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Possible lattices with imaginary gamma-transition for PS2: why and how?

The requirements to the absolute value of slip factor:

I. For many collective instabilities the threshold is proportional the slip factor

$$\eta = \alpha - 1/\gamma^2 \qquad \alpha = \frac{1}{\gamma_{tr}^2}$$

For instance, from the Landau damping theory the stability requires a minimum spread in incoherent frequencies for longitudinal motion

$$\left(\frac{\delta\omega}{\omega}\right)^2 = \eta^2\left(\frac{\sigma}{p}\right)^2 \geq \frac{eI_{peak}|\eta|}{2\pi m_0 c^2 \gamma\beta^2}\left|\frac{Z_L(n)}{n}\right|$$

So, **the absolute value of slip factor module** is desirable to have as large as possible.

The requirements to the absolute value of slip factor:

II. In the same time **the longitudinal beam size** is determined by the ratio

$$\Delta\phi_{\max} = \pm W\sqrt{\frac{2\pi h|\eta|\Omega_{rev}}{eVp_s R\cos\phi_s}}$$

and **the absolute value of slip factor** can be used as additional factor for the matching between two accelerators or/and control of beam sizes during acceleration.

III. The **transition energy crossing** $\gamma = \gamma_{tr}, \eta = 0$ has to be excluded, since the longitudinal stability disappears.

It means that γ_{tr} has to be moved away from PS2 energy region $\gamma \approx 5 \div 50$.

The requirements to the sign of slip factor:

IY. Many investigations devoted to the beam stability declare that the beam is more stable below the transition energy

$$\gamma < \gamma_{tr} \quad \Rightarrow \quad \eta < 0$$

Regular and Irregular lattices

Momentum Compaction factor (MCF):

$$\alpha = \frac{1}{2\pi}\int_0^C \frac{D(\vartheta)}{\rho(\vartheta)}d\vartheta$$

where the dispersion D is:

$$\mathrm{D}'' + \mathrm{K}(\vartheta)\mathrm{D} = \frac{1}{\rho(\vartheta)}$$

If in optics with eigen frequency ν the curvature is modulated with frequency ω

$$\sigma(\vartheta) = 1/\rho(\vartheta) \sim \boxed{Be^{i\omega\vartheta}} + 1/\overline{R}$$

the dispersion solution and MCF are:

$$D(\vartheta) \sim Ae^{i\nu\vartheta} + \boxed{\frac{B}{\nu^2 - \omega^2}e^{i\omega\vartheta}} + \overline{D} \qquad \alpha = \frac{\overline{D}}{\overline{R}} + \frac{\overline{\widetilde{D}(\vartheta)\cdot\widetilde{r}(\vartheta)}}{\overline{R}}$$

Regular lattice

In conventional regular FODO lattice $\omega >> \nu$.

Therefore the dispersion oscillates with eigen frequency (tune) ν:

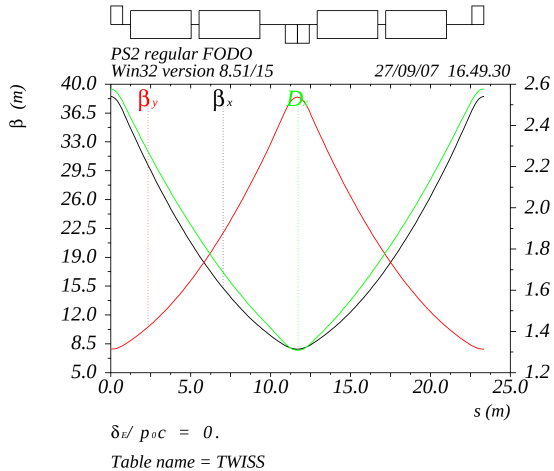
$$D(\vartheta) \approx A e^{i\nu\vartheta} + \overline{D}$$

Then Momentum Compaction Factor (MCF) is determined by average values ratio:

$$\alpha = \frac{\langle D(\vartheta) \rangle}{\langle \rho(\vartheta) \rangle} = \frac{\overline{D}}{\overline{R}} \approx \frac{1}{\nu^2}$$

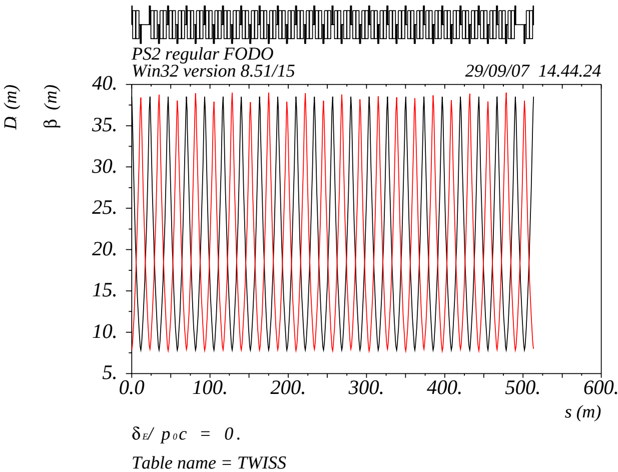
Regular lattice for PS2 based on FODO cells with real transition energy $\gamma_{tr} \sim 10$ (parameters taken from PS2 report by Wolfgang Bartman)

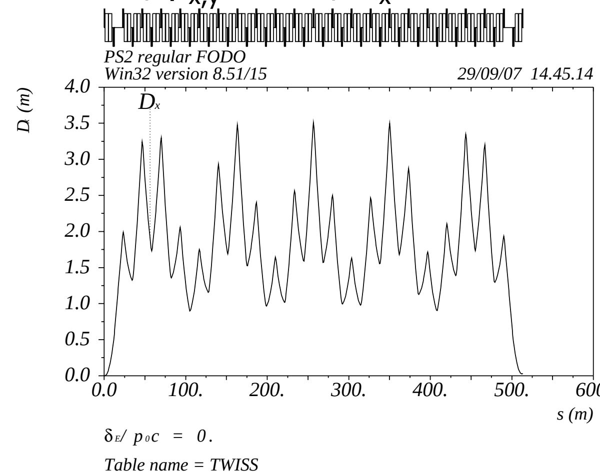
FODO cell with magnet length L=3.79m and drift 1.6m



Arc based on FODO lattice with tune per cell $\sim 84^0$

The total length (22 cells) 513.5 m; $\beta_{x,y}$~39 m, D_x~3.5m





To make higher γ_{tr} than 50 the total number of FODO cells has to be increased up to 110 per arc

Conclusion: The only possible solution is the imaginary gamma transition with the wide control of its absolute value

Irregular lattice with curvature modulation (missing magnet lattice)

In case of eigen frequency is enough close to the curvature oscillation with the superperiodicity frequency S=ν+δ, the dispersion oscillates with the forced frequency S:

$$D(\vartheta) \sim \frac{B}{\nu^2 - S^2} e^{i\vartheta S} + \overline{D}$$

In irregular structure MCF depends on the curvature modulation **B** and detuning δ= S- ν<< ν :

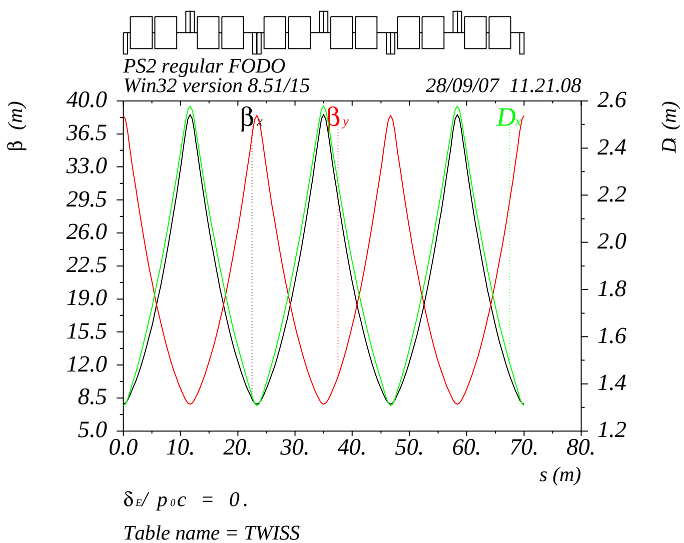
$$\alpha \approx \frac{1}{\nu^2} - \frac{B^2}{2\delta\nu}$$

Irregular lattice with curvature modulation ("missing" magnet lattice)

3 regular FODO cells with total length

3 x 23.21 m=69.63

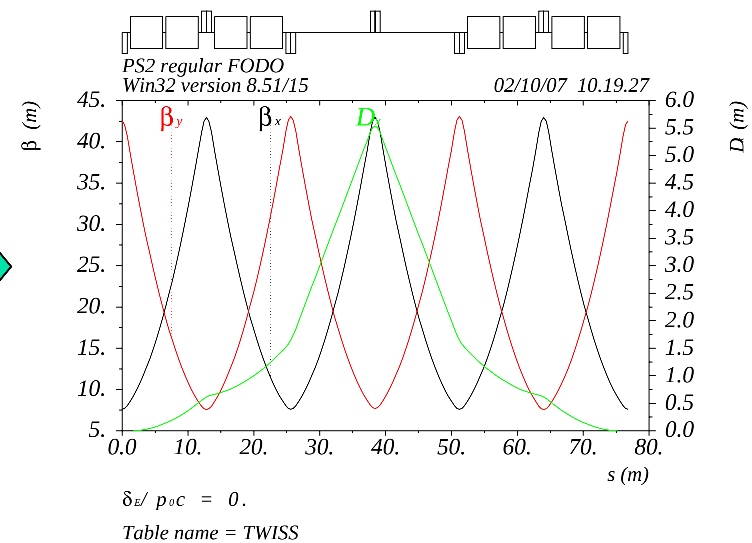
L_{mag}=3.7 m



3 irregular FODO missing magnet cells

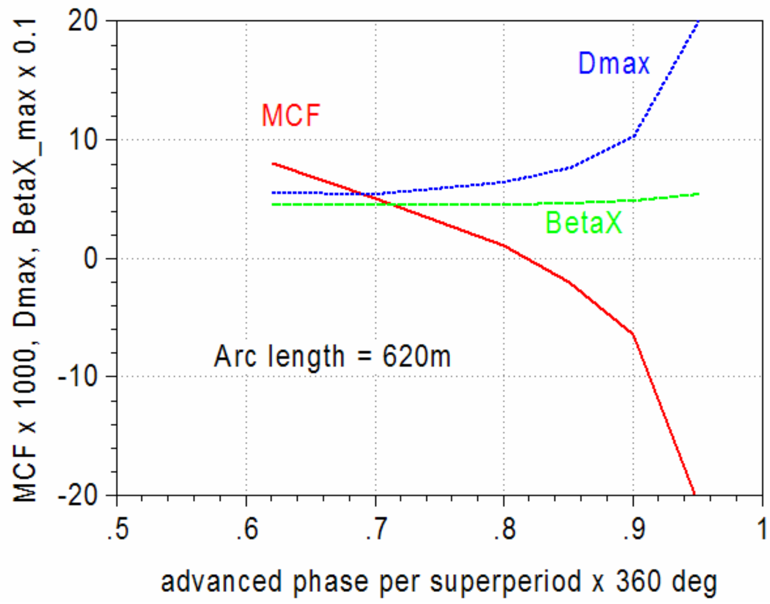
with total length 76.8 m

L_{mag}=4.9 m



Zero momentum compaction factor in the "missing" magnet lattice

In arc length~620 m MCF<0 at ν>0.82
In arc length~600 m MCF<0 at ν>0.875
In arc length~580 m MCF<0 is not reached



Conclusion

"Missing magnet" lattices has

advantages:

- practically does not perturbs β-functions;

disadvantages:

- requires the large phase advance value,
- significantly increases the arc length.

“Resonant” lattice
with resonantly correlated modulations of curvature and gradient

The first order optics lattice with gradient and curvature modulation was developed and used in the Moscow Kaon Factory project in 1987, N. Golubeva, A. Iliev, Yu. Senichev, The new lattices for the booster of Moscow Kaon Factory, International Seminar on Intermediate Energy Physics, Moscow 1987, INES-89, v.2, pp. 290-298.)

Later this lattice was adopted for:

- the **TRIUMF Kaon Factory**, A. Iliev and **Yu.Senichev**, Racetrack lattice study for Kaon booster, TRIUMF Kaon Factory Project definition study, TRI-DN-91-K193, 1991, 17 pages).
- applied as the most successful option in **the SSC Low Energy Booster**, E. Courant , A. Garen,U. Wienands, Low Momentum Compaction Lattice Study for the SSC Low Energy Booster, Proceeding IEEE PAC, 1991, p. 2829,
- the **CERN Neutrino Factory**; (B. Autin, R. Cappi, J. Gareyte, R. Garoby, M. Giovannozzi, H. Haseroth, M. Martini, E. Métral, W. Pirkl, H. Schönauer, CERN, Geneva, Switzerland, C.R. Prior, G.H. Rees, RAL, Chilton, Didcot, U.K., I. Hofmann, GSI, Darmstadt, **Yu. Senichev,** FZJ, Jülich, Germany, A Slow-Cycling Proton Driver for a Neutrino Factory, CERN-PS/2000-015(AE), June 26, 2000,
- the **Main Ring of the JPARC**, Y. Ischi, S. Machida, Y. Mori, S. Shibuya, Lattice design of JHF synchrotron, Proceeding of APAC, 2002;
- in the **High Energy Storage Ring of the FAIR**, Y. Senichev, et al., Lattice Design Study for HESR , Proceedings PAC, Lucerne 2004, p. 653

The theory was developed by Y. Senichev, **A “resonant” lattice for a synchrotron with a low or negative momentum compaction factor,** KEK Reprint 97-40A, June 1997

Results of "Resonant" lattice theory:

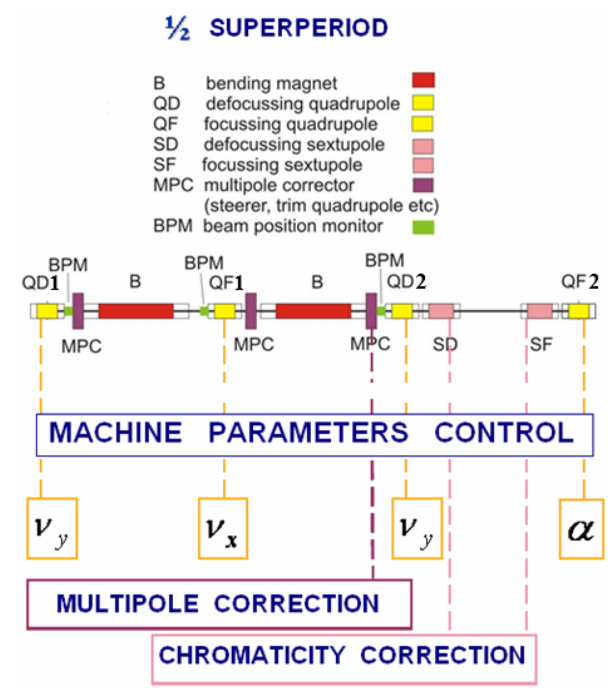
The solution of equation $\frac{d^2 D}{ds^2} + [K(s) + \varepsilon\, k(s)]D = \frac{1}{\rho(s)}$

with modulation of gradient and curvature:

$$\varepsilon \cdot k(\phi) = \sum_{k=0}^{\infty} g_k \cos\, k\phi\,; \quad \frac{1}{\rho(\phi)} = \frac{1}{\overline{R}}\left(1 + \sum_{n=1}^{\infty} r_n \cos n\phi\right)$$

gives the expression for MCF:

$$\alpha_s = \frac{1}{\nu^2}\left\{1 + \frac{1}{4 \cdot (1 - kS/\nu)} \cdot \left[\left(\frac{\overline{R}}{\nu}\right)^2 \frac{g_k}{[1 - (1 - kS/\nu)^2]} - r_k\right]^2\right\}$$



At development of “Resonant” structure we were guided by following 6 requirements:

1. Ability to achieve the negative momentum compaction factor with minimum circumference and control of gamma transition in a wide region;

2. Dispersion-free straight section without special suppressor;

3. Low sensitivity to multipole errors and sufficiently large dynamic aperture.

4. Minimum families of focusing and defocusing quadrupoles and separated adjustment of gamma transition, horizontal and vertical tunes;

5. Convenient sextupole chromaticity correction scheme;

6. Independent optics parameters of arcs and straight sections

1. Negative momentum compaction factor with minimum circumference and control of gamma transition in a wide region

The lattice has the remarkable feature:
The gradient and the curvature modulation amplify each by other if they have opposite signs,

$$g_k \cdot r_k < 0$$

The ratio between them is desirable to have:

$$|r_k| \le \left(\frac{\overline{R}}{\nu}\right)^2 \left|\frac{g_k}{1-(1-kS)^2}\right|$$

and

$$\frac{1}{4(kS/\nu-1)} \cdot \left(\frac{g_k}{[1-(1-kS/\nu)^2]} - r_k\right)^2 \approx 2$$

On the contrary they can compensate each other when they have the same sign.

Then gamma transition varies in a wide region from $\gamma_{tr}=\nu_x$ to $\gamma_{tr}=i\nu_x$

with quadrupole strength variation only!!!

2. Dispersion-free straight section without special suppressor;
3. Low sensitivity to multipole errors and sufficiently large dynamic aperture

First condition:

To provide a **dispersion-free straight section**, the arc consisting of S_{arc} superperiods must have a 2π integer phase advance.

Second condition:

In order to drive the momentum compaction factor, the horizontal betatron tune ν_{arc} must be less than the resonant harmonic of perturbation kS_{arc}, and the difference between them has to be of a minimum integer value. We take $\nu_{arc} - kS_{arc} = -1$

.

Third condition:

The arc superperiodicity S_{arc} has to be even and ν_{arc} is odd.

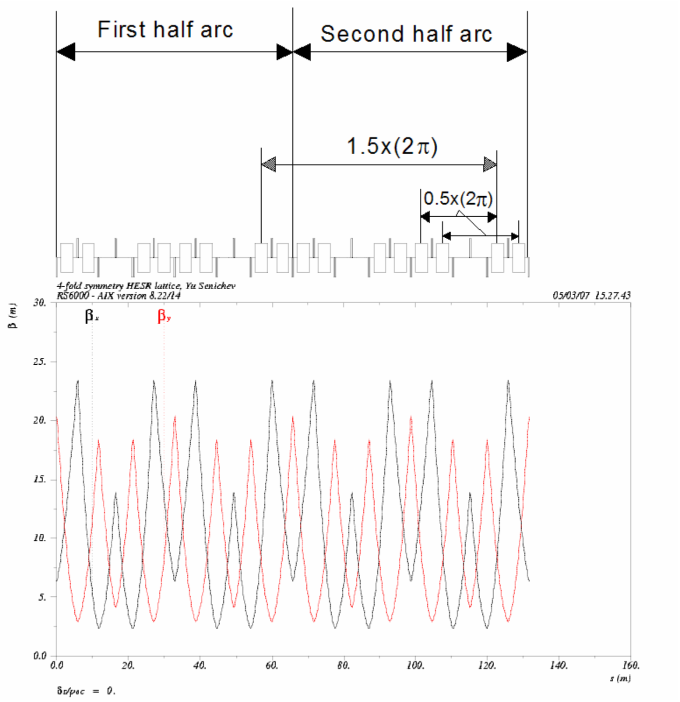
Compensation of sextupole non-linearity

- In that case the phase advance between any two cells located in the different half arcs and separated by $\frac{S_{arc}}{2}$ number of superperiods is then equal to

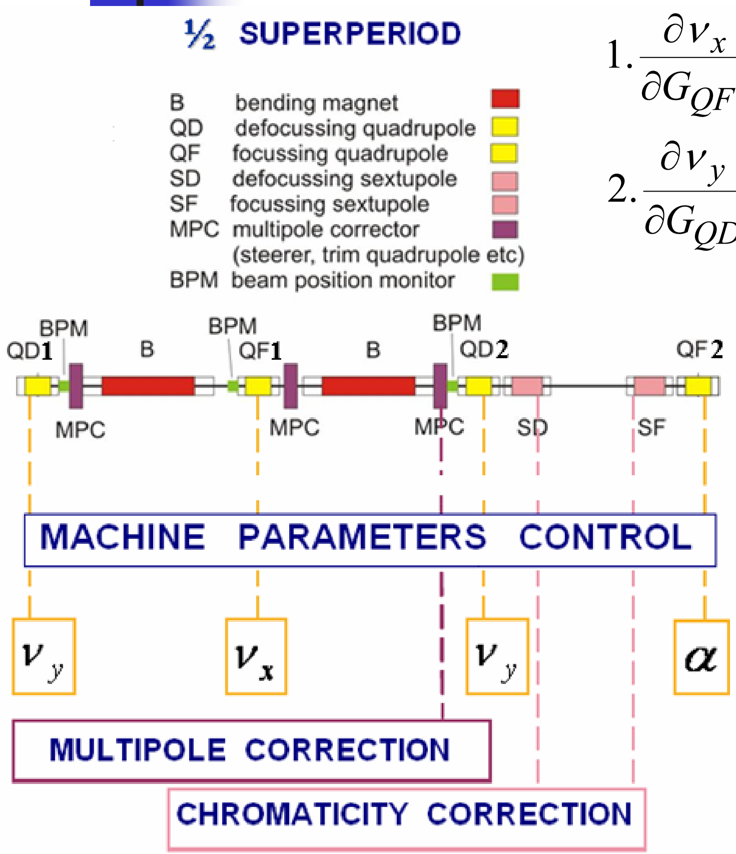
$$\frac{\nu_{arc}}{S_{arc}} \cdot \frac{S_{arc}}{2} = \frac{\nu_{arc}}{2} = \pi + 2\pi n \quad .$$

- the total multipole of third order is canceled:

$$M_3^{total} = \sum_{n=0}^{N} S_{x,xy} \beta_x^{l/2} \beta_y^{m/2} \exp in(l\mu_x + m\mu_y) = 0$$



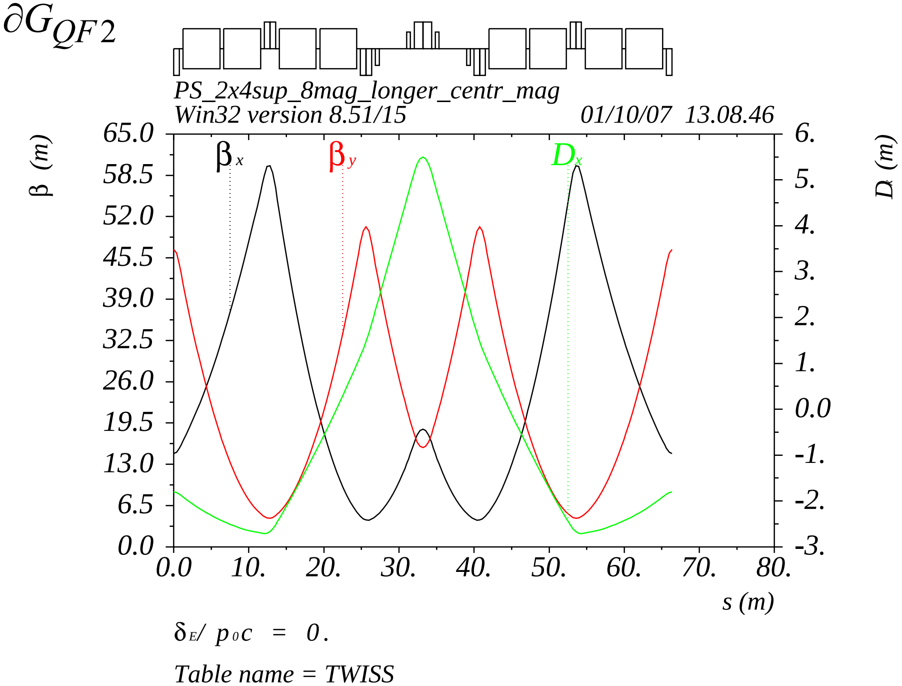
4. Minimum families of focusing and defocusing quadrupoles and separated adjustment of gamma transition, horizontal and vertical tunes



$$1.\ \frac{\partial\nu_x}{\partial G_{QF1}} > \frac{\partial\nu_x}{\partial G_{QF2}} >> \frac{\partial\nu_x}{\partial G_{QD}}$$

$$2.\ \frac{\partial\nu_y}{\partial G_{QD}} >> \frac{\partial\nu_y}{\partial G_{QF1}} \approx \frac{\partial\nu_y}{\partial G_{QF2}}$$

$$3.\ \frac{\partial\alpha}{\partial G_{QF2}} >> \frac{\partial\alpha}{\partial G_{QF1}} \approx \frac{\partial\alpha}{\partial G_{QD1}}$$



5. Convenient sextupole chromaticity correction scheme

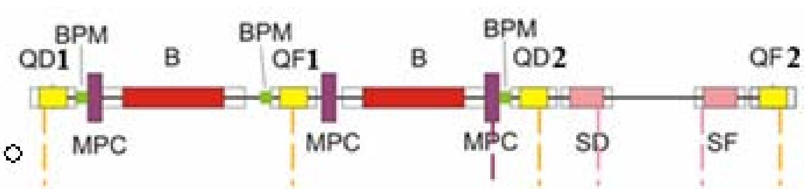
Total chromaticity

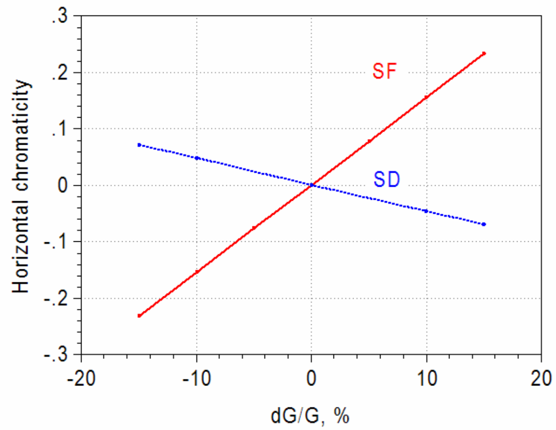
$$\frac{\partial \nu_{x,y}}{\partial \delta} = -\frac{1}{4\pi}\int_0^C \beta_{x,y}(s)K_{x,y}(s)ds$$

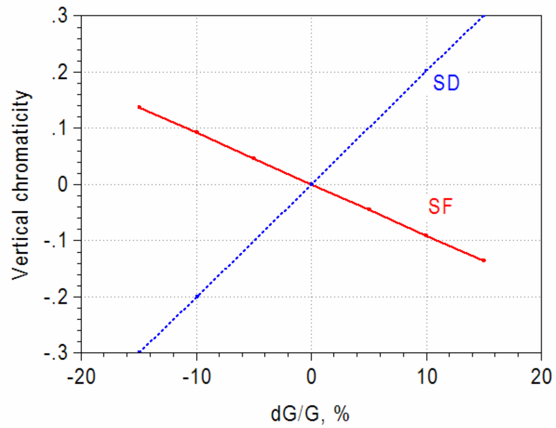
- Sextupole compensation

$$\frac{\partial \nu_{x,y}}{\partial \delta} = \pm\frac{1}{4\pi}\int_0^C \beta_{x,y}(s)\cdot D(s)\cdot S(s)ds$$

Half-superperiod







6. Independent optics parameters of arcs and straight sections

- Tune arc does not depends on the transition energy and is kept constant;
- Special insertion on the straight section allows to match the $\beta_{x,y}$-functions between arcs and straight sections;
- Dispersion-function on the straight sections always equal zero;
- All high order non-linearities are compensated inside each arc.

The "golden" ratio between S_{arc} and ν_{arc}

To fulfill all mentioned conditions we have to have the strictly fixed sets of $\mathbf{S_{arc}}$ and $\boldsymbol{\nu}_{\mathbf{arc}}$:

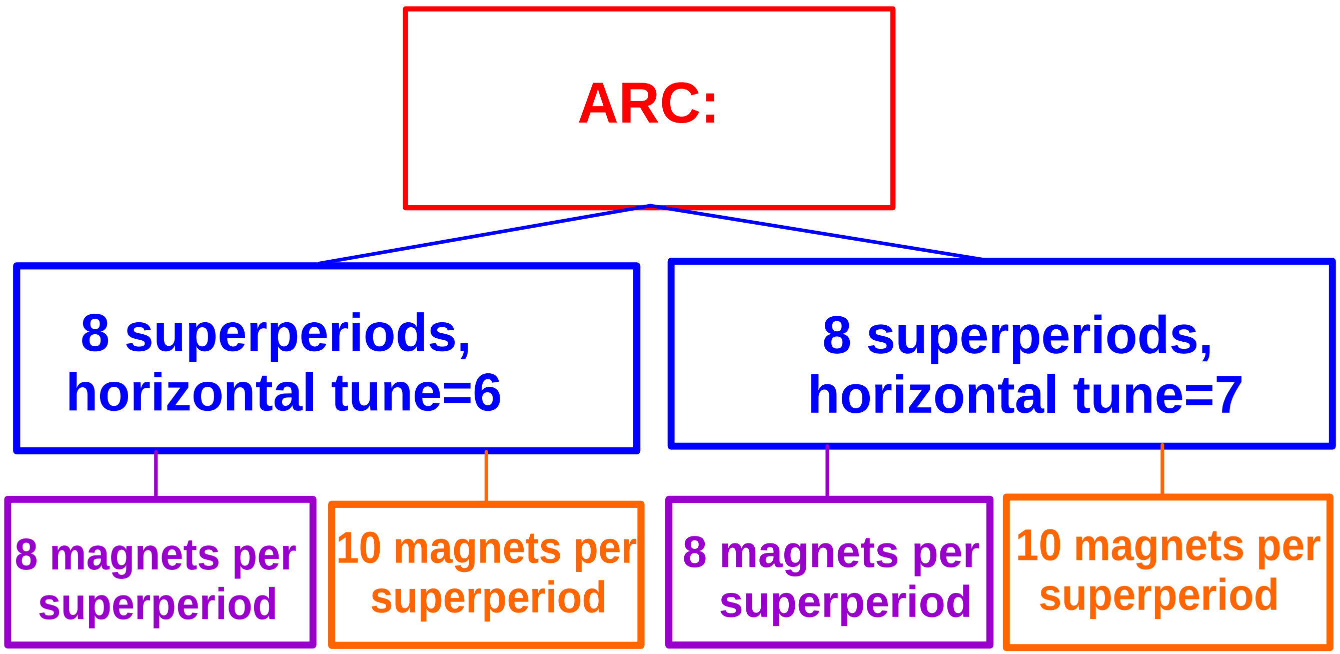
4:3; 6:5; 8:6; 8:7,…. and so on.

↓

4:3 + 4:3

Possible options for PS2: B_{max}=1.7 T; G_{quadr}=16T/m; tech. gap=0.5 m; Arc length=513.5 m and MCF=-0.01

PS2 with imaginary gamma transition: racetrack lattice with two arcs



8 superperiodical lattices with ν_{arc}=6 and ν_{arc}=7

At ν_{arc}=7 the multiplier 1/(1-kS/ν) is larger by factor $\left(1-\frac{8}{7}\right):\left(1-\frac{4}{3}\right)=2.333$ than at ν_{arc}=6 :

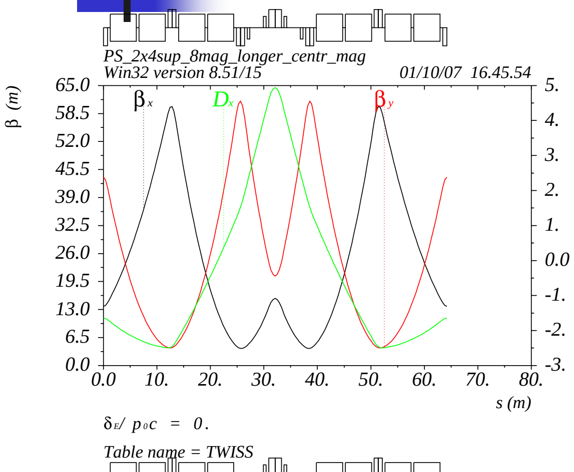
$$\alpha_s=\frac{1}{\nu^2}\left\{1+\frac{1}{4\cdot(1-kS/\nu)}\cdot\left[\left(\frac{\bar{R}}{\nu}\right)^2\frac{g_k}{[1-(1-kS/\nu)^2]}-r_k\right]^2\right\}$$

Therefore the smaller modulation of curvature and gradient modulation is required. It provides the smaller β-function.

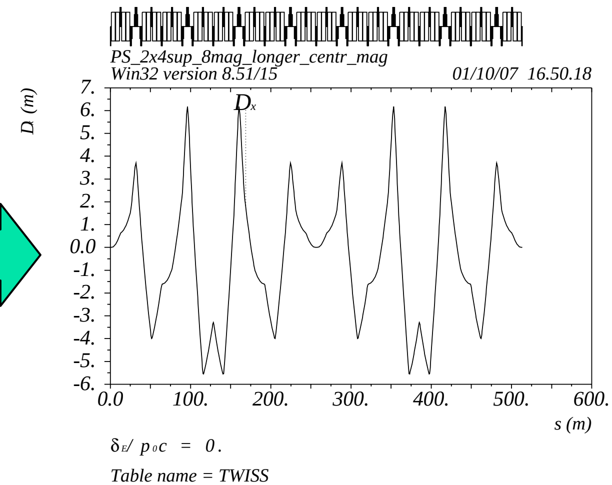
But due to the same multiplier 1/(1-kS/ν) the maximum dispersion in option with ν_{arc}=7 is even higher.

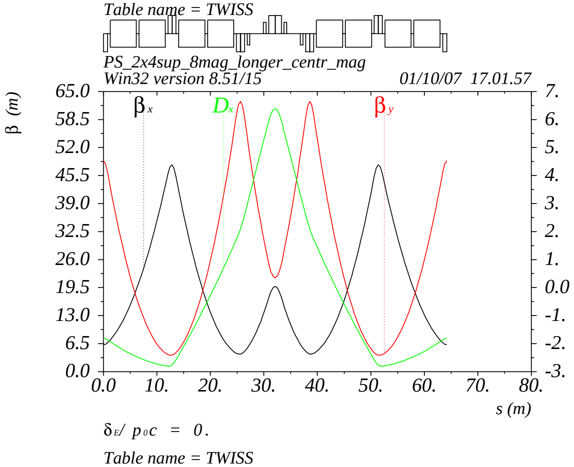
$$D_{\max}=\frac{\bar{R}}{\nu^2}\hat{f}\cdot\left\{1-\frac{1}{2}\left(\frac{\bar{R}}{\nu}\right)^2\frac{g_k}{(1-kS/\nu)[1-(1-kS/\nu)^2]}+\frac{1}{2}\frac{r_k}{1-kS/\nu}\right\}$$

8 superperiodical lattices with ν_{arc}=6 and ν_{arc}=7

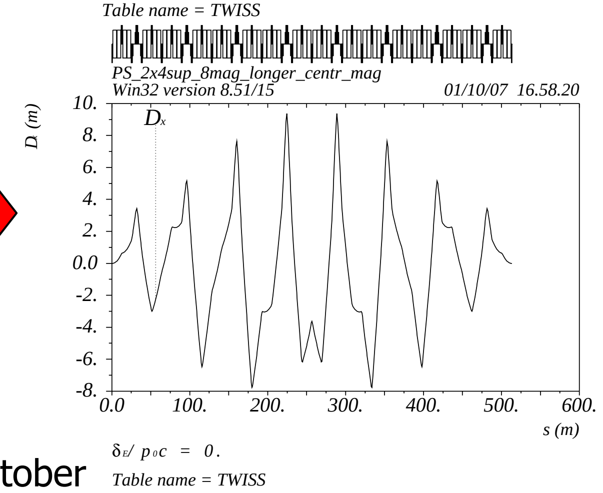


ν_{arc}=6:

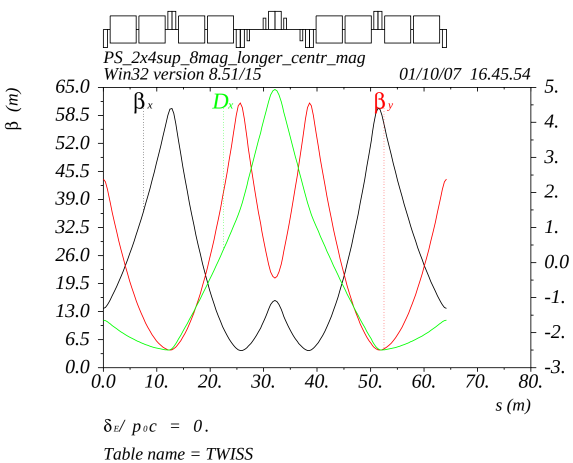


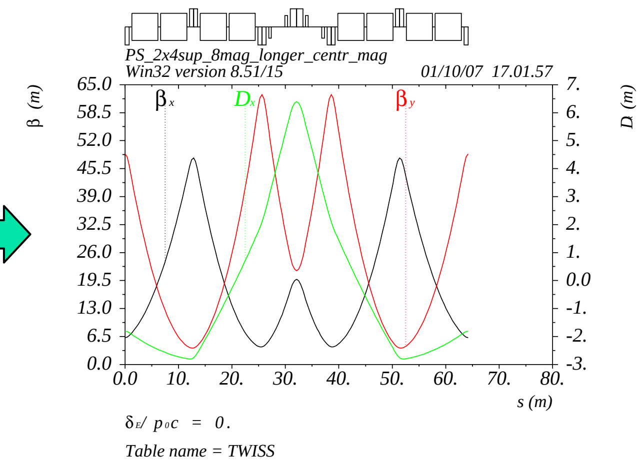


ν_{arc}=7:



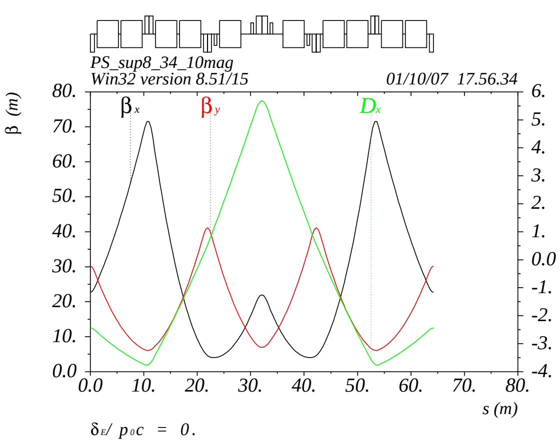
8 superperiodical lattices with 8 and 10 magnets per superperiod

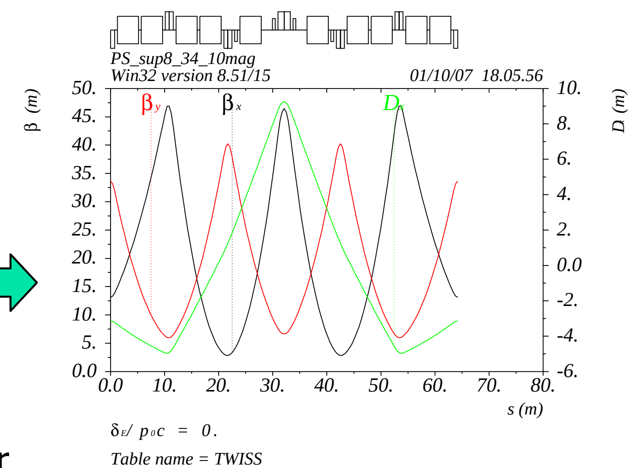




8 magnets

ν_{arc} =6 and ν_{arc} =7



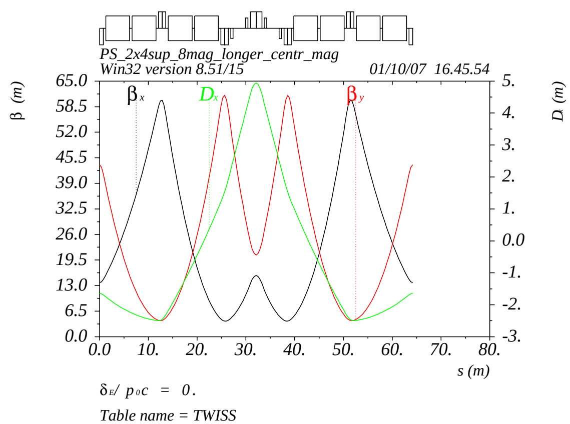


10 magnets

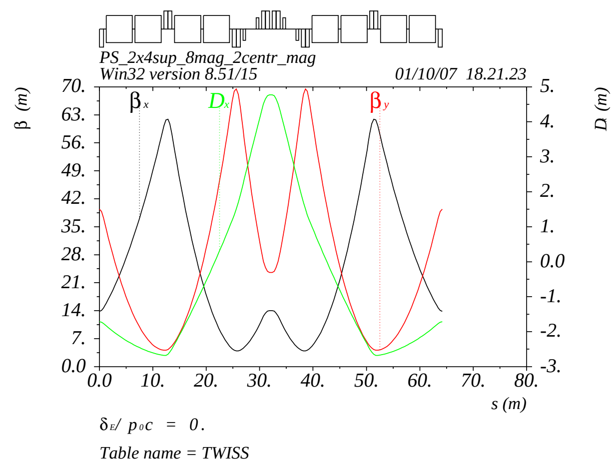
ν_{arc} =6 and ν_{arc} =7

Optics with different central quadrupole

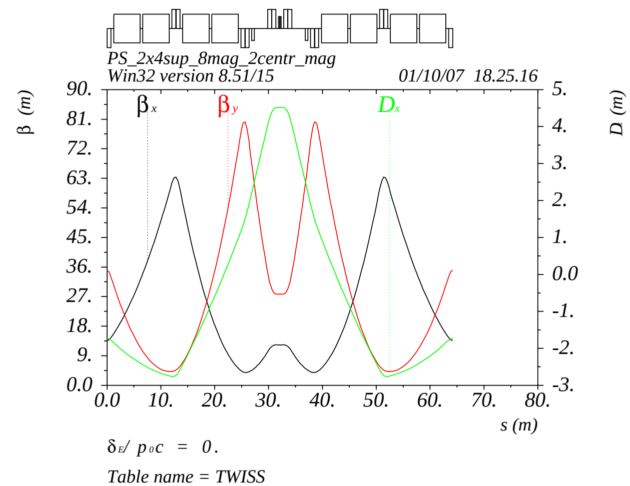
with one longer central quadrupole



with two short central quadrupoles



with two short central quadrupoles and one central sextupole



TWISS parameters of regular FODO lattice and Resonant lattices with 8 superperiods per arc

Options(arc length=513.5 m)	γ_{tr}	β_x^{max}	β_y^{max}	D_x^{max}
Regular with 22 FODO cells	10	39	39	3.5
Resonant with ν_x=6; ν_y=6; **longer central quadrupole**; **8 magnets per superperiod**	i10	60	61	6.2
Resonant with ν_x=6; ν_y=6; **two central quadrupoles**; **8 magnets per superperiod**	i10	62	69	6.0
Resonant with ν_x=7; ν_y=6; **longer central quadrupole**; **8 magnets per superperiod**	i10	48	62	9.4
Resonant with ν_x=7; ν_y=6; **two central quadrupole**; **8 magnets per superperiod**	i10	49	71	9.0
Resonant with ν_x=6; ν_y=6; **longer central quadrupole**; **10 magnets per superperiod**	i10	71	41	7.9
Resonant with ν_x=7; ν_y=6; **longer central quadrupole**; **10 magnets per superperiod**	i10	47	40	14.5

Magnito-optic elements of regular FODO lattice and irregular lattices with 8 superperiods on arc

One arc: length=513.5 m	N_{mag}	L_{mag}	N_{quad}	L_{quad}	N_{sext}	L_{sext}
Regular with 22 FODO cells (84 magnets per arc)	84	3.7	44	1.5	44	0.5
Resonant with ν_x=6; ν_y=6; **longer central quadrupole**; **8 magnets per superperiod**	64	4.9	48	1.5;2.3	32	0.5
Resonant with ν_x=6; ν_y=6; **two central quadrupoles**; **8 magnets per superperiod**	64	4.9	56	1.5	32	0.5
Resonant with ν_x=7; ν_y=6; **longer central quadrupole**; **8 magnets per superperiod**	64	4.9	48	1.5;2.3	32	0.5
Resonant with ν_x=7; ν_y=6; **two central quadrupole**; **8 magnets per superperiod**	64	4.9	56	1.5	32	0.5
Resonant with ν_x=6; ν_y=6; **longer central quadrupole**; **10 magnets per superperiod**	80	3.9	48	1.5;2.3	32	0.5
Resonant with ν_x=7; ν_y=6; **longer central quadrupole**; **10 magnets per superperiod**	80	3.9	48	1.5;2.3	32	0.5

After chromaticity correction sextupoles are main source of non-linearity

- In the variables “action-angle”:

$$H(I_x,\vartheta_x,I_y,\vartheta_y)=\nu_x I_x+\nu_y I_y+\frac{1}{2}\sum_{j,k,l,m}E_{lm}^{jk}\cdot I_x^{j/2}\cdot I_y^{k/2}\exp i(l\vartheta_x+m\vartheta_y)$$

- the non-linear part of Hamiltonian is:

$$V=\frac{1}{2}\sum_{j,k,l,m}\sum_{p=-\infty}^{\infty}h_{jklm}\cdot I_x^{j/2}\cdot I_y^{k/2}\exp i(l\vartheta_x+m\vartheta_y-p\theta)$$

- with the Fourier coefficients:

$$h_{jklm}=\frac{1}{2\pi}\int_0^{2\pi}E_{lm}^{jk}\exp ip\theta$$

The second order non-linearity

- After some canonical transformation we can get the second order approach of Hamiltonian in the next view:

$$H(J_x, \vartheta_x, \theta_x, I_y, \vartheta_y, \theta_y) =$$
$$\nu_x J_x + \nu_y J_y + \sum g(M, N, n_1, n_2, p) J_x^{M/2} J_y^{N/2} \exp i\left(n_1 \vartheta_x + n_2 \vartheta_y - p\theta\right)$$

- Now let us suppose that we are some where around of the third order resonance:

$$3\bar{\nu}_x = p_0,$$
$$\bar{\nu}_x = \nu_x + \Delta$$

- the Hamiltonian takes a view

$$H_1(J, \psi, \theta) = \nu_x J_x + \nu_y J_y + \frac{1}{2} J_x^{3/2} \left\{h_{3030p_0} \exp i(3\psi_x - p_0\theta) + c.c.\right\} +$$
$$\zeta_x J_x^2 + \zeta_{xy} J_x J_y + \zeta_y J_x^2$$

The higher order resonance excitation and non-linear tune shifts

- the coefficients $\zeta_x, \zeta_y, \zeta_{xy}$ are the non-linear tune shifts:

$$\zeta_x = \zeta_x^{sex} + \zeta_x^{oct}$$

$$\zeta_{xy} = \zeta_{xy}^{sex} + \zeta_{xy}^{oct}$$

$$\zeta_y = \zeta_y^{sex} + \zeta_y^{oct}$$

- as example

$$\zeta_x^{sex} = -\frac{3}{4}\left[\sum_{p=-\infty}^{\infty} \frac{\left|h_{3010p}\right|^2}{\nu_x - p} + \sum_{\substack{p-=-\infty \\ p \neq p_0}}^{\infty} \frac{3\left|h_{3030p}\right|^2}{3\nu_x - p}\right]$$

$$\zeta_x^{oct} = \frac{1}{32\pi\Delta^2} \int_0^{2\pi} \beta_x^2 O_x R d\theta$$

Nekhoroshev's criterium:

the non-linearity in both planes have to have the same sign and $4\zeta_x\zeta_y \geq \zeta_{xy}^2$

- The influence of the non-linearity in specified by the discriminant in the expression:

$$\hat{I}_x^{1/2} = -\frac{3h_{30p}\cos 3\hat{\vartheta}_x}{8\zeta_x} \pm \frac{1}{4\zeta_x}\sqrt{\frac{9}{4}h_{30p} - 8\zeta_x\left(\Delta + \zeta_{xy}I_y\right)}$$

- The lattices with $\zeta_x >> h_{30p}$ have to be classified as a special lattice, since it is a case, when the value of h_{30p} is effectively suppressed, but the non-linearity remain to be under control and strong.
- If the sign of the detuning Δ coincides with the sign of the tune shift ζ_x, the discriminant is negative and the system has only one centre at $I_x = 0$
- The quasi-isochronism condition by Nekhoroshev is ful- filled, when

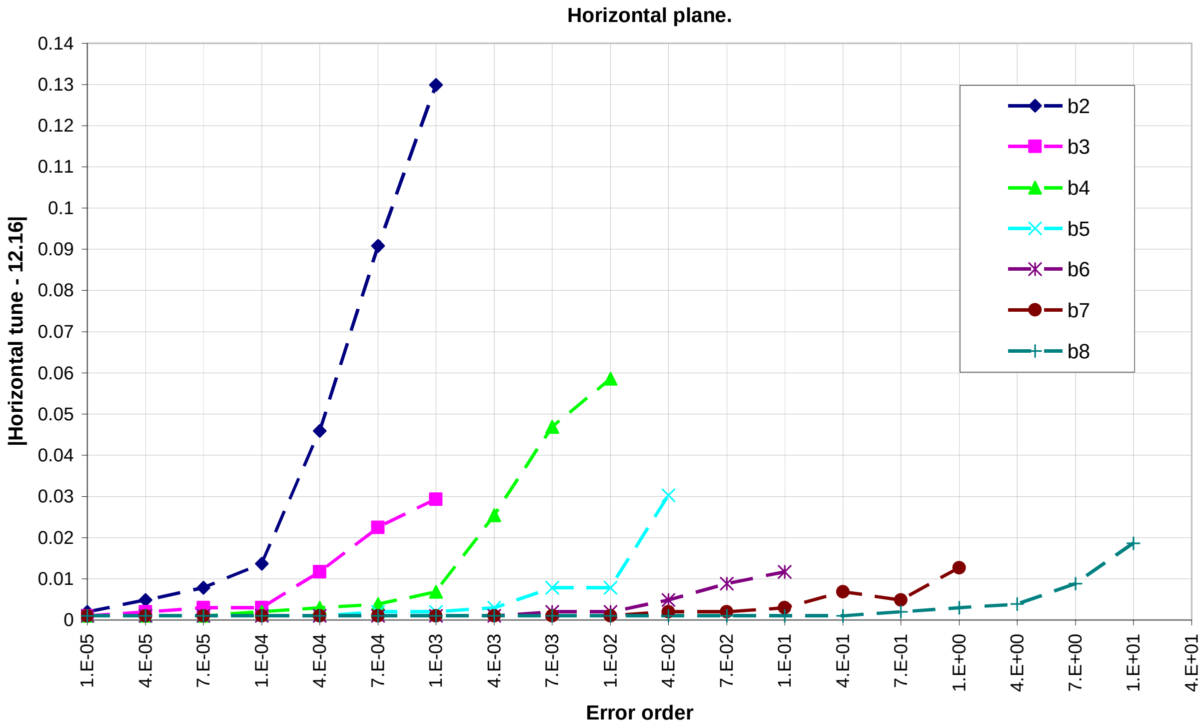
$$k_x\left(2\zeta_x I_x^r + \zeta_{xy} I_y^r\right) + k_y\left(2\zeta_y I_y^r + \zeta_{xy} I_x^r\right) = 0$$

$$\zeta_x k_x^2 + \zeta_{xy} k_x k_y + \zeta_y k_y^2 = 0$$

⟶ Convex or concave resonant surface with maximum stable region

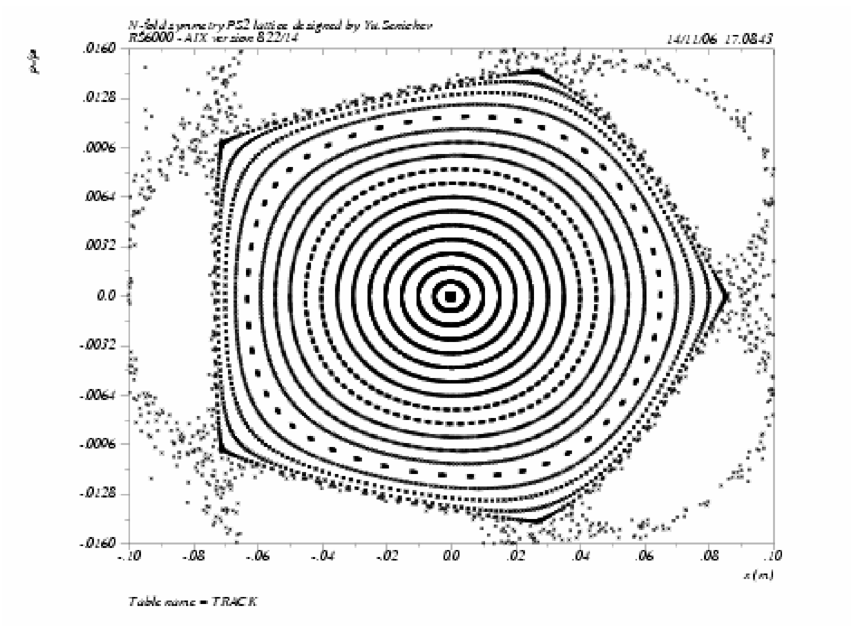
The numerical simulation of the influence of random errors in bend magnets

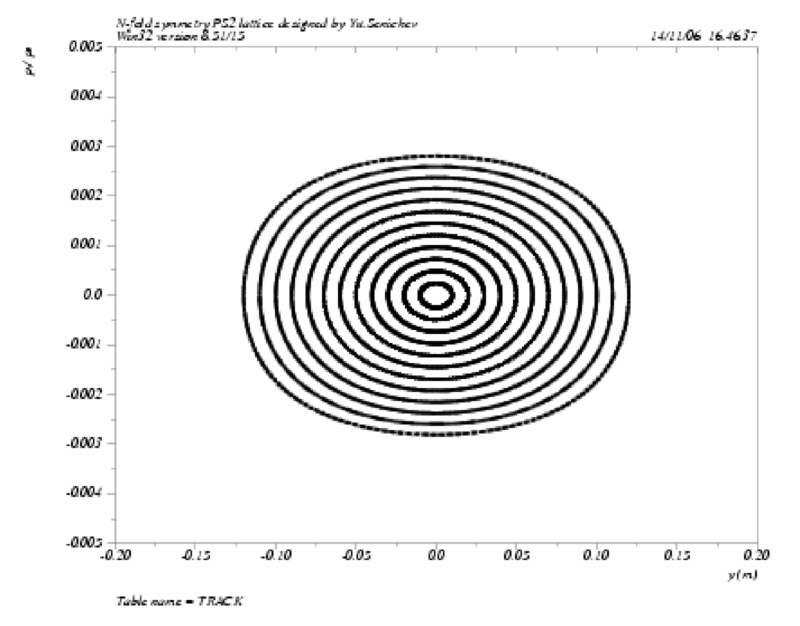
- Results:



Dynamic aperture after chromaticity compensation

- We calculated the dynamic aperture by the numerical tracking for one of options using MAD. It is ~Hor.=600 mm mrad and Ver.=400 mm mrad





Pro and Con for two types of lattices: resonant and regular FODO

	Resonant lattice with ρ and gradient modulation		Regular FODO lattice with suppressors	
	Advantages	disadvantages	advantages	disadvantages
Crossing $W_{transit}$	No			Yes, at $\gamma\sim10$
Variability and controll of $W_{transit}$	Yes			No
Necessity of dispers. suppressor	No			Yes
Decouping between arc and str. section	Yes			No
Free space on arcs	~16 x 3 m			2 x 8 m
Sextupole comp. on arc	Yes			No

Pro and Con for two types of lattices: resonant and regular FODO

	Resonant lattice with ρ and gradient modulation		Regular FODO lattice with suppressors	
	Advantages	disadvantages	advantages	disadvantages
Sensitivity to high multipoles	Low			High
Sextupoles on str. section	Yes			No
Quadr. families number		3	2	
Max dispersion		~6÷10 m, depends on var.	~3.5 m	
Max $\beta_{x,y}$ function		48÷70/40÷70 depends on var.	40/40	
$7\sqrt{\beta x \varepsilon rms}+5Dx\Delta p/p$ at $\varepsilon rms=0.68$; $\Delta p/prms=1x10^{-3}$		~75÷88 mm, depends on var.	~55 mm	

Conclusion

PS2 imaginary lattice was developed with features:

- ability to achieve the negative momentum compaction factor using the resonantly correlated curvature and gradient modulations;
- gamma transition variation in a wide region from $\gamma_{tr}=\nu_x$ to $\gamma_{tr}=i\nu_x$ with quadrupole strength variation only;
- integer odd 2π phase advance per arc with even number of superperiod and dispersion-free straight section;
- independent optics parameters of arcs and straight sections;
- two families of focusing and one of defocusing quadrupoles;
- separated adjustment of gamma transition, horizontal and vertical tunes;
- convenient chromaticity correction method using sextupoles;
- first-order self-compensating scheme of multipoles and as consequence low sensitivity to multipole errors and a large dynamic aperture