The Integrated Jet Mass Distribution With a Jet Veto At Two Loops And Beyond

Robert M. Schabinger

with Andreas von Manteuffel and Hua Xing Zhu

The PRISMA Cluster of Excellence and
Mainz Institute of Theoretical Physics

Outline

Thrust in Soft-Collinear Effective Theory

Thrust,

$$T = \max_{\mathbf{x}} \left\{ \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{x}|}{\sum_i |\mathbf{p}_i|} \right\}$$

is a well-studied e^+e^- event shape variable that requires resummation in the end-point region, $1 - T = \tau \to 0$.

(see *e.g.* Schwartz Phys. Rev. **D77** (2008) 014026, Becher and Schwartz JHEP **07** (2008) 034)

- The framework of soft-collinear effective theory is a convenient one in which to discuss factorization and resummation.
- In the context of thrust in the end-point region, the hard scale is simply Q and one defines the scaling behavior of a soft or collinear momentum by

$$p_{\eta_{\text{collinear}}} \approx Q\,(\tau, 1, \sqrt{\tau}) \quad p_{\text{soft}} \approx Q\,(\tau, \tau, \tau)$$

$$p = (p^+, p^-, p_\perp) \qquad p^2 = p^+p^- - p_\perp^2$$

Factorization For a Thrust-Like Observable Defined Using a Jet Algorithm With a Jet Veto

Ellis et. al. Phys. Lett. **B689** (2010) 82, Kelley et. al. arXiv:1102.0561

$$\frac{1}{\sigma_{tree}}\frac{d\sigma}{d\tau_\omega} = H(Q,\mu)\int dk_L dk_R dM_L^2 dM_R^2 J_{\rm alg}(M_L^2 - Q\ k_L,\mu)$$
$$\times J_{\rm alg}(M_R^2 - Q\ k_R,\mu) S_{\rm alg}(k_L,k_R,\omega,r,\mu)\delta\left(\tau_\omega - \frac{M_L^2+M_R^2}{Q^2}\right)\Theta\left(\omega-\lambda\right)+\cdots$$

It matters what jet algorithm is used to determine how the radiated partons are clustered into jets!

Factorization For a Thrust-Like Observable Defined Using a Jet Algorithm With a Jet Veto

Ellis et. al. Phys. Lett. **B689** (2010) 82, Kelley et. al. arXiv:1102.0561

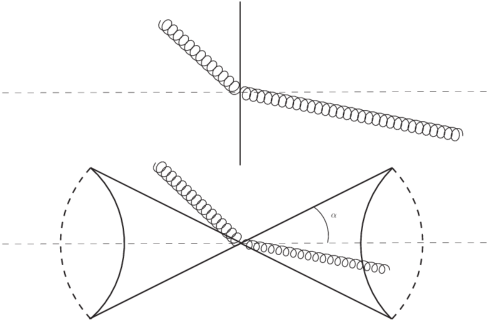
$$\frac{1}{\sigma_{tree}}\frac{d\sigma}{d\tau_\omega} = H(Q,\mu)\int dk_L dk_R dM_L^2 dM_R^2 J_{\rm alg}(M_L^2 - Q\ k_L,\mu)$$
$$\times J_{\rm alg}(M_R^2 - Q\ k_R,\mu) S_{\rm alg}(k_L,k_R,\omega,r,\mu)\delta\left(\tau_\omega - \frac{M_L^2+M_R^2}{Q^2}\right)\Theta\left(\omega-\lambda\right) + \cdots$$

It matters what jet algorithm is used to determine how the radiated partons are clustered into jets!

For example, the k_T and C/A algorithms are plagued by soft clustering logarithms in the C_F^L terms.

(see *e.g.* Kelley et. al. JHEP **1209** (2012) 117)

Jet Algorithms (Hemispheres vs. Cones)



$$S_{TC}(k_L, k_R, \omega, r, \mu) = \int_0^{\omega} d\lambda\, S(k_L, k_R, \lambda, r, \mu)$$

$$r = \tan^2\left(\frac{\alpha}{2}\right)$$

Our jet definition is similar to that of Sterman-Weinberg (Phys. Rev. Lett. 39 (1977) 1436)

To What Extent Can We Reduce, Reuse, Recycle the Hemisphere Calculation?

Actually, we can learn a lot without doing any hard calculations at all!

To What Extent Can We Reduce, Reuse, Recycle the Hemisphere Calculation?

Actually, we can learn a lot without doing any hard calculations at all!

In some cases, it is technically easier to work with fully integrated quantities:

$$K_{hemi}(\tau, \mu) = \int_0^\tau \mathrm{d}\tau' \int_0^\infty \mathrm{d}k_L \mathrm{d}k_R S_{hemi}(k_L, k_R, \mu) \delta\left(\tau' - \frac{k_L + k_R}{Q}\right)$$

$$\Sigma(X, Y, \mu) = \int_0^X dk_L \int_0^Y dk_R \; S_{hemi}(k_L, k_R, \mu)$$

$$K_{TC}(\tau_\omega, \omega, r, \mu) = \int_0^{\tau_\omega} \mathrm{d}\tau'_\omega \int_0^\infty \mathrm{d}k_L \mathrm{d}k_R S_{TC}(k_L, k_R, \omega, r, \mu) \delta\left(\tau'_\omega - \frac{k_L + k_R}{Q}\right)$$

Mapping the Hemisphere Plane Onto One of the Cones

The analysis is based on a suggestion made in Kelley et. al. Phys. Rev. **D86** (2012) 054017

Consider a Lorentz boost along the thrust axis acting on spacetime points which lie on the hemisphere plane, perpendicular to the thrust axis at the collision point:

$$\begin{pmatrix} x_c^0 \\ x_c^{thr} \end{pmatrix} = \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} x_h^0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh y \\ -\sinh y \end{pmatrix} x_h^0$$

We can easily find y such that the hemisphere plane gets mapped onto, say, the right cone (of half-angle α):

$$y = -\ln\left(\tan\left(\frac{\alpha}{2}\right)\right) = \ln\left(\frac{1}{\sqrt{r}}\right)$$

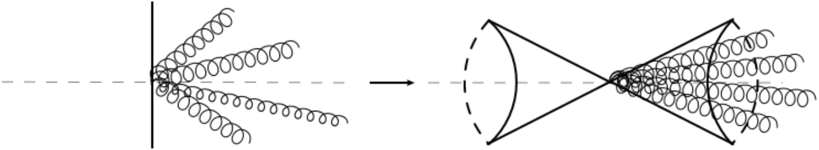
Transformation of the Light Cone Coordinates

$$
\begin{aligned}
x_c^+ &= x_c^0 + x_c^{thr} = (\cosh y - \sinh y)x_h^+ = e^{-y}x_h^+ = x_h^+\sqrt{r} \\
x_c^- &= x_c^0 - x_c^{thr} = (\cosh y + \sinh y)x_h^- = e^{y}x_h^- = x_h^-/\sqrt{r}
\end{aligned}
$$

In particular, these relations imply that

$$
\overset{\Rightarrow}{S}{}^{(L)}_{hemi}\left(k_L\sqrt{r}, k_R/\sqrt{r}, \mu\right) = \overset{\Rightarrow}{S}{}^{(L)}_{TC}\left(k_L, k_R, \omega, r, \mu\right),
$$

where the $\Rightarrow$ projects out those contributions to the soft functions where all soft radiation goes into the right hemisphere/cone.



What Does This Buy Us?...

After integrating over the transverse components of the momenta of the soft partons, we have

$$\overset{\Rightarrow}{S}{}^{(L)}_{hemi}(k_L, k_R, \mu) = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon L} \left(\frac{-i}{4(2\pi)^{3-2\epsilon}}\right)^L$$
$$\times \prod_{i=1}^{L} \left(\int \mathrm{d}q_i^- \mathrm{d}q_i^+ \Theta\left(q_i^+\right) \Theta\left(q_i^-\right) \Theta(q_i^- - q_i^+) \left(q_i^- q_i^+\right)^{-\epsilon} \right)$$
$$\times C_\Omega(\epsilon) \int \mathrm{d}\mathbf{\Omega}_\epsilon I^{(L)}\left(q_i^+, q_i^-, \mathbf{\Omega}\right) \delta(k_L) \delta\left(k_R - \sum_{i=1}^{L} q_i^+\right)$$
$$= k_R^{-1-2\epsilon L} \delta(k_L) \mu^{2\epsilon L} g^{(L)}_{hemi}(\epsilon)$$

by dimensional analysis.

...Quite a Bit, Actually

$$K_{TC}^{\mathrm{all-in}\,(L)}(\tau_\omega, \omega, r, \mu) = 2\int_0^{\tau_\omega} \mathrm{d}\tau'_\omega \int_0^\infty \mathrm{d}k_L \mathrm{d}k_R$$
$$\times \overset{\Rightarrow}{S}{}^{(L)}_{hemi}\left(\sqrt{r}k_L, \frac{k_R}{\sqrt{r}}, \mu\right)\delta\left(\tau'_\omega - \frac{k_L + k_R}{Q}\right)$$
$$= 2\int_0^{\tau_\omega} \mathrm{d}\tau'_\omega \int_0^\infty \mathrm{d}k_L \mathrm{d}k_R \left(\left(\frac{k_R}{\sqrt{r}}\right)^{-1-2\epsilon L}\delta(\sqrt{r}k_L)\mu^{2\epsilon L} g^{(L)}_{hemi}(\epsilon)\right)$$
$$= 2r^{\epsilon L}\overset{\Rightarrow}{K}{}^{(L)}_{hemi}(\tau_\omega, \mu)$$

This also tells us something very useful about the soft function integrand at $\mathcal{O}(\alpha_s^L)$

A Derivation of the Relation By Changing Variables

Consider the change of variables $q_i^- = p_i^-/r$:

$$K_{TC}^{\text{all-in}\,(L)}(\tau_\omega, \omega, r, \mu) = 2C_\Omega(\epsilon)\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{L\epsilon}\left(\frac{-i}{4(2\pi)^{3-2\epsilon}}\right)^L \int_0^{\tau_\omega} \mathrm{d}\tau'_\omega \int_0^\infty \mathrm{d}k_L \mathrm{d}k_R$$

$$\times \int \mathrm{d}\boldsymbol{\Omega}_\epsilon \prod_{i=1}^L \left(\int \mathrm{d}q_i^- \mathrm{d}q_i^+ \Theta\left(q_i^+\right)\Theta\left(q_i^-\right)\Theta(rq_i^- - q_i^+)\left(q_i^- q_i^+\right)^{-\epsilon}\right) I^{(L)}\left(q_i^+, q_i^-, \boldsymbol{\Omega}\right)$$

$$\times \delta\left(k_R - \sum_{i=1}^L q_i^+\right)\delta\left(\tau'_\omega - \frac{k_L + k_R}{Q}\right)$$

$$\overset{q_i^- \to q_i^-/r}{=} 2r^{(-1+\epsilon)L} C_\Omega(\epsilon)\left(\frac{-i\mu^{2\epsilon}e^{\gamma_E \epsilon}\pi^\epsilon}{4(2\pi)^3}\right)^L \int_0^{\tau_\omega Q} \mathrm{d}\tau''_\omega \int \mathrm{d}\boldsymbol{\Omega}_\epsilon \prod_{i=1}^L \left(\int \mathrm{d}q_i^- \mathrm{d}q_i^+\right)$$

$$\times \prod_{i=1}^L \left(\Theta\left(q_i^+\right)\Theta\left(q_i^-\right)\Theta(q_i^- - q_i^+)\left(q_i^- q_i^+\right)^{-\epsilon}\right) I^{(L)}\left(q_i^+, \frac{q_i^-}{r}, \boldsymbol{\Omega}\right)\delta\left(\tau''_\omega - \sum_{i=1}^L q_i^+\right)$$

The Hemisphere Integrand Transforms Homogeneously Under The Rescaling $q_i^- \to q_i^-/r$!

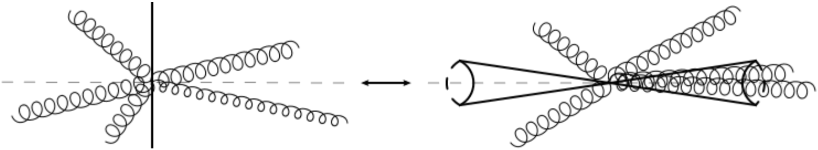
For this analysis to be consistent with the relation we found before, we must have

$$I^{(L)}\left(q_i^+, \frac{q_i^-}{r}, \mathbf{\Omega}\right) = r^L I^{(L)}\left(q_i^+, q_i^-, \mathbf{\Omega}\right)$$

This relation actually allows one to show that *any* other contribution to the integrated hemisphere soft function corresponds in a precise way to an analogous contribution to the integrated jet thrust distribution, provided r is sufficiently small!

Mixed In-Out Contributions to $K_{TC}^{(L)}(\tau_\omega, \omega, r, \mu)$

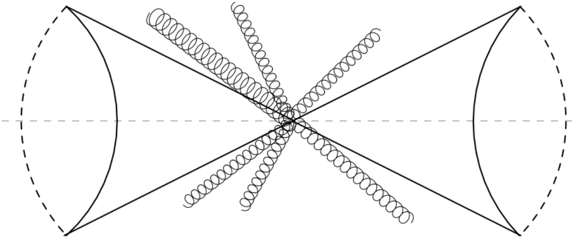
For instance, for r sufficiently small, a contribution to the integrated jet thrust distribution with n_L soft partons out of all jets and n_R soft partons in the right jet can be easily shown to be equivalent to a contribution to the integrated hemisphere soft function with n_L left-going and n_R right-going soft partons in the final state.



One simply needs to integrate k_L up to $2r\omega$ and k_R up to $\tau_\omega Q$.

What About the All-Out Contributions?

It is not immediately obvious from what has been discussed so far that we will be able to say anything about the all-out contributions



But let's see...

What Configurations of the q_i Dominate For Small r?

$$K_{TC}^{\text{all}-\text{out}\,(L)}(\tau_\omega,\omega,r,\mu)=\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{L\epsilon}\int_0^{\tau_\omega}\mathrm{d}\tau'_\omega\int_0^{\omega}\mathrm{d}\lambda\int_0^{\infty}\mathrm{d}k_L\mathrm{d}k_R$$

$$\times\prod_{i=1}^{L}\left(\int\frac{\mathrm{d}^d q_i}{(2\pi)^d}(-2\pi i)\delta(q_i^2)\Theta\left(q_i^+\right)\Theta\left(q_i^-\right)\Theta(q_i^- - r q_i^+)\Theta(q_i^+ - r q_i^-)\right)$$

$$\times I^{(L)}\left(q_i^+,q_i^-,\mathbf{q}_T^{(i)}\right)\delta(k_L)\delta\left(k_R\right)\delta\left(\lambda-\sum_{i=1}^{L}\frac{q_i^- + q_i^+}{2}\right)\delta\left(\tau'_\omega-\frac{k_L+k_R}{Q}\right)$$

What Configurations of the q_i Dominate For Small r?

$$K_{TC}^{\mathrm{all-out}\,(L)}(\tau_\omega,\omega,r,\mu) = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{L\epsilon} \int_0^{\tau_\omega} \mathrm{d}\tau'_\omega \int_0^{\omega} \mathrm{d}\lambda \int_0^{\infty} \mathrm{d}k_L \mathrm{d}k_R$$

$$\times \prod_{i=1}^{L} \left(\int \frac{\mathrm{d}^d q_i}{(2\pi)^d} (-2\pi i)\delta(q_i^2)\Theta\left(q_i^+\right)\Theta\left(q_i^-\right)\Theta(q_i^- - r q_i^+)\Theta(q_i^+ - r q_i^-) \right)$$

$$\times I^{(L)}\left(q_i^+, q_i^-, \mathbf{q}_T^{(i)}\right) \delta(k_L)\delta\left(k_R\right)\delta\left(\lambda - \sum_{i=1}^{L}\frac{q_i^- + q_i^+}{2}\right)\delta\left(\tau'_\omega - \frac{k_L + k_R}{Q}\right)$$

Answer: Either very large q_i^+ or very large q_i^-

The Logs of r in the All-Out Contributions Can Be Extracted By Considering Collinear Limits of the q_i!

By symmetry, taking the $q_i^+ >> q_i^-$ limit of the integrand is equivalent to the taking the extreme small r limit:

$$K_{TC}^{\text{all-out}\,(L)}(\tau_\omega, \omega, r, \mu)\Big|_{r \Rrightarrow 0} = 2K_{TC}^{\text{all-out}\,(L)}(\tau_\omega, \omega, r, \mu)\Big|_{q_i^+ >> q_i^-} = 2\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{L\epsilon}$$

$$\times \int_0^{2\omega} \mathrm{d}\lambda' \prod_{i=1}^{L}\left(\int \frac{\mathrm{d}^d q_i}{(2\pi)^d}(-2\pi i)\delta(q_i^2)\Theta\left(q_i^+\right)\Theta\left(q_i^-\right)\Theta(q_i^- - r q_i^+)\right)$$

$$\times I^{(L)}\left(q_i^+, q_i^-, \mathbf{q}_T^{(i)}\right)\delta\left(\lambda' - \sum_{i=1}^{L} q_i^+\right)$$

$$= 2r^{-\epsilon L}\overset{\Rightarrow}{K}{}^{(L)}_{hemi}(\tau_\omega, \mu)\Big|_{\tau_\omega Q \to 2\omega}$$

The Logs of r in the All-Out Contributions Can Be Extracted By Considering Collinear Limits of the q_i!

By symmetry, taking the $q_i^+ >> q_i^-$ limit of the integrand is equivalent to the taking the extreme small r limit:

$$K_{TC}^{\text{all-out}\,(L)}(\tau_\omega, \omega, r, \mu)\Big|_{r\Rrightarrow 0} = 2K_{TC}^{\text{all-out}\,(L)}(\tau_\omega, \omega, r, \mu)\Big|_{q_i^+>>q_i^-} = 2\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{L\epsilon}$$

$$\times \int_0^{2\omega} \mathrm{d}\lambda' \prod_{i=1}^{L}\left(\int \frac{\mathrm{d}^d q_i}{(2\pi)^d}(-2\pi i)\delta(q_i^2)\Theta\left(q_i^+\right)\Theta\left(q_i^-\right)\Theta(q_i^- - r q_i^+)\right)$$

$$\times I^{(L)}\left(q_i^+, q_i^-, \mathbf{q}_T^{(i)}\right)\delta\left(\lambda' - \sum_{i=1}^{L} q_i^+\right)$$

$$= 2r^{-\epsilon L}\overset{\Rightarrow}{K}{}^{(L)}_{hemi}(\tau_\omega, \mu)\Big|_{\tau_\omega Q\to 2\omega}$$

The logs of r are again fixed by the integrated hemisphere function!

Motivation and Recap of the $\mathcal{O}(\alpha_s)$ Results

$$K_{TC}^{(1)}(\tau_\omega, \omega, r, \mu) = C_F \left(-8\text{Li}_2(-r) - 4\ln^2(r) - \frac{\pi^2}{3} -8\ln^2\left(\frac{\mu}{Q\tau_\omega}\right) - 8\ln(r)\ln\left(\frac{\mu}{Q\tau_\omega}\right) + 8\ln(r)\ln\left(\frac{\mu}{2\omega}\right)\right)$$

$$K_{TC}^{(1)}(\tau_\omega, \omega, r, \mu)\Big|_{r\to 0} = C_F \left(-8\ln^2\left(\frac{\mu}{Q\tau_\omega}\right) - 8\ln(r)\ln\left(\frac{\mu}{Q\tau_\omega}\right) +8\ln(r)\ln\left(\frac{\mu}{2\omega}\right) - 4\ln^2(r) - \frac{\pi^2}{3}\right)$$

$$K_{hemi}^{(1)}(\tau, \mu) = C_F \left(\frac{\pi^2}{3} - 8\ln^2\left(\frac{\mu}{Q\tau}\right)\right)$$

The Small r $C_A C_F$ Terms at $\mathcal{O}\left(\alpha_s^2\right)$

$$K_{TC}^{(2)}(\tau_\omega,\omega,r,\mu)\Big|_{r\to 0} = C_A C_F \Bigg(-\frac{176}{9}\ln^3\left(\frac{\mu}{Q\tau_\omega}\right) + \left(-\frac{88\ln(r)}{3}+\frac{8\pi^2}{3}-\frac{536}{9}\right)\ln^2\left(\frac{\mu}{Q\tau_\omega}\right) + \left(-\frac{44}{3}\ln^2(r)+\frac{8}{3}\pi^2\ln(r)-\frac{536\ln(r)}{9}+56\zeta_3+\frac{44\pi^2}{9}-\frac{1616}{27}\right)\ln\left(\frac{\mu}{Q\tau_\omega}\right) + \left(-\frac{44}{3}\ln^2(r)-\frac{8}{3}\pi^2\ln(r)+\frac{536\ln(r)}{9}-\frac{44\pi^2}{9}\right)\ln\left(\frac{\mu}{2\omega}\right) + \frac{88}{3}\ln(r)\ln^2\left(\frac{\mu}{2\omega}\right) - \frac{8}{3}\pi^2\ln^2\left(\frac{Q\tau_\omega}{2r\omega}\right) + \left(-16\zeta_3-\frac{8}{3}+\frac{88\pi^2}{9}\right)\ln\left(\frac{Q\tau_\omega}{2r\omega}\right) + \frac{4}{3}\pi^2\ln^2(r) - \frac{268\ln^2(r)}{9} - \frac{682\zeta_3}{9} + \frac{109\pi^4}{45} - \frac{1139\pi^2}{54} - \frac{1636}{81}\Bigg)$$

The Small r $C_F n_F T_F$ Terms at $\mathcal{O}\left(\alpha_s^2\right)$

$$+C_F n_f T_F \left(\frac{64}{9}\ln^3\left(\frac{\mu}{Q\tau_\omega}\right) + \left(\frac{32\ln(r)}{3} + \frac{160}{9}\right)\ln^2\left(\frac{\mu}{Q\tau_\omega}\right)\right.$$
$$+\left(\frac{16\ln^2(r)}{3} + \frac{160\ln(r)}{9} - \frac{16\pi^2}{9} + \frac{448}{27}\right)\ln\left(\frac{\mu}{Q\tau_\omega}\right)$$
$$-\frac{32}{3}\ln(r)\ln^2\left(\frac{\mu}{2\omega}\right) + \left(\frac{16\ln^2(r)}{3} - \frac{160\ln(r)}{9} + \frac{16\pi^2}{9}\right)\ln\left(\frac{\mu}{2\omega}\right)$$
$$\left.+\left(\frac{16}{3} - \frac{32\pi^2}{9}\right)\ln\left(\frac{Q\tau_\omega}{2r\omega}\right) + \frac{80\ln^2(r)}{9} + \frac{248\zeta_3}{9} + \frac{218\pi^2}{27} - \frac{928}{81}\right)$$

It appears that, at $\mathcal{O}\left(\alpha_s^L\right)$, we have $\ln(r)$ terms

$$-\Gamma_{L-1}\ln^2(r) + \cdots$$

that cannot be naturally absorbed into the NGLs.

Outlook

We have shown that the thrust cone algorithm, investigated here in the context of a jet mass observable, has some very nice theoretical properties.

- Can we better understand the structure of the $\ln(r)$ terms that appear in the small r limit of the soft integrated τ_ω distribution?
- Is the general NGL resummation problem any easier for observables defined using a hemisphere jet algorithm? If so, our results would facilitate a resummation of the NGLs that appear in (integrated) jet mass distributions.
- Technical developments, which will appear in the paper upon which this talk is based, strongly indicate that the door to the calculation of the hemisphere NGLs at $\mathcal{O}\left(\alpha_s^3\right)$ is now wide open.
- Can we reproduce/improve upon large N_c analyses of NGLs?

(see *e.g.* Dasgupta and Salam Phys. Lett. **B512** (2001) 323, Banfi et. al. JHEP **08** (2010) 064)

Operator Definition of the Hemisphere Soft Function

$$S_{hemi}(k_L, k_R, \mu) =$$
$$\frac{1}{N_c}\sum_{X_s} \delta\Big(k_L - \bar{\eta}\cdot P^L_{X_s}\Big)\delta\Big(k_R - \eta\cdot P^R_{X_s}\Big)\langle 0|Y_\eta Y_{\bar{\eta}}|X_s\rangle\langle X_s|Y^\dagger_{\bar{\eta}} Y^\dagger_\eta|0\rangle$$

- $P_s^{L(R)}$ is the total soft momentum of final state X_s entering the left(right) hemisphere.
- The $Y's$ are Fourier transformed soft Wilson lines encapsulating the interaction of the "frozen" collinear quark and anti-quark with the soft gluon background.
- At $\mathcal{O}(\alpha_s^2)$, there are two soft partons emitted which can either travel into the same hemisphere or into opposite hemispheres.
- In earlier work, Hornig et. al. (JHEP **08** (2011) 054) and our group (Phys. Rev. **D84** (2011) 045022) calculated $S_{hemi}(k_L, k_R, \mu)$ to $\mathcal{O}(\alpha_s^2)$.

Operator Definition of the Thrust Cone Soft Function

$$S(k_L, k_R, \lambda, r, \mu) =$$
$$\frac{1}{N_c} \sum_{X_s} \delta\Big(k_L - \bar{\eta} \cdot P^L_{X_s}\Big) \delta\Big(k_R - \eta \cdot P^R_{X_s}\Big) \delta\Big(\lambda - E_{X_s}\Big) \langle 0 | Y_\eta Y_{\bar{\eta}} | X_s \rangle \langle X_s | Y^\dagger_{\bar{\eta}} Y^\dagger_\eta | 0 \rangle$$

At $\mathcal{O}\left(\alpha_s^2\right)$, the phase-space of the two soft partons naturally splits up into four different contributions:

- Both soft partons clustered into the same jet.
- One soft parton clustered into the $\mathbf{n}$ jet and one soft parton clustered into the $\bar{\mathbf{n}}$ jet.
- One soft parton clustered into a jet and the other out of all jets.
- Both soft partons out of all jets.

The Integrated Two-Loop Hemisphere Soft Function

$$\Sigma^{(2)}(X,Y,\mu) = \int_0^X dk_L \int_0^Y dk_R\; S^{(2)}_{hemi}(k_L,k_R,\mu) = \Sigma^{(2)}_\mu\left(\frac{X}{\mu},\frac{Y}{\mu}\right) + \Sigma^{(2)}_f\left(\frac{X}{Y}\right)$$

$$\Sigma^{(2)}_\mu\left(\frac{X}{\mu},\frac{Y}{\mu}\right) = \left[\frac{88}{9}\ln^3\left(\frac{X}{\mu}\right) + \frac{4\pi^2}{3}\ln^2\left(\frac{X}{\mu}\right) - \frac{268}{9}\ln^2\left(\frac{X}{\mu}\right) - \frac{11\pi^2}{9}\ln\left(\frac{XY}{\mu^2}\right) + \frac{404}{27}\ln\left(\frac{XY}{\mu^2}\right) - 14\zeta_3\ln\left(\frac{XY}{\mu^2}\right) + X\leftrightarrow Y\right]C_F C_A + \left[-\frac{32}{9}\ln^3\left(\frac{X}{\mu}\right) + \frac{80}{9}\ln^2\left(\frac{X}{\mu}\right) + \frac{4\pi^2}{9}\ln\left(\frac{XY}{\mu^2}\right) - \frac{112}{27}\ln\left(\frac{XY}{\mu^2}\right) + X\leftrightarrow Y\right]C_F T_F n_f$$

$$\Sigma_f^{(2)}\left(\frac{X}{Y}\right) = \left[-88\mathrm{Li}_3\left(-\frac{X}{Y}\right) - 16\mathrm{Li}_4\left(\frac{1}{\frac{X}{Y}+1}\right) - 16\mathrm{Li}_4\left(\frac{\frac{X}{Y}}{\frac{X}{Y}+1}\right) + 16\times\right.$$

$$\times\mathrm{Li}_3\left(-\frac{X}{Y}\right)\ln\left(\frac{X}{Y}+1\right) + \frac{88\mathrm{Li}_2\left(-\frac{X}{Y}\right)\ln\left(\frac{X}{Y}\right)}{3} - 8\mathrm{Li}_3\left(-\frac{X}{Y}\right)\ln\left(\frac{X}{Y}\right) - 16\zeta_3\times$$

$$\times\ln\left(\frac{X}{Y}+1\right) + 8\zeta_3\ln\left(\frac{X}{Y}\right) - \frac{4}{3}\ln^4\left(\frac{X}{Y}+1\right) + \frac{8}{3}\ln\left(\frac{X}{Y}\right)\ln^3\left(\frac{X}{Y}+1\right)$$

$$+\frac{4\pi^2}{3}\ln^2\left(\frac{X}{Y}+1\right) - \frac{4\pi^2}{3}\ln^2\left(\frac{X}{Y}\right) - \frac{4\left(3\left(\frac{X}{Y}-1\right)+11\pi^2\left(\frac{X}{Y}+1\right)\right)\ln\left(\frac{X}{Y}\right)}{9\left(\frac{X}{Y}+1\right)}$$

$$\left.-\frac{154\zeta_3}{9} + \frac{4\pi^4}{3} - \frac{335\pi^2}{54} - \frac{2032}{81}\right]C_F C_A + \left[32\mathrm{Li}_3\left(-\frac{X}{Y}\right) - \frac{32}{3}\mathrm{Li}_2\left(-\frac{X}{Y}\right)\times\right.$$

$$\left.\times\ln\left(\frac{X}{Y}\right) + \frac{8\left(\frac{X}{Y}-1\right)\ln\left(\frac{X}{Y}\right)}{3\left(\frac{X}{Y}+1\right)} + \frac{16\pi^2}{9}\ln\left(\frac{X}{Y}\right) + \frac{56\zeta_3}{9} + \frac{74\pi^2}{27} - \frac{136}{81}\right]C_F n_f T_F$$

The Small r Non-Global Contribution To The Integrated Jet Thrust Distribution

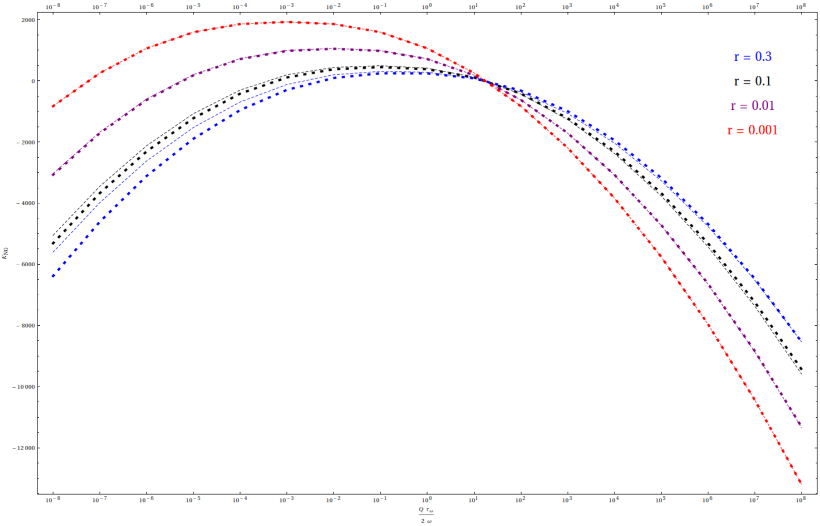
Kelley et. al. Phys. Rev. **D86** (2012) 054017

Remarkably, taking the small r limit before integrating in the way suggested captures not only the extreme small r asymptotics but also the dominant power-corrections to them!

$$K_{TC}^{(2)}(\tau_\omega,\omega,\mu)\Big|_{\mathrm{NG};\ r\to 0} = 2\,\Sigma_f^{(2)}\left(\frac{\tau_\omega Q}{2r\omega}\right) + C_A C_F\left(\frac{8}{3}\pi^2\ln^2(r) + 16\zeta_3\ln(r) - \frac{88}{9}\pi^2\ln(r) - \frac{4\ln(r)}{3} - \frac{16\pi^4}{5} + \frac{1012\zeta_3}{9} + \frac{871\pi^2}{27} + \frac{4064}{81}\right) + C_F n_f T_F\left(\frac{32}{9}\pi^2\ln(r) + \frac{16\ln(r)}{3} - \frac{368\zeta_3}{9} - \frac{308\pi^2}{27} + \frac{272}{81}\right)$$

(see Kelley et. al. Phys. Rev. **D84** (2011) 045022 and Hornig et. al. JHEP **08** (2011) 054)

Robustness Of The Small r Approximation: $C_A C_F$



Robustness Of The Small r Approximation: $C_F n_f T_F$