Impact of Noise in Hadron Colliders

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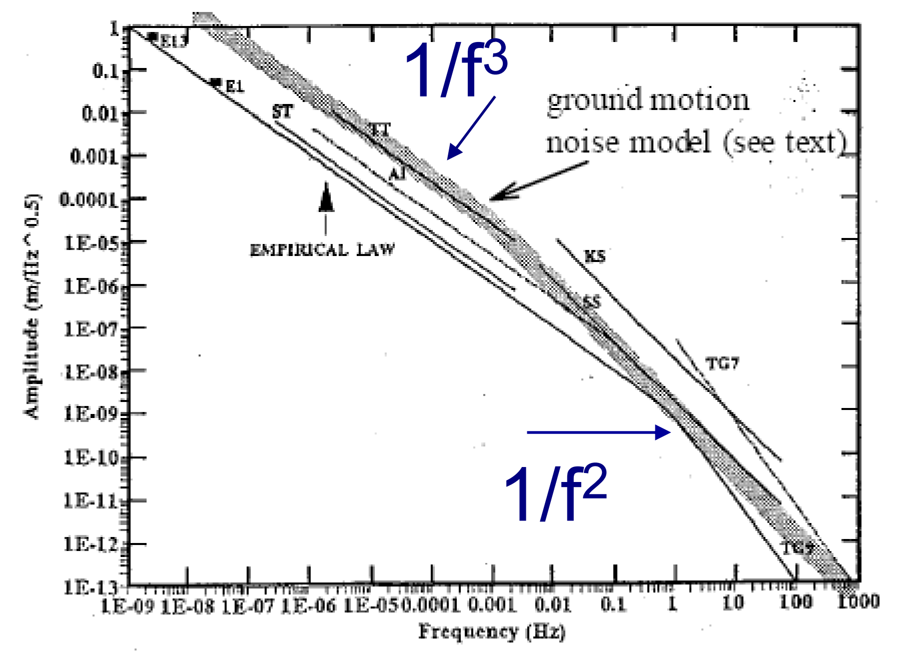
FNAL

CARE-HHH Beams 07

Different scenarios

- Transverse noise with linear motion
- Transverse noise with beam-beam interactions
  - offset fluctuations
  - tune fluctuations etc
- Longitudinal noise
- Diffusion model
- Open questions

Ground motion spectrum in the LHC



A.Verdier and L. Vos, LHC project Note 444 (2000)

- Spectral density of ground motion near betatron tunes ~ 10^{-20} mm^2/Hz
- Orbit amplification R^2 ~ 10 [E. Keil(1997)]
- S_{orbit} ~ 10^{-19}mm^2/Hz
- Spectral density ~ $1/f^2$ near betatron tunes
- Ornstein-Uhlenbeck process has a $1/f^2$ fall off.
- Correlation function

  $K(t_1,t_2)=|\eta|^2 \exp[-|t_1-t_2|/\tau_c]$
- S_{OU} ~ 10^{-19}mm^2/Hz near betatron tune if $\eta=10^{-4}$, $\tau_c=100$

High frequency magnet vibrations

Vibrations near the betatron frequency

- Quadrupole vibrations
- Thin lens model leads to

$$y'' + K(s)y = \frac{\Delta(t)}{f_q}\delta(s - s_0)$$

$$\Delta < y^2(t) >= \frac{\pi\beta\beta_q f^2{}_{rev}}{2 f_q^2} t\Sigma_{n=-\infty}^{\infty} S[\Omega(\nu - n)]$$

G. Stupakov, SSCL (1992)

- Example: Tevatron IR quad Q2 (f_q = 4 m)

  5% change in σ* after 10hrs

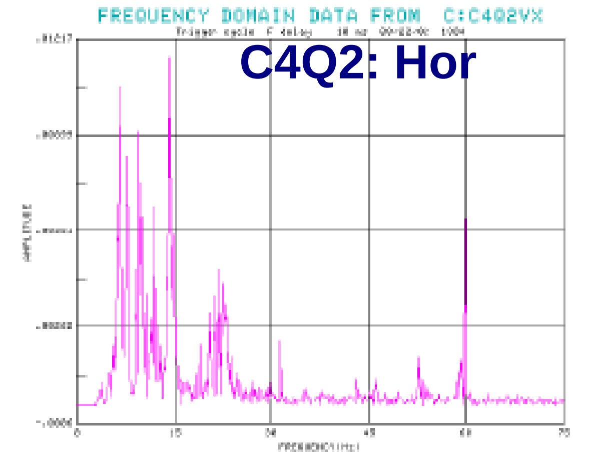
$$\Longrightarrow \quad S[27.7kHz] \le 10^{-21} mm^2 / Hz$$

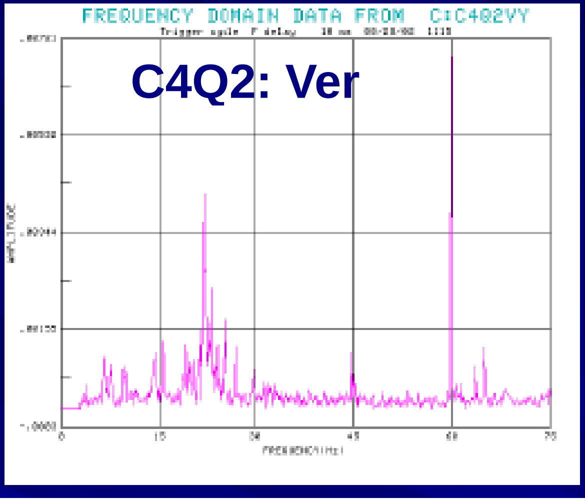
- Example: LHC IR quad Q3 (f_q=18.5m)

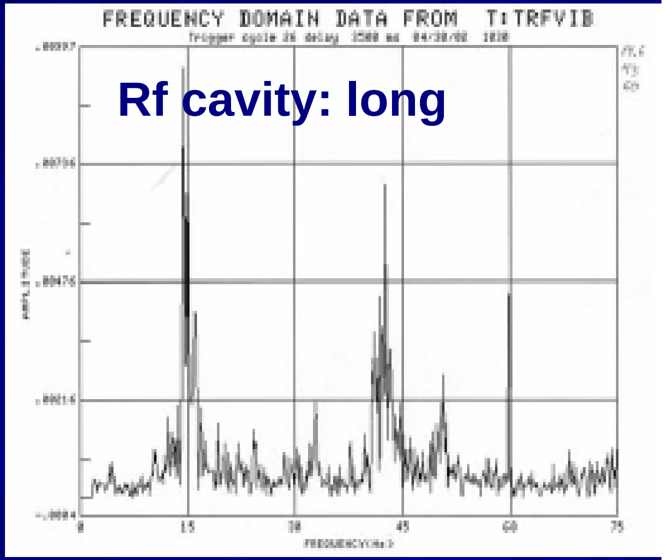
  5% change in σ* after 10hrs

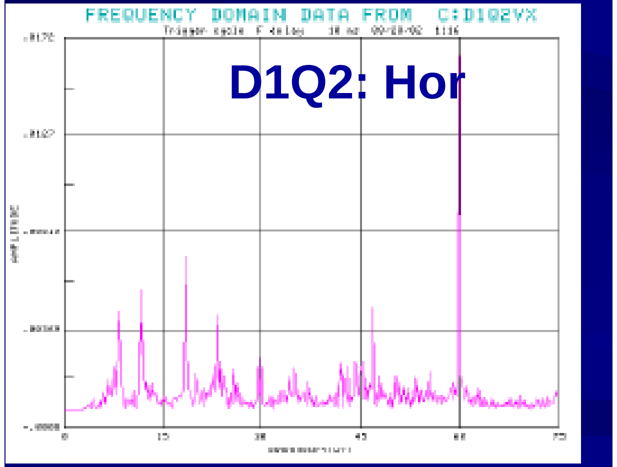
$$\Longrightarrow \quad S[3.5kHz] \le 2 \times 10^{-20} mm^2 / Hz$$

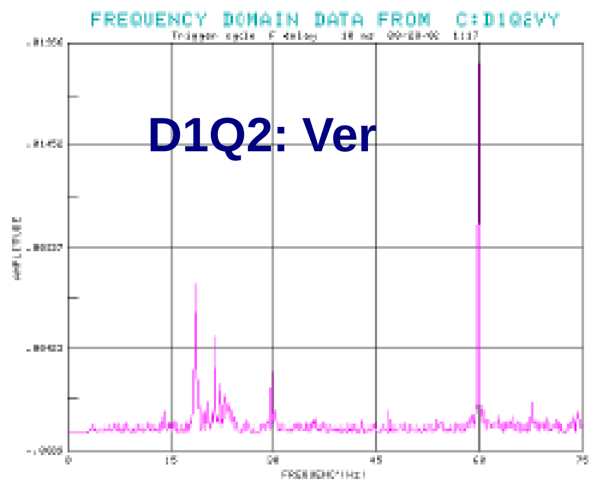
Tevatron: Magnet & cavity vibrations







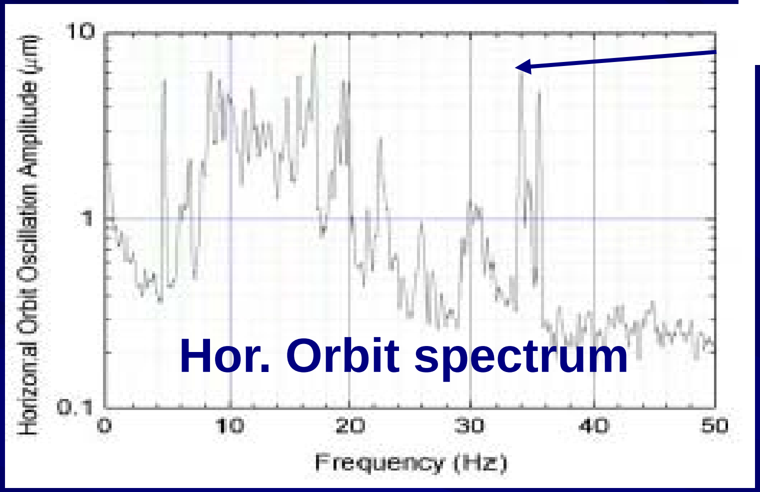




- Quad vibrations strongest at mechanical resonances and sensitive to liquid helium plant
- RF cavity mechanical vibrations strongest around 15 Hz and 43Hz

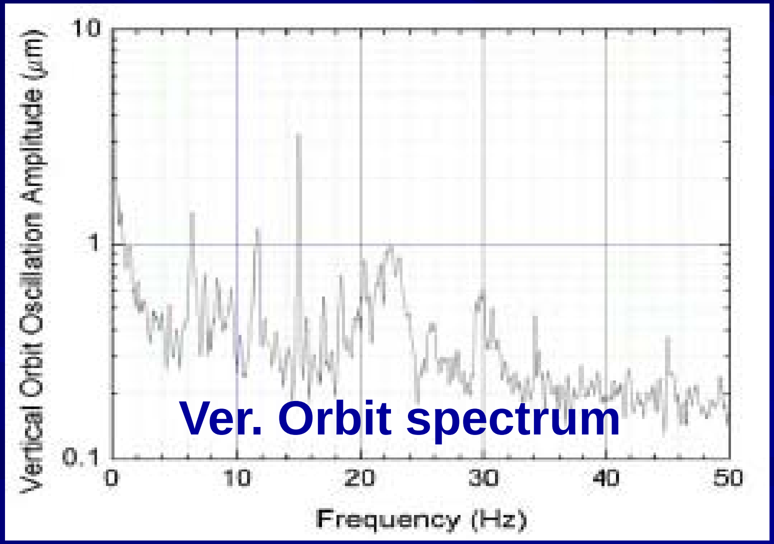
V. Shiltsev, T. Johnson, X.L. Zhang (2002)

Tevatron: Orbit motion



Hor. Orbit spectrum

Synchrotron frequency



Ver. Orbit spectrum

- Orbit spectra has lines from ground motion due to liquid helium plants
- Lines from resonances of support structures
- Synchrotron frequency lines in horizontal spectra
- Low-beta quad motion is amplified ~20-40 times in orbit spectra

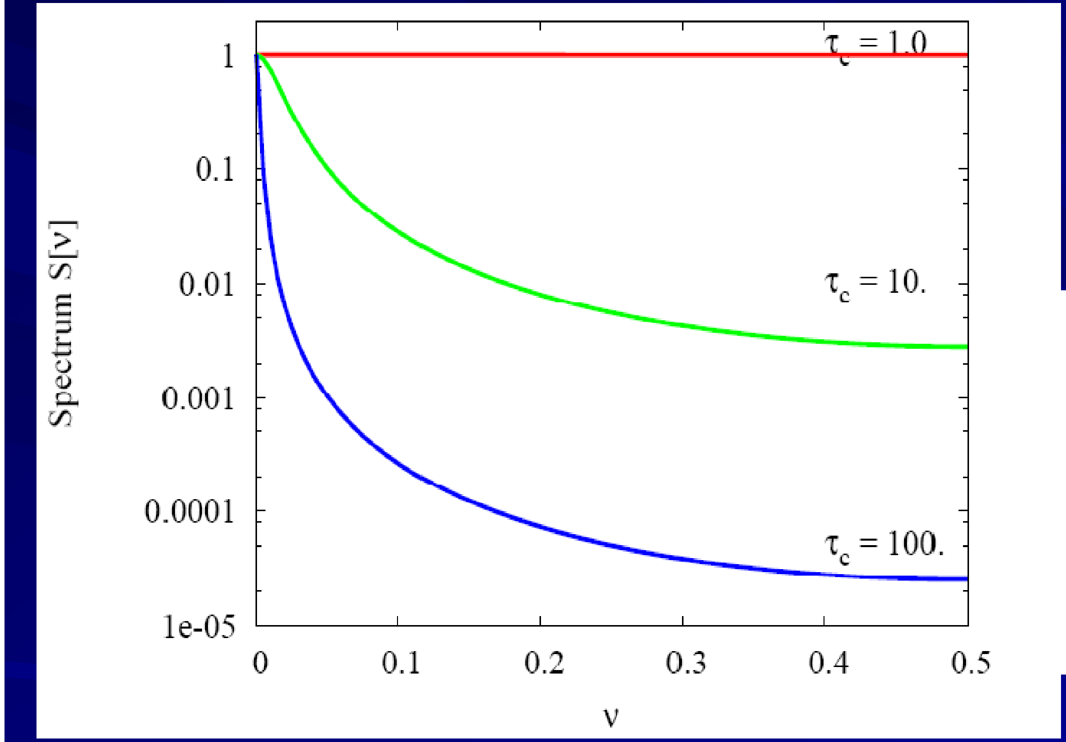
Offset fluctuations at the IPs

Possible sources include

- Triplet vibrations
- Power supply noise in triplets and beams offset in these magnets
- Noise in feedback kickers, bpm errors
- Crab cavity noise
- Wire compensator current jitter
- Ground motion

These can lead to emittance growth and loss of luminosity

Ornstein-Uhlenbeck spectrum

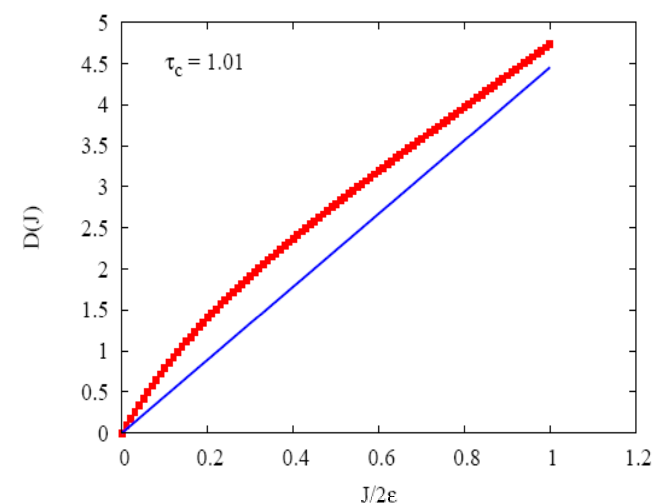


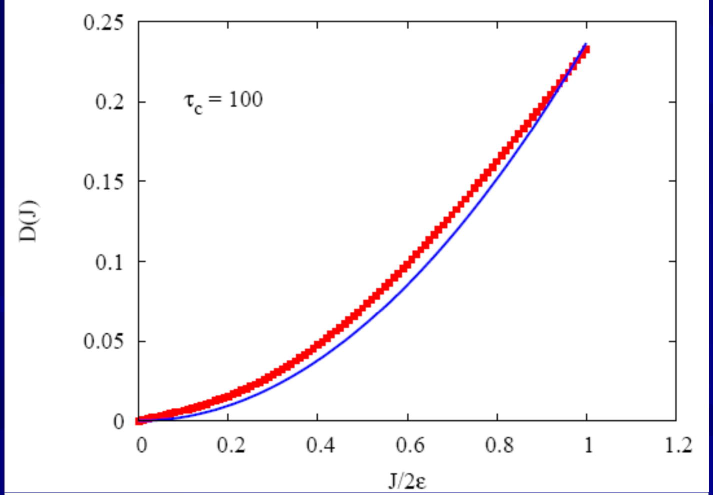
- OU process is the only stationary Markov process
- Spectral density

$$S(\nu) = \frac{|\eta|^2 T_{rev}}{\pi\tau_c} \frac{1}{1 - 2\alpha\cos[2\pi\nu] + \alpha^2}$$

$$\alpha = 1 - \frac{1}{\tau_c}$$

Diffusion due to a fluctuating offset





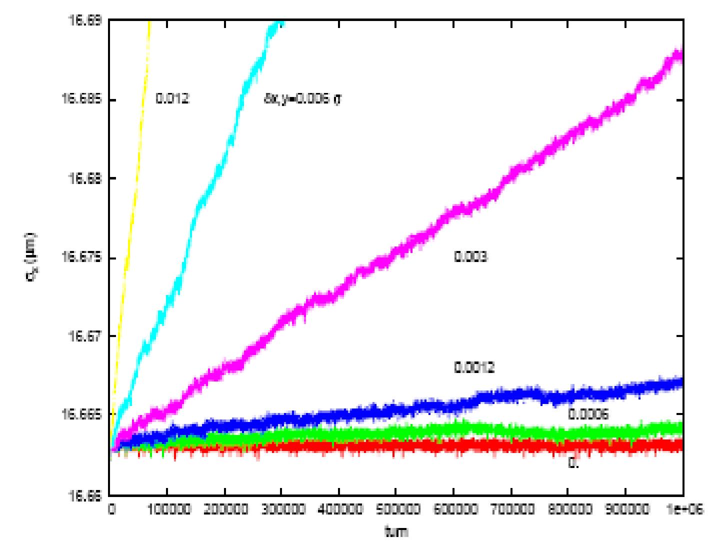
- Diffusion coefficients due to fluctuating offsets calculated analytically for any stationary stochastic process [T. Sen and J. Ellison, PRL (1996), T. Sen LHC Project Note 90 (1997)]
- Only noise at odd harmonics of the tune matters
- The dependence of D(J) on J changes with the correlation time
- τ = 1.001 (white noise): D(J) ~ J
- τ = 100: D(J) ~ J^2

Emittance growth

The average action follows from $< J >= \int_0^{J_A} J \rho(J,t)\, dJ$

If the density follows the diffusion equation, the aperture is far away and the density falls sufficiently rapidly (no long tails) then

1) $D(J) = D_1 J \Longrightarrow \frac{d}{dt} < J >= D_1$



For τ_c=1.001, ν=0.31, random offset Δdr

$D_1 = 4.7\, \pi^2 \xi^2 \varepsilon\, (\Delta d_r)^2$

Growth time of 10^9 turns (1 day)

$\Longrightarrow \Delta d_r = 1.4 \times 10^{-3}$ [units of σ]

Ohmi's estimate $\Delta d_r = 1 \times 10^{-3}$

2) $D(J) = D_2 J^2 \Longrightarrow \frac{1}{<J>}\frac{d}{dt} <J> = 2D_2$

For τ_c=100, ν=0.31,

$D_2 = 0.23\, \pi^2 \xi^2\, (\Delta d_r)^2/2$

Growth time of 10^9 turns (1 day)

$\Longrightarrow \Delta dr = 6.1 \times 10^{-3}$ [units of σ]

Ohmi's estimate $\Delta dr \sim 1 \times 10^{-2}$

K. Ohmi: Weak-strong simulations
Lumi 06 Proceedings

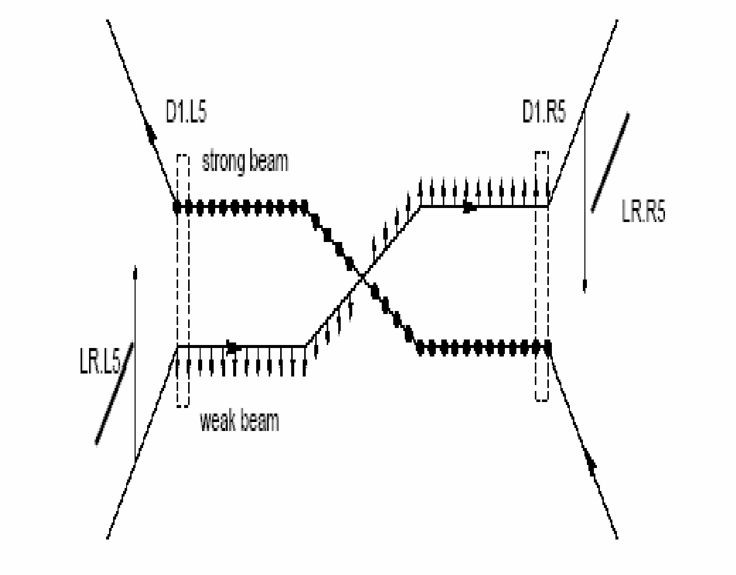
Crab cavity errors

- Cavity-cavity phase error

$$\Delta\phi \leq \frac{4\pi}{\lambda_{rf}\,\theta_c}\Delta x\,|_{\max}$$

- If phase noise modeled as white-noise, then $\Delta x|_{max} \sim 10^{-3}\sigma \rightarrow \Delta\Phi \leq 0.05$ degrees, $\Delta t \leq 0.37$ps for 400MHz cavity, 285μrad crossing angle
- If phase noise is OU noise with $\tau_c = 100$, then $\Delta x|_{max} \sim 6\times10^{-3}\sigma, \rightarrow \Delta\Phi \leq 0.32$ degrees, $\Delta t \leq 2.3$ps
- ILC crab cavity tolerance on timing jitter ~ 0.05ps (Burt, Dexter, Goudket; 2006)

Wire compensator tolerance



J.P. Koutchouk, 2000

- Nominal kick from wire

$$\theta_W = \frac{\mu_0}{2\pi} \frac{I_W L}{(B\rho)} \frac{1}{n\sigma}$$

- Current jitter in wire will cause a position fluctuation at the IP

$$\frac{\Delta\sigma_{IP}}{\sigma_{IP}} = 2 \times \left| \frac{\cos[\pi\psi(s) - \pi\nu]}{\sin[\pi\nu]} \right| \frac{\theta_w \sqrt{\beta_w}}{\sqrt{\varepsilon}} \frac{\Delta I_W}{I_W}$$

(for 2 wires)

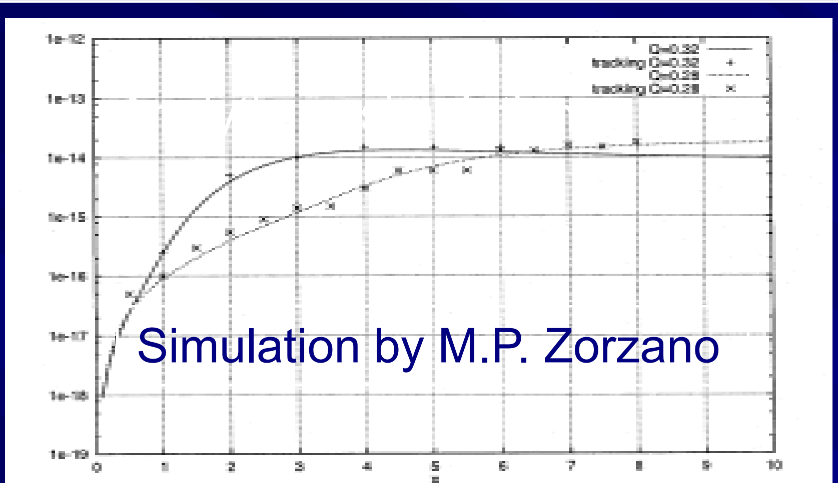
- If Δσ/σ < 10^{-3},

$$\Rightarrow \quad \frac{\Delta I_W}{I_W} \leq 3 \times 10^{-3}$$

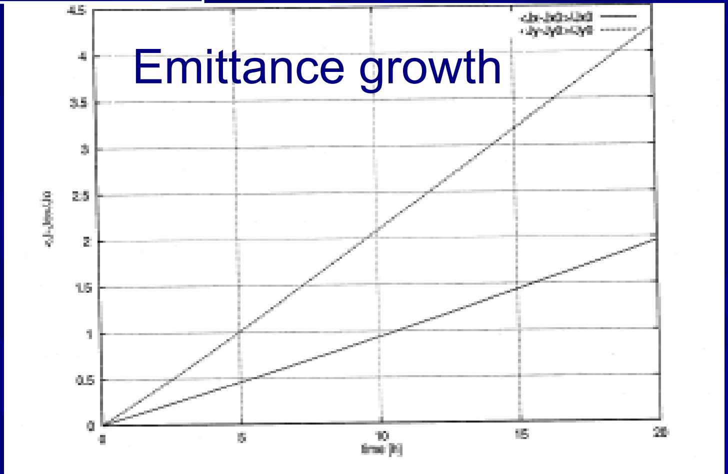
Wire parameters
Strength = 80A-m
Phase advance from IP=94°
Beam-wire separation=9.5σ

Offset fluctuations due to ground motion

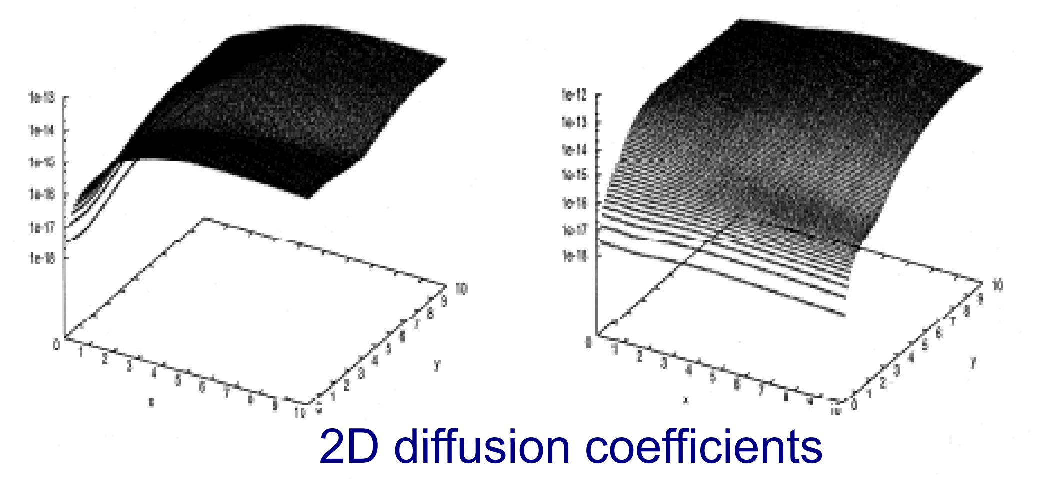
M.P. Zorzano & T. Sen, LHC Project Note 222 (2000)



Simulation by M.P. Zorzano



Emittance growth



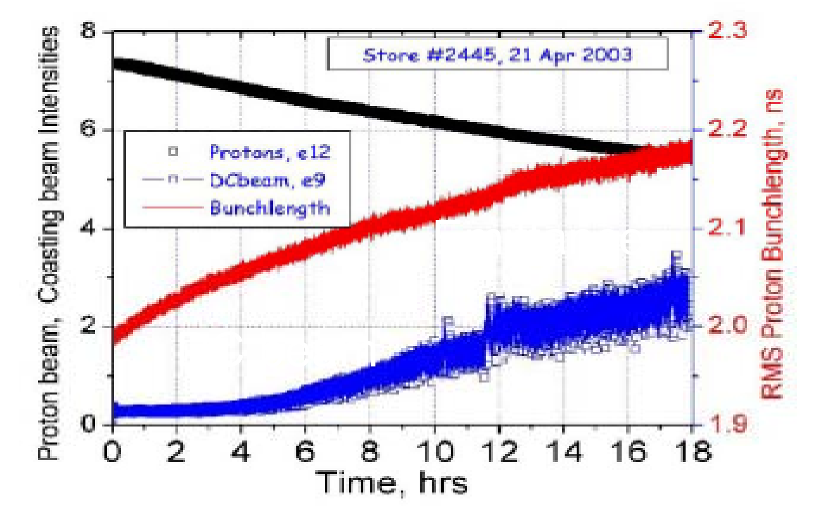
2D diffusion coefficients

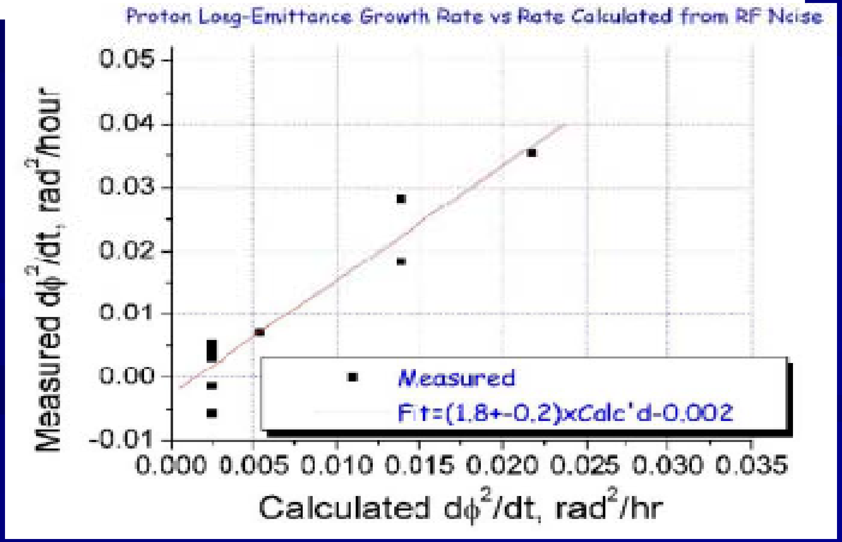
Estimated emittance doubling times of $(11\times10^4, 5\times10^4)$ hrs in the (H,V) planes with offset amplitudes of $10^{-4}\sigma$

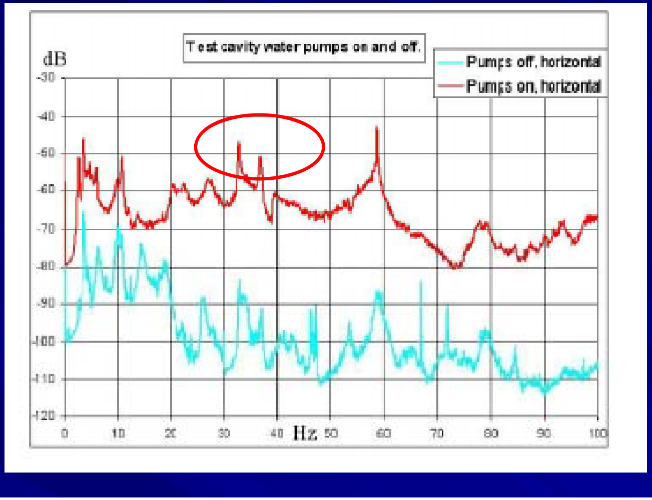
$(S[f_\beta] \sim 10^{-19}\mathrm{mm}^2/\mathrm{Hz})$

Longitudinal noise in the Tevatron

J. Steimel et al (PAC 2003)

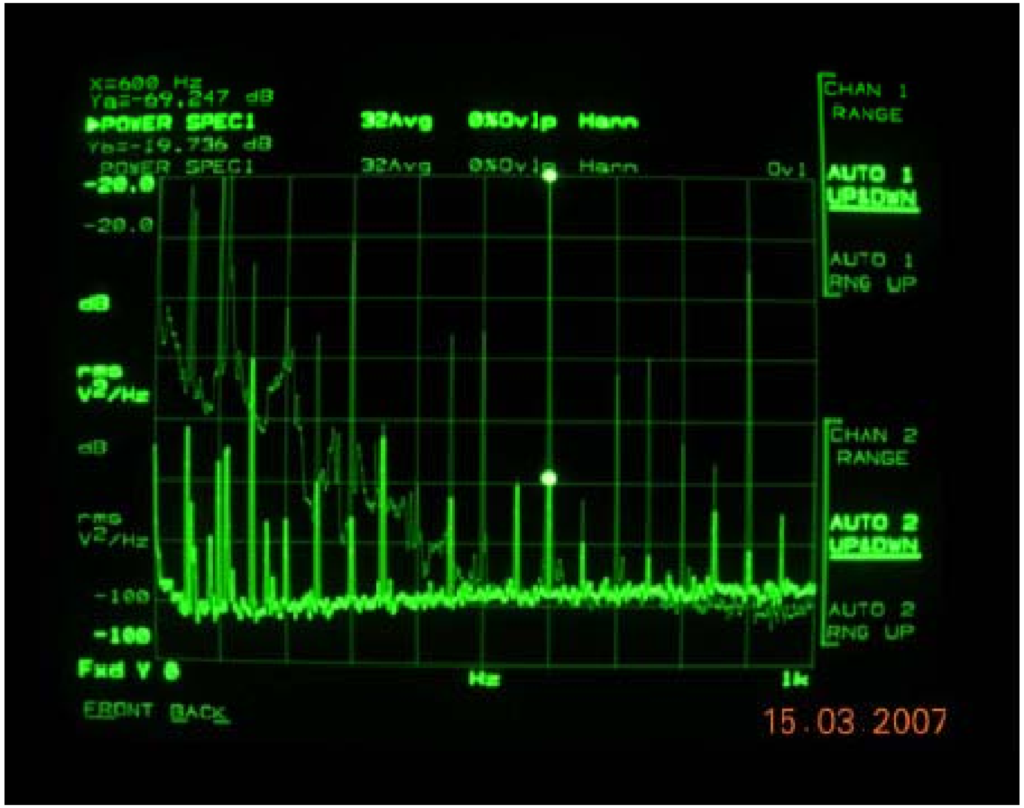






- Longitudinal emittance growth was accompanied by growth of DC beam
- Measured phase noise had peaks between 32-38 Hz (ν_s=37Hz)
- Emittance growth consistent with level of phase noise
- Water pumps drove vibrations of the cavities

LHC rf cavity spectrum

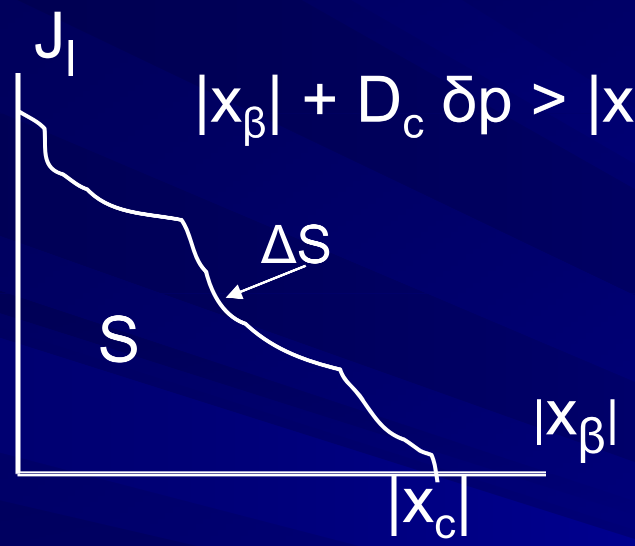


J. Tuchmantel, LHC Project Note 404(2007)

- Phase noise measured in tests is very low, σ_Φ~0.003degrees
- Several strong coherent lines at 50Hz and multiples
- Simulations of only longitudinal dynamics show (1) 50Hz lines cause slight emittance blow-up during ramp

  (2) During a store these lines do not have much impact
- Would IBS and synchrotron radiation make a difference?

Rf noise -> transverse diffusion



$|x_\beta| + D_c\, \delta p > |x_c|$

ΔS is the boundary in longitudinal-transverse space

$$\frac{dN}{dt} = -\iint_S \frac{dp(J_l, t \,||\, x_\beta|)}{dt} p(|x_\beta|) dJ d|x_\beta|$$

Newberger, Ellison & Shih, PRL 71 (1993)

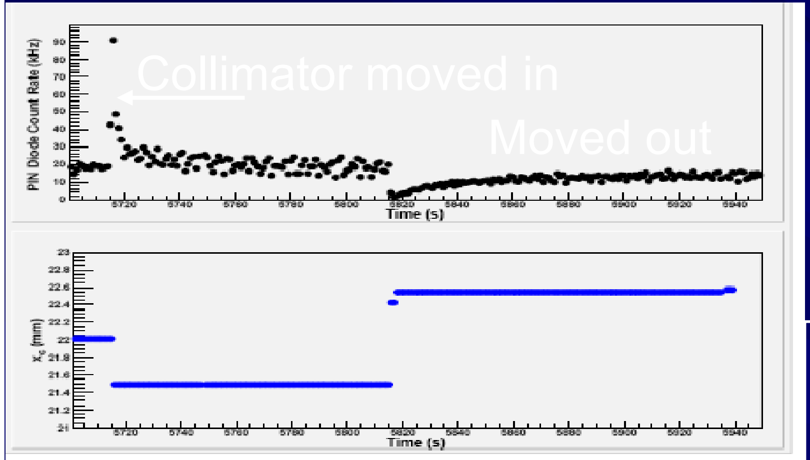
- Longitudinal diffusion couples to transverse diffusion via dispersion
- Conditional probability density satisfies the diffusion equation – known solutions for rf phase and amplitude noise
- Example: Phase noise

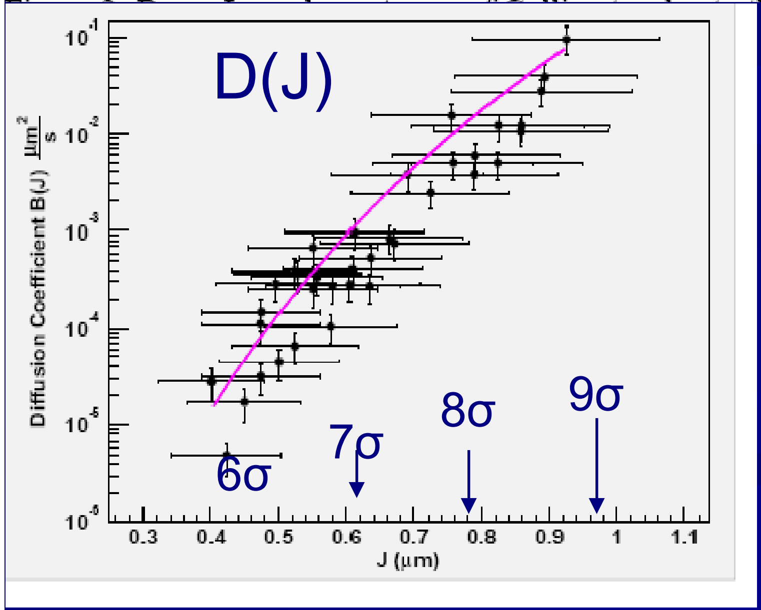
Initial loss rate at dispersion location ~

$$\frac{D_c^2 Q_s^4 \sigma_\phi^2}{(|x_c| - |x_\beta|)^2}$$

Diffusion in RHIC

Loss Rates





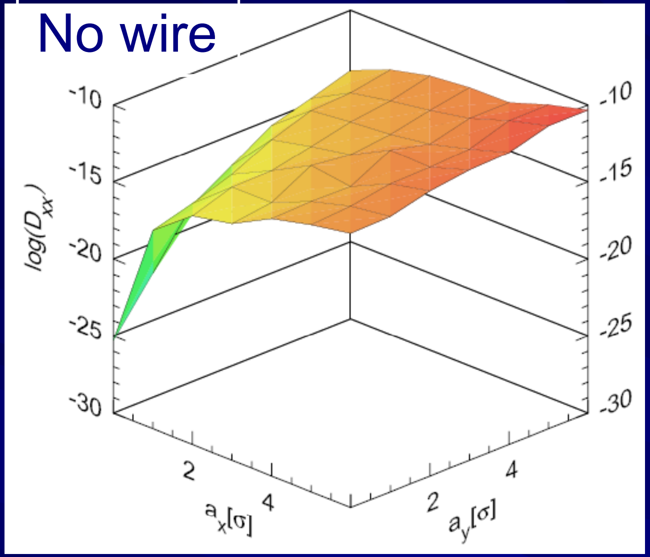
$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial}{\partial J}[D(J)\frac{\partial f}{\partial J}]$$

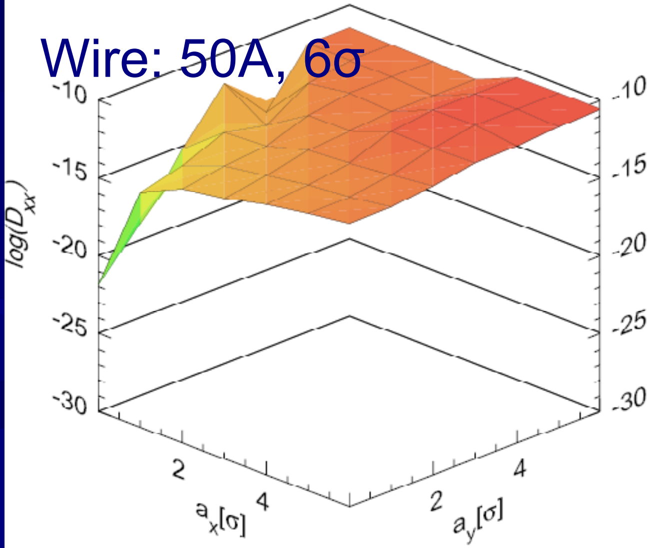
$$D(J) \sim bJ^n$$

- 3 stores gave <n> = 7.5 ± 0.5
- 1 store gave n ~ 3
- Halo producing mechanisms include IBS, triplet nonlinearities, magnet vibrations, beam-beam modulation, intensity related pressure rise
- Similar values of n for gold and protons suggests IBS not the dominant source
- Longitudinal loss occurred in some stores

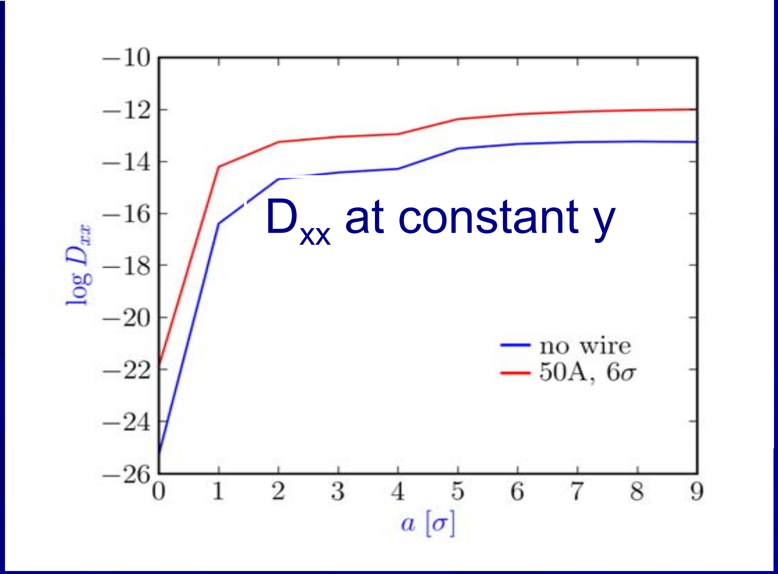
R. Fliller et al, EPAC 2002

RHIC simulations: diffusion w/wo wire


No wire


Wire: 50A, 6σ

H.J. Kim – simulations with BBSIM

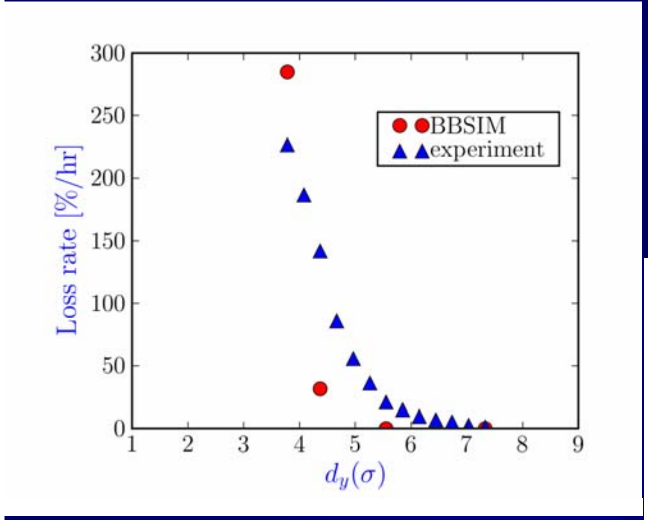

D_{xx} at constant y

- Diffusion coefficients found by tracking with BBSIM
- The wire increases diffusion by ~ 2 orders of magnitude at all amplitudes
- Similar changes in Dxy and Dyy with amplitude

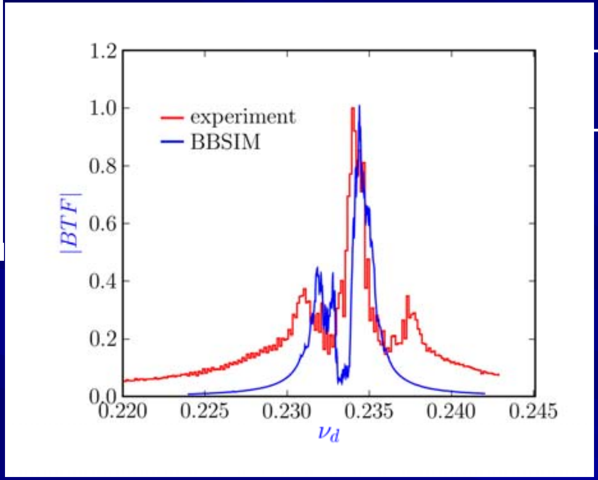
RHIC simulations

Loss rates with wire at Injection

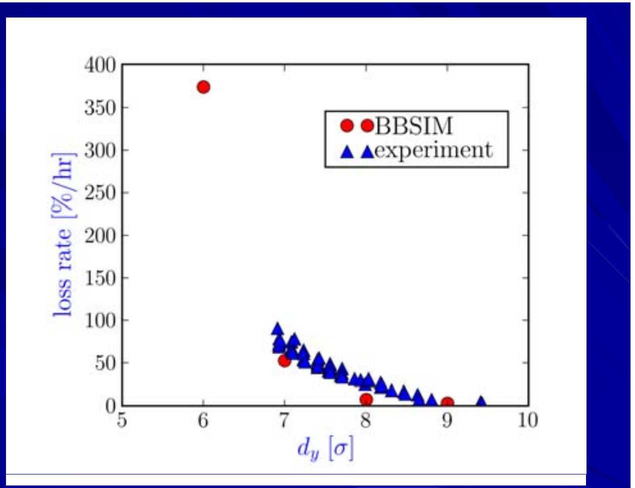
BBSIM



BTF comparison at 100 GeV



Loss rates with wire at 100 GeV



- RHIC simulations appear to be approaching reality

Open Questions

There are several – this is just a short sample

- Does a diffusion model describe transverse emittance growth?
- Is halo development described by a diffusion model? Probably not ?

  Solve the diffusion equation for the density and its moments & compare with measured emittance growths and lifetimes
- How do the long-range interactions affect the tolerances set? e.g on crab cavity phase noise, wire current jitter etc
- What is the impact of longitudinal jitter and cavity harmonics at 50Hz (near $2\nu_s$) on beam-beam interactions?
- Is it realistic to expect that all major noise sources are known and can be modeled?

Diffusion due to noise & resonances

- F. Ruggiero's thesis: section on "Renormalized Fokker-Planck equation with beam-beam interactions"
- The equation is for a steady-state distribution equation averaged over the phase (applicable to e+e- rings). Equation has two operators:

  - Diffusive operator

  - Matrix operator for the nonlinear part of the Hamiltonian

Remarks

- Noise changes the amplitude stochastically, hence induces a faster decay of phase correlations in a non-linear system.
- Diffusion "flattens" the distribution over phase space contaning low-order resonances. If the distribution is flattened over a region where resonances are near-overlapping, then beam size can blow up ->

  beam-beam threshold

Summary

- LHC: Ground motion near betatron frequencies may cause emittance growth at ~1%/hr
- Tevatron: ground motion at low frequencies evident in orbit motion spectra
- Simulations of emittance growth due to fluctuations in offsets at IPs consistent with theory
- Tolerances on crab cavity phase errors, wire current jitter can be set
- Transverse losses at high dispersion locations may help set limits on rf noise tolerances
- Diffusion model to describe the core and beam halo needs validation – RHIC experiments and simulations will help.

Dedicated to the memory of
Francesco Ruggiero