Transverse-momentum resummation for top-quark pair production at hadron colliders

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with

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and work in progress

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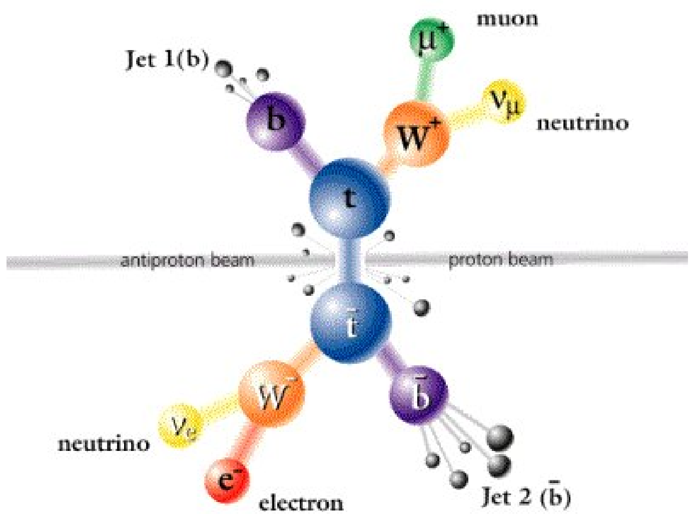
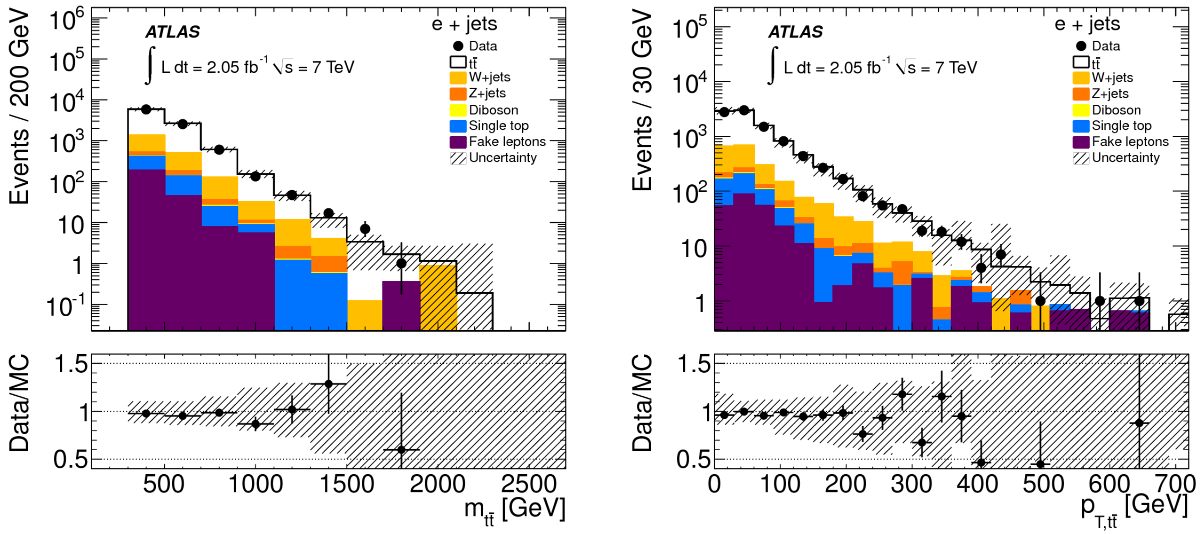
Motivation

- heaviest particle in the Standard Model
- plays special role in many new physics model
- large production cross sections at the LHC; background to many BSM physics signal
- Dominant production mechanism at hadron colliders: pair production



Motivation

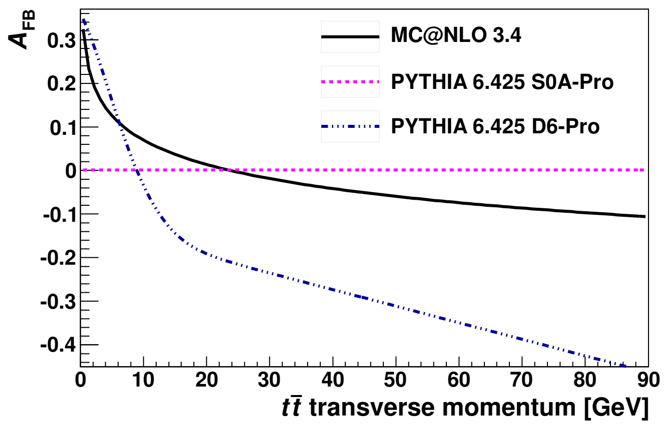
- Inclusive cross sections agree well with NNLO prediction (Baernreuther, Czakon, Mitov, PRL 109 (2012) 132001)
- More exclusive cross sections are being measured:

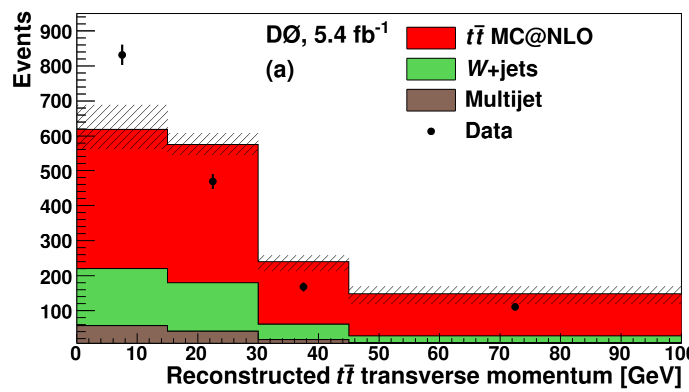


- Exclusive distribution accessible from fixed order or fixed order + parton shower
- Invariant mass distribution studied in SCET (Bauer, Fleming, Luke, Pirjol, Stewart, et.al.) by (Ahrens, et.al., 10'-11'; see also Beneke et.al. 10'; Czakon et.al. 09'; Kidonakis 10'; Cacciari et.al., 11')
- Transverse momentum of top quark pair: this talk! (C.S. Li, H.T. Li, S.D. Shao, L.L. Yang, HXZ, PRL 110, 082001 (2013)) (see also (Berger, Meng, PRD49, 3248 (1994); Mrenna, Yuan, PRD55, 120 (1997)) for LL resummation.)

Motivation

- Q_T of $t\bar{t}$ is interesting
  - intriguing Q_T dependence of forward-backward asymmetry @ Tevatron

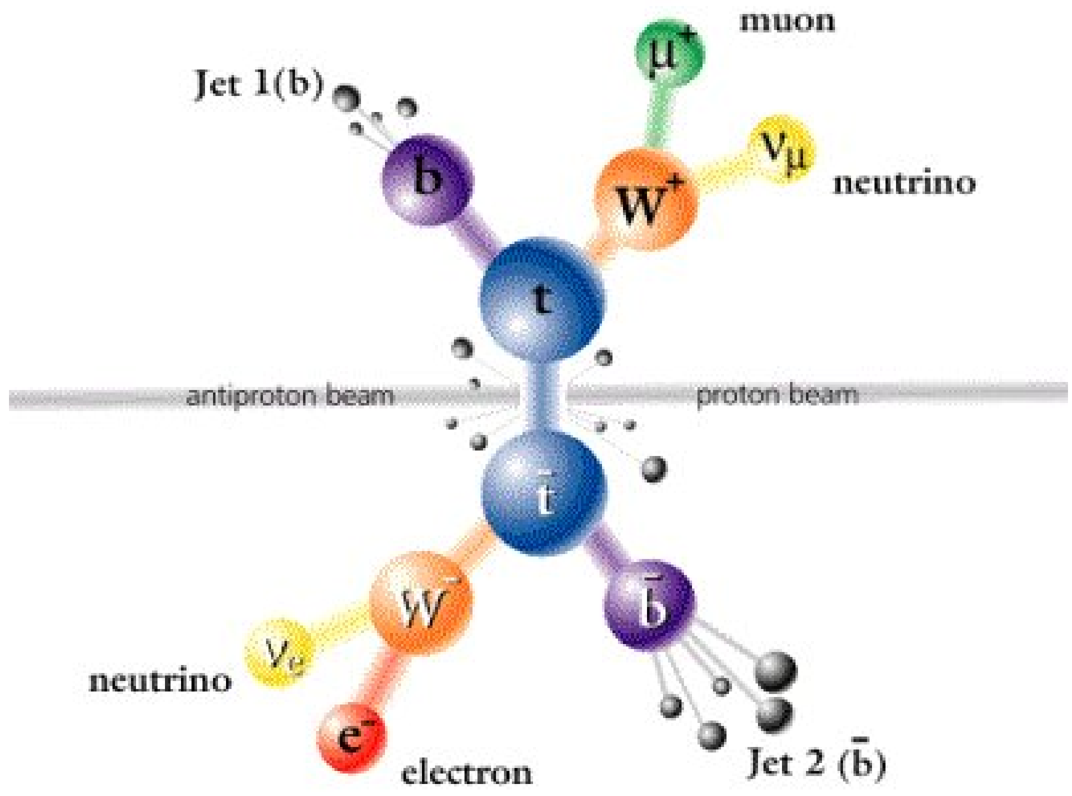




  - $\sigma_{t\bar{t}}$ Dominates at small Q_T
    - fixed order prediction is invalid at small Q_T, require resummation
    - New physics usually couples to $q\bar{q}$; to isolate new physics contribution need to probe even smaller Q_T: $C_F < C_A$
  - Used as a NNLO subtraction scheme for $t\bar{t}$ production, following the lines of (Catani, Grazzini, PRL 98 (2007) 222002) . Might be useful check to existing NNLO calculation

Definition of the observable

- Interested in the Q_T of “on-shell” top-quark pair
- Not an experimentally well defined observable



- But is a relatively clean observable theoretically

Theoretical problems with Q_T of $t\bar{t}$

- Unfortunately standard factorization doesn't hold (Collins, Qiu, PRD75, 114014; Collins, 0708.4410; Mitov, Sterman, PRD86, 114038)

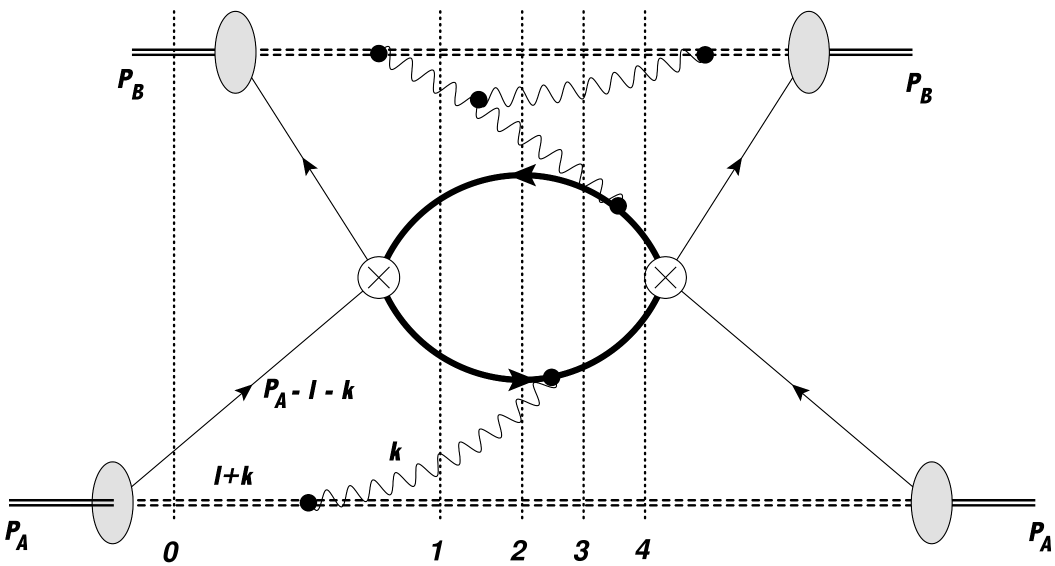


Figure 1: See George Sterman's talk

- At partonic level similar problem show up at NNNLO (Qiu, Collins, PRD75, 114014)
- In this talk only concentrate on partonic cross section, and leave these factorization violating effects aside

Outline of the factorization

- For Q_T resummation, expansion parameter $\lambda = \frac{Q_T}{m_t}$, relevant effective theory is $\mathrm{SCET}_{\mathrm{II}}$
- Collinear modes

$$n - \text{collinear} : (\lambda^2, 1, \lambda) \quad \bar{n} - \text{collinear} : (1, \lambda^2, \lambda)$$

- soft mode

$$(\lambda, \lambda, \lambda)$$

- SCET Operators describing $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ and LO/NLO hard functions are the same as in threshold resummation for $t\bar{t}$ production Ahrens et.al., JHEP 1009 (2010) 097
- 2(3) independent color structures for $q\bar{q}(gg)$ channel

$$c_1^{q\bar{q}} = \delta_{a_1 a_2}\delta_{a_3 a_4}, \quad c_2^{q\bar{q}} = t^c_{a_1 a_2} t^c_{a_3 a_4}$$

$$c_1^{gg} = \delta^{a_1 a_2}\delta_{a_3 a_4}, \quad c_2^{gg} = i f^{a_1 a_2 c} t^c_{a_3 a_4}, \quad c_3^{gg} = d^{a_1 a_2 c} t^c_{a_3 a_4}$$

Outline of the factorization

- No soft-collinear mixing at leading power
- Matrix elements naturally factorize into different part, each can be calculated separately ($q\bar{q}$ channel as an example)

$$
\begin{aligned}
\frac{d^4\sigma}{dM\,dQ_T^2\,dy\,d\cos\theta} = \; & \frac{\pi\beta_t}{4\pi sM}\int d^2x_\perp e^{-iq_\perp x_\perp}\mathrm{Tr}[\underbrace{\boldsymbol{H}_{q\bar{q}}(M,m_t,\cos\theta,\mu)}_{\texttt{hard function}} \\
& \times \underbrace{\boldsymbol{S}(x_\perp,M,m_t,\cos\theta,\mu)}_{\texttt{soft function}}] \\
& \times \sum_q [\underbrace{\mathcal{B}_{q/N_1}(\xi_1,x_T^2,\mu)\mathcal{B}_{q/N_2}(\xi_2,x_T^2,\mu)}_{\texttt{beam function}} + q \leftrightarrow \bar{q}]]
\end{aligned}
$$

- Technical complication: color evolution in color space, but method is identical as in threshold resummation
- Below will concentrate on new elements related to Q_T

The beam function

- Defined similar to ordinary PDF but with a transverse displacement

$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \langle N(p)|\bar{\xi}(t\bar{n} + x_\perp)\frac{\bar{\not{n}}}{2}\xi(0)|N(p)\rangle$$

- OPE on ordinary PDF: $\mathcal{B}_{q/N}(\zeta, x_T^2, \mu) = \sum_j \int_\zeta^1 \frac{dz}{z} \mathcal{I}_{q\leftarrow j}(z, x_T^2, \mu) f_{j/N}(\zeta/z, \mu)$.
- Beam func contain rapidity divergence (Collins, hep-ph/0304122)
- Different treatments in SCET
  - Fully unintegrated PDFs (Mantry, Petriello, PRD81:093007,2010)
  - η regulator, (Chui, et.al., PRL108 (2012) 151601)
  - EIS method, (Garcia-Echevarria, Idilbi, Scimemi, JHEP 1207 (2012) 002)
  - Analytic regulators (Becher, Neubert, EPJC71:1665,2011; Becher, Bell, PLB713 (2012) 41-46)
- We use the analytic regulators of (Becher, Bell, PLB713 (2012) 41-46)

The soft function

$$S'_{i\bar{i}}(x_\perp, M, m_t, \cos\theta, \mu) = \frac{1}{d_i}\sum_{X_s}\int dq_\perp e^{ix_\perp\cdot q_\perp} \times\langle 0|S_n S^\dagger_{\bar{n}} S^\dagger_t S_{\bar{t}}|X_s\rangle \delta^{(2)}(q_\perp + \hat{P}_\perp)\langle 0|S^\dagger_n S_{\bar{n}} S_t S^\dagger_{\bar{t}}|X_s\rangle$$

- At one-loop wrtten as a set of eikonal integral weighted by appropriate color factor

$$S^{(1)'} = \sum_{j,k} \boldsymbol{\omega}^{jk}_{i\bar{i}} I'_{jk}$$

$$I'_{jk} = -\frac{(4\pi\mu^2)^\epsilon}{\pi^{2-\epsilon}}\int d^d q (2\pi)\delta_+(q^2)\frac{v_j\cdot v_k e^{-ix_\perp\cdot q_\perp}}{v_j\cdot q v_k\cdot q}$$

- The problem with this definition is
  - Depends on $x_\perp \cdot v_\perp$ in an intricate way, therefore complicate the inverse Fourier transformation

The refined soft function

- Refined the definition of soft function by averaging over the azimuthal angle

$$
\begin{aligned}
S'_{i\bar{i}}(x_\perp, M, m_t, \cos\theta, \mu) \quad &= \quad \frac{1}{d_i} \sum_{X_s} \int \frac{d\phi_t}{2\pi} \int dq_\perp e^{ix_\perp \cdot q_\perp} \\
&\quad \times \langle 0| S_n S^\dagger_{\bar{n}} S^\dagger_t S_{\bar{t}} | X_s \rangle \delta^{(2)}(q_\perp + \hat{P}_\perp) \langle 0| S^\dagger_n S_{\bar{n}} S_t S^\dagger_{\bar{t}} | X_s \rangle
\end{aligned}
$$

$$
I'_{jk} = -\frac{(4\pi\mu^2)^\epsilon}{\pi^{2-\epsilon}} \int_0^{2\pi} \frac{d\phi_t}{2\pi} \int d^d q (2\pi) \delta_+(q^2) \frac{v_j \cdot v_k e^{-ix_\perp \cdot q_\perp}}{v_j \cdot q v_k \cdot q}
$$

- Four independent integral: I_{12}, I_{13}, I_{33}, I_{34}.
  - I_{12} vanishes
  - I_{13} contains rapidity divergence, regulated using analytic regulator $d^d q \to d^d q (\nu/q^+)^\alpha$ (Becher, Bell, PLB713 (2012) 41-46)
  - Rapidity divergences in I_{33} and I_{34} are automatically regulated by top quark mass

The refined soft function

- Calculated in momentum space

$$\tilde{I}_{jk}(Q_T,\theta,\mu) = -\frac{(4\pi\mu^2)^\epsilon}{\pi^{1-\epsilon}}\int\frac{d\phi_t}{2\pi}\int d^dk(2\pi)\delta_+(k^2) \times\left(\frac{\nu}{n\cdot k}\right)^\alpha \frac{v_j\cdot v_k\delta^{(2)}(\boldsymbol{Q}_\perp+\boldsymbol{k}_\perp)}{v_j\cdot kv_k\cdot k} \tag{1}$$

- results are

$$I_{13} = \frac{1}{q_T^2}\left(\frac{\mu^2}{q_T^2}\right)^{\epsilon+\alpha/2}\left(\frac{\nu^2}{\mu^2}\right)^{\alpha/2}\left(\frac{2}{\alpha}\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}-2\ln\frac{t_1}{m_tM}+\epsilon f_{13}\right),$$

$$I_{33} = \frac{1}{q_T^2}\left(\frac{\mu^2}{q_T^2}\right)^{\epsilon}(-2+\epsilon f_{33})$$

$$I_{34} = \frac{1}{q_T^2}\left(\frac{\mu^2}{q_T^2}\right)^{\epsilon}\left(\frac{1+\beta_t^2}{\beta_t}\ln\frac{1+\beta_t}{1-\beta_t}+\epsilon f_{34}\right)$$

The refined soft function

- Scale dependent term easily obtained from RG invariance consideration. Scale independent terms calculated by brute force: differential equation plus some guess

$$
\begin{aligned}
f_{13} &= \mathrm{Li}_2\left(\frac{(4m_t^2 - M^2)\sin^2\theta}{4m_t^2}\right) \\
f_{33} &= 2\ln\left(\frac{1-\beta_t^2}{1-\beta_t^2\cos^2\theta}\right) \\
f_{34} &= \frac{1+\beta_t^2}{\beta_t}\left[4\ln\frac{1-\beta_t}{1+\beta_t}\ln\cos\frac{\theta}{2} - \mathrm{Li}_2\left(-\frac{1-\beta_t}{1+\beta_t}\tan^2\frac{\theta}{2}\right)\right. \\
&\quad \left. + \mathrm{Li}_2\left(-\frac{1+\beta_t}{1-\beta_t}\tan^2\frac{\theta}{2}\right)\right]
\end{aligned}
$$

- Interestingly, all the finite terms vanish when $\theta = 0$ or π
- Might be useful to J/ψ production...

Resummed Q_T distribution

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q_T^2\mathrm{d}M^2\mathrm{d}Y\,\mathrm{d}\cos\theta} = \frac{4\pi\beta_t}{3SM}\sum_{i,j=q,\bar{q},g}\int_{\xi_1}^1\frac{\mathrm{d}z_1}{z_1}\int_{\xi_2}^1\frac{\mathrm{d}z_2}{z_2}f_{i/N_1}(\xi_1/z_1)f_{j/N_2}(\xi_2/z_2)$$
$$\times C_{R\leftarrow ij}(z_1,z_2,Q_T^2,\hat{s},\hat{t},m_t^2,\mu)$$

For $q\bar{q}$-channel

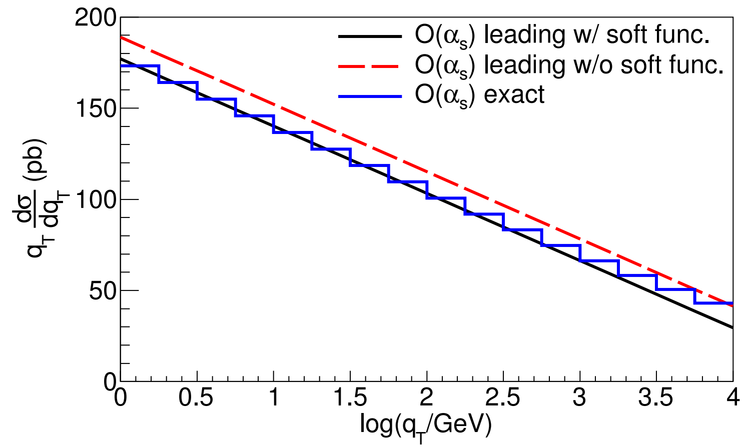
$$C_{q\bar{q}\leftarrow ij} = \frac{1}{2}\int_0^\infty \mathrm{d}b\,bJ_0(bQ_T)\exp\left[g_F(\eta,L_\perp,\alpha_s)\right]\bar{I}_{q\leftarrow i}(z_1,L_\perp,\alpha_s)\bar{I}_{\bar{q}\leftarrow j}(z_2,L_\perp,\alpha_s)$$
$$\times\mathrm{Tr}\left[\boldsymbol{H}_{q\bar{q}}(\hat{s},\hat{t},m_t^2,\mu)\boldsymbol{S}_{q\bar{q}}(\hat{s},\hat{t},m_t^2,L_\perp)\right]$$

- Distribution expanded to NLO

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q_T^2\mathrm{d}M^2\mathrm{d}Y\,\mathrm{d}\cos\theta} = \frac{4\pi\beta_t}{3SMd_R}\sum_{i,j=q,\bar{q},g}\frac{\alpha_s}{2\pi}\left\{\mathrm{Tr}\left[\boldsymbol{H}_R^{(0)}\left(\boldsymbol{A}_R\ln\frac{M^2}{Q_T^2}+\boldsymbol{B}_R\right)\right]f_{i/N_1}(\xi_1)f_{j/N_2}(\xi_2)\right.$$
$$\left.+\mathrm{Tr}[\boldsymbol{H}_R^{(0)}\boldsymbol{S}_R^{(0)}]\left[\left(P_{i\leftarrow a}^{(1)}\otimes f_{a/N_1}\right)(\xi_1)f_{j/N_2}(\xi_2)+f_{i/N_1}(\xi_1)\left(P_{j\leftarrow b}^{(1)}\otimes f_{b/N_2}\right)(\xi_2)\right]\right\}\frac{1}{Q_T^2}$$

Compared with fixed order prediction (MCFM)

- Exact agreement for the logarithmic terms



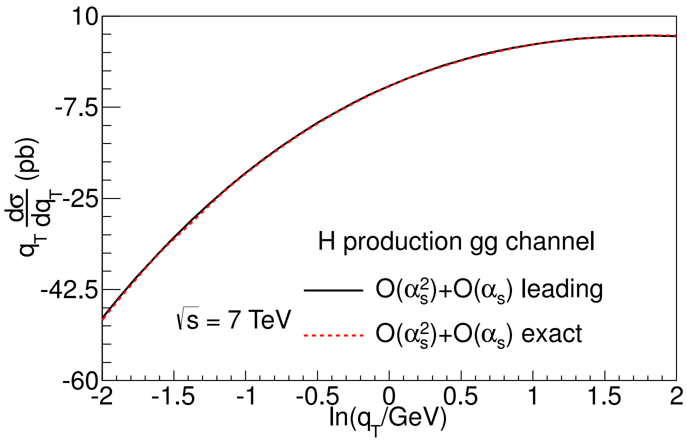
- Scale independent terms checked by reproducing total cross section at NLO

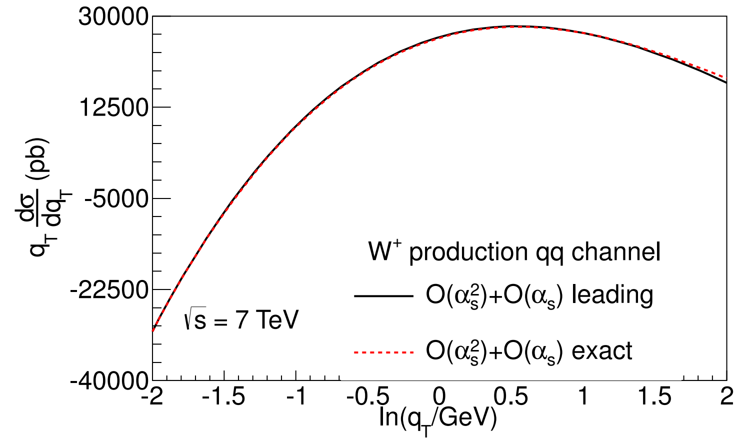
$$\sigma_{\mathrm{NLO}} = \int_0^{q_t,\mathrm{cut}} \mathrm{d}q_t \frac{\mathrm{d}\sigma_{\mathrm{NLO}}}{\mathrm{d}q_t} + \int_{q_t,\mathrm{cut}}^{\infty} \mathrm{d}q_t \frac{\mathrm{d}\sigma_{\mathrm{NLO}}}{\mathrm{d}q_t} = \sigma_I + \sigma_{II}$$

$q_{t,\mathrm{cut}}$	σ_I	σ_{II}	σ_{NLO}	MCFM
0.1	-884.06	1045.99	161.93	161.94
0.5	-478.60	640.52	161.91	161.94

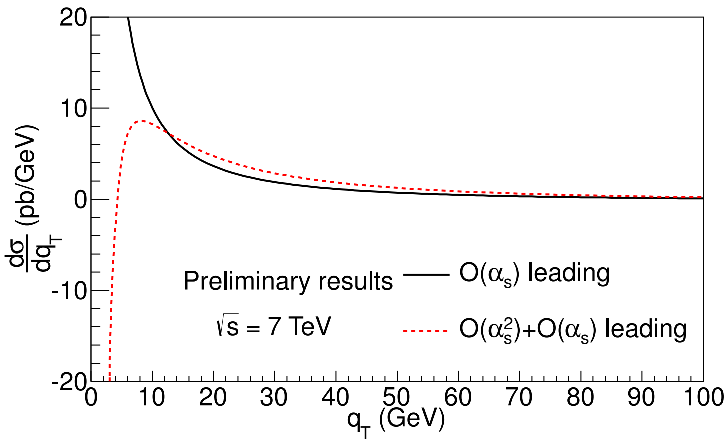
Q_T distribution expanded to NNLO

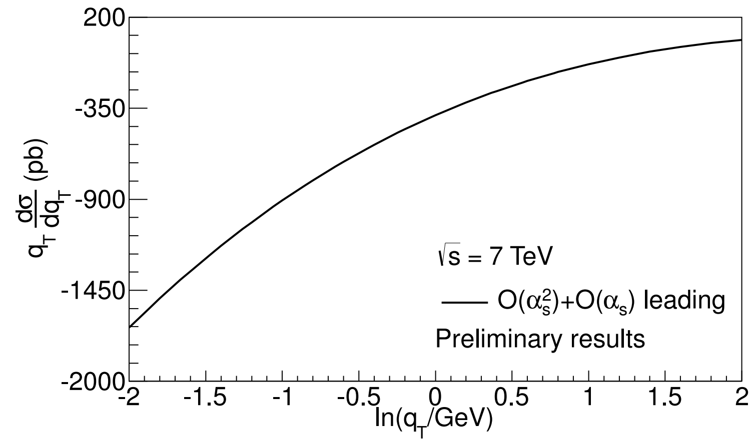
- Q_T distribution can be expanded to $\mathcal{O}(\alpha_s^2)$, can compared with fixed order code
  - Higgs production and Drell-Yan as examples





  - $t\bar{t}$: need calculation using NLO $t\bar{t}$+jet to verify this result



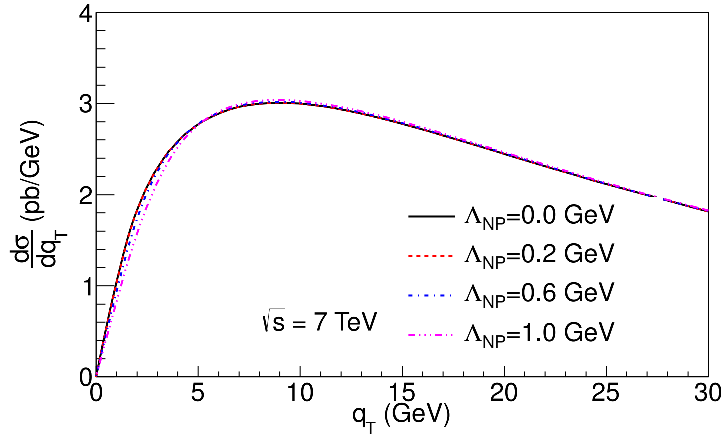


Detailed set up for resummation

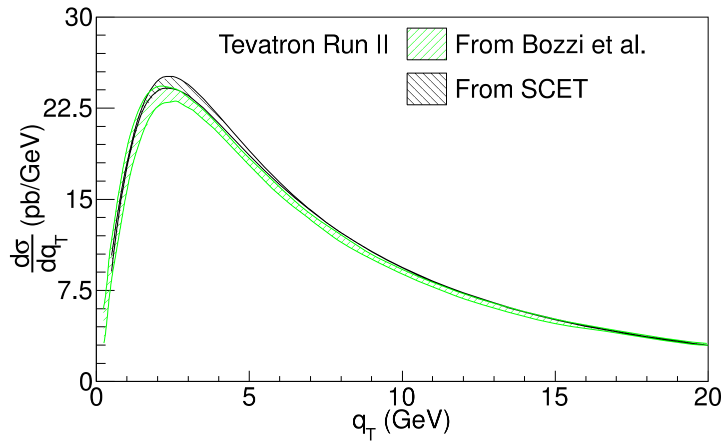
- We follow the approach of (Becher, Neubert, Wilhelm, JHEP 1202 (2012) 124) for numerical calculation
  - Running hard function from $\mu_h = m_t$ to $\mu \sim q^* + Q_T$
  - Beam function and soft function evaluated at μ
  - Modified power counting: $\alpha_s L_\perp^2$ is $\mathcal{O}(1)$
  - A Gaussian nonperturbative factor $\exp(-\Lambda_{\mathrm{NP}}^2 x_T^2)$ for $q\bar{q}$ channel
  - Estimate of higer order uncertainties by varying μ and μ_h independently by a factor of 2
  - Match on the exact NLO prediction (MCFM)

Detailed set up for resummation

- Results have mild dependence on nonperturbative parameter

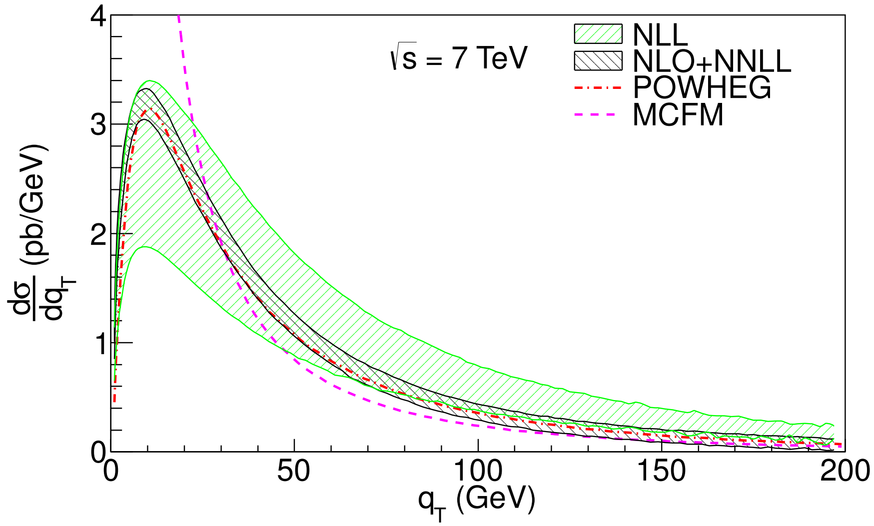


- Using similar set up we compare with (Bozzi, et.al., PLB696:207-213,2011) for Z production

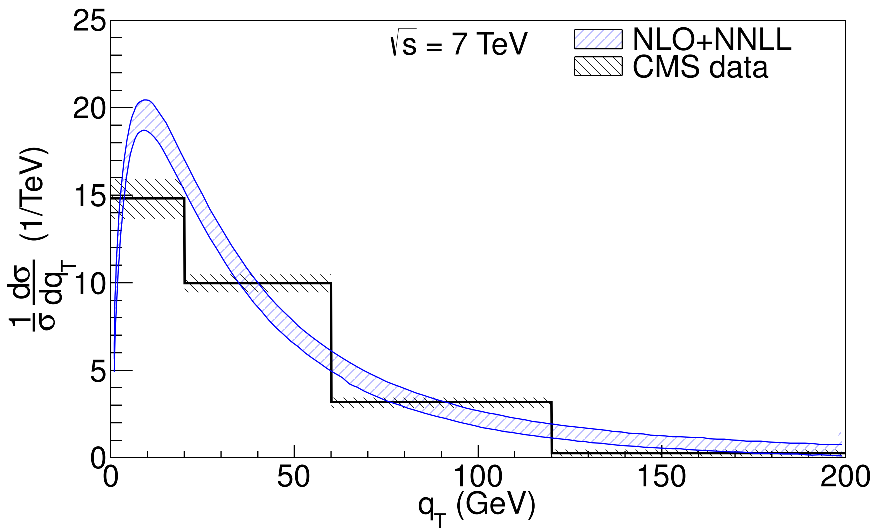


The resummation distribution

- Observe significant reduction of scale dependence from NLL to NNLL.

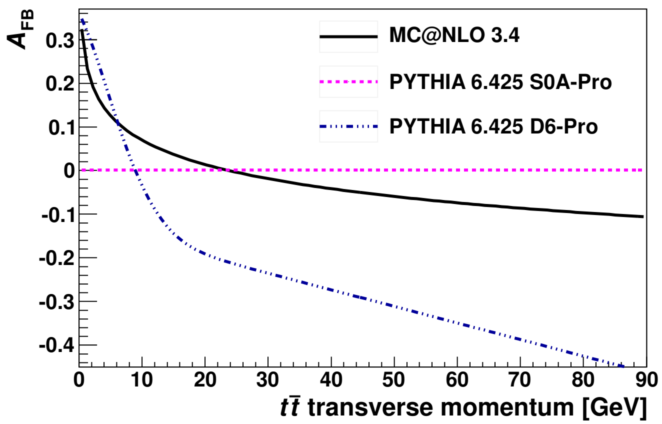


- Good agreement between resummation prediction and experiment

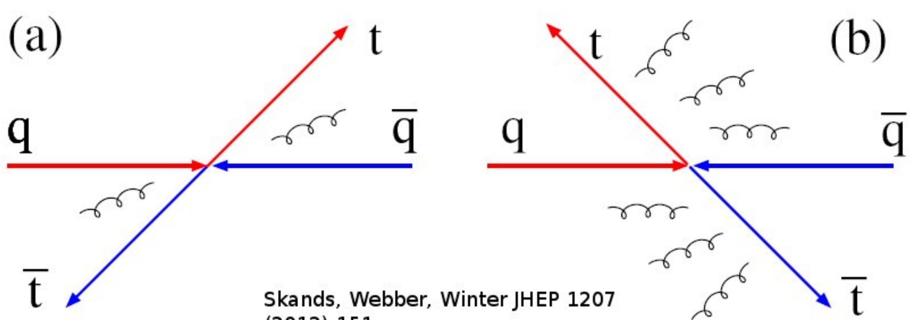


top quark charge asymmetry

- A_{FB} is sensitive to QCD radiation at small p_T
- It is observed that even a LO event generator like PYTHIA can generate non-zero A_{FB}



- Due to color coherence, even a LO event generator could produce non-zero A_{FB} (Skands, Webber, Winter, 12')



Skands, Webber, Winter JHEP 1207 (2012) 151

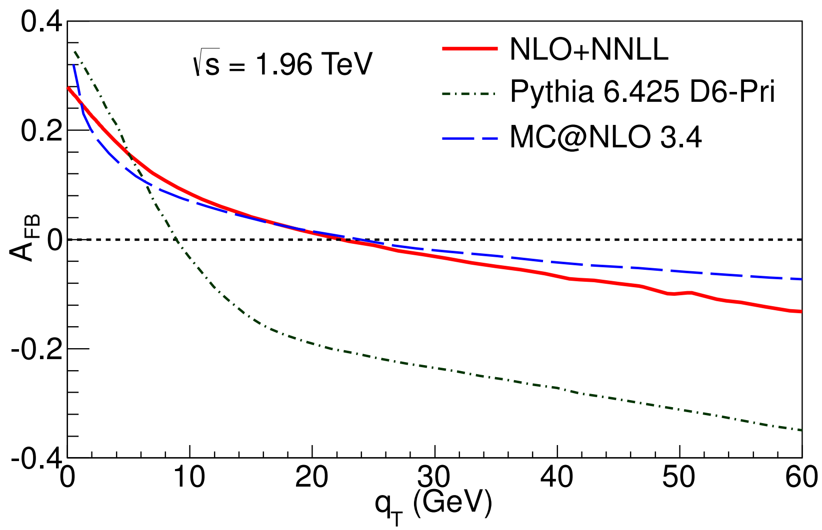
top quark charge asymmetry

- In resummation prediction $A_{\rm FB}$ arises from initial-final inteference

$$I_{13} = \frac{1}{q_T^2}\left(\frac{\mu^2}{q_T^2}\right)^{\epsilon+\alpha/2}\left(\frac{\nu^2}{\mu^2}\right)^{\alpha/2}\left(\frac{2}{\alpha}\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} - 2\ln\frac{t_1}{m_t M} + \epsilon f_{13}\right)$$

$$I_{14} = I_{13}(t_1 \to u_1)$$

- Both resummation and MC@NLO predict a crossover ~ 25 GeV.



Fixed order from resummation

- Resummation results can be used as a tool to calculate fixed order cross section
- The easiest way to do so is to implement a simple Q_T cutoff

$$\sigma_{\text{NLO}} = \int_0^{q_t,\text{cut}} \mathrm{d}q_t \frac{\mathrm{d}\sigma_{\text{NLO}}}{\mathrm{d}q_t} + \int_{q_t,\text{cut}}^{\infty} \mathrm{d}q_t \frac{\mathrm{d}\sigma_{\text{NLO}}}{\mathrm{d}q_t} = \sigma_I + \sigma_{II}$$

- For complete NNLO prediction need both beam function (Gehrmann, Lübbert, Yang, PRL109 (2012) 242003) and soft function to NNLO
- It seems almost impossible to calculate soft function to NNLO
- However for specific channel only NLO soft function is required

$$qq,\, qq',\, q\bar{q}',\, qg \to t\bar{t} + X$$

- It might be a useful check to existing calculation Czakon, Mitov, 12'

Summary

- Using SCET, We have studied the resummation of transverse momentum logarithms of $t\bar{t}$ pair, built upon the q_T resummation for Drell-Yan and Higgs production
- Both singular distribution and finite remainder checked at NLO explicitly
- Resummation to NNLL; significant scale reduction from NLL to NNLL; good agreement between theory and experiment
- Resummation prediction for Q_T dependent $A_{\rm FB}$. Crossover at ~ 25 GeV
- In the future, hope to check singular distribution at NNLO, and calculate some subprocesses for $t\bar{t}$ production at NNLO

Thank you!