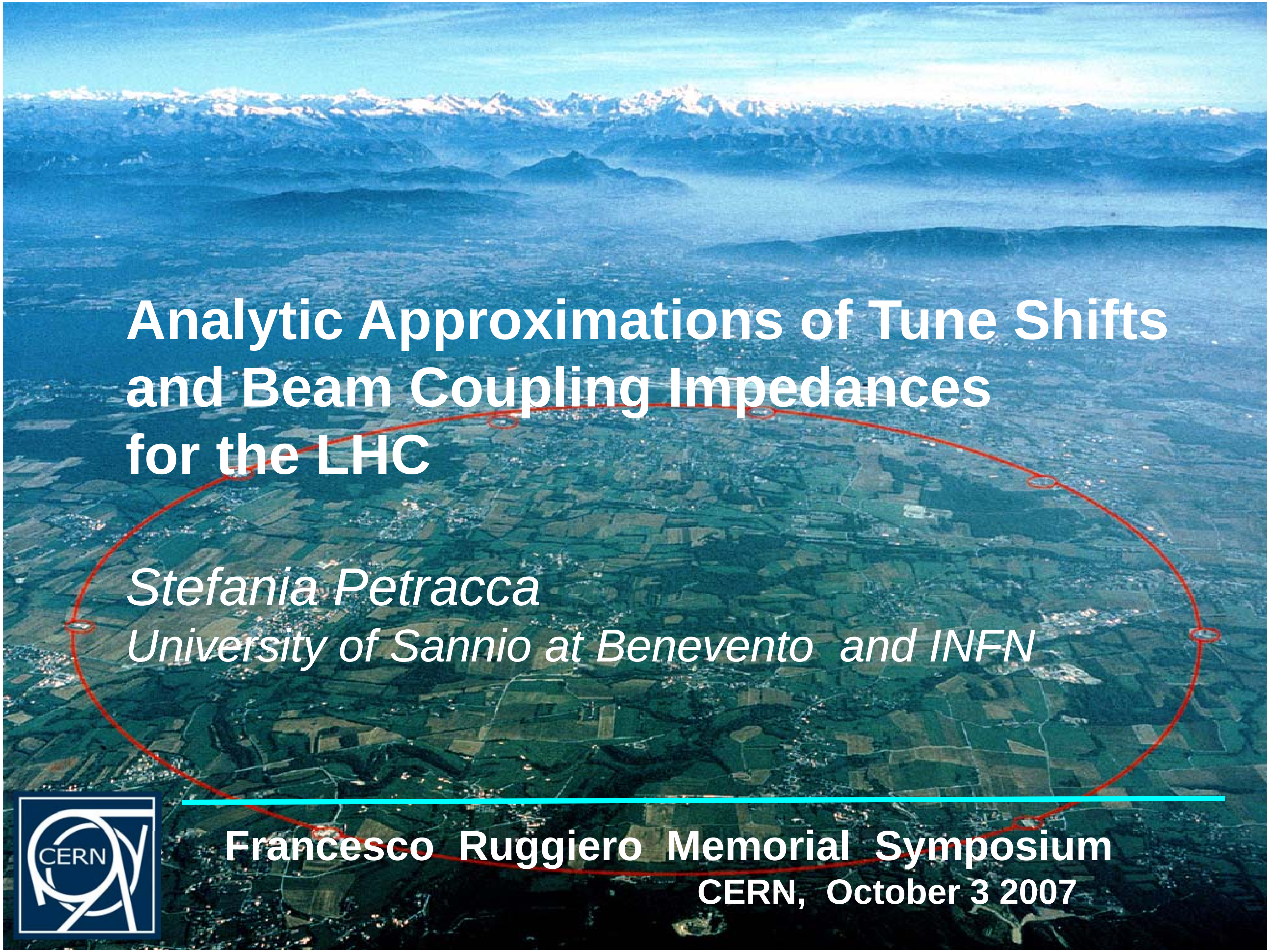


Analytic Approximations of Tune Shifts and Beam Coupling Impedances for the LHC

Stefania Petracca

University of Sannio at Benevento and INFN

Francesco Ruggiero Memorial Symposium

CERN, October 3 2007

Outlook

- Laslett coefficients for Betatron Normal Modes

  Square vs Circular Pipes
  A Toy Twin-Liner Model
  Real-World Gometries: MoMs & Random Paths

- Beam Coupling Impedances from Reciprocity Theorem

  Rounded corners and more

- Leontòvich Boundary Conditions and Beyond

  Lossy Walls, Coated Walls, Pumping Holes, etc.

…What's it all for ?

Betatron Oscillations

Forces acting on beam

$$\vec{f} = \vec{f}^{(sp.ch.)} + \vec{f}^{(im.)} + \vec{f}^{(g.f.)}$$,

vanishes at $\vec{\rho} = \vec{\rho}_b$

Nominal beam equilibrium position

$$\vec{f}^{(g.f.)}|_{\vec{\rho}=\vec{\rho}_{eq.}} + \vec{f}^{(im.)}|_{\vec{\rho}=\vec{\rho}_b=\vec{\rho}_{eq.}} = 0$$

Linearized forces :

$$\begin{cases} \vec{f} = (\vec{\rho} - \vec{\rho}_{eq.}) \cdot \nabla_{\vec{\rho}} \left[\vec{f}^{(im.)} + \vec{f}^{(sp.ch.)} + \vec{f}^{(g.f.)} \right]_{\vec{\rho} = \vec{\rho}_b = \vec{\rho}_{eq.}} \\ \vec{f} = (\vec{\rho} - \vec{\rho}_{eq.}) \cdot \left[(\nabla_{\vec{\rho}} + \nabla_{\vec{\rho}_b}) \vec{f}^{(im.)} + \nabla_{\vec{\rho}} \vec{f}^{(g.f.)} \right]_{\vec{\rho} = \vec{\rho}_b = \vec{\rho}_{eq.}} \end{cases}$$

Incoherent regime
$\vec{\rho}_b = \vec{\rho}_{eq.}, \vec{\rho} \neq \vec{\rho}_{eq.}$
(single particle dyn.)

Coherent regime
$\vec{\rho} = \vec{\rho}_b \neq \vec{\rho}_{eq.}$
(whole beam dyn.)

Nominal tune, H-V simmetry no H-V coupling
under guiding field (can be relaxed)

angular circulation freq.

$$\frac{1}{m\gamma} \left(\nabla_{\vec{\rho}} \vec{f}^{(g.f)} \right)_{\vec{\rho}=\vec{\rho}_{eq.}} \equiv \Omega_c^2 \nu_o^2 \bar{\bar{I}} \quad , \quad \Omega_c = \frac{\beta c}{R}$$

nominal orbit radius

Betatron Oscillations, contd.

Betatron oscillations ($\delta = \vec{\rho} - \vec{\rho}_{eq.}, \ \tau = s/c$)

$$\frac{d^2\vec{\delta}}{d\tau^2} + \Omega_c^2 \nu_0^2 \,\bar{\bar{U}} \cdot \vec{\delta} = 0$$

tune-shift tensor **Laslett tensor**

$\bar{\bar{U}}$ is a non-diagonal tensor, $\bar{\bar{U}} = \bar{\bar{I}} + \frac{2}{\nu_0} \Delta\bar{\bar{\nu}}$, $\Delta\bar{\bar{\nu}} = -\frac{N\ R\, r_0}{\pi \nu_0 \beta_0^2 \gamma_0 L^2} \bar{\bar{\epsilon}}$.

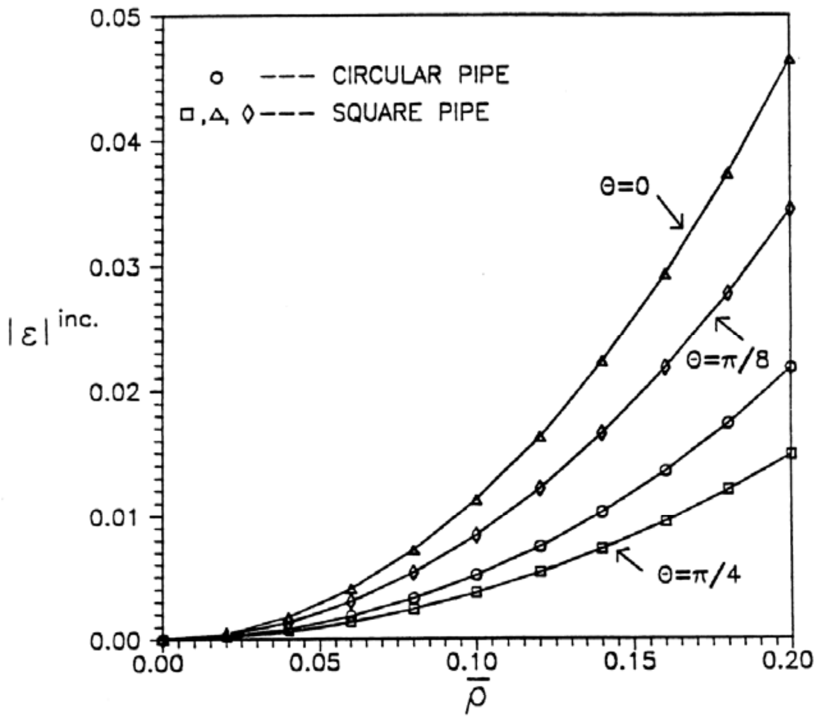
$$\bar{\bar{\epsilon}} = \frac{L^2}{4\Lambda} q^{-1} \begin{cases} \nabla_{\vec{\rho}} \left[\vec{f}^{(im.)} + \vec{f}^{(sp.ch.)} \right]_{\vec{\rho}=\vec{\rho}_b=\vec{\rho}_{eq.}} & incoherent, \\ (\nabla_{\vec{\rho}} + \nabla_{\vec{\rho}_b}) \vec{f}^{(im.)} \Big|_{\vec{\rho}=\vec{\rho}_b=\vec{\rho}_{eq.}} & coherent \end{cases}$$

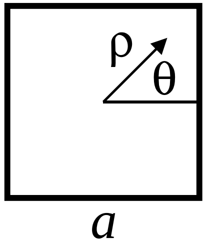
Normal-mode Lasletts:

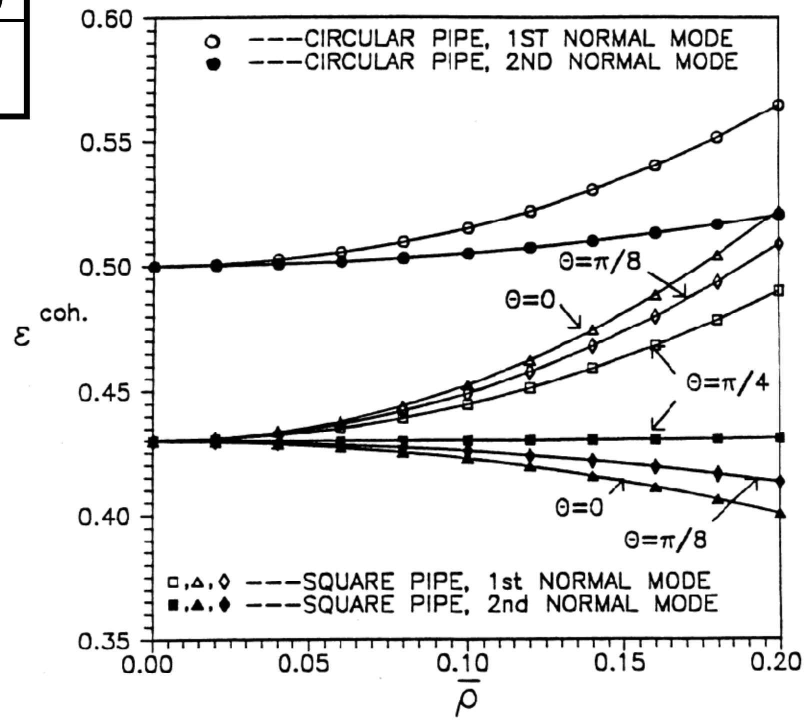
$$\epsilon_{1,2} = \frac{\epsilon_{11} + \epsilon_{22}}{2} \pm \left[\left(\frac{\epsilon_{11} - \epsilon_{22}}{2} \right)^2 + \epsilon_{12}\epsilon_{21} \right]^{1/2}$$

(for the two incoherent normal modes one has always $\epsilon_1 = -\epsilon_2$).

Square vs. Circular Liner







$\bar{\rho} = \rho/a$ *(square)*, $\bar{\rho} = \rho/R$ *(circle)*

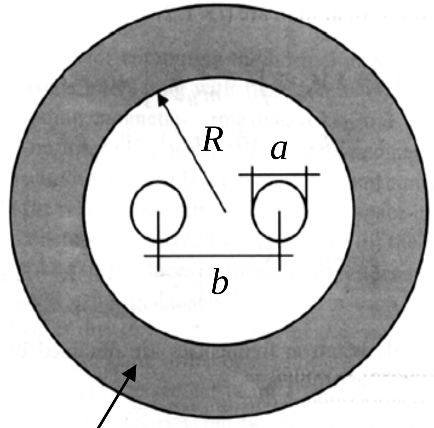
Twin-Beam Toy Model

Several possible regimes :

Incoherent-incoherent : $\vec{r}_b^{(i)} = \vec{r}_{eq.}^{(i)}, \vec{r}^{(i)} \neq \vec{r}_{eq.}^{(i)}, i = 1,2$

Coherent-coherent : $\vec{r}^{(i)} = \vec{r}_b^{(i)} \neq \vec{r}_{eq.}^{(i)}, i = 1,2$

Mixed : $\vec{r}_b^{(I)} = \vec{r}_{eq.}^{(i)}, \vec{r}^{(i)} \neq \vec{r}_{eq.}^{(i)}, \vec{r}^{(j)} = \vec{r}_b^{(j)} \neq \vec{r}_{eq.}^{(j)}$



High μ_r yoke

Different boundary conditions to be imposed on the electric (ϕ) and magnetic (A_z) potential static (=) and dynamic (~) parts :

$$\phi\big|_{pipe\ wall} = 0,\ \ \partial_n A_{=}\big|_{r=R^-\ (yoke)} = \mu_r^{-1} \partial_n A_{=}\big|_{r=R^+\ (yoke)}$$

$$\begin{cases} A_{\sim}\big|_{pipe\ wall} = 0 \\ \partial_n A_{\sim}\big|_{r=R^-\ (yoke)} = \mu_r^{-1} \partial_n A_{\sim}\big|_{r=R^+\ (yoke)} \end{cases}$$

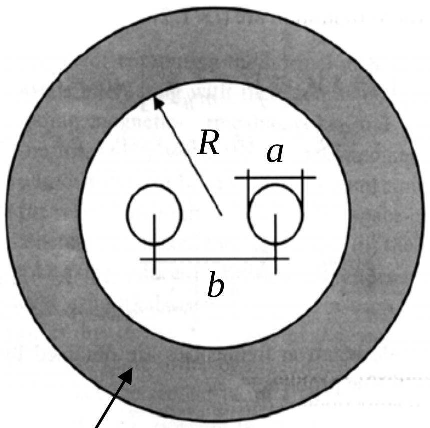
- first condition: high-frequency, *non penetrating* modes (skin depth << pipe wall thickness)
- second condition: low-frequency, *penetrating* modes (skin depth >> pipe wall thickness)

…yielding *different* coherent and mixed dynamics.

[S. Petracca, Part. Accel., 62 (1999) 241]

Twin-Beam Toy Model, contd.

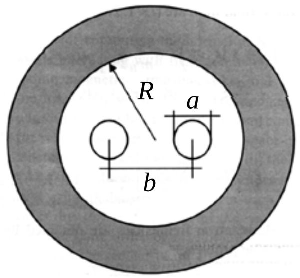
The Al-frame holding the beam pipes in the LHC design will prevent the dynamic magnetic field from coupling the beams, even at the lowest frequency associated with collective beam oscillations. As a result, the two beams are dynamically uncoupled. Neglecting space-charge effects, all regimes (incoherent, coherent, & mixed) merge together in the limit as $\beta \rightarrow 1$, yielding (both pipes):



High μ_r yoke

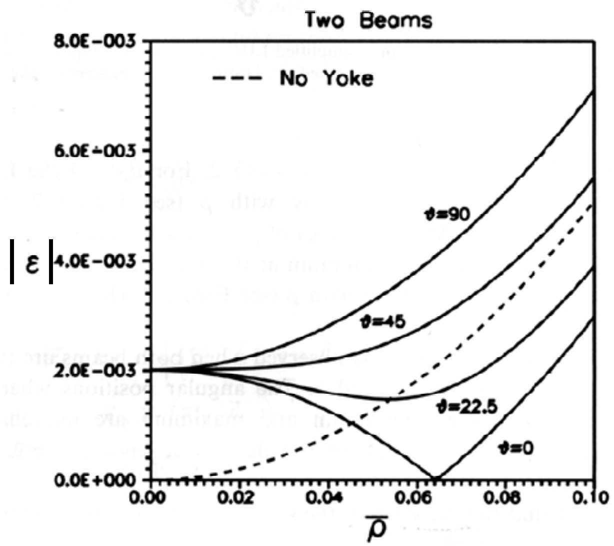
$$|\epsilon_{1,2}| = \pm \frac{1}{2}\left| -\left(\frac{\bar{z}^*_{eq} - \bar{z}^*_{c1}}{|\bar{z}_{eq} - \bar{z}_{c1}|^2 - 1}\right)^2 - \beta^2 \frac{1}{(\bar{z}_{eq.1} - \bar{z}_{eq.2})^2} + \beta^2 \frac{\mu_r - 1}{\mu_r + 1}\left[\left(\frac{\bar{z}^*_{eq.1}}{|\bar{z}_{eq.1}|^2 - \bar{R}^2}\right)^2 - \frac{\bar{z}_{eq.1}\bar{z}^*_{eq.2}}{(|\bar{z}_{eq.1}\bar{z}^*_{eq.2}|^2 - \bar{R}^2)^2}\right]\right|$$

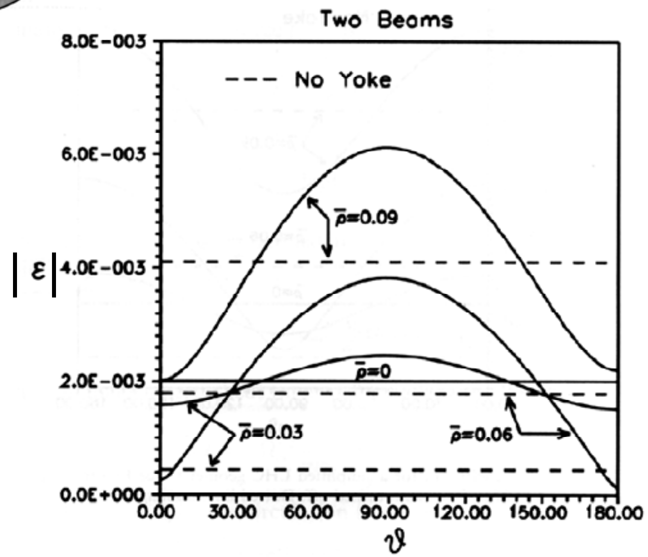
Twin-Beam Toy Model, contd.



$R=b=2a$

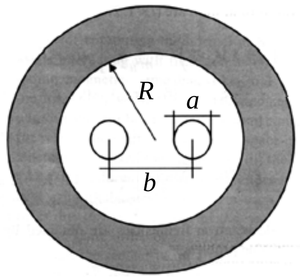
$\mu_r = 10^4$





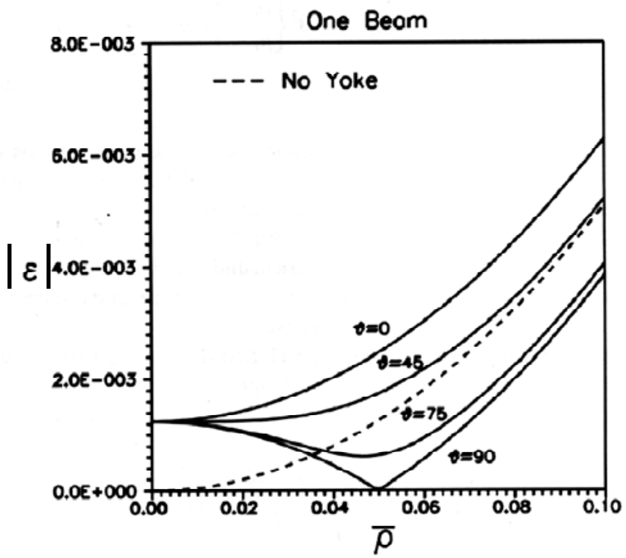
(ρ, θ) = local (pipe) polar coordinates

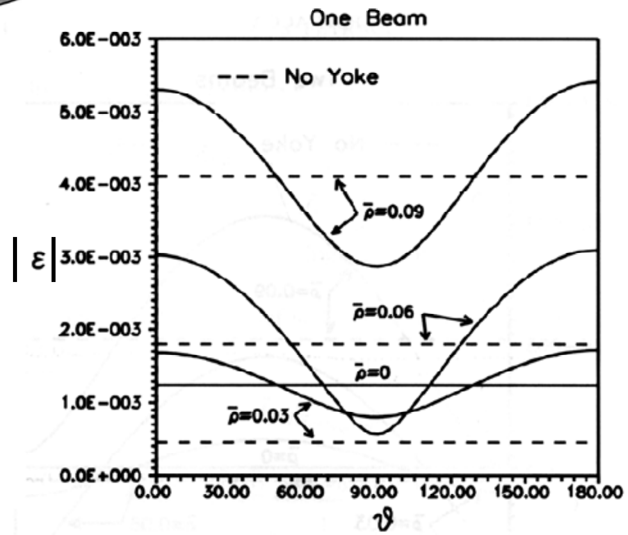
Twin-Beam Toy Model – One Beam



$R=b=2a$

$\mu_r = 10^4$





(ρ, θ) = local (pipe) polar coordinates

Real-World Geometries: Numerics

MoM : Rounded Corners & Stadium Shape Variations

Based on efficient representation of the (exact) Green's function for rectangular and circular domains, allowing to shrink the unknown charge density support (and related number of unknown charge expansion coeffs) to a min.

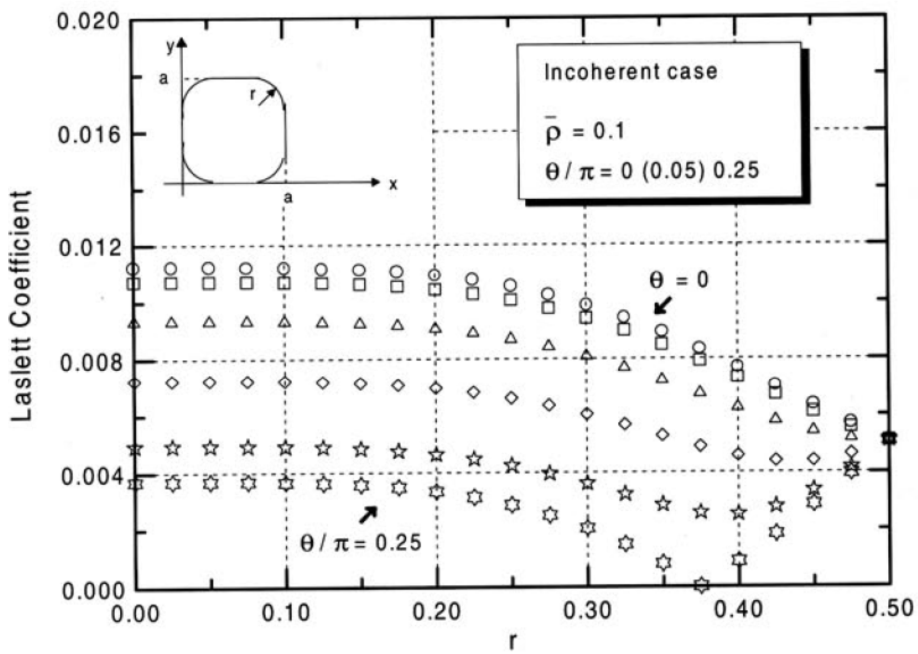
[S. Petracca, et al. Part. Accel. 63 (1999) 37]

Random Paths

Computes the (complex) potential only *on a circle*, using stochastic calculus, and then uses Cauchy integral formula for computing the Lasletts *without the need* of approximating derivatives with finite differences.

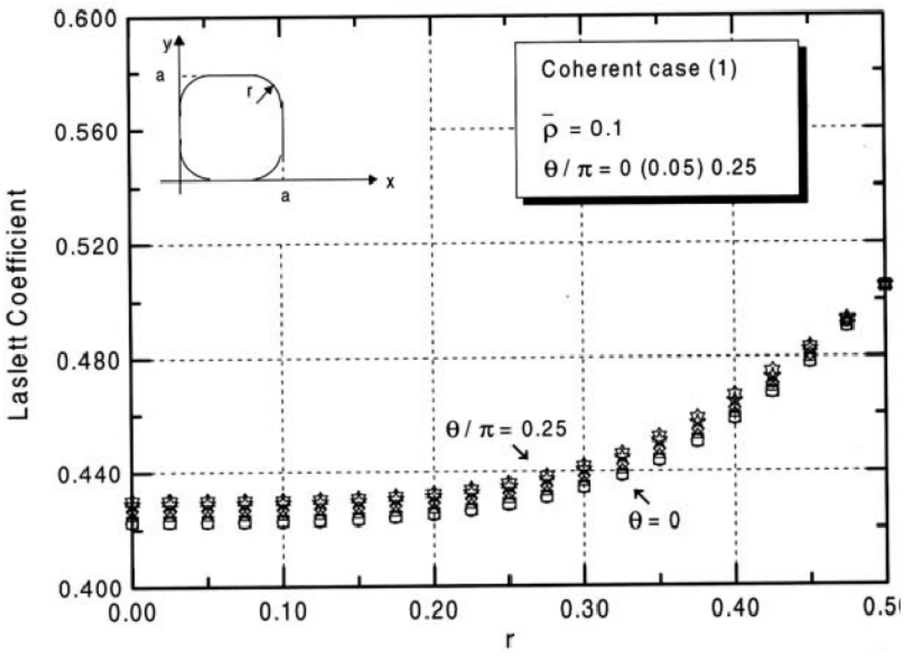
[S. Petracca, F. Ruggiero, et al., Proc. PAC-97, 1753]

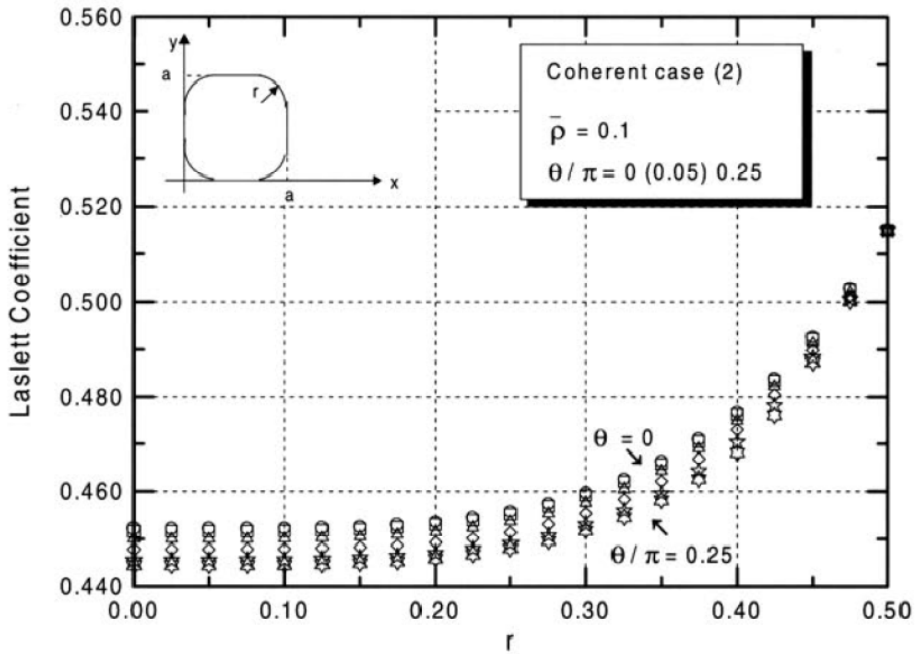
MoMs : Lasletts vs. Round-Corner Radius



(fixed off-axis distance; several angular position)

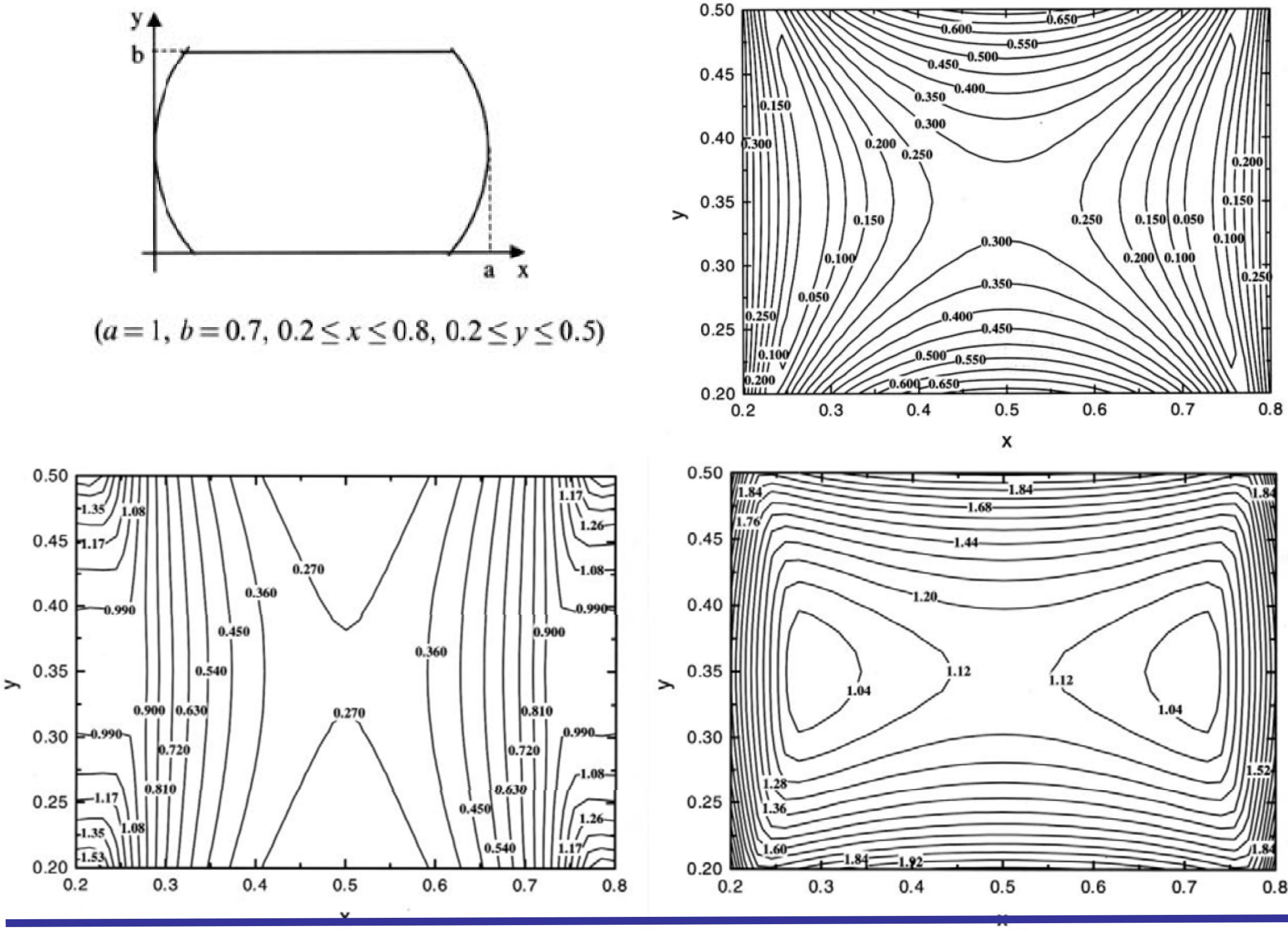
MoMs : Lasletts vs. Round-Corner Radius



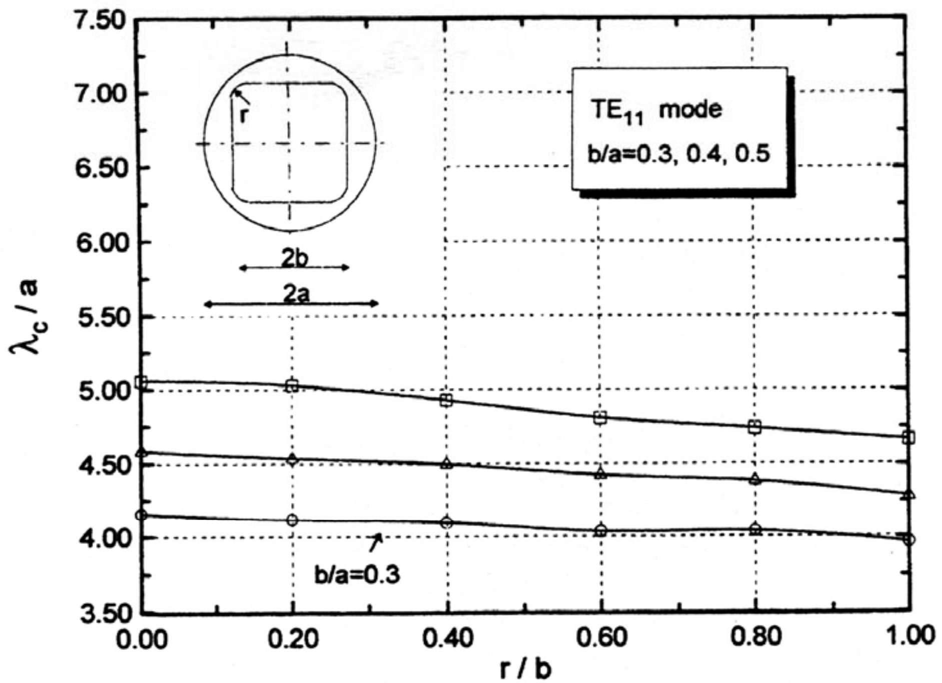


(fixed off-axis distance; several angular position)

MoMs : Hard-Cut Circle



Random Paths : Cold Bore HOM



Scaled cutoff wavelength of 1st HOM in the coax guide between the beam screen and the cold bore vs. (scaled) corner radius for several beam screen (scaled) sizes

Outlook

- Laslett coefficients for Betatron Normal Modes

  Square vs Circular Pipes
  A Toy Twin-Liner Model
  Real-World Gometries: MoMs & Random Paths

- Beam Coupling Impedances from Reciprocity Theorem

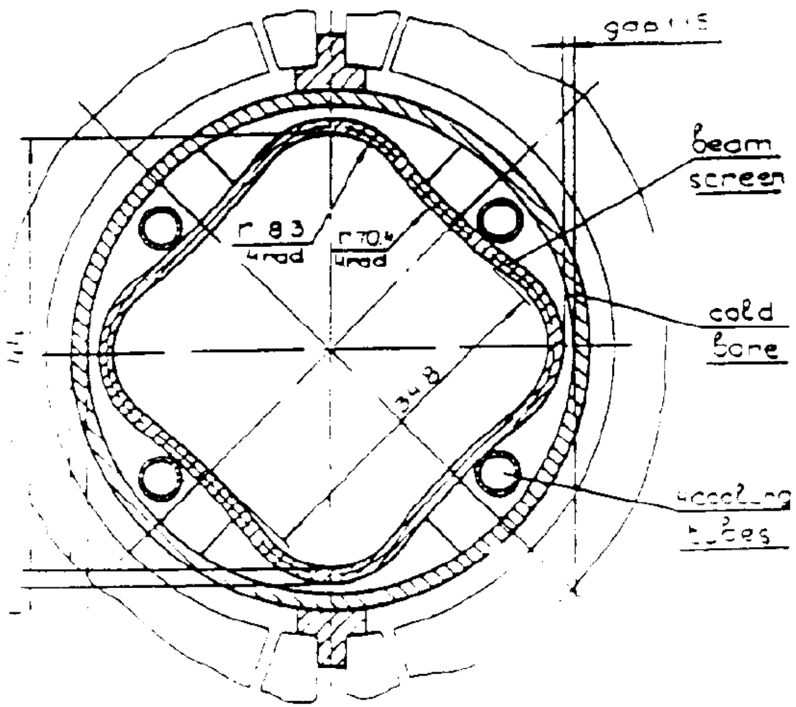
  Rounded corners and more

- Leontòvich Boundary Conditions and Beyond

  Lossy Walls, Coated Walls, Pumping Holes, etc.

…What's it all for ?

LHC Pipe Progress Design (1995)



Rounded corners
(manufacturing limitations)

Stainless steel (lossy) pipe
(pure Cu would not sustain the quenching forces due to magnetic field penetration (& parasitic currents) in case of failure of the cryogenics)

Copper-coated surface
(uncoated SS would give excessive parasitic losses; coating restricted to flat faces, where fields and loss would be largest)

Pumping holes
(removal of gas desorbed tue to synchrotron radiation)

Beam Coupling Impedances from Reciprocity Theorem

source (transverse) position

$$Z_{\|}(\omega) = jk\frac{(1-\beta_0^2)}{\beta c Q}\;\Phi(\vec{r}_0, \vec{r}_1)|_{\vec{r}_0=\vec{r}_1=0}$$

field (transverse) position

$$Z_{\perp}(\omega) = j\frac{(1-\beta_0^2)}{\beta c Q}\nabla_{\vec{r}_0}\nabla_{\vec{r}_1}\;\Phi(\vec{r}_0, \vec{r}_1)|_{\vec{r}_0=\vec{r}_1=0}$$

Beam coupling impedances from potentials…

$$\int_{\partial S}\left(\Phi\frac{\partial \Phi_0^*}{\partial n} - \Phi_0^*\frac{\partial \Phi}{\partial n}\right)d\ell = \int_S \left(\Phi\nabla_t^2\Phi_0^* - \Phi^*\nabla_t^2\Phi\right)dS$$

Reciprocity…

$$\nabla_t^2\Phi - k^2(1-\beta_0^2)\Phi = -\frac{Q}{\epsilon_0}\delta(\vec{r}-\vec{r}_1) \qquad \nabla_t^2\Phi_0^* - k^2(1-\beta_0^2)\Phi_0^* = -\frac{Q}{\epsilon_0}\delta(\vec{r}-\vec{r}_0)$$

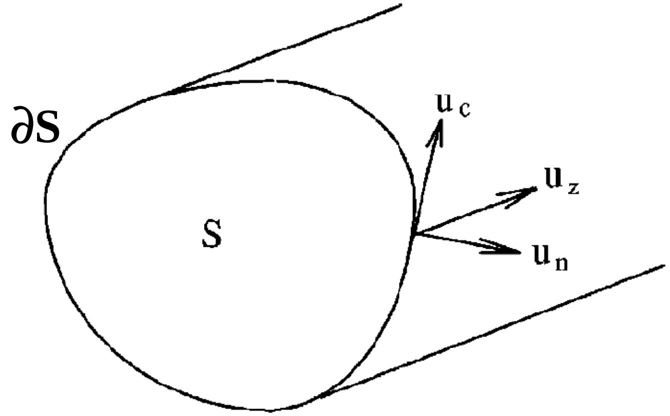
Potential – perturbed pipe (unknown) Potential – unperturbed pipe (known)

$$\int_{\partial S}\left(\Phi\frac{\partial \Phi_0^*}{\partial n} - \Phi_0^*\frac{\partial \Phi}{\partial n}\right)d\ell = \frac{Q}{\epsilon_0}\left[\Phi_0^*(\vec{r}_1, \vec{r}_0) - \Phi(\vec{r}_0, \vec{r}_1)\right]$$

Beam Coupling Impedances from Reciprocity Theorem, contd.

$$\left.\frac{\partial \Phi}{\partial n}\right|_{\partial S} = \nabla_t \Phi \cdot \hat{u}_n = -E_n^{(irr.)}$$

$$\left.\frac{\partial \Phi}{\partial n}\right|_{\partial S} = \nabla_t \Phi_0^* \cdot \hat{u}_n = -E_{0n}^{*(irr.)}$$



$$\Phi_0^* = -\frac{E_{0z}^*}{jk(1-\beta_0^2)}$$

Impedance (Leontóvich) b.c., non-perfectly conducting wall

$$\Phi|_{\partial S} = \frac{E_z|_{\partial S}}{jk(1-\beta_0^2)} = \frac{Z_{wall}\;H_c|_{\partial S}}{jk(1-\beta_0^2)} =$$

$$= -\frac{Y_0 Z_{wall}}{jk(1-\beta_0^2)}\left(\beta_0 E_n^{(irr.)} + \beta_0^{-1} E_n^{(sol.)}\right)_{\partial S}$$

Beam Coupling Impedances from Reciprocity Theorem, contd.

$$Z_{\|} - Z_{0,\|} = \frac{\epsilon_0}{\beta_0 c Q^2}\left\{\oint_{\partial S}\left[\beta_0 E_n^{(irr.)}(\vec{r},0) + \frac{1}{\beta_0} E_n^{(sol.)}(\vec{r},0)\right]\cdot\frac{Z_w}{Z_0} E_{0n}^{(irr.)*}(\vec{r},0)\,d\ell\right.$$

$$\left. + \oint_{\partial S} E_{0z}^*(\vec{r},0) E_n^{(irr.)}(\vec{r},0)\,d\ell\right\}$$

constitutive **wall perturbation**

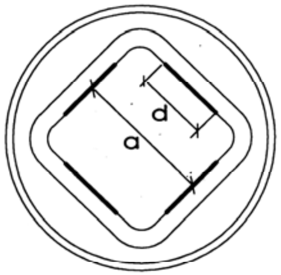
geometric **wall perturbation** ($E_{0z} = 0$ on ∂S_0)

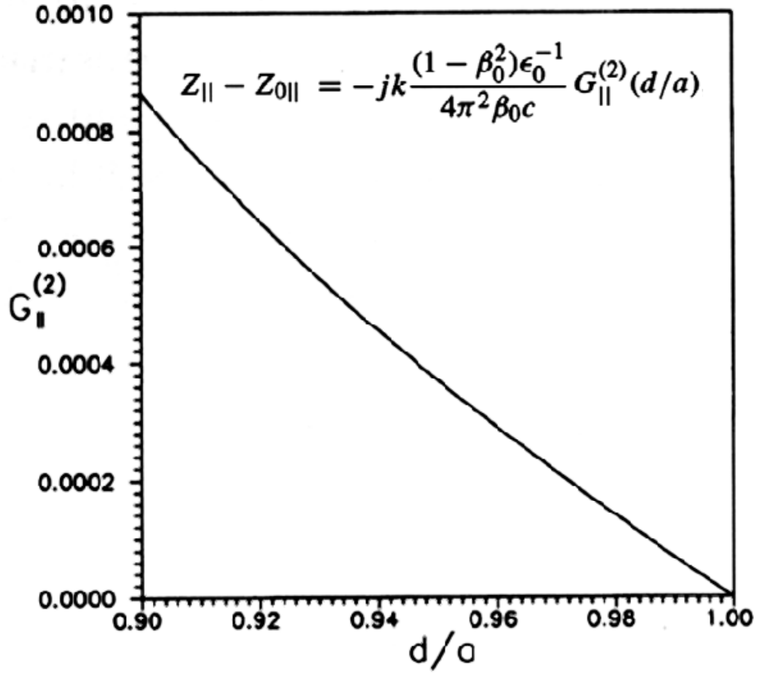
$$\overline{\overline{Z}}_\perp(\omega) - \overline{\overline{Z}}_{0,\perp}(\omega) = \frac{\epsilon_0}{\beta_0 c Q^2 k_0}\left\{\oint_{\partial S}\frac{Z_w}{Z_0}\nabla_{\vec{r}_0} E_{0n}^{(irr.)*}(\vec{r},\vec{r}_0)\otimes\left[\beta_0 E_n^{(irr.)}(\vec{r},\vec{r}_1) + \beta_0^{-1} E_n^{(sol.)}(\vec{r},\vec{r}_1)\right] d\ell\right.$$

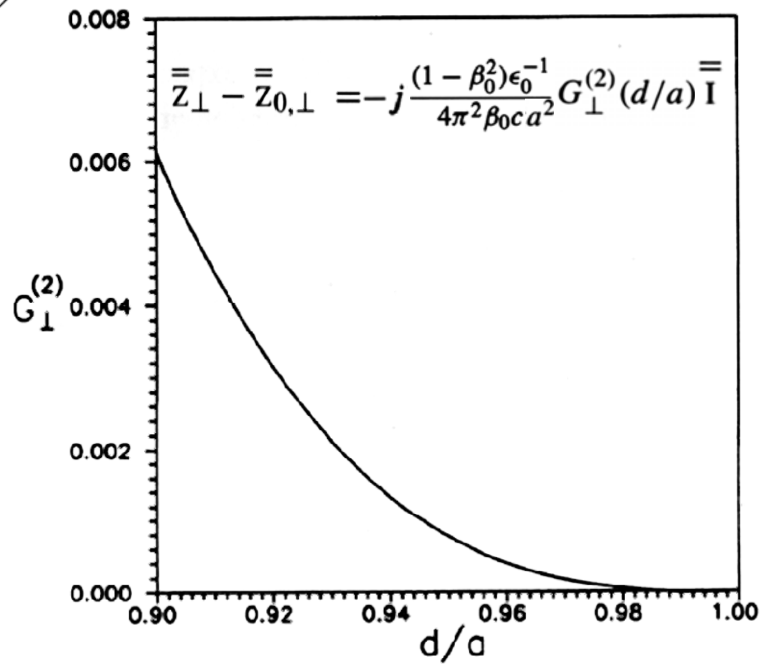
$$\left. + \oint_{\partial S}\nabla_{\vec{r}_0} E_{0z}^*(\vec{r},\vec{r}_0)\otimes\nabla_{\vec{r}_1} E_n^{(irr.)}(\vec{r},\vec{r}_1)\,d\ell\right\}_{\vec{r}_1=\vec{r}_0=0}$$

[S. Petracca, Part. Accel., 50 (1995) 211.]

Example: Rounded Square Liner







Outlook

- Laslett coefficients for Betatron Normal Modes

  Square vs Circular Pipes
  A Toy Twin-Liner Model
  Real-World Gometries: MoMs & Random Paths

- Beam Coupling Impedances from Reciprocity Theorem

  Rounded corners and more

- Leontòvich Boundary Conditions and Beyond

  Lossy Walls, Coated Walls, Pumping Holes, etc.

…What's it all for ?

Leontóvich B.C. and Beyond

$$|(\bar{\bar{I}}-\hat{u}_n\hat{u}_n)\cdot\vec{E}-Z_{\text{wall}}\hat{u}_n\times\vec{H}|_{\partial V}=0$$

Originally formulated for a *planar surface bounding some highly reflecting transversely homogeneous lossy half-space*…

…but in fact much more versatile than this. Can be applied, e.g., to:

-**lossy stratified media** (e.g., by repeated use of the TL impedance transport formula)

$$Z_{in} = Z_c \frac{Z_\ell + jZ_c \tanh(jk\ell)}{Z_c + jZ_\ell \tanh(jk\ell)}$$

-**curved surfaces**, provided e.g., $n \gg 1, \quad \mathrm{Im}(n)kR \gg 1$

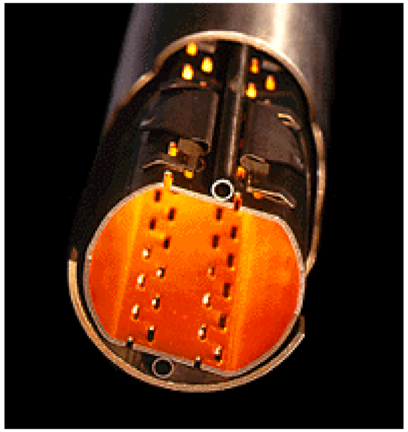
local radius of curvature

-**inhomogeneous media**, since

$$|(\bar{\bar{I}}-\hat{u}_n\hat{u}_n)\cdot\vec{E}-Z_{\text{wall}}\hat{u}_n\times\vec{H}|_{\partial} = O\left(\frac{1}{kZ_0}\frac{\partial Z_{\text{wall}}}{\partial n}\right)+O\left(\frac{1}{kZ_0}|\nabla_t Z_{\text{wall}}|\right)^2$$

[T. Senior, J. Volakis, *Approximate Boundary Conditions in EMtics*, IEE Press, 1995]
[L.N. Trefethen, L. Halpern, Math. Comput. **47** (1986) 421]

Leontòvich B.C. and Beyond : Perforated Walls



By solving a canonical b.v.p. we shew that perforated walls can be modeled using

$$Z_w = -jk_0 Z_0 n_\sigma (\alpha_m + \alpha_e) \quad , \quad n_\sigma = \text{surface density of holes}$$

Reproduces Kurennoy's result for $Z_{||}$ [S. Kurennoy, Part Accel. **50**, 167, 1995], when used in our perturbative formula for $Z_{||}$).

Axially aligned holes in a pipe surrounded by lossy co-axial shield (LHC cold bore)

skin depth, cold bore

TEM field, cold bore

$$\alpha_{e,m} = \alpha^{(i)}_{e,m} + F\alpha^{(e)}_{e,m} \quad , \quad F = \frac{-(\alpha^{(e)}_e + \alpha^{(e)}_m)}{(\alpha^{(i)}_e + \alpha^{(i)}_m) + jN_\lambda^{-1}\delta^*_S \dfrac{\oint_{\partial S_c} |e_{cb}|^2 d\ell}{|e_{cb}(\vec{l}_h)|^2}},$$

holes' position

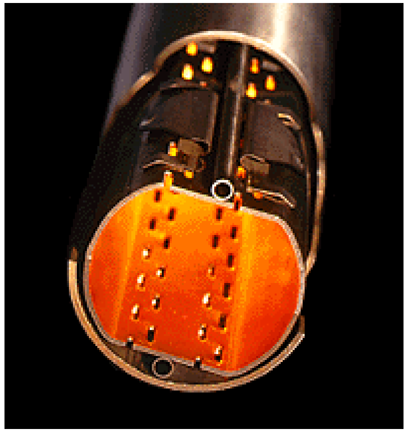
external hole polarizabilities

linear hole density

reproduces Gluckstern's formula (circular pipe) [R. Gluckstern, J. Diamond, IEEE Trans. **MTT-39**, 274, 1991] when used in our perturbational formula for $Z_{||}$)

[S. Petracca, Phys. Rev. **E60**, 6041, 1999]

Leontòvich B.C. and Beyond : Perforated Walls, contd.



Hole polarizabilities available for a variety of hole-shapes - see [F. de Meulenaere, J. Van Bladel, IEEE Trans., **AP-25** (1977)198; also R. de Smedt, J. van Bladel, IEEE Trans., **AP-28** (1980) 703]

Corrections for hole - hole couplings also worked out [S. Petracca, Phys. Rev. **E60** (1999) 6030] in the quasi-static approximation [R.E. Collin, *Field Theory of Guided Waves*, IEEE-McGraw-Hill, 1998]

Also, see [Van Bladel, Radio Sci. **14**, 319, 1979] for corrections to Bethe's formula for polarizabilities beyond the underlying quasi-static (kD << 1) assumption (very short bunches)

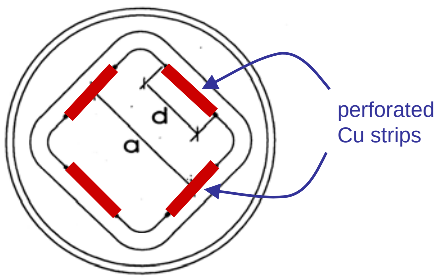
Example: LHC Parasitic Loss Budget

$$P^{(\text{ohmic})}_{\text{Cu,ss}} = (1-\alpha_{\text{Cu,ss}}) \cdot N_b \nu_r \frac{Q^2 c Z_0}{8\pi^3 a^2} W^{(\text{ohmic})}_{\text{Cu,ss}}\left(\frac{\sigma_z}{a}\right) G_{\text{Cu,ss}}\left(\frac{d}{a}\right)$$

surface fraction not covered by holes

bunch length (gaussian)

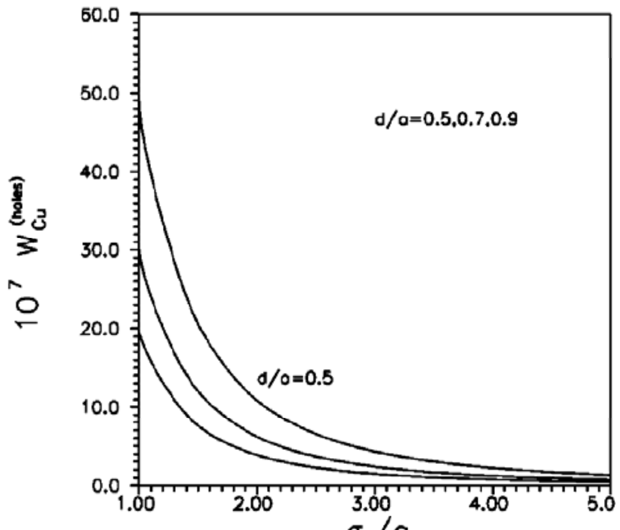
$$P^{(\text{holes})}_{\text{Cu}} = N_b \nu_r \frac{Q^2 c Z_0}{8\pi^3 a^2} W^{(\text{holes})}_{\text{Cu}}\left(\frac{\sigma_z}{a}\right) G_{\text{Cu}}\left(\frac{d}{a}\right)$$



$$W^{(\text{ohmic, holes})}\left(\frac{\sigma_z}{a}\right) = 2\int_0^{+\infty} e^{-(\sigma_z^2/a^2)(y^2/\beta_0^2)} \,\text{Re}\left[Y_0 Z^{(\text{ohmic, holes})}_{\text{wall}}\left(\frac{yc}{a}\right)\right] dy$$

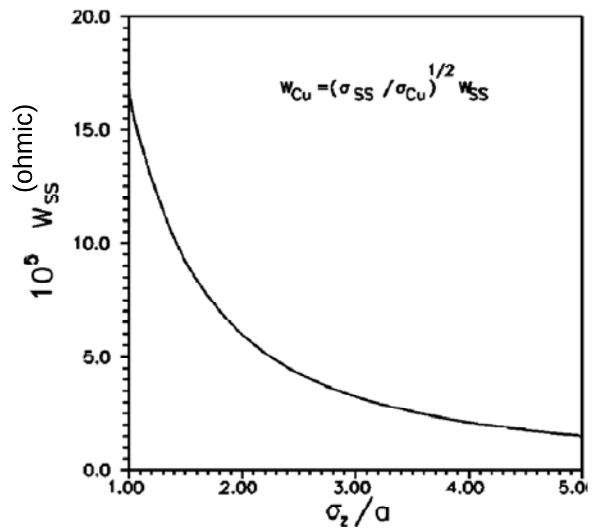
$$Z^{(\text{holes})}_w = -jk_0 Z_0 n_\sigma (\alpha_m + \alpha_e) \;, \quad n_\sigma = \frac{N_\lambda}{4a}\left(\frac{d}{a}\right)^{-1}$$

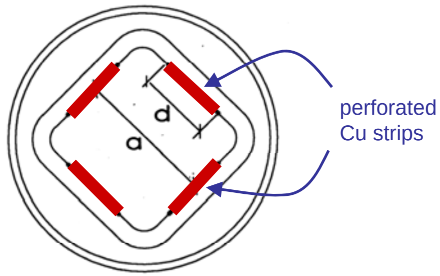
Example: Parasitic Loss Budget, contd.

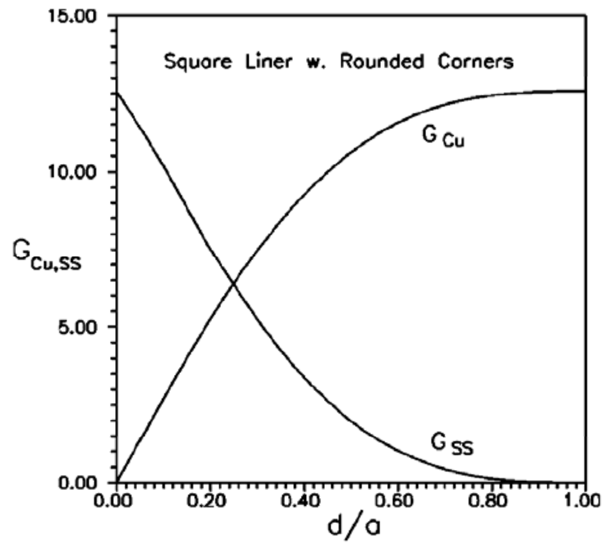


(a)

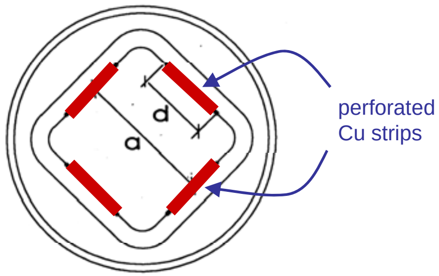
Stainless-steel resistivity ρ_{ss}	5×10^{-7} Ω m
Copper plating resistivity ρ_{Cu}	5.5×10^{-10} Ω m
Number of particles per bunch	10^{11}
Number of bunches N_b	2835
Revolution frequency ν_r	11.245 kHz
Hole radius r_0	0.75 mm
Wall thickness	0.75 mm
Liner diameter a	3.48 cm
Bunch length σ_z	7.5 cm







Example: Parasitic Loss Budget, contd.



Stainless-steel resistivity ρ_{ss}	5×10^{-7} Ω m
Copper plating resistivity ρ_{Cu}	5.5×10^{-10} Ω m
Number of particles per bunch	10^{11}
Number of bunches N_b	2835
Revolution frequency ν_r	11.245 kHz
Hole radius r_0	0.75 mm
Wall thickness	0.75 mm
Liner diameter a	3.48 cm
Bunch length σ_z	7.5 cm

	$d/a=0.5$	$d/a=0.7$
P_{Cu}	54 mW/m	63 mW/m
P_{ss}	326 mW/m	72 mW/m
$P_{Cu}^{(holes)}$	30 mW/m	19 mW/m
P_{total}	410 mW/m	154 mW/m

in good agreement with measurements
(Caspers, Morvillo and Ruggiero)

...What’s it all for ?

"Beauty is truth, truth beauty,“
that is all Ye know on earth,
and all ye need to know.

[John Keats, 1795-1821]

Thanks, Francesco.