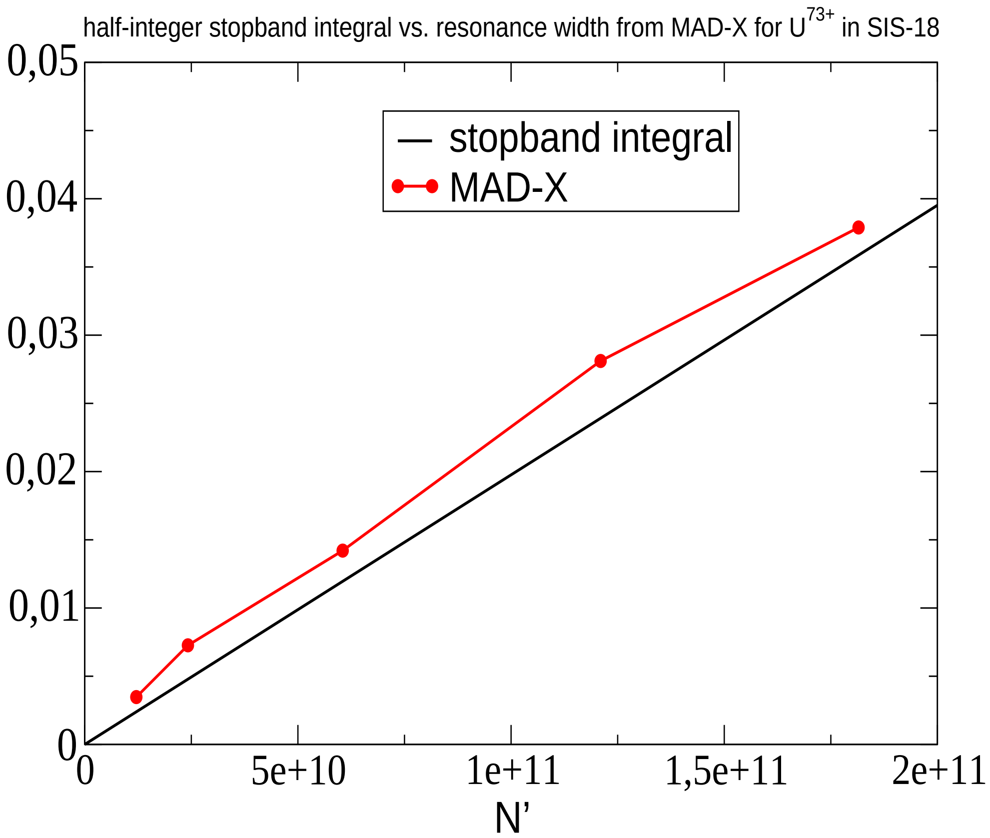
Half-integer resonance with constant width.

⇓

Possibility to determine width and compare it to analytical half-integer stopband width.

Width of half-integer resonance vs. half-integer stopband integral

$$\nu_x = 4.2, \sigma_{i,x} = \sigma_{i,y} = 0.01\ b$$



Good agreement between widths of half-integer resonance and stopband integral for different electron numbers

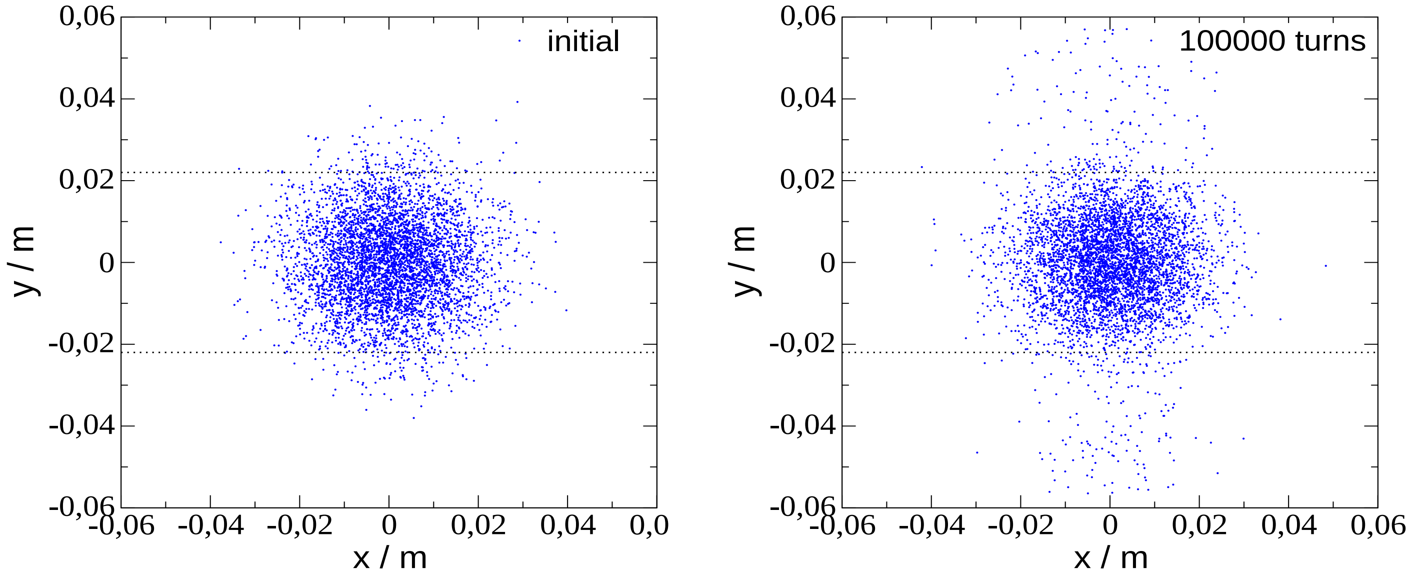
Possible constraint due to resonance crossing

Tune shift due to electron cooler $\Delta\nu_z$	
U^{73+} ions	$0.0066, 0.012$
Protons	$0.020, 0.037$
Eigen space charge tune shift $\Delta\nu_{z,sc}$	up to -0.25

$$\Delta\nu_{z,sc} = -\frac{q^2 N}{4\pi^2 \varepsilon_0 m_0 c^2 \beta^2 \gamma^3 \sqrt{\epsilon_z(\epsilon_x + \epsilon_y)} B_f}$$

Tune shift due to cooler is too small, but tune spread due to eigen space charge will cross resonances.

Beam behaviour at $\nu_x = 4.1,\ \nu_y = 3.245,\ 5000$ test particles



- **Beam profile after 0 and 100000 revolutions, $\sigma_{i,x} = \sigma_{i,y} = b/2$**
- **Close to resonance $(p, m, n) = (13, 0, 4)$ → only vertical beam widening**
- **Apparently, only particles from edge drift outwards, whereas beam core seems to be not touched.**

Possible constraint due to resonance crossing

Time evolution of beam properties at $\nu_x = 4.1,\ \nu_y = 3.245$. 5000 test particles

Particles with betatron amplitude $R_{\max} > b$

revolutions / σ_i	0	1000	10000	100000
$0.5\ b$	2049	2059	2051	2056
b	4515	4524	4523	4523

Vertical relative beam width $w_{y,\text{rel}}$

revolutions / σ_i	0	1000	10000	100000
$0.5\ b$	1	1.13	1.17	1.20
b	1	1.13	1.15	1.17

- Growth of beam width **without** increase of number of particles with $R_{\max} > b$
- Coincides with assumption, that resonance influences only particles, which feel non-linearity.

Number of those particles is given by initial conditions.

Summary and outlook

- Calculation of resonances driven by the electron cooler of SIS-18 for U^{73+} ions
- Tracking method applied to simplified model for SIS-18 using MAD-X
- Growth of beam width due to resonance excitation
- Identification of resonances up to 6th order possible
- Strongest resonances in tune range considered those of order 4
- Quantitative reproduction of resonance width with analytic model only for half-integer resonances

Summary and outlook

- Investigation of possible constraints due to resonance crossing
  - tune spread due to eigen space charge large enough to cross resonances
    $\longrightarrow$ possible decrease of space charge limit
  - resulting beam blow up yields decrease of cooling rate because of increase of beam width and possible limit for eigen space charge tune shift
  - beam blow up only due to particles being initially outside of electron beam excepted for half-integer resonances
    $\longrightarrow$ number of those particles can be chosen by proper initial conditions.
- Further investigations of interplay of cooler induced resonances, eigen space charge tune shift, and lattice generated resonances necessary
- Experiment on cooler induced resonances in SIS-18 under discussion.